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Global demographic trends and social security reform ☆

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Abstract

How sustainable are the current social security systems in the developed economies, given the projected demographic trends? The most recent literature has answered this question through dynamic general-equilibrium models in a *closed*-economy framework. This paper provides a new quantitative benchmark of analysis for this question represented by a *two-region model* (South and North) of the world economy where capital flows across regions. The timing and the extent of the demographic transition—and the associated economic forces shaping capital accumulation and equilibrium factor prices—are very different in the two regions. Thus, the projected paths of interest rate and wage rate in the North diverge substantially between closed and open economy. We perform a wide range of policy experiments under both scenarios. Our main conclusion is that if one is interested in quantifying the path of the fiscal variables (e.g., the value of the payroll tax) needed to keep the social security system viable or to finance a transition towards a fully funded system, then these two benchmarks yield similar results. However, if the focus is on quantifying the path of factor prices, aggregate variables and, ultimately, welfare, then the two approaches can diverge significantly.

JEL classification: F21; F41; J11; H55

Keywords: Capital flows; Demographic trends; Social security; Two-region model

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1. Introduction

The developed world will experience dramatic demographic changes throughout the 21st century. The most important projected "demographic events" are three: (i) a significant increase in longevity which will increase life expectancy at 65 by 1.5 years per decade; (ii) a decline in fertility which will induce negative rates of population growth for the next 50 years; (iii) the retirement of the baby-boom generations, born in the 1950s, which will accelerate the rise of the old-dependency ratio (population 60+ as a fraction of the total) after 2010.

These demographic changes raise a number of crucial public policy issues. The one at the forefront of the current debate in the economic and political arena is the "sustainability" of the Pay-As-You-Go (PAYG) pension systems which, since their inception in the 1930s, represent one of the main pillars of social insurance policies in many countries across the developed world.[1]

When the PAYG system was introduced people lived beyond retirement age, on average, for many fewer years than now. As a consequence of the changes in longevity (and parallel trends in fertility), the ratio of retirees to active workers has constantly increased. The remaining two trends highlighted above will further accelerate the ageing of the population structure in the north of the planet and will put the PAYG system under severe strain. For example, in the United States (where the trends are not as daunting as in Japan or Europe), the social security administration, which is currently running a large surplus cumulated thanks to the contributions of the baby boomers, is projected to experience a deficit by 2016 and to exhaust the trust fund entirely, barring reforms, by 2042.

The absence of a structural adjustment in the medium run is an unlikely scenario. The relevant question is, rather, how big should the changes in the current tax/benefits parameters be to ensure that the PAYG system will be in equilibrium in the long run? A vast literature has attacked this question using general equilibrium overlapping-generations (OLG) models, in the tradition of Auerbach and Kotlikoff (1987). For example, one can quantify the necessary long-run increase in the payroll tax, in absence of any change in the current level of benefits. De Nardi et al. (1999) and Kotlikoff et al. (2002) predict an increase of around 15% in the next 100 years to keep the U.S. system solvent.

Clearly, the quantitative results of these experiments are very sensitive to the dynamic path of the rate of return on capital and the wage rate which are predicted for the next century. At least since Diamond (1965), economists have recognized that these factor prices are affected, in general equilibrium, both by the demographic trends and by the particular (pension) policy option in place during the demographic transition. Moreover, if the demographic trends around the world are not fully synchronized, the evolution of factor prices depends crucially on whether one assumes a closed or an open economy. The set of issues arising when considering an open economy are very rarely addressed. For instance, the calculations mentioned above are typically performed under the assumption of *closed* economy.

[1]The economic consequences of such demographic changes are not limited to pension systems. The changes in the age structure are expected to also deeply affect the health insurance system, as the demand for health services rises steeply with age (see Bohn, 2003) and asset prices, since retired baby boomers will strengthen the relative demand of particular financial products that preserve the asset value and its liquidity (see Brooks, 2000; Poterba, 2001; Abel, 2003).

In this paper, we argue that an equally interesting—but surprisingly overlooked in the literature—benchmark of analysis for social security reform is a two-region (South and North) open-economy model, where unobstructed capital flows across regions equalize the rate of return on capital. Every country in the developed world (the North) faces quantitatively similar demographic trends and the same thorny issue of how to reform a strained PAYG pension system. In contrast, in the developing world (the South), large-scale social security systems are absent and the demographic trends are markedly different from those of the North. In particular, old-age dependency ratios are less than half than in the North: 8% compared to 18% in 2000, and are projected to converge to the level of the North only after 2100. Roughly speaking, the demographic transition in the South lags the one in the North by seven or eight decades.

This lack of synchronization in the demographic trends between North and South generates, in a two-region open-economy model, major economic forces that have not been fully explored in this literature.

The objective of the paper is to study whether the quantitative implications of various social security reforms for policy variables, factor prices, macroeconomic aggregates, and welfare of different cohorts in the North are sensitive to the benchmark adopted, i.e., closed vs. open economy.[2] The two-region model is a relevant alternative framework, especially in light of the ever-increasing magnitude of global linkages in the world economy.

We perform two types of policy experiments. First, we assume that the North will retain a PAYG scheme and we examine several options to finance the system through the demographic transition. In particular, we look, in turn, at the effects of financing the imbalance of the current system by increasing payroll taxes, and consumption taxes; by issuing debt; by reducing benefits, and by increasing retirement age. Second, we assume the PAYG will be gradually transformed into a fully funded system, and we study alternative ways of financing this privatization. We perform all these experiments under both open and closed economy.

In the first set of experiments, somewhat surprisingly, we find that the evolution of the policy variables used to finance the PAYG system (e.g., payroll tax, consumption tax, debt, benefits) is remarkably similar in closed and open economy. However, this similarity hides important discrepancies in the dynamics of the aggregate capital stock, aggregate output, and prices. In the two-region model, thanks to the inflow of resources from the South, in the long run the North accumulates capital faster than in closed economy: interest rates decrease (and wages rise) faster than in closed economy. From the point of view of the government budget constraint, in the open-economy equilibrium the gains in the labor income tax revenues due to the higher wage offset almost exactly the losses in the capital income tax revenues due to the lower interest rates. As a result, the equilibrium path of the fiscal variables is almost identical in the two benchmarks.

In the second set of experiments, households in the North massively accumulate savings for retirement, as the PAYG system is slowly phased out. This rapid capital accumulation in the North parallels the demographic forces in the South in generating a similar decline in the interest rates in both regions. The path of factor prices in closed and open economy

[2] An equally interesting question is calculating the welfare effect of the demographic transition (and how it differs in an open vs. closed economy) for various cohorts. We make these calculations, from the perspective of households living in developing economies (the South), in a companion paper. See Attanasio et al. (2006).

turns out to be quite close. As in the first set of experiments, but for very different reasons, the policy variables end up changing similarly in the two benchmarks.

With respect to welfare, the main result that holds across experiments is that in open economy the welfare effects of the reforms in the North are significantly smaller in absolute value than in closed economy. The key reason is that, in open economy, social security reforms have a lighter impact on wage rates and interest rates in the North. Capital can flow into the North when domestic savings are low, keeping wages of the future generations high, independently of the reform. In closed economy, instead, the nature of the reform greatly affects capital accumulation and the future path of wages and interest rates.

The chief conclusion of our exercise is, perhaps, that if one is interested in quantifying the dynamics of the fiscal variables (e.g., the payroll tax) needed to keep the PAYG viable, or needed to finance a transition towards a fully funded system, then it does not matter too much whether the closed or open economy view is taken. However, if the focus is on quantifying the evolution of factor prices and aggregate variables, then the two approaches diverge significantly. In terms of welfare, in virtually every policy experiment, the welfare effects of the reform in closed economy are larger in absolute values than those in open economy.

There are several branches of the literature related to our paper. First, there is a vast literature on equilibrium OLG models that evaluates quantitatively different scenarios for social security reform in closed economy. A far from exhaustive list includes Geanakoplos et al. (1998), De Nardi et al. (1999), Huggett and Ventura (1999), Kotlikoff et al. (1999), Abel (2001a, 2003), Bohn (2003), Diamond and Geanakoplos (2003), and Krueger and Kubler (2005).

Second, some authors have argued, within structural models, that the unsynchronized demographic trends across more and less developed countries can shape the dynamics of current accounts. See, among others, Henriksen (2002), Brooks (2003), Domeij and Floden (2004), and Attanasio et al. (2006).[3]

Third, some authors have explicitly recognized that the closed-economy benchmark may not be the right one to study the implications of reforming the PAYG system. Huang et al. (1997) analyze reforms in the U.S. under the "small open-economy" assumption, with fixed interest rates. Borsch-Supan et al. (2003) and Fehr et al. (2004a) proposed multi-region models of the *developed* world (i.e., a subset of OECD countries), where the focus is on the effects of pension reform in U.S. and Germany, respectively, in open economy. Fehr et al. (2006) extend their multi-country model to analyze the role of China and conclude that capital flowing from China to the more developed world will significantly contribute to the rise in labor productivity and wages, counteracting the payroll tax hike needed to sustain the PAYG systems.

Our paper lies right at the center of these contributions, since it combines all these various approaches by studying quantitatively social security reform in the developed world through a North–South equilibrium OLG model with demographics-induced capital flows.

The rest of the paper is organized as follows. Section 2 provides a description of the data on demographic trends in the North and the South. Section 3 outlines the economic

[3] Higgins (1998), Helliwell (2004), and Luhrmann (2005) have documented an empirical association in the data between capital flows and age structure of the population, via reduced-form regressions.

environment of our two-region open-economy model and defines the equilibrium. Section 4 describes the calibration of the model. Section 5 contains the results of our policy experiments. Section 6 discusses how robust our results are to the relaxation of various assumptions. Section 7 concludes the paper. The Appendices contain: (i) a detailed description of the data sources and the methodology chosen to model the demographic transition in the two regions, (ii) an outline of the computational algorithm, and (iii) an explanation of the solution of the extended model with endogenous labor supply.

2. Global demographic trends

We begin by documenting one of the building blocks of our analysis, i.e. the fact that the demographic trends across regions in the world are unsynchronized. We focus our description on the evolution of fertility rates and life expectancy.

Our main sources of demographic data are two publications of the United Nations (UN) (2003): *World Population Prospects: The 2002 Revision Population Database (2003)*, and *World Population in 2300*. The first database provides historical and projected demographic data across countries for the period 1950–2050. The second document projects world population by country until 2300. The projections include four variants, diverging with respect to the assumptions made about the future course of fertility. We use the so-called "medium scenario".

In preparation for our two-region model, we divide the world into two regions which we call the North and the South. The North corresponds to the set of "More Developed Regions" according to the UN's classification (i.e., North America, Europe, Japan, Australia, and New Zealand). The South corresponds to the UN group of "Less Developed Regions", which includes Africa, Asia (except for Japan), Latin America and the Caribbeans, plus Melanesia, Micronesia, and Polynesia.[4]

Fertility transition: Age-specific fertility rates are only available from UN data for 1995–2050. For the years before 1995, only the *total* fertility rates for each region are available, hence we extrapolate the available age-specific fertility rates backward and compute age-specific rates for 1950–1995 to match the data on total fertility rates. For the periods beyond 2050, we use the projected total fertility rates based on the study in United Nations (2004) and extrapolate forward the age-specific fertility rates using the projected total fertility rates for the years 2100 and 2200 as targets. See Appendix A for details.

Fig. 1 reports the age-specific fertility rates and the top panel of Fig. 2 reports the data on total fertility rates and the model's fit. The most interesting facts in Fig. 1 are two: first, in the North the rebound in fertility rates occurs for the age classes 25–39, but fertility before age 25 keeps declining; second, in the South, it is projected that after 2050, the women in the age group 25–34 will overcome women in the age group 20–24 as those more likely to have children, exactly like what happened in the North since 2000. In terms of total fertility rates, Fig. 2 shows that fertility dropped from 3.0 to 1.6 in the North

[4]The UN projections are necessarily undermined by some degree of statistical uncertainty, especially at long time horizons. However, we have two reasons to be fairly confident. First, past estimates have been very accurate: for example, in 1957 the projection overestimated the world population in 2000 by less than 4%. Second, most of the errors are made at the country level, so aggregation into two large regions helps substantially. In Section 6 we discuss the sensitivity of our results to using different scenarios.

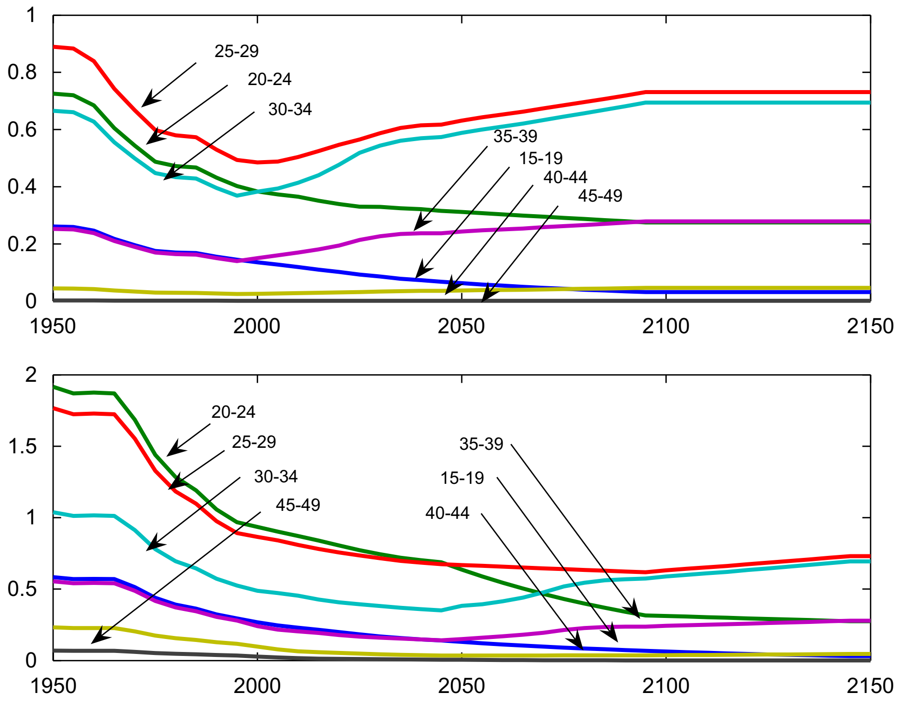


Fig. 1. Changes in fertility rates by age group. The top panel represents the North, the bottom panel the South.

in the past 40 years and is now rising back towards its "replacement value". In the same period, the number of children per woman in the South dropped from 6.0 to 3.0 and is forecasted to converge to its replacement value by 2100.[5] Note that in both regions fertility remained high for a decade or so in the 1950s, marking the so-called baby boom.

Life-expectancy transition: Age-specific surviving probabilities in the two regions for the period 1950–2050 are computed based on the actual and projected data on population shares by age group in the UN database. We compute the surviving rates of the two regions beyond 2050 to match the UN projections (see United Nations, 2004) for three key population moments: median age in 2100 and 2200, old-dependency ratio (proportion of persons aged 60+) in 2100 and 2200, and the average population growth rate for 2100–2150 and 2150–2200. We make an adjustment in the surviving rates for the South so that they converge smoothly to the same demographic steady state as the North by the year 2200. See Appendix A for details.

The second, third, and fourth panels of Fig. 2 plot the evolution of median age, old-dependency ratio, and population growth in the data together with the statistical model's fit. Note that until 2050, an individual in the North is, on average, 10 year older than a

[5]The sharp decline in fertility in the South is caused by a combination of three forces: (1) health improvements that reduced infant mortality, (2) the diffusion of family planning services, and (3) new labor market opportunities for women. See Schultz (1997).

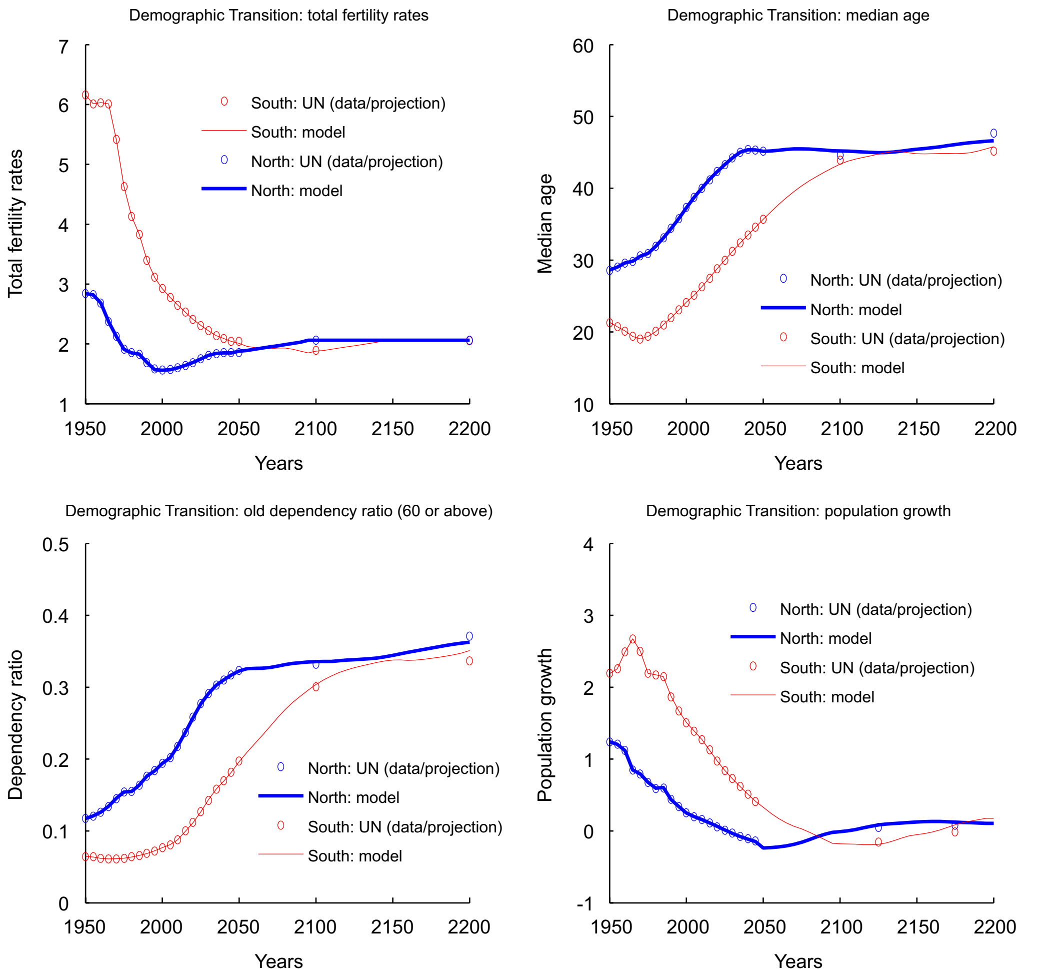


Fig. 2. Demographic transition in the North and the South.

typical one in the South; then, the age structure converges quickly in the following 50 years. The old-dependency ratio, a rough index of the (un-) sustainability of the PAYG system grows from 10% in 1950 to over 30% in 2050 in the North. Population growth in both regions converges towards 0.05% per year in the long run.

Another major demographic trend is the rise in female labor force participation. Our main data sources here come from the U.S. Census Bureau (2004): *The International Data Base* contains historical data for United States, Brazil, India, Korea, and Mexico. *Labor Force*, *Employment and Earnings* contains projections for the United States.

Female participation transition: The top panel of Fig. 3 plots historical data on women's participation rates from U.S. Census Bureau (2003) for the U.S. (representing the North) and the average of four countries in the South. Female participation rates in 2000 were roughly 42% in the South and 67% in the North, so the South is expected to substantially increase its stock of productive labor over the next few decades. Based on the projections

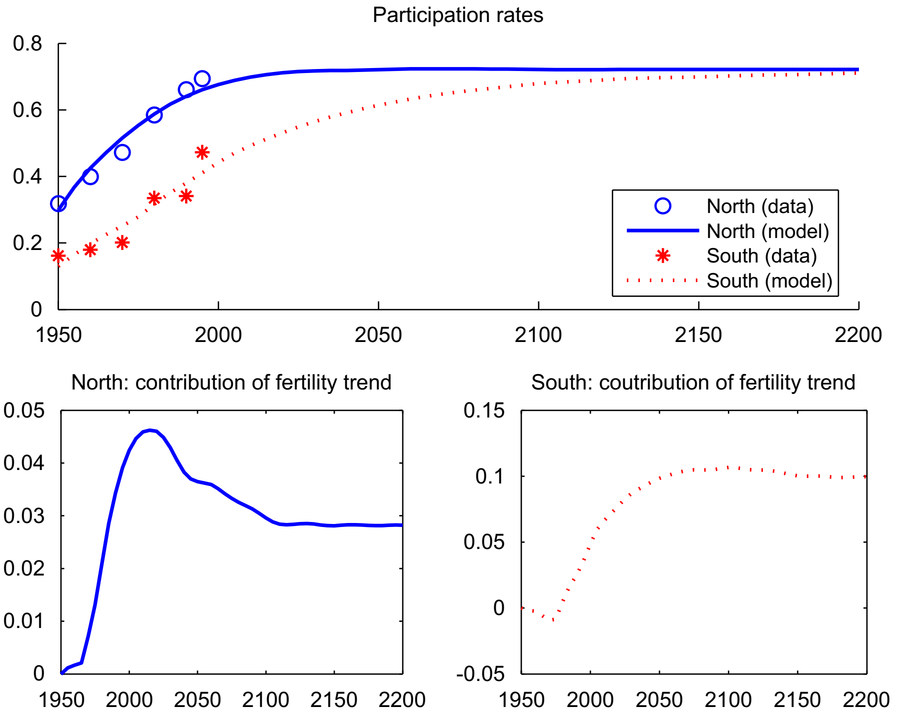


Fig. 3. Dynamics of female participation rates. The bottom two panels depict the contribution of the decline in fertility to the rise in participation in the two regions.

in U.S. Census Bureau (2003), the female participation rate will reach 72% in the North in the next 20 years and settle at that level for the long run. We assume that the level of women's participation to the labor market in the South converges to the North's level by 2200, like every other demographic variable.

