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U.S. DEMOGRAPHICS AND SAVING: PREDICTIONS OF THREE SAVING MODELS

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Abstract

This paper compares the predictions of three different saving models with respect to the impact of projected U.S. demographic change on future U.S. saving rates. The three models are the life-cycle model, the infinite horizon altruism model, and a reduced form econometric model. The findings for the different models indicate a great range of possible paths of future U.S. saving. However, the three models concur in predicting a peak in the U.S. national saving rate within the next fifteen years, followed by a significant decline in the saving rate thereafter. In fact, the findings suggest the strong possibility of negative U.S. saving rates beginning after 2030.

I. INTRODUCTION

The United States, like most other developed economies, is aging. Currently 12 percent of our population is 65 or over.[1] During most of the baby-boom generation's retirement years a fifth of the population will be over 65

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[1]See Table 1.

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Table 1:
Population Age Distributions for the United States by Decade

Age Group	1950s	1960s	1970s	1980s	1990s	2000s
0-17	329	.356	.318	.268	.256	.239
18-25	.109	.114	.143	.140	.111	.110
25-34	.133	.109	.125	.156	.146	.120
35-54	.256	.240	.220	.228	.227	.299
55-64	.090	.088	.092	.092	.083	.103
65 PLUS	.084	.093	.102	.116	.126	.129

Age Group	2010s	2020s	2030s	2040s
0-17	.22	.216	.210	.207
18-24	.106	.097	.097	.096
25-34	.123	.117	.111	.113
35-54	.269	.253	.255	.249
55-64	.132	.128	.113	.120
65 PLUS	.148	.188	.214	.215

(see Table 1). While dependent children will decline as a share of the population, the overall dependency ratio (defined as the ratio of those under 18 plus those 65 and over to those 18 to 64) will rise by 2050 from its current value of .616 to .730.

More dependents relative to workers suggests the likelihood of more consumption relative to income and, therefore, less national saving. For the United States, which already has a critically low saving rate, the prospect of even less national saving in the future is quite worrisome. Last year's (1989's) national rate of saving out of net national product was only 3.7 percent. This figure is only two-fifths of the 8.9 percent rate observed on average from 1950 to 1979, and the 1989 saving rate is not atypically low with respect to our recent saving behavior. Since 1985 the U.S. net national saving rate has averaged only 3.3 percent.

These national saving rates are based on National Income and Product Accounts data which, as is well-known, mismeasure private and government consumption of durables and mismeasure Net National Product by omitting

Table 2:
Postwar National Saving Rates in the United States

Years	Uncorrected	Corrected
1950-1959	9.2	13.3
1960-1969	8.9	13.0
1970-1979	8.5	11.8
1980	6.8	8.4
1981	7.4	8.9
1981	7.4	8.9
1982	3.2	4.7
1983	3.3	5.5
1984	5.7	8.6
1985	3.6	7.1
1986	2.8	na
1987	2.7	na
1988	3.7	na
1989	3.7	na

na - not available.
Sources:
Uncorrected Saving Rates: Economic Report of the President, 1990;
Corrected Saving Rates: Imputed rent on an asset is calculated as annual depreciation plus 3 percent times the stock of the asset. Annual depreciation of consumer durables and government nonmilitary tangible assets as well as the stocks of consumer durables and government tangible assets as well as the stocks of consumer durables and government tangible assets are reported in the U.S. Department of Commerce's Fixed Reproducible Tangible Wealth in the United States, 1925–85.

depreciation of private and government stocks of durables. As indicated in Table 2, which provides corrected and uncorrected measures of the net national saving rate, adjusting for these measurement problems significantly raises the saving rate, but does not appear to make much of a difference with respect to the percentage decline in U.S. saving in the 1980s.

Given the growing concern about U.S. saving, it seems important to understand the role of demographics in saving if only to identify what portion of saving rate changes may be attributable to demographic change. This paper is an empirical approach to understanding how demographics will affect U.S. saving over the next 60 years. The study complements our previous research on the subject.

In Auerbach and Kotlikoff (1987) and Auerbach *et al.* (1989), we used our 75-period life-cycle general equilibrium simulation model (augmented in the latter paper to include age-specific government consumption, technical change and utility for bequests) to analyze the effects of the demographic transition. The results of both studies predict a gradual decline in U.S. saving rates over the next 60 years, with the 2050 saving rate less than half the 1990 saving rate.

In contrast to these simulation findings, our more recent paper on the subject (Auerbach and Kotlikoff, 1990) used a different approach and obtained different predictions. Instead of simulating a complex general equilibrium model based on empirically estimated preference and production parameters, we used an *ad hoc* partial equilibrium simulation methodology that incorporated more information about actual age-specific saving patterns. We simply asked how U.S. saving would develop through time as the age-sex composition changes, assuming the shapes and levels of age-earnings and age-consumption profiles observed in the 1980s remained unchanged through time. The results suggested higher saving rates during the next 60 years, with significantly higher saving rates over the next 30 years. While this procedure provides a rough sense of the potential importance of demographic composition of U.S. saving, it is not, strictly speaking, consistent with standard models of consumption behavior; one would not expect the age profile of consumption to remain constant.

