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Population Changes and Capital Accumulation: The Aging of the Baby Boom

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Abstract

In this paper I explore the quantitative implications for savings of population aging. In doing so, I pay particular attention to some features that have been partially over-looked in the literature. These features include the details of the population aging process, the initial conditions with respect to assets holdings, and the relation between age and household size. In order to do so, I develop recursive methods capable of dealing with overlapping generations environments where the population is stochastic.

The main findings are: i) If population patterns revert to the averages of the last 50 years, the reduction in aggregate savings due to changes in the age structure of the population is small. ii) If, however, the demographic process is such that fertility patterns remain at their current low levels, then the effects of the aging of the baby boom are very large. iii) Initial conditions matter: both the choice for initial assets and the choice for the mechanism through which current fertility reverts to its long run average have implications for the economic allocations. And iv) The contribution of general equilibrium effects is to exacerbate the reduction of savings since population aging tends to make labor relatively scarce, and, therefore, to reduce rates of return of capital, which in turn reduces savings even further.

KEYWORDS: Savings, Population Aging, Wealth Distribution

1 Introduction

The demographic structure of most western countries is changing. The key feature of this process is that the population is getting increasingly older. The smooth but continuous decrease in mortality together with the reduction in fertility after the huge number of births in the "baby boom" of the 50's and early 60's are the main culprits of this process. The aging of the population brings forth a variety of very important social issues since so many features of individual behavior are age-dependent. Economists have been especially concerned with the fiscal implications of aging, mainly social security. There is a large literature devoted to study the implications of alternative policies that started with the seminal work of Auerbach and Kotlikoff (1987).[1] In general this literature has concentrated its effort in the implications of the policies themselves.

The purpose of this paper is to investigate how certain details in the modelling of populations affected by severe demographic changes shape the answers provided by explicit general equilibrium models of the overlapping–generations variety. The items that I look at in detail are the demographic process that generates the aging of the population and the initial conditions that are used to propagate the economy forward. I find that these features are quantitatively very important and that we should perhaps devote more resources to better understand the demographic processes that are behind the aging of the population.

This paper fills a void in the literature that so far has looked mostly at steady states, or at the transition between steady states. In the few cases that the demographic details have been looked at in some detail (Huang et al. (1997), De Nardi et al. (1998), and Auerbach et al. (1989)) by using official population projections, the initial assets used when computing the transition have been those corresponding to a steady state that is different from the final one to which the economy is converging. Moreover, the literature has taken population projections at face value and has not explored in any detail what would happen under different demographic assumptions (except the occasional differences in growth rates of the population).

[1] Auerbach and Kotlikoff (1984), Auerbach et al. (1991), Auerbach et al. (1989), Hubbard and Judd (1987), Danthine and Surchat (1991), Chauveau and Loufir (1993), Imrohoroğlu et al. (1995), Huang et al. (1997), Bohn (1998), Bütler (1998), De Nardi et al. (1998), Huggett and Ventura (1998), Storesletten et al. (1998), Conesa and Krueger (1999), Fuster (1999).

To make matters concrete I look at the implications of the demographic changes for savings rates because they are a clear indicator of the economic implications of the demographic process. I could have looked at other properties of the allocation, but savings rates nicely summarize the economic behavior of the system. In this paper I make three important modelling choices. The first one is that I assume that all assets are held for life cycle reasons. There is an very good case against this assumption since the seminal work of Kotlikoff and Summers (1981).[2] The reason for this assumption is in part to be in line with the literature of population aging that have always used overlapping generations models with little role for bequests. The reason is also that we do not have yet models that integrate a suitable theory of wealth inequality and a sophisticated demographic structure, although I have no doubt that those models will appear soon. The second assumption that I make is to completely abstract from the government and hence from social security. The last assumption that I make is to have leisure enter the utility function. The first two choices simplify the analysis (actually the first makes it feasible), while the last assumption complicates it. The reason for the last two choices lies in the fact that I am not interested per se in the actual value of the saving rate but in an exploratory investigation of the relevance of certain details of the demographic process and of the initial conditions in shaping the properties of the economic allocations. That these details have fiscal implications is sort of obvious, but what I want to know is whether they also matter for the properties of the equilibrium allocations. The reason to account explicitly for leisure is that I want to have another margin through which agents can adjust to the changes in prices. I believe that by making these assumptions, it is harder for the demographic details to be quantitatively important.

I build a general equilibrium overlapping generations model where agents choose to work, consume, and save and where the demographic process is modeled in some detail. Then, I solve the model and I calibrate a baseline model economy to Spanish data.[3] The nature of the exercise is to propagate

[2]See Quadrini and Ríos-Rull (1997) and Castañeda et al. (2000) for further arguments of what is a good theory of the wealth distribution.

[3]To look at Spain rather than at the U.S. given the data shortage on some dimensions that Spain has is due to the fact that migratory movements have not been very important in the period under study. Spain presents a standard pattern of a baby boom in the late 50's and early 60's together with a sharp reduction of fertility in the eighties and nineties. Compared to the rest of the western countries the demographic process in Spain might be

the model from 1998 onwards and to look at the savings rates along the equilibrium path. To do this, I have to specify not only the calibration details, but also the initial conditions that have to be mapped to 1998 Spain, and that in all cases specify the age structure of the Spanish population in 1998.

It is in the calibration process and in the specification of the initial conditions that the paper focuses on. In particular, I look at various possibilities for the initial distribution of assets and for the future path of fertility and mortality. The initial conditions of the model are the age distribution of the population that is always set to the one in 1998 Spain, an initial age-wealth distribution and the initial values for shocks that are used to forecast fertility. I look at various age–wealth distributions: those of the steady state of the deterministic version of the model, the age–wealth distribution in the U.S. and some age–wealth distributions generated endogenously by the model through simulations. The reason for the latter is that the 1998 age-structure of the population is very different from the steady state age structure, and it does not seem very interesting to use the age-wealth distribution associated to a very different age structure. To get around this I run many simulations of the model until its age–structure is similar to that in 1998 Spain and then use the age-wealth distribution of the simulations as initial conditions. In order to do this a stochastic model is needed, capable of having fertility being subject to shocks. Another contribution of this paper is to develop such a model. In addition, I perform standard robustness analysis on some of the parameter choices and, by looking at a partial equilibrium version of the baseline model economy, I separate the direct demographic effects from those that appear through changes in prices.

The key findings of the paper can be summarized by:

1. If the current drop in fertility is temporary and the demographic process is such that fertility will return to the average of the last fifty or sixty years, then there will be a small reduction in savings rates of around 2% of GNP until the latter part of the twenty first century.

2. The specific set of initial conditions that we choose matter for the allocations. Both the choice for initial assets and the choice for the mechanism through which current fertility reverts to its long run average have implications for the economic allocations.

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seen as slightly delayed, and, perhaps, sharper in its features.

3. If, however, the current drop in fertility is permanent, then there will be dramatic implications for savings rates that will suffer a huge and steady reduction towards to almost zero.

4. The contribution of the general equilibrium effects is to exacerbate the reduction of savings. The reason for this result is that the aging of the population generates a relative scarcity of labor relative to capital. This reduces rate of return of capital (interest rates) and make people to save less.

There is a large literature on the interaction between population and capital accumulation. Among the work that considers fertility as endogenous, Barro and Becker (1989) develop a model where parents are altruistic and there is capital accumulation. They also perform comparative–statics experiments when different parameters of their model change. They discuss the dynamic behavior of their economy and its stability and are able to partially characterize non steady-state behavior. However, their approach to study the dynamics does not easily extend to economies with more than two periods. In Eckstein and Wolpin (1985), parents are not altruistic but their utility depends on their children's consumption. Razin and Ben-Zion (1975) analyze steady states in their formulation that makes the number of children and the utility per child separable in their parents' preferences. Another theoretical analysis of steady states is Willis (1988), who investigates comparative statics under different rates of population growth in a model with exogenous fertility. None of these papers is quantitative in nature, and it is hard to think of suitable extensions of their specifications capable of accommodating the analysis of economies were agents live a large number of periods, and where the initial conditions of the economy do not lead to a steady state. None of this works is quantitative in nature.

There is some work that addresses the interrelations between economics and demographics quantitatively. For instance, Lee and Lapkoff (1988) carefully study of intergenerational transfers empirically both within and outside the family with special attention to the allocation of time. Their study assumes a steady–state equilibrium with constant relative prices. They also compare properties of the transfers in steady states that differ in the rate of population growth and consider some specifications that endogenize fertility.

Among the work that treats fertility as exogenous, but is quantitative in nature, Auerbach and Kotlikoff (1984), Auerbach and Kotlikoff (1987), Auerbach et al. (1991) and, in particular, Auerbach et al. (1989), consider the effects of demographic transitions. Their work is similar in spirit to the work in this paper. They use OEDC population projections to propagate the population for a certain number of years. After that, population is reset to replacement levels by fixing birth rates equal to death rates. The key differences between these papers and the present study are (*i*) how the population predictions are made, and (*ii*) the assumptions of the nature of the economy. In their world, population patterns include first a drastic aging of the population as predicted by the OEDC, but, after that, they assume that population reverts to a replacement regime, with a population pyramid shaped as a perfect rectangle. Their model is also deterministic, which, as we will see, prevents the use of simulations to generate endogenously assets distributions.[4] They also find a rise in wages associated to the aging of the population. As they are interested in the relative welfare of different generations, the increase in wages for the relatively scarce young people in the near future, will counteract the reduction in their welfare associated to the higher burden of social security. An excellent survey on other recent work on computational models of social security isImrohoroğlu et al. (1999). [5]

[4]Their procedure consists on calibrating a set of parameters so that the steady state matches certain properties of what is thought of as initial conditions. Next, another set of parameters is also chosen so that they have a steady state that corresponds to what is thought of as the long run of the economy. A change in the initial parameters is then postulated. This change might include the number of births for a hundred years, or a vector of policy parameters. After a certain number of periods the model's parameters become those associated with the final steady state. A large number of periods is taken to be associated with the transition, after which the economy is assumed to have converged to the new steady state. This procedure is ingenious, as it reduces the computation of the equilibrium to solving three nonlinear systems of equations: the two steady states, and the transition.

[5]Other work with computational overlapping generations models includes Danthine and Surchat (1991) also study the quantitative implications of population changes for the case of Switzerland. However, in their model agents are naive–they are incapable of properly predicting prices. Ríos-Rull (1996) analyzes quantitatively the business cycle behavior of large overlapping generations economies where the population is in steady state but the economy is not (both fertility and mortality are exogenous and constant). The model economies studied are stochastic in nature and the methods used enable the computation of equilibria that are not converging to deterministic steady states. Finally, another important contribution is Laitner (1990) computes transition paths from one steady state to another after a surprising change in one of the model's exogenous parameters by linearizing around the steady state. Although he only applies his methods to changes in the tax structure, they could be adapted to changes in fertility that change the age structure of the population slowly towards a new steady state.

Section 2 describes the model, with special emphasis on its demographics. Section 3 defines the equilibrium, and discusses the advantages of a recursive definition. In Section 4 I discuss the nature of the computational experiments, the choices for initial conditions and for the specific versions of the model that I use. Section 5 describes the calibration of the baseline model economy, both its demographics (mainly that mortality stays constant and that fertility reverts to its historical average) and its economics, that is the one with the preferred parameterization and that acts as a reference with respect to the other experiments. Section 6 describes the properties of the baseline model economy as is propagated towards the future. Section 7 discusses the implications of a different demographic scenario than that of the baseline, that fertility will remain at its current low levels. The robustness exercises are performed in Section 8 while Section 9 concludes. The Appendix includes details of the computational procedures (Section A) and some tables with the parameters chosen for the baseline model economy and with interesting statistics referred to in the text (Section B).

2 The Model

2.1 Demographic Structure

The model consists of overlapping generations of agents that live up to I periods. The probability of surviving between age i and age $i+1$ is s_i. Therefore, the unconditional probability of reaching age i is $s^i = \prod_{j=1}^{i-1} s_j$. These probabilities are assumed to be constant over time. The number of births is, however, stochastic. Let $x_t \in \mathbf{R}^{\mathbf{I}}$ be the age distribution across age groups in period t. Then $x_{1,t+1}$, the number of newborns the following period, is the product of x_t and the vector of age specific fertility rates $\phi_{i,t}$. Following Lee (1974), this product can be approximated by $x_{1,t+1} = \sum_i \phi_i x_{i,t} + z_t$, where ϕ_i are average age specific fertility rates, and where z_t is an error term that follows some *ARMA(p,q)* process. The population changes through changes in fertility, which subsequently induce changes in the age distribution. Let $\hat{\mathbf{\Gamma}}$ be the matrix representing the law of motion of a population with the same survival and fertility rates as above but with deterministic fertility. Then,

$$\hat{\mathbf{\Gamma}} = \begin{pmatrix} \phi_1 & \phi_2 & \phi_3 & \dots & \phi_I \\ s_1 & 0 & 0 & \dots & \dots \\ 0 & s_2 & 0 & \dots & \dots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & s_I & 0 \end{pmatrix}$$

It is a well-known mathematical property that the biggest eigenvalue of this matrix determines the long run rate of growth of the population regardless of the initial conditions, and that its associated eigenvector is the age distribution to which the population converges (note that the Perron-Frobenius Theorem guarantees that both are non-negative), often denoted the stable population. If the largest eigenvalue is different from one, the population will be non-stationary. To achieve stationarity, fertility and survival parameters are renormalized by dividing every element by the largest eigenvalue of $\hat{\mathbf{\Gamma}}$. The normalized matrix has a biggest eigenvalue of one and its associated eigenvector is its stationary population. We can think of this eigenvector as the deterministic steady state of our stochastic population.

