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Projected U.S. Demographics and Social Security*

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Without policy reforms, the aging of the U.S. population is likely to increase the burden of the currently unfunded Social Security and Medicare systems. In this paper we build an applied general equilibrium model and incorporate the population projections made by the Social Security Administration (SSA) to evaluate the macroeconomic and welfare implications of alternative fiscal responses to the retirement of the baby-boomers. Our calculations suggest that it will be costly to maintain the benefits at the levels now promised because the increases in distortionary taxes required to finance those benefits will reduce private saving and labor supply. We also find that the "accounting calculations" made by the SSA underestimate the required fiscal adjustments. Finally, our results confirm that policies with similar long-run characteristics have very different transitional implications for the distribution of welfare across generations. *Journal of Economic Literature* Classification Numbers: D52, D58, E21, E62. © 1999 Academic Press

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1. INTRODUCTION

Even with two scheduled increases in the normal retirement age in 2008 and 2026, the Social Security Administration projects that the dependency ratio (the ratio of workers entitled to Social Security retirement benefits to those paying payroll taxes) will more than double between 1997 and 2050. Figure 1 shows four projected paths of the dependency ratio, corresponding to alternative eligibility rules: perpetuating the current 65 age qualification, adhering to the two legislated postponements to age 66 in 2008 and to age 67 in 2026, adding two additional postponements beyond those two, or with 11 postponements—eventually leaving the retirement eligibility age at 76 (Appendix B describes how we construct this graph). The demographic transition will require fiscal adjustments to finance our unfunded Social Security system, with one possibility being further increases in the normal retirement age. Although the demographic projections contained in Fig. 1 have inspired public discussion of Social Security reforms, they have rarely been used in general equilibrium computations designed to inform that discussion.

Besides the issue of financing our unfunded Social Security system, the aging population contributes to what, according to the President's Council

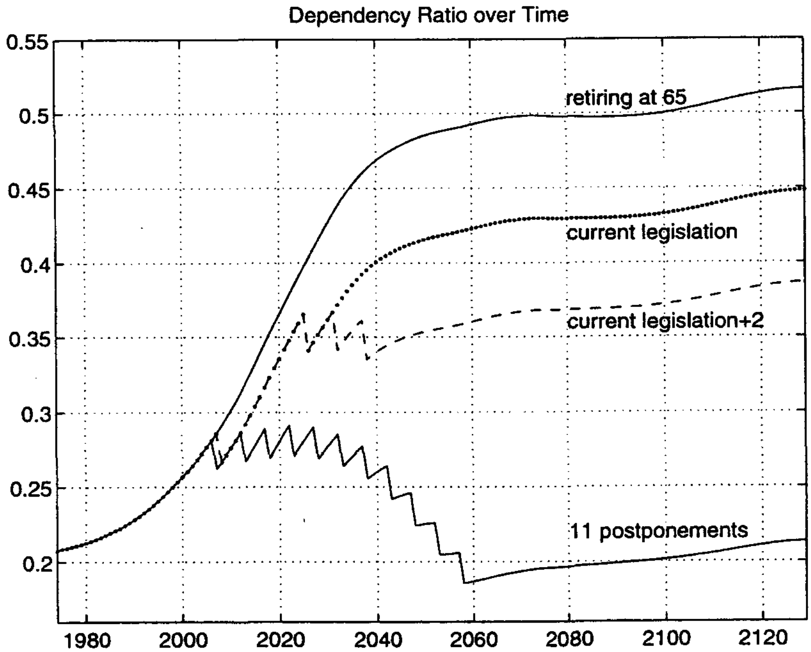

FIG. 1. Projected dependency ratios.

of Economic Advisors (1997), is an even larger cause for fiscal adjustment: it is projected that Medicare and Medicaid spending will increase from 2.7% and 1.2% of GDP in 1996 to 8.1% and 4.9% of GDP in 2050, respectively. This paper uses projected increases in the dependency ratio (associated with the current legislation) and Medicaid and Medicare to create a benchmark and then studies the economic consequences of eight alternative fiscal adjustment packages (see Fig. 2). These packages either (1) throw all of the fiscal burden onto the labor income tax rate; (2) raise a consumption tax rate; (3), (4), and (5) reduce benefits in various ways while also adjusting taxes; (6) and (7) increase the linkage of benefits to cumulative earnings while also adjusting either the labor income or the consumption tax rate; or (8) implement privatization by gradually phasing out benefits while adjusting the labor income tax rate. Except for (8), the experiments abstain from privatization and leave the Social Security system unfunded.

In the tradition of Auerbach and Kotlikoff (1987), we use a general equilibrium model of overlapping generations of long-lived people. As in İmrohoroğlu *et al.* (1995), our agents face uncertain lifetimes and endowments. Huang *et al.* (1997) extended the İmrohoroğlu *et al.* framework

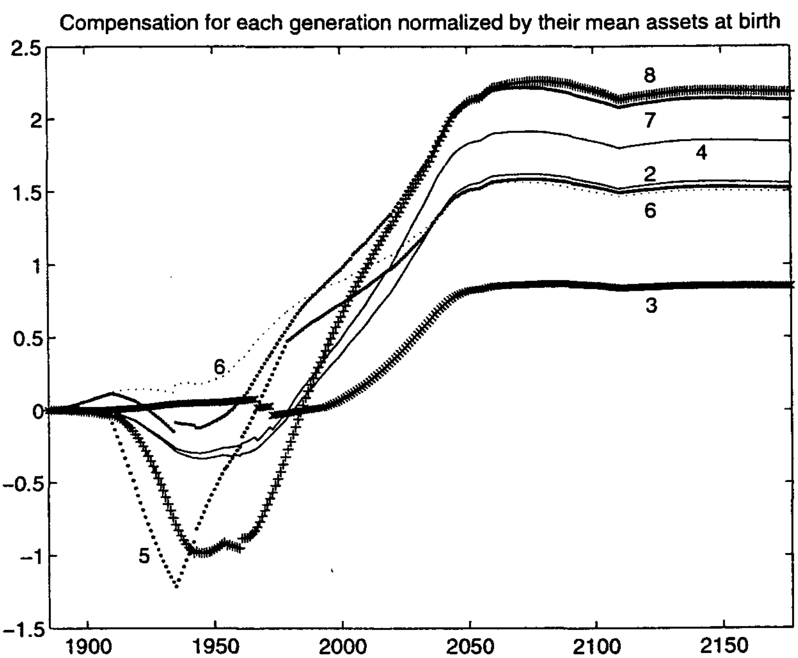

FIG. 2. Compensation in terms of fraction of assets to be given to a person born in year t living under experiment 1 to make him or her indifferent between experiment j and experiment 1.

to handle the aggregate time-variation occurring during transitions across steady states. We in turn extend Huang *et al.* (1997) work in four ways: (1) we modify the technology to incorporate labor-augmenting technical progress; (2) we assume time-varying survival probabilities and demographic patterns; (3) we change the household's preferences by activating a life-long bequest motive; (4) we let labor supply choices respond to how retirement benefits are related to past earnings. Innovation (1) introduces the growth rate as a key parameter affecting the efficiency of an unfunded retirement arrangement. Innovation (2) lets us study transitions induced by demographic changes. Innovation (3) allows us to boost savings above what would be produced by pure life-cycle households and thereby helps us calibrate the model to realistic capital–output ratios and age–savings profiles. Innovation (4) not only allows labor supply to respond to policy and price changes but also incorporates Auerbach and Kotlikoff's (1992a, 1992b) and Kotlikoff's (1997) stress on earnings relatedness as a key parameter governing the distortions generated by the Social Security retirement system.

Our main findings are these:

- In the face of projected demographics, it will be costly to maintain benefits at levels now promised. Large increases in distorting taxes will arrest capital accumulation and labor supply. Our work indicates that back-of-the-envelope accounting calculations made outside a general equilibrium model are prone to be overly optimistic. The Social Security Administration states that a 2.2 percentage point addition to the 12.4% OASDI payroll tax will restore the financial balance in the Social Security trust fund over the 75-year horizon, given intermediate projections of demographics and other key variables. According to Goss (1998), Deputy Chief Actuary of the Social Security Administration, a 4.7% immediate increase of the existing OASDI payroll tax is necessary to finance the existing Social Security System in perpetuity. Injecting the same projections of demographics into our calibrated general equilibrium model gives results that diverge from that official assessment. We compute an additional 17.1 percentage points in the payroll tax rate and large welfare losses associated with maintaining our current unfunded system. The projected increase in Medicaid and Medicare payments adds a further 12.7 percentage points to the required tax on labor income and increases distortions even more.

- Reducing retirement benefits through taxation of benefits and consumption or through postponing the retirement eligibility age results in a significant reduction of the fiscal adjustment required to cope with the aging of the population.

- Policies with similar long-run outcomes can have vastly different transient intergenerational distributional implications. With one exception, all our experiments impose welfare losses on transitional generations. Policies that partially reduce retirement benefits (by taxing benefits, postponing retirement, or taxing consumption) or gradually phase them out without compensation yield welfare gains for future generations but make most of the current generations worse off. The only experiment that raises the welfare of all current and future cohorts switches from the current system to a defined contribution system. Evidently, eliminating the distortion associated with the Social Security payroll tax by linking benefits to contributions is very important, confirming arguments by Kotlikoff (1999).

A sustainable Social Security reform seems to require reduced distortions in labor/leisure and consumption/saving choices and some transition policies that compensate current generations.

Besides the papers we mention above, many others have studied the U.S. Social Security system. Among those, the following seem closest to our work. Kotlikoff *et al.* (1997) use a general equilibrium, long-lived overlapping generations model to study the consequences of various ways of privatizing the U.S. Social Security system. They focus on both intergenerational and intragenerational heterogeneity (the individuals belong to different exogenous earnings-ability classes) and devote particular attention to matching current U.S. fiscal institutions. They incorporate deductions, exemptions, and progressive benefits schedules. Altig *et al.* (1999) use the model of Kotlikoff *et al.* (1996) to study the consequences of different tax reforms. Their model does not incorporate uncertainty. They assume constant population growth.

Cooley and Soares (1996) study the design and implementation of a pay-as-you-go social insurance system as a problem in political economy. They are particularly interested in the sustainability of such a system in a world with stochastic population growth. They consider a model with four-period-lived agents, no life-span uncertainty, and exogenous labor supply. They calibrate their population shares up to 1995. They do not study the substantial aging of the population after that date.

2. THE MODEL

The model economy consists of overlapping generations of individuals who live no longer than $T+1$ years, and an infinitely lived government. During the first t_R+1 periods of life, a consumer supplies labor in exchange for wages that she allocates among consumption, taxes, and asset accumulation. During the final $T-t_R$ periods of life, the consumer receives Social Security benefits. In addition to life-span risk, agents face different income shocks that they cannot insure. They can smooth consumption by accumulating two risk-free assets: physical capital and government bonds. The government taxes consumption and income from capital and labor, issues and services debt, purchases goods, and pays retirement benefits. There is a constant returns to scale Cobb–Douglas aggregate production function, constant labor-augmenting technical progress, and no aggregate uncertainty. Equilibrium factor prices are time varying but deterministic.