Increased women's participation is at least partially associated to the fertility decline. How can we capture this link? We use Consumers Expenditures Survey (CEX) data (sample of married household headed by males, aged 17–69, for 1982–1999) on female hours worked and the number of children of different age groups (0–4, 5–9, 10–14) and estimate, through a Probit regression, the marginal effect of having dependent children of a particular age on hours worked measured as a fraction of the time endowment. As expected, we find that the newborn child has the strongest negative impact on participation and, as the child ages, the effect is smaller.

Given our estimated fertility trends in both regions, we then use the estimated coefficients to calculate how much the decline in fertility contributes to the observed rise in participation. The bottom panels of Fig. 3 show that only a small fraction—roughly one-tenth—of the observed postwar increase in female participation in the North and in the South can be explained by the decline in fertility.

This finding, consistent with some of the evidence on female labor supply (see, for instance, Attanasio et al., 2004), makes a second step necessary to fit the data: we introduce a region-specific polynomial time trend that accounts for the residual increase documented above.[6] The top panel of Fig. 3 shows the fit of the model. See Appendix A for more details.

To conclude, while the North is in the last phase of a long demographic transition, the South is right in the midst of it. It suffices to think that virtually the entire world population growth, between now and 2050, is projected to occur in the South.[7] The presence of economic linkages among the two economies, due to capital mobility, means that the demographic changes in one region will be transmitted to the other region and, through this interaction, an equilibrium path for interest rates will arise that is likely to be very different from the closed-economy equilibrium path. In turn, different dynamics of factor prices can lead to diverging macroeconomic and welfare effects of social security reform in the North. In the next section, we outline a model where this idea is properly formalized.

3. The model

3.1. Economic environment

Preliminaries: The world economy is composed by two regions, the North and the South. The two regions differ by demographic structure, total factor productivity level, and fiscal institutions. In what follows these differences are spelled out more in detail. There is no aggregate or region-specific uncertainty, but since we will model a deterministic transition across two steady states, equilibrium factor prices will be time varying in a deterministic way. The only source of individual risk is related to the uncertain life span. We let t denote time, i individual's age, and r the two regions (North and South), with $r = n, s$.

Technology: In each region r, a constant returns to scale, aggregate production function $F(Z_t^r, K_t^r, H_t^r)$ produces output of a final good Y_t^r which can be used interchangeably for consumption C_t^r and investment X_t^r. Among the arguments of the production function, Z_t^r denotes the total factor productivity level in region r at time t, H_t^r is the stock of human capital (i.e., the aggregate efficiency units of labor), and K_t^r is the aggregate stock of physical capital used in production in region r. Physical capital depreciates geometrically at rate δ each period. The level of technology in region r grows exogenously at rate λ_t^r between t and $t+1$, but in the long run both regions grow at the same constant rate λ.

Demographics: Each region in the economy is populated by OLG of ex ante identical "pairs of individuals" who may live for a maximum of $\bar{I}$ periods indexed by $i = 1, 2, \ldots, \bar{I}$. Pairs of individuals are dependent children for the first I^d periods of their life and then they turn adult and form a household. For a pair of individuals born in region r, denote by $s_{i,t}^r$

[6] We remain agnostic on the economic forces underlying the trend. Attanasio et al. (2004) have focused on the role of the reduction in child-care cost; Jones et al. (2003) on the reduction of the gender gap; Olivetti (2001) on the rise in the return to experience; Greenwood et al. (2005) on technological progress in the household sector.

[7] For tractability, we have assumed homogeneity of demographics within the two regions. Even though the between-region divergence is an order of magnitude larger, there are some notable within-region differences. Within the North, the most important one is the distinction between the recent trends in population growth in the U.S. and the rest of the developed world. Fertility is, for example, hovering at replacement value in the U.S., while it is much lower in Europe and Japan. Within the South, Africa is at the early stages of the demographic transition compared, say, to Latin America. At the other end of the spectrum, China witnessed a dramatic decline in fertility rate in the past two decades, mostly due to family planning government policies.

the probability of surviving until age i at time t, conditional on being alive at time $t-1$ (with age $i-1$). Hence, in region r, the unconditional probability of surviving i periods up to time t is simply

$$S^r_{i,t} = \prod_{j=1}^{i} s^r_{j,t+(j-i)},$$

where $S^r_{1,t} = s^r_{1,t} \equiv 1$ for all t by definition. In each period t, pairs of age i in region r have an exogenously given fertility rate (i.e., a probability of giving birth to another pair of individuals) equal to $\phi^r_{i,t}$. During childhood, i.e. until age I^d, fertility is assumed to be zero. For what follows, it is useful to define $d^r_{i,t}$ as the total number of (pairs of) dependent children living in a (adult) household of age i at time t, i.e.

$$d^r_{i,t} = \begin{cases} 0 & \text{for } i \leqslant I^d, \\ \sum_{k=i-I^d+1}^{i} \phi^r_{k,t-(i-k)} S^r_{i-k+1,t} & \text{for } i > I^d. \end{cases}$$

We denote by $\mu^r_{i,t}$ the size of the population of age i at time t in region r and by $\boldsymbol{\mu}^r_t$ the $(\overline{I} \times 1)$ vector of age groups. Thus, in each region the law of motion of the population between time t and $t+1$ is given by $\boldsymbol{\mu}^r_{t+1} = \Gamma^r_t \boldsymbol{\mu}^r_t$ where Γ^r_t is a time varying $(\overline{I} \times \overline{I})$ matrix composed by fertility rates and surviving probabilities for households of region r described by

$$\Gamma^r_t = \begin{bmatrix} \phi^r_{1,t} & \phi^r_{2,t} & \cdots & \cdots & \phi^r_{\overline{I},t} \\ s^r_{2,t+1} & 0 & \cdots & \cdots & 0 \\ 0 & s^r_{3,t+1} & 0 & \cdots & 0 \\ 0 & 0 & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & s^r_{\overline{I},t+1} & 0 \end{bmatrix}.$$

The first row of this demographic transition matrix contains all the age-specific fertility rates, the elements $(i+1, i)$ contain the conditional surviving rates, whereas all the other elements are zeros. Lee (1974) shows that the largest eigenvalue of Γ^r_t is the growth rate of the population between time t and $t+1$, which we denote as γ^r_t (see also Rios-Rull, 2001).

Since we are interested in the economically active population, we reshape the matrix Γ^r_t and the vector $\boldsymbol{\mu}^r_t$ down to size $I = \overline{I} - I^d$ and we normalize the first period of adulthood (and economically active) life to be period 1 of life for households. We also restrict the parameters of the two matrices Γ^r_t to converge across regions as t becomes large, in order to generate a common long-run growth rate of the population γ.[8]

Household preferences: Households of age i at time t in region r are composed by a pair of adults and a number $d^r_{i,t}$ of pairs of dependent children living with their parents. The adults in the household jointly make consumption allocation decisions for themselves and their dependent children based on the intra-period utility function

$$u^r(c^a_{i,t}, c^d_{i,t}) = \frac{(c^a_{i,t})^{1-\theta}}{1-\theta} + d^r_{i,t}\omega(d^r_{i,t})\frac{(c^d_{i,t})^{1-\theta}}{1-\theta}, \tag{1}$$

[8]This restriction, similar to the one we impose for productivity growth, is necessary to achieve a long-run growth path where neither region is negligible in terms of output and population compared to the other one.

where $c^a_{i,t}$ denotes consumption for the adults, $c^d_{i,t}$ consumption per dependent child, and $\omega(d^r_{i,t})$ is a positive function that weighs consumption of children in households' utility. The intertemporal elasticity of substitution for consumption is $1/\theta$. This preference specification is convenient, because it permits to express utility only as a function of the total consumption of the household $c_{i,t} = c^a_{i,t} + d^r_{i,t}c^d_{i,t}$. From the optimality condition of the household with respect to $c^d_{i,t}$ one obtains

$$c^d_{i,t} = c^a_{i,t}\omega(d^r_{i,t})^{1/\theta}, \tag{2}$$

which sets optimally the consumption of children to a fraction of the consumption of parents proportional to their weight in the utility function. Using (2) into (1), together with the definition of the total consumption of the household $c_{i,t}$ one obtains

$$u^r(c_{i,t}) = \Omega^r_{i,t}\frac{c^{1-\theta}_{i,t}}{1-\theta}, \tag{3}$$

where $\Omega^r_{i,t} = [1 + \omega(d^r_{i,t})^{1/\theta}d^r_{i,t}]^\theta$ and acts like an age- and time-dependent preference shifter. To conclude, the intertemporal preference ordering for households born (adult of age $i = 1$) at time t is given by

$$U^r = \sum_{i=1}^{I} \beta^{i-1}S^r_{i,t+i-1}\Omega^r_{i,t+i-1}\frac{c^{1-\theta}_{i,t+i-1}}{1-\theta}, \tag{4}$$

where β is the discount factor. There is no explicit altruistic motive.

Household endowments: Households derive no utility from leisure. They have a fixed time endowment, normalized to one unit, that they can devote either to productive activities in the labor market or to child care at home. We denote by $\mathbf{d}^r_{i,t}$ the $(I^d \times 1)$ vector of pairs of children's by age groups for a household of age i at time t. Labor supply for households of region r at age i at time t is given by

$$l^r_{i,t} = \begin{cases} \Lambda^r_t(\mathbf{d}^r_{i,t}) & \text{if } i < I^R, \\ 0 & \text{otherwise}, \end{cases} \tag{5}$$

where $1 - \Lambda^r_t(\mathbf{d}^r_{i,t})$ is an exogenous fraction of time that needs to be devoted to child care. The function $\Lambda^r_t(\mathbf{d}^r_{i,t})$ is decreasing in the number of dependent children and captures the rise in labor force participation of women. At age I^R, households are subject to compulsory retirement from any working activity. Households of age i at time t in region r are endowed with $\varepsilon^r_{i,t}$ efficiency units of labor for each unit of time worked in the market. Finally, we assume that the initial asset holdings of each household is zero, i.e. $a_{1,t} = 0$ for any t in both regions.

Household budget constraint: Let $a^r_{i,t}$ be the net asset holding of individual i at time t in region r. We assume that there are annuity markets to cover the event of early death. Every household has the right to keep the share of assets of the deceased in the same cohort, thus we can write the budget constraint as

$$(1 + \tau^r_{c,t})c^r_{i,t} + s^r_{i+1,t+1}a^r_{i+1,t+1} = y^r_{i,t} + [1 + (1 - \tau^r_{a,t})r_t]a^r_{i,t}. \tag{6}$$

We require households to die with non-negative wealth once they reach age I, but otherwise we impose no borrowing constraint during their life. Net income $y^r_{i,t}$ accruing to households of age i in region r at time t is defined as

$$y_{i,t}^r = \begin{cases} (1-\tau_{w,t}^r) w_t^r \varepsilon_{i,t}^r l_{i,t}^r = (1-\tau_{w,t}^r)\tilde{y}_{i,t}^r & \text{if } i < I^R, \\ p_{i,t}^r & \text{if } i \geqslant I^R, \end{cases} \tag{7}$$

where w_t^r is the wage rate, $\varepsilon_{i,t}^r$ is the efficiency units of labor of an individual of age i, and $p_{i,t}^r$ is pension income. $\tilde{y}_{i,t}^r$ is the before-tax labor income. Households pay taxes $\tau_{c,t}^r$ on consumption, $\tau_{a,t}^r$ on capital income, and $\tau_{w,t}^r$ on labor income. Residents of region r pay capital income taxes in region r, independently of where capital was invested. Social security benefits are given by the formula

$$p_{i,t}^r = \kappa_t^r \frac{W_{i,t}^r}{I^R - 1},$$

where κ_t^r is the replacement ratio of average past earnings. Cumulated past gross earnings $W_{i,t}^r$ are defined recursively as

$$W_{i,t}^r = \begin{cases} \tilde{y}_{1,t}^r & \text{if } i = 1, \\ \tilde{y}_{i,t}^r + W_{i-1,t-1}^r & \text{if } 1 < i < I^R, \\ W_{i-1,t-1}^r & \text{if } i \geqslant I^R. \end{cases} \tag{8}$$

Government budget constraint: In each region r, public expenditures and social security program are administered by the government under a unique consolidated intertemporal budget constraint. The government can raise revenues through its fiscal instruments $(\tau_{c,t}^r, \tau_{a,t}^r, \tau_{w,t}^r)$ and can issue one-period risk-free debt B_t^r. Government borrowing and tax revenues finance a stream of expenditures G_t^r and the PAYG social-security program described above. The consolidated government budget constraint reads as

$$G_t^r + (1+r_t)B_t^r + \sum_{i=I^R}^{I} p_{i,t}^r \mu_{i,t}^r = \tau_{w,t}^r w_t^r \sum_{i=1}^{I^R-1} \mu_{i,t}^r \varepsilon_{i,t}^r \Lambda_{i,t}^r + \sum_{i=1}^{I} \mu_{i,t}^r (\tau_{a,t}^r r_t a_{i,t}^r + \tau_{c,t}^r c_{i,t}^r) + B_{t+1}^r. \tag{9}$$

Commodities, *assets and markets*: There are three goods in the world economy: a final good which can be used either for consumption or investment, the services of labor and the services of capital. The price of the final good (homogeneous across the two regions) is used as the world numeraire. Labor is immobile, thus wages are determined independently in regional labor markets. Physical capital is perfectly mobile across the two regions, so there is one world market for capital. We denote as N_t the *external wealth* of the North, i.e. the stock of capital productive in the South which is owned by households of the North, with the convention that a negative value denotes ownership of capital used for production in the North held by households of the South. Finally, in every region there is a financial market for government debt. The markets where these goods and assets are traded are perfectly competitive. An intuitive no-arbitrage condition between assets and the absence of aggregate uncertainty imply that the return on both regional bonds is equal to the return on physical capital, as we have already implicitly assumed when we wrote the budget constraints of the government and households.

3.2. Equilibrium

Before stating the definition of equilibrium, it is useful to point out that, without further restrictions, the equilibrium path of the fiscal variables $\{G^r_t, \kappa^r_t, \tau^r_{w,t}, \tau^r_{a,t}, \tau^r_{c,t}, B^r_t\}_{t=1}^{\infty}$ is indeterminate, as there is only one budget constraint we can operate on. In what follows, we define an equilibrium for the case where the paths of all fiscal variables are given, except for $\{\tau^r_{w,t}\}_{t=1}^{\infty}$. This case corresponds to our baseline experiment. It is straightforward to extend this definition to the case where the path of a different set of government policies is given exogenously. Finally, for brevity we omit the definition of the closed-economy equilibrium and state directly the equilibrium conditions for the open economy.

A *competitive equilibrium of the two-region economy*, for a given sequence of region-specific demographic variables $\{\Gamma^r_t, \Lambda^r_t\}_{t=1}^{\infty}$, TFP levels $\{Z^r_t\}_{t=1}^{\infty}$, and fiscal variables $\{G^r_t, \kappa^r_t, \tau^r_{a,t}, \tau^r_{c,t}, B^r_t\}_{t=1}^{\infty}$, is a sequence of: (i) households' choices $\{\{c^r_{i,t}, a^r_{i,t}\}_{i=1}^{I}\}_{t=1}^{\infty}$, (ii) labor income tax rates $\{\tau^r_{w,t}\}_{t=1}^{\infty}$, (iii) wage rates $\{w^r_t\}_{t=1}^{\infty}$, (iv) aggregate variables $\{K^r_t, H^r_t, X^r_t, C^r_t\}_{t=1}^{\infty}$ in each region r, (v) world interest rates $\{r_t\}_{t=1}^{\infty}$, and (vi) external wealth of the North $\{N_t\}_{t=1}^{\infty}$such that:

1. Households choose optimally consumption and wealth sequences $\{\{c^r_{i,t}, a^r_{i,t}\}_{i=1}^{I}\}_{t=1}^{\infty}$, maximizing the objective function in (4) subject to the budget constraint (6), the income process (7), and the time allocation constraint (5).
2. Firms in each region maximize profits by setting the marginal product of each input equal to its price, i.e.
$$w^r_t = F_H(Z^r_t, K^r_t, H^r_t) \quad \text{for } r = n, s, \tag{10}$$
$$r_t + \delta = F_K(Z^n_t, K^n_t, H^n_t) = F_K(Z^s_t, K^s_t, H^s_t). \tag{11}$$
3. The regional labor markets clear at wage w^r_t and aggregate human capital in each region is given by
$$H^r_t = \sum_{i=1}^{I^R-1} \mu^r_{i,t} \varepsilon^r_{i,t} \Lambda^r_{i,t}. \tag{12}$$
4. The regional bond markets and the world capital market clear at the world interest rate r_t, and the aggregate stocks of capital in the two regions satisfy
$$K^n_t + N_t + B^n_t = \sum_{i=2}^{I} \mu^n_{i-1,t-1} a^n_{i,t},$$
$$K^s_t - N_t + B^s_t = \sum_{i=2}^{I} \mu^s_{i-1,t-1} a^s_{i,t}. \tag{13}$$
5. The tax rates $\{\tau^r_{w,t}\}_{t=1}^{\infty}$ satisfy the consolidated budget constraint (9) in each region.

6. The allocations are feasible in each region, i.e., they satisfy the regional aggregate resource constraints

$$K^n_{t+1} - (1-\delta)K^n_t + N_{t+1} - (1+r_t)N_t = F(Z^n_t, K^n_t, H^n_t) - C^n_t - G^n_t,$$

$$K^s_{t+1} - (1-\delta)K^s_t - N_{t+1} + (1+r_t)N_t = F(Z^s_t, K^s_t, H^s_t) - C^s_t - G^s_t. \tag{14}$$

Before concluding, it is useful to recall that aggregate gross investments in region r are given by

$$X^r_t = K^r_{t+1} - (1-\delta)K^r_t, \tag{15}$$

whereas aggregate (private plus public) savings in the North and in the South are, respectively,

$$S^n_t = F(Z^n_t, K^n_t, H^n_t) + r_t N_t - C^n_t - G^n_t,$$

$$S^s_t = F(Z^s_t, K^s_t, H^s_t) - r_t N_t - C^s_t - G^s_t. \tag{16}$$

As a result, the current account surplus of the North (or, the net capital outflow from the North into the South) is given by

$$S^n_t - X^n_t = CA^n_t = N_{t+1} - N_t, \tag{17}$$

and it equals the change in the net foreign asset position of the North. Moreover, in this two-region economy, $CA^n_t = -CA^s_t$.

Appendix B contains a detailed description of the algorithm we use to compute the equilibrium transitional dynamics. It is worth remarking that our solution of the consumption-saving problem of the household in each period is *exact*. The only numerical approximation comes from the search for the sequence of market clearing prices and policy variables that make the government budget constraints hold with equality along the transition.

3.3. Discussion

The two-region model allows to quantify the importance of some factors that have been largely overlooked in the current debate on social security reform. In particular, the extent to which the existence of unsynchronized demographic trends, together with the presence of integrated capital markets, affects the comparison of alternative policy reforms usually performed in a closed-economy setting. Computing numerically the transitional dynamics of this model economy is a non-trivial task, thus inevitably there are some aspects of reality that, albeit potentially important, play no role in our stylized model.

We remain agnostic about the nature of the world development process, which we model through exogenous TFP growth. In the benchmark simulations, we let productivity growth in the South to fully catch up to the growth rate of the North, in the long run. The speed of the catch up is important as the role of the South on world factor prices depends on its size, measured by the total number of labor efficiency units.

Our assumption of frictionless world capital market is extreme: frictions in capital flows, or capital account restrictions in large countries like China and India could lead us to different conclusions.

We abstract from the effects of tax reforms on labor supply. That is we do not allow for the distortionary effects that increased taxation might have, and that has received some attention in the literature (see Feldstein, 2005). However, we model the major long run trend in labor supply, rising female participation, which we link, albeit mechanically, to the fall in fertility.

We also ignore intra-cohort heterogeneity and idiosyncratic labor market risk. As such, we are silent on the within-cohort insurance role of social security discussed, for example, by Conesa and Krueger (1999) in closed economy, and by Krueger and Ludwig (2006) in open economy. Our model does not display aggregate uncertainty, thus we do not speak to the issue of diversification and risk of retirement savings, as addressed, for instance, by Geanakoplos et al. (1998), Abel (2001a), and Diamond and Geanakoplos (2003).

With respect to labor migration towards developed countries, even though we do not model explicitly the migration decision, we do take into account the way migration affects the demographic structure, since the forecasted demographic trends include the migration flows projected by the UN for the next 300 years.[9] Storesletten (2000) and Fehr et al. (2004b), among others, also consider the potential of labor migration for mitigating the pressures on the PAYG system.

Rather than modelling explicitly altruism, we assume the existence of annuity markets. This assumption has the convenient property that it eliminates accidental bequests and greatly simplifies the numerical solution. Abel (2001b) discusses the potential importance of a bequest motive and concludes that its presence is not quantitatively important for the dynamics of factor prices following the retirement of the baby-boom generation.

We assume that, once the demographic steady state is (unexpectedly) perturbed in 1950 by the baby boom and by the subsequent baby bust, households anticipate these events and, consequently, have perfect foresight on the path of future prices and taxes. While probably unrealistic, this is a common assumption when solving for transitional dynamics. See Lucas (2003) for a discussion on how to model expectations about long-run future policies in OLG economies.[10]

In Section 6, we return on many of these assumptions (endogenous labor supply, TFP growth, population growth, degree of frictions in international capital markets, openness of China and India, etc.), and perform a robustness analysis. At this point, it is useful to recall that the objective of the paper is not determining the welfare maximizing reform, or the macroeconomic effects of a given reform in "absolute terms". Rather, the contribution of the paper is to contrast a set of PAYG reforms in closed and open economy, providing a relative comparison between two benchmarks, and all these assumptions are made in both benchmarks.

4. Calibration

Preliminaries: We calibrate the initial steady-state using demographic and economic variables for the period 1950–2000 in the two regions, and we assume that all demographic

[9]Broadly speaking, in the UN projections, the future path of international migration is set on the basis of past international migration estimates and an assessment of the policy stance of countries with regard to future international migration flows.