This paper reexamines the question of saving and demographics in a manner that is consistent with three theories of consumption choice: the infinite horizon altruism model, the life-cycle model, and a reduced form model intended to capture effects not present in the other two models. The data we use are the 1980–85 BLS Consumer Expenditure Surveys, the Social Security Administration's past and projected population totals by age and sex, and the National Income and Product Accounts. In addition to providing different perspectives on future U.S. saving, the different predictions of the three models represent a potential means for testing which model best describes U.S. saving behavior.

While we provide projections based on three models, the findings are still limited in that they are based on partial equilibrium assumptions; we do not consider the effects that changes in the capital-labor ratio or the age-composition of the labor force may have on the distribution or composition of income in the future. Further, the only type of risk we consider is mortality risk, and here we assume the presence of perfect annuity markets or the equivalent. It would be useful to add to our analysis the types of general equilibrium feedback we considered in our earlier simulation work (based on a much simpler model of household behavior) and the impact of types or risk other than lifetime uncertainty; still, we believe our analysis is a useful first step in the study of a very complex problem.

Our findings for the different models indicate a great range of possible paths for the saving rate over the next several decades. However, all three models concur in predicting a peak in the national saving rate in the near future, followed by a significant decline in the saving rate thereafter. In fact, with one exception, *every* simulation we present forecasts a negative national saving rate for the decades beginning in 2030 and 2040. While the same pattern of rising and then falling saving rates was also found in our recent paper, the sharpness of the drop in saving rates during the next century is here predicted to be much greater.

The paper begins in Section II with a presentation of our methodological approach to prediction under each of the three saving models. Section III discusses the data and our construction of empirical counterparts to the theoretical magnitudes identified in Section II. Section IV presents our findings, and Section V summarizes and concludes the paper.

II. METHODOLOGY

This section of the paper reviews the three alternative models used to predict the effects of demographics on the rate of national saving and the procedures necessary to implement these models.

A. The Altruistic Family Model

Perhaps the simplest model, from a theoretical perspective, is the model of the "dynastic" altruistically-linked family, in which the consumption of different cohorts of individuals is determined by a single optimization plan. In this model, the shocks to the economy associated with changes in population structure will be spread across members of different generations.

We consider the optimization problem of a representative family, with an age structure that mirrors the population as a whole. The family planner chooses consumption for each current and future member of the family in each year, subject to the budget constraint that the family's full resources be no less than the presnt value of its future consumption. These full resources include current tangible wealth plus the present value of all labor earnings, less the present value of resources absorbed by the government. Imposition of the government budget constraint—that the present value of resources extracted from the private sector equals the present value of its consumption—leads to this measure of private resources, namely, the sum of economy-wide net assets (tangible wealth) plus human wealth less the present value of government consumption.

The family planner maximizes the expected utility function over consumption of each surviving family member at different dates:

$$U=\sum_{t=0}^{\infty}\sum_{a=1}^{120}\theta_a P_{at}\frac{C_{at}^{1-\gamma}}{1-\gamma}(1+\beta)^{-t} \tag{1}$$

where θ_a is the weight in the family utility function given to an age a individual, P_{at} is the surviving population of age a in year t, C_{at} is the individual's consumption, β is a pure rate of time preference, and γ is the inverse of the intertemporal elasticity of substitution. We let a run from 1 to 120, in keeping with our population data.

The objective function in (1) is consistent with the maximization of expected utility by identical altruistic families, each having the same population structure and facing the same mortality risk and a perfect annuities market. In this case, the population totals at each age automatically incorporate aggregate survival probabilities.

The corresponding budget constraint is:

$$\sum_{t=0}^{\infty}\sum_{a=1}^{120}\frac{P_{at}C_{at}}{(1+r)^t}\leq R_0 \tag{2}$$

where R_0 is full family resources at time t and r is the real rate of interest.

Maximization of (1) subject to (2) provides a solution for the consumption path for each individual in the family. This solution has the distinctive property that the cross-section age-consumption profile is constant over time, and that consumption at each age grows over time at a rate determined by the after-tax interest rate, the rate of time preference, and the intertemporal elasticity of substitution. One can summarize the solution in terms of the ratio of the consumption of an individual of a particular age, say j, at a specific date, say zero, to full family resources at time zero, R_0:

$$C_{j0}=R_0/H_0, \tag{3}$$

a rate of growth of age-j consumption:

$$\frac{C_{jt+1}}{C_{jt}}=\left(\frac{1+r_n}{1+\beta}\right)^{\frac{1}{\gamma}}=\zeta, \tag{4}$$

and a stationary age-consumption profile:

$$\frac{C_{jt}}{C_{it}} = \left(\frac{\theta_i}{\theta_j}\right)^{\frac{1}{\gamma}}, \tag{5}$$

where r_n is the after-tax interest rate, H_0 is a function of the interest rate r, the rate of consumption growth ζ, the population shares P_{at}, and the age-specific utility function parameters θ_j:

$$H_0 = \sum_{t=0}^{\infty}(1+r)^{-t}\zeta\sum_{a=0}^{120}P_{at}\left(\frac{\theta_a}{\theta_j}\right)\frac{1}{\gamma} \tag{6}$$

Note that these conditions properly adjust for mortality risk. In particular, note that equation (4) does not include survival probabilities because of the existence of perfect annuity insurance; the probabilities do enter into expressions for R_0 and H_0 because these variables are influenced by the size of surviving populations.

