Ambiguous Business Cycles

By Cosmin Ilut and Martin Schneider

This paper studies a New Keynesian business cycle model with agents who are averse to ambiguity (Knightian uncertainty). Shocks to confidence about future TFP are modeled as changes in ambiguity. To assess the size of those shocks, our estimation uses not only data on standard macro variables, but also incorporates the dispersion of survey forecasts about growth as a measure of confidence. Our main result is that TFP and confidence shocks together can explain roughly two thirds of business cycle frequency movements in the major macro aggregates. Confidence shocks account for about 70% of this variation.

JEL: D81, D84, E20, E32, E52

An open question in business cycle analysis is what kind of shocks drive fluctuations. Recent literature has investigated the role of intangible information that is not contained in current innovations to TFP or policy. For example, models with “news shocks” have examined how the arrival of information about future TFP can induce booms or recessions. A common feature of existing models with intangible information is that uncertainty is constant. It is plausible, however, that intangible information also affects agents’ confidence about the future. For example, conflicting news reports or disagreement among experts about future growth is likely to affect the way households and businesses plan ahead.

This paper studies a business cycle model that allows for shocks to confidence. We start from a standard New Keynesian model along the lines of Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007). A key new feature is that the representative household is averse to ambiguity (Knightian uncertainty). We model shocks to confidence as changes in ambiguity. To assess the size of those shocks, our estimation uses not only data on standard macro aggregates, but also incorporates the dispersion of survey forecasts about growth as a measure of confidence. Our main result is that TFP and confidence shocks together can account for the majority of business cycle frequency movements in the major macro aggregates. Confidence shocks account for about 70% of this variation.

We find that confidence shocks matter because they move output and hours

* Ilut: Duke University and NBER, Department of Economics, Duke University, 213 Social Sciences Bldg., Durham, NC, 27708, E-mail: cosmin.ilut@duke.edu. Schneider: Stanford University and NBER, Department of Economics, Stanford University, 579 Serra Mall, Stanford, CA 94305, E-mail: schneidr@stanford.edu. We would like to thank George-Marios Angeletos, Francesco Bianchi, Lars Hansen, Nir Jaimovich, Alejandro Justiniano, Christian Matthes, Fabrizio Perri, Giorgio Primiceri, Bryan Routledge, Juan Rubio-Ramirez, Chris Sims and Rafael Wouters, as well as many seminar participants for helpful discussions and comments. Cosmin Ilut gratefully acknowledges financial support from the National Science Foundation through grant SES-3331805.
more than labor productivity. Indeed, a loss of confidence induces households and firms to act more cautiously. Nominal rigidities dampen relative price adjustment that could otherwise maintain high spending and employment. Economic activity can thus contract even if current labor productivity is high. In existing studies, this feature of the data is typically accounted for by unobservable shocks that drive a countercyclical “labor wedge” between the marginal rate of substitution of consumption for labor and the marginal product of labor. Confidence shocks not only generate a countercyclical labor wedge, but also closely track movements in forecast dispersion. This leads our estimation to prefer confidence shocks as a driver of business cycles even if we also allow for unobservable shocks to labor supply.

We describe preferences by the multiple priors utility axiomatized by Gilboa and Schmeidler (1989). Ambiguity averse agents lack the confidence to assign probabilities to all relevant events. Instead, they act as if they evaluate plans using a worst case probability drawn from a set of multiple beliefs. A loss of confidence is captured by an increase in that set. It could be triggered, for example, by news that experts now disagree more about the future. Conversely, an increase in confidence is captured by a shrinkage of the set of beliefs. For example, as agents observe experts moving towards consensus on the course of the economy, they move closer towards thinking in terms of a single probability.

Our focus is on confidence about TFP. At every date, agents have a set of beliefs about innovations to TFP. The belief set is parametrized by an interval of means centered around zero. A loss of confidence corresponds to an increase in the width of that interval. In particular, it makes the “worst case” mean worse. An agent who loses confidence thus responds as if he had received bad news about the future. Conversely, an agent who gains confidence is described by a narrowing of the interval, and responds as if he had received good news.

Although our model allows for uncertainty shocks, it can be solved and estimated using standard linear tools. This works because ambiguity is about the mean and the worst case mean that supports agents’ equilibrium choices can be written as a linear function of the state variables. Decision rules can then be well-approximated by standard linearization around a steady state. In contrast to the case of risk shocks, the study of ambiguity shocks therefore does not require any special techniques beyond those of standard business cycle modeling without uncertainty shocks.

We estimate our model with data on five macro variables – consumption, investment, hours, inflation and the nominal interest rate – for the period 1985-2011. Our sixth observable is the interdecile range of growth forecasts in the Survey of Professional Forecasters (SPF); its model counterpart is the interval of growth forecasts implied by agents’ belief sets. The idea here is that the representative agent samples experts’ opinions and aggregates them when making decisions. Since he is ambiguity averse, stronger disagreement among experts generates lower confidence in probability assessments of the future.
Our baseline estimation uses only TFP and confidence shocks. Our Bayesian estimation strategy allows for measurement error on all variables. It employs standard Kalman filtering techniques; this is possible because the model solution takes the familiar form of a linear state space system. Confidence shocks emerge as a major source of persistent joint movements in consumption, investment and hours. Historical decompositions show how the role of shocks differs across key episodes in our sample. In particular, the 1990s boom saw high confidence paired with moderately high productivity realizations. In contrast, the most recent recession is almost entirely explained by a large drop in confidence.

There are two reasons why our estimation assigns a major role to confidence shocks. One is empirical: forecast dispersion is countercyclical. In particular, it shows a drawn out low spell in the 1990s as well as a sharp upward spike during the 2008 recession. The other reason is that, in our model, confidence shocks induce recessions in which major macro aggregates fall together. The key intuition here is that cautious spending behavior by households and firms cannot be met by enough of a decline in the relative price of current consumption. Instead, nominal prices increase as firms fear cost runups and the monetary authority responds by increasing the nominal interest rate. As real interest rates do not decline, output is determined by weaker demand for goods, regardless of the level of productivity.

In our baseline model, confidence shocks not only generate the majority of quantity fluctuations, but also track movements in forecast dispersion. It is natural to ask whether we might gain by including unobservable shocks to labor supply. We thus consider two models that allow for shocks to the disutility of work. We first estimate a model with only TFP and disutility-of-work shocks on data for only macro variables. We verify that this model can also explain the majority of business cycle fluctuations in major aggregates. Disutility-of-work shocks now explain 80% of the variation of hours worked, as expected in light of previous studies. Interestingly, the estimated disutility shock series is very similar to the confidence shock series we found in the baseline estimation.

We then estimate a “combination model” with three shocks: we again introduce confidence shocks and also include forecast dispersion as an observable. The estimation results show that the share of variance explained by labor supply shocks drops to close to zero. At the same time, the fit is very close to that of the baseline model which is estimated on the same set of variables. In fact, a Bayesian model selection criterion favors the baseline model over the model that also allows for disutility-of-work shocks: the additional parameters introduced by disutility-of-work shocks do not sufficiently increase fit.

There is now a fair number of applications of ambiguity aversion, especially to financial markets (for surveys, see Epstein and Schneider (2010) or Guidolin and Rinaldi (2013)). In the business cycle literature, several authors have studied

\footnote{In the linearized model, this shock works much like the “wage markup” shock in Smets and Wouters (2007). There are several other candidates that similarly generate a countercyclical labor wedge. See Chari, Kehoe and McGrattan (2009) and Shimer (2009) for a discussion.}
models of ambiguity aversion described by “multiplier” preferences (for example, Hansen, Sargent and Tallarini Jr (1999), Cagetti et al. (2002), Bidder and Smith (2012)). They assume that agents act as if they evaluate plans using a worst case belief that minimizes the sum of expected utility and a smooth function that penalizes deviations from a reference belief. In contrast, we use a recursive version of the multiple priors model (Epstein and Wang (1994), Epstein and Schneider (2003)). Multiple priors utility is not smooth when belief sets differ in means, a fact we use in our solution strategy.

Since models of ambiguity aversion imply that agents behave as if they hold worst case beliefs, equilibrium actions are the same as in models with expected utility maximizers who are endowed with those worst case beliefs. However, while observational equivalence results between those two classes of models can be helpful, for example in computations, they are limited to equilibrium actions and do not imply that the models themselves are identical. Indeed, in models of ambiguity aversion the worst case belief is endogenous and therefore not policy invariant.\(^2\)

The use of worst case beliefs to model cautious behavior also recalls models of pessimism. For example, the asset pricing studies of Abel (2002) and Lam, Cecchetti and Mark (2000) assume investors make forecasts that are systematically biased with respect to those of an econometrician. Our model does not make such an assumption. Instead, agents in our model contemplate alternative beliefs that cannot be distinguished by the data. Agents’ actions are then rational responses to their lack of confidence in what the true data generating process looks like.

Rational expectations models allow for intangible information by introducing signals (or “news”) about future “fundamentals” (for example Beaudry and Portier (2006), Christiano et al. (2008), Jaimovich and Rebelo (2009), Christiano et al. (2010), Barsky and Sims (2012), Blanchard, L’Huillier and Lorenzoni (2013) and Schmitt-Grohé and Uribe (2012)). The difference between a change in confidence and a signal is that the latter is followed, at least on average, by a realization that validates the signal. For example, bad news about TFP is on average followed by low TFP. A loss of confidence about TFP, however, need not be followed by low TFP. Information that triggers a loss of confidence affects agents’ perception of future shocks, as described by their (subjective) set of beliefs. It does not directly affect the distribution of realized shocks (such as the direction or magnitude of the shocks).\(^3\)

Recent work on changes in uncertainty in business cycle models has focused on

---

\(^2\)For a stark example, assume that the government introduces a policy of wasteful spending in response to a productivity increase. If a positive productivity shock made agents worse off, then a loss of confidence would stoke fears of government overreach. Equilibrium would look very different from our baseline case in which agents fear low productivity.