[10]1950 is a natural date to "shock" the economy through a new set of demographic variables since, to a large extent, the baby boom was unexpected.

and TFP parameters in the two regions converge to the same values by 2200, thus the two regions converge to the same balanced growth path some time after 2200. We then let our world economy transit between the two steady-states, by imposing the path of mortality, fertility and female participation rates modelled in Section 2. The model's period is set to 5 years.[11]

The two regions: The North in our model corresponds to the UN's set of "more developed regions" which includes North America, Europe, Japan, plus Australia and New Zealand. The South corresponds to the set of "less developed regions", which combines the rest of the world.

Technological parameters: We choose a Cobb–Douglas specification

$$F(Z_t^r, K_t^r, H_t^r) = Z_t^r (K_t^r)^{\alpha} (H_t^r)^{1-\alpha},$$

for the production function with capital share $\alpha = .30$ in both regions. Based on the World Bank's *World Development Indicators* (WDI), we obtain growth rates of output per capita in the two regions from 1950–2000.[12] The growth rate of TFP λ^n in the North is set at the constant value of 1.78% so that the region achieves the target average per-capita output growth rate of 2.7% during 1950–2000, as computed from the WDI. We let the TFP in the South grow at rate $\lambda_t^s = 1.50\%$ so that output per capita grows at 2.2% during 1950–2000, as in the data. After 2000, we let λ_t^s converge smoothly to the TFP growth rate of the North.

We set the initial value of TFP $Z_0^n = 1.802$ in order to normalize income per capita in the North to 1 in the first steady state. Based upon the WDI, income per capita in the North in 1950–2000 was approximately 7 times larger than that of the South, requiring $Z_0^s = 0.677$. The depreciation rate of capital is set to 5% per year in both regions, implying a value of $\delta = 0.2262$.

Demographic parameters: Since each model-period corresponds to 5 years, we set $I^d = 3$ so that agents become adults and economically active at age 17 and we set $I = \bar{I} - I^d = 24 - 3 = 21$, so that households can live a maximum of 24 periods (120 years). We also set the retirement age $I^R = 11$ which corresponds to age 67. All these parameters are common in both regions.

We normalize the total population in the South in 1950 to one and set the initial population size for the North to 0.476, based on the UN data. During the transition away from the initial steady-state, the population size in both regions is determined by the evolution of age-specific fertility rates $\phi_{i,t}^r$ and survival rates $s_{i,t}^r$ described in Section 2. By 2200, the relative size of the population in the North falls to 0.164.

Preferences and endowments parameters: Preferences are common between the two regions. Following the bulk of the literature on consumption (for a survey, see Attanasio, 1999), we set $\theta = 2$. We set $\beta = 1.036$ to match the target capital–output ratio of 2.5 in the North on an annual basis in 2000. The weight parameter of children in the utility of adult parents is set to match the commonly used consumption adult-equivalent scales. The micro-evidence on equivalence scales summarized in Fernandez-Villaverde and Krueger

[11]The calibration strategy matches a set of moments in the data with the model's counterparts in the *closed-economy* equilibrium. The open-economy equilibrium has the exact same parameterization, thus, for example, different levels of output, and capital stock.

[12]For the North, we used the aggregate of individual countries called "high-income countries" in the WDI, corresponding to the UN definition of "more developed regions". For the South, we used the aggregate called "low- and middle-income countries" in the WDI, the equivalent of the UN's "less developed regions".

(2004, Table 3.2.1) points at a ratio between the consumption of a household with 1, 2, and 3 children compared to a household without children of 1.231, 1.470, and 1.694, respectively. Using Eq. (2), it is easy to see that our function $\omega(d^r_{i,t})$ should satisfy the three moment conditions

$$\omega(0.5)^{1/\theta} = (1.231 - 1)/0.5,$$

$$\omega(1)^{1/\theta} = (1.470 - 1),$$

$$\omega(1.5)^{1/\theta} = (1.694 - 1)/1.5.$$

Note that we need to make an adjustment for the fact that in our model children come in pairs. Given $\theta = 2$, setting $\omega = 0.216$ independently of the number of children yields an excellent fit.

The calibration of the age profile of efficiency units is done separately for the North and the South. The age-efficiency profile for the North is estimated on weekly wage data from the U.S. *Consumer Expenditure Survey* (CEX) for the period 1982–1999. For the South, we have estimated an age-efficiency profile on Mexican data—precisely from the *Encuesta Nacional de Ingreso y Gasto de los Hogares* (ENIGH), which is the equivalent of the U.S. CEX, using the 1989, 1992, 1994, 1996, 1998, and 2000 waves.[13] The sample, across both surveys, is the universe of married couples headed by males and aged 17–69 and the derived "household wage" is an average of male and female wage weighted by hours worked.

The estimated efficiency profile for the U.S. shows a twofold rise from the initial value to its peak (reached at 47 years old) and then it settles at 20% below the peak before retirement. We found that in Mexico the profile rises up to 90% above the initial value and it settles roughly 30% below the peak. This flatter pattern is exactly what we expected on the grounds that households in the South have lower educational levels and work in occupations with higher content of physical labor.

The function $\Lambda^r_t(\mathbf{d}^r_{i,t})$ measures the participation rate of the households, i.e. its time endowment net of time spent for child care. We normalize the time endowment of the household to 1 unit and assume that males work full time. Hence, $\Lambda^r_t(\mathbf{d}^r_{i,t}) = 0.5[1 + P^r_t(\mathbf{d}^r_{i,t})]$, where $P^r_t(\mathbf{d}^r_{i,t}) \in (0, 1)$ denotes the fraction of time a female worker of age i supplies to the labor market in region r at time t. The model for the dynamics of $P^r_{i,t}$ was discussed at length in Section 2.

Government policy parameters: Government debt and expenditures as a fraction of GDP for the North and the South are computed from the World Bank's *World Development Indicators* (WDI) as time-averages over the period 1970–2000. For the North, we obtain a ratio of government debt B^n_t to GDP equal to 35.5%, and a ratio of government expenditures G^n_t to GDP equal to 26.5%. For the South, the WDI yields a government expenditures–GDP ratio in the South around 20% over the sample period, and a debt–GDP ratio of 50%.[14]

[13] See Attanasio and Szekely (1999) for a detailed description of the Mexican survey data.

[14] The government expenditures data for the South are only available starting from 1981. The government debt data are not available for the low-income countries, but only for the middle-income countries. Luckily, they are reported separately for lower-middle-income countries and upper-middle-income countries. The debt–GDP ratio is 50% for the former region, and 40% for the latter. One would expect the low-income countries to be even more heavily indebted, hence we chose 50% for the overall aggregate of the South.

Based on the study in Whitehouse (2003), which reports replacement rate of average past earnings in nine OECD countries, we set $\kappa_t^n = 46.7\%$.[15] From the tax data collected by Mendoza et al. (1994) for seven OECD countries, we set $\tau_c^n = 9\%$ for the consumption tax, and $\tau_a^n = 38\%$ for the capital income tax. In the benchmark experiment, the labor income tax τ_w^n adjusts in the equilibrium of the model to balance the government budget. The equilibrium tax rate implied by the model is 26.3% over 1950–2000, which is very close to the 27.9% reported for the labor income tax by Mendoza et al. (1994).[16]

Unfortunately, similar systematic studies on the tax structure and the replacement rates for developing countries are not available. The WDI reports, for low- and middle-income countries, that social security tax revenues represent only 0.3% of income. Using the calibrated labor share of 70%, one obtains an average social security tax of 0.5%. Assuming that the social security system is self-financing, the model delivers a replacement ratio for average past earnings for the South of $\kappa_t^s = 10.1\%$ in the first steady-state.

This much lower value for the replacement ratio, compared to the North, is mainly due to two factors. First, the disproportionate role of self-employment and informal production means that a vast part of the working population is not covered by a pension system. Second, the involvement of Southern governments in the pension sphere is limited: in Asia, only Korea and Taiwan operate a defined benefits PAYG scheme with universal coverage; Latin America is the region with the largest number of pension system already reformed towards substantial privatization (see Mohan, 2004, for the Asian experience; see Corbo, 2004, for the Latin American experience).

Heady (2002) argues that, because of the importance of the informal sector, governments in the South rely much more on indirect taxation, compared to the developed world. From his estimates, we conclude that an appropriate value for the consumption tax in the South could be $\tau_c^n = 15\%$. Finally, we set the capital income tax at the same level of the North and let the wage tax adjust in equilibrium. The model implies $\tau_w^s = 6.04\%$ as the average labor income tax in the South during 1950–2000.[17]

5. Computational experiments on social security reform

We run two types of policy experiments. In the first set of experiments, the North maintains the current PAYG social security system, and we present alternative ways to finance this transfer system during the demographic transition. In the second set of policy simulations, the North privatizes social security by phasing out benefits and gradually moving towards a fully funded system. We study alternative privatization schemes announced unexpectedly by the government in 2005. Across all these experiments, we allow no policy change in the South.

[15]The countries in the study by Whitehouse (2003) are: U.S., Canada, U.K., Germany, Italy, Japan, Finland, Netherlands, and Sweden. The data refer to the mid to late 1990s. Our replacement rate for the North is a GDP-weighted average.

[16]The countries in the study by Mendoza et al. (1994) are: U.S., Canada, U.K., France, Germany, Italy, and Japan. The original data refer to the period 1965–1988. We used an unpublished extension up to 1996 available on Mendoza's web page. Our tax rates are averages over the sample period and over countries, weighted by GDP.

[17]The information on tax rates in the South is scant and often parameters are set based on priors. Fehr et al. (2006), for example, set the capital income tax rate for China to zero, for lack of better knowledge. In Section 6 we calibrate the capital income tax rate in the South to match capital flows from 1950 to 2000, and verify the robustness of our conclusions.

The experiments are chosen as a representative sample of those run in the literature in closed economy (e.g., De Nardi et al., 1999; Huang et al., 1997; Kotlikoff et al., 1999).

Welfare calculation: In every welfare calculation, our benchmark is the experiment where the additional benefit payments required by the PAYG during the demographic transition are financed by a rise in the wage tax. We compute the welfare change, cohort by cohort, associated to each one of our alternative social security reforms compared to this benchmark. Our welfare comparison is based on ex ante lifetime utility, i.e. for each cohort we compute lifetime utility at birth under the benchmark transition and under the alternative policy reform, and calculate the consumption equivalent variation, i.e. the amount by which lifetime consumption under the benchmark reform needs to be rescaled in order to make agents in that cohort indifferent between the benchmark reform and the alternative reform. Negative numbers mean that a cohort strictly prefers the benchmark reform. We always present two welfare computations, one for the closed economy and one for the open economy.

5.1. A first look at the results

The crucial difference between closed and open economy is the equilibrium path of factor prices and, in particular, the path of the interest rate that households and government take as given. Behind different paths for the interest rate lie different domestic saving rates and, clearly, capital flows of diverse magnitudes. We begin by analyzing the dynamics of saving rate, interest rate, and capital flows.

5.1.1. Domestic savings

The top panel of Fig. 4, which refers to the benchmark PAYG experiment, shows that the saving rate follows a U-shape pattern. The decline in the saving rate in the medium run is due to the retirement of the baby-boom generations, which starts around 2010. Over the long-run, savings increase again due to the rise in life expectancy, the lower fertility rates, and the decline in the interest rate, a general-equilibrium effect: given our choice of the intertemporal elasticity of substitution, the income effect of changes in the interest rate dominates the substitution effect. In open economy, the saving rate rises more steeply in the long run due to the general-equilibrium effect of a sharper decline in the interest rate.

The bottom panel refers to the benchmark privatization scenario. Note the sharp difference with the top panel: under the privatization of the PAYG system, households are forced to save more for retirement and capital accumulation increases dramatically. In particular, the saving rate is projected to increase by roughly 1.5% per decade.

Going back in time, the model generates a rise in the saving rate in the North from 17.5% to 20% over the period 1975–2000. Official data on the personal saving rate display a sharp decline, in contrast to the model's prediction. In the U.S., for example, the personal saving rate measured from the Flow of Funds Accounts (FFA) decreases from 15% in 1975 to 5.3% in 1999. However, the FFA (as well as the NIPA) concept of saving rate does not incorporate stock market capital gains, which are particularly important over this period. Lusardi et al. (2001) construct a series for the personal saving rate that includes omitted capital gains measured from the changes in financial asset balances from the National Balance Sheets published by the Federal Reserve Board. This series increases from 25% to 40% over the same time span. The model sits right in the middle of this plausible range of definitions for the domestic saving rate.

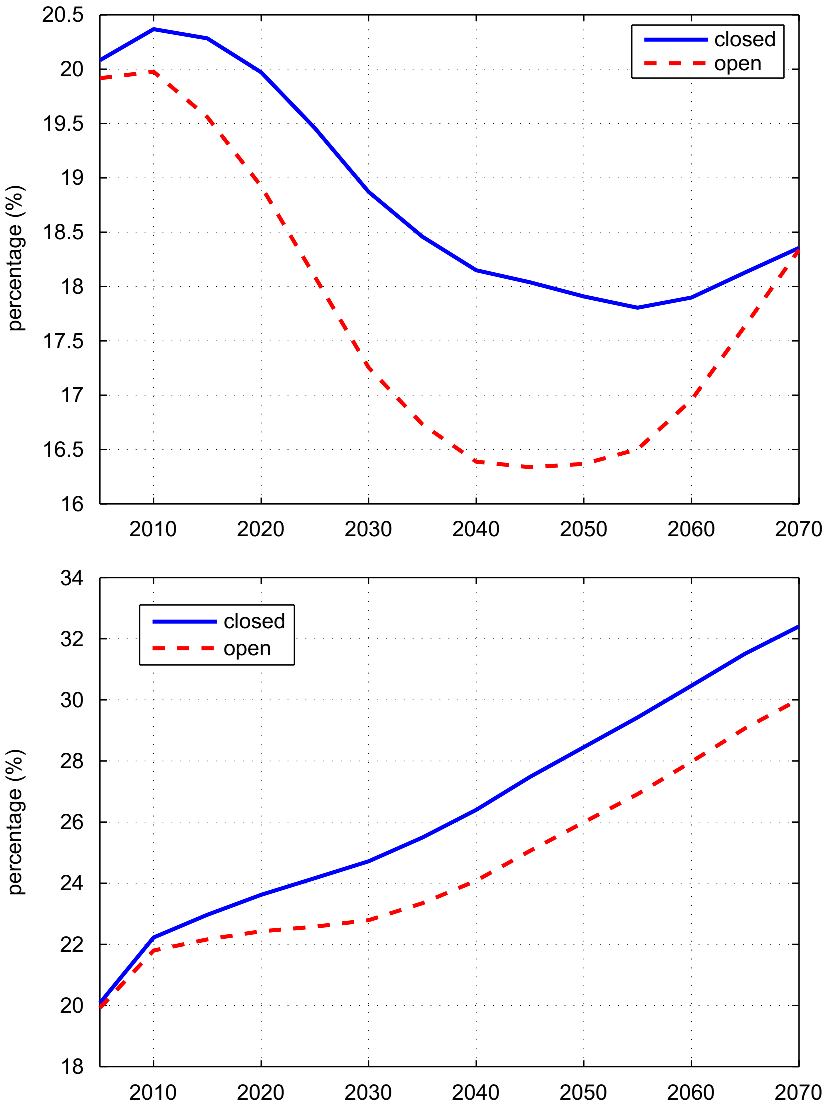


Fig. 4. The path of equilibrium saving rates in closed and open economy. The two panels depict the dynamics of the saving rate in the two regions in closed economy, and in open economy. The top panel refers to the benchmark PAYG experiment (FW); the bottom panel to the benchmark privatization experiment (PW).

5.1.2. *Interest rate paths*

The first panel of Fig. 5 plots the equilibrium interest rates in the closed-economy simulations for the North, and the open-economy interest rate in our benchmark policy experiment. In the closed economy, the demographics of the North tend to stabilize around 2040, and this is reflected in the path of the interest rate which falls by 2 points between 2000 and 2040 and levels off thereafter.[18] Why are both the interest rate and the

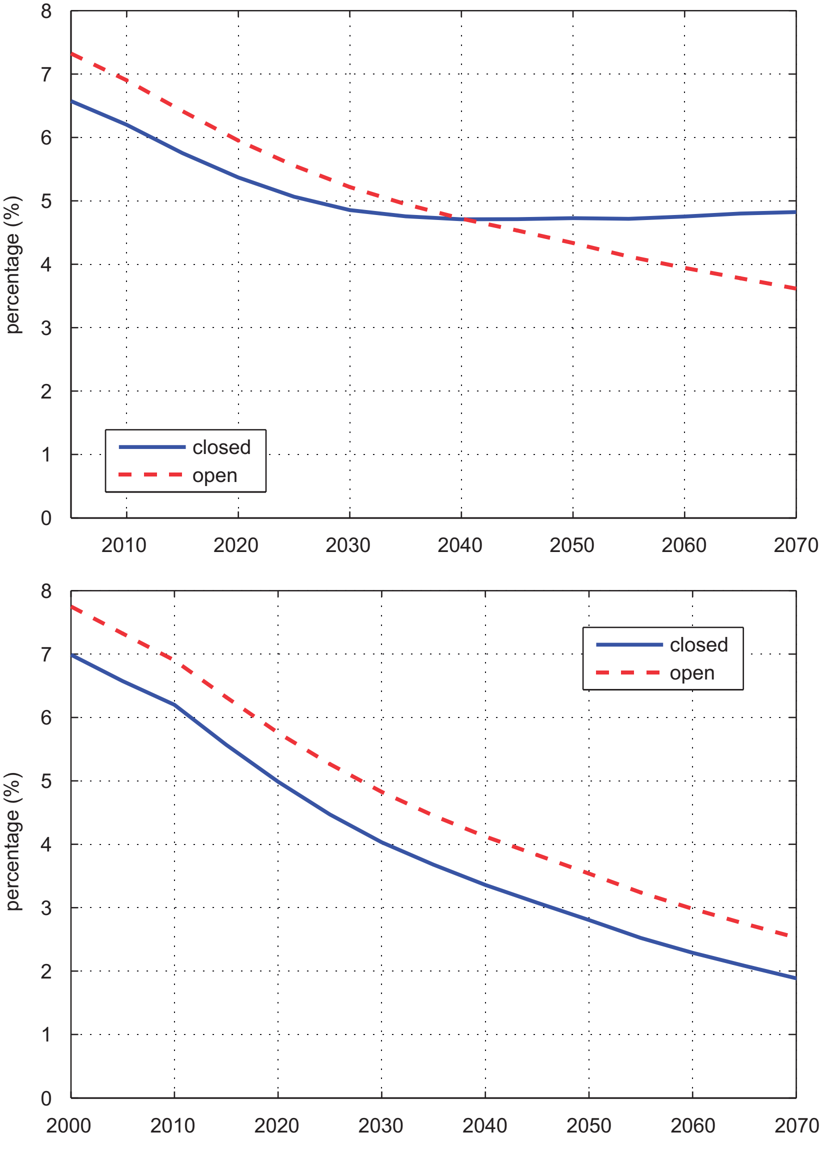


Fig. 5. The path of equilibrium interest rates in closed and open economy. The two panels depict the dynamics of the interest rate in the two regions in closed economy, and in open economy. The top panel refers to the benchmark PAYG experiment (FW); the bottom panel to the benchmark privatization experiment (PW).

saving rate falling until 2050? The answer lies in the fact that the equilibrium interest rate is determined by the capital–labor ratio. Besides reducing the capital stock, the retirement of

[18]Domeij and Floden (2004, Figure 1) report a fall of the equilibrium interest rate induced by demographic trends in the developed countries from 6.5% in 2000 to 5% in 2040, and then only a decline by half percentage point from 2040 to 2100. Thus, the path of our closed-economy North interest rate (the right one to compare to their model) is extremely similar to theirs.

the baby boomers generates a relative shortage of productive labor: the second effect dominates the first one and the capital–labor ratio rises.

The open-economy interest rate lies just above the closed-economy interest rate until 2035. Then, while the latter stabilizes due to the exhaustion of the demographic transition, the former keeps falling rapidly, by 1.3 additional percentage points until 2070. The reason is that over time the South keeps growing both in terms of population size and in terms of capital accumulation. In particular, the divergence in capital accumulation is large because of faster productivity growth in the South, and because in the North the PAYG crowds out a large amount of life-cycle savings which, in contrast, are made in the South where retirement benefits are minor. Indeed, in the South, the closed-economy rate of return is projected to fall by over 5 percentage points between 2000 and 2070.

The second panel of Fig. 5 plots the closed- and open-economy interest rates in our benchmark privatization experiment. Even in closed economy, the interest rate is projected to fall rapidly, due to the massive private capital accumulation to finance consumption after retirement once the PAYG system is dismantled.

To conclude the discussion on the interest rate, two criticisms can be moved based on the plots in Fig. 5: the level of the interest rates is too high, and they decline too much.

As for the level, the calibration strategy targets the capital–output ratio, the labor share and the depreciation rate. The equilibrium interest rate (6.6% in 2005) follows directly.[19] This value may be judged as high if compared to historical returns to Treasury bonds, but it is below the postwar real return on U.S. equity (around 8%). Recall that there is only one asset in our model economy. Interestingly, Caselli and Feyrer (2006) use the Penn World Tables to compute the marginal product of capital in domestic prices and conclude that, during 1970–2000, it averages 13% among high-income countries, and 16% among low-income countries. Given our assumed depreciation rate of 5%, these estimates imply real returns to capital of 8% in the North and 11% in the South. For comparison, over the period 1970–2000, in our model, the average interest rate is around 8% in the North, and 10% in the South.

With respect to the dynamics, the model does predict large movements in the interest rate: a decline of 2.0% from 1980 to 2000, and a further fall of 2 percentage points over the next 50 years. Even though historically, the interest rate has been remarkably stable (one of the "Kaldor facts"), in the medium run also the data show significant swings. For example, the real (quarterly) T-Bill rate was at 5% in the early 1980s, and fell to just 1% in 2000. In general, one may wonder whether, in the model, equilibrium interest rates are too sensitive to demographic changes. Demographic and intertemporal considerations are the only driving forces of saving behavior in frictionless models. In a framework with precautionary saving motives, as Krueger and Ludwig (2006), this sensitivity would be lowered because of the additional saving motive disconnected from demographics. One would expect the interest rate to vary less over time. Some of the results in Krueger and Ludwig (2006) seem to confirm it: given roughly the same demographic shocks, their model predicts the interest rate to fall by 1% in the next 50 years (Fig. 3) whereas our model predicts a decline of 2% over the same period.

