THE ALLOCATION PUZZLE IS NOT AS BAD AS YOU THINK

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ABSTRACT. The standard infinitely-lived representative agent model predicts that countries with fast aggregate income growth should experience net capital inflows. This is because the representative household with an increasing income path is willing to borrow to smooth lifetime consumption. However, this prediction is at odds with international capital flows data. This is called the allocation puzzle. We find that faster aggregate income growth benefits young cohorts more than older cohorts. A high aggregate income growth rate is always accompanied by flatter cross-sectional age-income profiles. Therefore in the faster growth periods there is a stronger cohort effect or a weaker age effect. These two effects mitigate the effect of fast aggregate income growth on each individual's income growth and lower their willingness to borrow. This is because even if aggregate income is growing fast, each individual's lifetime income path might be relatively flat. Our quantitative results show that the flatter age-income profiles can explain 41 percent of the Korean capital outflows puzzle.

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1. INTRODUCTION

Standard theory predicts that countries with fast aggregate income growth should experience net capital inflow. However, this prediction is at odds with data. This paper provides a potential solution to this puzzle.

In the textbook representative agent growth model, if a country's aggregate income grows quickly and the representative household foresees a future income increase, they augment borrowing to increase current consumption and pay this back later in the future. The inter-temporal consumption smoothing motive leads to net capital inflows. Even if the representative household cannot perfectly foresee the future pace of income growth, as long as they believe that the fast growth is highly likely to sustain,[1] it is still optimal for the representative household to borrow today and pay back in the future. However, despite high growth rates, it is known that countries like China and Korea have experienced net capital outflows instead of net capital inflows. Gourinchas and Jeanne (2011) investigate a panel of developing countries' growth rates of total factor productivity (TFP) and capital movements from 1980 to 2000. They find that there is a surprisingly negative correlation between average TFP growth rates and net capital flows from 1980 to 2000. Gourinchas and Jeanne (2011) call this the allocation puzzle.

We argue that the textbook infinitely-lived representative agent model provides a biased prediction of net capital flows. The reason is that the representative agent model only captures aggregate income growth. It does not capture the heterogenous effects of aggregate income growth on different age groups.

Using data from the Luxembourg Income Study, we first document that fast aggregate income growth benefits younger cohorts more than elder cohorts. In other words, in periods with faster aggregate income growth, a larger fraction of the increased income is distributed to younger cohorts. This finding implies that there is a stronger cohort effect profile or a weaker age effect profile when the aggregate income of a country is growing faster than usual.

The varying cohort effect profile or age effect profile that accompanies faster aggregate income growth implies that we might overestimate the size of net capital inflows if we only use aggregate income growth data. On one hand, other things equal, a faster growth of aggregate income steepens each individual's lifetime income path. Hence aggregate net capital

[1]e.g. It is hard to believe that aggregate income of China and Korea will grow slower than the United States in the near future, given the their fast growth in the last three decades.

inflows should go up. On the other hand, when paired with faster growth of aggregate income, a stronger cohort effect profile or a weaker age effect profile make each individual's lifetime income path flatter. This makes aggregate net capital inflows go down. The varying cohort and age effect profiles offset part of the impact of fast aggregate income growth on the slope of each individual's lifetime income path. Therefore, without internalizing the heterogenous effects of fast aggregate income growth on different age groups, the textbook growth model overestimates the size of net capital inflows.

First we estimate the change of age-income profile in response to the change of growth rate of aggregate real gross domestic product (GDP) per capita. Then we build an overlapping generation (OLG) model to predict the net capital flows within a 20-year fast aggregate income growth episode as in Gourinchas and Jeanne (2011). Despite the well known fact that time, age and cohort effect profiles cannot be identified separately,[2] we show that for the purpose of predicting net capital flows, we have to know only two statistics: first, the difference between age and cohort effect coefficients, which captures the slope of the snap shot of cross sectional age-income profile of given year; and second, the sum of time effect and cohort effect coefficients, which captures the slope of each individual's lifetime income path. In the benchmark model, we shut down the effect of time varying age-income profile. Thus, the benchmark model should be viewed as the counterpart of the textbook infinitely-lived representative household model in our OLG framework. Using Korea real GDP per capita data, the benchmark model predicts that the size of net capital flows into Korea between 1981 and 2000 should be about 4.3 times Korean real GDP in 1980. However, the actual net capital inflows in data during the same episode is about -0.3 times Korean real GDP in 1980. Therefore, the benchmark predicts that Korean households should have borrowed a lot from abroad although in fact they are net creditors.

We then extend the benchmark model by allowing for time varying age-income profiles. We incorporate time varying age-income profiles in two ways. In model a, we allow for a time-varying cohort effect profile but a constant age effect profile. In model b, we allow for a time-varying age effect profile but a constant cohort effect profile. A faster aggregate income growth is accompanied by a stronger cohort effect profile in model a and a weaker age effect

[2]See Heckman and Robb (1985).

profile in model b. In both models, each individual's lifetime income path is flatter than that in the benchmark model. Consequently, compared with the benchmark model, both model a and model b predict a much smaller scale of net capital inflows. Our quantitative results show that both models can explain about 41% of the difference between the predictions of the benchmark model and data.

The literature explaining international capital flows anomalies goes back to the classic paper by Lucas (1990). Regarding the negative relationship between TFP or income growth and net capital inflows, there are two strands of literature. One strand of literature focuses on the reason of low investment in fast growing countries.[3] Our paper is closer to the other strand of literature focusing on the reason of high saving or low borrowing in fast growing countries.

In particular, our paper is closely related to the literature that explores the heterogenous effects of aggregate income growth on different age groups and their macroeconomic implications. Song and Young (2010) use data from Urban Household Survey of China to show that the change in age-income profiles[4] of Chinese households and the change in replacement rates can account for the surge of Chinese household saving rate. Compared to their study, we provide a full analysis of age effect, time effect and cohort effect. Although it is well-known that those three effects cannot be identified jointly, we show that for the purpose of predicting aggregate net capital flows, we only need to know the difference between the age and cohort effects and the sum of time and cohort effects. More importantly, we use data from a panel of countries to illustrate the relationship between aggregate income and each individual's age-income profile, which might have further implications on exploring the reason of fast growth and varying age-income profiles.

Alternative explanations include Aghion et al. (2006), who study the the fast growing periods in Japan, Korea and Taiwan. They suggest that fast growing countries are incentivized to save in order to attract more productive foreign investment. Carroll et al. (2000) suggest that households with fast income growth cannot increase consumption immediately due to habit formation, which results in high saving. Rothert and Short (2010) build a model with complementary tradable and non-tradable goods. They argue that if tradable sectors grow faster than non-tradable sectors in a fast growing country, the representative household would only

[3]e.g. Song et al. (2011).

[4]They assume that an individual's income depends on age, cohort and aggregate business cycle disturbance.

increase consumption moderately until there is a catch-up in non-tradable sectors. Therefore, we can observe high savings and net capital outflows when an economy's income starts to take off. Feroli (2003) finds that demographic differences can explain the current account imbalance in G-7 countries.[5]

The remainder of the paper is organized as follows: in section 2 we use a simple example to illustrate that given the same aggregate income growth rate, different cross-cohort distributions of the benefit from aggregate growth growth can result in dramatically different capital flow patterns. Section 3 introduces individual income processes. It also shows the connection between aggregate income growth and the age-income profile in the data. Section 4 and 5 contain the benchmark model and its extensions. Section 6 presents our conclusion. There is a data appendix at the end of the paper.

