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Comment

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A common criterion for evaluating forecasting methodologies is the accuracy with which the forecasts match the eventual realizations of the actual data. Users of forecasts must choose among alternative forecasting models before such facts become available, however. Unwilling to wait for several decades to pass before writing this comment, I analyze the methodology of Lee and Carter using criteria suggested by Keyfitz (1981), Long (1984), and Rogers and Woodward (1991): (a) the transparency of assumptions used to generate the forecasts, (b) the ability of the model to generate measures of forecast uncertainty, (c) the extent of disaggregation (by age, sex, and race) permitted by the model, and (d) the quality of the data on which the forecasts are based.

Lee and Carter have presented a methodology for forecasting mortality that falls squarely within the extrapolative tradition of demographic forecasting. They intend their forecasts to be more than illustrative, presenting projections that clearly trace the implications of the continuation of historical trends. These projections can serve as a benchmark for the comparison and evaluation of official forecasts. As a set of benchmark forecasts, these will be most useful if we clearly understand the process generating the forecasts.

The singular value decomposition (SVD) is applied to the matrix of age-specific central death rates for each calendar year in this century. This decomposition yields an index of mortality, $\mathbf{k}_t$, and a set of age-specific constants, $\mathbf{b}_x$, which relate the central death rates, $\mathbf{m}_{x,t}$, to the index of mortality. In particular, $d(\ln \mathbf{m}_{x,t})/dt = \mathbf{b}_x dk_t/dt$. Because $\mathbf{k}_t$ is modeled as a random walk with drift, it is projected to decline at a constant linear rate. Therefore, each age-specific mortality rate is predicted to decline at its own constant exponential rate, as determined by the individual $\mathbf{b}_x$ parameters. In fact, as stated at several points in the article and as shown in their Figure 6, each central death rate is forecasted to decline "at its own specific historical rate" (p. 665). *From this analysis it becomes clear that forecasts of mortality identical to Lee and Carter's will be produced by directly projecting each age-specific mortality rate at its own historical rate of exponential decline.* Because the forecasts produced by straight extrapolation of individual mortality rates are the same as those generated indirectly from Lee and Carter's mortality index, the two methods are equivalent, despite the authors' statements to the contrary (p. 661).

Viewed in these terms, the Lee and Carter forecasts can be given high marks in terms of transparency of assumptions. Their projections capture the implications of a continuation of past exponential trends in age-specific mortality rates, uncomplicated by expert opinion or assumptions about medical advances, delay of deaths by cause, or ultimate levels of life expectancy.

Lee and Carter claim that "variation in a single parameter can generate the main outlines of the observed pattern" (p. 660), meeting Keyfitz's (1981) criterion of parsimony in representing the mortality profile. Actually their methodology involves 23 parameters—the $\mathbf{b}_x$ coefficients that capture the rates of change in each age-specific mortality rate, relative to changes in the mortality index $\mathbf{k}_t$.

There are disadvantages to mortality forecasts that are straight projections of individual age-specific central death rates. One concern is that if each age-specific rate is allowed to change at its own individual rate, the projected age profile of mortality may depart from plausible, historically observed patterns (Keyfitz 1981). Evidence of this outcome is seen in Lee and Carter's Figure 4, in which the projected profiles for 2030 and 2065 are characterized by several irregularities: mortality rates that are constant across ages 1–14, a very sharp rise to a prominent accident peak for young adults, and a strong upward curvature of senescent mortality rates

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for ages 50–70. Forecasts of mortality can be made to adhere to standard age profiles while still operating within the extrapolative tradition by using parameterized model schedules, as in McNown and Rogers (1989). By not imposing the regularity of the age pattern of mortality on their forecasts, Lee and Carter have ignored important information that could be exploited to improve forecast accuracy.