Cast of Characters

For easy reference, we summarize our notation in Table I. For any variable z, a subscript t denotes age, and an argument s in parentheses denotes calendar time. There is an exogenous gross rate $\rho > 1$ of labor-augmenting technical progress. We let $\epsilon_t(s) = \epsilon_t \rho^s$ be an exogenous time-dependent age–efficiency index. The number of people of age t at time s is $N_t(s)$; the total population alive at time s is $N(s) = \sum_{t=0}^{T} N_t(s)$; $k_t(s-1)\rho^{s-1}$ is physical capital held by an age-t person at the end of time $s-1$; $K(s-1)\rho^{s-1} = \sum_{t=0}^{T} k_t(s-1)\rho^{s-1} N_t(s-1)$ is total physical capital at the end of period $s-1$. Where δ is the physical rate of depreciation of capital, we let $R(s-1) = 1 + r(s-1) - \delta$ be the rate of return on asset holding; $r(s-1)$ is the gross-of-depreciation rate of return on physical capital from time $s-1$

TABLE I
Cast of Characters

	Demography		Government
$N_t(s)$	Population, age t time s	$g(s)\rho^s$	Gov't purchases
$\alpha_t(s)$	One-period survival probability	$\tau_a(s)$	Tax rate on assets
$\lambda_t(s)$	Survival prob to age t	$\tau_l(s)$	Labor tax rate
$f_t(s)$	Fraction of age-t people	$\tau_b(s)$	Tax rate on bequests
$T+1, t_R+1$	Max life span, work life span		
	Household		Production and Information
$c_t(s)\rho^s$	Consumption at age t	$w(s)$	Wage
$\ell_t(s)$	Labor at age t	$\epsilon_t(s)$	Age–efficiency index
$a_t(s)\rho^s$	Asset holdings at age t	$r(s)$	Rate of return
$b_t(s)\rho^s$	Bonds at age t	$K(s)\rho^s$	Aggregate physical capital
$k_t(s)\rho^s$	Physical capital at t	$L(s)$	Aggregate labor
$e_t(s)\rho^s$	Cumulative labor earnings	z_t	Information
$S_t(s)\rho^s$	Retirement benefits	d_t	Endowment shock
$\Upsilon_t(s)\rho^s$	Total tax payments	x_t	Household state vector
$\text{Beq}(s)\rho^s$	Aggregate bequests		

to time s; $g(s)\rho^s$ is per capita government purchases of goods at time s; $w(s)$ is a base wage rate at time s; $c_t(s)\rho^s$ and $\ell_t(s)$ are consumption and labor supply at time s for someone of age t; $\Upsilon_t(s)\rho^s$ denotes total tax payments, $S_t(s)\rho^s$ social security payments, and $e_t(s)\rho^s$ the cumulative labor earnings of a household of age t at time s. We let $a_{t-1}(s-1)\rho^{s-1}$ be a consumer's asset holdings at the beginning of age t at time s; d_t is a random component of a household's endowment, described by $d_t = U_{d,t}z_t$, where z_t is an exogenous first-order vector stochastic process used to model the flow of information, and $U_{d,t}$ is an age-dependent selection vector.

Factor Prices

We assume a constant returns Cobb–Douglas aggregate production function with labor and capital arguments $\rho^s \sum_{t=0}^{t_R} \epsilon_t \ell_t(s) N_t(s)$ and $K(s-1)\rho^{s-1}$, respectively. From the firm's problem in a competitive equilibrium, the rentals $r(s-1)$ and $w(s)$ are determined from marginal productivity conditions:

$$r(s-1) = r\left(\frac{K(s-1)}{\rho L(s)}\right) = \tilde{\alpha} A\left(\frac{K(s-1)}{\rho L(s)}\right)^{\tilde{\alpha}-1}$$

$$w(s) = w\left(\frac{K(s-1)}{\rho L(s)}\right) = (1-\tilde{\alpha}) A\left(\frac{K(s-1)}{\rho L(s)}\right)^{\tilde{\alpha}}.$$

The presence of ρ in the denominator is due to our timing convention. $L(s) = \sum_{t=0}^{t_R} \epsilon_t \ell_t(s) N_t(s)$. The wage of an age-$t$ worker at time s is $\epsilon_t \rho^s w(s)$. $\tilde{\alpha} \in (0,1)$ is the income share of capital and A is total factor productivity.

Economywide Physical Resource Constraint

Using the firm's first-order conditions and constant returns to scale, we can write the economywide physical resource constraint at time s as

$$g(s)N(s) + \sum_{t=0}^{T} c_t(s)N_t(s) + K(s) = \frac{R(s-1)}{\rho}K(s-1) + w(s)\sum_{t=0}^{t_R} \epsilon_t \ell_t(s) N_t(s).$$

Demographics

At date s, a cohort of workers of measure $N_0(s)$ arrives. The luckiest live during $s, s+1, \dots, s+T+1$, but many die before age $T+1$. As a cohort ages, mortality is described by $\alpha_t(s)$, the conditional probability of surviving from age t to age $t+1$ at time s. Let $N_t(s)$ be the number of age-t people alive at time s. It moves according to

$$N_{t+1}(s+1) = \alpha_t(s) N_t(s). \tag{1}$$

Iterating on 1 gives

$$N_t(s) = \alpha_{t-1}(s-1)\alpha_{t-2}(s-2)\cdots\alpha_0(s-t)N_0(s-t). \tag{2}$$

The probability that a person born at $s-t$ survives to age t is given by

$$\lambda_t(s) \equiv \prod_{h=1}^{t} \alpha_{t-h}(s-h). \tag{3}$$

We assume a path $n(s)$ of the rate of growth of new workers, so that $N_0(s) = n(s)N_0(s-1)$, which implies $N_0(s) = \prod_{h=1}^{s} n(h)N_0(0)$. Let $\nu(s) = \prod_{h=1}^{s} n(h)$. Then the fraction $f_t(s)$ of age-t people at time s is

$$f_t(s) = \frac{\lambda_t(s)\nu(s-t)}{\sum_{t=0}^{T} \lambda_t(s)\nu(s-t)}. \tag{4}$$

The total population alive at time s is

$$N(s) = \sum_{t=0}^{T} N_t(s).$$

We take the paths $n(s)$ and $\alpha_t(s)$ for $s = 1970, \ldots, 2060+3T$ as parameters.

The people that enter the model at $t=0$ are 21-year-old workers. New retirees are 65 years old and agents can live up to age 90.

Households

We assume the one-period utility function for an age-t person

$$u(c_t(s), \ell_t(s)) = -\tfrac{1}{2}\left[(c_t(s)-\gamma)^2 + \left(\pi_2 \ell_t(s)\right)^2\right],$$

where π_2 and γ are preference parameters. Conditional on being alive, the household discounts future utilities by a constant β.

We adopt "warm glow" altruism, which was first introduced by Andreoni (1989, 1990). It asserts that the agent derives utility from leaving a bequest, independent of the prospective consumption stream of the beneficiary. We adopt this formulation mainly for computational manageability. In our setup, agents are long-lived and face a large state space. We compute long transitions. Considering a model in which one agent's utility depends on the other agent's state variables would substantially increase the computational burden. However, Andreoni (1989, 1990) argues that there is empirical evidence against "pure altruism models" that make consumption by parent and heir independent of the distribution of income between them (Barro, 1974). Becker (1974) suggests that "warm-glow" preferences may arise because perhaps people have a taste for giving: they receive status or acclaim or simply experience utility from having done their bit.

We use this device not only to get more capital accumulation than in a pure life-cycle model (see Jones and Manuelli (1992) for a discussion of the difficulties in matching capital accumulation in a pure life-cycle model) but also to reconcile our model with Kotlikoff and Summers's (1981) computations, according to which intergenerational transfers account for 70–130% of the current value of U.S. capital stock. The fact that we do not allow for inter vivos transfers in our model is not a restrictive assumption: since we do not have borrowing constraints, the timing of bequests or inter vivos transfers is not relevant.

Let the *state* of an age-t person at the start of time s be denoted $x_t(s) = [a_{t-1}(s-1), e_{t-1}(s-1), z_t']'$. We formulate preferences recursively. We impute to an age-$t-1$ person a particular type of bequest motive via a "terminal value" function $V_t(x_t(s)|\text{ dead at } t) = V_{T+1}(x_t(s)) = x_t(s)'P_{T+1}x_t(s)$, where P_{T+1} is a negative semidefinite matrix with parameters that determine the bequest motive (Appendix A describes the value functions more explicitly). Our formulation gradually activates the bequest motive, intensifying it with age as the mortality table makes the household think more about the hereafter. For $t = 0, \ldots, T$, let $V_t(x_t(s))$ be the optimal value function for an age t person. The household's Bellman equations are

$$\begin{aligned} &V_t(a_{t-1}(s-1), e_{t-1}(s-1), z_t) \\ &= \max_{\{c_t(s),\, \ell_t(s),\, a_t(s)\}} \Big\{u\big(c_t(s),\, \ell_t(s)\big) + \beta\alpha_t(s)E\big[V_{t+1}\big(a_t(s),\, e_t(s),\, z_{t+1}\big) \mid J_t(s)\big] \\ &\qquad + \beta\big(1-\alpha_t(s)\big)E\big[V_{T+1}\big(a_t(s),\, e_t(s),\, z_{t+1}\big) \mid J_t(s)\big]\Big\}, \end{aligned} \tag{5}$$

where $J_t(s)$ is the information set of an age-t agent at time s and the maximization is subject to the constraints

$$c_t(s) + a_t(s) = \frac{R(s-1)}{\rho}a_{t-1}(s-1) + w(s)\epsilon_t\ell_t(s) + S_t(s) - \Upsilon_t(s) + d_t \tag{6}$$

$$\begin{aligned} \Upsilon_t(s) &= \tau_1(s)\big[w(s)\epsilon_t\ell_t(s) + d_t\big] \\ &\quad + \tau_a(s)\left[\frac{R(s-1)}{\rho} - 1\right]a_{t-1}(s-1) + \tau_c(s)c_t(s) \end{aligned} \tag{7}$$

$$e_t(s) = \begin{cases} e_{t-1}(s-1) + w(s)\epsilon_t \ell_t(s) & \text{for } t \le t_R + 1 \\ \rho^{-1} e_{t-1}(s-1) & \text{for } t > t_R + 1 \end{cases} \tag{8}$$

$$S_t(s) = \begin{cases} 0 & \text{for } t \le t_R + 1 \\ \text{fixben}_t(s) + \text{rrate}_t(s) \cdot \rho^{-1} \cdot e_{t-1}(s-1) & \text{for } t > t_R + 1 \end{cases} \tag{9}$$

$$z_{t+1} = A_{22} z_t + C_2 \tilde{w}_{t+1} \tag{10}$$

$$\begin{bmatrix} d_t \\ \gamma_t \end{bmatrix} = \begin{bmatrix} U_{d,t} \\ U_\gamma \end{bmatrix} z_t. \tag{11}$$

The right side of 6 is the household's after-tax income, the sum of wages, earnings on assets, a possibly serially correlated idiosyncratic mean-zero endowment shock d_t, and retirement benefits (if any), minus tax payments. Equation 7 decomposes total tax payments Υ_t into taxes on labor income, assets, and consumption. Equation 8 updates $e_t(s)$, the cumulated, wage-indexed, labor earnings of the household that, depending on the parameter rrate in 9, affects the household's eventual entitlement to retirement benefits. The worker's past contributions are indexed to wage productivity growth; the pension he or she receives during retirement is not—as in the U.S. Social Security system.

Formula 9 tells how retirement benefits are related to past earnings. Part of Social Security payments ($\text{fixben}_t(s)$) is independent of past earnings, and part ($\text{rrate}_t(s) \cdot e_{t-1}(s)$) responds to past earnings.

We compute fixben as follows. For people living within a steady state,

$$\text{fixben}_t(s) = \rho^{t_R+1-t} \cdot \text{fixrate} \cdot \text{AV}(s), \tag{12}$$

where AV records the average earnings of a worker who has survived to retirement age:

$$\text{AV}(s) = \frac{1}{t_R + 1} \sum_{t=0}^{t_R} \epsilon_t \ell_t(s) w(s). \tag{13}$$

To mimic current U.S. benefits, Eq. 12 computes average earnings to account for changes in the average wages since the year the earnings were received; but once a worker retires, her pension is no longer indexed to productivity growth.

For people living during the transition, we make fixben a linear combination of the contribution in the initial steady state and that in the final steady state. This simplifies the computations.

Bequests are distributed only to newborn workers: each agent born at time s begins life with assets $\rho^{s-1} a_{-1}(s-1)$, which we set equal to a per capita share of total bequests from people who died at the end of period

$s-1$. This distribution scheme implies that within a steady state, per capita initial assets equal per capita bequests adjusted for population and productivity growth. However, during either policy or demographic transitions between steady states, this distribution scheme implies that what a generation receives in bequests no longer equals what it leaves behind.

In 10, $\tilde{w}_{t+1}$ is a martingale difference process, adapted to the history of z_τ's up to age t, driving the information flow z_t, and U_γ, U_{dt} are selector vectors determining the preference shock process γ_t and the endowment shock process d_t. In the experiments reported in this paper, we set the preference shock to a constant but specify d_t to be a random process with mean zero: $d_t = \psi d_{t-1} + \tilde{w}_{1t}$, with $\psi = 0.8$. The martingale difference sequence $\tilde{w}_{t+1}$ is adapted to $J_t = (\tilde{w}_0^t, x_0)$, with $E(\tilde{w}_{t+1} \mid J_t) = 0$, $E(\tilde{w}_{t+1}\tilde{w}'_{t+1} \mid J_t) = I$.

