1 Model Economy

To quantify the effects demographic dynamics have on savings, investment, rate of return on the factors of production, and, finally, current account, we calibrate a two-country general equilibrium model with fine demographic dynamics.

1.1 Demographics

The economy consists of overlapping generations of ex ante identical agents who live up to I periods, with ages denoted by $i \in \mathscr{I} \equiv \{1, \dots, I\}$. At every point in time, there are I different cohorts alive. Individuals remain children for I_0 periods. As children they neither consume, accumulate capital nor supply labor. After I_0 periods the agents enter the economy as autonomous decision makers.

The survival probability between age i and $i+1$ is denoted $s_{i,t}$ and varies with ages i and time. The unconditional probability of reaching age i is denoted s^i and is the product of conditional survival probability rates; $s^i = \prod_{j=1}^{i-1} s_j$.

Let $x_t \in \mathbb{R}^I$ be the vector of number of members in each cohort in period t. The demographic structure of the population changes through changes in fertility, mortality and immigration. According to time and age specific fertility rates $\varphi_{i,t}$, in each period these individuals give birth to a certain number of new individuals, and the number of newborns in period $t+1$, $x_{1,t+1}$, is the product of x_t and the vector of fertility rates φ_t. Then the law of motion of a population with survival rates as given above, but deterministic fertility, can be described by a simple $(I \times I)$ matrix[1]

$$\hat{\Gamma} = \begin{pmatrix} \varphi_1 & \varphi_2 & \varphi_3 & \cdots & \varphi_I \\ s_1 & 0 & 0 & \cdots & 0 \\ 0 & s_2 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & s_{I-1} & 0 \end{pmatrix},$$

where the diagonal elements $(s_1, \dots, s_{I-1})$ are the conditional survival probabilities.

Let $m_t \in \mathbb{R}^I$ be a vector with each element representing the cohort specific number of net immigrants at time t. Denoting $\hat{\Gamma}_t$ the matrix of deterministic fertility and mortality rates at time t, the law of motion for the population may be written

$$x_{t+1} = \hat{\Gamma}_t x_t + m_t.$$

[1] The largest eigenvalue of the matrix Γ is the rate of growth of the population in steady state regardless of the initial condition. The eigenvector corresponding to this eigenvalue describes the share of each age group in the steady state population.

1.2 Preferences and technology

1.2.1 Individuals' optimization

Preferences of an agent born in period t may be summarized by a standard time-separable utility function with age specific weight β^i

$$\mathrm{E}_{t+I_0}\left\{\sum_{i=I_0+1}^{I}\beta^i s^i u_i\left(c_{i,t}\right)\right\}, \tag{1}$$

where u_i is the instantaneous utility function, and $c_{i,t}$ is consumption and leisure of an agent of age i in period t.

The instantaneous utility function has the standard isoelastic specification

$$u\left(c_{i,t}\right)=\frac{\left(c_{i,t}\right)^{1-\sigma}-1}{1-\sigma},$$

Each individual supplies labor inelastically to the market. The productivity and the rate of return on labor supplied changes with age according to a deterministic pattern. The vector of age specific efficiency units of labor is denoted $\{\epsilon_i\}_{i=1}^{I}$. An easy way to exogenously capture childhood inactivity and old age retirement is to set labor efficiency for those cohorts equal to zero.

There are at least two ways of taking care of accidental bequests and by that closing the model. On alternative is to do like Storesletten, Telmer and Yaron (2000), Rios-Rull (2001) and others, where each agent has written a contract with the members of it's own cohort that make the survivors share the wealth or debts of those who die prematurely. This way of closing the model do, however, eliminate the risk element by disturbing the intertemporal price of capital and the result is that the Euler equation and policy function makes the model very close to a model with no surivial/death probability.[2]

An alternative way to close the model where the individuals still face mortality risk is by dividing total aggregate bequest equally on all inhabitants. Agents may accumulate assets in positive and negative amounts. The agent maximizes utility subject to the budget constraint

$$c_{i,t}+a_{t+1,i+1}=a_{i,t}\left(1+r_t\right)+\epsilon_i w_t+h_{i,t}, \tag{2}$$

and the constraints following from the absence of a bequest motive

$$a_{1,t}=a_{I+1,t}=0. \tag{3}$$

$a_{i,t}$ is wealth, r_{t+i-1} is the spot market net rate of return of one unit of capital, w_{t+i+1} is the spot market price of one efficiency unit of labor, and accidental