Our approach to estimating annual saving rates for this model utilizes the above equations. First, we estimate the relative weights θ_i by age and sex in equation (5) (letting $j = 40$ and setting $\theta_{40} = 1$) using data from the 1980-85 Consumer Expenditure Surveys. Then, using these weights and population data for the period 1950-1988, we calculate the consumption of a 40-year-old in each year during this historical period that would have been consistent with aggregate consumption in that year. We estimate ζ as the average annual growth rate of the time series of estimated consumption for an age-40 individual in each year of the sample period. We take 1988 as the benchmark year and the value of the 40-year-old consumption that is consistent with aggregate consumption in 1988. Using this benchmark consumption level and ζ, we then estimate future values of consumption at age 40 and the contemporaneous consumption levels of individuals of other ages. Summing together these predicted values of C_{at} over all ages gives us total predicted consumption in year t. All that remains for the calculation of annual saving rates is to estimate future levels of NNP and government consumption.

The estimated 1988 consumption of a 40-year-old that we project using the age-specific consumption weights and populations provides an estimate of the ratio R_0/H_0, using expression 3. The denominator of this ratio, H_0, depends on population sizes, which are data, and ζ and the vector θ, which we estimate. The only unknown determinant of H_0 is the pretax real rate of interest, r. The numerator of the ratio, R_0, equals the present value of future earnings from labor and capital, less the present value of government consumption. It depends not only on the interest rate but also on the projected growth rates of government consumption and labor earnings, which we base on information from the National Income and Product Accounts as described in the next section. In all the simulations presented, we assume that these growth rates are the same, so that government spending and labor earnings grow at the same rate.

Given growth and interest rates, we obtain solutions for both H_0 and R_0. Imposing the requirement that the resulting ratio equals our estimate of C_{40} in 1988 yields consistent combinations of interest and growth rates which provide the basis for alternative sets of projections of NNP presented below. For each assumption about growth and interest rates, we solve recursively for each successive year's NNP and capital stock, starting from 1988 levels of income and capital, using the national income identity and the future consumption levels already projected. Subtracting these consumption projections from the future income estimates provides a projection of each year's national saving rate. By construction, these saving-rate projections are consistent with current and past saving behavior under the assumptions of the model.

B. The Life-Cycle Model

The life-cycle model differs from that of the altruistic family in assuming that there is no connection among generations of adults. Each nuclear family chooses consumption for itself and its minor children, subject to its own budget constraint. This gives rise to distributional effects on saving that do not occur in the previous model. Changes in the ratio of children to adults and, more generally, changes in the age structure of the population will be associated with shifts in the intergenerational allocation of the burden of government spending, and will certainly affect the proportion of family resources that must be spent on child-rearing.

We assume that there are a large number of identical life-cycle households born in each year and index each generation's representative life-cycle household by the year in which the adult turns 21. As before, we assume that the family planner seeks to maximize expected utility and has access to perfect annuity insurance.

For the cohort whose representative household reaches adulthood in year t, the objective function[2], analogous to expression (1) for the dynastic family,

[2]Note that, as in the previous model, population sizes equal the product of initial population totals and survival rates, consistent with our assumption of identical families and perfect annuity insurance. Stated differently, perfect insurance implies that the cohort's

is:

$$U_t = \sum_{a=21}^{120} \theta_a P_{a,t-21+a} \frac{C_{a,t-21+a}^{1-\gamma}}{1-\gamma} 1 + \beta^{-t} + \sum_{a=1}^{20} \theta_a P_{a,t-1+a} \frac{C_{a,t-1+a}^{1-\gamma}}{1-\gamma} \quad (7)$$

where the notation is the same as we used above. The first part of expression (7) represents the direct lifetime expected utility of generation t's adult, while the second part of the expression is the utility derived from the children of this family, who are assumed to be those born 20 years later.[3]

Combined with family t's budget constraint, expression (7) yields the family's optimal consumption path, beginning at date t. For $s \geq t$, successive periods of the family's adult consumption are related by the expression:

$$\frac{C_{a+1,s+1}}{C_{a,s}} = \left(\frac{\theta_{a+1}(1+r_n)}{\theta_a(1+\beta)} \right) \frac{1}{\gamma} = \xi_{a+1} \quad (8)$$

Comparing (4) and (8), we note that while the former compares individuals of *different* generations at different dates, the latter compares individuals of the same generation at different dates. Combined with expression (5), expression (4) yields expression (8), permitting a comparison of consumption for the same generation across different dates in the altruistic family model; however, there is no such condition connecting the consumption of different generations in the life-cycle model.