Aggregates and Distributions across People

In addition to life-span risk, individuals face different sequences of random labor income shocks, which they cannot insure. People smooth consumption across time and labor income states only by accumulating two risk-free assets—physical capital and government bonds; they use these, together with Social Security retirement benefits, to provide for old-age consumption. Let $\mathcal{D}_t(s) = [c_t(s)\ \ell_t(s)\ a_t(s)]'$ be the vector of decisions made by an age-t worker at time s. Our specification makes $\mathcal{D}_t(s)$ a linear time-and-age-dependent function of $x_t(s)$,

$$\mathcal{D}_t(s) = L_t(s)x_t(s),$$

and makes the state vector follow the linear law of motion $x_{t+1}(s+1) = A_t(s)x_t(s) + C_t(s)w_{t+1}$. Our model imposes restrictions on the matrices $L_t(s)$, $A_t(s)$, and $C_t(s)$. Individuals have rational expectations and make $L_t(s)$, $A_t(s)$ and $C_t(s)$ depend on the sequence of prices and government fiscal policies over their potential life-span $s, s+1, \ldots, s+T+1$.

We can compute probability distributions across workers for the state and decision vectors. Let $\mu_t(s) = Ex_t(s)$, $\Sigma_t(s) = E(x_t(s) - \mu_t(s))(x_t(s) - \mu_t(s))'$. Given a mean and covariance for the state vector of the new workers $(\mu_0(s), \Sigma_0(s))$, the moments follow the laws of motion $\mu_{t+1}(s+1) = A_t(s)\mu_t(s)$ and $\Sigma_{t+1}(s+1) = A_t(s)\Sigma_t(s)A_t(s)' + C_t(s)C_t(s)'$.

Aggregate quantities of interest such as aggregate per capita consumption and aggregate per capita physical capital can be easily computed by obtaining weighted averages of features of the distributions of quantities across individuals alive at a point in time. Aggregate quantities are deterministic functions of time because all randomness averages out across a large number of individuals. Only these aggregate quantities appear in the government budget constraint and the model's market-clearing conditions.

The Government

An age-t person divides his or her time s asset holdings $a_t(s)$ between government bonds and private capital: $a_t(s) = b_t(s) + k_t(s)$, where $b_t(s)$ is government debt. The government's budget constraint at s is

$$\begin{aligned} g(s)N(s) &+ \sum_{t=t_R+1}^{T} S_t(s)N_t(s) + \frac{R(s-1)}{\rho}\sum_{t=0}^{T} b_t(s-1)N_t(s) \\ &= \tau_b \frac{R(s-1)}{\rho}\mathrm{Beq}(s) + \sum_{t=0}^{T} b_t(s)N_t(s) \\ &\quad + \sum_{t=0}^{T} N_t(s)\Bigg\{\tau_a(s)\left[\frac{R(s-1)}{\rho} - 1\right]a_{t-1}(s-1) \\ &\qquad + \tau_l(s)w(s)\epsilon_t \ell_t(s) + \tau_c(s)c_t(s)\Bigg\}, \end{aligned} \tag{14}$$

where

$$\mathrm{Beq}(s) = \sum_{t=0}^{T}(1-\alpha_t(s))a_t(s-1)N_t(s-1) \tag{15}$$

and

$$a_{-1}(s-1) = \frac{\mathrm{Beq}(s)\cdot(1-\tau_b)}{N_0(s)}. \tag{16}$$

The amount $a_{-1}(s-1)$ of assets is inherited by each new worker at time s. We assume that in administering the bequest tax, the government acquires capital and government bonds in the same proportions in which they are held in the aggregate portfolio. Consistent with this specification, the per-new-worker inheritance $a_{-1}(s-1)$ is divided between physical capital and government bonds as follows:

$$\begin{aligned} k_{-1}(s-1) &= \frac{\sum_{t=0}^{T}(1-\alpha_t(s))k_t(s-1)N_t(s-1)}{N_0(s)} \\ b_{-1}(s-1) &= \frac{\sum_{t=0}^{T}(1-\alpha_t(s))b_t(s-1)N_t(s-1)}{N_0(s)}. \end{aligned} \tag{17}$$

3. THE ALGORITHM

We first compute the initial steady state. We use backward induction to compute an agent's value functions and policy functions, taking as given government policy, bequests, and prices. We then iterate until convergence on

(i) the Social Security benefits, to match the desired replacement rate;

(ii) bequests, so that planned bequests coincide with received ones;

(iii) the labor income or consumption tax to satisfy the government budget constraint;

(iv) factor prices, to match the firms' first-order conditions.

To compute the final steady state we use the same procedure described for the initial steady state, and we also iterate on the government debt-level-to match the debt-to-GDP ratio we have in the initial steady state. In the initial steady state the debt-to-GDP ratio was calibrated; in the second steady state we fix it.

Last, we compute the transition dynamics by solving backward the sequence of value functions and policy functions, taking as given the time-varying transition policies, prices, and bequests. We iterate until convergence on

(i) a parameterized path for the tax rate to match the final debt-to-GDP ratio;

(ii) factor prices.

Prices are allowed to adjust for a phase-out period after the changes in the demographics and policies have ended. Though the model economy would converge to a new steady state only asymptotically (because prices are endogenous) we "truncate" this process and impose convergence in $2T$ periods.

4. CALIBRATED TRANSITION DEMOGRAPHICS

We calibrate and compute an initial steady state, associated with constant pre-1975 values of the demographic parameters α_t, n. We then take time-varying $\alpha_t(s)$, $n(s)$ parameters from 1975 to 2060, so that

$$\alpha_t(s) = \begin{cases} \alpha_t^0 & \text{if } s \leq 1974; \\ \hat{\alpha}_t(s) & \text{if } 1975 \leq s \leq 2060; \\ \alpha_t^1 & \text{if } s > 2060, \end{cases}$$

where $\alpha_t^0 = \alpha_t(1970)$ from the mortality table and $\alpha_t^1 = \alpha_t(2060 + t)$, the SSA numbers for the cohort to be born in 2060; the $\hat{\alpha}_t(s)$ are taken from Bell *et al.* (1992). We calibrate the growth rate $n(s)$ to match the SSA's forecasts of the dependency ratio. According to the SSA, the dependency ratio

TABLE II
Parameters

		Production	
$A = 2$	$\bar{\alpha} = 0.33$	$\{\epsilon_t\}$ Hansen (1993)	$\delta = 0.055$
$\rho = 1.016$			
		Household	
$\gamma = 11$	$\pi_2 = -1.7$	$\beta = 0.994$	$\mathrm{JG} = 0.032$
$\mathrm{JB} = 60$	P_T see text	d_t see text	
		Demography	
$n(s)$ assumed path	$\alpha_t(s)$ from life tables	$T = 69$	$t_R = 43$

was about 18% in 1974 and will increase to about 50% in 2060. The population of new workers continues to grow at its initial steady-state value of 1.3% until 1984. After 1984, we gradually diminish $n(s)$ to 0.8% per year so that the dependency ratio becomes roughly 50% in 2060 and then stabilizes. Our calculations begin by assuming that prior to 1975, the economy was in a steady state and that people behaved as if they expected their survival probabilities to be those experienced by people alive in 1970; but in 1975, people suddenly realized that the survival probability tables were changing over time and switched to using the "correct" ones. After the conditional survival probabilities attain a steady state in 2060, the demographic structure changes for another $T+1$ years, until it reaches a new steady state in $2060 + (T+1)$. The departure of the demographic parameters $\alpha_t(s)$, $n(s)$ from their values at the initial steady state requires fiscal adjustments.

Initial Steady State

All of our experiments start from a common initial steady state. We set $T = 69$, $t_R = 43$ (Table II). Since our age-0 people work immediately, we think of them as 21-year-olds, of new retirees as 65-year-olds, and of age-$(T+1)$ workers as 90-year-olds. We calibrated the parameters A, γ_t, β, π_2, JG, JB so that in this initial steady state the capital-to-GDP ratio is 3.0, the government-purchases-to-GDP ratio is 0.21, the debt-to-GDP ratio is 0.46, and the mean age–consumption profile resembles the observed data. For the initial steady state, we set the tax rate on income from capital at 30%, the tax rate on bequests at 10%, and the tax rate on consumption at 5.5%. Given government purchases, steady-state debt, and these tax rates, the steady state equilibrium tax rate on labor income turns out to be 29.7%. In the initial steady state, the interest rate is 5.9% and the marginal productivity of labor (w) is 3.2. Each new worker receives a bequest worth about 52% of the average per capita capital in the economy. Throughout the paper, the rate of technical progress is kept constant at its

TABLE III
Eight Experiments

Experiment	Benefits	Tax Adjustment
1	Benchmark:	
	Postpone in 2008, 2026	Gradually raise $\tau_l(s)$
2	Benchmark	Gradually raise $\tau_c(s)$
3	Also postpone in 2032, 2038	Gradually raise $\tau_l(s)$
4	Also postpone in 2032, 2038	Gradually raise $\tau_c(s)$
5	Tax benefits	Gradually raise $\tau_l(s)$
6	Link benefits to earnings	Gradually raise $\tau_l(s)$
7	Link benefits to earnings	Gradually raise $\tau_c(s)$
8	Gradual privatization	Gradually raise $\tau_l(s)$

initial steady-state level of $\rho = 1.016$. In Appendix C we perform some sensitivity analysis on JG to gauge the sensitivity of our results to the bequest motive.

Alternative Fiscal Responses

We computed eight equilibrium transition paths associated with alternative government responses to the demographic parameters $\alpha_t(s)$, $n(s)$, $s = 1975, \ldots, 2060$. In addition to the change in these demographic variables, there are two changes that are common to all eight computations. First, to reflect projected increases in Medicare and Medicaid, we gradually increase government purchases so that they eventually become 25% higher than their initial steady-state level. Second, current legislation on the postponement of the retirement age is implemented, raising the mandatory retirement age by one year in 2008 and by another year in 2026 for the cohort that qualifies for retirement then.

We name the computations 1, 2, 3, 4, 5, 6, 7, and 8. For easy reference, they are summarized in Table III. In computations 1 and 2, Social Security benefits are kept at their levels in the first steady state (i.e., the benefit rate parameters fixben are left intact); the entire burden of adjusting to the demographic changes is absorbed by scheduled increases in the tax on labor income alone (in experiment 1) or in the tax on consumption alone (in computation 2). Computations 3 and 4 impose reductions on benefits in the form of announced increases of $t_R + 2$, the mandatory retirement age, by two additional years, one in 2032 and the other one in 2036, to eventually raise it to 69. The remaining burden of adjustment is absorbed by scheduled increases in the labor income tax rate (experiment 3) or the consumption tax rate (experiment 4). Computation 5 also imposes a reduction in benefits, not by increasing retirement age, but by exposing all Social Security retirement benefits fully to the labor income tax rate τ_l; it schedules increases in the labor income tax rate to complete the fiscal adjustments. Computations 6 and 7 schedule adjustments in the formula for benefits, fully linking them to past earnings for people retiring in year 2000 or later. Thus, while in the first five experiments and experiment 8 rrate $= 0$ and fixrate $= 0.6$, in experiments 6 and 7, rrate $= \frac{0.6}{t_R+1}$ and fixrate $= 0$ for people retiring in year 2000 and after. In experiment 6, the labor income tax is raised to pick up the residual tax burden, while in experiment 7, the consumption tax is increased. Finally, computation 8 is an uncompensated phase-out of the current system, in which benefits are phased out to zero over a 50-year horizon, starting in the year 2000.

Government Tax Policy during Transitions

In steps, the government increases one tax rate (either τ_l or τ_c) during a transition, leaving all other tax rates constant. These tax changes are scheduled and announced as follows. In 1975 the government announces that starting in year 2000, it will increase the tax on labor income (in experiments 1, 3, 5, 6, and 8) or on consumption (in experiments 2, 4, and 7) every 10 years in order to reach the terminal steady state with the desired debt to GDP ratio. Starting in 2060, that tax rate is held constant at its new steady-state level, but the wage rate and interest rate continue to vary for another $2(T+1)$ periods, after which time we fix them forever. We then enter a new phase of $T+1$ periods, during which the wage rate and interest rate are pegged at their terminal steady-state values. As cohorts born during the transition period die, new ones are born into the terminal steady state.