[2]If $h=(1-s_{-1})(1+r)a_{-1}$, then envelope condition wrt. the state variable a_{-1}, updated one period, is

$$\partial v'(a)/\partial a=\left(1+r'\right)(2-s)\left[\left(1+r'\right)a+w\epsilon'-a'\right]^{-\sigma},$$

and the intertemporal marginal rate of substitution between consumption in two consecutive periods is equal to $\beta(1+r)(2-s)s$, with $(2-s)s>s$ for all $0<s<1$. So "cohort specific insurance" is to close to entirely canceling out the effect of the death probability.

bequests, h_t, is the fraction of total inheritance or bequests that devolve on each individual alive at time t

$$h_t = \frac{H_t}{\sum_{i=I_0}^{I} x_t},$$

where

$$H_t = \sum_{i=I_0}^{I} (1 - s_i)\, a_{i,t}.$$

In order to close the model and limit the heterogeneity, immigrants who enter the economy are assumed to have the same stocks of human and physical capital stock as the existing members of their cohort. The physical capital stock is assumed transferred from the members of their own cohort.

Hence, individuals maximize (1) subject to (2) and (3).

1.2.2 Production

Country j time t

$$y_{j,t} = \theta_{j,t} K_{j,t}^{\alpha} N_{j,t}^{1-\alpha}.$$

1.3 Equilibrium

1. Individuals in each country choose optimal quantities of capital supplied (saved) given prices.

   Combining the individuals' intratemporal optimality condition with the period-by-period budget constraint gives the following second-order difference equation

$$a_{i+1,t+1} = a_{i,t} R_t + \epsilon_i w_t + h_t - \left(\frac{1}{\beta s_{i-1,t-1} R_t} \right)^{-\frac{1}{\sigma}} \left(a_{i-1,t-1} R_{t-1} + \epsilon_{i-1} w_{t-1} + h_{t-1} - a_{i,t} \right).$$

   Combined with the initial and terminal conditions, eq. (3), this uniquely defines the life-cycle savings (and consumption) sequence for given prices.

2. Firms in each country choose optimal quantities of capital demanded given prices

$$K_{j,t}^{d} = \left(\frac{r_{j,t}}{\alpha \theta_{j,t}} \right)^{\frac{1}{\alpha-1}} N_{j,t}.$$

3. Quantities are aggregated across countries and prices are set such that aggregate quantity demanded equals aggregate quantity supllied. For country j at time t