\(^3\)Confidence shocks can affect agents’ actions (and hence the business cycle) even if they are uncorrelated with shocks to “fundamentals” (such as productivity) at all leads and lags. They share this feature with sunspots (for example Farmer (2012)), stochastic bubbles (Martin and Ventura (2012)) and shocks to higher order beliefs (Angeletos and La’O (2013)). They differ from those other shocks in that they alter agents’ perceived uncertainty of fundamental shocks.
changes in realized risk – looking either at stochastic volatility of aggregate shocks (see for example Fernández-Villaverde and Rubio-Ramírez (2007), Justiniano and Primiceri (2008), Fernández-Villaverde et al. (2011), Basu and Bundick (2011) and the review in Fernández-Villaverde and Rubio-Ramírez (2010)), time-varying probability of aggregate disaster (Gourio (2012)) or at changes in idiosyncratic volatility in models with heterogeneous firms (Arellano, Bai and Kehoe (2010), Bloom et al. (2012), Bachmann, Elstner and Sims (2013) and Christiano, Motto and Rostagno (2013)). We view our work as complementary to these approaches. In particular, confidence shocks can generate responses to uncertainty – triggered by news, for example – that is not connected to later realized changes in risk.

The rest of this paper is structured as follows. Section I describes the model. Section II analytically studies a simple case to illustrate the role of ambiguity in an economy with nominal rigidities. Section III explains our estimation strategy and quantitative results.

I. Model

This section describes a New Keynesian business cycle model along the lines of Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007). Many elements of our model are standard in the literature (for example in Del Negro et al. (2007), Christiano et al. (2008), Fernández-Villaverde, Guerrón-Quintana and Rubio-Ramírez (2010) and Justiniano, Primiceri and Tambalotti (2011)). What is new is that decision makers are ambiguity averse and experience shocks to confidence.

Technology

There are two types of perishable goods. Output of a final good $Y_t$ is made by combining a continuum of intermediate goods $Y_{j,t}$. Each intermediate good is made using capital $K_{j,t}$, labor $H_{j,t}$ and final goods. The production functions are

$$
Y_t = \left[ \int_0^1 Y_{j,t} \frac{1}{\lambda_f} \frac{1}{\lambda_f} dj \right]^{\lambda_f}
$$

$$
Y_{j,t} = \max\{Z_t K_{j,t} \gamma_t \Phi \gamma_t, 0\}
$$

For intermediate goods production, the parameter $\lambda_f$ regulates the substitutability between intermediate goods; the elasticity of substitution is $\lambda_f / (\lambda_f - 1)$. In final goods production, $\gamma$ is the growth rate of labor augmenting technical progress. Uncertainty enters via TFP shocks $Z_t$. There is a fixed cost of production in terms of final goods that grows at the trend rate, $\gamma$, which ensures that profits are zero in steady state.

There are also two types of labor. Production uses homogeneous labor services $H_t$ that in turn combine a continuum of specialized labor services. The latter are
aggregated according to

$$H_t = \left[ \int_0^1 (h_{i,t})^{-\lambda_w} \, di \right]^{\lambda_w},$$

where the parameter $\lambda_w$ regulates the substitutability between different types of specialized labor. Specialized labor is supplied by households who experience disutility from working. There is a continuum of households, each of whom has the ability to produce a particular type of specialized labor.

Capital accumulation is subject to adjustment costs. Given a path of investment $I_t$, the capital stock evolves according to

$$K_{t+1} = (1 - \delta)K_t + \left[ 1 - \frac{1}{2} \kappa (\gamma - I_t/I_{t-1})^2 \right] I_t,$$

where $\kappa > 0$ is a parameter.

**Technology shocks**

We follow common practice in business cycle analysis by assuming that TFP follows a persistent AR(1) process. We do not assume, however, that the innovations to TFP are IID. Instead, the data generating process for the TFP shock $Z_t$ is

$$\log Z_{t+1} = \rho_z \log Z_t + \mu_t^* + u_{t+1}$$

Every innovation to TFP $z_{t+1}^x = \log Z_{t+1} - \rho_z \log Z_t$ consists of two components: $u_t$ is an iid normal sequence of innovations with mean zero and variance $\sigma_u^2$ whereas $\mu_t^*$ is a deterministic sequence. We restrict the long run behavior of $\mu_t^*$ by assuming that its empirical moments converge to those of an iid normal stochastic process that has mean zero and variance $\sigma^2_z - \sigma_u^2 > 0$ and that it is independent of $u$.

To an observer who samples TFP data, the innovations $z^x$ thus “look like” a realization of an iid process with mean zero and variance $\sigma^2_z$. It follows that the sequence $\mu_t^*$ cannot be learned: even with a large amount of data, it is impossible to disentangle the parameter sequence $\mu^*$ and the shock sequence $u$. In other words, (2) describes a large family of possible processes for TFP, all indistinguishable even with a large amount of data, that can have rather different implications in the short run, for example because they differ in the conditional mean $\mu_t^*$. Agents in our model respond to this uncertainty by treating $\mu^*$ as ambiguous; their perception of ambiguity changes over time with the arrival of information as described below.

Alternatively, an observer might deal with uncertainty about $\mu^*$ by forming a probabilistic prior that is iid normal with mean zero and variance $\sigma^2_z - \sigma^2_u$. This is what we will do ourselves when we empirically evaluate the model below.
From our own perspective as econometricians, we will treat uncertainty about \( \mu^* \) and uncertainty about \( u \) in the same way, so the distinction between the two components is not important. Moreover, since the empirical moments of TFP (mean, variance and covariances) are independent of the particular sequence \( \mu^* \), it is not necessary to learn what \( \mu^* \) is in order to study the long run predictions of the model.

**Preferences**

To define utility, we collect the exogenous state variables in a vector \( s_t \in S \). We use \( s^t = (s_1, ..., s_t) \) to denote history up to date \( t \). A consumption plan \( C \) says, for every history \( s^t \), how many units of the final good \( c_t(s^t) \) a household consumes and for how many hours \( h_{i,t}(s^t) \) a household works. The subscript \( i \) indicates that each household supplies specialized labor services. For a given consumption plan \( C \), utility is defined recursively by

\[
U_t(C; s^t) = \log c_t(s^t) - \psi_L \frac{h_{i,t}(s^t)}{1 + \sigma_L} + \beta \min_{p \in \mathcal{P}_t(s^t)} E^p[U_{t+1}(C; s^t, s_{t+1})],
\]

where \( \mathcal{P}_t(s^t) \) is a set of conditional probabilities about next period’s state \( s_{t+1} \in S \). The recursive formulation ensures that preferences are dynamically consistent. Details and axiomatic foundations are in Epstein and Schneider (2003).

Utility conditional on history \( s^t \) equals felicity from current consumption plus discounted expected continuation utility. The conditional probability \( p \) is drawn to minimize expected continuation utility, subject to the constraint that \( p \) must lie in the set \( \mathcal{P}_t(s^t) \). The primitives of the utility representation are thus the parameters governing felicity (\( \psi_L \) and \( \sigma_L \)), the discount factor \( \beta \), and the entire process of one-step-ahead conditional probability sets \( \mathcal{P}_t(s^t) \). The standard model is obtained as a special case if all sets \( \mathcal{P}_t(s^t) \) contain only one belief. More generally, a nondegenerate belief set means that agents are not confident in probability assessments. A larger set at history \( s^t \) describes an agent who is less confident, perhaps because he has only poor information about what will happen at \( t + 1 \).

The multiple priors functional form (3) captures a strict preference for knowing probabilities. To illustrate, fix two complementary events \( \tilde{S}_1 \subset S \) and \( \tilde{S}_2 = S \setminus \tilde{S}_1 \) that may occur at \( t + 1 \). Consider an agent who is indifferent, as of date 1, between two plans \( i = 1, 2 \), where plan \( i \) pays \( c \) if \( s_{t+1} \in \tilde{S}_i \) and \( c < \bar{c} \) otherwise. Expected utility captures this situation by a single belief under which the events \( \tilde{S}_i \) both have probability one half. The multiple priors model allows for a range of probabilities about \( \tilde{S} \). A bet on \( s_{t+1} \in \tilde{S} \) is then evaluated using a belief under which the probability of \( \tilde{S} \) occurring is low, whereas a bet on \( s_{t+1} \notin \tilde{S} \) is evaluated using a belief under which the probability of \( \tilde{S} \) not occurring is low. Since the worst case probability moves endogenously with the bet, both probabilities can be lower than one half. Both bets can therefore be ranked worse than a fair bet.
with known probability one half. This strict preference for knowing probabilities cannot be captured by the standard expected utility model.

Our baseline model assumes that the felicity function does not depend on the state of the world. For comparison with other literature, we also consider below an extension that allows for shocks to the disutility of work. For that case, we assume that the parameter $\psi_L$ is replaced by $\hat{\psi}_L\zeta_t$, where the disutility shock $\zeta_t$ follows an AR(1) process

$$\log \zeta_t = \rho \zeta_{t-1} + \sigma \epsilon_{\zeta_t}$$

and where $\epsilon_{\zeta_t}$ is a standard normal iid innovation.