5. NUMERICAL RESULTS

Table VI compares outcomes across steady states for the eight experiments. Comparing the steady states we see only the positive aspects of taxing or reducing pensions and increasing savings (the savings and capital increase is also linked to the increased life span). When we do welfare comparisons, it will become apparent how distinct policies affect members of different generations in the transition.

Table IV refers to variables that are normalized by the exogenous productivity growth. Therefore, in column 1 for example, GDP −17.4% mean that in the final steady state for experiment 1, GDP is 17.4% lower than it would have been, should the economy have grown at the constant, exogenous productivity growth rate. This is the convention that we have in mind when discussing the results.

TABLE IV
Alterations of Steady States in Eight Experiments

Experiment	1	2	3	4	5	6	7	8
Variable								
τ_l	29.7%↑59.5%	29.7%	↑52.9%	29.7%	↑42.8%	↑51.3%	29.7%	↓26.0%
τ_c	5.5%	↑36.9%	5.5%	↑31.2%	5.5%	5.5%	↑30.5%	5.5%
Interest rate	5.9%↓5.0%	↓4.1%	↓4.9%	↓4.2%	↓4.2%	↓4.9%	↓4.2%	↓3.0%
Wage	+4.2%	+8.8%	+4.8%	+8.5%	+8.2%	+4.6%	+8.5%	+15.2%
GDP	−17.4%	−4.6%	−10.7%	−2.25%	−3.4%	−2.4%	+3.7%	+8.7%
Mean asset holdings	−12.5%	+9.2%	−4.4%	+11.4%	+9.4%	+4.0%	+18.1%	+38.0%
Mean capital	−11.7%	+11.3%	−3.4%	+13.5%	+11.3%	+5.0%	+20.3%	+42.5%
Mean consumption	−28.4%	−13.4%	−19.1%	−9.8%	−11.1%	−7.0%	−1.4%	−0.2%
Mean eff. labor	−20.8%	−12.4%	−14.8%	−10.0%	−10.6%	−6.6%	−4.4%	−5.6%
Bequests	+35%	+105%	+57%	+108%	+64%	+100%	+137%	+63%
K/GDP	3.0↑3.2	↑3.5	↑3.2	↑3.4	↑3.4	↑3.2	↑3.4	↑3.9
G/GDP	20.6%↑31.2%	↑27.1%	↑28.9%	↑26.4%	↑26.7%	↑26.4%	↑24.9%	↑23.7%

We can summarize our main results as follows:

• When the government uses the labor income tax rate to finance the fiscal burden (experiment 1), the tax rate goes from 29.7% to eventually 59.5%, the labor supply falls by 20.8%, the capital stock decreases by 11.7%, and output falls by 17.4%. The decline in the aggregate labor input owes much to the projected demographics and the increased distortionary taxation of labor income. Experiment 6 involves a similar computation except that now retirement benefits are linked to past earnings, which removes a distortion in the leisure/labor choice as far as Social Security contributions are concerned. However, these contributions deliver a rate of return equal to the growth rate of output in the economy, which is less than the return on private capital. Overall, the results from experiment 6 are far better than those in experiment 1. τ_l rises to 51.3%, the labor input falls by only 6.6%, the capital stock rises by 5.0%, and the GDP decreases by only 2.4%.

• When the government uses the consumption tax to finance the fiscal burden created by the retirement of the baby-boom generation (experiment 2), the consumption tax rate rises from 5.5% in the initial steady state to 36.9% in the final steady state. Aggregate labor input falls by 12.4%, capital rises by 11.3% and GDP falls by 4.6%. Experiment 7 links the retirement benefits to the agent's past average earnings (as in experiment 6) and uses the consumption tax increase to finance government expenses. With the linkage of benefits and contributions and the use of the consumption tax, the labor supply distortion is the smallest among all experiments. The tax rate on consumption rises to 30.5%; GDP and mean capital rise 3.7% and 20.3%, respectively; and consumption decreases by 1.4%. Aggregate labor input falls by 10.0%.

• Taxing benefits at the labor income tax rate and using a higher labor income tax rate to finance the residual burden (experiment 5) delivers results that are similar to those of the second experiment in many respects, since taxing benefits is like taxing the consumption of the old. For example, aggregate labor input falls by 10.6%, capital rises by 11.3%, and GDP falls by 3.4%.

• Postponing the retirement age by two additional years, to age 69, and then using either τ_l (experiment 3) or τ_c (experiment 4) to finance the remaining burden significantly reduces the size of the fiscal burden and therefore the size of the additional tax required to finance it. When the labor income tax is used, it rises to 52.9% in the final steady state compared to 59.5% in experiment 1. Aggregate labor input falls only by 14.8% (20.8% in experiment 1), capital stock falls by 3.4%, and GDP falls by 10.7%. When the consumption tax is used, it rises to 31.2% compared with 36.9% in experiment 2; labor supply falls by 4.4%, capital stock rises by 13.5%, and GDP decreases by 2.25%.

• When we compare experiments 3–5 to experiment 1, it should be noted that the key difference in the former is the reduction in Social Security benefits through using the consumption tax instead of the labor income tax, postponing the retirement age, or taxing Social Security benefits. All three alternative fiscal policies yield a higher work effort, higher consumption, and larger saving and capital, relative to the experiment 1 policy of using the labor income tax to finance the fiscal burden. As a result, the economy achieves a softer landing to a final steady state after the demographic transition.

• The gradual (and uncompensated) phase-out of the unfunded Social Security system (experiment 8) delivers a final steady state in which consumers can invest only in capital or government debt to provide for retirement. This yields a substantial increase of 38% of mean asset holding and a rise of 42.5% for capital. As a result the interest rate decreases from 5.9% to 3.0%. The labor supply falls by 5.6%, wages increase by 15.2%, and the labor income tax rate falls to 26%. Consumption decreases by 0.2% and GDP rises by 8.73%.

6. STEADY-STATE PROFILES

The discipline of using an applied general equilibrium model manifests itself by generating several forces that act on individuals' choices over the life cycle and along the transition to a final steady state. There are income and intertemporal substitution effects from changes in the real interest rate; there are incentive effects stemming from changes in tax rates, reductions in benefits, and retirement age postponements; and there are demographic changes prompting individuals to save more (for both precautionary and life-cycle reasons) as they face increased life expectancy. In our discussion of steady-state profiles below, we will highlight those factors that we think are most responsible for the outcomes.

Age–Labor Supply Profiles

Figures 3 and 4 show the age–labor supply profile in our experiments (the graphs depict both the cross-section and the life-cycle profiles in the steady state: labor efficiency increases exogenously as a result of the technological progress, but labor supply does not). The profile labeled "0" belongs to the initial steady state. Labor supply rises with age, peaks around age 40, then falls and drops to zero at the mandatory retirement age of 65. Experiments 1, 2, 5, 6, 7, and 8 have the same mandatory retirement age of 67. Experiments 3 and 4 postpone retirement to age 69. Apart from this dif-

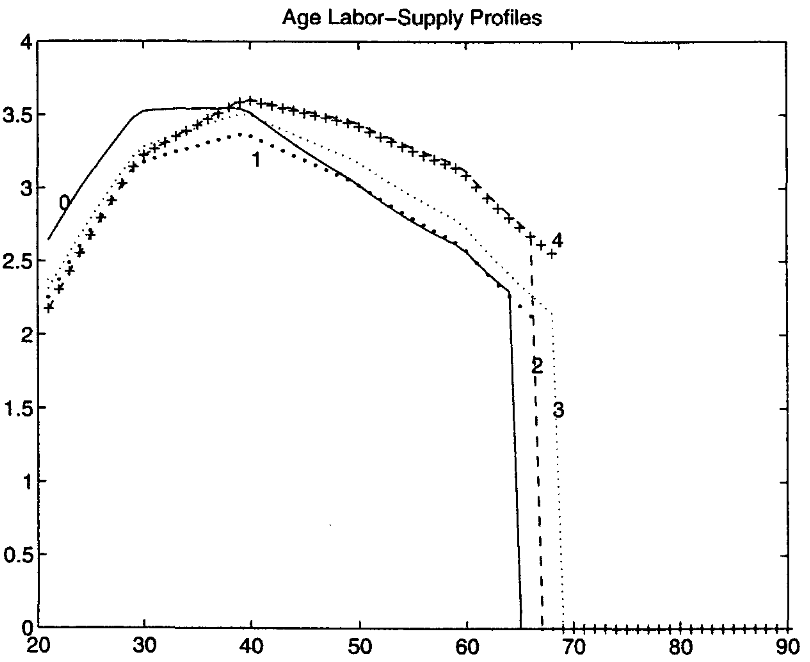

FIG. 3. Age–labor supply profiles in steady states; 0 denotes the initial steady state.

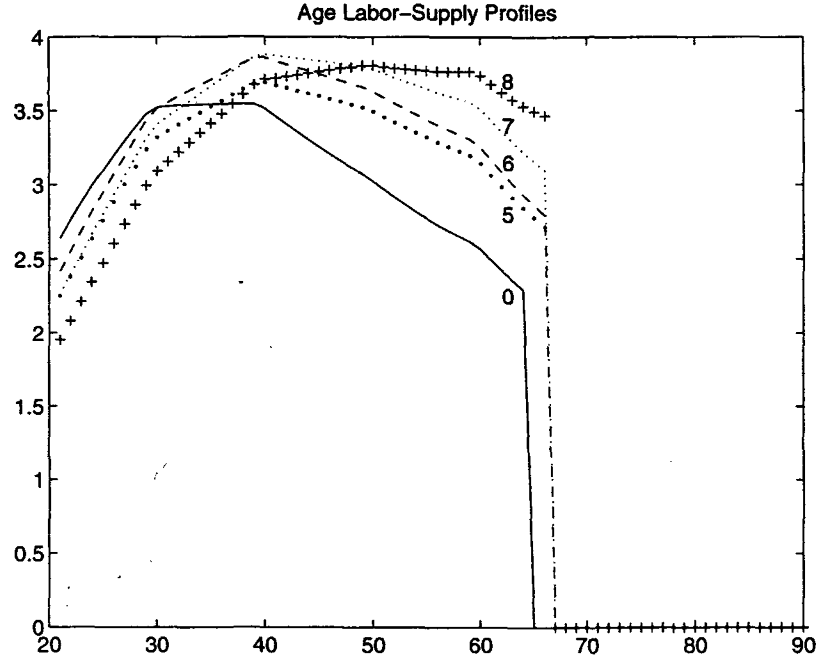

FIG. 4. Age–labor supply profiles in alternative steady states; 0 denotes the initial steady state.

ference, all experiments tilt the age–labor supply profile in the final steady state in the counterclockwise direction. Although there are several forces at work, two in particular seem to be responsible for the reallocation of work effort over the life cycle. First, the real interest rate in all of the final steady states is lower than that in the initial steady state. Second, the postponement of retirement by at least two years provides an incentive to postpone work effort since efficiency in these "later" years is still higher than efficiency in the "very young years."

For the initial steady state and for each worker age, we calculated compensated and uncompensated one-period labor supply elasticities. We calculated them by taking an average age-t agent at the stationary equilibrium price and tax rates and raising his time t wage by 1%, leaving wages for all other periods fixed. For the compensated elasticity, we deducted from the worker's income a lump sum transfer equal to the efficiency index of age-t workers times the wage change. The uncompensated elasticities ranged from 0.97 to 1, depending on age. The compensated elasticities are mostly around 1.7 except near the end of career, when they fall to 0.69.