As discussed in more detail below, we estimate the terms ξ_{a+1} by regressing the ratio of a cohort's (distinguished by sex) per capita consumption in year $t+1$ to its per capita consumption in year t on a polynomial in age and sex. Given our estimates of the $\xi's$, we estimate the time path of per capita consumption of each existing adult age-sex cohort by multiplying its initial 1988 consumption (benchmarked so that total consumption corresponds to the NIPA total) by the appropriate product of the $\xi's$. We follow the same procedure for projecting the consumption of existing children until the years in which they reach age 21. Children born after 1988 are assumed to consume the same amount as 1988 newborns, adjusted for a productivity growth factor which we describe below, with their subsequent childhood consumption governed by the ξ terms.

For all cohorts reaching age 21 after 1988, we must make some assumption about the level of their consumption profiles. This is because, unlike the

objective function is the same as that of any representative family.

[3]The exact timing of births is not crucial to the analysis.

model of the altruistic family, the life-cycle model imposes no connections in consumption levels across generations. We experimented with a variety of assumptions about the consumption of individuals reaching adulthood in the future. In each case, having specified the consumption rules for each generation, we then adjusted the productivity factor, the interest rate, and the assumed rate of growth of government spending to ensure that the economy's intertemporal budget is in balance, i.e., that the present value of the economy's private plus government consumption equals the present value of its resources.

One assumption about the initial consumption levels of individuals reaching age 21 after 1988, that their consumption equals the consumption of 1988 21-year-olds, adjusted for the same productivity growth factor as used to adjust the initial consumption of newborns after 1988, leads to fairly unreasonable results. Such levels of future consumption appear to be consistent with intertemporal budget balance only if government grows substantially faster than productivity, but such a large difference in growth rates leads to implausible results after a few decades. Hence, in the empirical section below, we use an alternative assumption about the relationship of the consumption of future generations and present ones.

We assume that all those reaching age 21 after 1988 have consumption levels (adjusted for productivity growth) equal to those of 1988 21-year-olds, multiplied by some factor $(1 + x)$, and choose x so that the economy's intertemporal budget constraint is balanced for equal growth rates of productivity and government. That is, rather than imposing the assumption that $x = 0$ and adjusting the productivity growth rate to ensure that the resource constraint is met, we let x vary and impose equality on the growth rates of productivity and government spending, an historically reasonable constraint.

C. The Reduced Form Model

Our last model is a reduced form model. Given our lack of certainty about the "true" model of consumption behavior, we believe it is useful to include in the analysis the projections of a model incorporating few structural restrictions. By not restricting the relationship of consumption and current income, taxes and government spending, our reduced form model implicitly allows for market imperfections, such as liquidity constraints, that could lead to "excess sensitivity" of consumption to cash flow income. The previous two models ignored restrictions on the ability of families to transfer resources across time, leaving no possibility for consumption to depend on the predictable component of current income, given the present value of resources. There is, however, evidence that a portion of U.S. households have little tangible wealth and face liquidity constraints (e.g., Hayashi, 1987, Zeldes, 1989). Since the importance of such constraints is likely to be related to age, changes in the age structure of the population may alter the level of saving in ways not picked up by our previous models.

For example, if individual families attempt to follow the life-cycle model but face liquidity constraints until a certain age, at which labor earnings become sufficiently high to eliminate the constraint, younger households would face liquidity constraints. The shift in population away from the young would therefore suggest a reduction in the importance of liquidity constraints and a decline in the national saving ratio, *ceteris paribus.*

Unfortunately, a model of consumption based on utility maximization in the presence of liquidity constraints is much more complicated than the other models we have considered. It is necessary to specify the exact nature of the constraints, and allow for regime switches in consumption behavior. Rather than attempt such an ambitious exercise here, we use a reduced form regression approach that relates aggregate consumption to the variables that one would expect to matter if households faced liquidity constraints (or were for other reasons sensitive to current disposable income in their cosumption behavior). Interacting such variables with demographic variables enables us to include in our projections of future saving rates the effects of changes in the importance of liquidity constraints.

Our reduced form regression for aggregate consumption divided by NNP, C_t, has the form:

$$C_t = F\left[G_t, T_t, S_{1t}, \ldots, S_{5t}\right], \tag{9}$$

where S_{jt} is the share of age group j in the total population in year t (these age-group divisions are discussed below), and G_t and T_t are total government consumption and taxes (net of transfers) in year t.

In order to predict future values of the consumption-income ratio C, we need to forecast not only the evolution of the S_{it}'s (available from actuaries of the Social Security Administration) but also future government consumption and taxes (relative to NNP). In the spirit of this exercise, we predict each of these variables using a vector autoregression, including lagged values of each variable plus current values of the age shares as explanatory variables.

III. DATA AND IMPUTATIONS

To predict future saving rates for each of the years 1990–2050, we need to predict each year's Net National Product (NNP), government consumption, and private consumption. NNP is the sum of annual labor income plus annual capital income. We discuss, in turn, our predictions for each model of future labor income, capital income, government consumption, and private consumption.