$$K_{j,t}^{s} = \sum_{i=I_0}^{I} a_{i,j,t}.$$

and

$$R_t : \sum_j K^d_{j,t} = \sum_j K^s_{j,t}$$

2 Calibration / parametrization

Parameter	Target	Value
β	...	1.0
σ	...	4
α	...	0.33
δ	...	$5 \cdot .025$

3 Results

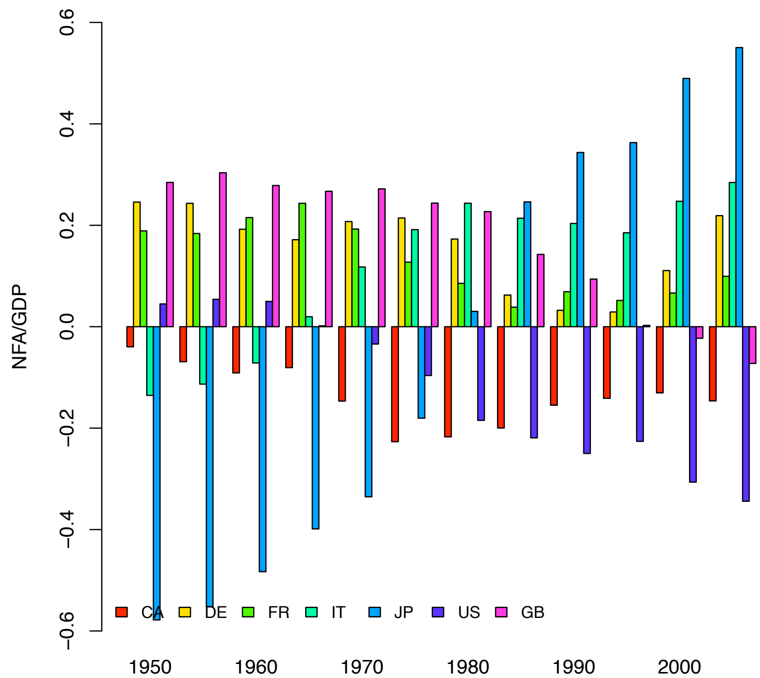
3.1 Benchmark

3.2 Differences in TFP

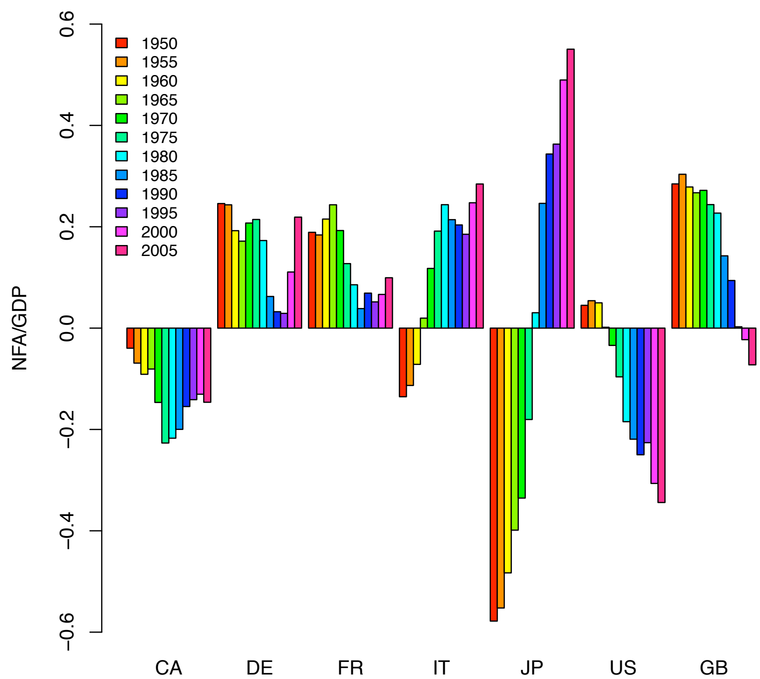
– matching relative GDP

3.3 Differences in capital wedges

– matching K/Y ratios



NFA/GDP
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
CA
DE
FR
IT
JP
US
GB
1950
1960
1970
1980
1990
2000


1950
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
NFA/GDP
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
CA
DE
FR
IT
JP
US
GB