**Belief sets and uncertainty shocks**

We now specify belief sets that describe the evolution of confidence about future TFP. The idea is that agents gather intangible information about the ambiguous component of TFP $\mu^*_t$. For example, they consult experts and sample news reports. In some periods, such information gathering leaves them relatively confident that the correct forecast of future TFP $\log Z_{t+1}$ is $\rho_z \log Z_t$. In other periods, various pieces of information might contradict each other, and agents are less confident about their forecast. We summarize the ambiguity perceived at time $t$ by a number $a_t$. The belief set is parametrized by an interval of means centered around zero:

$$\log Z_{t+1} = \rho_z \log Z_t + \mu_t + u_{t+1}; \quad \mu_t \in [-a_t, -a_t + 2|a_t|]$$

where $u$ is normal with mean zero and variance $\sigma_u^2$. When $a_t$ is higher, there is more ambiguity and the set of belief is larger – in particular, a wider interval implies a lower worst case mean.

We think of $-a_t$ as an indicator of the quality of intangible information available at date $t$ about TFP at $t+1$. It is assumed to follow an AR(1) process

$$a_{t+1} - \bar{a} = \rho_a(a_t - \bar{a}) + \sigma_a \epsilon_{a_{t+1}}$$

where $\epsilon_{a_{t+1}}$ is a standard normal iid innovation. This specification allows for persistence in the quality of information. Agents know the law of motion for $a_t$ and thus take the evolution of information quality – and hence ambiguity – into account when making decisions. The exogenous state of the economy in our model can thus be summarized by the pair $(Z_t, a_t)$. Agents form expectations about endogenous variables in the standard way using knowledge of the equilibrium mapping from exogenous to endogenous variables. As a result they also perceive ambiguity about endogenous variables such as consumption.

We assume that $a_t$ is an exogenous persistent process, interpreted as the cumulative effect of news that affect confidence. Its key property is that confidence moves slowly with signals. Models of learning under ambiguity derive details of
the dynamics of confidence endogenously from a description of the learning process (see for example Epstein and Schneider (2007), Epstein and Schneider (2008) and Ilut (2012)). In this paper, we start from the functional form (6) and infer the dynamics of confidence from data on forecast dispersion. The idea is that the degree of confidence reflected in representative agent actions is higher if experts, whose opinions agents observe as signals, are in closer agreement about the future path of TFP.

**Firms**

Production is organized as follows. Final output is produced by competitive one shot firms. Profit maximization and the zero profit condition imply a demand for intermediate good $j$

\[
Y_{j,t} = Y_t \left( \frac{P_t}{P_{j,t}} \right)^{\lambda_f - 1}; \quad P_t = \left[ \int_0^1 P_{j,t}^{1/\lambda_f} \, dj \right]^{(1-\lambda_f)}
\]

where $P_t$ is the ideal index of intermediate goods prices. Intermediate good $j$ is produced by price-setting monopolistically competitive firms that sell at sticky prices. Below we consider two models of price setting. In the simple model of section II we assume that all firms must set prices one period in advance. This is a minimal setup for studying the effect of ambiguity on output through an intertemporal pricing decision.

In the estimated model of section III, we follow Calvo (1983) in assuming that a random fraction $1 - \xi_p$ of firms can reoptimize their price every period. The remaining $\xi_p$ firms cannot reoptimize and set $P_{it} = \Pi P_{i,t-1}$, where $\Pi$ is steady state inflation. There is a complete set of contingent claims on price setting events. Let the random variables $M_{t,t+s}$ denote the date $t$ prices of contingent claims to date $t + s$ consumption goods, normalized by the relevant conditional probabilities. Whenever the $j^{th}$ firm has the opportunity to set its price, it maximizes the expected present discounted value of future profits:

\[
E_t \sum_{s=0}^{\infty} (\xi_p)^s M_{t,t+s} \left[ P_{j,t+s}^s Y_{j,t+s} - W_{t+s} H_{j,t+s} - P_{t+s}^s r^k_{t+s} K_{j,t+s} \right],
\]

subject to the demand function (7). Here $P_{t+s}^s$ is the rental rate on capital services and $W_t$ is the wage rate for homogeneous labor services. Both inputs are purchased by the intermediate goods firms in competitive factor markets.

We now turn to the labor market. Homogeneous labor services are produced by competitive “employment agencies”. The agencies’ demand for specialized labor is analogous to (7):

\[
h_{i,t} = H_t \left( \frac{W_t}{W_{i,t}} \right)^{\lambda_w - 1}
\]
Here $W_{i,t}$ is the wage rate for the $i$th type of specialized labor. We follow Erceg, Henderson and Levin (2000) in assuming that there is a continuum of households who are monopolistically competitive suppliers of specialized labor. Wages are sticky in a Calvo fashion: with probability $1 - \xi_w$, household type $i$ sets the wage $W_{i,t}$ optimally; with $\xi_w$ he sets $W_{i,t} = \bar{\Pi} W_{i,t-1}$. There is also a complete set of contingent claims on wage setting events.

**Households**

The $i$th household’s budget constraint is:

\begin{equation}
P_tC_t + P_tI_t + B_t = B_{t-1}R_{t-1} + P_t^kK_t + W_{i,t}h_{i,t} + X_{i,t} - T_tP_t
\end{equation}

where $B_t$ are holdings of government bonds, $R_t$ is the gross nominal interest rate, $X_{i,t}$ is the net cash inflow from participating in state contingent securities at time $t$ and $T_t$ denotes net lump-sum taxes. Households maximize utility subject to the budget constraint, labor demand (9) and capital accumulation (1). The presence of contingent claims for price and wage setting events imply that both types of idiosyncratic shocks are perfectly shared across firms and households, respectively. This standard argument is independent of the presence of ambiguity about aggregate shocks to TFP.

**The government**

The monetary policy authority cares about deviations of inflation from a target $\bar{\Pi}$ as well as deviations of output from the balanced growth path. Let $\bar{Y}$ denote output relative to trend and let $\bar{R} = \bar{\Pi} \gamma / \beta$. The nominal interest rate is set as

\begin{equation}
\frac{R_t}{\bar{R}} = \left( \frac{R_{t-1}}{\bar{R}} \right)^{\rho_R} \left[ \left( \frac{\Pi_t}{\bar{\Pi}} \right)^{\alpha_y} \left( \frac{Y_t}{\bar{Y}^{(1-\alpha)\bar{\gamma}} t} \right)^{\alpha_y} \right]^{1-\rho_R}
\end{equation}

where the coefficient $\alpha_y$ on inflation is strictly greater than one (the Taylor principle), the coefficient on output is positive and the policy rule is potentially inertial if $0 < \rho_R < 1$.

**Equilibrium**

An equilibrium consists of stochastic processes for prices ($R_t$, $P_t$, $r^k_t$, $(M_{t,t+s})$, $(W_{i,t})$) as well as quantities ($C_t$, $I_t$, $K_t$, $B_t$, $Y_t$, $(Y_{j,t}, H_{j,t}, K_{j,t})$, $(h_{i,t})$) such that (i) individual households choose wages as well as consumption and labor to maximize utility given the demand for their own type of labor as well as prices for other goods and assets, (ii) individual intermediate goods firms choose the price for their own product as well as production to maximize the present value of profits given demand for their own type of goods as well as prices for other goods and assets, (iii) final goods firms optimize given input prices, and (iv) markets for final goods, capital and all types of intermediate goods and labor clear.
II. A simple case

In this section we illustrate the interaction of confidence shocks and nominal rigidities in a simplified version of the model that has no capital and no labor market frictions. All types of specialized labor are perfect substitutes, so households effectively supply homogeneous labor services $H_t$ in a competitive labor market with flexible wages. There is no growth; intermediate goods firms have production functions $Y_{j,t} = Z_t H_{j,t}$. We focus on the case of iid productivity and confidence shocks ($\rho_a = \rho_z = 0$). Firms set prices one period in advance. The $j$th firm thus maximizes

$$E_t M_{t,t+1} [P_{j,t+1} Y_{j,t+1} - W_{t+1} H_{j,t+1}]$$

subject to the demand function (7). Finally, we simplify utility by making felicity linear in hours: we set $\sigma_L = 0$ and normalize $\psi_L = 1$ in (3).

We solve for an equilibrium following the same four steps as in the case of the larger estimated model below. First, we guess a worst case belief: we conjecture the form of the minimizing $\mu_t$ that supports the optimal choice in (3) after every history given the endogenous equilibrium prices. The natural guess here is that agents behave as if innovations to productivity are low, that is, $\mu_t = -a_t$ at all histories. If the guess is correct, then agents' decision rules are the same as in a model with fixed worst case beliefs.

The second step is to find a first order approximation of equilibrium around the steady state under the guessed worst case belief. We then verify our guess for the worst case belief by checking that the value function is indeed increasing in the ambiguous variable, here the innovation to productivity. Finally, we characterize the dynamics of the model under the true data generating process.

Equilibrium under the worst case belief

We denote conditional moments under the guessed worst case belief $\mu_t = -a_t$ by stars. The intertemporal tradeoffs faced by households and firms are summarized by

$$1 = R_t E_t^* \left[ \beta \frac{C_t}{C_{t+1}} P_t \right]$$

subject to the demand function (7). Finally, we simplify utility by making felicity linear in hours: we set $\sigma_L = 0$ and normalize $\psi_L = 1$ in (3).