2. AN EXAMPLE

In this section we will use a simple example to show that the heterogeneity of age-income profiles across countries is an important factor in determining the capital flow pattern.

Suppose that there are two countries with the same aggregate income growth rate. They also have the same preference, initial asset position and risk free interest rate. If households in each country are identical so that each country can be summarized by a representative household, the standard theory predicts that they should have the same capital flow patterns.

Now suppose each individual lives for three periods. Thus, at any given time there are three generations (young, middle and old). Suppose the aggregate income growth rate is 50% per period. Figure 1 and Figure 2 illustrate the age-income profiles in each country. In this small open economy example, we assume the exogenous interest rate $R = \frac{1}{\beta}$.

Figure 1 illustrates the income of each cohort in Country A. In Figure 1, each generation's income is growing at 50% per period (which is the same as the aggregate income growth rate). Therefore, aggregate income growth benefits all cohorts evenly. This is the analogue of the infinitely-lived representative agent model in our OLG model environment. In this economy, at any given time t, the young cohort borrows to smooth his lifetime consumption while the old cohort saves. Since the young cohort's lifetime income level is higher than that of the old cohort, the young cohort's borrowing dominates the aggregate capital flow pattern. Current

[5]Other explanations include Sandri (2010), Caballero et al. (2008) and Mendoza et al. (2009).

account is negative and aggregate asset position goes down. The country should experience net capital inflows.

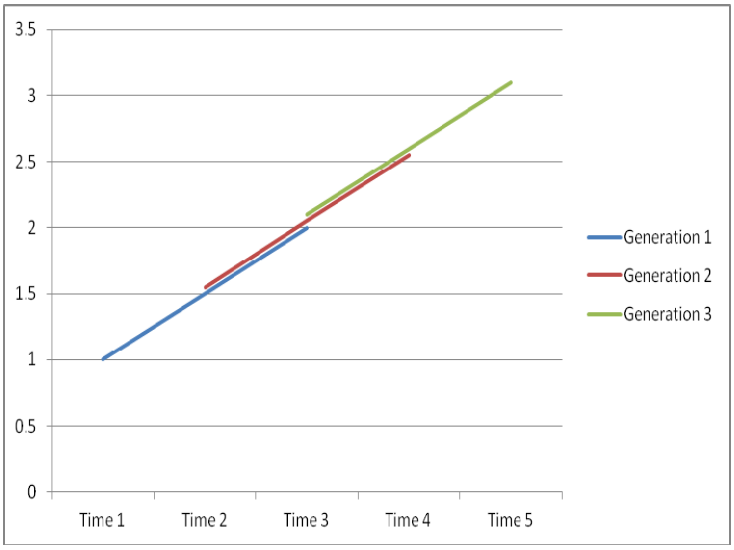

FIGURE 1. The Income of Each Generation in Case I
The horizontal axis shows the time. The vertical axis is the income.

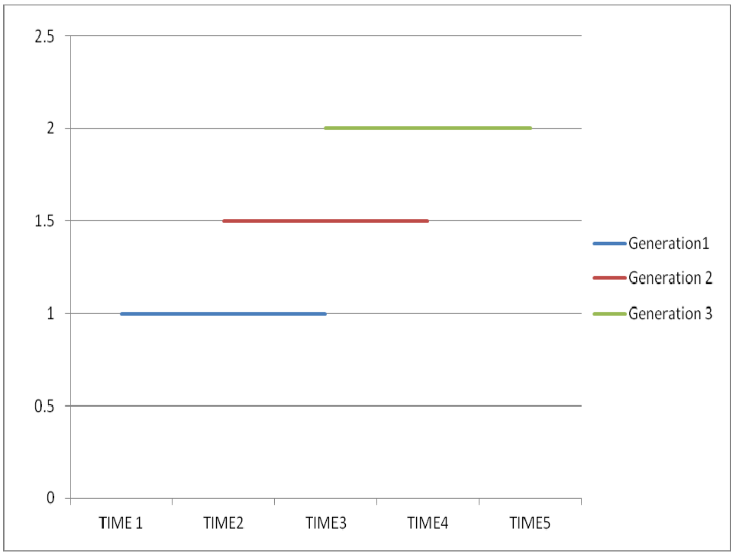

FIGURE 2. The Income of Each Generation in Case II
The horizontal axis shows the time. The vertical axis is the income.

Figure 2 illustrates the income of each cohort in Country B. In Figure 2, each generation's lifetime income path is flat. However, there is a jump between any two consecutive cohorts' initial income levels. Therefore, the growth rate of aggregate income is still 50%. In contrast to Country A, in Country B all the benefit of the aggregate growth is distributed to the young cohort. Due to the flat individual income path, each individual optimally consumes his endowment at any time and there is no intertemporal borrowing or lending. Consequently, in each period aggregate consumption equals aggregate income. The size of net capital flows is zero in this economy.

The same aggregate income growth rate can be compatible with different patterns of capital flows. Both Country A and Country B have the same aggregate growth rate (50% per period). But Country A has positive net capital inflows (which is the same as the capital flow pattern in the infinitely-lived representative agent model) while Country B has zero net capital flows. The reason is that the same speed of aggregate income growth can benefit each cohort differently. As we point out that in Country A each individual's income growth rate is exactly the same as the aggregate income growth rate, which leads to positive net capital inflows. However, in Country B each individual's income growth rate is zero, which leads to zero net capital inflows. From this example it's obvious that using only aggregate growth rate data might result in significant mis-prediction of international capital flow patterns.

3. The General Model

Gourinchas and Jeanne (2009) build a model using the framework of the infinitely lived representative agent model. However, as we mentioned in the introduction, the key of our paper is to consider the effect of the change of the age-income profile while a country is growing. Therefore it's necessary for us to analyze the net capital inflow problem with the overlapping generation model framework with a proper choice of log income process for each individual. In this section we first introduce a linear income process. Then we show that a simple linear income process can poorly fit the data. Eventually we move to its extension: the time-varying cohort/age effect process.

3.1. **Linear Log Income Process.** Suppose each individual's log income process follows

$$y_{a,t,c} = \alpha a + \beta t + \gamma c + constant. \tag{1}$$

Here $y_{a,t,c}$ is the log income. a, t and c are age, time, and cohort, respectively. Substitute $c = t - a$ into (1) and we have

$$y_{a,t,c} = (\alpha - \gamma)a + (\beta + \gamma)t + constant. \tag{2}$$

Therefore by running OLS regression of y on a and t, we can get the estimate of $\alpha - \gamma$ and $\beta + \gamma$. A natural question is how we should interpret $\alpha - \gamma$ and $\beta + \gamma$?