[19] Krueger and Ludwig (2006, Figure 3) calibrate their model similarly and obtain an interest rate just above 7% in 2005.

5.1.3. Capital flows

The top panel of Fig. 6 displays the equilibrium ratio of external wealth to total wealth in the South implied by the model under the benchmark experiment. We chose this

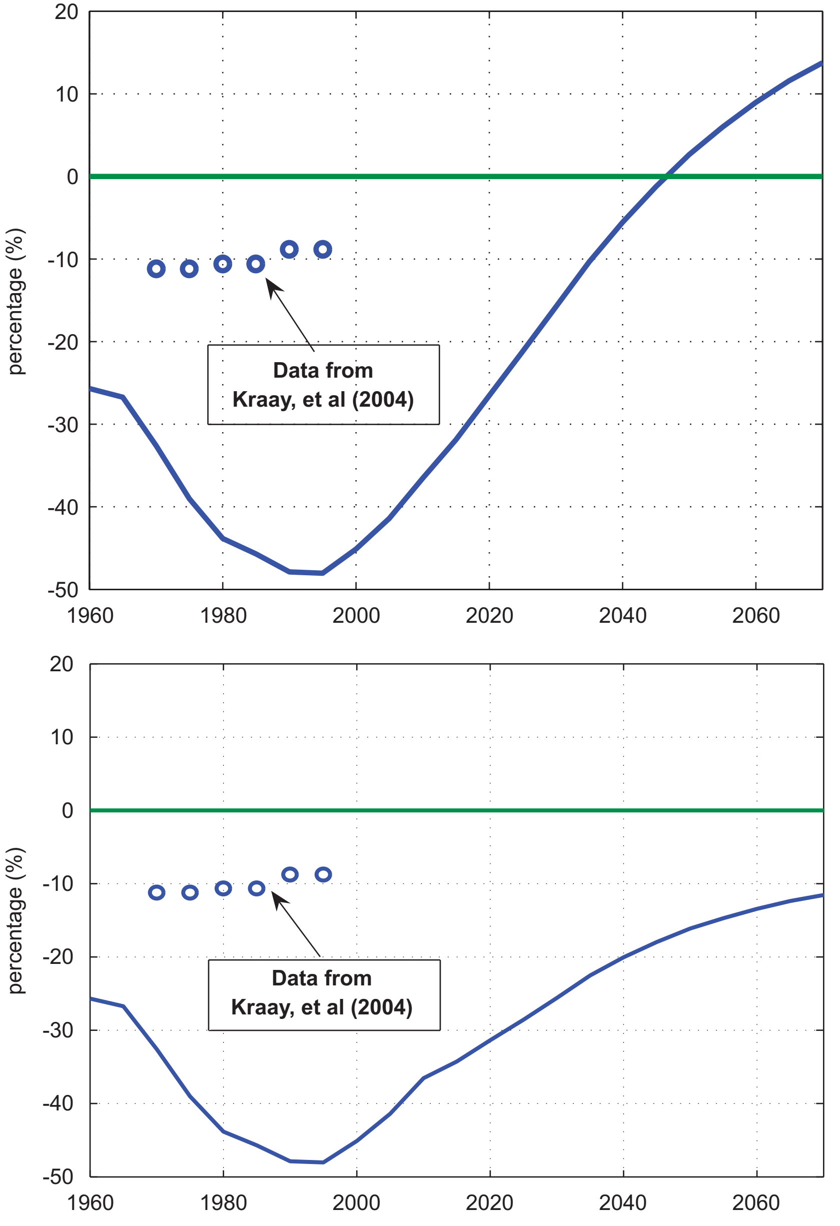


Fig. 6. The path of equilibrium external wealth in open economy. The two panels depict the dynamics of the value of external wealth as a fraction of total wealth in the South (negative numbers mean that the North owns assets of the South). The dots represent the data points documented by Kraay et al. (2004). The top panel refers to the benchmark PAYG experiment (FW); the bottom panel to the benchmark privatization experiment (PW).

statistics because it is the one Kraay et al. (2004) report after aggregating data on foreign assets across countries. They document that this ratio averaged −10% from 1960–1990. Our frictionless open-economy model generates −35% over the same period. The plot clearly shows the reversal in the sign of the stock of external wealth around 2045 and predicts a large positive external wealth of the South, of the order of 14% of total wealth, by 2070. In terms of current account dynamics, the North runs a surplus around 1% of GDP until 2020, when the capital flow reverse. The current account deficit of the North peaks in 2060 at around −4% of GDP, and since then it slowly returns towards zero.[20]

Under the privatization scenario, in contrast, there will be no reversal: capital is projected to flow from rich to poor countries throughout the 21st century, and the ratio is forecasted to shrink to −10% in 2070 (see the bottom panel of Fig. 6).

The model yields implicitly a measure of how sizeable the capital flows need to be, in absence of frictions, in order to equalize differences across regions stemming only from different demographic structure of the population. Taylor and Williamson (1994) have argued that $\frac{3}{4}$ of the net capital inflows of Canada, Australia and Argentina in the early 20th century could have demographic origins. More recently, Domeij and Floden (2004) have shown that this demography-based approach can explain a non-negligible part of the postwar low-frequency current account dynamics among OECD countries. With respect to this debate, our contribution is to point out that structural reforms to the PAYG system will have a first-order effect on the sign and magnitude of these flows in the next 50 years.

Compared to the last 20 years of data from Kraay et al. (2004), our model generates too much capital flowing between the North and the South. This is not surprising, given the assumption of frictionless global financial markets which is an extreme benchmark in the same way a pure closed-economy model without any capital flow is at the opposite end of the spectrum. Country-level risk associated to political uncertainty, the likelihood of sovereign default, and expropriation are paramount in explaining why historically capital did not flow towards poor countries as much as predicted by the standard growth model (Lucas, 1990).

However, when thinking about the future, two considerations become important. First, important steps have been taken in the past few years with respect to "financial engineering" of emerging markets, so one may expect capital to flow freely and more safely in the years to come. Second, recall that under all the policy experiments where the PAYG is preserved, there is a reversal in the direction of flows from South to North. Arguably, country-specific risks are smaller in the North, since political uncertainty is lower, financial institutions are more developed, and property rights better protected. As a consequence, one may expect the frictionless benchmark to better predict future capital flows compared to past flows.

[20] Brooks (2003) solves a multi-region model of the world to make long-run predictions about capital flows generated by different demographic dynamics across regions. His model predicts a reversal in the direction of flows precisely around 2020. Helliwell (2004) argues that the empirical results of several reduced-form exercises also imply that in a few decades the South will transfer resources to the North.

5.2. Financing the PAYG system during the demographic transition

We consider five different options to finance the PAYG pension system during the demographic transition:

(FW) The government covers all the additional social security expenditures associated with the demographic changes by adjusting the wage income tax τ_w^n. Every other policy variable is fixed at its 2005 level. This will be our benchmark for the welfare calculations (Fig. 7).

(FC) The government covers all the additional expenditures by adjusting the consumption tax. Every other policy variable is fixed at its 2005 level (Fig. 8).

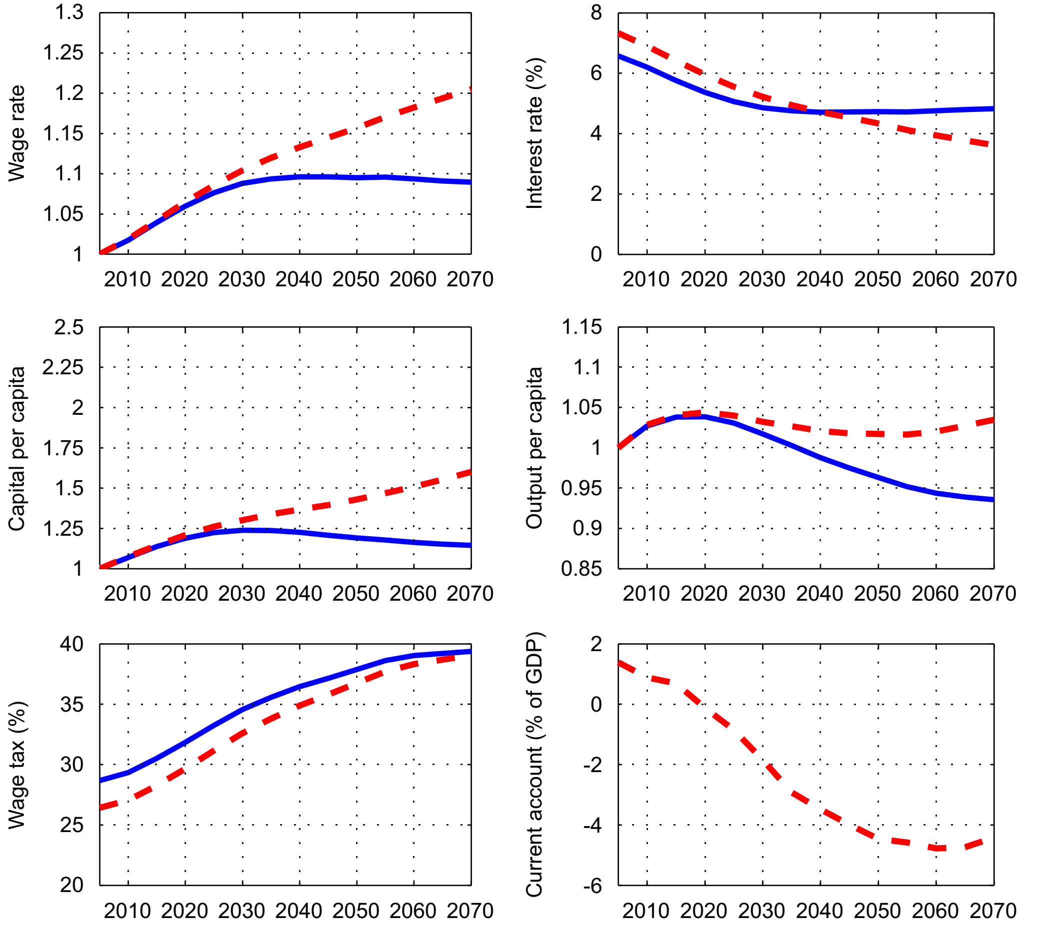


Fig. 7. Results of the benchmark experiment where PAYG is financed through the payroll tax. Solid lines represent closed economy and dotted lines represent open economy.

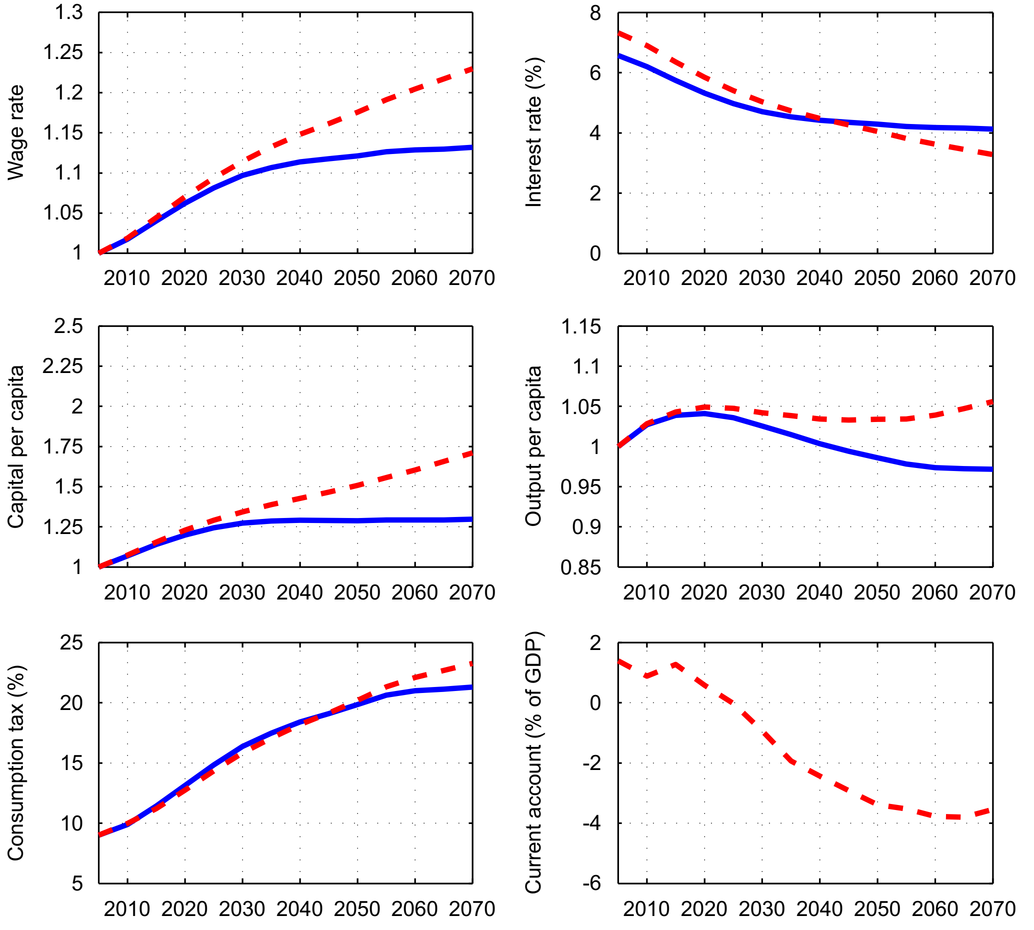


Fig. 8. Results of the experiment where PAYG is financed through the consumption tax. Solid lines represent closed economy and dotted lines represent open economy.

(FD) The government raises public borrowing to finance the expenditures. The debt to GDP ratio increases linearly from the current level of 35.5% to 71% by 2050. The wage tax adjusts residually to balance the budget (Fig. 9).

(FR) The government in 2005 announces that in 2020 it will raise the compulsory retirement age I^R by 5 years, from 67 to 72 years-old. The wage tax τ_w^n adjusts to balance the budget residually (Fig. 10).

(FB) The government reduces pension benefits by lowering the replacement rate κ^n in order to keep the budget balanced. Every other policy variable is fixed at its 2005 level (Fig. 11).

Table 1a and b reports the initial and final steady-state values of the key aggregate and policy variables in each experiment. The figures report the dynamics of various detrended aggregate variables along the transition between steady states.

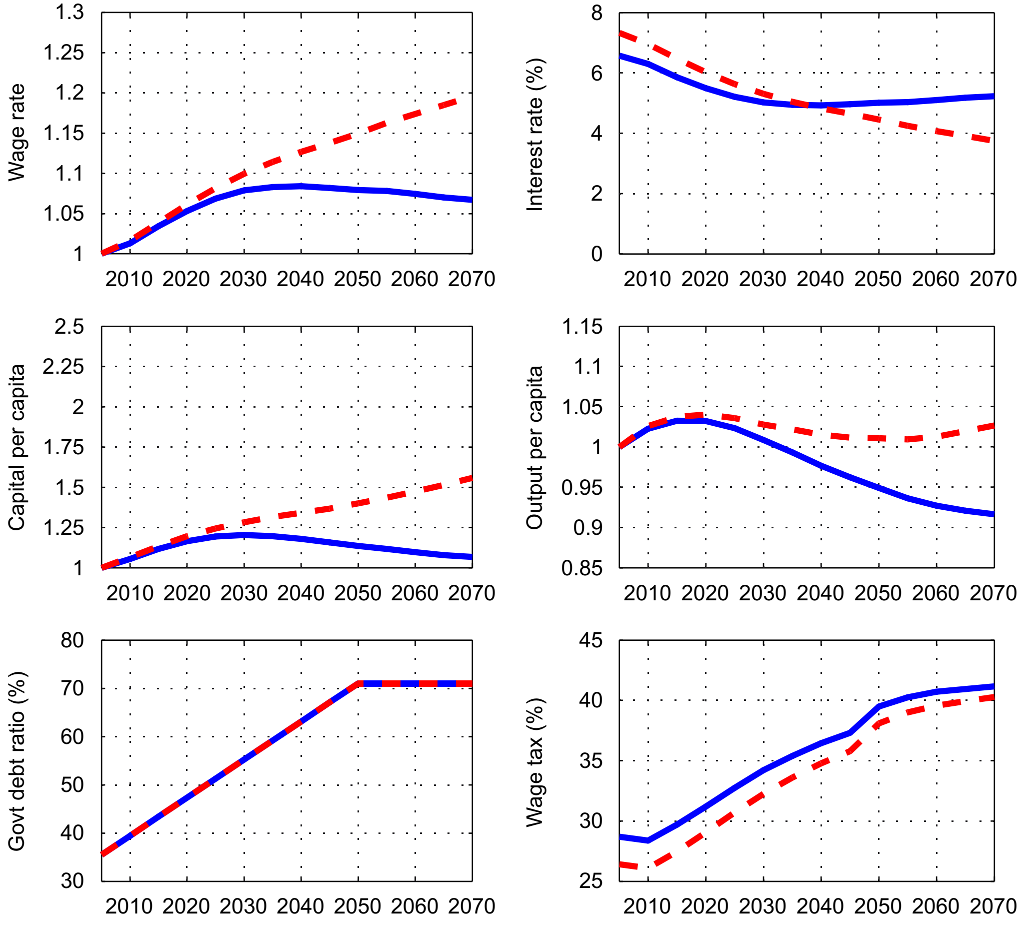


Fig. 9. Results of the experiment where PAYG is financed through a combination of higher debt and higher payroll taxes. Solid lines represent closed economy and dotted lines represent open economy.

5.2.1. Results I: macroeconomic aggregates and policy variables

Benchmark (*FW*): In the open-economy model, the capital stock in the North grows faster in the long run because domestic saving rates are higher and because of the capital inflows from the South. As a consequence, both wages and output per capita grow at a more rapid rate in open economy. Actually, income per capital falls below trend in the closed economy, due mainly to the decline in the employment population ratio. Surprisingly, there is very little difference in the time path of the payroll tax between the closed economy and the two-region model. The wage tax τ_w^n rises by 11.3% in closed economy (from 28.0% to 39.4%) and by 13.2% in open economy, from 2000 to 2070.[21] Why such similar evolutions of our key policy variable in spite of large differences in

[21]To be precise, in closed economy, from 2070 to the final steady-state the wage tax keeps increasing, slowly, by an additional 4%, making the total rise since 2005 equal to 14.7% (see Table 1). De Nardi et al. (1999) report, for

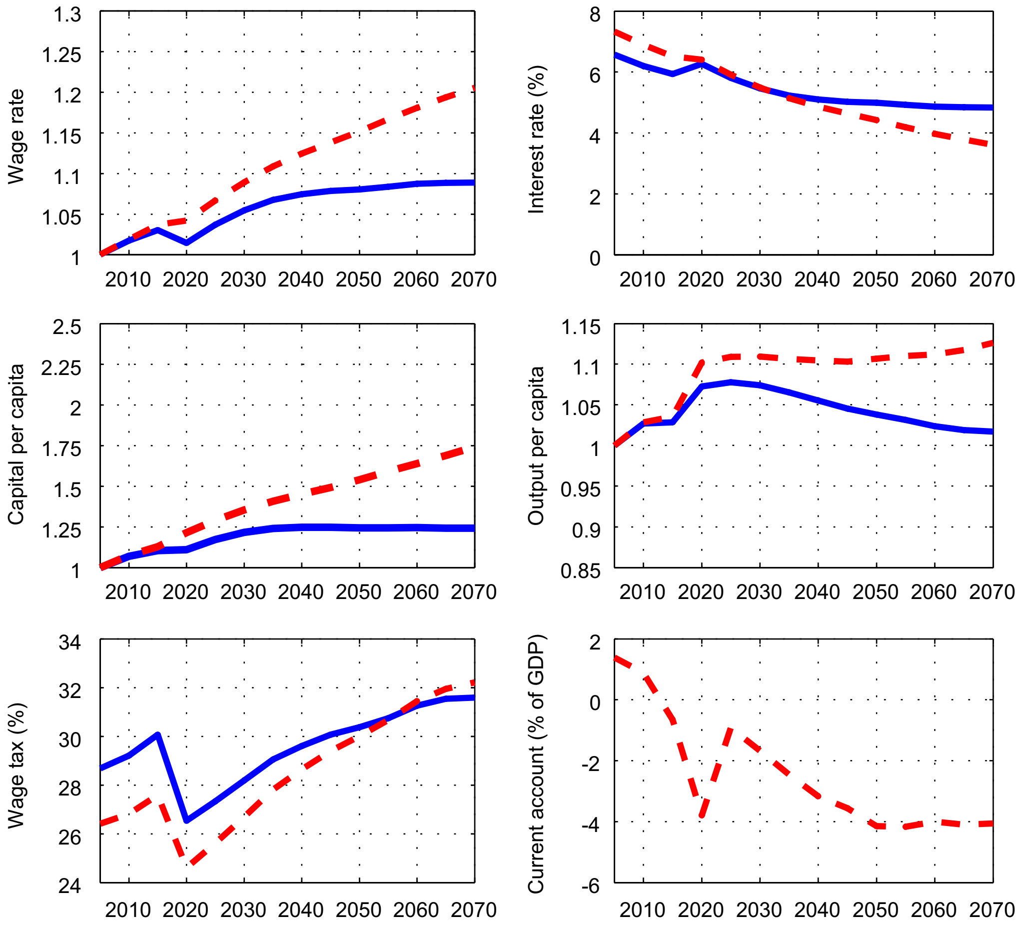


Fig. 10. Results of the experiment where PAYG is financed through a rise in retirement age. Solid lines represent closed economy and dotted lines represent open economy.

macroeconomic aggregates? The reason is that in open economy there is a sharper fall in the interest rate and a larger rise in the wage rate: the fraction of fiscal revenues raised from the capital income tax τ_k^n declines by 7 percentage points and the fiscal revenues from the payroll tax τ_w^n rise by a comparable amount.[22]

Along the demographic transition, the world goes through a decline in fertility, a rise in life expectancy, and a rise in participation rates. Which force is mostly responsible for the changes in the aggregate variables of interest. Table 2 answers this question. The demographic force that accounts for the bulk of the divergence in the evolution of the

(*footnote continued*)
the U.S. economy, a slightly higher long-run rise in the payroll tax (+17.1%) in order to finance the system throughout the demographic transition.