The first equation is the household Euler equation for nominal bonds: it relates the marginal rate of substitution and the inflation rate to the policy interest rate $R_t$. Ambiguity aversion implies that asset prices are governed by worst case expectations. The second equation reflects optimal price setting of intermediate goods producers: firms set prices equal to worst case nominal marginal cost multiplied by a markup. Worst case expectations are relevant here because firms compute marginal revenue and marginal cost on an expected uncertainty-adjusted basis.
To characterize equilibrium, we further use the policy rule (11) and the resource constraint $C_t = Z_t H_t$. Moreover, the household’s marginal rate of substitution of consumption for labor must be equal to the real wage. Independently of how prices are set, nominal marginal cost is thus always
\[
\frac{W_t}{Z_t} = P_t H_t
\]

Two features of preferences and technology are important here. First, even when prices are sticky, nominal wages are flexible and so nominal marginal cost moves one for one with the price level. Second, reductions in real marginal cost from higher productivity are exactly offset by an increase in real wages. Real marginal cost thus depends only on hours – it is higher when workers must be enticed to work harder.

The equilibrium with flexible prices provides a simple benchmark. Since firms equate marginal revenue and marginal cost, employment is constant at
\[
(13) \quad H_t = \frac{1}{\lambda_f}
\]
If productivity increases, firms bid up the real wage. Consumption increases and the real interest rate declines to discourage savings. Offsetting wealth and substitution effects on labor supply imply that hours remain unchanged. If ambiguity increases, households behave as if future productivity (and hence consumption) is expected to fall. State prices increase and the real interest rate declines. Offsetting wealth and substitution effects imply that both consumption and labor supply remain unchanged.

*Loglinearization around the worst case steady state*

Consider the steady state under the worst belief. This is a convenient point around which to linearize the equilibrium law of motion, since agents act as if the economy converges there in the long run. We thus compute the solution to the equilibrium conditions under the assumption that $Z_t = e^{-\bar{a}}$ for sure. From (12), hours are constant at the flexible price equilibrium level (13) and output is $e^{-\bar{a}}/\lambda_f$. The real interest rate is $1/\beta$ and the nominal interest rate is therefore $\bar{R}$ as defined above. Note that neither growth rates nor the level of hours depends on $\bar{a}$; however, ambiguity does affect agents’ perception of the level of productivity, and hence output, relative to trend.

We write the equilibrium law of motion in terms of log deviations from the worst case steady state. We use letters in lower case (to indicate logs) and with both hats (to indicate deviations) and stars (to indicate deviations are from the worst case steady state). We obtain a three-equation loglinear New Keynesian
model:

\[ E_t^* \hat{h}_{t+1} = 0 \]
\[ i_t^* - \hat{\pi}_{t+1} = E_t^* (z_t^* + \hat{h}_t^*) - (\hat{z}_t^* + \hat{h}_t^*) \]
\[ i_t^* = \alpha_\pi \hat{\pi}_t^* + a_y (\hat{z}_t^* + \hat{h}_t^*) \]

The first equation shows that optimal price setting is purely forward-looking.\(^4\) Firms equate nominal marginal revenue and cost under worst case expectations and thus stabilize expected hours. The second equation is an intertemporal Euler equation; it simplifies here because expected hours are zero and inflation is determined one period in advance. The third equation is the policy rule.

It is convenient to initially restrict the policy rule so \( \alpha_\pi = 1 + a_y \). This restriction holds for example in Taylor (1993) who found \( a_\pi = 1.5 \) and \( a_y = 0.5 \). The method of undetermined coefficients then delivers the solution

\[ \hat{h}_t^* = -\hat{z}_t^* - \hat{a}_{t-1} \]
\[ \hat{\pi}_{t+1} = a_t^* \]

(14)

Under the worst case belief the conditional mean of \( \hat{z}_{t+1}^* \) is \( -\hat{a}_t^* \) so that \( \hat{z}_t^* + \hat{a}_{t-1}^* \) is an innovation to productivity. Its effect is to lower hours. Moreover, a loss of confidence – an increase in \( \hat{a}_t^* \) leads to a contemporaneous increase in inflation.

A negative effect of productivity shocks on hours is typical of models with sticky prices. Suppose firms hire to serve demand at a predetermined price. Demand depends on the wage and real interest rate faced by households, which are also affected by price stickiness. In particular, suppose that, in response to the productivity shock, the real interest rate cannot fall as much as it would with flexible prices. Demand is then lower and firms respond to higher productivity by using less labor to produce the same output, rather than by increasing production. The stickiness of real interest rates is due to the interaction of price setting and monetary policy. On the one hand, price setting is forward looking so inflation does not respond to productivity. On the other hand, the Taylor rule calls for an increase in the nominal rate when output rises.

The solution (14) describes – to first order – how endogenous variables respond to the state variables, namely current and past ambiguity \( (a_t, a_{t-1}) \) as well as current productivity \( z_t \). This response does not depend on the true data generating process (2). All that matters is the worst case belief under which agents’ decisions are computed. To verify our guess for the worst case belief, consider household utility at the optimal loglinear policy and equilibrium prices. The Bellman equation at the optimal policy is

\(^4\)Since all firms set prices one period in advance, we do not obtain a Phillips Curve equation as is typically derived from staggered price setting a la Calvo. The model is therefore easier to solve since inflation is not a state variable.
\[
V (a_t, a_{t-1}, z_t) = \log \left(e^{-a_t/\lambda_f} \right) - \lambda_f^{-1} e^{-z_t-a_{t-1}} + \\
+ \beta \min_{\mu_t \in [-a_t, -a_t + 2|a_t|]} E_t^* [V (a_{t+1}, a_t, \mu_t + u_{t+1})]
\]

Innovations to productivity \( \hat{z}_t^* \) do not change consumption but decrease the disutility of work. The value function is therefore increasing in \( z_t \). It follows that the minimizing conditional mean is indeed always \( \mu_t = -a_t \).

**Steady state & equilibrium dynamics**

We are now ready to characterize the equilibrium dynamics. To this end, we combine the equilibrium law of motion (14) and the true data generating process (2). We first substitute \( \hat{z}_t^* = z_t + \bar{a} \) and \( \hat{a}_t^* = a_t - \bar{a} \) into (14). We then collect constant terms to define the true steady state of the model. Log deviations from the true steady state are denoted by hats. For ambiguity, we have \( \hat{a}_t = \hat{a}_t^* \) since its distribution is known. For hours and inflation, we obtain

\[
\hat{h}_t := h_t - (\log (1/\lambda_f) - \bar{a}) = -\hat{a}_{t-1} - (\hat{\mu}_{t-1} + u_t) \\
\hat{\pi}_{t+1} := \pi_{t+1} - \log \bar{\Pi} = \hat{a}_t
\]

The deviation of productivity from steady state depends on the ambiguous component \( \hat{\mu}_t^* \). By construction, this sequence fluctuates around zero, as does \( \hat{h}_t \) itself.

Consider first the steady state effects of ambiguity. Equilibrium hours fluctuate around \( e^{-\bar{a}/\lambda_f} \), which is strictly lower than hours in the flexible price case. This reduction in mean economic activity reflects cautious price setting behavior on the part of firms and precautionary savings on the part of households. On the one hand, firms know that if productivity falls then hours and hence real marginal cost rise. On average, ambiguity about productivity leads firms to set prices as if productivity is lower by \( \bar{a} \). On the other hand, households also worry about future low productivity. As a result, although actual productivity is higher than what households fear, the high steady state real interest rate holds back demand.

Consider next how the economy responds to a loss of confidence. If confidence drops below the steady state, firms fear cost runups and therefore increase prices. The following period, productivity is not unusually low. However, the monetary authority responds to inflation by raising the nominal, and thereby the real, interest rate. As demand shrinks, employment and output jointly contract. At the same time, the “labor wedge” between the marginal product of labor and

\[5\] The Bellman equation here clarifies not only that our guess is correct, but also that it is the only viable guess. Indeed, if we had guessed, for some period \( t \), some other worst case mean \( \hat{\mu}_t \in [-a_t, -a_t + 2|a_t|] \), the dependence of the value function on innovations to \( z \) would look the same, so the worst case mean would be \(-a_t \), a contradiction.
the household marginal rate of substitution – here equal to $1 - H_t$ – increases.\footnote{The labor wedge is the tax rate $\tau$ that solves the equation $MRS = (1 - \tau) MPL$, where $MRS$ is the marginal rate of substitution of consumption for labor (here simply equal to $C_t = Z_t H_t$) and $MPL$ is the marginal product of labor, here equal to $Z_t$.}

Confidence shocks thus generate comovement of hours and consumption, together with a countercyclical labor wedge.\footnote{The effect of confidence shocks on hours occurs with a delay of one period, because all firms set prices one period in advance. With Calvo price setting, there will be a contemporaneous effect as well, since price adjusters who fear cost runups already set higher prices on impact.}

The solution (15) shows that a confidence shock does not have a contemporaneous effect on hours – the recession arrives with a lag of one period. This is because we have restricted the policy rule so $\alpha = 1 + a_y$. Monetary policy then responds sufficiently to output so as to stabilize demand in the face of an uncertainty shock, at least on impact. With general coefficients $a_y$ and $a_{\pi}$, the steady state remains unchanged, but the dynamics become

$$\hat{h}_t = - \left( (1 + a_y)^{-1} - a_{\pi}^{-1} \right) \hat{a}_t - \hat{a}_{t-1} - (\mu^*_t + u_t)$$

$$\hat{\pi}_{t+1} = a_{\pi}^{-1} (1 + a_y) \hat{a}_t$$

If $a_{\pi} > 1 + a_y$ as in many estimated New Keynesian models, a confidence shock immediately induces a recession. For example, if $a_y = 0$ the current nominal interest rate does not respond to confidence at all and firms increase prices by less as they anticipate future antinflationary actions. Both effects imply a smaller drop in the real rate that fails to encourage spending in the face of ambiguity. In contrast, with $a_y > 0$ lower nominal rates and higher expected inflation combine to stabilize demand.