A. Predicting Future Labor Income

For the family and life-cycle models we use the same method for predicting annual future labor income. Specifically, we calculate benchmarked average earnings by age and sex in 1988, where benchmarking refers to the fact that the sum of our estimates of average earnings by age and sex multiplied by the population by age and sex equals total national labor income in 1988. These benchmarked average values of annual earnings by age and sex are assumed to grow through time at a constant rate equal to the rate of labor productivity growth. Estimated future total annual real labor income in year t then equals the sum of projected average labor income by age and sex in year t multiplied by the projected population by age and sex in year t.

The benchmarking procedure is based on the following equation:

$$E_{88} = e_{40,m,88} \sum_{a=1}^{120} (R_{eam} P_{am88} + R_{eaf} P_{af88}) \tag{10}$$

where E_{88} stands for total real labor earnings in 1988, e_{40m88} stands for the average earnings of a 40-year-old male in 1988, R_{eam} and R_{eaf} stand, respectively, for the earnings of a male age a and a female age a relative to a male age 40, and P_{am88} and P_{af88} stand for the number of males and females, respectively, who are age a in 1988. Given the population data, the terms R_{eam} and R_{eaf}, and E_{88}, we can use this equation to solve for the value of e_{40m88}. Multiplying e_{40m88} by the relative earnings profile (the terms R_{eam} and R_{eaf}) gives an estimate of average earnings by age and sex in 1988. Average values of earnings by age and sex in year t in the future are these 1988 age- and sex-specific average earnings values multiplied by the appropriate growth factor.

The terms R_{eam} and R_{eaf} are determined from the CES data. We form the weighted average of reported annual wages plus salaries by age and sex for each calender year, using the CES January survey participants in each year. These survey respondents were asked about their labor income over the previous 12 months. For each year we formed the ratio of the weighted average of earnings by age and sex to the weighted average of the earnings of 40-year-old males. We then pooled these age- and sex-specific relative earnings ratios across our five years of data and ran a regression of the ratios against a polynomial in age and sex. The purpose of the regression was to smooth the predicted R_{eam} and R_{eaf} profiles; i.e., we used the values of R_{eam} and R_{eaf} predicted by this regression in our calculations.

The term E_{88} equals the sum of (1) NIPA (National Income and Product Accounts) wages and salaries, (2) a portion of NIPA proprietor's income, and (3) a portion of indirect business taxes. We allocated proprietor's income and indirect business taxes to labor, assuming that labor's share of these quantities was equal to labor's share of NNP.

B. Predicting Future Capital Inome

The U.S. wealth accumulation identity is used to calculate capital income for each year from 1990 through 2050.

$$A_{t+1} = A_t(1 + r) + E_t - C_t - G_t \tag{11}$$

In (11) A_t stands for national net wealth in year t, r_t is the pretax interest rate in year t, E_t is aggregate labor earnings in year t, C_t is aggregate private consumption in year t and G_t is government consumption in year t. Given the value for A_{88}, national net wealth in 1988, the values of E_{88}, C_{88}, and G_{88}, and the interest rate r we can use equation (11) to calculate A_{89}. Given A_{89}, E_{89}, C_{89} and G_{89} we can derive A_{90}, and so forth obtaining all the values of A_t through 2050. Multiplying A_t times r gives us aggregate capital income in year t. The values of A_{88} was determined by subtracting E_{88} (adjusted for proprietors' income and indirect business taxes) from NNP_{88} and dividing by r.[4]

C. Predicting Future Government Consumption

In predicting future government consumption in year t, we assume that the level of per capita government consumption observed in 1988 stays constant through time adjusted for a constant rate of government consumption growth. Hence, total predicted government consumption in year t equals our growth-adjusted level of per capita consumption multiplied by the population predicted in year t.[5]

[4]We use this approach of capitalizing returns to capital rather than using directly available estimates of the stock of tangible private wealth (such as are available from the Federal Reserve Board) in order to ensure internal consistency of our wealth and interest-rate assumptions.

[5]An alternative assumption, considered in Auerbach *et al.* (1989), is that, in addition to overall trend growth, age-specific components of government spending shift to maintain a fixed level *relative* to the population served; e.g., the level of education spending per capita would increase with an increase in the share of children in the population. Although this might be an interesting extension, our earlier study found the patttern of predicted saving rates to be relatively unresponsive to whether expenditures were normalized by the population as a whole or by affected subgroups of the population.

D. Using Consumption Data to Predict Future Private Consumption

The Infinite Horizon Family

We used the CES data to find the consumption growth factor (see eq. (4)) and the age-specific relative consumption weights (see eq. (5)) for the infinite-horizon family model. Specifically, we allocated each CES household's nondurables plus services consumption expenditures reported in each of their interviews to the particular members of the household. In this allocation we were able to identify certain expenditures as expenditures on adult males, adult females, and children. Adult male expenditures were divided equally among each of the household's adult males. Adult female expenditures were divided equally among each of the household's adult females. Children's expenditures were divided equally among each of the household's children. Expenditures that could not be classified as adult-male-, adult-female-, or children-specific expenditures were divided among all household members with children weighted as the equivalent of half an adult.