[22]The revenues of the consumption tax τ_c^n are very stable across the two experiments.

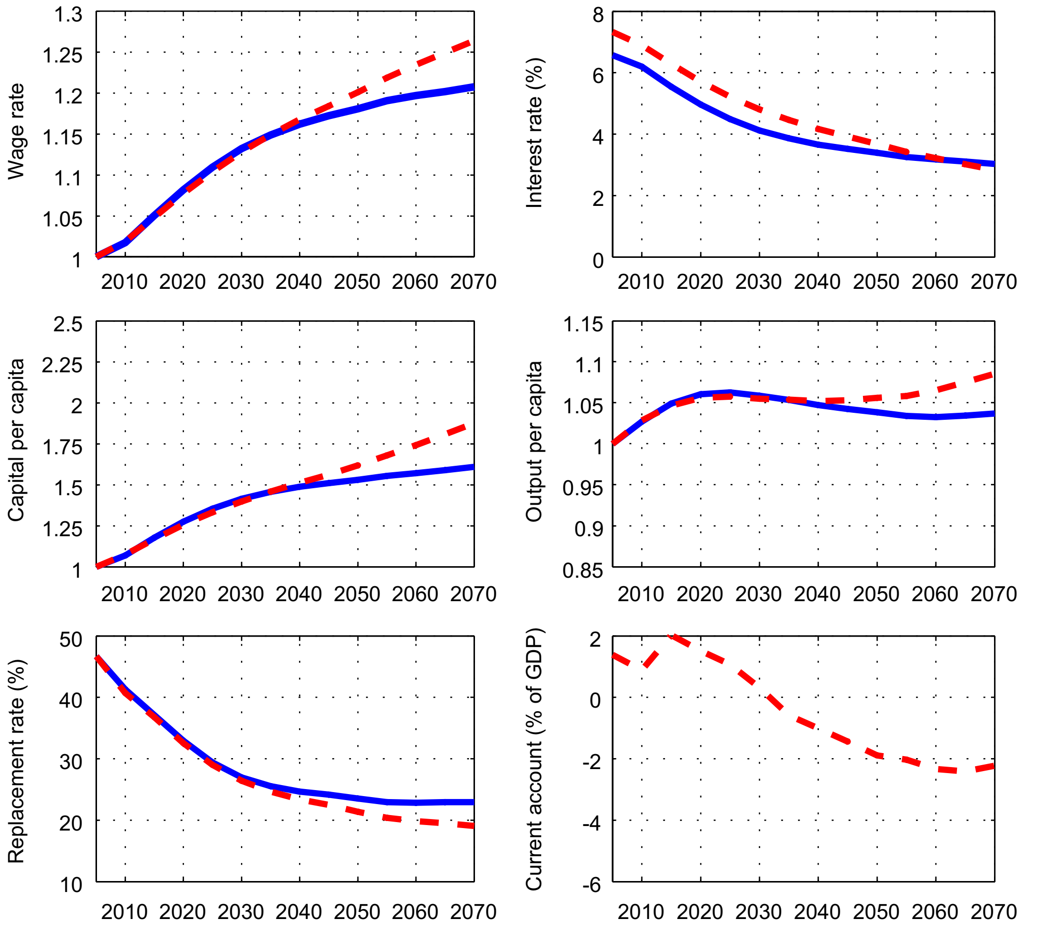


Fig. 11. Results of the experiment where PAYG is financed through a decline in benefits. Solid lines represent closed economy and dotted lines represent open economy.

aggregates between closed and open economy is the fertility transition. The reason is that the fertility trend is the only one, among the three demographic trends examined, that moves in opposite directions between North and South, after 2000: while the North slowly rebounds towards replacement rates, the South rapidly falls towards them. The sharp reduction in fertility in the South generates a strong rise in the capital stock, while in the North the light rebound in fertility determines a decline in savings (Fig. 4). As a result, in open economy, the fertility transition induces a big inflow of capital from the South (see the last column in Table 2) which reverses the implications for the closed-economy dynamics in the capital–output ratio in the North.[23]

It is also informative to study the dynamics of the same aggregate and policy variables in the South. We report our findings in Table 2. The rise in capital and output per capita, due

[23]We add that the rise in longevity affects the population structure very slowly, while the participation transition is virtually exhausted in 2005 in the North.

Table 1
Long-run steady states: (a) closed economy, (b) open economy

	Wage	Interest rate (%)	Capital/ capita	Output/ capita	Saving rate (%)	Wage tax rate (%)	Policy variable
(a)							
Benchmark							
1950	0.860	10.23	0.433	0.614	14.74	26.24	–
2005	1.000	6.57	1.000	1.000	20.08	28.69	–
Long run	1.109	4.50	1.104	0.868	20.20	43.34	–
PAYG system							
Consumption tax	1.189	3.28	1.395	0.931	23.80	28.69	27.02
Government debt	1.082	4.96	1.018	0.847	19.09	44.74	71.00
Retirement age	1.120	4.32	1.241	0.952	20.69	35.69	–
Retirement benefit	1.307	1.84	1.910	1.023	29.66	28.69	18.25
Privatization							
Wage tax	1.461	0.43	2.772	1.144	38.48	21.17	–
Consumption tax	1.412	0.84	2.472	1.105	35.52	28.69	−3.54
Government debt	1.122	4.28	1.150	0.879	20.79	28.69	425.67
(b)							
Benchmark							
1950	0.890	10.23	0.484	0.635	14.68	24.61	–
2005	1.000	7.32	1.000	1.000	19.92	26.41	–
Long run	1.322	2.16	1.986	1.035	24.22	45.01	–
PAYG system							
Consumption tax	1.363	1.73	2.198	1.067	27.51	26.41	33.30
Government debt	1.313	2.26	1.939	1.028	23.47	45.44	71.00
Retirement age	1.326	2.12	2.177	1.128	24.53	37.78	–
Retirement benefit	1.433	1.07	2.599	1.122	33.33	26.41	12.49 –
Privatization							
Wage tax	1.477	0.70	2.875	1.157	37.20	20.84	–
Consumption tax	1.463	0.81	2.784	1.145	35.93	26.41	−0.00
Government debt	1.284	2.60	1.800	1.005	21.10	26.41	686.00

Wage, capital, and output are normalized by their 2005 levels. Policy variables from the top entry: consumption tax rate, the ratio of government debt to GDP, replacement ratio, consumption tax rate, and the ratio of government debt to GDP in the long-run steady state.

to demographic forces is substantial: demographics are projected to account for more than half percentage point of annual income growth in the South, over the next 50 years. In the open-economy scenario, output growth is slower because capital in the long run transits towards the North. Due to the rise in the old-dependency ratio (see Fig. 2), even in the South a rise in the wage tax will be necessary to finance the social security. The increase, however, given the minute size of the system, is less than half than in the North.[24]

Financing by consumption tax (*FC*): The macroeconomic aggregates behave very similarly to the benchmark in this experiment. Capital and output per capita grow slightly

[24]In Attanasio et al. (2006), the focus is on the South. There, we study how the policy choices of the North with respect to the future of social security will affect the rate of development and the distribution of income in the South.

Table 2
Benchmark experiment: (a) decomposition by demographic trend, (b) South

		Wage	Interest rate (%)	Capital/capita	Output/capita	Wage tax rate (%)	Current account/GDP (%)
(a)							
Benchmark							
2005	Closed	1.0000	6.5740	1.0000	1.0000	28.6859	–
	Open	1.0000	7.3228	1.0000	1.0000	26.4097	1.3883
2040	Closed	1.0964	4.7086	1.2245	0.9878	36.4563	–
	Open	1.1332	4.7205	1.3666	1.0209	34.8830	−3.4783
2070	Closed	1.0898	4.8230	1.1435	0.9356	39.3660	–
	Open	1.2053	3.6146	1.5996	1.0347	39.0560	−4.4144
Fertility							
2005	Closed	1.0000	8.1333	1.0000	1.0000	27.7518	–
	Open	1.0000	9.3139	1.0000	1.0000	24.1828	1.6390
2040	Closed	1.0464	7.0770	1.0835	0.9746	31.7371	–
	Open	1.0608	7.8569	1.1340	0.9880	27.5166	−2.3337
2070	Closed	1.0157	7.7633	0.9641	0.9297	31.3398	–
	Open	1.0826	7.3829	1.1926	0.9910	29.6425	−2.8508
Life expectancy							
2005	Closed	1.0000	7.8146	1.0000	1.0000	28.1182	–
	Open	1.0000	8.0612	1.0000	1.0000	27.0072	−0.0896
2040	Closed	1.0241	7.2621	1.0688	1.0110	29.3026	–
	Open	1.0260	7.4588	1.0752	1.0128	28.4361	0.0233
2070	Closed	1.0561	6.5780	1.1673	1.0277	30.8340	–
	Open	1.0570	6.7871	1.1705	1.0286	30.0557	0.1241
Participation							
2005	Closed	1.0000	9.8617	1.0000	1.0000	26.2130	–
	Open	1.0000	10.3086	1.0000	1.0000	24.5291	0.1774
2040	Closed	1.0132	9.5173	1.0711	1.0388	26.5047	–
	Open	1.0126	9.9726	1.0689	1.0382	24.6698	0.0290
2070	Closed	1.0162	9.4400	1.0862	1.0462	26.6771	–
	Open	1.0155	9.8971	1.0835	1.0454	24.7883	0.0076
(b)							
2005	Closed	1.0000	8.4788	1.0000	1.0000	6.6976	–
	Open	1.0000	7.3228	1.0000	1.0000	10.1852	−1.7282
2040	Closed	1.1919	4.6873	2.0406	1.3548	9.8710	–
	Open	1.1332	4.7205	1.7242	1.2880	10.9869	2.5010
2070	closed	1.3226	2.8905	2.8969	1.5086	13.0092	–
	Open	1.2053	3.6146	2.1253	1.3748	12.0948	2.6030

Wage, capital, and output are normalized by their 2005 levels. Current account is in % of GDP.

more than in the benchmark, a feature that will affect our welfare calculations. For the same reasons explained above, the rise in the consumption tax τ_c^n is similar across closed- and open-economy simulations. In the closed economy, the required increase is roughly from 9% to 21% in 2100. Even though the paths for the consumption tax overlap each other until 2050, in the long run the rise in consumption taxation is 6 percentage points larger in open economy, suggesting that the loss in capital income tax revenues outweighs the additional revenues from labor income taxation.

Financing by debt (*FD*): The rise in government debt severely displaces private capital accumulation in the closed economy. In the long run, output per capita falls by 15% below trend. The inflow of capital from the South guarantees the rise in wages, and prevents average income to decline in open economy. Even though debt to GDP ratio doubles, the wage tax has still to climb by 13 percentage points to balance the budget. This magnitude is as large as in the benchmark (FW) experiment since now the government must also finance the higher interests on debt through the labor income tax. The same general-equilibrium effect on the government budget discussed earlier explains why the path of the wage tax remains similar in closed and open economy.

Financing by a rise in the retirement age (*FA*): The dynamics of the macroeconomic variables in this experiment are similar to the benchmark, except for a discrete jump in 2020 corresponding to the year where a whole new cohort (those aged 65–69) remains active in the labor market for the first time. From a fiscal perspective, the simulation yields an important number: postponing retirement by 5 years translates into an increase in the payroll tax around 7 percentage points smaller, compared to the benchmark, in both closed and open economy in the long run. Once again, the overall change in the tax rate is similar across the two scenarios.

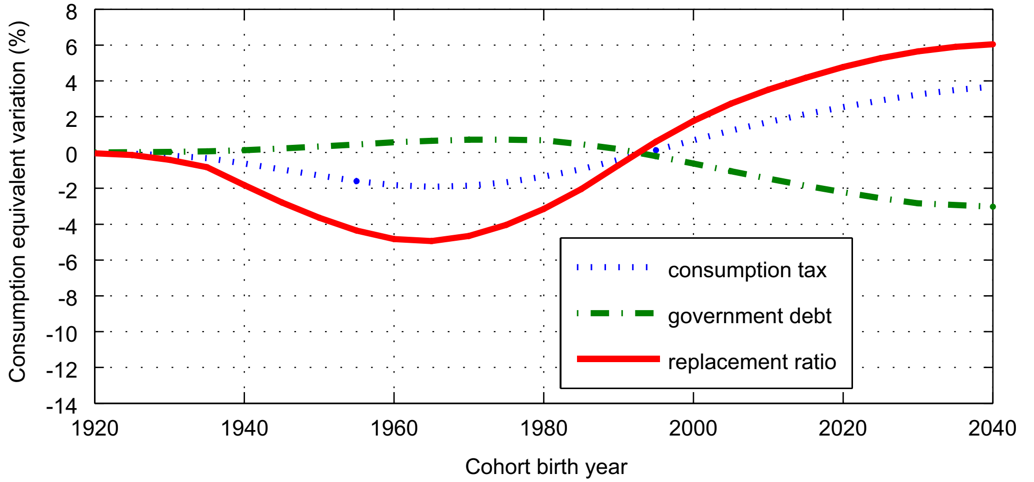
Financing by a fall in benefits (*FB*): The reduction in benefits in the North triggers huge capital accumulation in the form of retirement savings, similarly to the effects of a privatization (see below). The path of the interest rate in the closed-economy equilibrium falls more rapidly, compared to the other experiments, but still not as much as in the open-economy equilibrium. Both the closed- and open-economy simulations suggest that if the government were to leave payroll taxes unchanged, the replacement rate would have to decline sharply, from over 40% to 20%. Consistently with the other experiments, we find that in open economy benefits need to fall somewhat more to balance the budget.

We conclude that, in spite of the strikingly different paths of macroeconomic aggregates, surprisingly, the evolution of the policy variables between closed- and open-economy models is almost identical, quantitatively. This is due to general-equilibrium effects. Capital accumulation and output grow much more in open economy: wages grow by 10–15% more and interest rates fall by 2 extra points, in the long run, with most of the divergence occurring after 2030 in concurrence with the reversal of capital flows towards the North. However, the effects of a broader wage income tax base and a smaller capital income tax base, in the open economy, offset each other almost exactly in equilibrium, and the paths of policy variables end up being very similar.

5.2.2. Results II: welfare effects

The top and bottom panels of Fig. 12 report welfare changes of each experiment in closed and open economy, respectively, relative to the benchmark (BW). Note that the benchmark happens to be a policy simulation where the "baby boomers" fare relatively well: the benefits implied by the current system are preserved and their cost is passed onto the future cohorts that will be in the labor market over the next 30 years.

Compared to a rise in the payroll tax, financing the PAYG through the demographic transition via higher consumption taxes (FC) or via lower benefits (FB) hurts the older generations at the time of the reform (who consume most of their income and are retired) and yields a gain to the future generations who find themselves with a higher capital stock



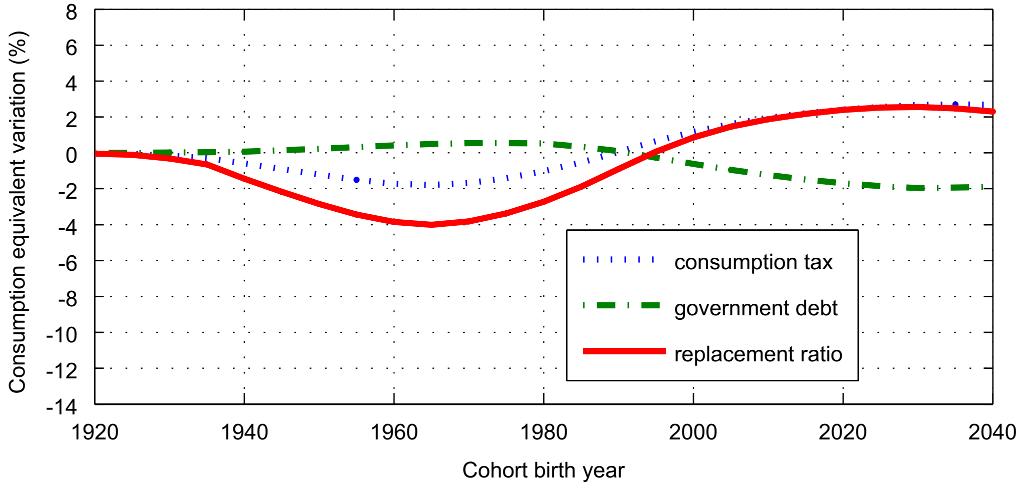


Fig. 12. Welfare effects of different policy options to finance the PAYG system. The top panel summarizes the welfare changes, cohort by cohort, in closed economy; the bottom panel refers to the open-economy simulations.

and higher wages. Instead, using public debt (FD) hurts, in relative terms, more the future generations: public debt crowds out private capital, lowering future wages compared to the benchmark.[25]

The comparison between closed and open economy shows that the quantitative differences in welfare can be very large, especially for cohorts appearing in the next two–three decades. For example, in the case of the benefits experiment (FB), the maximum welfare gain, occurring for the generations born between 2030 and 2040, is 2.6% in open economy and 6% in closed economy. In the case of public debt (FD), the maximum

[25]We do not report welfare calculations for the experiment where we increase the retirement age I^R because they are not too meaningful, given the absence of disutility from leisure.

welfare loss is −3.0% for the cohort born around 2040 in closed economy and only −2.0% in open economy.

In general, the *open economy dampens both welfare gains and welfare losses*. The key reason is that, in open economy, social security reform has a smaller impact on wage rates and interest rates in the North, since capital can flow into the North when domestic savings are low, keeping wages of the future generations high.

5.3. Social security privatization during the demographic transition

We assume that the government announces the reform in 2005 and begins its implementation in 2015. In 2015, the intergenerational transfer system implicit in the PAYG scheme is dismantled completely, except for those who are already retired in 2015 who will keep receiving the promised benefits as long as they survive.

The government acknowledges that the households not yet retired in 2015 (i.e. with age $i<I^R$) have contributed to the system throughout the years and have accumulated pension rights. To compensate these cohorts, the government credits a "recognition bond" to these workers upon retirement.[26]

Every year $t>t^* = 2015$, a bond Ω_t is paid to the cohort that at t^* has age $i = t^* + I^R - t$. For example, in year $t+1$ the bond is paid to the cohort that at t^* was 1 year from retirement (age $i = I^R - 1$); in the last year of the reform $t = t^* + I^R - 1$, the bond is paid to the workers who had age $i = 1$ at t^*. The value of the cumulated pension wealth in the old system for a cohort that retires (and receives the bond) at time t is given by the formula

$$\Omega_t = \rho\left[\kappa^{\text{old}} \frac{W_{t^*+I^R-t,t^*}}{I^R-1}\right]\left[1 + \sum_{i=1}^{I-I^R} \prod_{j=1}^{i}\left(\frac{s_{I^R+j,t+j}}{R^{\text{old}}_{t+j}}\right)\right],$$

where ρ is a parameter measuring how much of the cumulated pension rights the government is willing to pay back; κ^{old} is the replacement rate under the PAYG system; $\{R_t^{\text{old}}\}$ is the sequence of equilibrium interest rates computed in the benchmark transition (the economy without privatization and with the payroll tax adjusting). Under the privatization, the path of equilibrium interest rates will be different from the benchmark—we expect returns to capital to be lower—so it would be inappropriate to use the new interest rates, as we need to compute the value of pension rights households cumulated under the "old system".

To maintain the government budget balanced during the transition, we consider three alternatives.

(PW) The government changes the wage tax τ_w^n during the privatization to balance the budget. Every other policy variable (except retirement benefits, of course) remains constant at its 2005 level (Fig. 13).

(PC) The government changes the consumption tax τ_c^n during the privatization to balance the budget. Every other policy variable remains constant at its 2005 level (Fig. 14).

[26] The recognition bond is a scheme that was used, successfully, in the Chilean reform of 1981 to finance the transition towards a fully funded system. See Edwards (1998) for a detailed description of the Chilean experience.

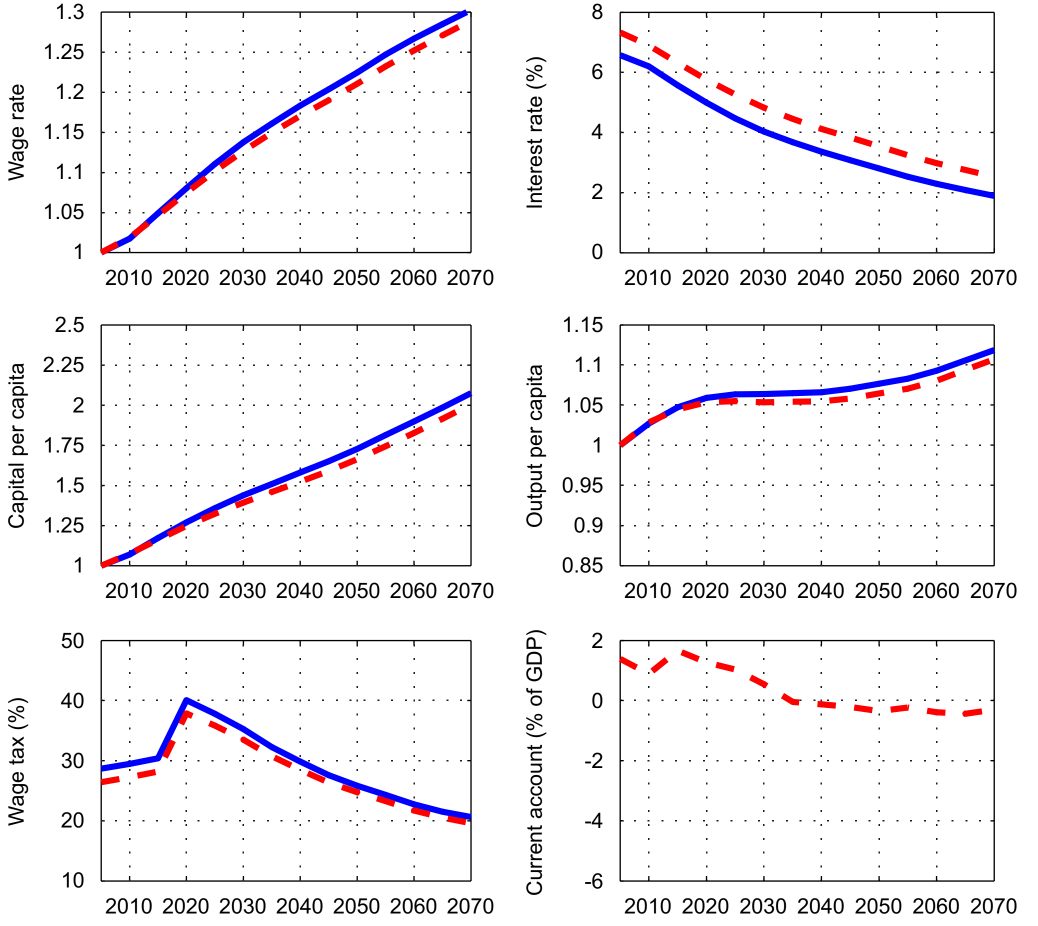


Fig. 13. Results of the benchmark privatization experiment where the transition towards a fully funded system is financed through the payroll tax. Solid lines represent closed economy and dotted lines represent open economy.