First, $\beta + \gamma$ is the growth rate of average income. To see this, suppose each individual lives for A periods. At any time t, there are A different age groups. Their log incomes[6] are (from $a = 1$ to $a = A$): $\alpha + \beta t + \gamma(t-1)$, $\alpha \cdot 2 + \beta t + \gamma(t-2)$,..., $\alpha A + \beta t + \gamma(t-A)$. At time $t+1$, the log incomes of individuals from $a = 1$ to $a = A$ are: $\alpha + \beta(t+1) + \gamma(t+1-1)$, $\alpha \cdot 2 + \beta(t+1) + \gamma(t+1-2)$,..., $\alpha A + \beta(t+1) + \gamma(t+1-A)$. It is easy to see that for any a, we have

$$y_{a,t+1,t+1-a} = y_{a,t,t-a} + \beta + \gamma. \tag{3}$$

That is, an a-year old individuals at time $t+1$ earns more than an a-year old individuals at time t by $\beta + \gamma$ (in log income). Therefore $\beta + \gamma$ is the growth rate of the average income. Thus its data analogue should be the aggregate income growth rate (for example, the growth rate of GDP per capita from IFS or Penn Table).

Second, $\alpha - \gamma$ characterizes the log income difference of different age groups. To see this, suppose at time t there are two individuals i and j: $a_i = 1, c_i = t - 1$ and $a_j = 2, c_j = t - 2$. Then the log income differential between the older (individual j) and the younger (individual j) is $y_j - y_i = \alpha - \gamma$. Another way for us to understand it is to look at the snapshot of the age-income profile at a given time with age on the horizontal axis and log income on the vertical axis. The slope of the age-income profile line is exactly $\alpha - \gamma$. Therefore we define $\alpha - \gamma$ as the *age advantage*. Notice that we do not call it *age premium* because $\alpha - \gamma$ is a composition of the age premium α and the cohort premium $-\gamma$.

An individual's life-cycle income path depends on both $\beta + \gamma$ and $\alpha - \gamma$: $\beta + \gamma$ decides the starting point (intercept) of his/her income path over the life; $\beta + \gamma$ and $\alpha - \gamma$ jointly determine the slope of the income path. In order to see this, we normalize the log income of age group

[6]to simplify the notation, we eliminate the $constant$ part.

$a = 1$ at time t_0 to 0.

$$y_{1,t_0,t_0-1} = \alpha + \beta t_0 + \gamma(t_0 - 1) = 0; \tag{4}$$

Consequently the succeeding cohorts' log incomes at age $a = 1$ are determined by $\beta + \gamma$ as below:

$$y_{1,t_0+1,t_0} = \alpha + \beta(t_0 + 1) + \gamma(t_0) = y_{1,t_0,t_0-1} + \beta + \gamma;$$

$$\cdots$$

$$y_{1,t_0+j,t_0+j-1} = \alpha + \beta(t_0 + j) + \gamma(t_0 + j - 1) = y_{1,t_0,t_0-1} + j(\beta + \gamma).$$

The slope of each cohort's age-income profile is determined by the sum of $\beta + \gamma$ and $\alpha - \gamma$: for any i and c, we have

$$\begin{aligned} y_{i+1,c+i+1,c} - y_{i,c+i,c} &= [\alpha(i + 1) + \beta(c + i + 1) + \gamma c] - [\alpha i + \beta(c + i) + \gamma c] \\ &= (\alpha - \gamma) + (\beta + \gamma). \end{aligned}$$

Therefore, although we cannot estimate the age, time and cohort effect due to the identification problem (Heckman and Robb, 1985), we can still draw each cohort's life time income path from the estimations of $\alpha - \gamma$ and $\beta + \gamma$. Hence, the consumption, saving and international capital flow can be simulated in a small open economy in which each cohort's income path is determined by $\alpha - \gamma$ and $\beta + \gamma$.

3.2. **Time Varing Age/Cohort Effects.** From the last subsection, we know that the slope of the snapshot of a given year's age income profile, $\alpha - \gamma$, is constant over time (without measurement error or other shocks) if the log income process follows (1). However, in data this slope can vary dramatically over time. Figure 3 is the snapshot of Chinese age-income profiles in 1988 and 2002. We can see that the age advantage was much stronger in 1988 than 2002, which is at odds with the predictions following the linear log income models with constant age, time and cohort effects.

More generally, we collect data of age advantage and average GDP growth rate for a panel of 23 countries[7] and Figure 4 shows the results. Each point in the figure represents an observation $(\alpha - \gamma, g)_{i,t}$, the (age advantage, average GDP growth rate) pair of country i at time t. There

[7]See Appendix for the details about the data.

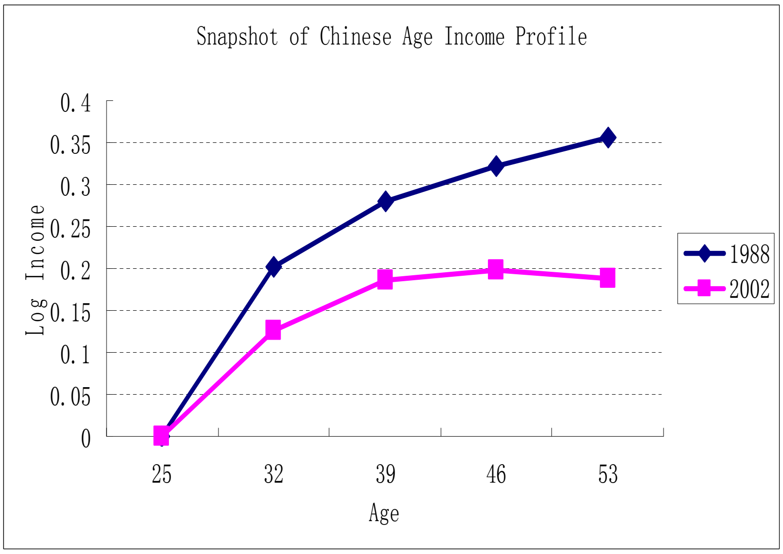

FIGURE 3. Snapshot of Age Income Profile in 1988 and 2002. Log income of the age 25 group is normalized to 0. Data source: Chinese Household Income Project.

is a negative correlation between the age advantage and the growth rate of GDP per capita. Controlling the country fixed effects, the negative correlation is more pronounced. This is shown in Figure 5.

Table 1 shows the results of regressions of the age advantage on the average growth rate of GDP per capita using the observations shown in Figure 4 and Figure 5. The regression results indicate that after controlling the heterogeneity of countries, one percent increase of the average growth rate of GDP per capita is accompanied by a 0.668 percent decrease in the age advantage. To see the magnitude of the regression results, from 1980 to 2000, the average age advantage in the United States is about 3.3 percent[8] and the average growth rate of GDP per capita is about 2.4 percent. The regression result implies if the United States were growing at an annual rate of 7.5 percent, an individual with age a would earn more or less the same as an individual with age $a + j(j > 0)$ at any given time.

The varying age advantage implies that we cannot simply assume that the log income process follows (1). We have to allow for either a time-varying age effect profile $\{\alpha_t\}$ or a time-varying cohort effect profile $\{\gamma_t\}$. Unfortunately, from the data we cannot identify whether the varying age advantage is from a time-varying age effect profile or a time-varying cohort effect profile.

[8]i.e. on average the income of a m-year-old person earn 3.5 percent more than a (m-1)-year-old person.