These allocated data then provided us with a data set of monthly nondurables plus services consumption expenditures by age and sex. We next determined the weighted average of these expenditures by age, sex, and month. Adding together for each age and sex group the 12 monthly weighted average expenditures in each calendar year gave us estimates of average annual nondurables plus services consumption expenditures by age and sex for each calendar year 1980–85. Next, we benchmarked these average values against the corresponding calendar year NIPA estimate of total private consumption. The benchmarking procedure involved, for each year, comparing the sum of the product of our average consumption expenditures (by age and sex) times the population (by age and sex) with aggregate private consumption. The difference between the two magnitudes was used to adjust our calendar-year age-sex average so that the product of our benchmarked calendar-year averages (by age and sex) times the population (by age and sex) equals total NIPA calendar-year private consumption.

We next formed for each calendar year the ratios of these age- and sex-specific weighted average consumptions to the weighted average consumption of 40-year-old males for the calendar year in question. These ratios for the six calendar years, 1980-1985, were pooled and used as observations of the dependent variable in a regression of relative consumption by age and sex against constant and sex-specific 5th order polynomials in age. This regression provided us with a smoothed cross-section age-sex consumption profile and, thus, the parameters indicated in (4).

To obtain the average growth rate of earnings (eq. (5)), we first used equation (12) to determine the average consumption of 40-year-old males for each year from 1950 through 1988, C_{40mt}.

$$C_t = C_{40mt} \sum_{a=1}^{120} (R_{cam} P_{amt} + R_{caf} P_{aft}) \tag{12}$$

In (12) the terms R_{cam} and R_{caf} refer, respectively, to the relative values of consumption of males age a and females age a predicted by our smoothed cross-section age-sex consumption profile. We then calculated the average value of C_{40ms+1}/C_{40ms} for $1950 \leq s \leq 1987$. This average, which equals .023, is used as the growth parameter defined in (5).

The Life-Cycle Model

To implement our prediction of future consumption under the life-cycle model, we need values of age- and sex-specific consumption growth rates (eq. (8)) as well as base year values of consumption by age and sex from which we extrapolate future consumption values as described above. We used the age- and sex-specific benchmarked average annual consumption data just described; specifically, we formed for males and females at each age the ratios of average consumption in one year to average consumption in the year before. These raw age- and sex-specific ratios were then regressed on 5th order polynomials in age. The regression was then used to produce smoother fitted growth rates of consumption by age and sex. We took as our base year values of consumption by age and sex the values of average consumption by age and sex computed from the CES for 1985, benchmarked to 1988 aggregate consumption. We chose 1985 because this is the latest year for which we had CES data.

IV. PREDICTED SAVING RATES, 1990–2050

This section provides forecasts of the next six decades saving rates, based on the three models described above.

A. The Altruistic Family

As indicated in the section describing the model, we use historical data to estimate ζ, the consumption growth factor. The result used in our simulations is $\zeta = 1.02$. Given, ζ and the projected population sizes, we consider a variety of combinations of interest rates and growth rates of earnings and government spending. Table 3 presents projected saving rates (averaged over

Table 3:
Saving Rates: The Family Model

Parameter Assumptions				
Rate of Growth:	2.0	2.4	2.4	2.4
Rate of Interest:	5.0	5.0	7.0	3.0
	Saving Rate			
1988	9.2	4.2	3.6	4.0
Decade Beginning:				
1990	10.2	4.8	4.3	4.3
2000	11.0	5.0	4.8	4.0
2010	10.3	3.7	4.1	2.0
2020	8.2	0.7	1.8	-1.6
2030	6.4	-2.1	-0.6	-4.6
2040	5.7	-3.6	-1.9	-6.0
Peak Saving Rate:	11.1	5.1	4.9	4.4
	(2006)	(2003)	(2007)	(1998)

decades) for four simulations, along with the estimated current value and the value and year in which the saving rate peaks before declining for the remainder of the period (a characteristic shared by all the simulations).

The first simulation sets the rate of growth of government spending per capita and labor productivity at 2.0 percent, and the real before-tax interest rate at 5.0 percent. We begin with this simulation because the growth rates seem to accord well with historical values. However, as is clear from the implied 1988 saving rate, this set of parameters predicts too small a value for C_{40} and hence too little aggregate consumption in 1988. Nevertheless, it is interesting to observe the pattern of saving that this simulation produces.

The simulation projects a smooth increase in national saving rates until the year 2006, with the national saving rate higher by 1.8 percentage points at its peak. Thereafter, the saving rate falls quite sharply, dropping by 4.5 percentage points from the 1990s to the 2040s. The increase and the drop each signal the movement of the large baby-boom cohort through life, first as its earnings relative to consumption peak and then as earnings fall with the cohort's aging and retirement. The timing of the saving rate peak, during the first decade of the next century, is consistent with the results of our earlier paper (Auerbach and Kotlikoff, 1990).