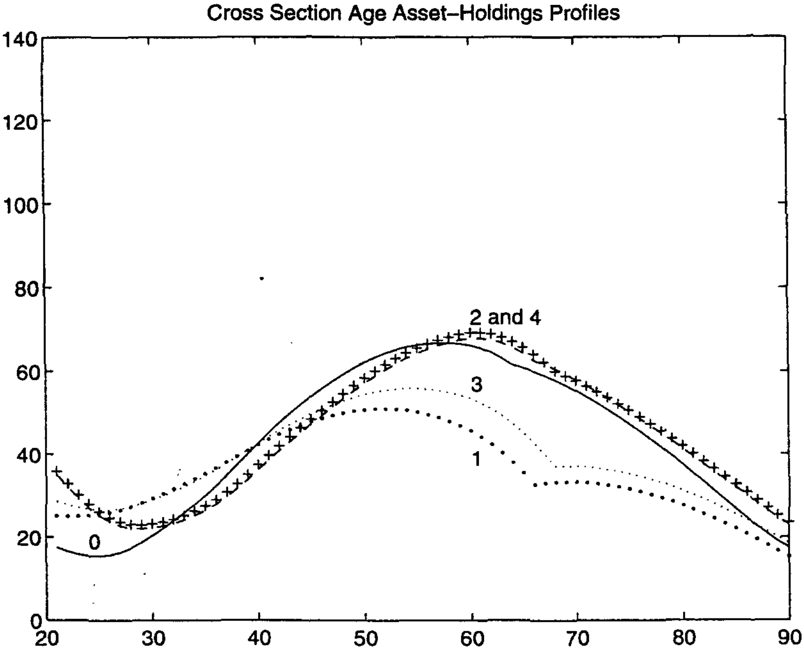

FIG. 5. Cross-section age–asset holding profiles in alternative steady states.

Age–Wealth Profiles

Figures 5 and 6 display the cross-section age asset-holding profiles: individuals in all the final steady states inherit higher wealth and decumulate faster early on in the life cycle. In some of the experiments $(2, 4, 5, 7)$, they accumulate wealth for a longer period of time or decumulate slower later in the life cycle $(1, 3, 6)$ and leave larger bequests. This behavior is consistent with a lower real interest rate in the final steady states combined with an increase in the incentive to save brought on by the increase in life expectancy and/or a reduction in benefits.

In the final steady state of experiment 8, the government no longer provides Social Security payments (nor collects taxes to finance them) and consumers lose the life-span insurance provided by pensions, which are paid as long as they live. Private saving becomes the only source of consumption during retirement. In this world, consumers between ages 50 and 67 save much more than in the other experiments and capital accumulation is much larger. After retirement, they run down their assets much faster to consume. Since there are no annuities markets in the model, asset accumulation also serves the purpose of self-insurance against life-span risk. Should the consumer live long enough, he or she will run down most of his

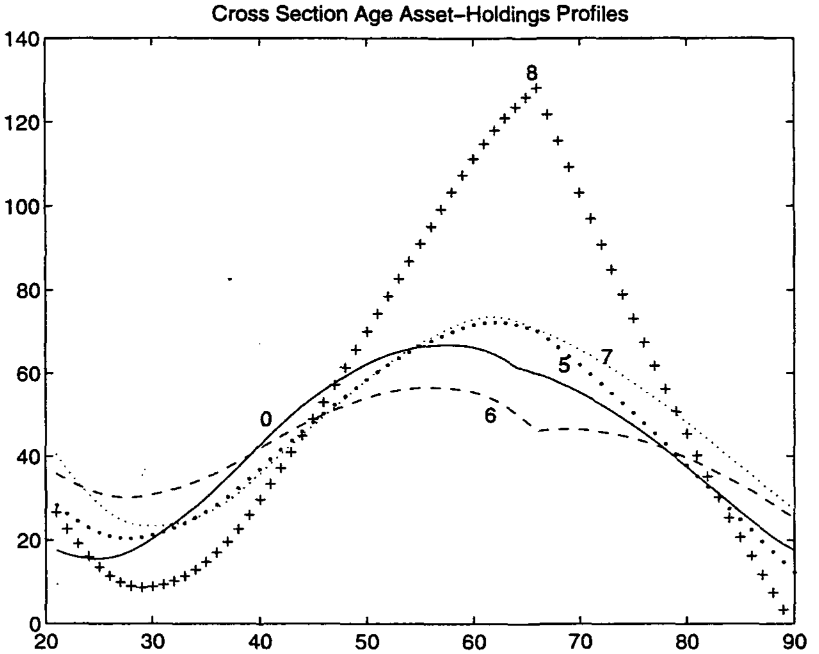

FIG. 6. Cross-section age–asset holding profiles in steady states.

or her assets and leave almost no bequest: heirs will share the "longevity risk."

Age–Consumption Profiles

Figures 7 and 8 plot the cross-section consumption profile. In the initial steady state consumption declines rather steeply for people past retirement age. This happens because older people retired in periods during which the technological progress was lower and Social Security benefits are not productivity-indexed after retirement. Therefore retired older consumers tend to be poorer than retired younger ones and can consume less.

The cross-section age–consumption profiles in the final steady states of experiments 1–7 are flatter than that in the initial steady state. The decline in the real interest rate and the enhanced desire to save due to the aging of the population are powerful forces in shaping these profiles. In experiment 8 an even lower interest rate, the necessity of financing retirement consumption out of accumulated assets, and the wealth effect we discussed above combine with an increased life expectancy to produce an even sharper decline for cross-sectional consumption past retirement age.

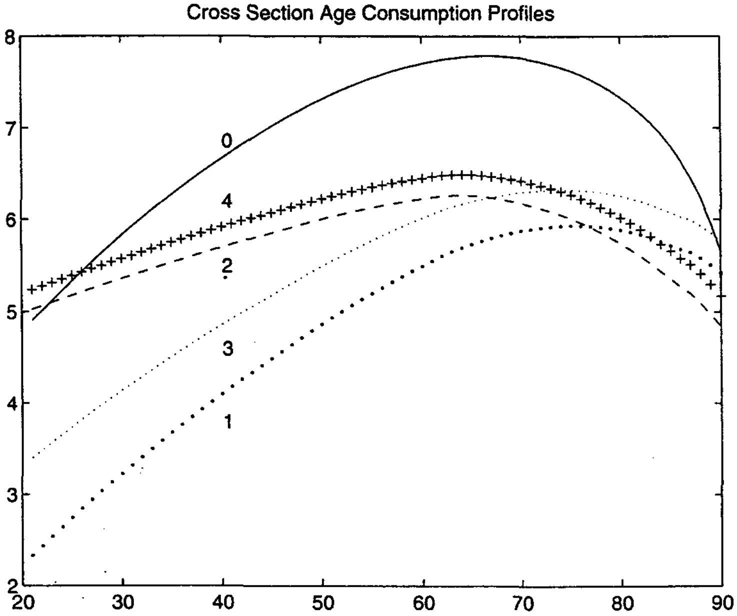

FIG. 7. Cross-section age–consumption profiles in steady states.

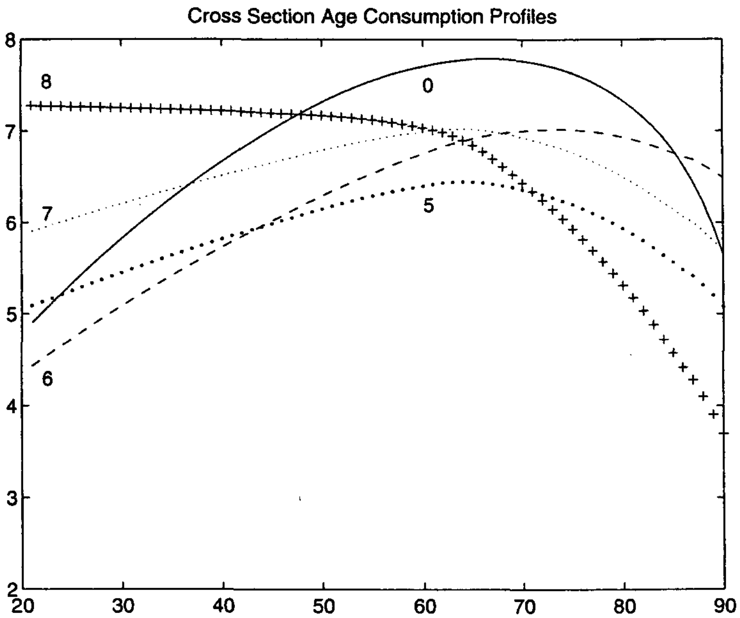

FIG. 8. Cross-section age–consumption profiles in steady states.

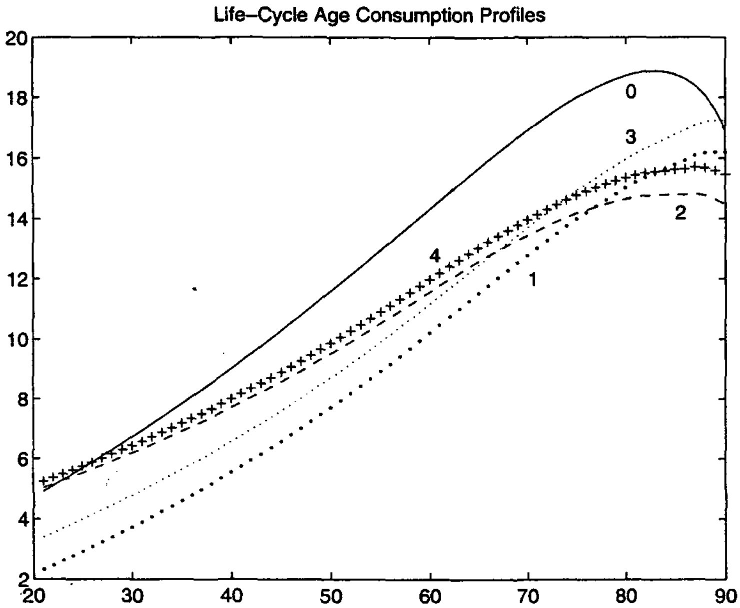

FIG. 9. Life-cycle age–consumption profiles in steady states.

Figures 9 and 10 depict the life–cycle consumption profile: consumption over time from the point of view of an individual born in the initial or final steady state, with exogenous technological progress increasing the worker's productivity.

7. TRANSITION PATHS

Figures 11 and 12 show the time path of labor income and consumption tax rates. As described in the previous section, the government is required to announce and raise the appropriate tax rate in five steps, each lasting 10 years, and keep these rates unchanged at the new steady-state levels. Note that in experiment 8, where Social Security is gradually phased out, the labor income tax rate needs eventually to fall for the government to maintain the target debt-to-GDP ratio. Figures 13 and 14 show the time path of interest rates.

The structure of preferences and our calibration combine to produce consumers who do not mind substituting intertemporally consumption and leisure: in Figures 15–18 we can see how average consumption and average labor supply are not smooth over time.

Figures 17 and 18 show the time path of aggregate labor along the transition in our experiments. For example, in experiment 1, the five sharp drops

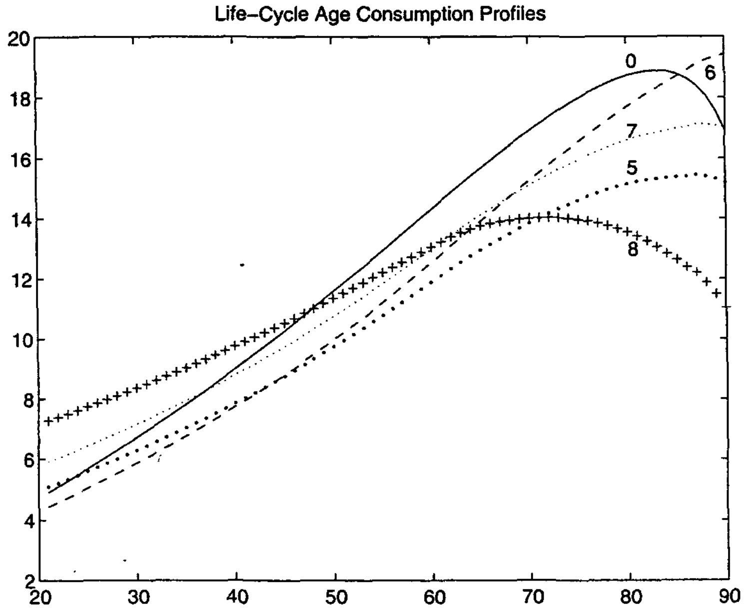

FIG. 10. Life-cycle age-consumption profiles in steady states.