The law of motion of the stochastic population together with the shock for fertility following, for example, an *AR(2)* , is defined in the following way:

$$\begin{pmatrix} x_{1,t+1} \\ x_{2,t+1} \\ \vdots \\ x_{I,t+1} \\ z_{t+1} \\ z_t \end{pmatrix} = \begin{pmatrix} & & \rho_1 & \rho_2 \\ & \hat{\mathbf{\Gamma}} & 0 & 0 \\ & & \vdots & \vdots \\ 0 & \dots & \rho_1 & \rho_2 \\ 0 & \dots & 1 & 0 \end{pmatrix} \begin{pmatrix} x_{1,t} \\ x_{2,t} \\ \vdots \\ x_{I,t} \\ z_t \\ z_{t-1} \end{pmatrix} + \begin{pmatrix} \nu_{t+1} \\ 0 \\ \vdots \\ 0 \\ \nu_{t+1} \\ 0 \end{pmatrix}, \tag{1}$$

where the ρ's denote the coefficients in the $AR(2)$ representation of z_t and where ν_{t+1} is the innovation to the process. I write this compactly, denoting next period's variables with primes, as $\binom{x'}{z'} = \Gamma \binom{x}{z} + \nu'$.

2.2 Preferences and Technology

After birth, the agents remain children for a number of periods I_0 in which they do not consume, nor work, nor make any decision. After these first periods, they become adults and have standard preferences over streams of consumption and leisure for the remainder of their life. Agents are endowed with an age–dependent profile of efficiency units of labor, denoted ϵ_i. Each period agents have one unit of time that the can transform into leisure or efficient units of labor.

Preferences of an agent born in period t can be summarized by the following utility function where the expectation is taken with respect to the moment where the agent is first capable of making decisions:

$$E_{t+I_0} \left\{ \sum_{i=I_0+1}^{I} \beta^i s^i \, u_i \left(c_{i,t}, l_{i,t}\right) \right\}, \tag{2}$$

where $(c_{i,t}, l_{i,t})$ are the consumption and leisure of an agent of age i born in period t. β^i can be given at least two interpretations; it could represent pure time preference, but it could also reflect changes in family size (so that a given consumption is enjoyed differently at different ages).

Capital and efficiency units of labor transform into output through a deterministic neoclassical production function. Capital depreciates at rate δ.

2.3 Market Structure

Every period there are spot markets for labor and the consumption good. Agents can also accumulate real assets, whose total is the amount of real capital in the economy. There are no state–contingent markets. If there were, agents could write contracts contingent on the period's fertility rate. The computational methods for finding the equilibrium in a complete–markets world are cumbersome, and recent research (Ríos-Rull (1994) in overlapping–generations models with aggregate shocks, Krusell and Smith (1997) in models with idiosyncratic as well as aggregate shocks, and Storesletten et al. (1999) in overlapping–generations models with idiosyncratic as well as aggregate shocks) indicate that the differences implied by these two market structures are quantitatively unimportant as the resulting risk premium is minute.

There are, however, annuities markets to cover the eventuality of early death. The simplest implementation of annuities is to allow agents to write a contract with the members in their own cohort that make the survivors share the wealth or debts of those who die prematurely. I make this assumption because closing this market implies choosing an arbitrary important alternative, and this paper does not investigate the implications of this assumption.

The real assets that agents accumulate and whose return they collect may be negative.[6]

These considerations imply that every period, the budget constraint of an age i agent born in period t is:

$$c_{i,t} + y_{i,t} = a_{i,t}(1 + r_{t+i-1}) + (1 - l_{i,t})\ \epsilon_i\ w_{t+i-1}, \tag{3}$$

$$a_{i+1,t} = y_{i,t}/s_i. \tag{4}$$

Here, $y_{i,t}$ is gross savings of the age i agent born in t, and $a_{i,t}$ is his wealth, while r_{t+i-1} is the rental price of capital (net of depreciation), and w_{t+i-1} is the price of one unit of efficient labor in period $t+i-1$. Note that the second constraint reflects the existence of the annuities markets introduced above.

3 Equilibrium

I define equilibrium recursively. This is particularly appropriate given that there are shocks to the system.[7] As an added bonus for using recursive methods, the objects that I define are precisely those objects that I calculate in the computational process. The literature typically either looks only at steady states, where the prices are constant, or it defines equilibria as sequences. Equilibria defined as sequences can only be computed when they converge to a steady state when it becomes a well–behaved system of finitely many equations (see Auerbach and Kotlikoff (1987) and Laitner (1990)).

At any point in time, the economy is characterized by a certain distribution of the population x, by the values of the fertility shock needed to make accurate predictions of its future values z, and by the assets owned by individuals in each age group k. The vector $(x, z, k) \in S \subset I\!R_+^I \times I\!R^M \times I\!R^{I-I_0}$ is the state of the economy, where **M** is the number of lags in the process for z required to predict its future values. It depends on the order of the *ARMA* process. The individual state is $(x, z, k, a) \in \tilde{S} = S \times I\!R$, where the last element is the agent's own asset holdings. The definition includes a set of age-specific value functions, decision rules, and the aggregate law of motion for capital. Formally:

[6] Negative assets can be thought of loans with returns that are perfectly correlated with the rate of return of capital.

[7] The notion of recursive equilibria is partly based on the work of Prescott and Mehra (1980) and Ríos-Rull (1996).

Definition 1 *A Recursive Competitive Equilibrium for a closed economy is a set of value functions* $v_i : \tilde{S} \mapsto I\!R$*, for* $i = I_0 + 1, \ldots, I$*, a set of decision rules for gross savings, consumption and leisure* $y_i : \tilde{S} \mapsto I\!R$*,* $l_i : \tilde{S} \mapsto [0,1]$*,* $c_i : \tilde{S} \mapsto I\!R_+$*, for* $i = I_0 + 1, \ldots, I$*, laws of motion for the population,* Γ*, and for the capital stock* $G : S \mapsto I\!R^{I-I_0}$*, and functions for prices* $w : S \mapsto I\!R_+$*, and* $R : S \mapsto I\!R_+$*, for aggregate capital* $K : S \mapsto I\!R_+$*, and for aggregate labor input* $N : S \mapsto I\!R_+$*, such that:*

(i) The allocation is feasible, i. e. for all$(x, z, k) \in S$*,*

$$\sum_i \left(y_i(x, z, k, k_i) + c_i(x, z, k, k_i)\right) x_i = f\left(K(x, z, k), N(x, z, k)\right).$$

(ii) Prices are competitively determined, i.e., they are the marginal productivities of the factors of production:

$$\begin{aligned} r(x, z, k) &= f_1\left(K(x, z, k) - \delta, N(x, z, k)\right), \\ w(x, z, k) &= f_2\left(K(x, z, k), N(x, z, k)\right). \end{aligned}$$

(iii) Given the laws of motion of the aggregate state variables, Γ*, and* G*, the decision rules of the agents* y_i*,* c_i*, and* l_i*, solve their maximization problem:* $\{y_i(x, z, k, a), c_i(x, z, k, a),\ l_i(x, z, k, a)\} \in$

$$\arg\max_{y,c,l} \qquad u_i(c, l) + \beta_i s_i\ E\left\{v_{i+1}\left(x', z', k', a'\right)|z\right\}$$

$$\begin{aligned} s.t. \qquad k' &= G(x, z, k), \\ \begin{pmatrix} x' \\ z' \end{pmatrix} &= \Gamma \begin{pmatrix} x \\ z \end{pmatrix} + \nu, \\ c_i, +y_i &= a\ \left[1 + r\left(x, z, k\right)\right] + \left(1 - l_i\right)\ \epsilon_i\ w(x, z, k), \\ a' &= y_i / s_i. \end{aligned}$$

(iv) The value functions are generated by the decision rules of the agents and by using the fact that $v_{I+1}(x, z, k, a) = 0$*:*

$$\begin{aligned} v_i(x, z, k, a) &= u_i\left(c_i(x, z, k, a), l_i(x, z, k, a)\right) + \\ &\qquad \beta_i s_i E\ \left\{v_{i+1}\left(\Gamma\begin{pmatrix} x \\ z \end{pmatrix} + \nu, G(x, z, k), y_i(x, z, k, a)/s_i\right)|z\right\}. \end{aligned}$$

(v) The law of motion of the capital stocks is generated by the decision rules of the agents:

$$G_{i+1}(x, z, k) = y_i(x, z, k, k_i)/s_i.$$

(vi) Aggregate functions K, and N, are generated by aggregation and the decision rules of the agents:

$$\begin{aligned} K(x, z, k) &= \sum_i x_i k_i, \\ N(x, z, k) &= \sum_i x_i \ (1 - l_i(x, z, k, k_i)) \,. \end{aligned}$$

Equilibrium conditions for a small open economy with internationally determined factor prices at levels $\bar{r}$ and $\bar{w}$ can be defined in a similar way with minor changes. These changes pertain to condition *(i)* that is not required now; the marginal productivities of condition *(ii)* which are required to equate the internationally determined prices; and in *(vi)*, where the first of the two conditions need not hold, as international capital movements will make prices be the same across countries.

4 The Nature of the Experiments

As stated above, this paper explores the implications of a variety of demographic and initial conditions for the study of life cycle savings. To do this, a number of experiments have to be performed. I start by constructing a baseline model economy designed to match some crucial features of the Spanish economy. I propagate this economy from different initial conditions and compare the implications of those initial conditions. All the initial conditions share the same population age distribution, that of Spain in 1998, but differ in the asset distribution and in the lagged values of the demographic shocks that affect the future path of fertility.

The different set of initial conditions for assets include those generated endogenously by the model in the steady state, the U.S. age–wealth distribution (there is no available data for Spain), and others that are obtained by simulating the model economy. The rationale for the latter require some explanation: we do not observe the actual wealth distribution in Spain. Moreover, the empirical wealth distribution is not consistent with a pure life-cycle model (see Quadrini and Ríos-Rull (1997), and Castañeda et al. (2000)), and the current population distribution is very far from the steady state, so it is hard to argue that the wealth distribution of the steady state is appropriate as an initial condition when the accompanying population distribution is so different. Therefore, I simulate the model for a large number of periods and use as initial conditions the asset distribution that prevails in the model economy when the simulated population is close to the actual age distribution of 1998 Spain. With respect to the initial values for the fertility shock, I use zero (its unconditional average) when using steady state or empirical asset distributions as initial conditions and I use those generated by the simulations themselves when using simulated asset distributions. Obviously, when propagating the model into the future, I shut off any further innovations to the shocks to the system.

I run additional experiments to ask a variety of other questions. The first one (Section 7) is particularly important as it asks a fundamental question that has to do with the nature of the current level of fertility. What are the implications of fertility not recovering and staying at its current low levels? I then run a variety of experiments under the heading of robustness analysis (Section 8) where I explore the implications of alternative assumptions both about demographics and economics. In Section 8.1 I ask what are the implications of a further decrease in mortality. In Section 8.2 I explore the implications of higher productivity growth, by looking at a model where the growth rate is the average since 1950 rather than the average of the last twenty years as it is assumed in the baseline model economy. Sections 8.3 and 8.4 explore how the answers to our main question depend on specific assumptions about preferences (intertemporal elasticity of substitution and relative value of leisure versus the consumption good). Finally, I run an additional experiment with constant exogenous prices. An interpretation of this experiment is that the demographic processes in the world as a whole compensate each other so that there are no price fluctuations.

Next, I describe the calibration of the baseline model economy.

5 The Baseline Model Economy

To determine the parameterization of the baseline model economy, I use Spanish demographic information, and as much as possible Spanish economic data, but when the latter is unavailable, I use their U.S. counterpart. I start describing the modelization of the demographics in Section 5.1 and then I follow with the economics in Section 5.2.

5.1 Demographics of the Baseline Model Economy

5.1.1 Mortality

The Spanish population has a very high life expectancy (second only to Japan): 77.93 years in 1994.[8] This is almost a year higher than in the rest of Europe. To properly model this, one has to use a very large number for the maximum life length, more than is standard in the literature.[9] I use 100 years. A period is set to be five years so that the maximum number of periods that an agent lives is 20. However, all numbers are presented in yearly terms for ease of comparison. Agents become adults at age 21, when they start making decisions about how much to work, consume, and save.[10] However, they can become parents before that age (as they do in the data). Probabilities of survival into the next age group are taken to be those of the Spanish population for 1994, the latest for which there are definitive numbers.