The remaining three simulations in Table 3 are based on a growth rate, 2.4 percent, that, for a wide range of interest rates, predicts 1988 aggregate consumption levels and saving rates that are close to observed values. Once one allows for the difference in 1988 saving rates, the simulated saving rate pattern is similar to that of the first simulation. However, the rise to peak saving early in the next century (in 2003) is milder, with the saving rate rising by just .9 percentage points, and the drop in saving thereafter sharper, with the last decade having a saving-rate average that is 8.7 percentage points below that at the peak.

Changing the assumed interest rate also alters the pattern of predicted saving. The third and fourth columns of the table present simulations for the same growth-rate assumptions as in the second simulation, but with the interest rate set equal to 7.0 percent and 3.0 percent, respectively. The higher interest-rate assumption leads to a delayed, but greater increase in the saving rate, which peaks in 2007 after increasing by 1.3 percentage points, and a smaller ultimate decline through 2050. For the lower interest-rate assumption, the saving rate rises only by .4 percentage points, peaks in 1998, and drops very sharply thereafter.

The basic reason for this difference is the higher level of consumption that can be supported, when interest rates are higher, by the current stock of assets plus those assets that are subsequently generated by the baby-boom's excess of earnings over consumption. This higher feasible consumption causes the initial 1988 saving rate to be lower, but over time is offset by the higher asset income being generated by the higher interest rate. Since a greater fraction of the consumption in the next century is being financed by capital *income* (as opposed to the decumulation of capital itself), the saving rate is higher when the assumed interest rate is higher.

Even these differences across simulations associated with the interest-rate assumption are not large relative to the saving-rate drop that each simulation predicts over the six decades, ranging from 6.2 percentage points to 10.3 percentage points. Even the first simulation, with its unrealistic 1988 saving rate, predicts a drop of 4.5 percentage points from the 1990s to the 2040s. By comparison, the low national saving rate in 1989, 3.7 percent, was 5.5 percentage points lower than the 1950s' peak average of 9.2 percent. Thus, all the simulations point to a relatively small increase in the national saving rate during the coming decade and a half, followed by an historically sharp decline through the year 2050.

B. The Life-Cycle Household

Table 4 presents saving rates based on simulations of the life-cycle model. Since each simulation is benchmarked to the actual 1988 aggregate consump-

Table 4:
Saving Rates: The Life-Cycle Model

Parameter Assumptions				
Rate of Growth:	2.0	2.4	2.4	2.4
Rate of Interest:	5.0	5.0	7.0	3.0
	Saving Rate			
1988	3.7	3.7	3.7	3.7
Decade Beginning:				
1990	7.3	7.7	7.6	7.2
2000	9.0	9.6	10.2	7.8
2010	5.3	6.0	7.7	2.7
2020	-0.6	0.2	3.0	-4.3
2030	-9.0	-7.8	-4.4	-12.3
2040	-21.1	-18.8	-16.9	-21.0
Peak Saving Rate:	9.6	10.2	10.7	8.9
	(2003)	(2003)	(2003)	(2001)
Percentage Increase in 21–year old consumption in 1989	98	106	114	104

tion level, all start from the actual 1988 saving rate. We present four simulations, based on the same combinations of assumed interest rates and government growth rates as were used in the previous case.

Our estimated longitudinal age-consumption profiles rise through age 35, but then, quite surprisingly, decline sharply over the remaining age range. Given that the population will be aging in the coming decades, this may help to explain why it is necessary for us to assume that future generations will consume more at age 21 than current generations in order to obtain sufficient aggregate consumption to exhaust the economy's resources. As the last entry of each column indicates, we find that the implied increase in consumption for individuals reaching adulthood after 1988, referred to above as x, ranges between 98 percent to 114 percent—essentially a doubling of consumption levels for generations reaching adulthood after 1988.

Because of the lack of linkage across generations, the predicted swings in saving rates are much larger for this model than the family model. The timing of the rise and fall in saving rates is in line with those of the family model, but the amplitude of the shifts is much larger. The year of peak saving predicted by the two models is never more than four years apart. The peak saving rate ranges from a low of 8.9 percent to a high of 10.7 percent. For the three simulations based on a 2.4 percent growth rate, the increase in saving from 1988 to the peak ranges from 5.2 percent to 7.0 percent, values that are consistent with our earlier findings (Auerbach and Kotlikoff, 1990), but much larger than the range of .4 percent to 1.3 percent predicted by the corresponding simulations of the family model. The predicted decade-average saving rates rise to historically high levels in all but the first simulation.