To sum up, the simple model of this section demonstrates how confidence shocks generate a business cycle in an economy with nominal rigidities. The mechanism we have described here is also at the heart of the richer estimated model below. In a nutshell, confidence shocks work like bad news ex ante, and therefore have a first order effect on intertemporal decisions such as household savings and firm price setting and savings, as well as, in the richer model, firm wage setting and investment. With nominal rigidities in place, the response of interest rates is sluggish and prevents large relative price adjustments. As a result, cautious behavior leads to quantity adjustments that drive the economy into recession.

### III. The quantitative exercise

In this section, we describe results from three estimation exercises. Our baseline case allows only for TFP shocks and shocks to confidence. The labor supply case allows for TFP shocks as well as shocks to the disutility of work. Finally, the combination case allows for all three shocks. The laws of motion for confidence and disutility-of-work shocks are given by (6) and (4), respectively.
As discussed above, we cannot identify the ambiguous component of TFP in (2). To efficiently communicate results in Bayesian language, we treat TFP as probabilistic — from our perspective, it is given by an AR(1) process with autocorrelation coefficient \( \rho_z \) and innovation standard deviation \( \sigma_z \). We thus resolve the uncertainty about TFP differently than the agents in our model whose decision rules reflect their perception that TFP shocks are ambiguous.

A. Estimation

To solve the model, we follow the steps described for the simple model in Section II; the equations describing the equilibrium conditions are presented in Appendix A and details on the solution method are in Appendix B. Linearity of the state space representation of the model and normality of the shocks allow us to estimate the model using standard Bayesian methods as discussed in e.g. An and Schorfheide (2007), Smets and Wouters (2007) and Fernández-Villaverde, Guerrón-Quintana and Rubio-Ramírez (2010).

Data

The sample period is 1985Q1-2011Q1. All estimation exercises use five macro variables: the log gross inflation rate, log gross short term nominal interest rate as well as the growth rates of investment, consumption and hours per capita. These variables are plotted as solid black lines in the first five panels of Figure 1; data sources are described in Appendix C. For estimations that allow for ambiguity, that is, in the baseline and combination cases, we also use survey dispersion, denoted \( D_t \). Keeping the same notation as in the description of the model above, and writing \( \Delta \) to indicate first differences, the full vector of observables is

\[
[D_t, \log \pi_t, \log R_t, \Delta \log I_t, \Delta \log C_t, \Delta \log H_t]
\]

The model counterparts of the macro variables follow directly from the description of the model.

The variable \( D_t \) is the interdecile range of one quarter ahead projections for Q/Q real GDP growth from the Survey of Professional Forecasters (SPF). It is plotted as a solid black line in the last panel of Figure 1. Expressed in annualized percentage points, it fluctuates roughly between one and four percentage points, with an average of 2.2 and a standard deviation of 0.72.

The model counterpart of \( D_t \) is the range of one quarter ahead real GDP growth forecasts implied by agents’ belief set. To compute it, we start from agents’ belief sets about one-quarter-ahead TFP shocks given by (5) and (6). Since agents know the structure of the economy, their set of forecasts for GDP growth follows from the solution of the model. Let \( \varepsilon_{gz} \) denote the elasticity of GDP growth with respect to the TFP innovation \( u_t \) in the linearized solution. The range of growth forecasts is then

\[
D_t = \varepsilon_{gz} 2|a_t|.
\]
Indeed, in the linearized solution, conditional forecasts of growth can differ only because forecasts of TFP – that vary in the range $[-a_t, a_t]$ – affect growth.

**Figure 1. Data and baseline model**

- **Investment Growth**
- **Hours Growth**
- **Consumption Growth**
- **Nominal rate**
- **Inflation**
- **Dispersion**

**Priors: standard parameters**

We fix a small number of parameters to values commonly used in the literature. We set the quarterly depreciation rate of capital ($\delta$) at 0.025 and the share of government expenditures in output to 0.2. We follow Christiano, Eichenbaum and Evans (2005) in choosing steady state markups $\lambda_w = 1.05$ for the labor market and $\lambda_f = 1.2$ for the product market.

The other structural parameters are estimated. The parameters governing fertility, technology and policy are present in most standard medium scale DSGE studies. Our choice of priors here follows previous work (e.g. Justiniano, Primiceri and Tambalotti (2011) and Christiano, Motto and Rostagno (2013)) in selecting...
dispersed priors. Details of the prior distribution are provided in Table 1. We also follow most of the literature in choosing identical and rather diffuse priors for the structural shock processes. In particular, the priors for the autocorrelation parameters $\rho_z$ and $\rho_\zeta$ are Beta distributions with mean 0.5 and standard deviation 0.1. The priors for the standard deviations $\sigma_z$ and $\sigma_\zeta$ are Inverse Gamma with mean of 0.005 and a standard deviation of 0.2.

All of our exercises use fewer structural shocks than observables. We thus allow for iid measurement error in the observation equation of the state-space representation that links observed variables and their model counterparts. The six standard deviations of the measurement errors are estimated jointly with the other parameters of the model. For an observed variable $X$ with variance $\sigma_X^2$, say, the prior for the standard deviation of measurement error on $X$ is an Inverse Gamma distribution with mean $\sqrt{0.01\sigma_X}$ and standard deviation $\sqrt{2\sigma_X}$. At the prior mean, measurement error thus explains 1% of variation in $X$, while at one standard deviation it explains 20%.

**Priors: ambiguity parameters**

Our choice of priors for ambiguity is guided by our view that agents’ ambiguity about TFP should be “small enough” relative to our own measured volatility of TFP. Indeed, if measured TFP shocks are small to begin with, there should be little room for intangible information that agents may plausibly worry about. We thus expect that the extreme forecasts from the implied belief set, that is, $a$ and $-a$, perform sufficiently well in forecasting the true DGP (2) in equilibrium. In particular, a level of ambiguity $a$ is “too high” if the extreme forecasts are too often worse than other less extreme forecasts. Our prior should then make such a level of ambiguity very unlikely.

The performance of forecasts in equilibrium depends on the true dynamics of the ambiguous component of TFP $\mu^*_t$. We assumed above that the sequence $\mu^*_t$ "looks like" an iid normal process that is independent of $u$. Formally, let $I$ denote the indicator function and let $\Phi(.,m,s^2)$ denote the cdf of a univariate normal distribution with mean $m$ and variance $s^2$. An iid normal looking sequence is such that, for any integers $k, \tau_1, \ldots, \tau_k$ and real numbers $\bar{\mu}_1, \ldots, \bar{\mu}_k$,

$$\lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} I \left( \left\{ \mu^*_t + \tau_j \leq \bar{\mu}_j; \ j = 1, \ldots, k \right\} \right) = \prod_{j=1}^{k} \Phi \left( \bar{\mu}_j; 0, \sigma^2_{\mu} \right).$$

We also assume that, for almost every realization of the shocks $u$, the empirical second moment $\frac{1}{T} \sum_{t=1}^{T} \mu^*_t u_t$ converges to zero. Together these assumptions imply that for almost every realization of the shocks $u$, the empirical mean of the

---

8Put differently, our choice of prior here imposes a model consistency criterion that relates beliefs and the true DGP. It is akin to the rational expectations assumption in models with expected utility which equates agents’ subjective equilibrium forecast to the conditional mean measured by the modeler. Here we derive a bound on the process $\alpha_t$ (and thereby on belief sets) that depends on the variance of TFP shocks measured by the modeler.
innovation to TFP $\frac{1}{T}\sum t \hat{z}_t$ converges to zero, the empirical variance $\frac{1}{T}\sum (\hat{z}_t)^2$ converges to $\sigma_z^2 > \sigma_u^2$, and the empirical autocovariances of $\hat{z}_t$ at all leads and lags converge to zero.

Consider now the performance of extreme forecasts. If $\mu^*_t < -a$, then the lowest forecast $-a$ is the best forecast in the set $[-a,a]$, in the sense that it is closest to the true conditional mean $\mu_t^*$. To bound ambiguity, we thus compute the frequency with which $-a$ is the best forecast in the set $[-a,a]$. Since the ambiguous component $\mu^*_t$ looks like the realization of an iid process with mean zero and variance $\sigma_z^2 - \sigma_u^2$, that frequency is

$$\Phi(-a;0,\sigma_z^2 - \sigma_u^2),$$

By symmetry, there is an analogous formula for the highest forecast $a$. It follows that if one of the extreme forecasts is the best forecast at least 5% of the time, then we have $a \leq 2\sqrt{\sigma_z^2 - \sigma_u^2}$. Even if the share of variation in $\hat{z}_t$ perceived to be ambiguous is large, we must still have $a \leq 2\sigma_z$.

We are now ready to formulate a specific prior for the ambiguity parameters. We know that the maximal forecast dispersion in our sample is about twice average forecast dispersion. We thus expect average ambiguity $\bar{a}$ to be smaller than $\sigma_z$. Larger values of $\bar{a}$ would imply that ambiguity would have to become "too high" to be reasonably explained by intangible information about TFP shocks. We thus reparametrize the ambiguity process by $(\rho_a, n, \sigma_n)$, so $\bar{a} = n\sigma_z$ and $\sigma_a = \sigma_n\sigma_z$. The prior for $n \in (0,1)$ is a Beta distribution with mean 0.5 and standard deviation 0.2. The prior for the standard deviation $\sigma_n$ is an Inverse Gamma with mean of 0.005 and a standard deviation of 0.2. Finally, the prior for $\rho_a$ is a Beta distribution with mean 0.5 and standard deviation 0.1.