5.1.2 Fertility

With respect to fertility, recall that following Lee (1974) the process for fertility can be written as:

$$x_{1,t+1} = \sum_i \phi_i \, x_{i,t} + z_t$$

where ϕ_i is an age–specific fertility rate. The ϕ's were taken to be average from the 1922-2000 period divided by the rate of population growth jointly implied by those same fertility rates and by the age specific mortality rates of 1994. Note that I have assumed that there is no migration.[11] z_t, the residual

[8] Source: INE.

[9] Auerbach and Kotlikoff (1987) use 74, Conesa and Krueger (1999) use 85, and Storesletten (2000) uses 90.

[10] This may seem too old. However, Spaniards are well known for living at home until a very high age.

[11] As it will be seen later in the context of endogenous assets, this assumption is not as unrealistic for the Spanish society as it would be for the U.S. The current Spanish age distribution can be obtained from simulations using the law of motion obtained from its historic fertility rates and its current mortality rates. This is not the case for the U.S.

process for fertility, was estimated as a univariate process, and it turned out that the best match was an AR(2), the same process that Lee (1974) found for the U.S. The values estimated for the parameters are $\rho_1 = 1.09$ and $\rho_2 = -0.72$, with a standard deviation of innovations of 0.495. This comes to no surprise: AR(2) processes with complex roots, as this one is, typically generate long cycles. This is exactly the behavior of fertility since World War II for most western countries as there is both a baby boom and a recent decay in fertility.

5.1.3 Population Path

The process of the population implies a rate of growth of the population of 0.83% annually. This demographic regime will be referred to as historic or average fertility.

Figure 1 shows the path of the population age distribution implied by this process starting with the actual 1998 age structure and setting the variance of the fertility shocks equal to zero. There are some important characteristics that should be noted. First, the fact that fertility switches dramatically to its historical average implies that the model forecasts fertility to be much higher than in the recent past. Consequently, the fraction of the population under 20 years of age, that now stands at around 23%, is forecasted to grow until its stable level of around 32%. Note that there is a small baby bust associated to the echo of the current low number of children, that oscillates in a vanishing wave kind of fashion as the model moves towards the future. The fraction of the population older than 65, currently set at around 16% will not grow much more than that. In fact, it will peak at just below 20% in 2045. It is interesting to note that the dependence ratio (population under 20 plus population over 65 over population in working age) will have an early peak in 2020 and will be high again after 2045. It is also interesting to note that the very low fertility of the last twenty years of the twentieth century has its influence on the following century through a small series of booms

where the large number of immigrants affects the age distribution to the extent that it is impossible to generate the current age distribution from simulations that only use data on fertility and mortality rates. The assumption of no immigration can be relaxed in a variety of ways. For instance, under the assumption of an existing infinitely sized set of prospective immigrants it can be relaxed by stating an immigration policy that depends linearly on the age distribution or on features of the wealth distribution. See Storesletten (2000) for an excellent study of the role of migrations with an extended version of the OLG model.

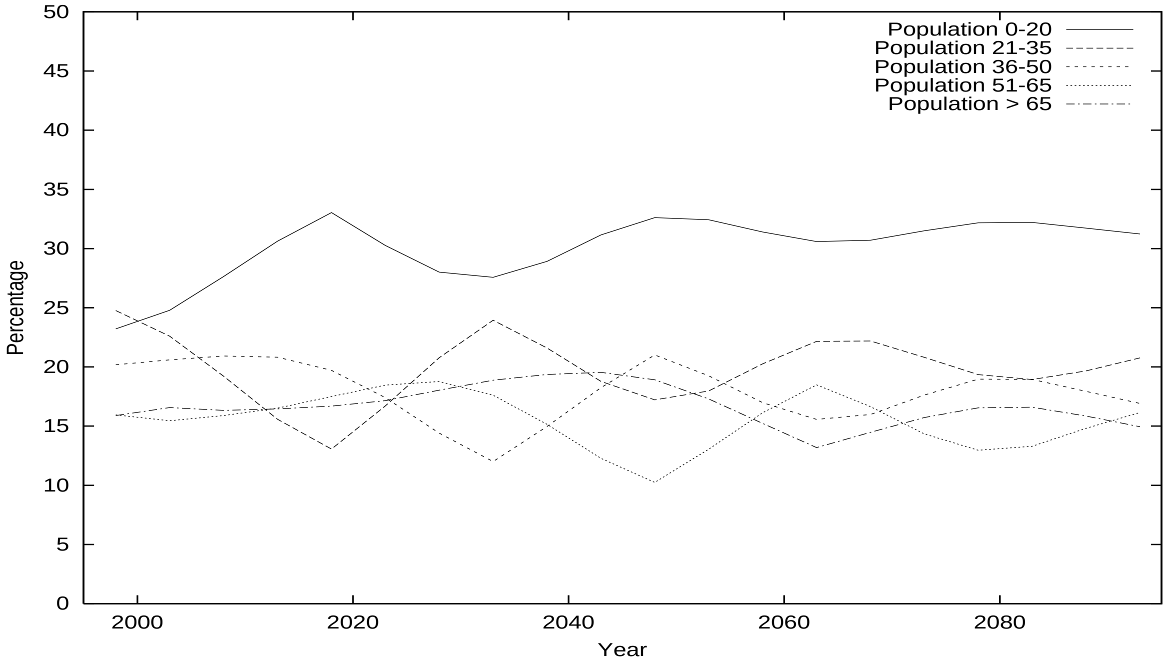


Figure 1: Population Structure in the Baseline Scenario.

and busts that show up more clearly in the middle age groups.[12] But this overall picture is of very little change. The population age structure is not bound to be dramatically different from the one today under the demographic assumptions of the baseline model economy.

5.2 Economics of the Baseline Model Economy

Calibration is not parameter picking. It is a process where the parameters are chosen so that certain statistics of the model economy have the values desired by the researchers. The target values are chosen so that the model economies have certain properties in common with the economies under study.

The key target for the calibration of the baseline model economy is the capital (wealth) to output ratio for which I choose a value of 3.0 (see Castañeda et al. (2000)). I chose an investment–to–output ratio of one fifth. Productivity growth is the average growth rate of the Spanish output to employment ratio since 1980, which is 2.09%.[13] The labor share of income is set at 0.64, larger than recent assessments of this statistic that include the public sector. When we abstract from public capital a value of labor share of .64 or even higher seems appropriate. The assessment of leisure is chosen so that households never work more than 30% of their time, even at their peak. I have not been able to obtain a series for age specific wages (that are needed to construct the age specific efficiency units of labor) of Spanish workers, so I have used American data. In particular, I used the series obtained by Gary Hansen from CPS data.[14] I also make some adjustments for family size that assumes that consumption enjoyed is an age-specific fraction of consumption spent. In particular, I adapt the coefficients used in Cubeddu and Ríos-Rull (1996) (who in turn obtains them from the household equivalence scales of the OEDC) and I adapt them to account for the fact that this is a one sex model.

As is traditional in growing economies, I use utility and production functions that are consistent with a balanced growth path. In particular, I use

[12] These oscillations occur for all ages. The middle age groups are the smallest in length so they show the oscillations more sharply.

[13] The value of this parameter is obtained from David Taguas who maintains this series, first published as Corrales and Taguas (1989).

[14] Hourly wages by age and sex groups are used to obtain an index of relative efficiency across individuals of different ages that is later aggregated by sex and decomposed into the age groups that I use in this paper. See Hansen (1986) and Ríos-Rull (1996) for details.

a constant relative risk aversion instantaneous utility function and a Cobb-Douglas production function with exogenous productivity growth. The specific utility function that I use is

$$u_i(c_{i,t}, l_{i,t}) = u\left(\frac{c_{i,t}}{\eta_i}, l_{i,t}\right) = \frac{\left(\left(\frac{c_{i,t}}{\eta_i}\right)^{\alpha} \; l_{i,t}^{1-\alpha}\right)^{1-\sigma} - 1}{1-\sigma}, \qquad \text{for all } i \in \{I_0, I_0{+}1, I\}.$$

where $\{\eta_i\}$ is the vector of adjustments for family size that account for the consumption of the children.

For the coefficient of risk aversion, also referred to as the inverse of the intertemporal elasticity of substitution,[15] the choice is not clear Auerbach and Kotlikoff (1987) describe the literature and choose a value of 4 as a compromise between different studies, which is also the value that I chose.[16]

Table 1 shows the parameter values used to calibrate the baseline model economy as well as the age structure of the population in 1998.

6 Findings: The Baseline Model Economy

I start describing the main features of the baseline model economy as it is propagated towards the future. I use mainly figures since they are a better tool to see what happens.[17] The figures show the paths of relevant variables as predicted by the model between 1998 and 2093.

Our main objects of interest are savings rates. Figure 2 shows the paths for savings rates for the baseline model economy under various initial conditions, all of which have patterns in common: the aging of the population associated to the low fertility of the last 25 years is such that current savings are slightly above their long run average, and the oscillations of the savings rate are not very large. The savings rates for most of the initial conditions oscillate between 7.5% and 10.5%. There will be also a continuous decrease of savings until 2010, and then, around 2035, savings will increase for the next 25 years, after which there is a slow decrease towards the long run level of 9.8%.

[15] In economies with leisure this convention of names is confusing. Note that for retired households the actual coefficient of risk aversion is not σ, but $\alpha\,\sigma$. I simply use those words to refer to the parameter σ.

[16] See Ríos-Rull (1996) for a more detailed discussion.

[17] The data upon which these figures are based are available in `http://www.ssc.upenn.edu/~vr0j/eicpsd/`.

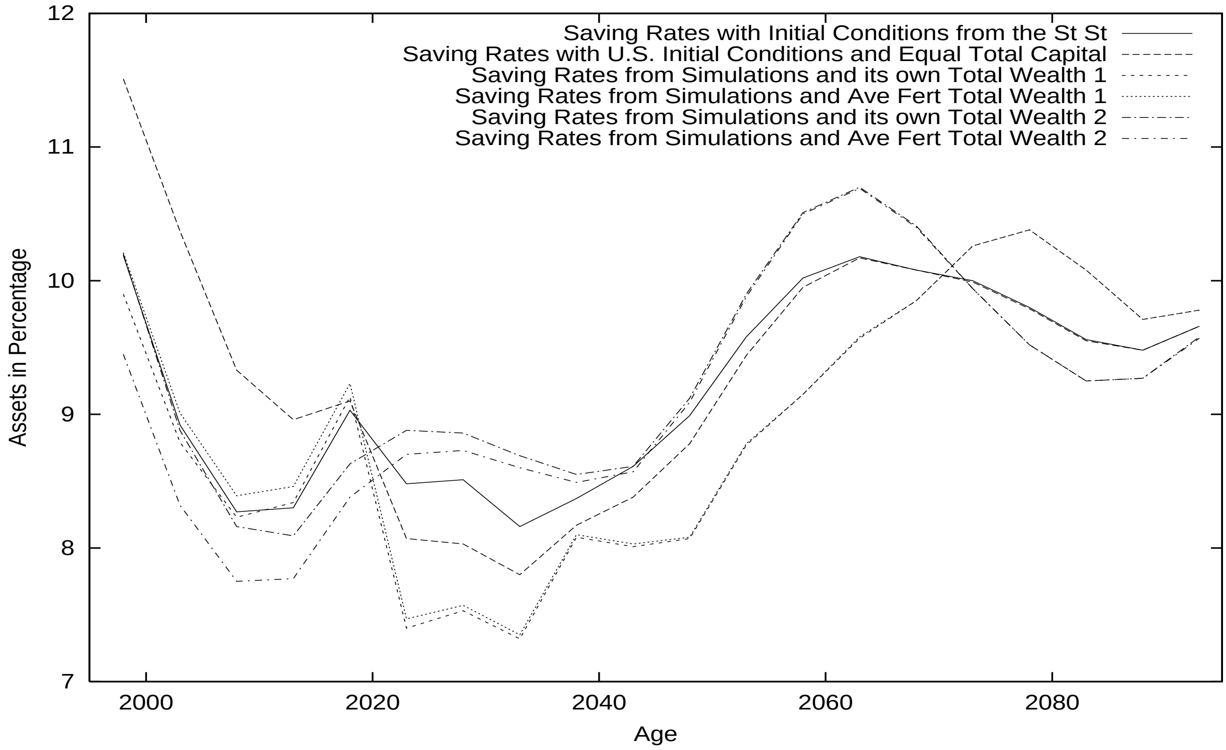


Figure 2: Savings rates for different initial wealth distributions for the baseline economy.

Besides those patterns that are common to all initial conditions there are some specificities depending on the initial conditions. To analyze them, we start by discussing those various initial conditions in detail; the asset distributions and the recent values for the fertility shocks.