(PD) The government changes the level of public debt B^n during the privatization to finance the promised payments of benefits. Every other policy variable remains constant at its 2005 level (Fig. 15).

In every experiment, we assume full replacement of the pension rights, i.e. $\rho = 1$.

5.3.1. *Results I: macroeconomic aggregates and policy variables*

Privatization with wage tax adjustment (*PW*): In the closed economy, the rapid phasing out of benefits leads to a huge increase in aggregate life-cycle savings (recall Fig. 4): capital per capita in both economies is projected to rise by a factor of 2.0 by 2070, and output per capita is expected to rise by 12% above trend in spite of the severe decline in the

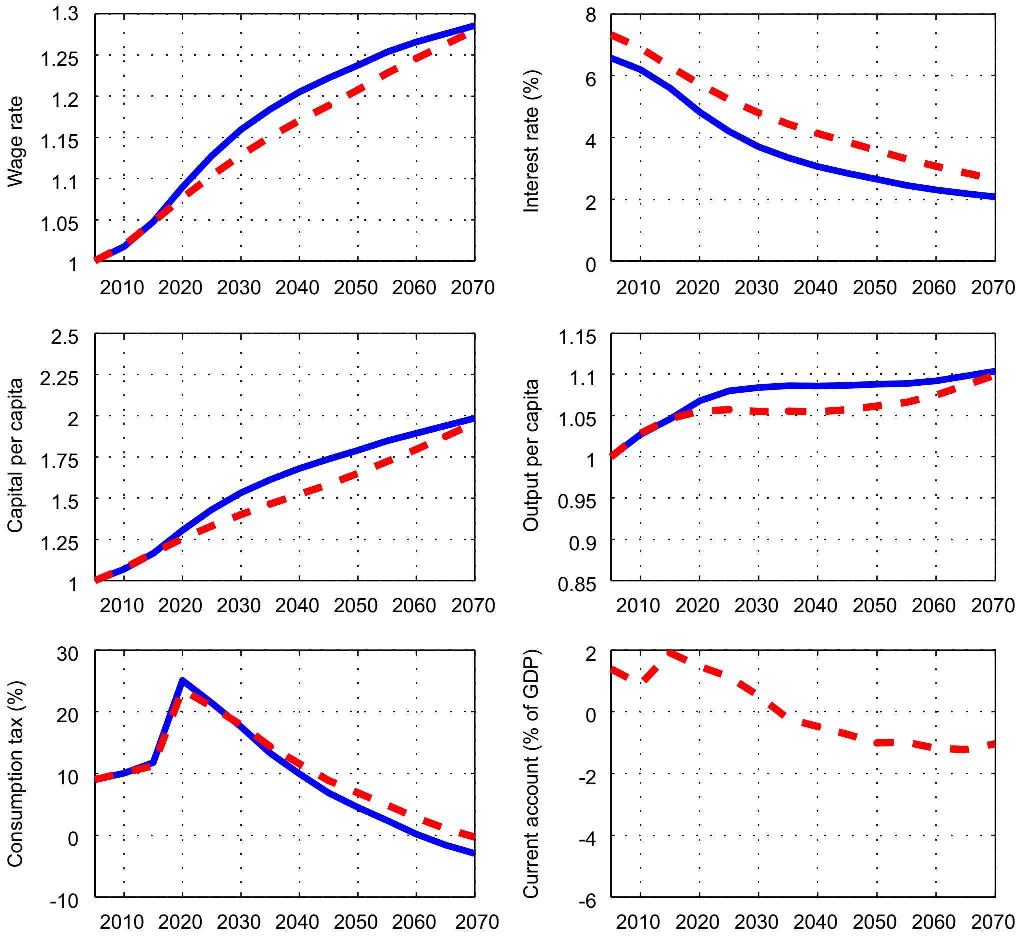


Fig. 14. Results of the benchmark privatization experiment where the transition towards a fully funded system is financed through the consumption tax. Solid lines represent closed economy and dotted lines represent open economy.

employment–population ratio.[27] This capital deepening is so pronounced that the privatization-induced decrease in the closed-economy interest rate (and the rise in wages) in the North has roughly the same magnitude as the demographics-induced changes in the South. In particular, there is only a very mild reversal of capital flows. Since aggregate variables and prices have such similar dynamics in closed and open economy, the wage tax behaves similarly in the two scenarios. It first increases dramatically to pay for the recognition bonds and then falls, relative to the initial level, by approximately 9 percentage points.

[27] Kotlikoff et al. (1999) project a rise in income per capita around 13.5% in the long run in a similar closed-economy experiment (Table V, p. 563).

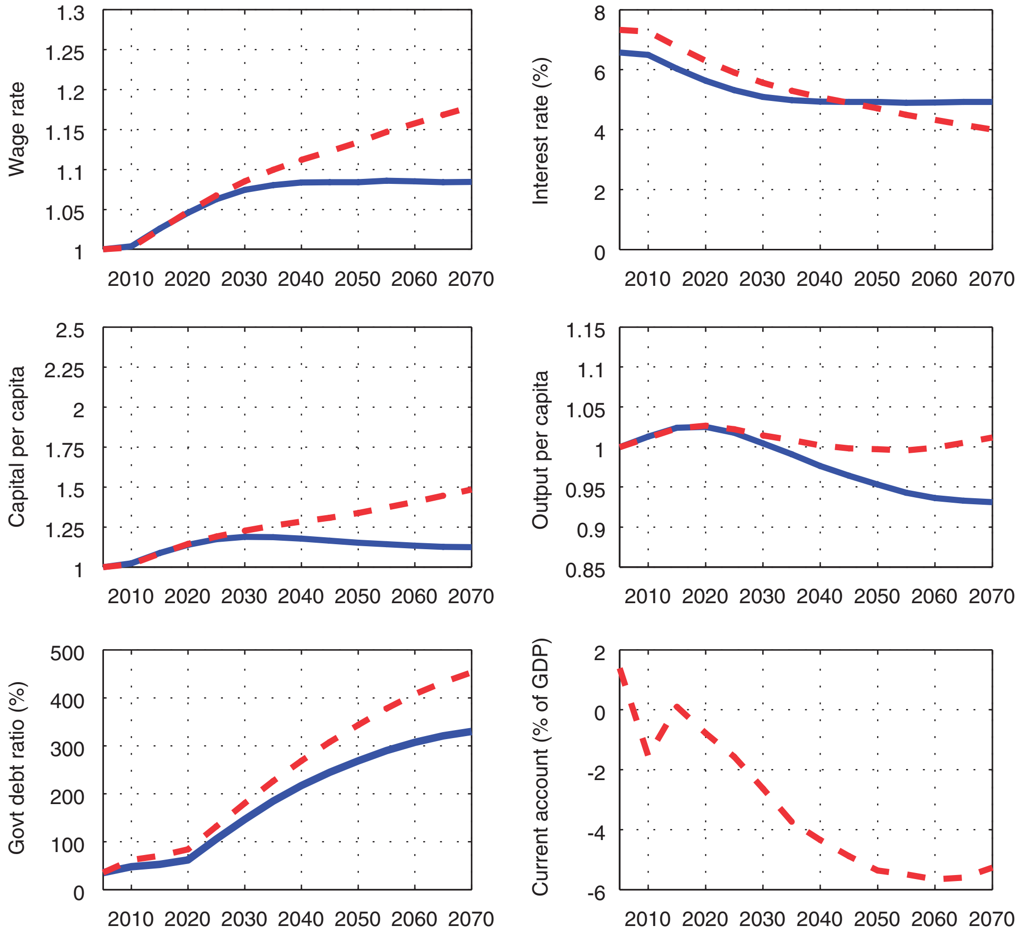


Fig. 15. Results of the benchmark privatization experiment where the transition towards a fully funded system is financed through debt. Solid lines represent closed economy and dotted lines represent open economy.

Privatization with consumption tax adjustment (*PC*): The qualitative dynamics of aggregate variables and prices are similar to the wage tax case. The consumption tax must rise from 9% to 25% by 2020 to finance the recognition bonds, and then it slowly declines towards the new long-run value near zero. Once again, closed an open-economy dynamics are very close.

Privatization with public debt adjustment (*PD*): In order to finance payment of recognition bonds, public debt to GDP ratio has to increase by a staggering factor of 9 in closed economy and by a factor of 13 in open economy by 2070. Huang et al. (1997) run a similar experiment in closed economy and also find that the ratio of debt to GDP should increase massively in order to finance the entitlement debt due to households. The induced crowding out of private capital accumulation is so extensive that capital in closed economy remains stagnant and output decreases by 10% in the long run. In the two-region model, even though the capital inflow from the South substantially mitigates this displacement effect, capital still grows much less than under the other privatization experiments. The surge in debt keeps the interest rate high, so in open economy around 2030, capital begins flowing from the poor to the rich countries, exactly like in the previous set of experiments. Finally note that this is the only experiment where there is a sizeable difference between the path of the policy variable of interest in the two scenarios: in open economy, the debt must rise even more.

In conclusion, when taxation is used to finance the PAYG privatization, most of the demographics-driven divergence in the interest rate between closed and open economy are offset by the extent of capital deepening in the North. What is due to the demographic transition in the South occurs in the North because of the policy transition towards a fully funded system.

5.3.2. *Results II: welfare effects*

The two panels of Fig. 16 report welfare changes of each experiment in closed and open economy compared to the benchmark (BW).

Notice first that when the system is privatized and taxes are used to finance the transition (PW and PC), the losses of the generations currently alive are much larger than in the first set of experiments (survival of PAYG), notwithstanding the payment of the recognition bond. The reason is twofold: first, those born between 1970 and 2000 will suffer particularly harshly from the jump in the taxes around 2020; second, as they age, they face a lower interest rate than in the benchmark.

The cohorts born in the 1980s experience a loss of −13% under the (PW) experiment, −8% under the (PC) experiment, and −3% in the (PB) experiment, in closed economy. In open economy, these welfare losses are reduced, on average, by 2%. The winners are the cohorts born around 2030–2040 who experience welfare gains of 2.5% and 5% under (PW) and (PC) in closed economy, thanks to the higher labor productivity and wages, and lower tax rates. In open economy, these gains are reduced by roughly 2–3%.

As in the previous set of policy simulations, in open economy the welfare effects are considerably dampened since capital accumulation (the rise in wages and the fall of interest rates) takes place independently of the pension reform.

The welfare effects of the (PD) experiment are particularly interesting. Here, the welfare change of every cohort born after 1980 in open economy is negative and larger than in closed economy. The crowding-out force of public debt in limiting capital accumulation has a more negative impact in open economy, where it jeopardizes the wage growth that can be achieved under the benchmark policy experiment.

6. Robustness analysis

We now investigate the sensitivity of our experiment results under alternative assumptions about some key parameters of the model.

Endogenous labor supply: We generalize the model to permit households to choose the fraction of their time endowment they supply to market production. In order to be consistent with balanced growth, we specify preferences over consumption and leisure as

$$u(c_{it}, l_{it}) = \frac{[c_{it}^{\eta}(\bar{L}_{it} - l_{it})^{1-\eta}]^{1-\theta}}{1-\theta},$$

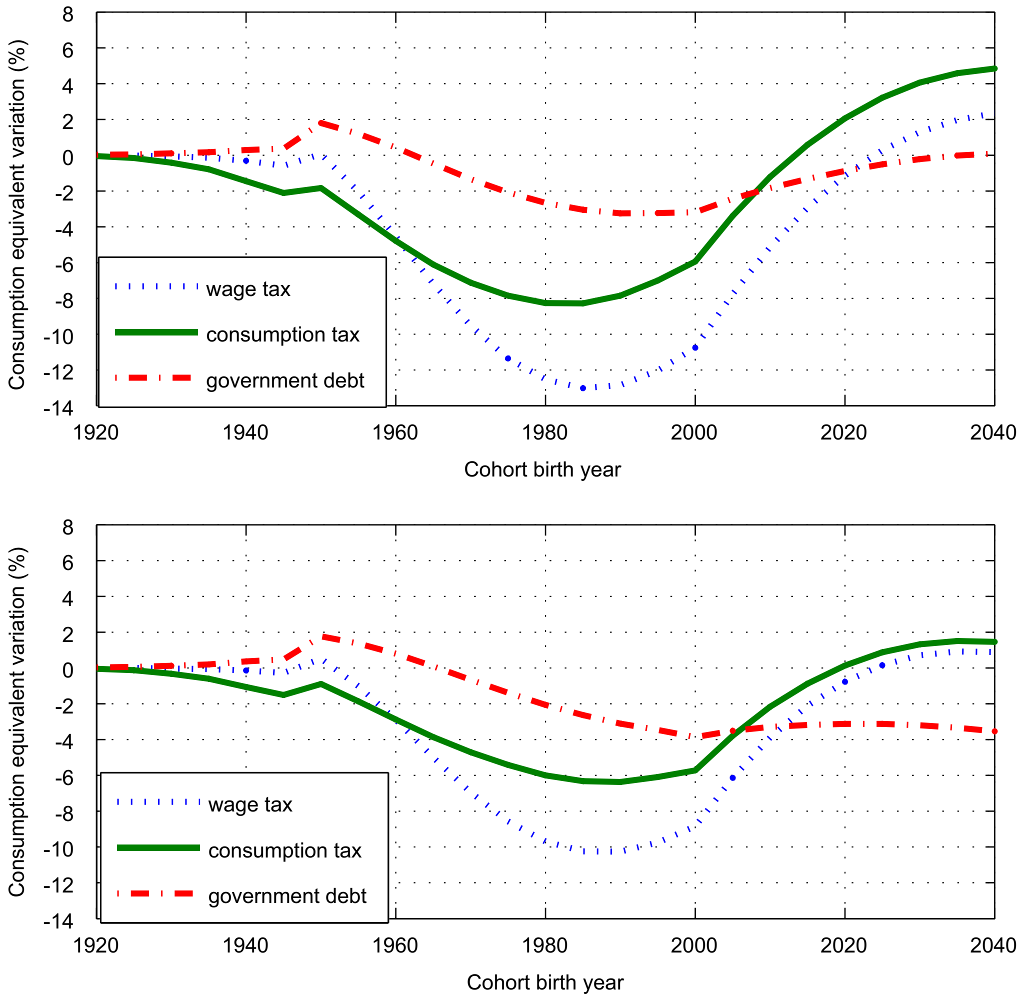


Fig. 16. Welfare effects of different policy options to finance the transition of the PAYG system towards full privatization. The top panel summarizes the welfare changes, cohort by cohort, in closed economy; the bottom panel refers to the open-economy simulations.

where $\bar{L}_{it}$ is the household time endowment, and l_{it} is the household's labor supply. We make the simplifying assumption that pension benefits are calculated according to the formula

$$p_{k,t} = \frac{\kappa_t}{I^R - 1} \sum_{j=1}^{I^R-1} w_{t-k+j} \varepsilon_{j,t-k+j} \phi \bar{L}_{j,t-k+j}, \tag{18}$$

where ϕ is a fixed fraction, independent of the time supplied by the household to the market at time t. Under this assumption, the choice of labor supply remains static, and the household problem has still an exact solution (see Appendix C for details).

We re-calibrate β and η jointly so that we get a capital–output ratio of 2.5 in 2005 (as in the benchmark), and average work hours equal to 40% of the time endowment (assumed

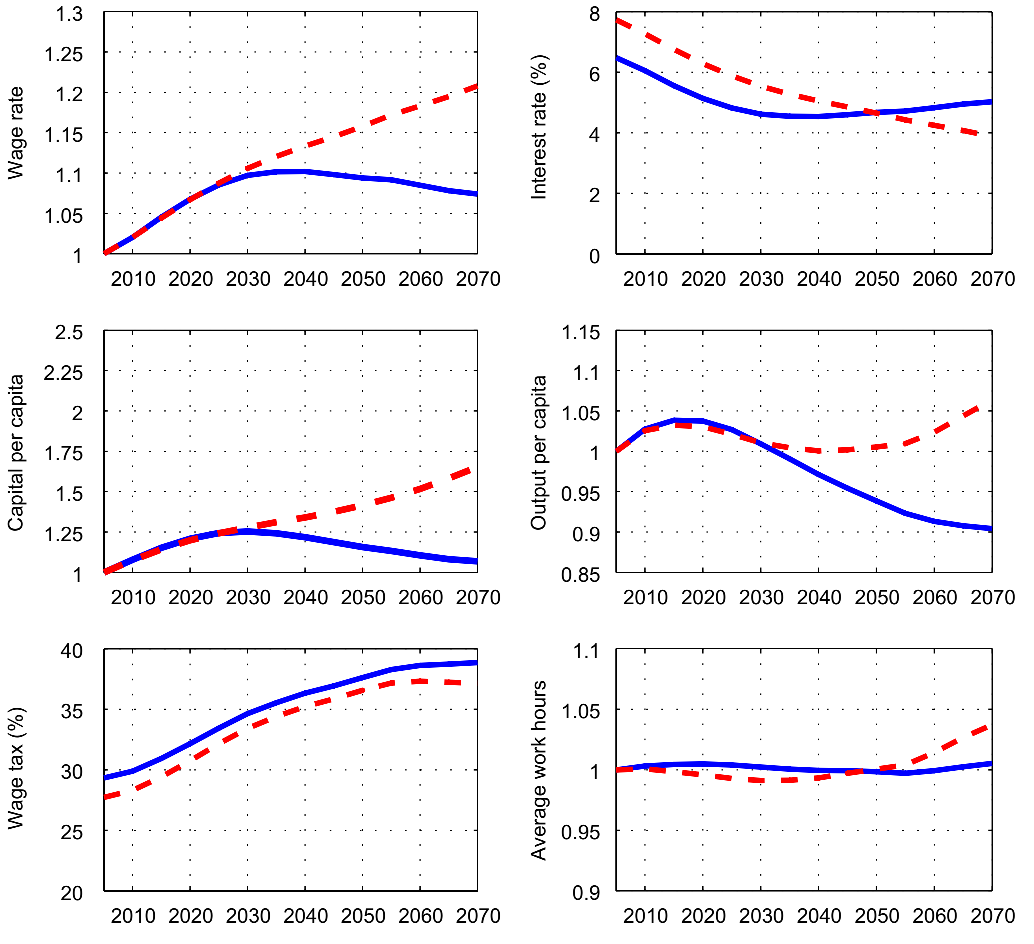


Fig. 17. Model with endogenous labor supply. Solid lines represent closed economy and dotted lines represent open economy. The simulation refers to the benchmark case.

to be time-invariant) during 1950–2000. Consistently, we set $\phi = 0.4$. The parameter θ is set at 4, so the parameterization displays an intertemporal elasticity of substitution of 0.45 and a Frisch elasticity of 0.8. The latter is an average between micro-estimates for men (between 0 and 0.5), and for women (typically, above 1).

We report our results for the benchmark experiment in Fig. 17. The main finding is endogenizing labor supply has little effect on the equilibrium (compare this plot with Fig. 7). Given the high Frisch elasticity, and the significant fluctuations in wages, one would expect stronger swings in average hours worked (lower right panel). However, the increase in the payroll tax largely offsets the intertemporal substitution effect coming from wages, since what matters for labor supply decisions are wages net of taxes, so households barely modifies their choices of hours worked.[28]

[28]Indeed, the only increase in labor supply occurs in open economy, towards the end of the period, when the payroll tax levels off, but wages keep rising.

Risk aversion: We depart from the benchmark calibration with respect to the coefficient of relative risk aversion θ, to which we assigned a value of 2 in the benchmark. We conduct two experiments with $\theta = 1.5$ and 2.5. To maintain the same capital–output ratio, we also adjust the subjective discount factor β to match the target capital–output ratio of 2.5 in the North, as in the benchmark.[29] All the other parameter values are unchanged.

A higher value of θ implies a lower elasticity of intertemporal substitution and induces more savings and higher output. With $\theta = 2.5$, interest rates are slightly lower than in the benchmark and wage rates slightly higher. The paths of the policy variables, however, are almost identical to the benchmark simulations. Similarly, when we lower the value of θ, experiment results are found to be robust.

Development process and TFP growth: In the benchmark experiment, during the transition, the average GDP per capita of the South rises from $\frac{1}{7}$ of GDP per capita of the North to almost $\frac{1}{5}$, and in the long run the two regions grow at the same rate. In the first robustness experiment, we let the TFP growth rate of the South converge very quickly to the level of the North in 2000, instead of having it converge gradually by 2050. This alternative assumption has almost no effect on factor prices and results remain unchanged.

In a second robustness experiment, we allow the South to grow and fully catch up with the North in terms of per-capita output (i.e., convergence is in levels, not in growth rates). We assume the South will experience high TFP growth rate during 2000–2200, large enough to converge to the value of the North after 2200. Under this alternative assumption, the "size" of the South economy relative to the North is much larger than in the benchmark. However, the difference does not generate any significant change in our experiment results. Faster growth of labor productivity in the South leads to a higher interest rate in the long run in its closed economy. Moreover, the path of the open-economy interest rate is much closer to that of the South due to its increased relative size. With these two effects offsetting each other, the dynamics of the interest rate faced by the North in the open economy closely resembles the one in the benchmark. As a result, our results are extremely robust to this alternative assumption on the TFP growth rate as well.