However, the simulations of the life-cycle model also predict enormous declines in the saving rate toward the end of the simulation period, and with saving rates dropping below those predicted by the family model to levels that are quite negative.[6]

C. The Reduced Form Model

As we have already indicated, our reduced form model is intended to allow for a variety of behavioral relationships not picked up by either of the other two models employed. We relate consumption (as a fraction of NNP) to its owned lagged value, concurrent levels of government spending and tax receipts (also relative to NNP), and shares of different age groups in the population. We predict government spending and taxes, using lagged values of each as well as the population shares. The estimated equations are:

$$\begin{aligned} C_t = & \underset{(6.55)}{-.64\, T_t} + \underset{(1.27)}{0.14\, G_t} + \underset{(1.56)}{0.23\, S1_t} + \underset{(2.58)}{0.39\, S2_t} + \underset{(2.44)}{0.99\, S3_t} + (13) \\ & \underset{(1.32)}{0.40\, S4_t} + \underset{(5.63)}{0.50\, C_{t-1}} \qquad R^2 = 0.99 \end{aligned}$$

$$\begin{aligned} T_t = & \underset{(2.71)}{0.48\, T_{t-1}} - \underset{(1.30)}{0.22\, T_{t-2}} + \underset{(0.94)}{0.21\, G_{t-1}} - \underset{(0.56)}{0.08\, G_{t-2}} + \underset{(0.28)}{0.08\, S1_t} - \\ & \underset{(0.42)}{0.12\, S2_t} + \underset{(1.00)}{0.65\, S3_t} + \underset{(0.12)}{0.06\, S4_t} \qquad R^2 = 0.99 \end{aligned}$$

[6] To test the sensitivity of our results to the unusual shape of the longitudinal consumption profile, we redid our simulations based on the profiles of expenditures on food and clothing rather than all consumption of nondurables and services. Although this led to a smaller implied jump in consumption (x), it led to very similar saving rates as those given in Table 4.

$$G_t = \underset{(2.04)}{0.22\, T_{t-1}} - \underset{(00.32)}{0.03\, T_{t-1}} + \underset{(5.67)}{0.76\, G_{t-1}} - \underset{(5.97)}{0.54\, G_{t-2}} - \underset{(0.31)}{0.05\, S1_t} -$$

$$\underset{(2.04)}{0.20\, S2_t} + \underset{(0.94)}{0.37\, S3_t} + \underset{(2.78)}{0.90\, S4_t} \qquad R^2 = 0.99$$

(t-statistics are in parentheses)

where consumption C, taxes T, and government spending G are all expressed relative to NNP. The share variables S_i indicate a considerable dependence of consumption on age. $S1$ is the share of the population younger than 21, $S2$ the share between 21 and 40, $S3$ the share betewen 41 and 60, and $S4$ the share over 60.

Unfortunately for our predictions, the aging of the population in the 1980s has coincided with a sharp drop in national saving. Given the model, this leads one to predict continued declines in saving as the population ages further in the coming decades, with the peak occurring immediately.[7] Because we find this pattern of saving implausible, we alter the model slightly by adding a dummy variable for the 1980s ($In80$), under the assumption that some unknown factor other than the aging of the population caused the decline during the 1980s. (See Table 5.) This leads to the revised consumption equation:

$$C_t = \underset{(6.40)}{-.63\, T_t} + \underset{(0.96)}{0.11\, G_t} + \underset{(2.67)}{0.27\, S2_t} + \underset{(2.67)}{0.41\, S2_t} + \underset{(2.49)}{1.02\, S3_t} + \quad (14)$$

$$\underset{(1.29)}{0.40\, S4_t} + \underset{(4.89)}{0.47\, C_{t-1}} + \underset{(0.85)}{0.004\, In80} \qquad R^2 = 0.99$$

and more sensible saving-rate predictions.

As the results indicate, the saving rate is predicted to peak sooner than the other two models had predicted. Even with the 1980s' dummy, the predicted saving rates do not rise very far above the current level and are predicted to be negative by the end of the first decade of the next century. By the last decade of the simulation period, the saving rate is similar to that predicted by the family model (Table 3). The main difference between the predicted saving rates of these two models is the earlier decline in the saving rate under the reduced form model.

[7]The 2.2 percent saving rate in 1988 is considerably less than the actual saving rate because it is the saving rate predicted by the regression. The size of the error indicates the difficulty the model has in explaining the behavior of recent years.

Table 5: Saving Rates: The Reduced-Form Model

1980s' Dummy Variable?	Yes	No
	Saving Rate	
1988	3.4	2.2
Decade Beginning:		
1990	3.5	-0.5
2000	0.6	-6.8
2010	-1.9	-11.7
2020	-3.6	-15.4
2030	-4.8	-17.8
2040	-5.0	-18.3
Peak Saving Rate:	4.4 (1992)	2.2 (1988)

V. CONCLUSIONS

In this paper, we have estimated the long-run saving rates that three different models of saving predict for the coming decades in the United States, during which a considerable demographic transition will occur. The results for the life-cycle and reduced-form models illustrate how difficult it is to extrapolate current behavior into the distant future when there are no natural connections between current and future consumption decisions. This difficulty is made more apparent by the absence from our analysis of any feedback effects on factor prices and the associated equilibrating mechanism that would cause consumption profiles to shift as interest rates and wage rates change.

In Auerbach *et al.* (1989), we found that real wages might increase substantially in the course of a demographic transition to an older population, due to an increased capital-labor ratio. In that paper, our simulations predicted that saving rates would begin to decline much sooner than in the first part of he next century. A potential explanation for the earlier commencement of the savings decline is that model's higher expected future wages. Combining the possibility of such general equilibrium effects with the richer descriptions of consumption behavior considered here presents a challenge for future research.

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