6.1 The different initial conditions

There are three alternative types of aggregate variables, the age structure of the population, the age-wealth distribution, and the lagged values of the shocks that affect future fertility through their role in the AR(2). So specifying initial conditions means choosing values for each of the three vectors. In all the experiments, the initial age structure of the population is the same, that observed in 1998 Spain. The initial conditions that I use differ in the choices of wealth structure and values for the lagged shocks.

The initial conditions that I use can be divided in three types: the steady state age-wealth distribution, the U.S. age-wealth distribution and those generated by the model through simulations. I now describe them in detail.

The steady–state age-wealth distribution as initial condition. I use the age-wealth distribution of the steady state of the economy as one of the possible initial conditions. The values of the shock are set to zero. There is no especial rationale for using this age-wealth distribution as initial condition except for the fact that it is very easy to calculate; researchers often claim that they do not have any particular bias when they use it (for example, see Auerbach et al. (1989) and Conesa and Krueger (1999)).

The age-wealth distribution of the U.S. as initial condition. Here we use the age–wealth distribution actually observed.[18] I normalize the U.S. age-wealth assets so that total wealth is the same as for the previous initial conditions. The differences between the U.S. asset distribution and the steady state assets distribution are, perhaps surprisingly, not very large, as shown in Figure 3.[19] The graph is constructed so that total wealth is the same for the two distributions.[20] The graph shows that the elderly in the

[18] In our case, the age wealth distribution observed in the U.S. is the one found in the 1998 Survey of Consumer Finances.

[19] This information is also available in Table 2.

[20] This means that the area below the curves weighted by the population in each age group is the same. Obviously, there are many more people below 65 than above 65 years

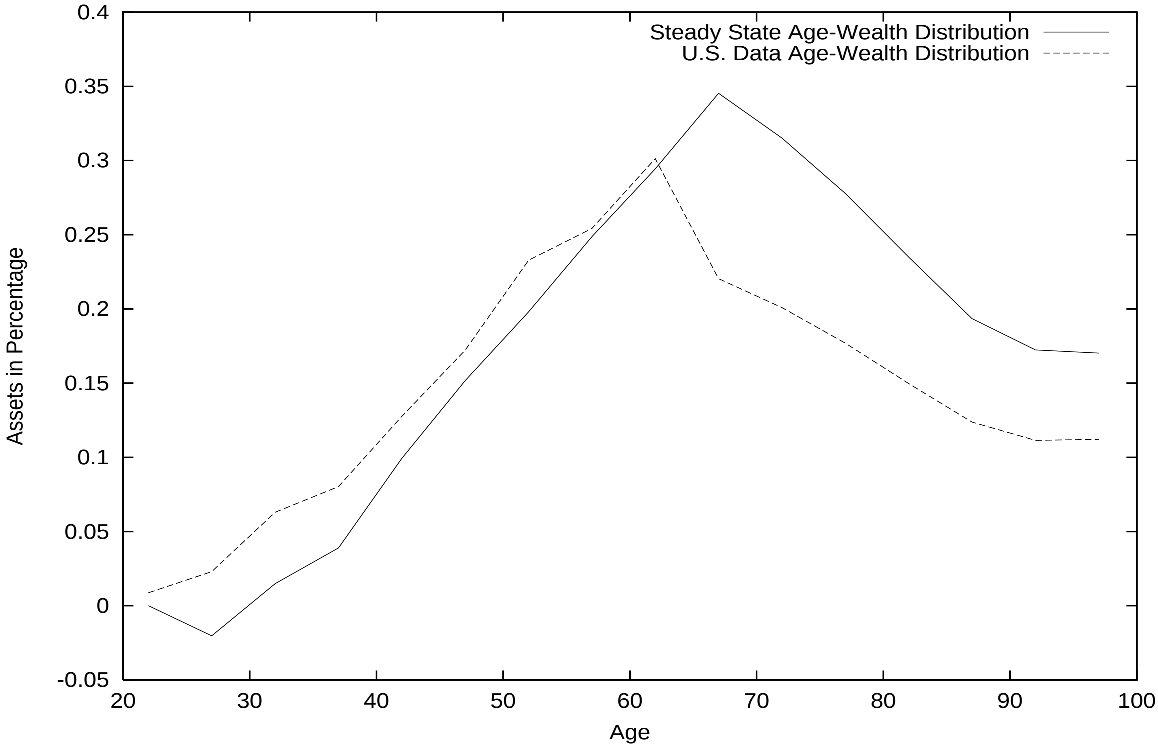


Figure 3: Steady State Asset Distribution for the baseline economy and U.S. Asset Distribution.

U.S. hold less assets than what is predicted by the pure life–cycle theory used in this paper, but overall, the two profiles are not very different. The main difference between the two distributions is that, in the model, assets are held later in life, but the steepness of the age-assets profile is quite similar, showing, perhaps the importance of the social security system in shaping the age-assets profile. This is relatively surprising because of the often argued fact that the U.S. asset distribution is not well accounted for by life cycle reasons. This point was argued pretty forcefully and convincingly by Kotlikoff and Summers (1981), and more recently by Castañeda et al. (2000) in their theory of the U.S. asset distribution. Of course, it is important not to forget that in the U.S. there are large differences in assets held within cohorts while in this OLG model there are none.

Initial conditions obtained through simulations of the model. The population age-structure in Spain in 1998 is very different from that implied by the model that we have used to model it. The reason is, as I have said above, that fertility in the last twenty–five years has been well below its long–run average and the model assumes that fertility will revert to this long run average. For this reason, perhaps the age-wealth distribution of the steady state is not the most appropriate one to associate with the population age structure of 1998. To obtain an age–wealth asset distribution predicted by the model but consistent with the 1998 age-structure of the population, I simulated the model a large number of periods until the age distribution of the simulation was similar to that of 1998 Spain. Once this is done I then recorded the age–wealth profiles at that point and then used them as initial conditions. These simulations do not necessarily yield the same total assets as in the previous sets of initial conditions. Therefore, for the sake of comparison, I looked at simulated wealth distributions both normalized and normalized to the same total wealth as that implied by the steady state age-wealth distribution. Moreover, because these simulations include sampling error, I looked at more than one simulation. Figure 2 includes two simulations, labeled 1 and 2, each generating two sets of initial conditions depending on whether or not total wealth is normalized. Finally, the actual simulations carry different initial conditions for the values of past shocks that are used to forecast fertility. Those used are the ones implied by the simulations themselves.

of age.

Differences in the initial values for the shocks turned out to be quite important: Figure 4 shows the predictions of the model for the shares of population under 20 and over 65 years of age starting from the same age structure that of 1998 Spain and differing in the value of the initial shocks that are used to propagate fertility. The system is stationary so the three forecasts will eventually converge, but, as Figure 4 shows, it takes a long time to do so. Notice that the population over 65 is quite smooth and this is partly due to the fact that it is the aggregate of a large number of ages. The population under twenty belongs to a more compact set of age groups and we can see clearly the oscillations in its size. It is also interesting to note that the two simulated paths of initial conditions are quite different. The reason is that the shocks have different signs: one of them starts forecasting a boom and then a bust of births while the other starts with the bust. In any case we see that there may be large differences in the path of the population.

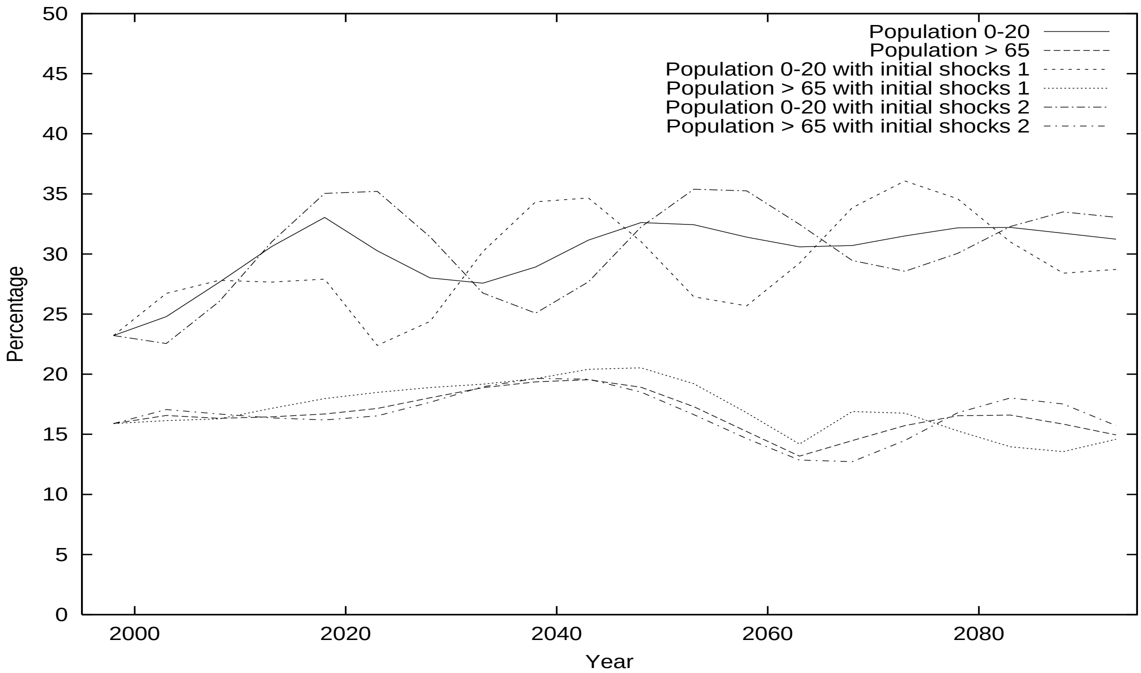


Figure 4: Shares of the youngest and the oldest without initial shocks and with the values of the initial shocks obtained from simulations.

6.2 Behavior of the Savings Rate under the Different Initial Conditions

When we compare the paths for savings rates of the different initial conditions we see that they all have an early dip with a short lived recovery around 2020, another dip with lowest point in 2035, and then a smooth recovery towards the steady state (which, recall, is the same for all economies) level of 9.8%. The first thing that we see is that there are large differences in between the different paths. Perhaps, the path that stands out is the one using initial conditions obtained from U.S. age-wealth data. In this case savings fall by around one third between the first period of the simulation and 2035. By the mid of the century this path has got very close to that with steady state assets, the only other path with whom it shares the population path. Among the paths that are generated by simulations we see that those coming from different simulated populations are quite different from each other, while those that have the same population but different total initial capital start within one percentage point of each other and end up converging. We see that both the steady state path and the U.S. wealth distribution path are in between the simulated paths, one of which shows a much larger dip than the other.

From these observations we can conclude that the mechanics of the age structure of the population induced by the very low recent fertility will imply a sharp reduction of the savings rate of up to one third during the next forty years even if there is a complete recovery of fertility. We also conclude that the form of the recovery, whether the long swing carries on or whether it suddenly reverts to its historic average also matters, but it is not clear in which way, it depends on whether one thinks of the current low fertility, as either a temporary development such as a baby bust, or whether one thinks of it as a permanent phenomenon.

Figure 5 describes the savings rates by age implied by the previous exercise in the case where the initial condition is that implied by the steady state. There are various observations to make here. First, unlike the aggregate savings rate changes that are due both to changes in the age structure of the population and to changes in response to prices, all responses depicted in this table are due to changes in prices. Second, the values of the savings rates vary quite a lot. Changes of more than 25 percentage points can be seen between contiguous cohorts. Finally, note that the movements in savings rates are not perfectly correlated across age groups as can also be seen