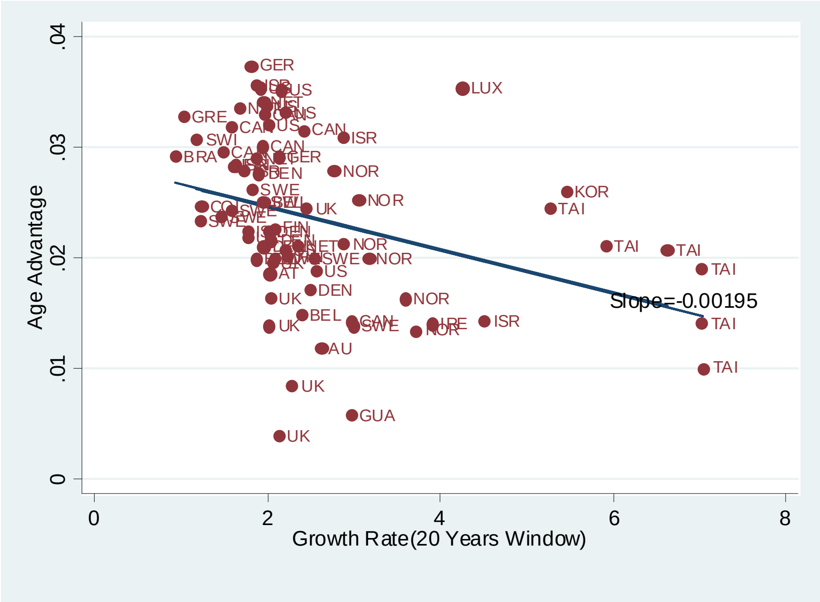

FIGURE 4. Average Growth Rate of GDP Per Capita and Age Advantage. Each observation is an (age advantage, growth rate) pair of a given county in a given year. Some countries have multiple observations because the data is available for multiple years. The average growth rate is defined as the average growth rate of GDP per capita within 20 years preceding the survey year. The age advantage is defined as the slope of the snapshot of the age-income profile within the survey year. Data source: Luxembourg Income Study, Penn World Table and authors' calculations.

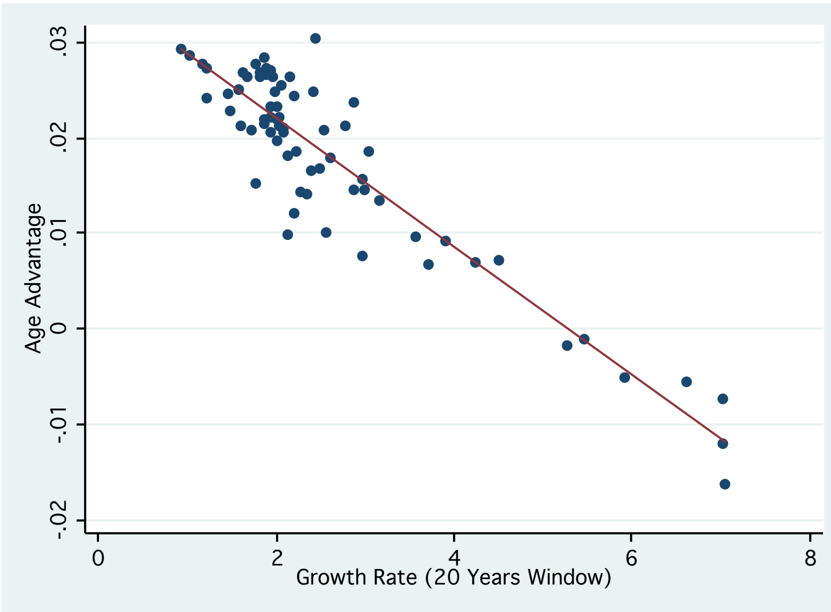

FIGURE 5. Average Growth Rate of GDP Per Capita and Age Advantage Adjusted by Country Fixed Effects. Data source: Luxembourg Income Study, Penn World Table and authors' calculations.

Therefore we look at two models: a model with a time-varying age effect profile and a model with a time-varying cohort effect profile. With each model we are interested in its implication

TABLE 1. Regression Results

	Regression I	Regression II
average growth rate	$-.00195^{*}$	$-.00668^{**}$
constant	$.0285^{**}$	$.035^{**}$
country dummies	No	Yes
R-Squared	0.13	0.73
No. of observations	73	73

$**$, $*$ indicates significance at the 1%, 5%, respectively.

on capital flows. As a benchmark, we also consider the model in which the age effect profile and the cohort effect profile are time-invariant. Notice that the benchmark is the analogue of the infinitely lived representative model in our OLG framework since they both ignore the declining age advantage in the fast growing periods. Table 2 summarizes the three models and the log income process we use in each model.

TABLE 2. Three Models

Model	Feature
Benchmark	Fixed α, γ
Model a	Time varying α
Model b	Time Varying γ

4. THE ALLOCATION PUZZLE

In this section we first use Korea as an example to show that the allocation puzzle is still a puzzle in our OLG benchmark model which is characterized by a fixed age effect and cohort effect profile. Then we show that if we add varying age advantage into our benchmark model, the amount of capital flows simulated by our model is much closer to data. In particular, the quantitative exercise we are doing here is that we use our small open OLG framework to predict the net capital flows of Korea from 1980 to 2000. We simulate the model for three times and in each time we use one of the three models of income process we mentioned earlier.

As in Gourinchas and Jeanne(2009), the background of the exercise is to explore the amount of aggregate capital flows during the fast growing periods. We assume that GDP per capita in Korea grows at g per year from 1981 to 2000[9]. These are the *catch-up* (with the United States) periods in Gourinchas and Jeanne(2009). We then assume that the *catch-up* is over after year 2000 and Korea grows at g_{us} per year after 2000. Here $g_{us} < g$ is the long run growth rate of GDP per capita of the United States. The purpose of the exercise is to see the effect of a twenty-year fast growth on Korea's national external asset position, excluding any other possible factors such as the fast growth beyond those 20 years. Thus we also assume that before 1980 Korea's average growth rate of real GDP per capita is g_{us}. Formally, that is,

$$g_t = \begin{cases} g & \text{if } 1981 \leq t \leq 2000 \\ g_{us}. & \text{otherwise.} \end{cases} \tag{5}$$

We use a small open endowment economy model. R is the exogenous world interest rate. Each individual works for 28 years: age $a = 23, ..., 50$. Individuals are retired from age 51 to 70. They die at age 70. After retirement the income source is pension. We assume the amount of the pension income is determined by the replacement rate ϕ and the average income of working population jointly:

$$Y_{a,t,c} = \phi \sum_{i=23}^{i=50} Y_{i,t,t-i} \text{ for } 70 \geq a \geq 51 \tag{6}$$

Before retirement each individual's log income follows the process:

$$Y_{a,t,c} = \alpha a + \beta_t + \gamma c = (\alpha - \gamma)a + (\beta_t + \gamma t). \tag{7}$$

Notice that we allow for time effect dummies β_t (instead of a linear time effect β) since we only fix age and cohort effect profile. It is easy to see that in this economy the aggregate log income increases by $(\beta_t - \beta_{t-1} + \gamma)$ annually at time t. Thus we have[10]

$$(\beta_t - \beta_{t-1} + \gamma) = g_t \tag{8}$$

where g_t is the annual growth rate of aggregate income.

[9]It will be clear later that $g = 6.4\%$, which is Korea's average growth rate of real GDP per capita from 1981 to 2000. Data source: Penn Table

[10]The proof is similar with the proof in the linear case.