**B. Results**

We begin with a detailed discussion of the estimation results for the baseline exercise.

**Parameter estimates**

Table 1 compares the posterior distribution for the baseline exercise to the prior. The posterior estimates of the structural parameters that are unrelated to ambiguity are in line with previous estimations of medium scale DSGE models (Del Negro et al. (2007), Smets and Wouters (2007), Justiniano, Primiceri and Tambalotti (2011), Christiano, Motto and Rostagno (2013)). They imply that there are significant ‘frictions’ in the economy: there is substantial price and wage stickiness as well as a significant cost to adjusting investment.\(^9\) The estimated policy rule is inertial and responds little to output but strongly to inflation. In

\(^9\)In particular, the estimated wage stickiness parameter is consistent with evidence reported in Barattieri, Basu and Gottschalk (2013) who find a quarterly probability of a wage change in US micro data between 5-18 percent.
cases other than the baseline exercise, the estimated parameter values are very similar and we do not report them separately.

### Table 1—Priors and Posteriors for structural parameters

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Type</th>
<th>Mean</th>
<th>St.dev</th>
<th>Mode</th>
<th>[ .5 , .95]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>α</strong> Capital share</td>
<td>B</td>
<td>0.3</td>
<td>0.05</td>
<td>0.21</td>
<td>0.189 0.249</td>
</tr>
<tr>
<td>100(β−1 − 1) Discount factor</td>
<td>G</td>
<td>0.25</td>
<td>0.05</td>
<td>0.175</td>
<td>0.135 0.248</td>
</tr>
<tr>
<td>100(γ − 1) Growth rate</td>
<td>N</td>
<td>0.4</td>
<td>0.1</td>
<td>0.47</td>
<td>0.4 0.537</td>
</tr>
<tr>
<td>100(σ − 1) Net inflation</td>
<td>N</td>
<td>0.6</td>
<td>0.2</td>
<td>0.518</td>
<td>0.45 0.622</td>
</tr>
<tr>
<td>ξp Calvo prices</td>
<td>B</td>
<td>0.375</td>
<td>0.1</td>
<td>0.725</td>
<td>0.665 0.764</td>
</tr>
<tr>
<td>ξw Calvo wages</td>
<td>B</td>
<td>0.375</td>
<td>0.1</td>
<td>0.889</td>
<td>0.877 0.91</td>
</tr>
<tr>
<td>κ Investment adj. cost</td>
<td>G</td>
<td>4</td>
<td>2</td>
<td>2.85</td>
<td>2.27 4.95</td>
</tr>
<tr>
<td>σL Disutility of labor</td>
<td>G</td>
<td>1</td>
<td>0.3</td>
<td>1.11</td>
<td>0.822 1.9</td>
</tr>
<tr>
<td>aσ Inflation response</td>
<td>N</td>
<td>1.75</td>
<td>0.3</td>
<td>2.762</td>
<td>2.577 3.275</td>
</tr>
<tr>
<td>aσ Output response</td>
<td>N</td>
<td>0.2</td>
<td>0.05</td>
<td>0.034</td>
<td>0.029 0.039</td>
</tr>
<tr>
<td>ρσ Interest smoothing</td>
<td>B</td>
<td>0.5</td>
<td>0.1</td>
<td>0.53</td>
<td>0.474 0.638</td>
</tr>
<tr>
<td>n Level ambiguity</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.955</td>
<td>0.86 0.983</td>
</tr>
<tr>
<td>ρσ TFP persistence</td>
<td>B</td>
<td>0.5</td>
<td>0.1</td>
<td>0.852</td>
<td>0.821 0.873</td>
</tr>
<tr>
<td>ρσ Ambiguity persistence</td>
<td>B</td>
<td>0.5</td>
<td>0.1</td>
<td>0.887</td>
<td>0.873 0.91</td>
</tr>
<tr>
<td>σz TFP</td>
<td>IG</td>
<td>0.005</td>
<td>0.2</td>
<td>0.038</td>
<td>0.003 0.0047</td>
</tr>
<tr>
<td>σa Ambiguity</td>
<td>IG</td>
<td>0.005</td>
<td>0.2</td>
<td>0.134</td>
<td>0.1 0.168</td>
</tr>
<tr>
<td>100σR,ε Meas. error int. rate</td>
<td>IG</td>
<td>0.063</td>
<td>0.28</td>
<td>0.02</td>
<td>0.02 0.04</td>
</tr>
<tr>
<td>100σr,ε Meas. error inflation</td>
<td>IG</td>
<td>0.025</td>
<td>0.11</td>
<td>0.24</td>
<td>0.21 0.27</td>
</tr>
<tr>
<td>100σf,ε Meas. error invest.</td>
<td>IG</td>
<td>0.19</td>
<td>0.88</td>
<td>1.54</td>
<td>1.37 1.78</td>
</tr>
<tr>
<td>100σC,ε Meas. error cons.</td>
<td>IG</td>
<td>0.053</td>
<td>0.24</td>
<td>0.34</td>
<td>0.29 0.47</td>
</tr>
<tr>
<td>100σH,ε Meas. error hours</td>
<td>IG</td>
<td>0.076</td>
<td>0.34</td>
<td>0.47</td>
<td>0.41 0.56</td>
</tr>
<tr>
<td>100σD,ε Meas. error dispersion</td>
<td>IG</td>
<td>0.018</td>
<td>0.08</td>
<td>0.15</td>
<td>0.13 0.17</td>
</tr>
</tbody>
</table>

**Note:** B refers to the Beta distribution, N to the Normal distribution, G to the Gamma distribution, and IG to the Inverse-gamma distribution. Posterior percentiles obtained from 4 chains of 600,000 draws generated using a Random walk Metropolis algorithm with an acceptance rate of 29%. We discard the initial 100,000 draws for each chain and retain one out of every 4 subsequent draws. The online appendix reports the Brooks-Gelman-Rubin Potential Scale Reduction Factor (PSRF) for each parameter. The PSRF has values below 1.001, with values below 1.1 regarded as indicative of convergence.

At the posterior mode, the average level of ambiguity is \( \bar{a} = nσ_a = .0036 \), more than three times the unconditional standard deviation of ambiguity \( σ_a / \sqrt{1 − ρ_a^2} = .0011 \). It follows that the probabilities that the process \( α_t \) either falls below zero or goes beyond the upper bound \( 2σ_z \) are both negligible. Ambiguity is persistent at \( ρ_a = .88 \). Intangible information that raises confidence thus appears to be clustered over time.

To put these numbers in perspective, we can also express ambiguity in terms of the model implied range of GDP growth forecasts. In annualized percentage points, the steady state range of forecasts is 1.63. A one standard deviation shock to ambiguity increases this range by 0.23 percentage points.

**Model fit**

How much of a business cycle do the two shocks of the baseline model – TFP and confidence – generate? Figure 1 compares our observables to the posterior...
estimates of their model counterparts, computed by a Kalman smoother. In each panel the dark line is the data and the lighter line comes from the model. The difference is the estimated measurement error. The model does a good job fitting the business cycle comovement of investment, hours and consumption growth, as well as the stability of inflation. It closely tracks movements in the nominal interest rate and also matches business cycle frequency movements in inflation and dispersion.

The top panel of Table 2 provides summary statistics that bear out the visual impression from the figure. The first two lines report, for each variable, the sample standard deviations for the data and model implied series, respectively. The third line shows the correlation of the two series. The lowest correlation coefficient of .49 is obtained for inflation – as the figure shows, the variation in inflation is for the most part small over our sample. The largest difference in volatilities is obtained for investment. However, the model still accounts for \( \rho^2 \approx .5 \) of the variation in investment, and more for the other aggregates.

The fourth line in the table shows, for each variable, the p-value for a Ljung-Box Q test of the null that the measurement error on the variable is serially uncorrelated. For hours and investment, the real variables for which there is relatively more measurement error, we cannot reject the null of zero correlation even at the 20% level. These results suggests that confidence shocks provide a source of persistent comovement in real variables.

### Table 2—Fit of observables

<table>
<thead>
<tr>
<th>Model</th>
<th>Statistic</th>
<th>Hours</th>
<th>Consumption</th>
<th>Investment</th>
<th>Inflation</th>
<th>Interest rate</th>
<th>Dispersion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \sigma ) (data)</td>
<td>0.76</td>
<td>0.53</td>
<td>1.96</td>
<td>0.25</td>
<td>0.63</td>
<td>0.18</td>
</tr>
<tr>
<td>Baseline</td>
<td>( \sigma ) (model)</td>
<td>0.56</td>
<td>0.45</td>
<td>1.08</td>
<td>0.23</td>
<td>0.63</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>( \rho ) (data, model)</td>
<td>0.84</td>
<td>0.91</td>
<td>0.71</td>
<td>0.49</td>
<td>0.99</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>p-val Q-test</td>
<td>0.22</td>
<td>0.01</td>
<td>0.41</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>Labor supply</td>
<td>( \sigma ) (model)</td>
<td>0.6</td>
<td>0.42</td>
<td>1.28</td>
<td>0.21</td>
<td>0.63</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( \rho ) (data, model)</td>
<td>0.9</td>
<td>0.84</td>
<td>0.71</td>
<td>0.47</td>
<td>0.99</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>p-val Q-test</td>
<td>0.96</td>
<td>0.01</td>
<td>0.86</td>
<td>0.01</td>
<td>0.01</td>
<td>-</td>
</tr>
</tbody>
</table>

**Note:** The first row reports the standard deviation of the observed variables in the data expressed in percentage points. The statistic \( \sigma \) (model) refers to the standard deviation of the model-implied (obtained from the Kalman smoother) observable. We report values for the baseline and labor supply versions. \( \rho \) (data, model) reports the correlation coefficient between the model and data. The reported p-val is for the Ljung-Box Q-test. The null hypothesis of the test is that the measurement error for each observable exhibits no autocorrelation for 20 lags, against the alternative that some autocorrelation coefficient is nonzero.