Keyfitz (1981), Land (1986), and Long (1984) have emphasized the need for statistically based interval forecasts in demographic projections. Because Lee and Carter's forecasts are grounded in the time series tradition, they are able to provide standard errors for their mortality forecasts. The mortality index, $\mathbf{k}_t$, is forecasted from an autoregressive integrated moving average (ARIMA) model, so that its forecast standard errors are given by conventional formulas. Because the mortality rates $\mathbf{m}_{x,t}$ and life expectancy $e_{0,t}$ are nonlinear functions of $\mathbf{k}_t$, involving additional estimated parameters, the computation of their standard errors requires the use of asymptotic approximations (Goldberger 1964) or a bootstrap procedure, as described in their Appendix B. The complexity of such computations is of the same order of magnitude as would be involved in the derivation of interval forecasts for mortality rates from the forecasted parameters of a model schedule, as in McNown and Rogers (1989).

By deriving forecasts of $\mathbf{m}_{x,t}$ from $\mathbf{k}_t$, Lee and Carter introduce an unnecessary element of complexity in computing forecast standard errors. Because each age-specific rate is in fact projected forward at its own historical exponential rate, the forecasts of $\mathbf{m}_{x,t}$ are equivalent to those generated by individual random walk models applied to the logarithm of each age-specific rate. The forecast standard errors for each $\mathbf{m}_{x,t}$ would follow directly from these models, avoiding the complications in their Appendix B. Because the other components of the life table ($e_{0,t}$ in particular) are nonlinear functions of the $\mathbf{m}_{x,t}$, computation of their interval forecasts still would require using asymptotic approximations or the bootstrap procedure. Lee and Carter use a bootstrap procedure to estimate the components of the variances of the life table elements, but it would seem more straightforward to compute the variance of $e_{0,t}$ by generating pseudo-observations on forecasted $e_{0,t}$ directly. (See Peters and Friedman [1985] for an example in the regression context.)

Lee and Carter present confidence intervals for the forecast of life expectancy that appear astonishingly narrow at first glance. We are accustomed to time series forecasts of other demographic phenomena (fertility and total population) with excessively wide confidence intervals (Land 1986). Among the explanations for Lee and Carter's small forecast standard errors is the decreasing entropy of the life table, so that errors in forecasting mortality rates have only small effects on forecasts of life expectancy. The same phenomenon accounts for the relatively small gains in life expectancy that can be realized from rather dramatic changes in mortality rates, as demonstrated by Olshansky, Carnes, and Cassel (1990). One lesson from the Lee and Carter projections is that time series forecasts need not necessarily have wide confidence intervals.

Another positive departure from most time series applications in demographic forecasting is the degree of disaggregation offered in Lee and Carter's projections. Early applications of time series methods in demography (McDonald 1979, 1981; Saboia 1974, 1977) focused on aggregates (e.g., population totals, numbers of births). Such models lack the age-specificity required of demographic forecasts for policy discussions. With forecasts of age-specific mortality rates, Lee and Carter provide projections of all elements of the life table. Combined with forecasts of birth rates and migration, their mortality projections allow the extraction of information on the aging of the population, dependency ratios, age-incidence of mortality, and other demographic changes of substantial policy significance. Disaggregation by sex would provide additional information relevant to policy addressing the needs of the elderly.

The attention to data quality is exemplary in the Lee and Carter study. The substantial aging anticipated for the U.S. population places greater importance on the size, age-distribution, and mortality patterns of the elderly population. Grouping all persons age 85 years and older into a single category, as in officially published data, leaves out important information about mortality patterns among the oldest old. Lee and Carter's use of the methods of Coale and Guo (1989) to extrapolate death rates to the 105–109 age group is particularly valuable for projections into years when a substantial proportion of the population will survive beyond age 85.

Examining the attributes desired of demographic forecasts, the Lee and Carter model is commendable in several areas. The projections are based on the transparent assumption that each age-specific mortality rate will continue to decline at its historical exponential rate. Valuable information is provided on forecast uncertainty, and we learn that life expectancy forecasts are considerably more certain than we might have expected. Projections are provided with the same detailed age disaggregation as official forecasts, providing a viable alternative to the official projections. Finally, they are to be commended for their attention to the issue of data quality at the highest ages.

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