There is no credit frictions. An individual born in year c maximizes his life-time utility:

$$Max \quad \sum_{a=23}^{a=70} \delta^{a-23} u(C_{a,a+c,c}) \tag{9}$$

s.t.

$$\sum_{a=23}^{a=70} \frac{C_{a,a+c,c}}{R^{a-23}} = \sum_{a=23}^{a=70} \frac{Y_{a,a+c,c}}{R^{a-23}} \tag{10}$$

The asset positions of an individual born in year c are:

$$A_{a,a+c,c} = \begin{cases} A_{a-1,a-1+c,c}R + Y_{a,a+c,c} - C_{a,a+c,c} & \text{if } 24 \leq a \leq 70, \\ 0. & \text{if } a = 23. \end{cases} \tag{11}$$

The aggregate asset position in year t is defined as:

$$A_t = \sum_{a=23}^{a=70} A_{a,t,t-a} \tag{12}$$

The aggregate income in year t is defined as:

$$Y_t = \sum_{a=23}^{a=70} Y_{a,t,t-a}. \tag{13}$$

Following Gourinchas and Jeanne(2009), an appropriate measure of international capital flow is:

$$CF = \frac{\Delta A}{Y_{1981}} = \frac{A_{2000} - A_{1981}}{Y_{1981}} \tag{14}$$

4.1. **Benchmark Model With Fixed Age Effect and Cohort Effect.**

4.1.1. *Parameters.* We set $g = 0.064$, which is the average growth rate of real GDP per capita of Korea from 1981 to 2000. We set $g_{us} = 0.024$, which is the average growth rate of real GDP per capita of the United States in the long run. In the benchmark we fix the age affect and cohort effect. Hence the age advantage, $\alpha - \gamma$, is also constant over time. We set $\alpha - \gamma = 0.033$, which is the average age advantage of the United States during the episode 1981-2000. Obviously, the average age advantage of Korea is different from that in the United States and a different value of age advantage would change the model predicted amount of capital flow. However, in this paper we try to explore the importance of the time varying age advantage instead of a different average age advantage. Therefore in the benchmark model we set $\alpha - \gamma$ to exactly match the age advantage in the United States. Table 3 lists the key variables in the benchmark model.

TABLE 3. Key Parameters

Parameter	Symbol	Value
world interest rate	R or $1 + r$	1.04
time discount factor	δ	0.967
risk aversion	σ	3
replacement rate	w	0.3
annual growth rate of U.S	g_{us}	2.4%
annual growth rate of Korea	g	6.4%
average age advantage of U.S	$\alpha - \gamma$	0.033

4.1.2. *Result.* We first use our model to predict the capital flow of the United States by assuming $g_t = g_{us}$ for any t. We choose the value of δ such that with the parameters listed in Table 3, the model predicted capital flow between 1980 and 2000 exactly matches the capital flow of the United States within the same period. Then we use our model to predict the capital flow of Korea within the same period by assuming g_t follows (5). Our benchmark economy predicts that between 1981 and 2000, the capital inflow of Korea should be 4.3 times the GDP in 1981. However, in the data we find that Korea's capital inflow during 1981 and 2000 is -0.3 times the GDP in 1981. Therefore, our benchmark OLG model predicts a huge capital inflow while the data shows the opposite. It implies that even in our benchmark economy with OLG structure, we still have the allocation puzzle: model predicted capital flow pattern is at odds with data.

Our benchmark economy is the analogue of the infinitely-lived representative agent model in our life cycle environment. Both of them do not consist of the feature that fast growth is accompanied with the decline of age advantage, which results in a misprediction of capital flow. In later subsections we will take into account the declining age advantage of fast growing countries and see its effect on the scale of capital flow. As we mentioned earlier we cannot identify whether the time varying age advantage is due to a time varying age effect profile or a time varying cohort effect profile. Therefore, we will shut down one kind of effect separately and consider the impact of the other kind of effect in Model a and Model b. As it will be clear later, both models will have similar predictions about the scale of capital flow.

5. Models With Time Varying Age Advantage

5.1. **Model a: Time Varying Cohort Effect Profile.** In this model each individual's utility maximization problem is the same as that in the benchmark model. The only difference lies in the log income process.

To replicate the fact that fast growth is accompanied with the decline of age advantage (see Figure 5), here we allow for a time varying cohort effect profile. In particular, we assume that before year 1981, the age, time and cohort effect profile is $\{\alpha, \beta_t, \gamma\}$ so that the age advantage is constant. Between year 1981 and 2000, the profile switches to $\{\alpha, \beta_t, \gamma'\}$. Here $\gamma' > \gamma$ because faster aggregate income growth (see equation (5)) is accompanied with stronger cohort effect in this model. We further assume that after 2001 the age, time and cohort effect profile returns to $\{\alpha, \beta_t, \gamma\}$ because in our experiment the aggregate growth rate drops to the pre-1980s level after 2000.

In order to compute the international capital flow between year 1981 and 2000, we have to know the saving and consumption of people born from 1911 to 1977. Cohort 1911 is the eldest cohort alive in year 1981. Cohort 1977 is the youngest cohort that are older than 23 years old in year 2000. However, cohort 1977's consumption decision depends on his pension income, which is part of the life time income. When cohort 1977 is 70 years old, the youngest cohort at that time (year 2047) is cohort 2024. Therefore we have to specify the income the levels of people born from 1911 to 2024. Denote $\Delta = \gamma' - \gamma$, Table 4 illustrates the levels log income of different age groups in multiple years.

We choose the values of $\{\alpha, \beta_t, \gamma, \gamma'\}$ to match two statistics from the data:

(1) The aggregate growth rate g_t.

(2) The rate of change of age advantage in response to the change of aggregate growth rate.

As in the linear case in Section 3.1, to compute the international capital flow, we do not have to know the values of α, β_t, γ separately. To characterize each individual's income path we only have to know the value of $\alpha - \gamma$, $\Delta\gamma$ and $\beta_t + \gamma t$ for every t. We set $\alpha - \gamma = 0.033$ as we do in the benchmark model. The regression result in Table 1 (Regression II) implies that age advantage increases by 0.67% in response to a 1% increase of aggregate income growth rate. Therefore, we choose $\Delta = \gamma' - \gamma = (0.064 - 0.024) \cdot 100 \cdot 0.0067$ since the aggregate growth rate increases from 2.4% to 6.4% from pre-1980s to 1981-2000 episode. The aggregate