**Variance decomposition**

To understand how the baseline model generates a business cycle, the top panel of Table 3 reports the contributions of confidence and TFP shocks. For each shock, the first line reports shares of business cycle variation explained, that is, variance contributed at frequencies between 6 and 32 quarters, computed by a bandpass
filter as in Stock and Watson (1999). The second line reports the share of overall variance explained by the shock.

Confidence shocks explain the majority of variation in quantities at any frequency. They also account for the majority of overall variation in inflation and the nominal interest rate. TFP shocks matter less for quantities, but they account for a large share of business cycle variation in inflation and interest rates.

<table>
<thead>
<tr>
<th>Table 3—Theoretical variance decomposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
</tr>
<tr>
<td>Baseline</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>TFP</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Labor supply</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>TFP shock</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Note: For each model and variable, we report the percent of variance attributed to each shock within than model. For each shock, the first row is the business cycle frequency (6-32 quarters) and the second row is the long-run decomposition.

**Impulse responses**

Consider the reaction of the economy to a confidence shock. The dark lines in Figure 2 are the impulse responses, in percentage deviations from the steady state, to a one standard deviation increase in ambiguity. We refer to the steady state of our linearized model as a ‘zero risk steady state’, in which the variances of the shocks only have an effect on the endogenous variables through ambiguity. Details on this computation are presented in Appendix B.

The shock increases ambiguity by about 14% from its steady state. Forecast dispersion – plotted in row one, column three of the figure – is proportional to the ambiguity shock and thus shows the same percentage increase. A loss of confidence generates a recession during which hours worked, consumption and investment fall.

The light lines in Figure 2 are impulse responses to the shock under the agent’s worst case belief. In other words, they represent the worst case expected path that agents worry about if there is a one standard deviation loss of confidence. The worst case response of technology is negative and U-shaped. Since agents know that worrisome intangible information clusters in time, they expect the worst case path to deteriorate for a number of quarters after the first piece of worrisome news.

To understand the U-shapes of the actual impulse responses for aggregates, recall the two effects of ambiguity on hours in the simple model solutions (15) and (16). On the one hand, a loss of confidence has an impact effect if rigid relative prices remain too high to accommodate more cautious spending behavior
in the impact period. On the other hand, more cautious price setting by firms in the impact period implies that rigid relative prices in later periods will be too high to accommodate even normal spending behavior. If the first effect is small, then we obtain U-shaped reactions. An extreme example is the model (15)) where the policy rule neutralizes the first effect and the recession occurs with a lag of one period.

An interesting feature of confidence shocks is that they generate large movements in quantities without strong reactions in inflation. Intuitively, a loss of confidence has two counteracting effects. On the one hand, price setting is forward looking, so firms who fear higher marginal cost set higher prices. In the simple model (16), where all prices are set one period in advance, this is the only effect and a loss of confidence always generates inflation. In the current model, where a random subset of firms changes prices each period, there is an additional effect: since cautious behavior by households lowers demand already on impact, marginal cost falls and pushes those firms who can adjust to lower prices. The impulse response shows that inflation is actually below steady state a few periods before inflation sets in.

Model comparisons: labor supply case

In existing studies of New Keynesian models, the labor wedge and the behavior
of hours is prominently affected by a shock to the disutility of work, specified as in (4). We now compare confidence and labor supply shocks. Consider first the labor supply model: it allows only for TFP shocks and disutility-of-work shocks. At the posterior mode, we find that the evolution of the labor supply shock is governed by $\rho_\zeta = 0.81$ and $\sigma_\zeta = 0.167$. The bottom panel of Table 2 shows that the fit of macro observables is remarkably similar to the baseline model.

The bottom panel of Table 3 shows that disutility-of-work shocks account for large shares of quantity fluctuations in the labor supply model. Those shares are similar in magnitude to the shares explained by confidence shocks in the baseline model. The top panel of Figure 3 provides a direct comparison by plotting the Kalman smoothed shock series from the respective models. The bottom panel shows the contribution of the shocks to hours, that is, the counterfactual evolution of hours if there had been only disutility-of-work shocks (in the labor supply model) versus only confidence shocks in the baseline model.

**Figure 3. Ambiguity versus labor supply shocks**

Disutility-of-work shocks are similar to confidence shocks in that they reduce current consumption demand. Indeed, if households lose confidence, they try to consume less and instead save more for a worrisome future. If households find work more onerous, they try to consume less and instead take more leisure. Importantly, disutility-of-work shocks alter the intratemporal choice between con-
sumption and leisure and thus generate comovement of consumption and hours, as well as a “labor wedge” even when prices and wages are flexible. In contrast, rigidities in nominal prices and the real interest rate (from the policy rule) are important for confidence shocks to generate such comovement. If those rigidities were not present, prices could adjust to offset the negative effect of confidence on demand.

In our model, disutility-of-work shocks generate U-shaped impulse responses of major aggregates as do confidence shocks. This is due to the role of rigidities in propagation. In particular, the increase in labor cost from a disutility-of-work shock increases firms’ marginal cost and leads adjusting firms to raise prices. In later periods, prices are then too high to accommodate “normal” demand and drawing out the slump, the same effect that occurs along the response to a confidence shock.

Model comparisons: combination case

We have seen that disutility-of-work shocks and confidence shocks can both generate persistent comovement of macro aggregates. A natural question is thus whether we can improve on the baseline model by including disutility-of-work shocks. We thus estimate a combination model that asks three shocks to account for six variables, now again including forecast dispersion.

The estimation strongly prefers confidence shocks as the driver of business cycle fluctuations. The posterior estimates for the labor supply shock are that \( \rho_\zeta = 0.5 \) and \( \sigma_\zeta = 0.0005 \) so that the share of variance explained by labor supply shocks drops close to zero. The posterior estimates of our other structural parameters, including those related to ambiguity, are very similar to our baseline model. Moreover, we find that Bayesian model selection criteria favor our baseline model over the combination model that also allows for labor supply shocks.\(^{10}\)

In principle, there could be two reasons why the combination model does worse than the baseline model. One is that the dynamics of the macro variables alone – excluding forecast dispersion – cannot be fit as well with suitably chosen disutility-of-work shocks as with confidence shocks. Since the impulse responses for the labor supply model above are quite similar to those of our baseline model, this reason cannot be particularly important.

The success of the baseline model over the combination model thus comes from the fact that both models must explain not only the macro aggregates, but also forecast dispersion. Confidence shocks must play some role so forecast dispersion is accounted for. Once they are given that role, they explain the fluctuations quite well, or at least well as what one could get with labor supply shocks. Further introduction of labor supply shocks, and the corresponding additional parameters, does not sufficiently increase fit to be beneficial.

\(^{10}\) The posterior odds over the two models, when we attach equal prior odds, is the Bayes factor \( p(Y|M_1)/p(Y|M_2) \), where \( Y \) is the data and \( M_1 \) and \( M_2 \) are the baseline and combination model, respectively. We compute the log of marginal data densities, \( \log p(Y|M_i) \), by the Geweke’s modified harmonic mean estimator. We find that the baseline model is favored by a Bayes factor of \( e^2 \).
Countercyclical excess returns on capital

A recession generated by an ambiguity shock is predictably followed by high excess returns on capital relative to the interest rate. Indeed, Figure 2 shows that an ambiguity shock leads to a countercyclical excess return. Intuitively, a loss of confidence about productivity makes capital less attractive to hold; as the price of capital falls, the expected excess return increases and thus compensates investors for holding a more ambiguous asset.

The resulting excess volatility of the price of capital is due to the fact that confidence shocks change agents’ perceptions of uncertainty. It cannot occur with labor supply shocks: asset valuation in a linear rational expectations model shuts down predictability of excess returns by construction. The common finding in empirical finance, that asset prices are excessively low in recessions (or equivalently, that excess returns are predictably higher after recessions), is thus consistent with the presence of confidence shocks. In contrast, other New Keynesian models cannot speak to this evidence.

Technically, we obtain predictable excess returns here even though we have solved the model by first order approximation. This is because the payoff from renting out capital is driven by the true data generating process, while the price of capital reflects agents’ worst case beliefs. Mechanically, linearization implies that the expected nominal return on capital $\mathbb{E}^*_t R^K_{t+1}$ must equal the nominal interest rate $R_t$, so the worst case expected excess return $\mathbb{E}^*_t R^K_{t+1} - R_t$ is constant at zero. However, the realized $R^K_{t+1}$ is on average higher than $\mathbb{E}^*_t R^K_{t+1}$, more so the more confidence decreases $\mathbb{E}^*_t R^K_{t+1}$.

IV. Conclusion

This paper has proposed and estimated a tractable business cycle model with nominal rigidities and ambiguity averse agents. Shocks to confidence are modeled as changes in ambiguity and identified by movements in the dispersion of experts’ growth forecasts. They can generate persistent comovement of major aggregates together with a countercyclical labor wedge. While confidence shocks have effects similar to disutility-of work shocks, they drive out the latter shocks in a joint estimation.