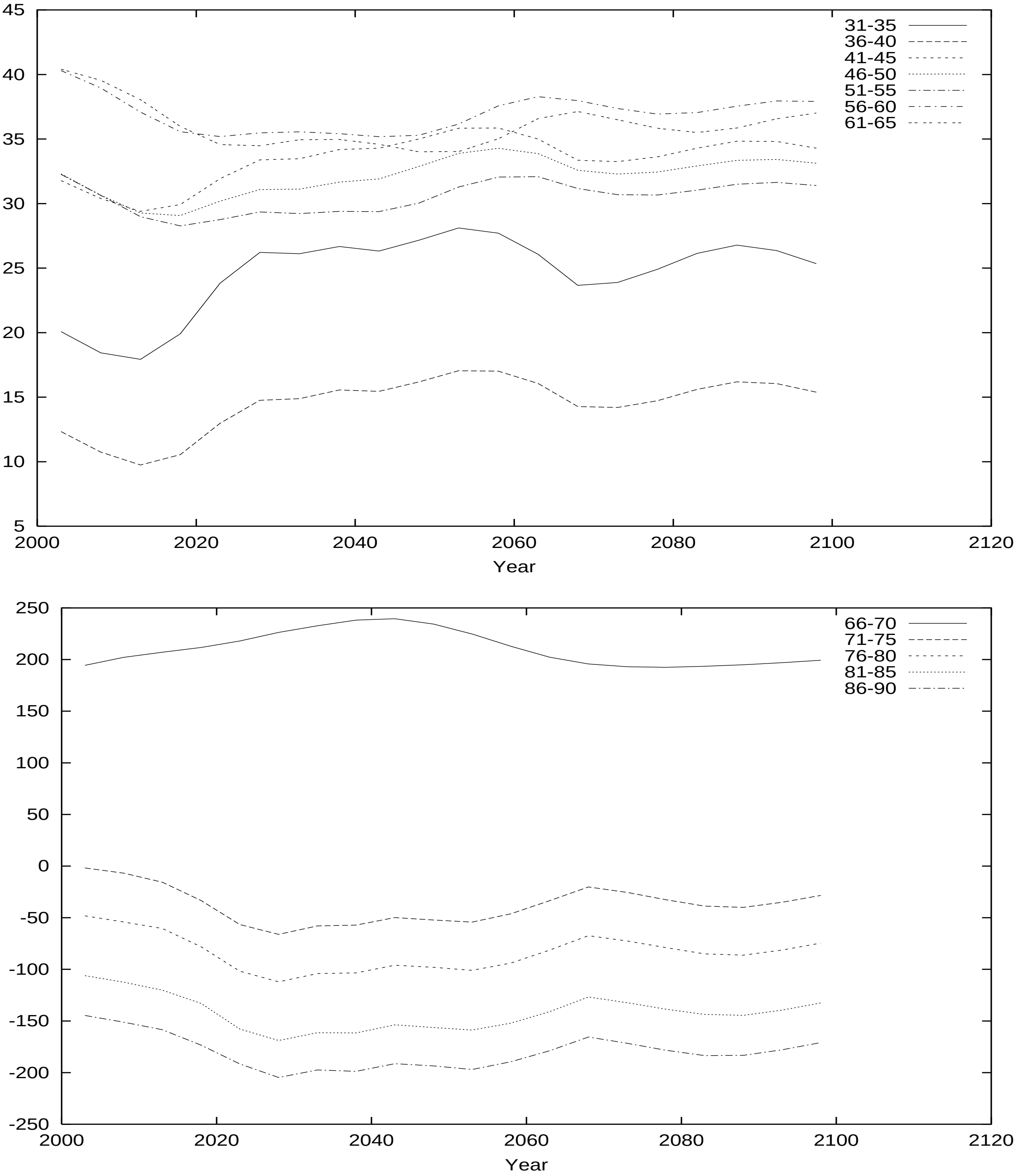


Figure 5: Savings rates in percentages for various age groups for the baseline economy with steady state assets.

in Table 3. Contiguous age groups have a high correlation between their savings rates, but the more distant these age groups, the more different they are: their savings respond not only to current interest rates, but also to the sequence of future interest rates.

In what we have done so far, fertility is supposed to remain at its historic average. This fertility is much higher than it has been in the last twenty–five years. For this reason, in the next section we describe the behavior of the model under the assumption that the age specific fertility of 1998 will be the value for the foreseeable future.

7 The low fertility regime

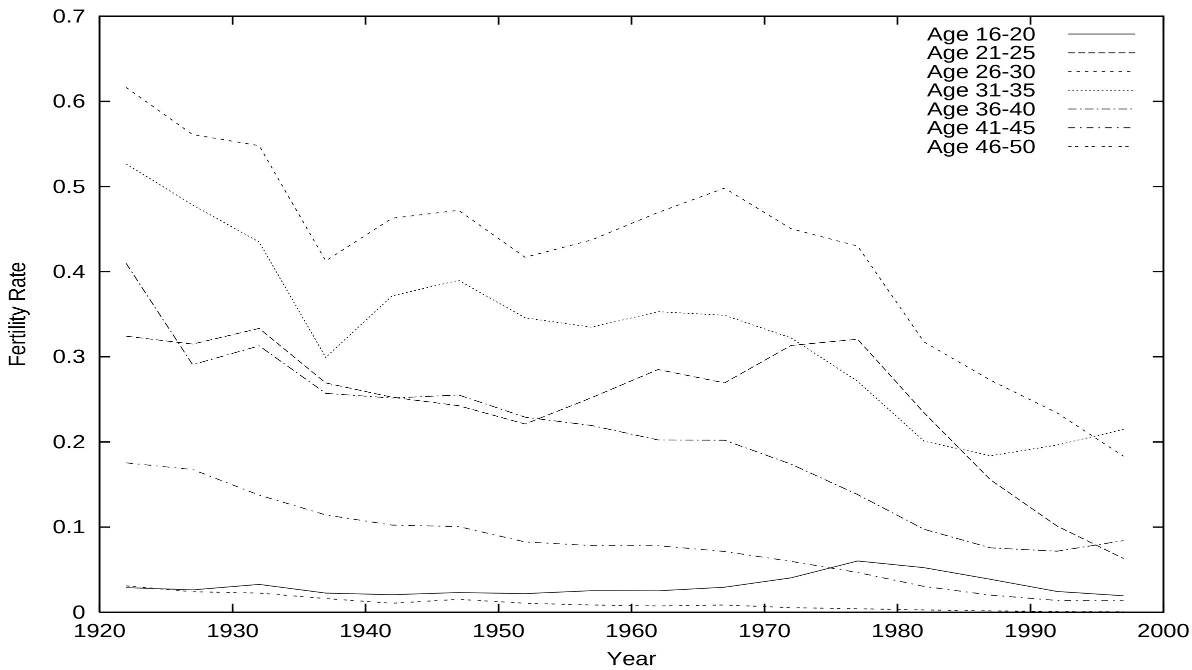


Figure 6: Age Specific Fertility Rates.

So far we have assumed that fertility follows an AR(2) and that the low values of recent years are just due to a cyclical swing. However, if we look at the actual paths of age–specific fertility rates (Figure 6) we see that there has been not only long swings that may justify the use of an AR(2) with complex roots, but also a severe drop in fertility that may justify the use of a different process. I, therefore, will now turn to a case when agents forecast fertility to stay constant forever at the very low level that it had in 1998.

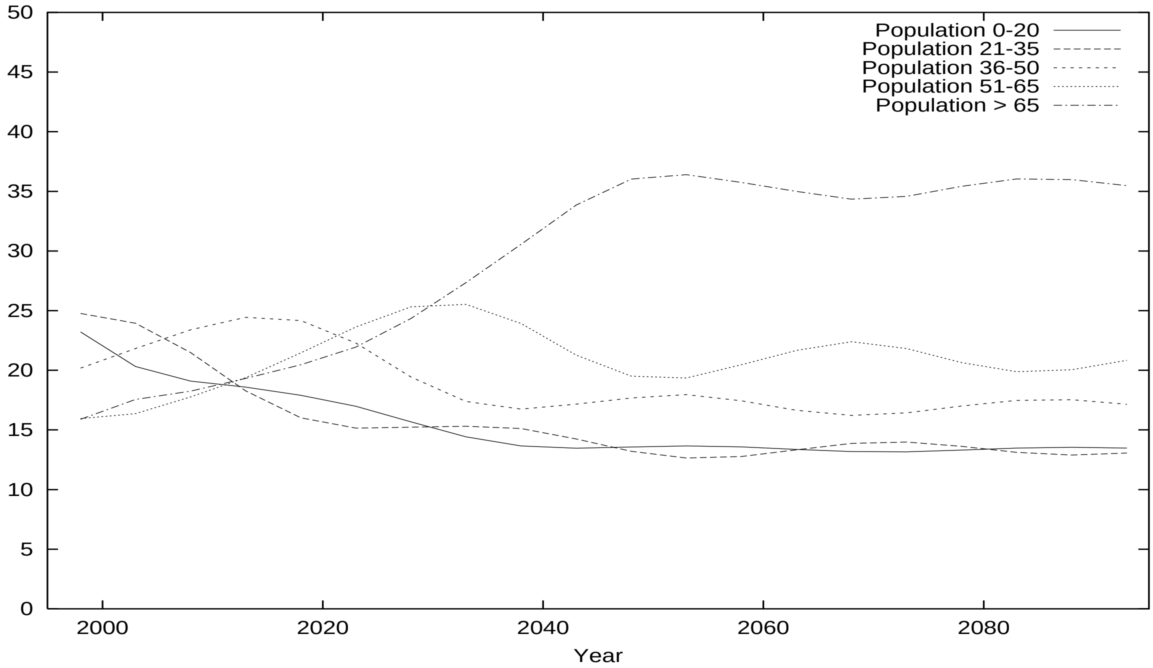


Figure 7: Population Structure with Low Fertility in percentages.

Figure 7 shows the path for the age structure of the population implied by the low fertility rates starting from the population structure in 1998 Spain. The first thing to note is how different the population structure becomes. The fraction of the population above 65 goes up from a little above 15% to more than 35% percent: it more than doubles. This is the type of population change has been worrying many in recent years. Because the recent drop in fertility is so large in Spain the numbers here are particularly large, but the picture in the rest of the western world is only a slightly less dramatic. We do not have a very good idea of what type of fertility scenario is more likely to prevail; this is one of the most important questions that the social sciences are facing. Note also that population growth becomes negative with this fertility scenario, at -1.7% per year.

With fertility changing towards a very low level, the explicit consideration of the initial conditions is even more important than for the case where

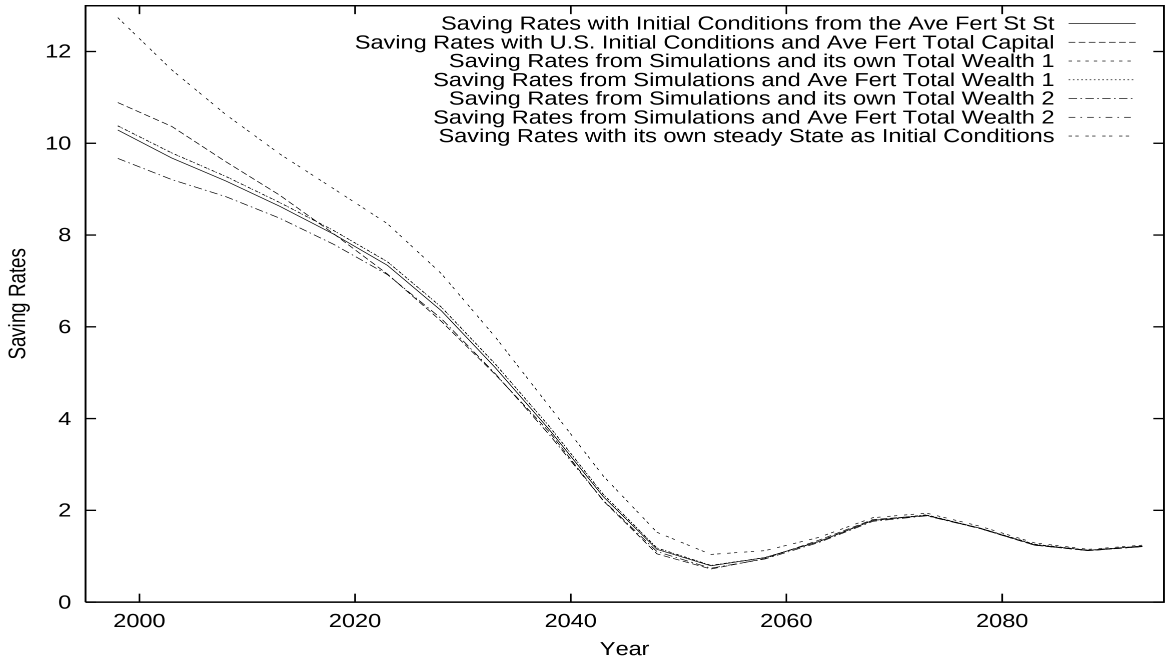


Figure 8: Savings rates for different initial wealth distributions for the low fertility economy in percentages.

the drop in fertility was just a temporary phenomenon. The reason is that now the initial conditions should certainly not be those associated to the steady state, since the low fertility has only been in place for very few years. Figure 8 shows the savings rates for the low fertility scenario. Its features are absolutely dramatic. Savings drop from over 10% to around 1%, albeit it does so in a slow, but steady, manner. The huge change with respect to the scenario where fertility reverts to its historic average can be seen very clearly in Figure 9 where the two sets of savings rates are depicted with the same scale.

With respect to initial conditions, there are some differences between them, but they are minute in the big picture of the dramatic reduction of savings that accompanies population aging. Note also that all in all the economies considered the value of the shocks plays no role. The fertility process is deterministic and it is the same for all initial conditions and this is what accounts for the similarity of the savings rates paths. There are some additional differences between the initial conditions that result from using the steady state of an economy with low fertility and those that we used before for the baseline model economy. However, this case is not very interesting since we should not think of 1998 as anything close to the long run allocation implied by the low fertility regime.

To summarize, the low fertility scenario shows dramatic differences with the baseline model economy and it also points to a very low level of savings.

8 Robustness

The next step in this paper is to do some robustness analysis to see what other features might affect the path of saving rates. I look at five: the path of mortality in the future years, the growth rate of productivity, the value of the coefficient of risk aversion, the valuation of leisure, and constant interest rates, also known as the partial equilibrium assumption.