TABLE 4. Log Income of Different Age Groups in Model a

Year/Age	$a = 23$	$a = 24$	...	$a = 50$
...	...	...	...	...
1979	$23(\alpha - \gamma)$ $+(\beta_{1979} + 1979\gamma)$	$24(\alpha - \gamma)$ $+(\beta_{1979} + 1979\gamma)$	... ...	$50(\alpha - \gamma)$ $+(\beta_{1979} + 1979\gamma)$
1980	$23(\alpha - \gamma)$ $+(\beta_{1980} + 1980\gamma)$	$24(\alpha - \gamma)$ $+(\beta_{1980} + 1980\gamma)$	... ...	$50(\alpha - \gamma)$ $+(\beta_{1980} + 1980\gamma)$
1981	$23(\alpha - \gamma)$ $+(\beta_{1981} + 1981\gamma) + \Delta\gamma$	$24(\alpha - \gamma)$ $+(\beta_{1981} + 1981\gamma)$	... ...	$50(\alpha - \gamma)$ $+(\beta_{1981} + 1981\gamma)$
1982	$23(\alpha - \gamma)$ $+(\beta_{1982} + 1982\gamma) + 2\Delta\gamma$	$24(\alpha - \gamma)$ $+(\beta_{1982} + 1982\gamma) + \Delta\gamma$	... ...	$50(\alpha - \gamma)$ $+(\beta_{1982} + 1982\gamma)$
1983	$23(\alpha - \gamma)$ $+(\beta_{1983} + 1983\gamma) + 3\Delta\gamma$	$24(\alpha - \gamma)$ $+(\beta_{1983} + 1983\gamma) + 2\Delta\gamma$	... ...	$50(\alpha - \gamma)$ $+(\beta_{1983} + 1983\gamma)$
...	...	...	...	...
2000	$23(\alpha - \gamma)$ $+(\beta_{2000} + 2000\gamma) + 20\Delta\gamma$	$24(\alpha - \gamma)$ $+(\beta_{2000} + 2000\gamma) + 19\Delta\gamma$	... ...	$50(\alpha - \gamma)$ $+(\beta_{2000} + 2000\gamma)$
2001	$23(\alpha - \gamma)$ $+(\beta_{2001} + 2001\gamma) + 20\Delta\gamma$	$24(\alpha - \gamma)$ $+(\beta_{2001} + 2001\gamma) + 20\Delta\gamma$	... ...	$50(\alpha - \gamma)$ $+(\beta_{2001} + 2001\gamma)$
2002	$23(\alpha - \gamma)$ $+(\beta_{2002} + 2002\gamma) + 20\Delta\gamma$	$24(\alpha - \gamma)$ $+(\beta_{2002} + 2002\gamma) + 20\Delta\gamma$	... ...	$50(\alpha - \gamma)$ $+(\beta_{2002} + 2002\gamma)$
...	...	...	...	...

growth rate g_t determines $\beta_t + \gamma t$. For $t \leq 1980$, since $g_t = 0.024$, we set $\beta_t + \gamma t = 0.024t$. For $t > 1980$, since age advantage is varing across time, we choose $\beta_t + \gamma t$ recursively. Given $\beta_{t-1} + \gamma(t-1)$, we choose $\beta_t + \gamma t$ such that $\frac{Y_t}{Y_{t-1}} = e^{g_t}$. Notice that in Model a, which is different from the benchmark model, $(\beta_t + \gamma t) - (\beta_{t-1} + \gamma(t-1))$ is not necessarily the same with the growth rate of aggregate income because the age advantage is changing over time. This can be easily seen after comparing the log income of different age groups in 1981 and 1982. From 1981 to 1982, the income of 23-year-old and 24-year-old age group increases by a proportion of $\beta_{1982} - \beta_{1981} + \gamma + \Delta\gamma$. The income of other age groups increases by a proportion of $\beta_{1982} - \beta_{1981} + \gamma$. Therefore, the growth rate of aggregate income between 1981 and 1982 should be larger than $\beta_{1982} - \beta_{1981} + \gamma$ but smaller than $\beta_{1982} - \beta_{1981} + \gamma + \Delta\gamma$.

Figure 6 illustrates the cross-section log income profile in year 1981, 1986, 1991 and 1996 in the benchmark model and model a. In model a, due to stronger cohort effect profile (from

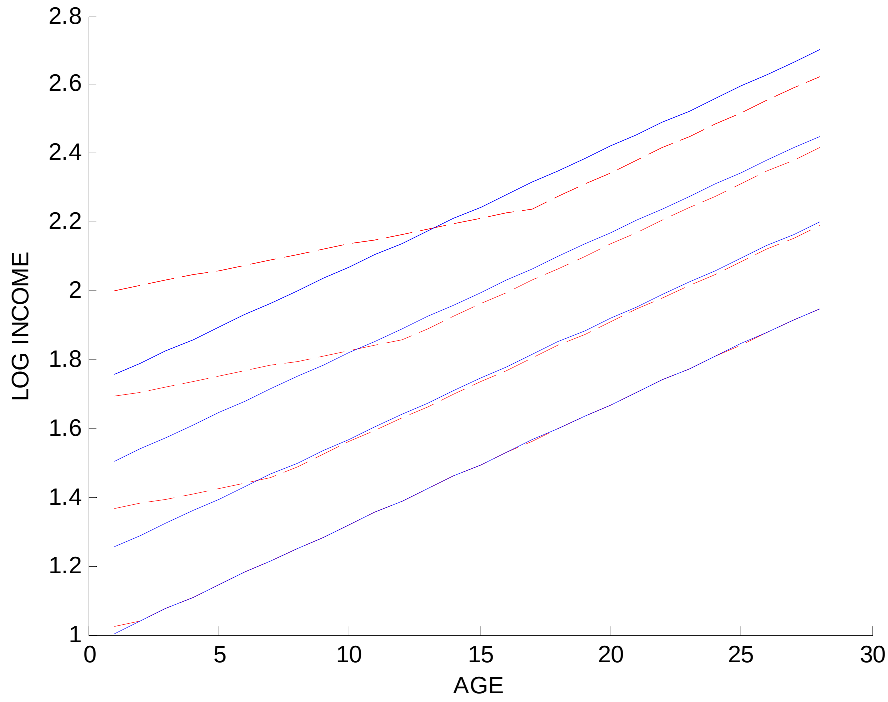

FIGURE 6. Cross section log income in 1981,1986,1991,1996. Blue line is model benchmark model; Red line is model a.

γ to γ') accompanied with faster aggregate income growth, the growth of aggregate income benefits young cohorts more than elder cohorts. Therefore in Figure 6 young people's income is relatively higher than that in the benchmark model. Consequently, for each individual, the lifetime income path in the benchmark model with constant age advantage is steeper than that in model a with time-varying cohort effect profile. This is illustrated in Figure 7.

5.2. **Model b: Time Varying Age Effect Profile.** In contrast to model a, the change of age advantage in model b comes from time varying age effect profile instead of time varying cohort effect profile. In particular, we assume that before year 1981, the profile of age, time and cohort effects is $\{\alpha, \beta_t, \gamma\}$ so that age advantage is constant before 1981. Between year 1981 and 2000, there is an adjustment of age effect profile $\Delta\alpha$. Here $\Delta\alpha < 0$ because faster aggregate income growth is accompanied with weaker age effect profile in this model. We further assume that after 2001 the profile returns to $\{\alpha, \beta_t, \gamma\}$ as in model a. Table 5 illustrates the levels log income of different age groups in multiple years.