Population projections: Throughout the analysis, we used the "medium variant" UN projection of population growth which is based on the presumption that total fertility in (almost) all countries will converge to a below-replacement level of 1.85 by 2050, remain there for about 100 years, and then gradually rebound to its replacement level. The UN supplies also demographic projections under two alternative variants. In the low-variant (high-variant) scenario, total fertility is reduced (increased) by 0.5 children per woman before 2050 and by 0.25 after 2050, relative to the medium variant. Figs. 18 and 19 plot the results for the benchmark experiment under the high- and low-variant projection, respectively. According to the high-variant, the demographic trends would not be too severe, thus the rise in the wage tax smaller than the baseline simulation done under the medium-variant. In the low-variant case, instead, all the dynamics are exacerbated. For example, the model predicts a jump in the payroll tax by over 18 points. In both scenarios, though, the comparison between closed- and open-economy simulations remains qualitatively similar to the baseline experiment of Fig. 7.

Role of China and India: In a recent paper, Fehr et al. (2006) found that the impact of the demographic transition on the dynamics of U.S. wages crucially depends on the inclusion

[29]Calibrated values of the discount factor β in the experiments are 0.954 (0.991 per year) when θ is 1.5 and 1.143 (1.027 per year) when θ is 2.5.

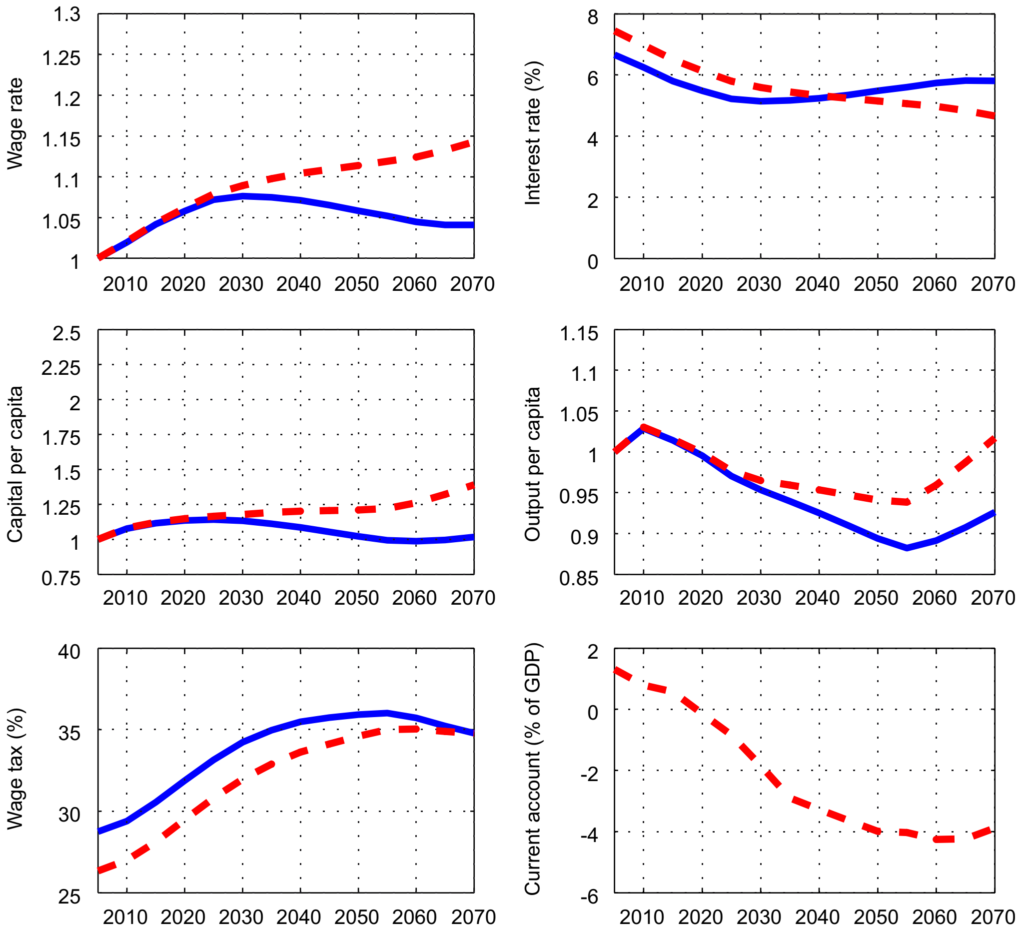


Fig. 18. Alternative population projections: high-variant. Solid lines represent closed economy and dotted lines represent open economy. The simulation refers to the benchmark case.

of China in their multi-country model: when capital is allowed to move between the U.S. and China, U.S. wages at the end of the 21st century are 8% higher than in the closed-economy model (since capital flows from China to the U.S. in the long run, as in our model). We run a similar experiment in our model, excluding from the South both China and India—the two largest countries. In Fig. 20 we report the simulations of the benchmark economy with and without China and India. The results confirm that these two countries play an important role in determining the growth of labor productivity in the North: in their absence, the capital flows towards the North would be halved and wages would be lower by 6% in 2070. Our findings confirm the conclusion of Fehr et al. (2006) that the degree of capital market liberalization in China and India is a decisive factor in determining the fortunes of the developed world. Coherently with the rest of the paper, we also find that the dynamics of the payroll tax in the North are unaffected by the exclusion of China and India.

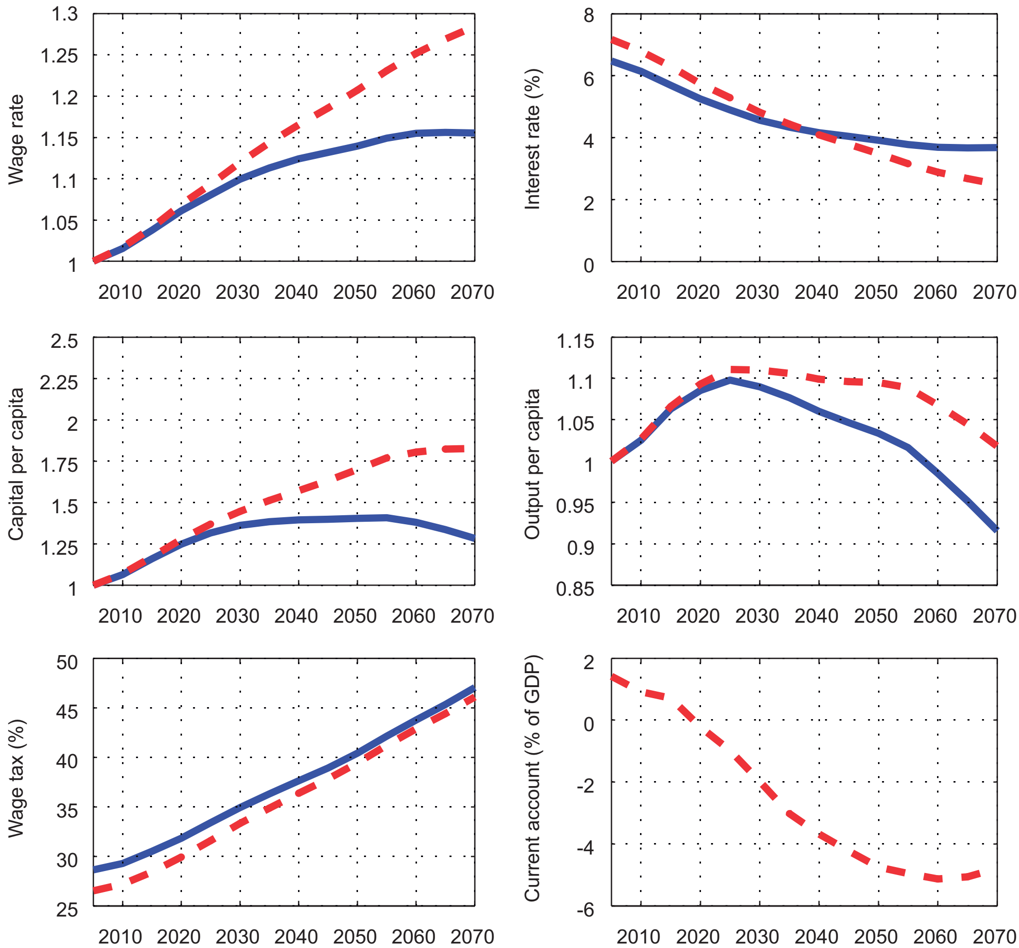


Fig. 19. Alternative population projections: low-variant. Solid lines represent closed economy and dotted lines represent open economy. The simulation refers to the benchmark case.

Frictional capital markets: As illustrated in Fig. 6, for the period 1970–1990, the model predicts capital flows across the two regions much larger than those observed. We have also solved the model by imposing the existence of a transaction cost in the world capital market that reduces the return to the capital *invested by the North in the South*. This is a simple reduced form to capture region-specific expropriation risk typical of less developed financial markets, and can be thought of as the probability that the interests on the foreign investment will not be paid back.

We model the transaction cost as a smooth concave function of the external wealth of the North as follows:

$$\chi(N_t) = \begin{cases} \bar{\chi} N_t^{0.5} & \text{if } N_t > 0, \\ 0 & \text{if } N_t \leqslant 0. \end{cases}$$

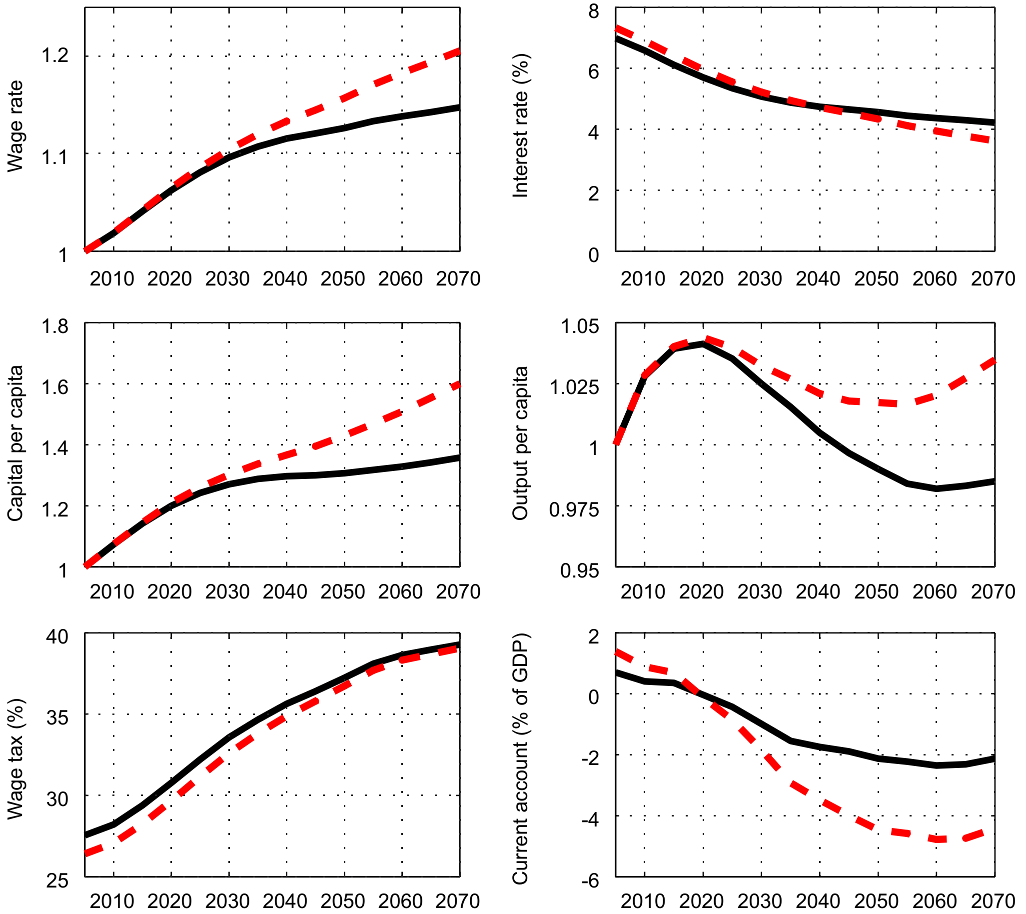


Fig. 20. The role of China and India in the world capital market. Solid lines depict the open economy where China and India are excluded from the South, dotted lines depict the economy where they are included. The simulation refers to the benchmark case.

and set the parameter $\bar{\chi}$ to 1.47 to match the Kraay et al. data of Fig. 6. This parameterization implies a transaction cost of 17%, on average, in the period 1970–1990. The new indifference condition for households in the North between investing domestically and in foreign assets becomes: $r_t^n = r_t^s(1 - \chi(N_t))$, where r_t^s and r_t^s are the interest rates paid to domestic investors in each region.

In Fig. 21 we contrast the benchmark economy with perfect and frictional capital markets. The differences in the dynamics of aggregate variables are rather small, mainly because after 2040 the stock of external wealth of the North turns negative and the capital market friction becomes irrelevant.

Capital income tax in the South: As discussed earlier, the data on capital income tax rates in the developing world is far from abundant. Here we follow an alternative strategy to calibrate τ_a^s. We set τ_a^s so that, in the period 1950–2000, the open-economy model predicts foreign assets of the South which match the Kraay et al. (2004) observation. Since the

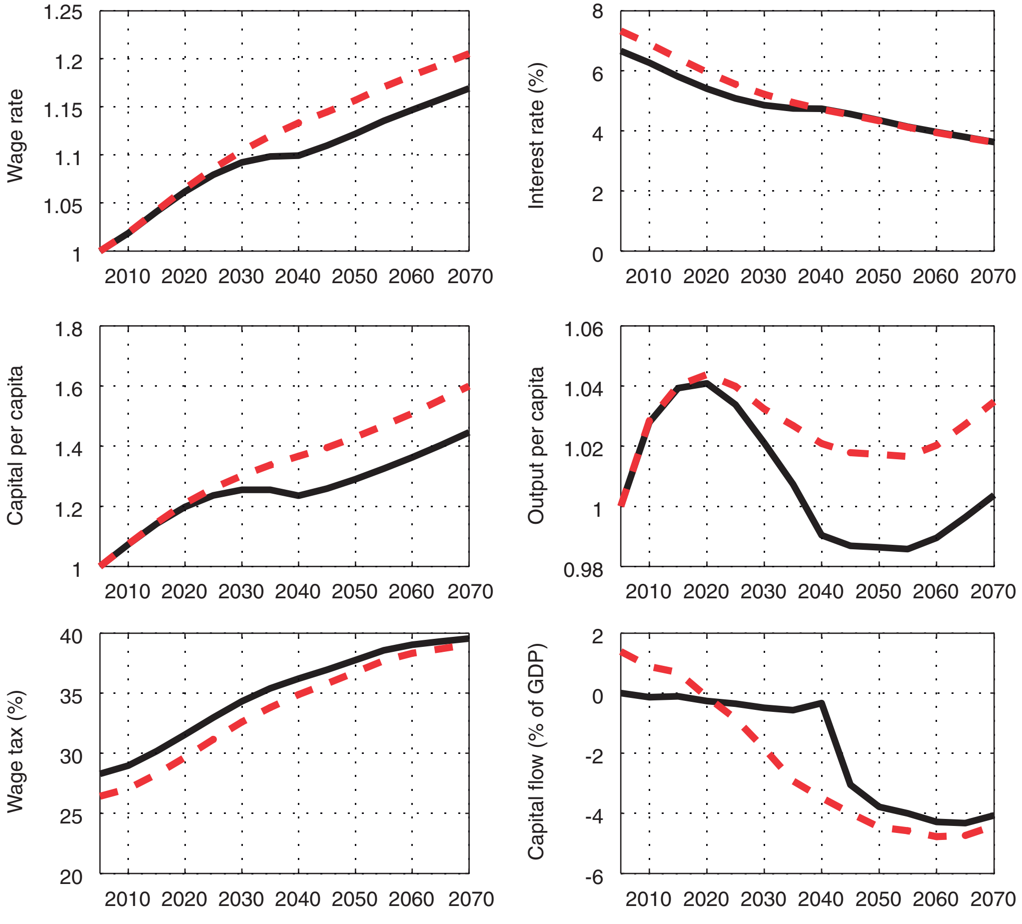


Fig. 21. Frictional capital markets. Solid lines depict the open economy with transaction costs when capital flows from the North to the South. Dotted lines depict the economy with perfect international capital markets. The simulation refers to the benchmark case.

baseline calibration with $\tau_a^s = 0.386$ overpredicts capital flows to the South, one should expect the new calibration to deliver a lower tax rates (hence, more capital accumulation in the South and smaller interest rate differential between North and South). It turns out that $\tau_a^s = 0.20$ allows to generate a ratio of external wealth to total wealth in the South of exactly -10%. The results of our benchmark simulation remain almost unchanged under this new parameterization (see Fig. 22).

7. Concluding remarks

The sustainability of PAYG pension systems in developed countries, in the face of the projected transformations in the demographic structure of the population, is at risk. The current political and economic debate is centered on the best way to reform the system in order to limit welfare losses for the cohorts alive today. The typical approach, when

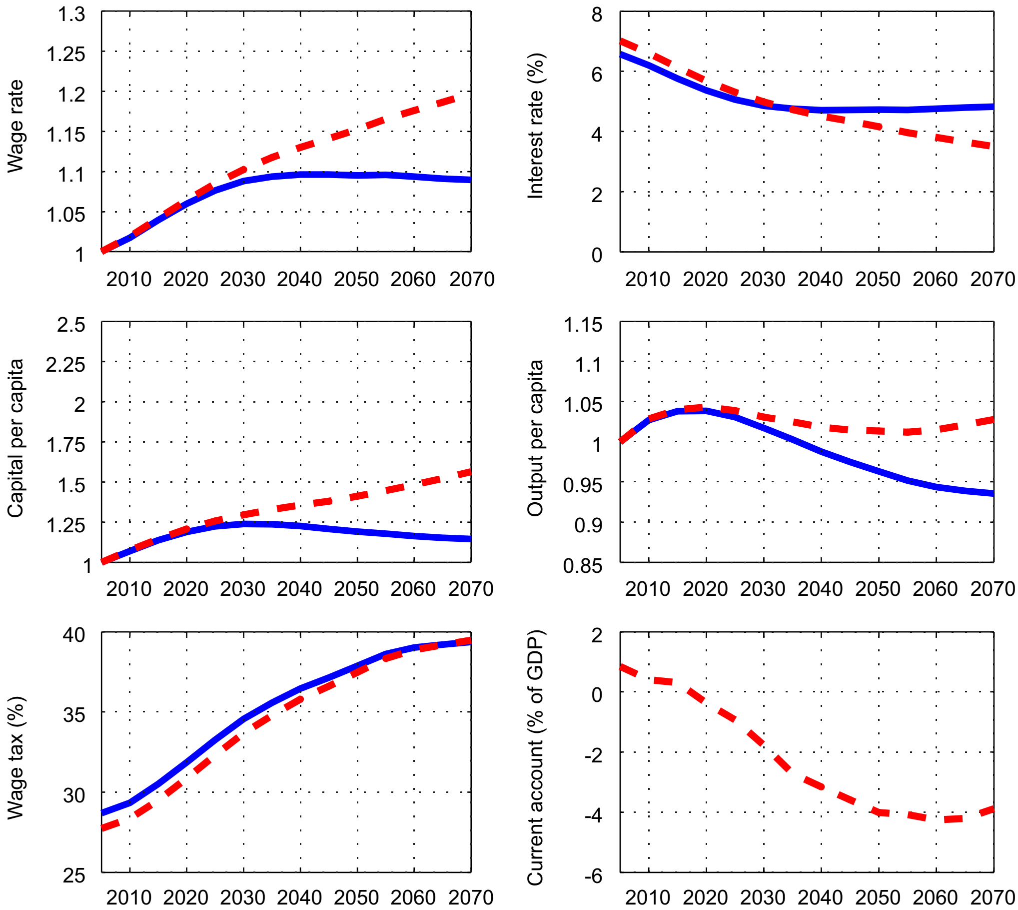


Fig. 22. Alternative parameterization of capital income tax rate in the South. Solid lines depict the closed economy. Dotted lines depict the open economy. The simulation refers to the benchmark case, where the tax rate on capital income in the South is set to $\tau_a^s = 0.20$ to match external wealth of the South in 1980.

quantifying these effects, is studying transitional dynamics within an OLG model, under the assumption of closed economy.

In this paper, we asked whether the results of these typical policy experiments differ if we consider a different benchmark: a two-region model of the world where the unsynchronized demographic patterns between the two regions lead to an adjustment through capital flows.

We argued that the answer depends on what the precise focus of the question is. If one is interested in forecasting the required changes in certain key policy variables (like the payroll tax, the consumption tax, etc.) needed to finance the transition, the answer is "no", due to general-equilibrium effects on the government budget constraint. If one is interested in computing the welfare effects of various policy reforms, then the answer is "yes", with the welfare impact in open economy being typically smaller in absolute value.

We do not necessarily believe that our two-region benchmark with frictionless capital flows is more appropriate than the closed-economy benchmark. It is just the opposite end of the spectrum, and actual economies lie somewhere in the middle. As such, it is at least as interesting as the closed-economy model. And one could argue that the framework we propose, is becoming progressively more relevant. Our model, as well as several other demographics-based model of current account dynamics, predict that capital will soon start flowing from poor to rich countries. Then the typical criticism of neoclassical open-economy models, i.e., that they overpredicts the size of external wealth, will become irrelevant since, arguably, sovereign risk, political uncertainty, and expropriation risk are much smaller in the developed world thanks to better institutions.

Appendix A. Construction of demographic variables

Fertility rates: The UN data provide age-specific fertility rates for 1995–2050. We need to construct age-specific fertility rates for the years before 1995 and after 2050, for which only *total* fertility rates are available. For the years before 1995, we adjust the age-specific rates available for 1995 proportionately in order to match the total fertility rate in each period. We use exactly the same rule for both regions. Details are available upon request.