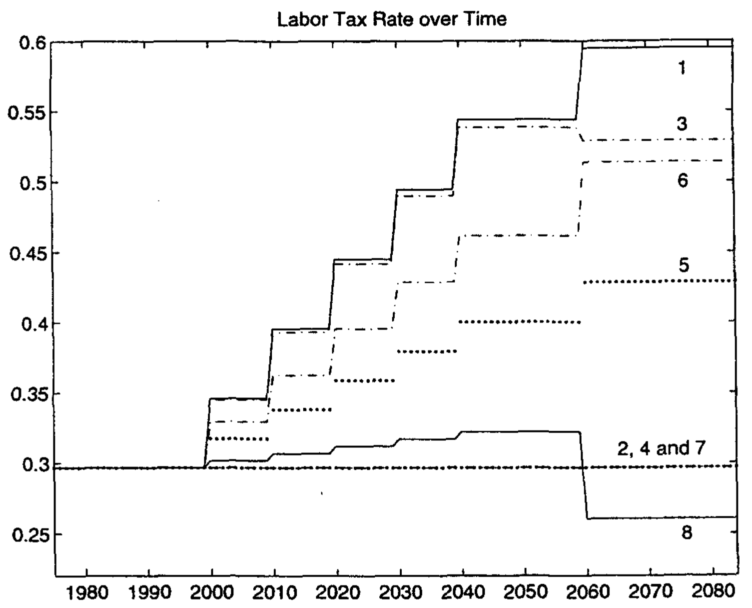

FIG. 11. Labor tax rate τ_t during transitions.

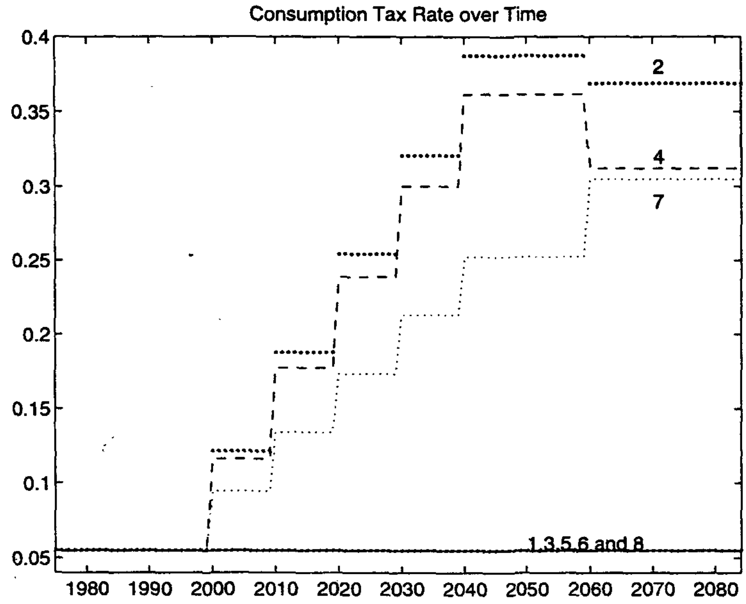

FIG. 12. Consumption tax rate τ_c during transitions.

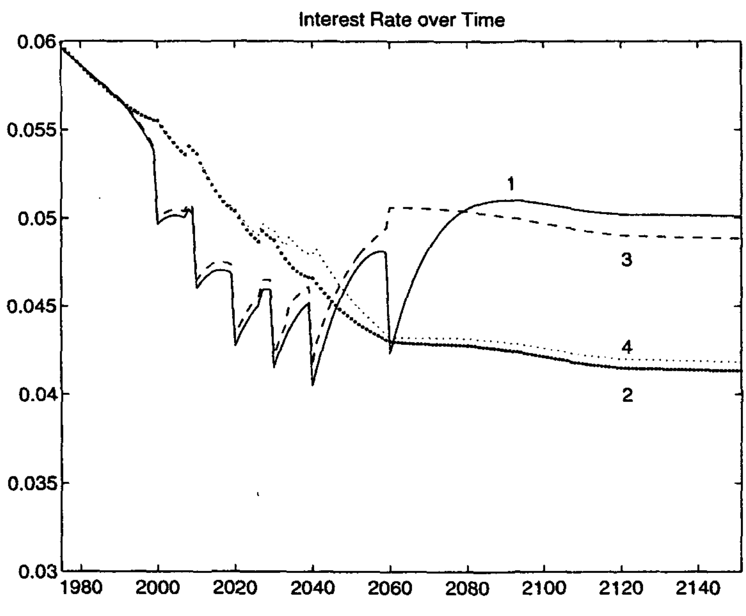

FIG. 13. Interest rate during transitions.

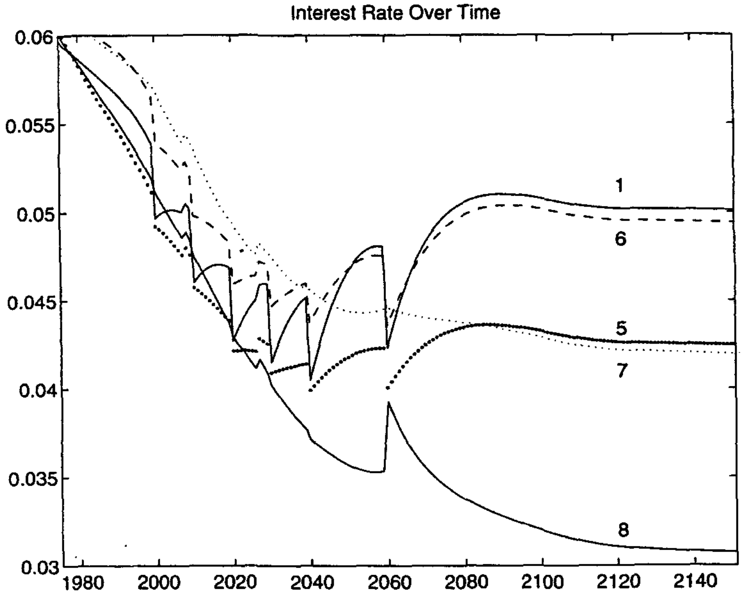

FIG. 14. Interest rate during transitions.

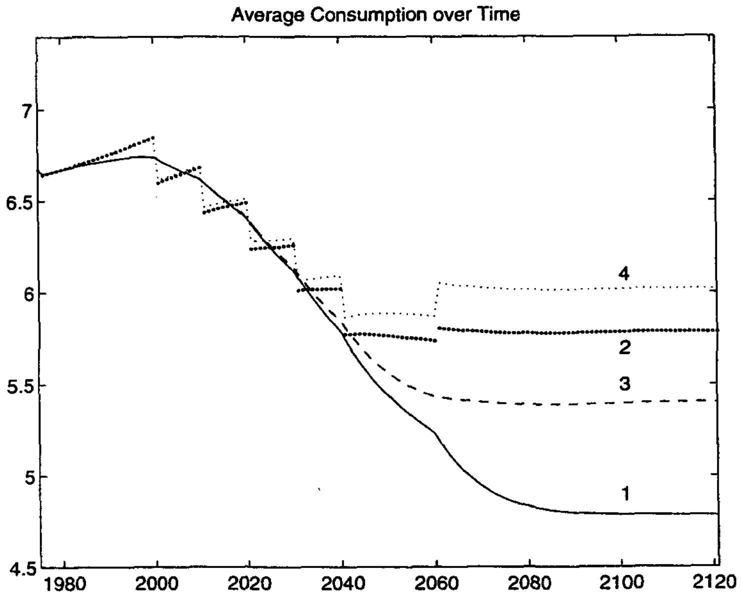

FIG. 15. Average consumption during transitions.

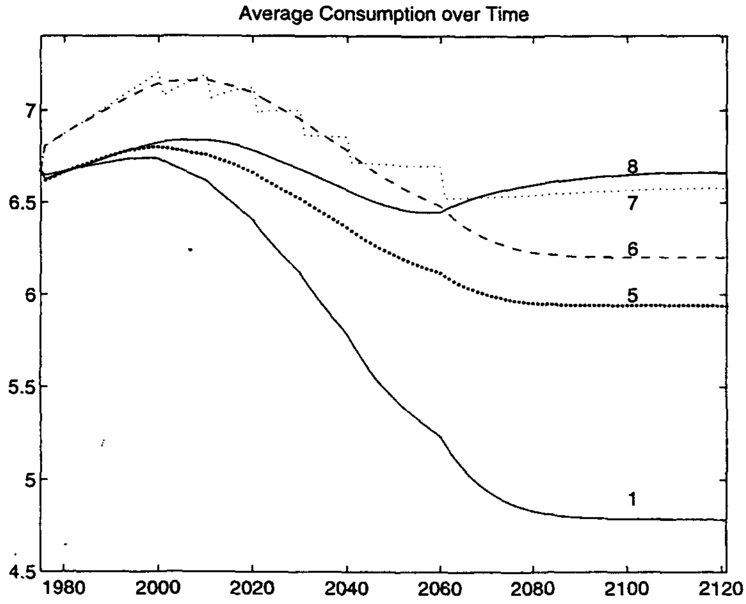

FIG. 16. Average consumption during transitions.

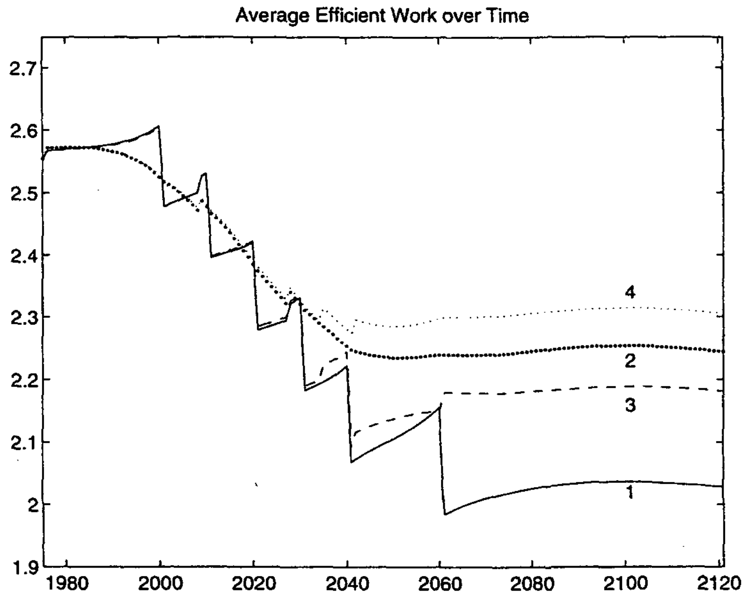

FIG. 17. Average labor supply during transition, in efficiency units.

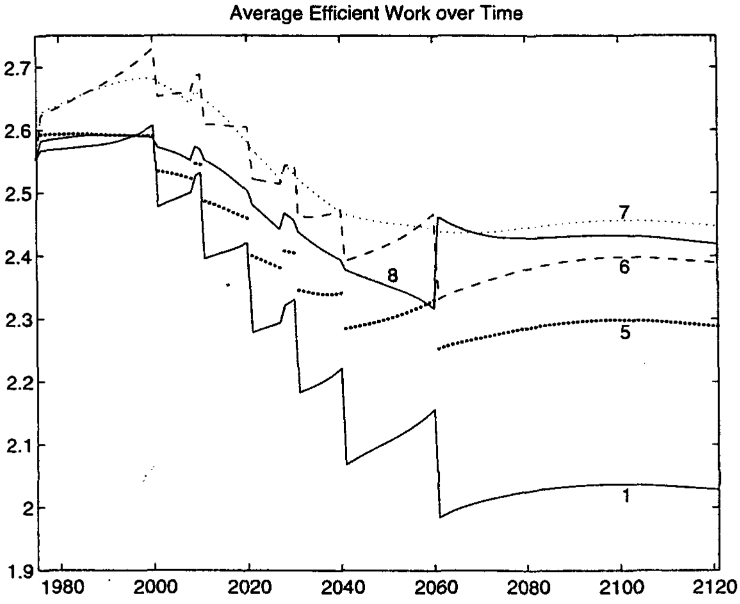

FIG. 18. Average labor supply during transition, in efficiency units.

in aggregate labor coincide with the implementation of the announced increases in the labor income tax rate. The two spikes that are smaller in size correspond to the scheduled increases in the mandatory retirement age (years 2008 and 2026 for all experiments, plus years 2032 and 2038 for experiments 3 and 4 only). Aggregate labor input declines much less under experiments 6 and 7 because labor income taxation is now less distortionary because of the linkage between benefits and contributions.