8.1 Lower Mortality

To investigate further decreases in mortality rates, I ran some experiments with the assumption that from 1998 on, age–specific mortality rates are half of those in 1998. These experiments are run in the same way as those for the low–fertility regime. We pose the same initial conditions as for the baseline

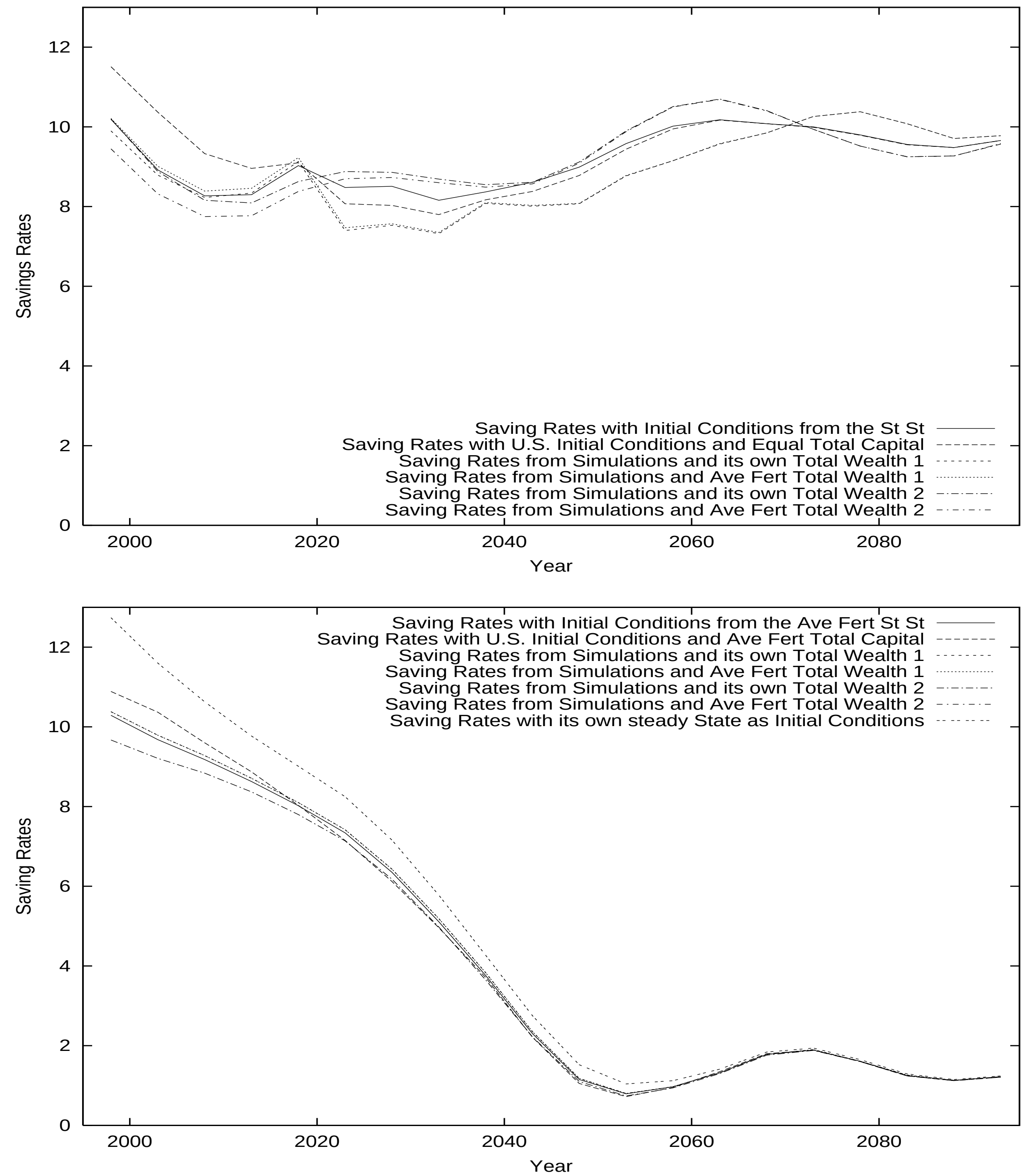


Figure 9: Savings rates in percentages for the baseline economy and for the low fertility economy with the same scale.

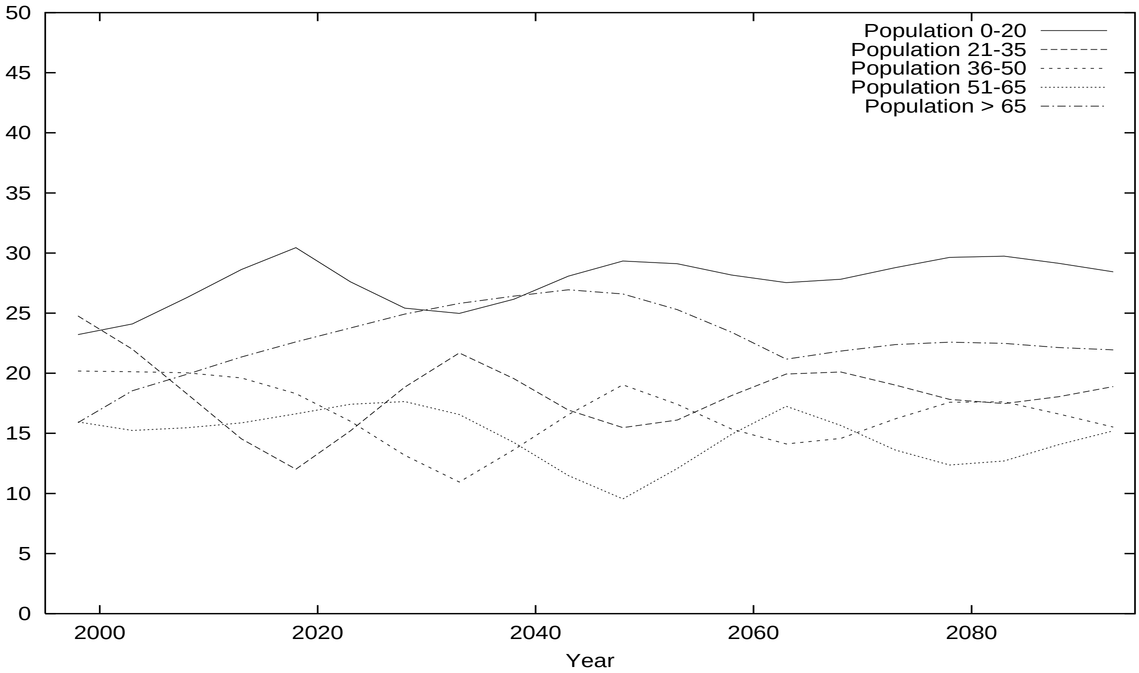


Figure 10: Population Structure if Mortality rates go down by 50% starting in 1998.

model economy and, for comparison, we also look at the initial conditions implied by the steady state.

Figure 10 shows the population structure associated to the lower mortality. Note the dramatic increase of the fraction of the population over 65 years, peaking in 2045. Then it starts going down as the smaller cohorts born in the last quarter of the twentieth century join this age group.

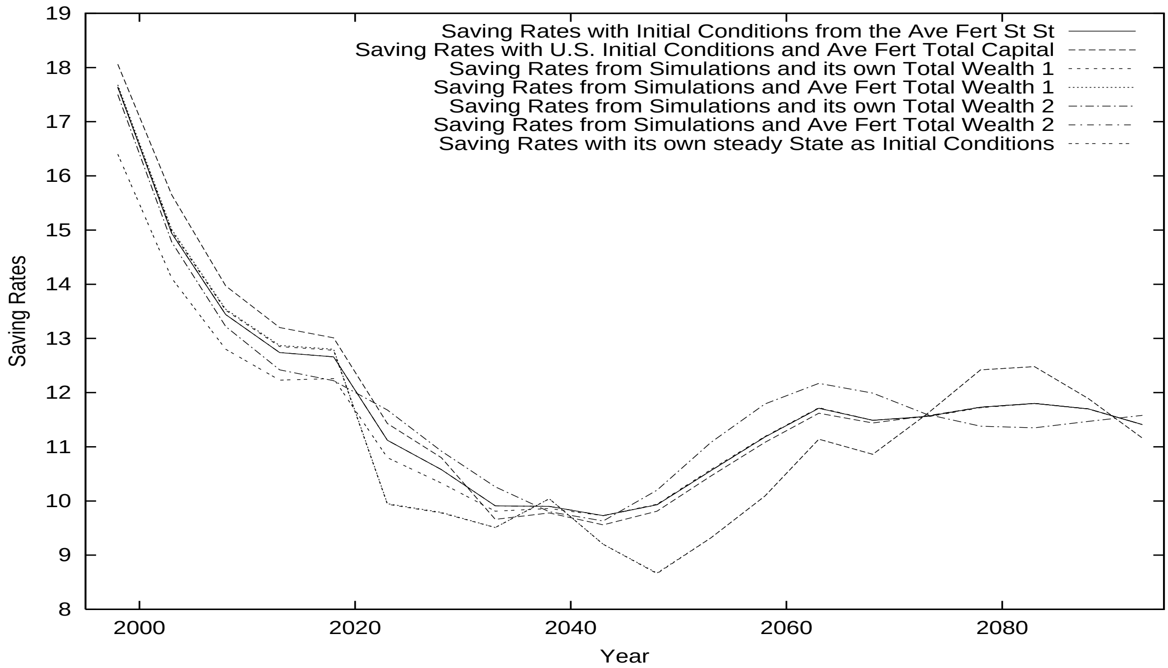


Figure 11: Savings rates for different initial wealth distributions for the low mortality economy.

In addition to a much older population associated to the lower mortality, there is a surprise effect: the population in 1998 learns its their higher life expectancy, inducing all agents to save more (note that the assets holdings in 1998 reflect previous beliefs of a shorter life span). These two effects imply that savings need to be higher than in the baseline model economy and that savings in the first few periods of the simulation have to be especially high. The surprise effect overpowers the differences in population forecasts implied by the different population forecasts implied by the different initial conditions. All this can be seen in Figure 11, especially clear is the decrease in savings throughout the first periods of the simulation from quite a high initial level.

8.2 High Productivity Growth

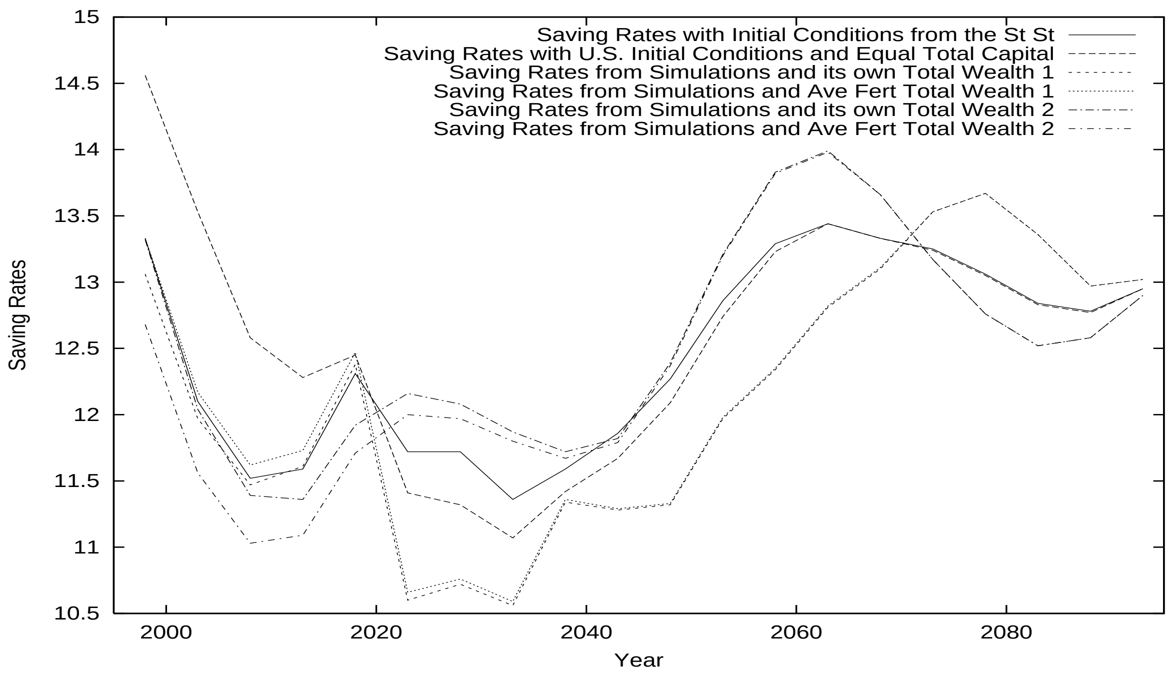


Figure 12: Savings rates for different initial wealth distributions for the high productivity growth economy.