A Code

NFAoverGDP.R

```r
rm(list = ls())

setwd("/Users/espenhenriksen/documents/research/global_economy")


law.of.motion <- function(a,R,w,h,beta,sigma,s,I0,I) {
  for(i in c((I0+2):I)){
    a[i+1] <- a[i]*R + w[i] + h - (1/(beta*s[i-1]*R))^(-sigma)*(a[i
        -1]*R + w[i-1] + h - a[i])
  }
  return(a)
}

load.sample.cohorts <- function(countries,year) {
  print("...loading and formatting cohort data...")
  n <- length(countries)
  namesabb <- data.frame(read.csv("abbreviations.csv", na.strings="
      ", skip = 0))
  demographics <- data.frame(read.csv("WPP2008_DB3_F1_AGE_BOTH_
      SEXES_estimates.csv", na.strings="\311", skip = 16, nrows
      =2977, header=TRUE))
  cohorts <- array(0, dim = c(21, n))
  for(i in 1:n){
    for(j in 1:nrow(namesabb)){
      if(namesabb[j,2]==countries[i]){
        country.code <- namesabb[j,3]
      }
    }
    for(j in 1:(nrow(demographics)-1)){
      if((demographics[j,5] == country.code) & (demographics[j,6]
          == year)) {
        cohorts.row <- j
        for(g in 1:22){
          if(g < 17){
            cohorts[g,i] <- demographics[cohorts.row,6+g]
          }
          if(g > 17){
            cohorts[g-1,i] <- demographics[cohorts.row,6+g]
          }
        }
      }
    }
  }
  row.names(cohorts) <- c("0-4","5-9","10-14","15-19","20-24","
      25-29","30-34","35-39","40-44","45-49","50-54","55-59","60-64
      ","65-69","70-74","75-79","80-84","85-89","90-94","95-99","
      100+")
  colnames(cohorts) <- countries

  return(cohorts)
}
```

```
load.sample.mortality <- function(countries,year) {
  print("...loading and formatting mortality data...")
  n <- length(countries)
  namesabb <- data.frame(read.csv("abbreviations.csv", na.strings="
      ", skip = 0))
  if(year == 2005){
    year <- 2006
  }
  load(paste("whodata",year,".RData",sep=""))
  s <- array(0, dim = c(21, n))

  for(i in 1:n){
     y <- get(paste("lt_",countries[i],year,sep=""))
     s[1:21,i] <- 1 - y[2:22,4]
  }
  row.names(s) <- c("1-4","5-9","10-14","15-19","20-24","25-29","
      30-34","35-39","40-44","45-49","50-54","55-59","60-64","65-69
      ","70-74","75-79","80-84","85-89","90-94","95-99","100+")
  colnames(s) <- countries

  return(s)
}

compute.asset.holdings <- function(R,w,h,beta,sigma,s,I0,I){
  alow <- 0
  ahigh <- 100

  a <- numeric(length=21)
  a[I0+2] <- alow
  a <- law.of.motion(a,R,w,h,beta,sigma,s,I0,I)
  flow <- a[I+1]

  a <- numeric(length=21)
  a[I0+2] <- ahigh
  a <- law.of.motion(a,R,w,h,beta,sigma,s,I0,I)
  fhigh <- a[I+1]

  a <- numeric(length=21)
  amid <-  (ahigh + alow)/2
  a[I0+2] <- amid
  a <- law.of.motion(a,R,w,h,beta,sigma,s,I0,I)
  fmid <- a[I+1]

  diff <- 1

  while (diff > 1E-6) {
    if (fmid*flow < 0) {
      ahigh <- amid
      fhigh <- fmid
    } else {
      alow <- amid
      flow <- fmid
    }
    a <- numeric(length=21)
    amid <-  (ahigh + alow)/2
    a[I0+2] <- amid
```

```
    a <- law.of.motion(a,R,w,h,beta,sigma,s,I0,I)
    fmid <- a[I+1]

    diff = abs(fmid)
  }
  return(a)
}

net.capital.demand <-  function(R,alpha,beta,delta,sigma,I0,I,
    countries,mortality,cohorts){
  n <- length(countries)
  w <- numeric(length=I)
  w[5:13] <- (1-alpha)*((R-(1-delta))/alpha)^(alpha/(alpha-1))
  h = 0
  K <- array(0, dim = c(3, n))
  colnames(K) <- countries
  row.names(K) <- c("Supply","Demand","Diff")
  for(i in 1:n){
    s <- mortality[2:21,i]
    a <- compute.asset.holdings(R,w,h,beta,sigma,s,I0,I)
    K[1,i] <- sum(a[1:20]*cohorts[1:20,i])
    N <- sum(cohorts[5:13,i])
    K[2,i] <- ((R - (1-delta))/alpha)^(1/(alpha-1))*N
    K[3,i] <- K[2,i] - K[1,i]
  }
  sumK <- sum(K[3,1:n])
  return(sumK)
}

extract.imf.gpd <- function(countries,years){
  print("...loading and formatting IMF GDP data...")
  nc <- length(countries)
  ny <- length(years)
  imfgdp  <- array(0, dim=c(ny, nc), dimnames=list(years, countries
      ))

  load("NFAdata.Rdata")
  namesabb <- data.frame(read.csv("abbreviations.csv", na.strings="
      ", skip = 0))

  for(j in 1:ny){
    for(i in 1:nc){
      for(imfctr in 1:nrow(namesabb)){
        if(namesabb[imfctr,2]==countries[i]) {
          imfcode <- namesabb[imfctr,4]
        }
      }
      for(nfactr in 1:(nrow(NFAdata)-1)){
        if((NFAdata[nfactr,2] == imfcode) & (NFAdata[nfactr,3] ==
            as.numeric(years[j]))){
          nfarow <- nfactr
        }
      }