For the years after 2050, we use the projection from United Nations (2004) of total fertility rates for 2100 and 2200 as the benchmark. For the North, we extrapolate the data using the average change in fertility rate over the past 20 years for each age group up to the year 2100 and obtain total fertility rates that closely match the UN projections. For the period beyond 2100, we impose that age-specific fertility rates are constant at their 2100 values. For the South, we use slightly different rules of adjustment. We assume that age-specific fertility rates in the South in 2100 converge to those projected for the North in 2050 and let them adjust smoothly during 2050–2100 starting from the actual data available for 2050. For 2100–2150, we apply the same extrapolation rule used for North in 2050–2100. From 2150 and onward, we impose that age-specific fertility rates are constant at their 2150 values (equal to the values for the North from 2100 onward). These assumptions are consistent with the ones underlying the "medium variant" (see Section 6).

Surviving probabilities: We use actual and projected population data from the UN database to compute surviving probabilities for 1950–2050. We do not model migration across regions, but we implicitly account for it in the demographic dynamics of two regions, insofar as they appear in the UN data. To obtain the conditional surviving probability between age $i-1$ and age i at time t, we divide the population of age i in period t by the population of age $i-1$ in period $t-1$.

For the years beyond 2050, we estimate the surviving rates of the two regions to match the following demographic moments based on the projections in United Nations (2004).

The population growth rates in the bottom rows are the UN projections of the average annual growth rate for the years 2100–2150 and 2150–2200 in each region.

We assume that after 2050 the age-specific surviving probabilities grow over time at a rate which is a given polynomial function of age and time. We estimate the function so to minimize the distance between the implied demographic moments and the above targets. For the South, we use the same method, and for the years closer to 2200, we let the surviving probabilities of each age group grow at a constant rate so that they converge smoothly to the same final steady state as the North by the year 2200. Remember that, while we assume convergence of fertility and mortality rates in two regions by 2200, it takes longer for the age structure of the population and other demographic moments to fully converge (Table A1).

Table A1
Target moments

	2100	2200
North		
Median age	44.63	47.65
Fraction of the old over 60 (%)	33.15	37.09
Population growth rate (%)	0.0525	0.0787
South		
Median age	43.89	45.15
Fraction of the old over 60 (%)	30.05	33.67
Population growth rate (%)	−0.1573	−0.0115

Participation function: Let $\Lambda^r_{i,t}$ be the fraction of the time endowment (normalized to one) worked by the household, i.e. $\Lambda^r_{i,t} = 0.5[1 + P^r_{i,t}]$, where the husband is assumed to work full time and where $P^r_{i,t} \in (0, 1)$ denotes the fraction of time a woman of age i supplies to the labor market in region r at time t. Let $d^r_{i,j,t}$ be the number of pairs of children of age j present in her households, and $\mathbf{d}^r_{i,t}$ be the vector with typical entry $d^r_{i,j,t}$.[30] As described in the main text, we use CEX data to estimate the marginal effects α_j of the presence in the household of a pair of dependent child in age group j (0–4, 5–9, 10–14 years old, in the data) on women's probability of participation $P_{i,t}$, controlling for several other observable individual characteristics (age, race, education). This Probit regression yields $\hat{\alpha}_{0-4} = -0.146$, $\hat{\alpha}_{5-9} = -0.0960$, and $\hat{\alpha}_{10-14} = -0.0464$. As expected, all coefficients are negative and significant and the newborn child has the strongest impact on the probability of participation.

Next, we enrich the female labor force participation function with an exponential trend to capture other forces that, beyond the decline in fertility, have contributed to the rise in women's hours worked. The statistical model becomes

$$P^r_{i,t}(\mathbf{d}^r_{i,t}) = \beta^r_0 + (\overline{P} + \overline{T}_i - \beta^r_0)\{1 - \exp[-\beta^r_1 * (t-1)]\} + \sum_{j=1}^{I^d} \hat{\alpha}_j d^r_{i,j,t}, \tag{19}$$

where the coefficient β^r_0 measures the participation rate for a female worker with no children in the initial period (1950), which we can see by substituting $t = 1$ and $d^r_{i,j,t} = 0$ in (19); $\overline{P} = 0.721$ is the long-run female labor participation rate (U.S. Census Bureau, 2003); $\overline{T}_i$ is the long-run value of the time devoted by a woman of age i to child care (common across the two regions) computed from the final steady-state value of the vector of dependent children and the estimated coefficients of the Probit regression, i.e. $\overline{T}_i = -\sum_{j=1}^{I^d} \hat{\alpha}_j \underline{d}^r_{i,j,\infty}$; the parameter β^r_1 regulates the speed of convergence towards the long-run rate $\overline{P}$: as $t \to \infty$, the average participation rate implied by the function $P^r_{i,t}$ converges to $\overline{P}$.

We estimate the parameters (β^r_0, β^r_1) for each region using historical and projected data on participation rates of the U.S. for the North, and on the average of four countries (Brazil, India, Korea, and Mexico) for the South. The estimated parameters are as follows: $\hat{\beta}^n_0 = 0.386$, $\hat{\beta}^s_0 = 0.290$, $\hat{\beta}^n_1 = 0.189$, $\hat{\beta}^s_1 = 0.077$.

[30] We express $d^r_{i,j,t}$ in terms of pairs, to remain consistent with the model.

Appendix B. Computation of the equilibrium

We start by stationarizing the economy using the long-run productivity growth λ and the long-run population growth rate γ, common to both regions. In all our experiments, we study the transition between two steady-states of the world economy induced by secular changes in the demographic structure (fertility rates, surviving rates, and participation rates) in the two regions.

We present the description of the computation of the equilibrium for an example where the only fiscal variable that needs to adjust to satisfy the government budget constraint (thus the only fiscal variable to be determined in equilibrium) is the sequence of labor income taxes $\boldsymbol{\tau}_w^{\mathbf{r}} \equiv \{\tau_{w,t}^r\}_{t=1}^{\infty}$. All the other policy variables either remain fixed throughout the transition or move deterministically, i.e. they can be thought of as parameters.

Step 1 (*Steady-state*): Compute the initial and final steady-states of the model. Set T, the length of the transition to a very large number. Truncating the transition at $T = 180$ or beyond does not affect the results.

Step 2 (*Initial guess*): Guess two T-dimensional vectors for the world interest rate and the labor income tax rate in both regions, and denote these initial guesses by $\{\mathbf{r}_0, \boldsymbol{\tau}_{w0}^{\mathbf{r}}\}$. The first and last entry of these vectors are the initial and final stationary equilibrium values computed in Step 1. Given the path for $\mathbf{r}_0$, using the CRS property of F and the optimization conditions for the firm (10) and (11), we can derive initial sequences of wages in each region $\mathbf{w}_0^r$. Given prices and the sequence of policies $\boldsymbol{\tau}_{w0}^{\mathbf{r}}$, one can solve the problem of the households in both regions.

Step 3 (*Household problem*): Consider the budget constraint of the agent at time t who, conditional on surviving, receives the savings of the deceased:

$$(1 + \tau_{c,t}^r)c_{i,t} + a_{i+1,t+1} = y_{i,t}^r + [1 + (1 - \tau_{a,t}^r)r_t]a_{i,t} + (1 - s_{i+1,t+1})a_{i+1,t+1}.$$

Using this expression for the annuity, we can rewrite the budget constraint as

$$(1 + \tau_{c,t}^r)c_{i,t} + s_{i+1,t+1}a_{i+1,t+1} = y_{i,t}^r + [1 + (1 - \tau_{a,t}^r)r_t]a_{i,t}. \tag{20}$$

For convenience, denote net-of-taxes gross interest rate as

$$R_t \equiv 1 + (1 - \tau_{a,t}^r)r_t.$$

From the FOC with respect to asset holdings next period, we obtain

$$\frac{c_{i+1,t+1}}{c_{i,t}} = \left[\beta \frac{\Omega_{i+1,t+1}}{\Omega_{i,t}} \frac{1 + \tau_{c,t}^r}{1 + \tau_{c,t+1}^r} R_{t+1}\right]^{1/\theta} \equiv g_{i+1,t+1}^c, \tag{21}$$

which is the optimal growth rate of consumption between age i and $i + 1$ and between time t and $t + 1$. Iterating backward over (21), we obtain that

$$c_{i+1,t+i} = c_{1,t} \prod_{j=1}^{i} g_{j+1,t+j}^c.$$

Thus, we can obtain the discounted present value of the total (gross of taxes) lifetime consumption expenditures of the household of age 1 at time t as

$$\bar{c}_{1,t} = c_{1,t}\left[(1 + \tau_{c,t}^r) + \sum_{i=1}^{I-1}(1 + \tau_{c,t+i}^r) \prod_{j=1}^{i} \frac{s_{j+1,t+j}}{R_{t+j}} g_{j+1,t+j}^c\right], \tag{22}$$

and, in general, the expression for the discounted present value of the total (gross of taxes) lifetime consumption expenditures of the household of age i^* at time t is

$$\overline{c}_{i^*,t} = c_{i^*,t}\left[(1+\tau^r_{c,t}) + \sum_{i=i^*}^{I-1}(1+\tau^r_{c,t+(i-i^*+1)})\prod_{j=i^*}^{i}\frac{s_{j+1,t+(j-i^*+1)}}{R_{t+(j-i^*+1)}}g^c_{j+1,t+(j-i^*+1)}\right]. \tag{23}$$

The discounted present value of the total (net of taxes) lifetime earnings of a household of age 1 at time t is

$$\overline{y}_{1,t} = y^r_{1,t} + \sum_{i=1}^{I-1}\left(\prod_{j=1}^{i}\frac{s_{j+1,t+j}}{R_{t+j}}\right)y^r_{i+1,t+i}, \tag{24}$$

where we are implicitly imposing the initial condition $a_1 = 0$. The discounted present value of the total (net of taxes) lifetime earnings of a household of age i^* at time t is

$$\overline{y}_{i^*,t} = y^r_{i^*,t} + \sum_{i=i^*}^{I-1}\left(\prod_{j=i^*}^{i}\frac{s_{j+1,t+(j-i^*+1)}}{R_{t+(j-i^*+1)}}\right)y^r_{i+1,t+(i-i^*+1)} + R_t a_{i^*,t}. \tag{25}$$

Since individual optimization requires $\overline{c}_{i^*,t} = \overline{y}_{i^*,t}$ for each age i^* and time t, from (27) and (25), we obtain $c_{i^*,t}$ as

$$c_{i^*,t} = \frac{\overline{y}_{i^*,t}}{[(1+\tau^r_{c,t}) + \sum_{i=i^*}^{I-1}(1+\tau^r_{c,t+(i-i^*+1)})\prod_{j=i^*}^{i}\frac{s_{j+1,t+(j-i^*+1)}}{R_{t+(j-i^*+1)}}g^c_{j+1,t+(j-i^*+1)}]}.$$

Note that $a_{i^*,t}$ in Eq. (25) is computed residually from $c_{i-1^*,t-1}$ and the budget constraint (6):

$$a_{i^*,t} = \frac{1}{s_{i^*,t}}[y^r_{i^*-1,t-1} + R_{t-1}a_{i^*-1,t-1} - (1+\tau^r_{c,t-1})c_{i^*-1,t-1}].$$

Step 4 (*Updating*): Aggregating asset holdings across age groups and using Eq. (13) for the North, we obtain the implied sequence for external wealth of the North $\mathbf{N}^n_0$. Using $\mathbf{N}^n_0$ into Eq. (13) for the South, we obtain the sequence of capital stock for the South $\mathbf{K}^s_0$. Using the labor market clearing condition for the South and the implied sequence of human capital stocks $\mathbf{H}^s_0$, together with $\mathbf{K}^s_0$ and the world capital market clearing condition (11), we arrive at a new guess for the world interest rate $\mathbf{r}_1$. Finally, we use the government budget constraints (9) in each region with price sequences $\mathbf{r}_0$ to update our guess for the tax rate to $\boldsymbol{\tau}^{\mathbf{r}}_{w1}$. If convergence is not reached, we restart from Step 3 with the new vector of guesses $\{\mathbf{r}_1, \boldsymbol{\tau}^{\mathbf{r}}_{w1}\}$.

Appendix C. Solution of the model with endogenous labor supply

From our preference specification (6), it follows that the intra-temporal FOC yields

$$l_{i,t} = \bar{L}_{it} - lrgy\left(\frac{1-\eta}{\eta}\right)\frac{(1+\tau^r_{c,t})}{(1-\tau^r_{w,t})w_t\varepsilon_{i,t}}c_{i,t}, \tag{26}$$

where $\bar{L}_{it}$ is the time endowment of a household of age i at time t. For convenience, denote net-of-taxes gross interest rate as $R_t \equiv 1 + (1-\tau^r_{a,t})r_t$.

From the FOCs with respect to asset holdings next period, we obtain, after rearranging,

$$\frac{c_{i+1,t+1}}{c_{i,t}} = \left[\beta \frac{\Omega_{i+1,t+1}}{\Omega_{i,t}} \frac{1+\tau^r_{c,t}}{1+\tau^r_{c,t+1}} R_{t+1}\right]^{1/\theta} \left[\frac{(1+\tau^r_{c,t+1})(1-\tau^r_{w,t})w_t\varepsilon_{i,t}}{(1+\tau^r_{c,t})(1-\tau^r_{w,t+1})w_{t+1}\varepsilon_{i+1,t+1}}\right]^{(1-\eta)(1-\theta)/\theta}$$
$$\equiv g^c_{i+1,t+1}$$

which is the optimal growth rate of consumption between age i and $i+1$ and between time t and $t+1$. Iterating backward over the previous equation, we arrive at

$$c_{i+1,t+i} = c_{1,t} \prod_{j=1}^{i} g^c_{j+1,t+j}.$$

Thus, the discounted present value of the total (gross of taxes) lifetime consumption expenditures of the household of age i^* at time t is

$$\overline{c}_{i^*,t} = c_{i^*,t}\left[(1+\tau^r_{c,t}) + \sum_{i=i^*}^{I-1}(1+\tau^r_{c,t+(i-i^*+1)})\prod_{j=i^*}^{i}\frac{s_{j+1,t+(j-i^*+1)}}{R_{t+(j-i^*+1)}}\, g^c_{j+1,t+(j-i^*+1)}\right]. \tag{27}$$

The discounted present value of the total (net of taxes) lifetime earnings of a household of age i^* at time t is

$$\begin{aligned}
\overline{y}_{i^*,t} &= y^r_{i^*,t} + \sum_{i=i^*}^{I-1}\left(\prod_{j=i^*}^{i}\frac{s_{j+1,t+(j-i^*+1)}}{R_{t+(j-i^*+1)}}\right)y^r_{i+1,t+(i-i^*+1)} + R_t a_{i^*,t}\\
&= (1-\tau^r_{w,t})w^r_t\varepsilon^r_{i*^*,t}l^r_{i^*,t} + \sum_{i=i^*}^{I^R-2}\left(\prod_{j=i^*}^{i}\frac{s_{j+1,t+(j-i^*+1)}}{R_{t+(j-i^*+1)}}\right)y^r_{i+1,t+(i-i+1)}\\
&\quad + \sum_{i=I^R-1}^{I-1}\left(\prod_{j=i^*}^{i}\frac{s_{j+1,t+(j-i^*+1)}}{R_{t+(j-i^*+1)}}\right)p^r_{i+1,t+(i-i^*+1)} + R_t a_{i^*,t}.
\end{aligned} \tag{28}$$

It is useful to suppose, for the moment, that pension benefits are zero. Since individual optimization requires $\overline{c}_{i^*,t} = \overline{y}_{i^*,t}$ for each age i^* and time t, from (27) and (28), we obtain $c_{i^*,t}$ as

$$\begin{aligned}
c_{i^*,t} &= \frac{\overline{y}_{i^*,t}}{[(1+\tau^r_{c,t}) + \sum_{i=i^*}^{I-1}(1+\tau^r_{c,t+(i-i^*+1)})\prod_{j=i^*}^{i}\frac{s_{j+1,t+(j-i^*+1)}}{R_{t+(j-i^*+1)}}\, g^c_{j+1,t+(j-i^*+1)}]} = \frac{\overline{y}_{i^*,t}}{\mu_{i^*,t}}\\
&= \frac{1}{\mu_{i^*,t}}\left[(1-\tau^r_{w,t})w^r_t\varepsilon^r_{i^*,t}\bar{L}_{i^*t} - \left(\frac{1-\eta}{\eta}\right)(1+\tau^r_{c,t})c_{i^*,t} + \sum_{i=i^*}^{I^R-2}\left(\prod_{j=i^*}^{i}\frac{s_{j+1,t+(j-i^*+1)}}{R_{t+(j-i^*+1)}}\right)\right.\\
&\quad \times\left[(1-\tau^r_{w,t+(i-i^*+1)})w^r_{t+(i-i^*+1)}\varepsilon^r_{i+1,t+(i-i^*+1)}\bar{L}_{i+1,t+(i-i^*+1)} - \left(\frac{1-\eta}{\eta}\right)\right.\\
&\quad \left.\left.\times(1+\tau^r_{c,t+(i-i^*+1)})c^r_{i+1,t+(i-i^*+1)}\right] + R_t a_{i^*,t}\right],
\end{aligned}$$

where the last row uses Eq. (26). Let

$$\begin{aligned}\bar{Y}_{i^*,t} &= (1-\tau^r_{w,t})w^r_t\varepsilon^r_{i^*,t}\bar{L}_{i^*t}\\ &+ \sum_{i=i^*}^{I^R-2}\left(\prod_{j=i^*}^{i}\frac{s_{j+1,t+(j-i^*+1)}}{R_{t+(j-i^*+1)}}\right)(1-\tau^r_{w,t+(i-i^*+1)})w^r_{t+(i-i^*+1)}\varepsilon^r_{i+1,t+(i-i^*+1)}\bar{L}_{i+1,t+(i-i^*+1)}\\ &+ R_t a_{i^*,t}. \end{aligned}\tag{29}$$

Then, we have

$$\begin{aligned}c_{i^*,t} = \frac{\bar{Y}_{i^*,t}}{\mu_{i^*,t}} - \frac{1}{\mu_{i^*,t}}\left(\frac{1-\eta}{\eta}\right)&\left[(1+\tau^r_{c,t})c_{i^*,t} + \sum_{i=i^*}^{I^R-2}\left(\prod_{j=i^*}^{i}\frac{s_{j+1,t+(j-i^*+1)}}{R_{t+(j-i^*+1)}}\right)\right.\\ &\left.\times(1+\tau^r_{c,t+(i-i^*+1)})c^r_{i+1,t+(i-i^*+1)}\right].\end{aligned}$$

Using the Euler equation,

$$\begin{aligned}c_{i^*,t} = \frac{\bar{Y}_{i^*,t}}{\mu_{i^*,t}} - \left(\frac{1-\eta}{\eta}\right)\frac{c_{i^*,t}}{\mu_{i^*,t}}&\left[(1+\tau^r_{c,t}) + \sum_{i=i^*}^{I^R-2}\left(\prod_{j=i^*}^{i}\frac{s_{j+1,t+(j-i^*+1)}}{R_{t+(j-i^*+1)}}\right)\right.\\ &\left.\times(1+\tau^r_{c,t+(i-i^*+1)})\prod_{j=i^*}^{i}g^c_{j+1,t+(j-i^*+1)}\right].\end{aligned}$$

In summary, we have derived that

$$c_{i^*,t} = \frac{\bar{Y}_{i^*,t}}{X_{i^*,t}},\tag{30}$$

where $\bar{Y}_{i^*,t}$ is defined as in (29) and

$$\begin{aligned}X_{i^*,t} = \mu_{i^*,t} + \left(\frac{1-\eta}{\eta}\right)&\left[(1+\tau^r_{c,t}) + \sum_{i=i^*}^{I^R-2}\left(\prod_{j=i^*}^{i}\frac{s_{j+1,t+(j-i^*+1)}}{R_{t+(j-i^*+1)}}\right)\right.\\ &\left.\times(1+\tau^r_{c,t+(i-i^*+1)})\prod_{j=i^*}^{i}g^c_{j+1,t+(j-i^*+1)}\right].\end{aligned}$$

We now introduce social security benefits again. Consumption is defined as in (30), except that $\bar{Y}_{i^*,t}$ in (29) is given by

$$\begin{aligned}\bar{Y}_{i^*,t} &= (1-\tau^r_{w,t})w^r_t\varepsilon^r_{i^*,t}\bar{L}_{i^*t}\\ &+ \sum_{i=i^*}^{I^R-2}\left(\prod_{j=i^*}^{i}\frac{s_{j+1,t+(j-i^*+1)}}{R_{t+(j-i^*+1)}}\right)(1-\tau^r_{w,t+(i-i^*+1)})w^r_{t+(i-i^*+1)}\varepsilon^r_{i+1,t+(i-i^*+1)}\bar{L}_{i+1,t+(i-i^*+1)}\\ &+ \sum_{i=I^R-1}^{I-1}\left(\prod_{j=i^*}^{i}\frac{s_{j+1,t+(j-i^*+1)}}{R_{t+(j-i^*+1)}}\right)p^r_{i+1,t+(i-i^*+1)} + R_t a_{i^*,t}.\end{aligned}\tag{31}$$

Using the definition of benefits in (18), we arrive at

$$\begin{aligned}\bar{Y}_{i^*,t} &= (1-\tau^r_{w,t})w^r_t\varepsilon^r_{i^*,t}\bar{L}_{i^*t} \\ &+ \sum_{i=i^*}^{I^R-2}\left(\prod_{j=i^*}^{i}\frac{s_{j+1,t+(j-i^*+1)}}{R_{t+(j-i^*+1)}}\right)(1-\tau^r_{w,t+(i-i^*+1)})w^r_{t+(i-i^*+1)}\varepsilon^r_{i+1,t+(i-i^*+1)}\bar{L}_{i+1,t+(i-i^*+1)} \\ &+ \sum_{i=I^R-1}^{I-1}\left(\prod_{j=i^*}^{i}\frac{s_{j+1,t+(j-i^*+1)}}{R_{t+(j-i^*+1)}}\right)\frac{\kappa_{t+(i-i^*+1)}}{I^R-1}\left(\sum_{j=1}^{I^R-1}w_{t-i^*+j}\varepsilon_{j,t-i^*+j}\phi\bar{L}_{j,t-i^*+j}\right) \\ &+ R_t a_{i^*,t}, \end{aligned} \tag{32}$$

which is a function of parameters and prices taken as given by households, exactly like $X_{i^*,t}$. As a result, we have a recursive consumption function that we can use in the algorithm to compute the equilibrium. The algorithm mimics the one outlined in Appendix B.

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