We still choose the values of $\{\alpha, \beta_t, \gamma, \Delta\alpha\}$ to match the aggregate growth rate g_t and the rate of change of age advantage in response to the change of aggregate growth rate in data. As in

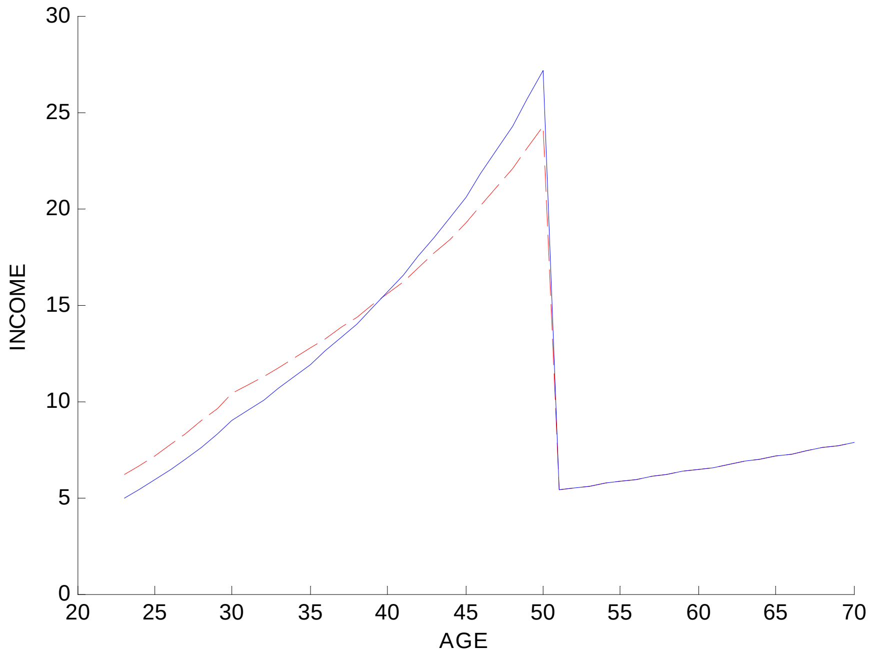

FIGURE 7. Cohort 1970's life time income path. Blue line is the benchmark model; Red line is model a.

model a, we set $\Delta\alpha = -(0.064 - 0.024) \cdot 100 \cdot 0.0067$ to generate the change of age advantage caused by a weaker age effect profile. We choose $\beta_t + \gamma t$ recursively. Given $\beta_{t-1} + \gamma(t-1)$, we choose $\beta_t + \gamma t$ such that $\frac{Y_t}{Y_{t-1}} = e^{g_t}$.

Figure 8 and Figure 9 are the counterparts of Figure 6 and Figure 7. Figure 8 shows that in model b the age advantage is weaker due to a weaker age effect profile. Figure 9 shows that each individual's life income path is steeper in the benchmark model than that in model b due to a weaker age effect profile.

5.3. **Capital Flows in Model a and b.** The model predictions of capital flows are listed in Table 6.

Table 6 shows that the scale of capital inflow in model a and b is much smaller than that in the benchmark model. The benchmark model predicts that the net capital inflow in Korea during 1981 and 2000 should be 4.31 times the real GDP in 1981, which is at odds with data. Model a and model b, however, can explain about 41 percent of difference between the prediction of benchmark model and data. The reason is clear. In a fast growing economy, an individual wants to borrow when he is young to smooth his life time consumption. With fast aggregate income

TABLE 5. Log Income of Different Age Groups in Model b

Year/Age	$a = 23$	$a = 24$	...	$a = 50$
...	...	...	...	...
1979	$23(\alpha-\gamma)$ $+(\beta_{1979}+1979\gamma)$	$24(\alpha-\gamma)$ $+(\beta_{1979}+1979\gamma)$	... ...	$50(\alpha-\gamma)$ $+(\beta_{1979}+1979\gamma)$
1980	$23(\alpha-\gamma)$ $+(\beta_{1980}+1980\gamma)$	$24(\alpha-\gamma)$ $+(\beta_{1980}+1980\gamma)$	... ...	$50(\alpha-\gamma)$ $+(\beta_{1980}+1980\gamma)$
1981	$23(\alpha-\gamma)$ $+(\beta_{1981}+1981\gamma)$	$24(\alpha-\gamma)$ $+(\beta_{1981}+1981\gamma)+\Delta\alpha$	... ...	$50(\alpha-\gamma)$ $+(\beta_{1981}+1981\gamma)+\Delta\alpha$
1982	$23(\alpha-\gamma)$ $+(\beta_{1982}+1982\gamma)$	$24(\alpha-\gamma)$ $+(\beta_{1982}+1982\gamma)+\Delta\alpha$	... ...	$50(\alpha-\gamma)$ $+(\beta_{1982}+1982\gamma)+2\Delta\alpha$
1983	$23(\alpha-\gamma)$ $+(\beta_{1983}+1983\gamma)$	$24(\alpha-\gamma)$ $+(\beta_{1983}+1983\gamma)+\Delta\alpha$	... ...	$50(\alpha-\gamma)$ $+(\beta_{1983}+1983\gamma)+3\Delta\alpha$
...	...	...	...	...
2000	$23(\alpha-\gamma)$ $+(\beta_{2000}+2000\gamma)$	$24(\alpha-\gamma)$ $+(\beta_{2000}+2000\gamma)+\Delta\alpha$	... ...	$50(\alpha-\gamma)$ $+(\beta_{2000}+2000\gamma)+20\Delta\alpha$
2001	$23(\alpha-\gamma)$ $+(\beta_{2001}+2001\gamma)$	$24(\alpha-\gamma)$ $+(\beta_{2001}+2001\gamma)$	... ...	$50(\alpha-\gamma)$ $+(\beta_{2001}+2001\gamma)+19\Delta\alpha$
2002	$23(\alpha-\gamma)$ $+(\beta_{2002}+2002\gamma)$	$24(\alpha-\gamma)$ $+(\beta_{2002}+2002\gamma)$	... ...	$50(\alpha-\gamma)$ $+(\beta_{2002}+2002\gamma)+18\Delta\alpha$
...	...	...	...	...

TABLE 6. Net Capital Inflows*

Data	Benchmark Model	Model a	Model b
0.3	-4.31 or -2.46	-2.43	

*Net Capital Inflows are defined as $\frac{A_{2000}-A_{1981}}{Y_{1981}}$.

growth, young cohorts' borrowing is larger than elder cohorts' saving. Therefore the change of aggregate asset position is negative. This prediction is the same with the prediction of infinitely lived representative agent model. In model a and b, the age advantage is weaker either due to a stronger cohort effect profile or a weaker age effect profile. The benefit of fast aggregate income growth is distributed more to young cohorts. Consequently each individual's life time income path is flatter (as shown in Figure 7 and Figure 9) compared with the benchmark model. A flatter life time income path implies weaker borrowing motive at young and hence results in a smaller scale of aggregate net capital inflows.

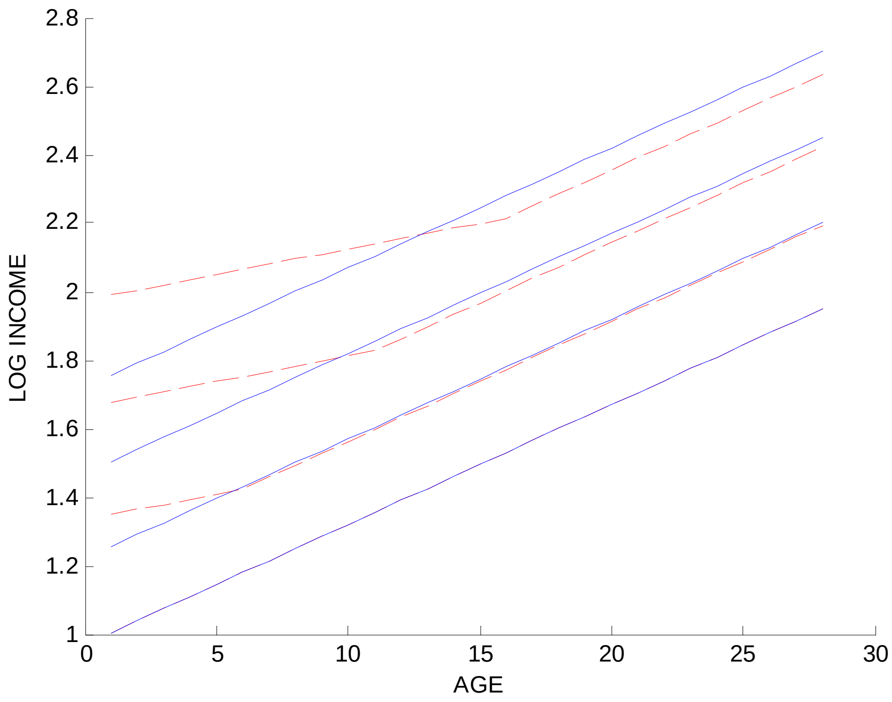

FIGURE 8. Cross section log income in 1981,1986,1991,1996. Blue line is the benchmark model; Red line is model b.