      imfgdp[j,i] <- (as.numeric(gsub(",","",NFAdata[nfarow,18])))
    }
  }
  return(imfgdp)
```

```
}



### MAIN PROGRAM ###

# Parametrize model

# Countries and no of countries
countries <- c("CA","DE","FR","IT","JP","US","GB")
nc <- length(countries)

years <- c("1990","2000","2005")
ny <- length(years)

# Preferences
beta  <- 1.0
sigma <- 4.0

# Technology
alpha = 0.33
delta = 5*.075

# Demographics
I0 <-  4
I  <- 20

K <- array(0, dim=c(3, nc, ny), dimnames=list(c("Supply","Demand","
    Diff"),countries,years))
NFA <- array(0, dim=c(3, nc, ny), dimnames=list(c("NFA","GDP","NFA/
    GDP"), countries,years))
NFAGDP <- array(0, dim=c(ny, nc), dimnames=list(years, countries))
imfgdp <- extract.imf.gpd(countries,years)

for(y in 1:length(years)){

  cohorts   <- load.sample.cohorts(countries,years[y])
  mortality <- load.sample.mortality(countries,years[y])

  # Solve for market-clearing prices

  Rlow  <- 1.02
  Rhigh <- 1.18
  Rmid  <- (Rhigh + Rlow)/2

  flow  <- net.capital.demand(Rlow, alpha,beta,delta,sigma,I0,I,
      countries,mortality,cohorts)
  fhigh <- net.capital.demand(Rhigh,alpha,beta,delta,sigma,I0,I,
      countries,mortality,cohorts)
  fmid  <- net.capital.demand(Rmid,alpha,beta,delta,sigma,I0,I,
      countries,mortality,cohorts)

  diff <- 1

  while (diff > 1E-6) {
    if (fmid*flow < 0) {
```

```
      Rhigh <- Rmid
      fhigh <- fmid
    } else {
      Rlow <- Rmid
      flow <- fmid
    }
    Rmid <- (Rhigh + Rlow)/2
    fmid <- net.capital.demand(Rmid,alpha,beta,delta,sigma,I0,I,
        countries,mortality,cohorts)
    print(fmid)
    diff <- abs(fmid)
  }


  # Display equlibrium quantities and ratios

  print(" ")
  print(Rmid)
  R <- Rmid
  w <- numeric(length=I)
  w[5:13] <- (1-alpha)*((R-(1-delta))/alpha)^(alpha/(alpha-1))
  h = 0


  for(i in 1:nc){
    s <- mortality[2:21,i]
    a <- compute.asset.holdings(R,w,h,beta,sigma,s,I0,I)
    K[1,i,y] <- sum(a[1:20]*cohorts[1:20,i])
    N <- sum(cohorts[5:13,i])
    K[2,i,y] <- ((R - (1-delta))/alpha)^(1/(alpha-1))*N
    K[3,i,y] <- K[2,i,y] - K[1,i,y]
  }
  sumK <- sum(K[3,1:nc,y])

  for(i in 1:nc){
    NFA[1,i,y] <- -K[3,i,y]
    N <- sum(cohorts[5:13,i])
    NFA[2,i,y] <- K[2,i,y]^alpha * N^(1-alpha)
    NFA[3,i,y] <- NFA[1,i,y]/NFA[2,i,y]
    NFAGDP[y,i] <- NFA[3,i,y]
  }
}
rm(fhigh,fmid,flow)
rm(Rhigh,Rmid,Rlow)

print(K)
print(NFA)
TAB <- rbind(NFAGDP)

pdf(file <- "nfaovergdp.pdf")
barplot(TAB, beside = TRUE,col=c("darkblue","red","darkgreen"),
    legend = rownames(TAB),ylab="NFA/GDP",ylim=c(-.6,.6))
dev.off()

pdf(file <- "nfaposition.pdf")
barplot(NFA[1,1:7,3]/abs(NFA[1,6,3]),col="navy",ylab="NFA position,
     normalized by the U.S. position",ylim=c(-1,1))
```

```
dev.off()

for(i in 1:nc){
  tmp <- array(0, dim=c(2, ny),dimnames=list(c("Model", "IMF"),
      years))
  for(j in  1:ny){
    tmp[1,j] <- NFA[2,i,j]/NFA[2,6,j]
    tmp[2,j] <- imfgdp[j,i]/imfgdp[j,6]
  }
  assign(paste(countries[i],sep=""),tmp)
  print(tmp)
}
rm(tmp,i,j)

# rbind() the tables together
TAB <- rbind(CA,DE,FR,IT,JP,GB)

pdf(file <- "gdpcomp.pdf")
# Do the barplot and save the bar midpoints
mp <- barplot(TAB, beside = TRUE, axisnames = FALSE,col=c("
     deepskyblue","indianred"),legend = c("Model","Data"),ylab="GDP
     relative to the U.S.")
# Add the individual bar labels
mtext(1, at = mp, text = c("M", "D"),line = 0, cex = 0.5)
# Get the midpoints of each sequential pair of bars
# within each of the four groups
at <- t(sapply(seq(1, nrow(TAB), by = 2),
function(x) colMeans(mp[c(x, x+1), ])))
# Add the group labels for each pair
mtext(1, at = at, text = rep(c("CA","DE","FR","IT","JP","GB"), 4),
     line = 1, cex = 0.75)
# Add the color labels for each group
mtext(1, at = colMeans(mp), text = c("1990", "2000", "2005"), line
     = 2)
dev.off()

rm(TAB,mp,at)


load("temp.Rdata")

NFAGDPtmp <- array(0, dim=c(ny, nc), dimnames=list(years, countries
     ))

for(y in 1 : ny){
  for(i in 1:nc){
    NFAGDPtmp[y,i] <- NFAtmp[5,i,y]
  }
}

TAB <- rbind(NFAGDPtmp)
pdf(file <- "nfaovergdp_tmp.pdf")
barplot(TAB, beside = TRUE,col=c("darkblue","red","darkgreen"),
     legend = rownames(TAB),ylab="NFA/GDP",ylim=c(-.6,.6))
dev.off()

pdf(file <- "tfp.pdf")
```

```
TAB <- rbind(exp(tfp))
barplot(TAB, beside = TRUE, col=c("darkblue","red","darkgreen"),
    legend = rownames(TAB), ylab="TFP (US normalized to 1)", ylim=c
    (0,1.5))
dev.off()
```