Next, I look at the implications of faster productivity growth, 3% rather than 2%. I adjust the discount rate so that the steady–state wealth–to–output ratio is the same as in the baseline model economy. As can be seen in Figure 12, there are is a major change compared to the baseline model economy: the level of the savings rate is now higher. This is logical as more resources are now devoted to keep capital growing at the higher rate. We see the same patterns as in the baseline model economy with respect to the different initial conditions.

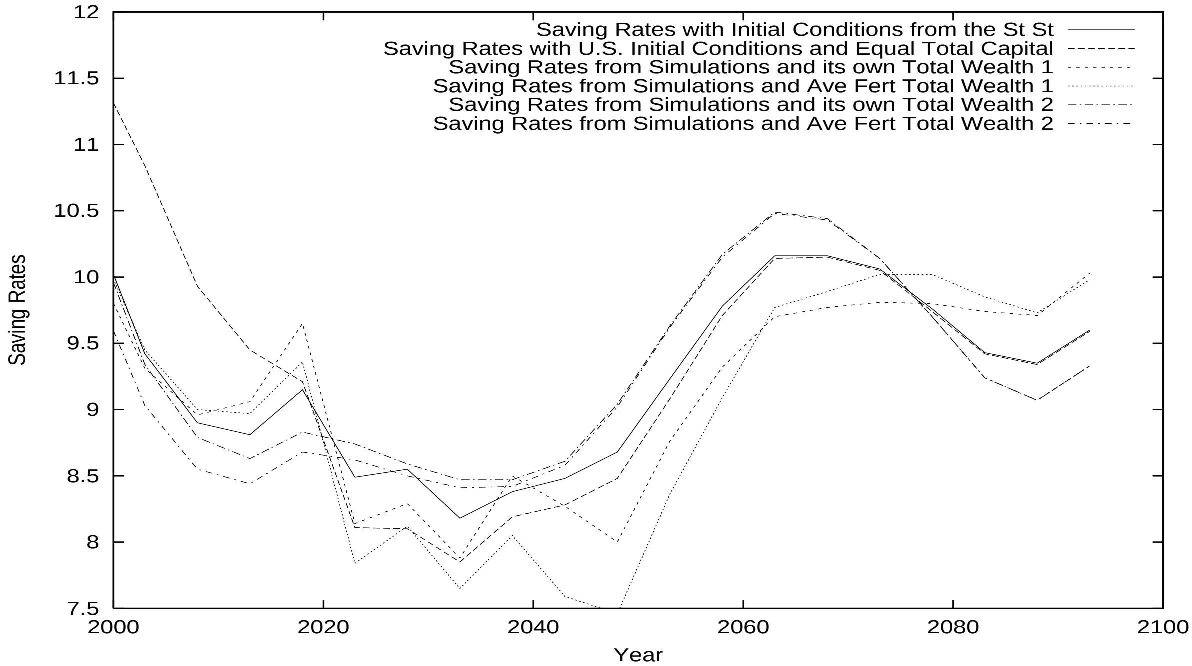


Figure 13: Savings rates for different initial wealth distributions for the low intertemporal elasticity of substitution economy.

8.3 High Coefficient of Risk Aversion

To assess the role of a higher value of the coefficient of risk aversion, I run the same experiments as for the baseline model economy changing the value of σ which indexes the risk aversion to 6 (it was 4). In a growing economy, this change also alters the steady–state wealth–to–output ratio. To keep the latter constant, I adjust again the discount factor. The findings are reported in Figure 13. As we can see, it is very similar to that of the baseline model economy, indicating that this parameter is not very important for determining the saving rates.

8.4 Low valuation of leisure

It is not clear what is the best way to measure the partition of time between work and leisure. For this reason, I also performed a robustness exercise with respect to the parameter that governs the relative value of consumption and leisure in the utility function, α. I run the experiments with a value of .4 for this parameter. Obviously, I have to adjust the discount rate to get the same wealth–to–output ratio than the other model economies. I do not adjust the coefficient of risk aversion. To see why this may be an issue, note that for retired agents the relevant coefficient of risk aversion is affected by the parameter α.

Figure 14 shows the paths for the savings rates for this scenario. Notice that they are almost identical to the baseline and to the high coefficient of risk aversion economy, indicating that the precise value of this parameter is not very important when assessing the implications for savings of population aging.

8.5 Partial Equilibrium

The final set of experiments that I run isolates the price effects from the pure demographic effects. I thus set the interest rate at the value for 1998 implied by the general equilibrium version of the model. Similarly, I set wages to grow at the constant productivity growth rate starting from the value that have in the general equilibrium version of the model.

Figure 15 shows the interest rates and the savings rates of the baseline model economy and this partial equilibrium economy. We see that the general equilibrium interest rate has a small drop and, as a consequence, the oscil-

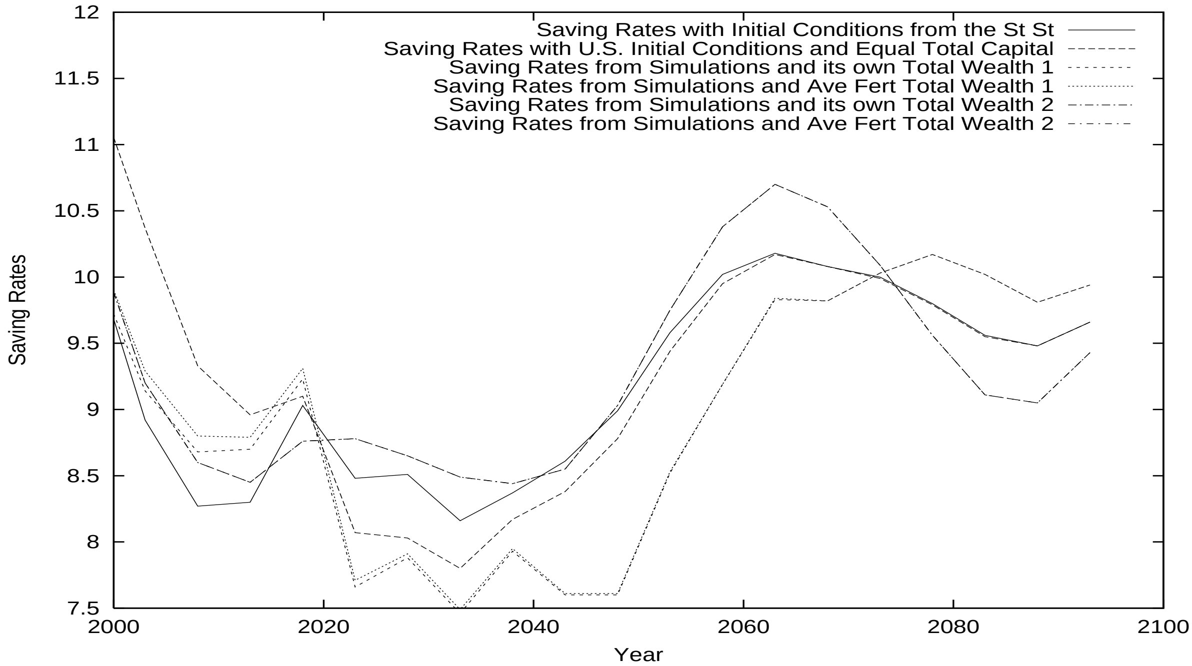


Figure 14: Savings rates for different initial wealth distributions for the low valuation of leisure economy.

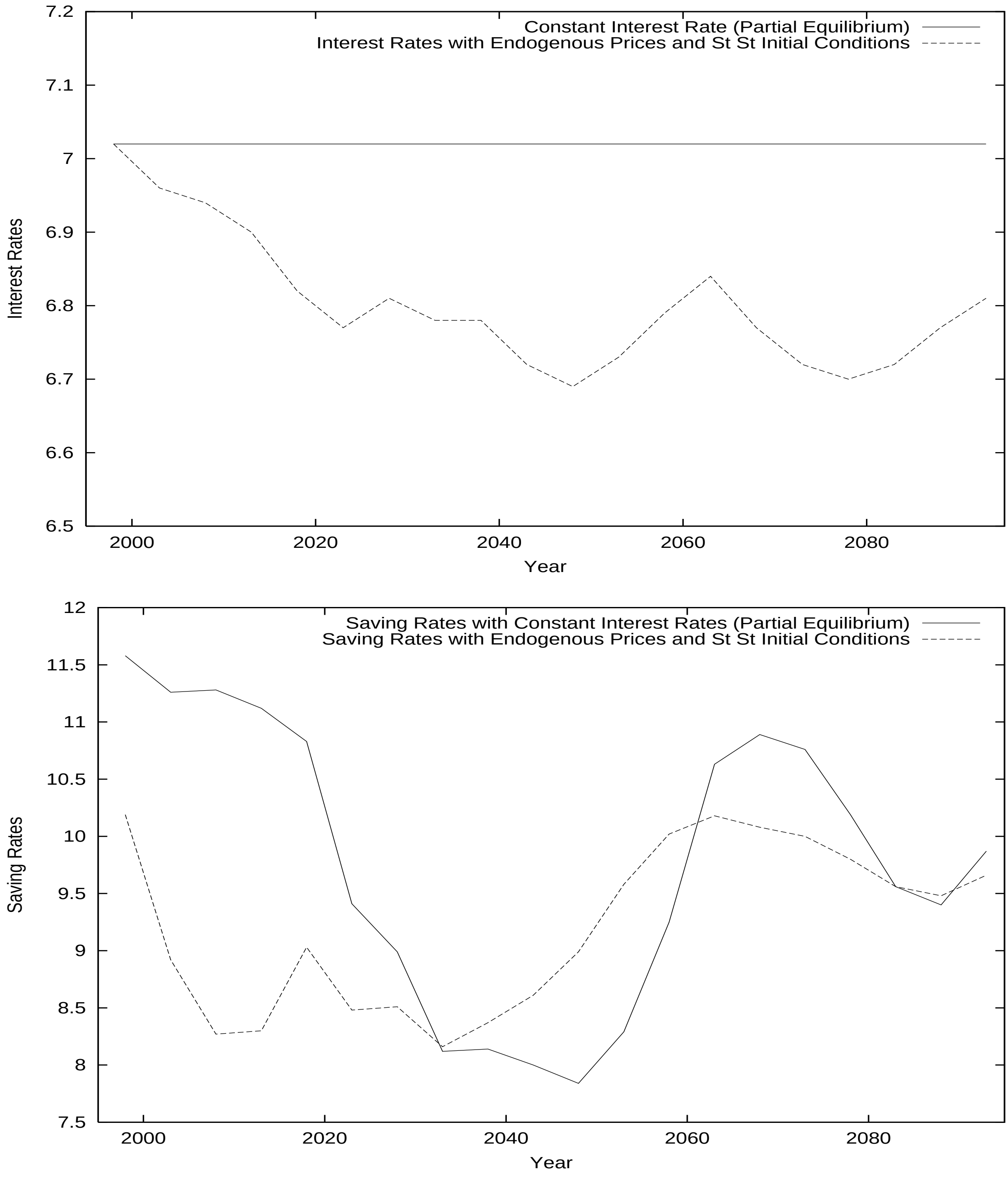


Figure 15: Interest rates and savings rates in percentages for the baseline model economy and its partial equilibrium counterpart with the closed economy steady state assets as initial conditions.

lations in savings are more dramatic since the response of prices smoothes the response of agents. The endogenous interest rate induces a reduction of savings until 2035 relative to its exogenous prices counterpart. The increase in savings from then on is also more smooth.

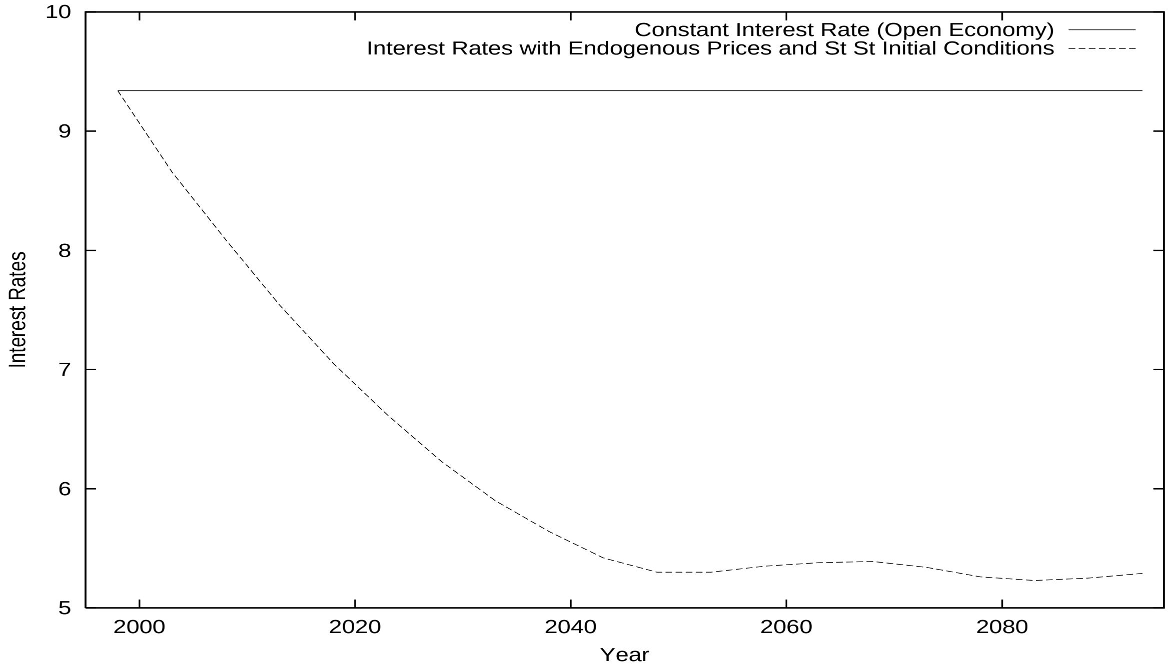
Figure 16 displays results from the same type of exercise but for the economy with low fertility. Here the reduction in the interest rate is much larger: the aging of the population means that there is more capital relative to labor. As a result, the behavior of prices magnifies the reduction in savings, which are lower until the middle of the twenty–first century.