Although we do not know whether the stronger age advantage comes from a stronger cohort effect profile (model a) or a weaker age effect profile (model b), both model a and model b imply the similar scale of net capital inflows. This is because each individual's life time income path are similar in both models (See Figure 7 and Figure 9).

6. CONCLUSION

This paper shows that in predicting the international capital flow of fast growing countries, using infinitely lived representative agent models might induce a large bias. The reason is, the infinitely lived representative agent model only makes use of the information of aggregate data, excluding information of each individual's income path. However, different life time income paths of each cohort can be consistent with the same aggregate data but generate quite different scale of international capital flows. Using the Luxembourg Income Study we find that a country tends to have a flatter cross sectional age-income profile when it is growing faster. We suggest that this flatter age-income profile come from either a time varying cohort effect profile or a time varying age effect profile. On one hand fast aggregate income growth implies that each

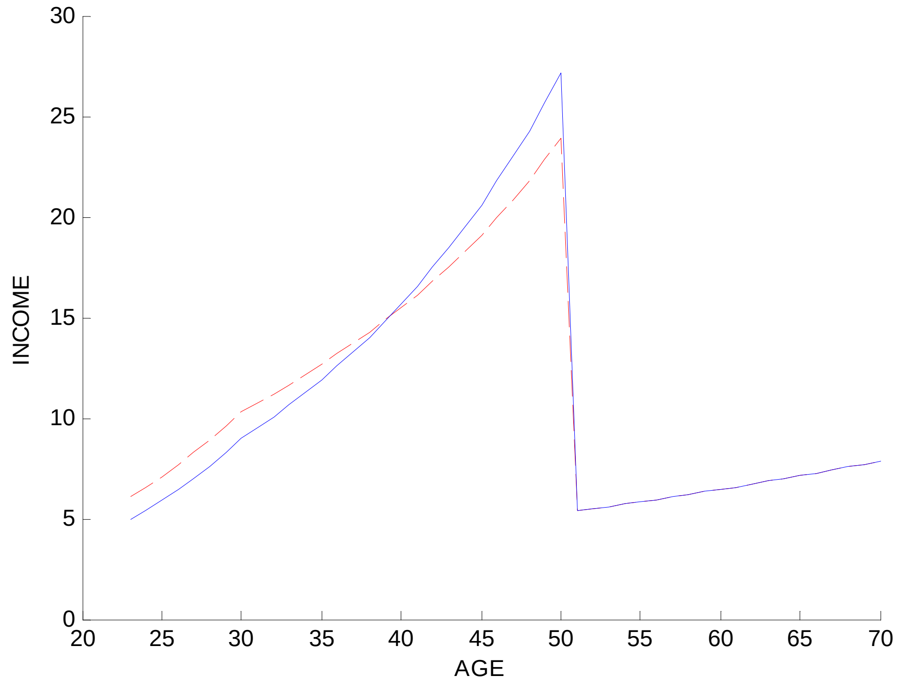

FIGURE 9. Cohort 1970's life time income path. Blue line is model a; Red line is model b.

individual is facing a steep upward sloping life time income path so that he should borrow from abroad to smooth his life time consumption. On the other hand the fast aggregate income growth benefits younger cohorts more so that each individual's life time income growth rate does not increase as much as the aggregate growth rate, which mitigates the borrowing motive. The quantitative result shows that after taking into account the flatter cross sectional age-income profile accompanied with the fast aggregate income growth, the model predicted scale of net capital inflows is substantially closer to data.

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7. Data Appendix

Luxembourg Income Study

Figure 2 is from the Luxembourg Income Study(LIS) data.

Survey Description

LIS data includes income, demographic and expenditure data on person and household levels. The data is adjusted to make different national data equivalent. Datasets are grouped in 5 year intervals(called a wave). There are six waves of data from around 1980 to around 2004. There are also some historical(pre-1980) databases for limited number of countries.

Personal Income

We use the variable PGWAGE in the LIS databases as personal annual income. PGWAGE is defined as the cash wage and salary income (including employer bonuses, 13th month bonus, etc.), gross of employee social insurance contributions/taxes but net of employer social insurance contributions/taxes. Conscript's pay is also included here.

Sample Selection

There are 23 countries in the sample:

Historical: Sweden(SWE), United Kingdom(UK), United States(US);

Wave 1 (around 1980): Australia(AU), Canada(CAN), Germany(GER), Israel(ISR), Norway(NOR), Sweden(SWE), Switzerland(SWI), Taiwan(TAI), United Kingdom(UK), United States(US);

Wave 2 (around 1985): Canada(CAN), Denmark(DEN), Germany(GER), Israel(ISR), Netherlands(NET), Norway(NOR), Sweden(SWE), Taiwan(TAI), United Kingdom(UK), United States(US);

Wave 3 (around 1990) Belgium(BEL), Canada(CAN), Denmark(DEN), Germany(GER), Israel(ISR), Netherlands(NET), Norway(NOR), Sweden(SWE), Sweden(SWE), Switzerland(SWI), Taiwan(TAI), United Kingdom(UK), United States(US);

Wave 4 (around 1995) Belgium(BEL), Canada(CAN), Denmark(DEN), Finland(FIN), Germany(GER), Israel(ISR), Netherlands(NET), Norway(NOR), Sweden(SWE), Sweden(SWE), Taiwan(TAI), United Kingdom(UK), United States(US);

Wave 5 (around 2000) Austria(AT), Belgium(BEL), Canada(CAN), Denmark(DEN), Finland(FIN), Germany(GER), Greece(GRE), Ireland(IRE), Israel(ISR), Netherlands(NET), Norway(NOR), Spain(ESP), Sweden(SWE), Sweden(SWE), Taiwan(TAI), United Kingdom(UK), United States(US);

Wave 6 (around 2004) Brazil(BRA), Canada(CAN), Columbia(COL), Denmark(DEN), Finland(FIN), Guatemala(GUA), Israel(ISR), Korea(KOR), Luxembourg(LUX), Norway(NOR), Sweden(SWE), Sweden(SWE), Taiwan(TAI), United Kingdom(UK), United States(US).

Each observation includes information of PGWAGE, gender, age and weight. We only use the observations in which PGWAGE> 0 and age is between 23 and 50.

Age Advantage

We compute age advantages from LIS databases in two steps:

Step 1: We compute the weighted average income by age for every given wave and given country.

Step 2: We run the linear regression of weighted average income by age(from Step 1) on age for every given wave and given country. The coefficient is the age advantage in that given wave and given country.