B Misc

$$c_{i,t} + a_{t,i+1} = a_{i,t}\left(1 + r_{t+i-1}\right) + \epsilon_i w_{t+i-1} + h_{i,t},$$

$$\beta s_{i,t} R_{t+1} \left(\frac{c_{t+1}}{c_t}\right)^{-\sigma} = 1$$

so

$$c_{t+1} = \left(\frac{1}{\beta s_{i,t} R_{t+1}}\right)^{-\frac{1}{\sigma}} c_t$$

and

$$a_{t+1}R_{t+1} + \epsilon_i w_{t+1} + h_{t+1} - a_{t+2} = \left(\frac{1}{\beta s_{i,t} R_{t+1}}\right)^{-\frac{1}{\sigma}} \left(a_t R_t + \epsilon_i w_t + h_t - a_{t+1}\right),$$

hence

$$a_{t+2} = a_{t+1}R_{t+1} + \epsilon_{i+1} w_{t+1} + h_{t+1} - \left(\frac{1}{\beta s_{i,t} R_{t+1}}\right)^{-\frac{1}{\sigma}} \left(a_t R_t + \epsilon_i w_t + h_t - a_{t+1}\right),$$

or

$$a_{t+1} = a_t R_t + \epsilon_i w_t + h_t - \left(\frac{1}{\beta s_{i-1,t-1} R_t}\right)^{-\frac{1}{\sigma}} \left(a_{t-1} R_{t-1} + \epsilon_{i-1} w_{t-1} + h_{t-1} - a_t\right),$$