Figures 19 and 20 show the time path of aggregate capital. In experiment 8, where Social Security is gradually phased out, the capital stock rises near-monotonically to a much larger value than in all of the other experiments.

8. WELFARE IMPLICATIONS

In this section we report our findings on the intergenerational redistribution of welfare. Figure 2 uses the value function of people in experiment 1 as a base from which to evaluate the other seven experiments. It measures one-time awards of assets to those people already working or retired in 1975 (the date when the transition from the initial demographics begins) and to those new workers arriving after 1975. The awards are designed to make people as well off under the policy parameters of ex-

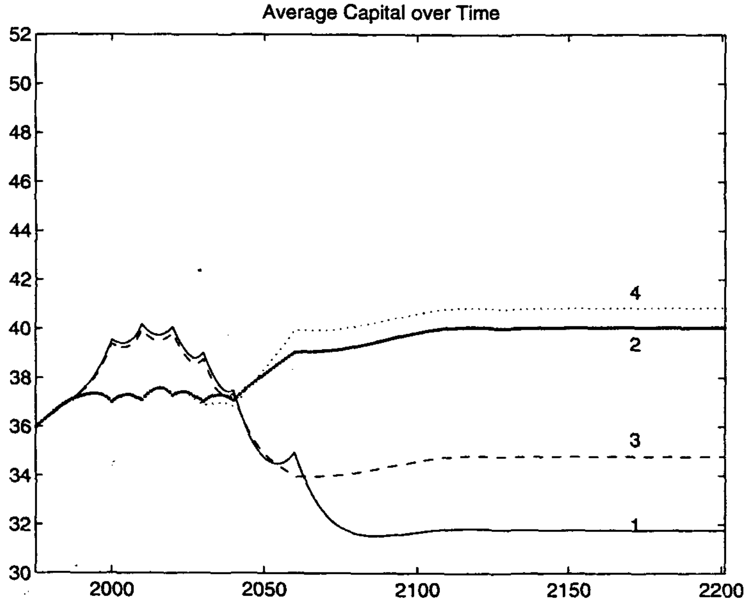

FIG. 19. Average capital holdings during transitions.

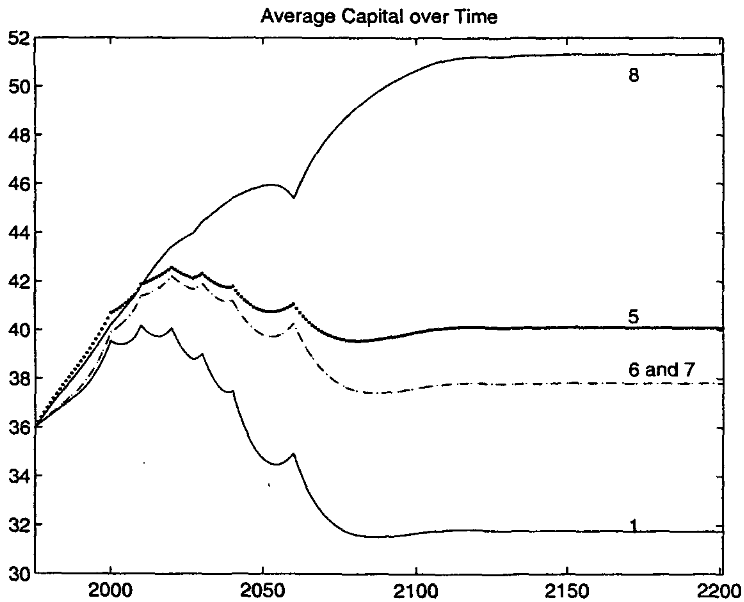

FIG. 20. Average capital holdings during transitions.

periment 1 (with compensation) as they would be under the parameters of experiment j (without the compensation). The awards are made as follows. To people already working or retired in 1975, we use the appropriate age-indexed value function of a person born in the year indicated, evaluate it at the mean assets of a surviving person of the relevant age, and express the award of assets as a ratio to the mean assets owed by people of that age at birth. For people entering the work force after 1975, we use the value function of a new entrant and express the award as a ratio of the assets inherited by a new entrant at that date. Thus, a positive number indicates that a positive award would be needed to compensate a person of the indicated birth date living in experiment 1 to leave him/her as well off as in experiment j. The figure reveals the different interests served by the different transition measures. For example, consider an average member of the cohort born in 1940. This individual would rather give up some wealth and stay under the experiment 1 fiscal policy of rising labor income taxation than accept the experiment 5 policy of taxing benefits.

Essentially all future generations are better off under experiments 2–8 than in experiment 1. In fact, when we compute an overall welfare measure by properly taking into account the welfare gains and losses of all generations, weighing them by their (time-varying) population shares, and discounting the future gains and losses by the after-tax real interest rate, all of the experiments deliver a welfare gain. Experiment 2 produces a welfare improvement of 54% of GDP (at the initial steady state) relative to experiment 1. Experiments 3, 4, and 5 yield overall welfare gains of 49%, 84%, and 56%, respectively. Experiments 6, 7, and 8 produce overall welfare gains of 197%, 189%, and 10.9% of GDP, respectively, relative to experiment 1.

Despite the fact that different fiscal policies have similar long-run and overall welfare consequences, existing generations fare quite differently under these policies. The only fiscal policy that benefits existing generations in addition to future generations is the policy of switching from the current defined benefit system to a defined contribution system and using a higher labor income tax rate to finance the residual fiscal burden (experiment 6). When a link is established between what an agent contributes to the system and what the agent eventually receives as benefits, much of the labor income tax no longer distorts labor supply decisions. Evidently, this particular reform of the (still) unfunded system goes a long way to produce economic benefits even for the generations that are currently alive. The experiment 3 policy of postponing retirement for two additional years and using the labor income tax to finance the remaining burden seems to benefit almost all of the existing generations; only the youngest generations, those that are 21 years old between 1970 and 1980, appear to experience small welfare costs under this policy. In general, the use of a higher consumption tax hurts existing generations, as experiments 2, 4, and 7 indicate. The magnitude of the welfare cost for the existing generations also depends on other components of the fiscal package. For example, use of a higher consumption tax and introduction of a linkage of benefits to contributions yield a smaller welfare cost for the existing generations compared to those produced by experiments 2 and 4. The largest welfare costs on the existing generations are generated under experiments 5 and 8. Experiment 5 makes retirement benefits taxable and uses a higher labor income tax rate to finance the residual fiscal burden. This policy simultaneously worsens the labor supply distortion and imposes a large cost on the retirees. A gradual and uncompensated privatization of the Social Security system makes all existing generations worse off relative to maintaining the unfunded system and relying on a higher labor income tax rate to provide for larger aggregate benefits.

These findings point to the importance of compensation schemes that will cushion the transition to a funded system and underline the significance of the distortionary taxation inherent in a defined contribution system.

Comparing a labor income tax versus a consumption tax (experiment 1 vs 2 and 3 vs 4), we see that the consumption tax significantly reduces distortions. This is partly due to the well-known public finance result that switching to a consumption tax is equivalent to taxing the initial capital. In our setup, this also derives from the fact that a consumption tax is also a tax on Social Security benefits, which are lump sum, and hence acts as a lump-sum tax on retirees. The consumption tax also has important redistributional aspects because a labor tax hits only the workers while the consumption tax hits both workers and retired agents.

As we have seen in comparing the steady states of experiments 1 vs. 2, the drop in GDP in experiment 2 is much less pronounced and savings, capital, and consumption are much higher. This is linked to several forces: the lump, sum component in the consumption tax, a smaller labor/leisure distortion (the labor tax rate is lower in experiment 2), and the fact that since the consumption tax reduces pensions, people save more for retirement. The resulting effect is that the interest rate decreases and real wage increases. Figure 2 shows that people born between 1920 and 1980 are those who lose in experiment 2: they are hit by the consumption tax while not benefiting from the reduction in the burden of pensions. As time passes, the second effect becomes stronger than the first one. In particular, baby boomers (born in 1947–1960) lose a lot in experiment 2: they retire when the consumption tax starts to hit. They worked and paid to finance the Social Security system and, when they retire, they pay additional taxes on consumption.

Postponing the retirement age also reduces distortions, allowing people to work an additional two periods. Comparing experiment 1 with experiment 3 we see that while labor increases because people work longer, savings increase even more because agents have to work for two more years in a region where their efficiency is quite low, and this could be a negative shock to their income in that period. The effect on savings is much smaller than the one we get with a consumption tax. Postponing the mandatory retirement age seems to leave most generations unhurt or better off relative to experiment 1. As benefits are reduced there is less taxation and this offsets the welfare loss associated with having to work two extra years until retirement. This explains why experiment 4 dominates experiment 2 and experiment 3 nearly dominates experiment 1. Experiments 2 and 4 roughly generate the same winners and losers.

Taxing Social Security benefits at the same rate as labor income (experiment 5) is a way of reducing benefits and making the retirees share the burden of an increased labor tax with the workers. Excluding privatization, experiment 5 is the policy that redistributes more across generations: it hits old people alive in year 2000 (older than baby boomers) hard, but asymptotically it is similar to experiment 2, where the consumption tax is raised. Again, this finding highlights the similarities in the economic incentives generated by reducing benefits through retirement age postponement, taxing benefits at the labor income tax rate, and taxing consumption.

Privatization through a gradual, uncompensated phase-out is the most welfare-enhancing policy in the long run. However, the transitional cohorts stand to suffer a great deal in the absence of any intertemporal redistribution of benefits and losses. This policy especially hurts the younger baby boomers and the children of the older baby boomers. These transitional generations not only see their benefits phased out but share in the burden of financing the retirement of a succession of larger-than-before cohorts.

One policy which is unambiguously beneficial to all generations and one with a sizable welfare gain for the future generations is a switch from the current system to a defined contribution system, namely experiment 6. Figure 2 reveals that even the transitional generations are quite better off under experiment 6 relative to going along the transition path under experiment 1. Evidently the reduction of the distortion in the labor income tax is economically quite important.

In the computation of cumulated earnings which defines contributions and determines benefits, we left out the individual's idiosyncratic income shock. As a sensitivity check, we computed an alternative experiment 6 transition in which we included the income shock in the formula for cumulated earnings. Note that this eliminates the insurance aspect implied by our previous formulation. We found that the policy functions of the agents and the aggregates of the economy were the same but the agents' welfare was slightly lower with respect to the "linkage with insurance against income risk." The effect on welfare was smaller than 1%.

9. ROLE OF MEDICARE AND MEDICAID INCREASES

The Social Security Administration calculates that a 2.2 percentage point increase to the 12.4% OASDI payroll tax will restore the financial balance in the Social Security trust fund. According to Goss (1998), Deputy Chief Actuary of the Social Security Administration, a 4.7% immediate increase of the existing OASDI payroll tax is necessary to finance the existing Social Security system in perpetuity. The 1997 Economic Report of the President argues that projected increases in Medicare and Medicaid expenditures will contribute a heavier burden than financing the Social Security system. In line with the perspective of the 1997 Economic Report of the President, all of our calculations up to now assume substantial increases in Medicare and Medicaid. Therefore, our computed fiscal adjustments are designed to fund both higher Social Security and higher Medicare and Medicaid expenses. In this section, we briefly describe two calculations designed to shed light on how much of the fiscal burden comes from our having projected increases in Medicare and Medicaid.

Thus, in our benchmark experiment 1 (which includes projected increases for Medicaid and Medicare expenditure), we computed that, in our general equilibrium setup, the tax on labor income should increase from 29.7% to 59.5%. We now consider two other experiments. Experiment 9 is a partial equilibrium version of experiment 1 where factor prices are held fixed at their values in the initial steady state and in which Medicaid and Medicare expenditures do not increase over time. Experiment 10 has factor prices adjusting to factor quantities, as in experiment 1, but keeps Medicaid and Medicare expenditures constant over time. In both experiments 9 and 10 the government gradually increases the tax on labor income to finance its expenditures.

Table V compares outcomes across steady states (the initial steady state is common to all experiments). The first column reports the results for experiment 1; the second and third columns describe the final steady state for experiments 9 and 10.