9 Conclusion

In this paper, I have investigated the implications for savings of the aging of the population. In particular, I have looked at how various modelling choices change the answers that the models give us. I have looked at both various forms of modelling initial conditions as well as other features of the model such as the actual demographic details of fertility and mortality, the productivity growth path, the intertemporal elasticity of substitution and the role of general equilibrium prices.

The main finding is that we should expect a relatively small (between 1% and 2%) reduction in the aggregate savings rate in the next fifty years even if fertility recovers dramatically to its twentieth century average. However, if the fertility rate does not recover and the 1998 rate becomes the norm, then the savings rate will fall dramatically and consistently to very low levels, below 2%, although the reduction will take place slowly. This decrease is more important when we take into account general equilibrium considerations than when we just propagate the behavior of households into the future, showing that population aging induces a scarcity of labor and hence a reduction of interest rates that pushes savings down even more.

In addition, in this paper I have developed recursive methods to deal with aging populations where the demographic process is affected by shocks, a development that may be fruitful in the study of dynamic systems and that allows us to look at stochastic environments.



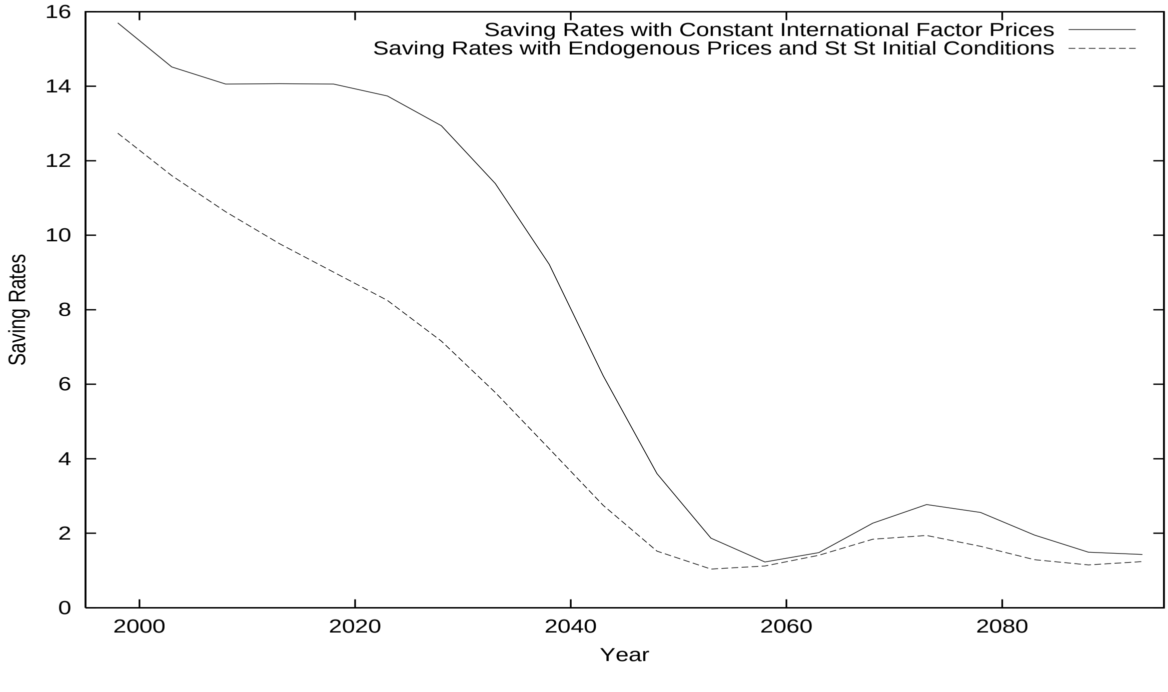


Figure 16: Interest rates and savings rates in percentages for the baseline model economy and its partial equilibrium counterpart with the closed economy steady state assets as initial conditions.

Colophon

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Appendix

A Computation of the Equilibrium

To compute the steady state of this economy, first the stable population has to be computed. This requires the computation of the eigenvalues and eigenvectors of the matrix $\hat{\Gamma}$. The eigenvector associated to the largest eigenvalue is the steady–state distribution of the population. With this population structure, a steady–state growth path (with aggregate variables growing at the sum of the rates of growth of population and productivity) is computed by solving for the zero of an equation in the capital–labor ratio. This equation is typically monotonic (typically here refers to economies that use production and utility functions of the Cobb-Douglas family, see Galor and Ryder (1989) for details). Next, I describe the steps used computing the law of motion of the population.

Let $R_i(x, z, k, a, \ell, y, l)$ be the quadratic approximations around the steady state to the current utility function for each age, where consumption has been substituted using the budget constraint. Here $\ell \in I\!R^{I_0}$ is a distribution of leisure across agents that yields, jointly with x and ϵ, the amount of labor input. Let $\mathbf{V}$ be the space of quadratic functions defined over values of (x, z, k, a) whose range is $I\!R^I$. We define an operator $T : \mathbf{V} \mapsto \mathbf{V}$ by:

$$(Tv)_i\ (x, z, k, a) = \max_{y,l} \quad R_i(x, z, k, a, \ell, y, l) + \beta_i\ s_i\ E\left\{v_{i+1}\left(x', z', k', y/s_i|z\right)\right\} \tag{5}$$

where $v_{I+1} = 0$, $\binom{x'}{z'} = \Gamma\binom{x}{z} + \nu'$, and where $k' = G_v(x, z, k)$, and $\ell = H_v(x, z, k)$ are taken as given but have to be generated by the decision rules of the agents. The specific steps within every iteration to obtain $T(v)$ are:

- **Step 1.** Given $v \in \mathbf{V}$, obtain $\hat{v}(v)$ and substitute it in (5) using the law of motion of the population

$$\hat{v}_i(x, z, k', a') = E\left\{v_{i+1}\left(\Gamma\begin{pmatrix}x\\z\end{pmatrix} + \nu', k', a'|z\right)\right\} \tag{6}$$

- **Step 2.** Solve (5), obtaining linear solutions $y_i(x, z, k, a, \ell, k')$ and $l_i(x, z, k, a, \ell, k')$.

- **Step 3.** Use conditions $v.$ and $vi.$ of the definition of equilibrium to obtain aggregate law of motion of assets holdings, G_v and aggregate leisure choices H_v. This step involves inverting some matrices. Denote as $\hat{l}_i^1$ the subset of coefficients of l_i that multiply $(1, x, z, k)$ where we added the coefficient of a to the coefficient of k_i it, effectively using the representative–agent condition.

Label $\hat{l}_i^2$, the rest of the coefficients (those that affect (ℓ, k')). Repeat for y_i^1 and y_i^2. Using representative–agent conditions, we obtain the following:

$$\begin{pmatrix} \ell_1 \\ \vdots \\ \ell_I \\ k'_2 \\ \vdots \\ k'_I \end{pmatrix} = \begin{pmatrix} l_1^1 \\ \vdots \\ l_I^1 \\ y_1^1 \\ \vdots \\ y_{I-1}^1 \end{pmatrix} \begin{pmatrix} 1 \\ x \\ z \\ k \end{pmatrix} + \begin{pmatrix} l_1^2 \\ \vdots \\ l_I^2 \\ y_1^2 \\ \vdots \\ y_{I-1}^2 \end{pmatrix} \begin{pmatrix} \ell_1 \\ \vdots \\ \ell_I \\ k'_2 \\ \vdots \\ k'_I \end{pmatrix}. \tag{7}$$

- From this point, obtaining functions G_v, and H_v is trivial, as it only involves solving the above linear system.

- **Step 4.** Substitute decision rules, y_i, and l_i, and functions H_v and G_v to obtain $T(v)$.

It is easy to check that, by construction, a fixed point of this operator, together with its decision rules, and functions for marginal productivities, satisfy the requirements of a recursive competitive equilibrium. Since the approximated utility functions are quadratic and since v_{I+1} is zero, T maps quadratic functions into quadratic functions, and policy functions are linear, insuring the computational feasibility of the computation of the image of T. Iterations on T are performed with starting values of $v_i = 0$, for all i.

B Parameter Values of the Baseline Economy and Other Tables.

In this section of the appendix I include the main parameter values for the experiments, and some other tables cited in the text.

Table 1: Parameter Values for the Baseline Model Economy (in yearly values).

Preferences		
Time discount factor	β	1.00006
Coefficient of Risk Aversion	σ	4.0
Relative share of consumption and leisure	α	0.3
Technology		
Capital share	θ	0.36
Capital depreciation rate	δ	0.04

Age	Probability of Survival, s_i	Fertility Rate, ϕ_i	Coefficient of family size, η_i	Efficiency Rate ϵ_i	1998 Age Structure
1	0.9939774	0.00000	0.00	0.00	1.0000000
2	0.9992093	0.00000	0.00	0.00	1.0348816
3	0.9991003	0.00009	0.00	0.00	1.2021154
4	0.9977037	0.03071	0.00	0.00	1.5173191
5	0.9971544	0.24705	1.10	0.78	1.7244444
6	0.9959642	0.42384	1.15	1.14	1.7002336
7	0.9939681	0.32948	1.20	1.14	1.6491199
8	0.9928539	0.20443	1.25	1.37	1.5265628
9	0.9909430	0.08077	1.30	1.37	1.3483522
10	0.9865209	0.01056	1.35	1.39	1.2584521
11	0.9802318	0.00187	1.25	1.39	1.1772499
12	0.9714237	0.00000	1.15	1.33	1.0013380
13	0.9525320	0.00000	1.10	1.33	1.0883646
14	0.9263057	0.00000	1.00	0.89	1.0471363
15	0.8782778	0.00000	1.00	0.00	0.8793682
16	0.7937459	0.00000	1.00	0.00	0.6284231
17	0.6261969	0.00000	1.00	0.00	0.4053717
18	0.3293028	0.00000	1.00	0.00	0.2216926
19	0.1000000	0.00000	1.00	0.00	0.0735304
20	0.0000000	0.00000	1.00	0.00	0.0100000

Table 2: Various Age-Wealth Distributions used in the Baseline Model Economy.

Age	Steady State Age–Wealth	US Wealth Age–Wealth
20– 24	0.00000	0.00870
25– 29	-0.02033	0.02298
30– 34	0.01491	0.06304
35– 39	0.03892	0.08026
40– 44	0.09927	0.12760
45– 49	0.15162	0.17212
50– 54	0.19800	0.23281
55– 59	0.24864	0.2544
60– 64	0.29439	0.30125
65– 69	0.34530	0.22038
70– 74	0.31513	0.20086
75– 79	0.27783	0.17696
80– 84	0.23493	0.14972
85– 89	0.19346	0.12373
90– 94	0.17238	0.11147
95– 99	0.17021	0.11216

Table 3: Cross Correlations between age specific savings rates 1998-2093.

Ages	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	–														
2	.99	–													
3	.99	.98	–												
4	.98	.99	.97	–											
5	.97	.99	.95	1.0	–										
6	.78	.83	.71	.86	.89	–									
7	.35	.43	.26	.47	.53	.85	–								
8	-.30	-.21	-.38	-.16	-.10	.33	.78	–							
9	-.72	-.65	-.77	-.61	-.56	-.17	.35	.86	–						
10	.23	.17	.34	.20	.16	-.21	-.58	-.78	-.64	–					
11	-.66	-.59	-.75	-.57	-.52	-.10	.39	.84	.94	-.73	–				
12	-.68	-.62	-.77	-.60	-.54	-.13	.36	.82	.94	-.73	1.0	–			
13	-.71	-.65	-.79	-.63	-.58	-.16	.33	.81	.94	-.71	1.0	1.0	–		
14	-.73	-.67	-.81	-.65	-.59	-.19	.32	.81	.96	-.69	.99	.99	1.0	–	
15	-.70	-.63	-.78	-.62	-.56	-.15	.34	.80	.94	-.72	1.0	1.0	1.0	.99	–

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