Experiment 9 shows that to finance the increased burden of Social Security in this environment, the tax rate on labor income should increase by 8.8 percentage points (from 29.7% to 38.5%). Even this partial equilibrium or "small open economy" environment produces a large jump in the labor income tax rate, much larger than the 2.2% computed by the SSA to balance the Social Security trust fund over the next 75 years and somewhat larger than the 4.7% projected by Goss (1998). The main difference from the computations by Goss is probably the timing of the tax increases: Goss assumes that the OASDI payroll tax is raised at once at the time of the computation (1996), while we assume that it is raised in six steps, every 10 years, starting from the year 2000. The discrepancy with the much

TABLE V
Comparing Steady States, in Partial or General Equilibrium, with or without Medicare and Medicaid Expenditure Increase

Steady state	1	9	10
Variable			
τ_l	29.7% ↑ 59.5%	↑38.5%	↑46.8%
Interest rate	5.9% ↓ 5.0%	0% ↓ 4.7%	
Wage	+4.2%	0%	+5.8%
GDP	−17.4%	+17.4%	−11.0%
Mean asset holdings	−12.5%	+88.5%	−3.1%
Mean capital	−11.7%	+99.4%	−1.9%
Mean consumption	−28.4%	+4.5%	−10.8%
Mean eff. labor	−20.8%	−24.8%	−15.8%
Bequests	+35%	+210%	+75%
K/GDP	3.0 ↑ 3.2	↑5.0	↑3.3
G/GDP	20.6%↑31.2%	↓17.6%	↑23.2%

lower 2.2% increase projected by the SSA to reestablish equilibrium of the Social Security trust fund is obviously due (besides their making the same assumption as Goss on the timing of tax increases) to the fact that they only consider a 75-year horizon, starting from 1996.

Experiment 10 acknowledges that the U.S. economy is a very large one and that changes in its economic scenario will affect the interest rate and the real wage. In this environment, the tax rate on labor income increases by 17.1 percentage points (from 29.7% to 46.8%), reflecting the fact that the interest rate declines to 4.7% and the wage increases by 5.8%. Not surprisingly, the decrease in labor supply is less than that in the "small open economy" (−15.8% instead of −24.8%) and capital accumulation decreases by almost 2%, instead of jumping up by 99%. The aging of the population and the increases in the labor income tax to finance the Social Security system have very large effects.

The 1997 Economic Report of the President argues that financing increased expenditures on Medicare and Medicaid will have a much larger impact on the economy than the fiscal burden due to maintaining the current unfunded Social Security system. A comparison of experiments 1 and 10 reveals, instead, that the distortions due to financing our unfunded Social Security system using a labor income tax will be large, given the SSA forecasts about the aging of the population.

Experiment 10, in which the distortions stem only from the necessity of financing the unfunded Social Security (no increase in government health expenditure here), shows that the labor income tax rate has to be raised from 29.7% to 46.8% to maintain retirement benefits at current levels and that average consumption and labor supply will eventually decrease by 11% and 16%, respectively.

Not surprisingly, however, our results confirm that adding to this burden the projected increase in health expenditure will make the impact on the economy even heavier. In experiment 1, in which the tax on labor income is raised to finance both our unfunded Social Security and the projected increase in government health expenditure, the tax on labor income eventually rises to 59.5% and average consumption and labor supply, respectively, decrease by 28% and 21% in the final steady state.

10. CONCLUDING REMARKS

We have studied some implications of the SSA-projected demographic dynamics under alternative fiscal adjustments. Our setup allows for exogenous productivity growth and the projected increase in Medicare and Medicaid spending. To the best of our knowledge, this is the first study to address the issue of the retirement of the baby-boom generation in a setting in which two important features inducing private saving coexist:

1. Life-span uncertainty: This feature of the model induces the households in our economy with no private annuity markets to save in order to insure against living longer than expected. An increase in life expectancy, ceteris paribus, generates higher private saving.

2. Life-long bequest motive: This motive not only helps us match the observed capital output ratio but also makes the capital stock more resilient to different ways of financing the fiscal burden.

Our results indicate that the projected demographic transition will induce a transition to a new stationary equilibrium at which a large fiscal adjustment in the form of a much higher labor income or consumption tax rate needs to be made. We find that reducing benefits (by taxing them or by postponing the normal retirement age) or imposing a consumption tax will go far toward reducing the rise in the rate of taxation of labor that will be required to sustain our unfunded social retirement system but will hurt some generations during the transition. An uncompensated phase-out of benefits toward eventual privatization delivers the largest welfare gains for future generations but at the same time imposes the largest welfare costs on current and transitional generations. We also find that a simplification of the Social Security structure that makes clear the linkage between the agent's past contributions and their future pensions eliminates a labor/leisure distortion and improves the welfare of all cohorts.

APPENDIX A

Preferences

For ease of exposition, we suppress the time subscript s, but it should be understood to be present. A person's Bellman equations are

$$V_t(x_t) = \max_{u_t,\, x_{t+1}} \left\{ u_t' Q_t u_t + x_t' R_t x_t + \beta E_t V_{t+1}(x_{t+1}) \right\},$$

where

$$\begin{aligned}
E_t V_{t+1}(x_{t+1}) &= \alpha_t(s) E_t\big(V_{t+1}(x_{t+1}) \mid \text{alive}\big) \\
&\quad + \big(1 - \alpha_t(s)\big) E_t\big(V_{t+1}(x_{t+1}) \mid \text{dead}\big) \\
V_t(x_t \mid \text{alive}) &= x_t' P_t x_t + \xi_t \\
V_t(x_t \mid \text{dead}) &= x_t' P_{T+1} x_t \\
x_t' P_{T+1} x_t &= -\text{JG}\big((1 - \tau_b) a_{t-1} - \text{JB}\big)^2.
\end{aligned}$$

This last term captures the bequest motive. Here JG is a parameter governing the intensity of the bequest motive and JB is an inheritance bliss point.

Riccati equations for P_t, F_t, and ξ_t are

$$\begin{aligned}
F_t &= \big(Q_t + \beta\alpha_t(s) B_t' P_{t+1} B_t + \beta(1 - \alpha_t(s)) B_t' P_{T+1} B_t\big)^{-1} \\
&\quad \times \big(\beta\alpha_t(s) B_t' P_{t+1} A_t + \beta(1 - \alpha_t(s)) B_t' P_{T+1} A_t\big) \\
P_t &= R_t + F_t' Q_t F_t + \beta\alpha_t(s)(A_t - B_t F_t)' P_{t+1} (A_t - B_t F_t) \\
&\quad + \beta(1 - \alpha_t(s))(A_t - B_t F_t)' P_{T+1} (A_t - B_t F_t) \\
\xi_t &= \beta\alpha_t(s)\big(\operatorname{trace}(P_{t+1} C'C) + \xi_{t+1}\big) + \beta(1 - \alpha_t(s)) \operatorname{trace} P_{T+1} C'C.
\end{aligned}$$

APPENDIX B

Projected Demographics

Figure 1 is constructed by taking the projections of the conditional survival probabilities from Bell *et al.* (1992) and assuming a growth rate for entrant workers such that we match the dependency ratio for 1975 and the one forecast for 2040. The forecast dependency ratio we match is the "medium" projection given by the Social Security Administration, under the current retirement age legislation. These are also the survival probabilities and the new workers growth rate that we use in our experiments. In particular, the line corresponding to "current legislation" is the dependency ratio implied in experiments 1, 2, 5, 6, 7, and 8; the one marked "current legislation +2" is the dependency ratio in experiments 3 and 4.

Starting from the early 1900s, each successive cohort has faced successively more favorable vectors of conditional survival probabilities as a result of improvements in exercise and nutrition habits, medical techniques, environmental practices, etc., which have become especially important by the 1950s. Combined with a drastic increase in fertility in the 1950s, the demographic dynamics have created the anticipation of future increases in the dependency ratio even before the time the baby boomers start to retire.

The Social Security Administration arrives at this gloomy picture of the future as follows. First, the SSA takes as its starting population for its projections the Social Security Area as of January 1, 1989, and its breakdown by age, sex, and marital status. Second, the SSA projects future (a) fertility (taking into account historical trends, future use of birth control methods, female participation in the labor force, divorce, etc.), (b) mortality (taking into account future development and applications of medical methods, environmental pollutants, exercise and nutrition trends, drug use, etc.), (c) net immigration, (d) marriage, and (e) divorce. The final step is to compute projections for future survival probabilities, fertility, immigration, marriage, and divorce, after adjusting the above "primitive objects" for a number of reasons. For example, instead of using "death rates," "death probabilities" are computed as the ratio of the number of deaths occurring to a group in a given year to the number of persons in this group at the beginning (as opposed to the middle) of the year.

The outcome is a series of tables that show the Social Security Area population by year, age, sex, and marital status under three alternative projections (optimistic, medium, pessimistic). Since we abstract from immigration, marriage, divorce, etc. in our model, we approximate the time variation in the cohort shares and therefore the time path of the dependency ratio by choosing a time path for the fertility rate $\{n(s)\}$ which is the ratio of new borns (model age 0 but real time age 21) at time s to those at time $s-1$ and using the cohort-specific conditional survival probabilities given in Bell, Wade, and Goss (1992).

APPENDIX C

Changing the Intensity of the Bequest Motive (JG)

To analyze the sensitivity of our results to the strength of the bequest motive, we compare the results of experiment 1, calibrated as described in the paper, with its results when either JG (the bequest motive intensity) or JG and β vary.

In the first sensitivity check, we lower the value of JG by 10% (from 0.0320 to 0.0288). In the second one, we decrease JG by 25% (from 0.0320 to 0.024) and increase β to obtain the same capital-to-GDP ratio as in the initial steady state of experiment 1 (β increases from 0.994 to 0.996). The latter parameter configuration is such that, should the consumer live up to 90 years of age, he or she would die with few assets.

Columns 1 and 2 in Table VI refer to the initial and final steady states of experiment 1 in the original calibration; columns 3 and 4 to its initial and final steady states with a lower JG, and columns 5 and 6 to its steady states when a much lower JG and a higher β are assumed.

In our original calibration, the effective average discount factor of an individual over his or her lifetime (taking the mean of β times conditional survival probability) is 0.9679 in the initial steady state and 0.9838 in the final one, reflecting the increase in life expectancy. In the second sensitivity check we run, it is 0.9702 and 0.9859, respectively.

Contrasting columns 1 and 2 with columns 3 and 4 in Table VI, we see that a 10% decrease of JG does not change the results substantially. Labor supply stays the same in both the initial and final steady states. In the run with a lower JG, asset holdings and consumption are slightly lower in both steady states. The interest rate and the tax rate on labor income are a little higher. The capital-to-GDP ratio is also pretty much unchanged.

Comparing of the first two columns with columns 5 and 6, it appears clear that, from the aggregate point of view, considering an environment in which people care less about leaving bequests but are more patient, does not greatly change the aggregates or even the behavior of the economy over

TABLE VI
Steady States Comparisons

Steady State	1	2	3	4	5	6
Variable						
τ_l	29.66%	59.48%	29.81%	61.19%	29.66%	59.92%
Interest rate	5.93%	5.01%	6.02%	5.17%	5.92%	5.17%
Wage	3.18	3.31	3.17	3.29	3.18	3.29
GDP	12.11	10.00	12.06	99.87	12.13	9.94
Mean asset holdings	41.55	36.35	41.08	35.42	41.62	35.68
Mean capital	36.00	31.77	35.53	30.87	36.07	31.13
Mean consumption	6.67	4.77	6.65	4.70	6.68	4.76
Mean eff. labor	2.55	2.02	2.55	2.01	2.56	2.02
Bequests	18.75	25.32	17.83	21.46	16.65	16.66
K/GDP	2.97	3.17	2.95	3.13	2.97	3.13
G/GDP	0.20	0.31	0.20	0.31	0.20	0.31

time. Not surprisingly, the variable most affected is the amount of bequests in the economy.

We choose to adopt the model with a stronger bequest motive, rather than the one with more patient agents, because on the age–asset accumulation profile, the model with a stronger bequest motive is more consistent with the empirical evidence. In fact, in the run with more patient agents and lower "altruism," people run down their assets faster in the second part of their lifes, much faster than observed in the data. This is due to the fact that their conditional survival probability decreases over time and the increase in their joy-of-giving, should they die, is lower. Moreover, we feel that the calibration we adopt in the paper is consistent with Kotlikoff and Summers' findings on the proportion of the present value of wealth which is transmitted from one generation to the next.

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