Our solution approach uses standard tools of business cycle analysis – linearization around a steady state and estimation by the Kalman filter – to study shocks to confidence about TFP. It is a natural next step to extend the analysis to allow for ambiguity about other variables, for example government policy or demographics. In addition, our finding of countercyclical excess returns on capital suggests that one could further explore the implications of confidence shocks for asset price dynamics.
REFERENCES


**Appendix A: Equilibrium conditions for the estimated model**

Here we describe the equations that characterize the equilibrium of the estimated model in Section I. To solve the model, we first scale the variables in order to induce stationarity. The real variables are scaled as follows:

\[
c_t = \frac{C_t}{\gamma_t}, \quad y_t = \frac{Y_t}{\gamma_t}, \quad k_{t+1} = \frac{K_{t+1}}{\gamma_t}, \quad i_t = \frac{I_t}{\gamma_t}, \quad \lambda_{z,t} = \lambda_t P_t \gamma_t.
\]

where \( \lambda_t \) is the Lagrange multiplier on the household budget constraint in (10). Let \( \xi_t \) be the Lagrange multiplier on the capital accumulation equation in (1) and define the nominal price of capital expressed in units of consumption goods as

\[
Q_{K,t} = \frac{\xi_t}{\lambda_t}.
\]
Price variables are then scaled as:

\[ q_t = \frac{Q_{K,t}}{P_t}, \quad \tilde{w}_t = \frac{W_t}{\gamma P_t} \]

We will also make use of other scaling conventions:

\[ p_t^* = (P_t)^{-1} \left( \int_0^1 P_{j,t}^{-\lambda_f/j} \, dj \right)^{1-\lambda_f/\lambda_f}, \quad w_t^* = (W_t)^{-1} \left( \int_0^1 W_{i,t}^{1-\lambda_w/\lambda_w} \, di \right)^{1-\lambda_w/\lambda_w} \]

where the index \( i \) refers to households and the index \( j \) to monopolistically competitive firms. The aggregate homogeneous labor, \( l_t \), can be written in terms of the aggregate, \( h_t \), of household differentiated labor, \( h_{i,t} \):

\[ l_t := \frac{1}{h_t} \int_0^1 H_{j,t} \, dj = (w_t^*)^{\lambda_w/\lambda_w} h_t, \quad h_t := \frac{1}{h_{i,t}} \int_0^1 h_{i,t} \, di. \]

We now present the nonlinear equilibrium conditions characterizing the model, in scaled form, where we also allow for the labor supply shocks \( \zeta_t \). We denote conditional moments under the worst case belief \( \mu_t = -a_t \) by stars.

The real marginal cost of producing one unit of output, \( m_t \):

\[ (A1) \quad m_t = \left( \frac{1}{1-\alpha} \right)^{1-\alpha} \left( \frac{1}{\alpha} \right)^{\alpha} \left( \frac{r_k^k}{Z_t} \right)^{1-\alpha} \]

Marginal cost must also satisfy another condition: namely, that \( m_t \) must equal the cost of renting one unit of capital divided by the marginal productivity of capital (the same is true for labor):

\[ (A2) \quad m_t = \frac{r_k^k}{\alpha Z_t \left( \frac{\gamma_Z}{\gamma_t} \right)^{1-\alpha}}, \]

where we used that the labor to capital ratios will be the same for all firms.

Conditions associated with Calvo sticky prices:

\[ (A3) \quad p_t^* = \left[ (1 - \xi_p) \left( \frac{\Psi_{p,t}}{P_{p,t}} \right)^{\lambda_f/\lambda_f} + \xi_p \left( \frac{\pi}{\pi_t} p_t^{*1-\lambda_f/\lambda_f} \right)^{\lambda_f/\lambda_f} \right]^{1-\lambda_f/\lambda_f} \]
\[ \Psi_{p,t} = \lambda_f \lambda_z \gamma_m + \beta \xi_p E_t^* \left( \frac{\pi}{\pi_{t+1}} \right)^{\lambda_f / (1 - \lambda_f)} \Psi_{p,t+1} \]

\[ \Phi_{p,t} = \lambda_z \gamma_t + \beta \xi_p E_t^* \left( \frac{\pi}{\pi_{t+1}} \right)^{1 / \lambda_f} \Phi_{p,t+1} \]

\[ \Psi_{p,t} = \Phi_{p,t} \left[ (1 - \xi_p)^{-1} \left(1 - \xi_p \left( \frac{\pi_t}{\pi_{t+1}} \right)^{1 / (1 - \lambda_f)} \right) \right]^{1 - \lambda_f} \]

Household’s marginal utility of consumption (FOC wrt \(c_t\)):

\[ \lambda_{z,t} = \frac{1}{c_t} \]

Capital accumulation decision (FOC wrt \(k_{t+1}\)):

\[ \lambda_{z,t} = E_t^* \frac{\beta}{\pi_{t+1} \gamma} \lambda_{z,t+1} R_{t+1}^k \]

where the return on capital is defined as:

\[ R_t^k = \frac{r_t^k + (1 - \delta)q_t}{\pi_t} \]

and capital accumulates following:

\[ k_{t+1} = \frac{(1 - \delta)k_t}{\gamma} + \left[ 1 - S \left( \frac{\tilde{i}_{t+1} \gamma}{\tilde{i}_{t-1}} \right) \right] \tilde{i}_t \]

where

\[ S \left( \frac{\tilde{i}_{t-1} \gamma}{\tilde{i}_{t-1}} \right) = \frac{1}{2} \kappa \left( \gamma - \frac{\tilde{i}_{t-1} \gamma}{\tilde{i}_{t-1}} \right)^2. \]

Investment decision (FOC wrt \(i_t\)):

\[ \lambda_{z,t} = \lambda_{z,t} q_t \left[ 1 - S \left( \frac{\tilde{i}_t \gamma}{\tilde{i}_{t-1}} \right) - S' \left( \frac{\tilde{i}_t \gamma}{\tilde{i}_{t-1}} \right) \tilde{i}_t \right] + \beta E_t^* \frac{\lambda_{z,t+1}}{\gamma} q_{t+1} S' \left( \frac{\tilde{i}_{t+1} \gamma}{\tilde{i}_t} \right) \left( \frac{\tilde{i}_{t+1} \gamma}{\tilde{i}_t} \right)^2. \]

Bond decision (FOC wrt \(B_t\)):

\[ \lambda_{z,t} = E_t^* \frac{\beta}{\pi_{t+1} \gamma} \lambda_{z,t+1} R_t \]
Conditions associated with Calvo sticky wages:

(A13) \[ w_t^* = \left[ (1 - \xi_w) \left( \frac{\psi_L \Psi_{w,t}}{\bar{w}_t F_{w,t}} \right) \right]^{1-\lambda_w(1+\sigma_L)} + \xi_w \left( \frac{\bar{\pi}_t}{\pi_{w,t}} \right) \left[ \frac{\bar{w}_t}{w_{t-1}} \right]^{1-\lambda_w} \]

(A14) \[ \pi_{w,t} = \pi_t \gamma \frac{\bar{w}_t}{\bar{w}_{t-1}} \]

(A15) \[ F_{w,t} = l_t \frac{\lambda_{z,t}}{\lambda_w} + \beta \xi_w \gamma \left( \frac{1}{\pi_{w,t+1}} \right) \frac{\lambda_w}{1-\lambda_w} \pi_t \left( \frac{1}{\gamma} \right) F_{w,t+1} \]

(A16) \[ \Psi_{w,t} = \xi_t \left[ (1+\sigma_L) \right] + \beta \xi_w \left( \frac{1}{\pi_{w,t+1}} \right) \frac{\lambda_w}{1-\lambda_w} \gamma \Psi_{w,t+1} \]

(A17) \[ \Psi_{w,t} = \bar{w}_t F_{w,t} \frac{1}{\psi_L} \left[ (1 - \xi_w) \left( \frac{\bar{\pi}_t}{\pi_{w,t}} \right) \right]^{1-\lambda_w} \left( 1 - \xi_w \left( \frac{\bar{\pi}_t}{\pi_{w,t}} \right) \right)^{\lambda_w} \]

Taylor rule

(A18) \[ \frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho^R} \left[ \frac{\pi_t}{\pi} \right]^{a_R} \left( \frac{y_t}{y} \right)^{a_R} \]^{1-\rho^R}

Production function:

(A19) \[ \left( p_t^L \right)^{\frac{\lambda_f}{\gamma - 1}} \left\{ Z_t \left( \frac{k_t}{\gamma} \right)^{\alpha} \right\}^{l_t^{1-\alpha} - \Phi} = y_t \]

Resource constraint:

(A20) \[ y_t = c_t + i_t + g \]

The 20 endogenous variables to be determined are:

\[ c_t, i_t, y_t, l_t, k_{t+1}, \lambda_{z,t}, q_t, r^k_t, R^b_t, m_t, R_t, p_t^s, \pi_t, F_{p,t}, \Psi_{p,t}, w_t^s, F_{w,t}, \Psi_{w,t}, \tilde{w}_t, \pi_{w,t}. \]

We have listed 20 equations above, from (A1) to (A20).
Appendix B: Solution method

Here we describe the solution method for the estimated model presented in Section I. The logic follows the general formulation in section II, in which the solution involves the following procedure. First, we solve the model as a rational expectations model in which the worst case scenario expectations are correct on average. Under the worst case belief

\begin{equation}
zt_{t+1} = \rho zt \cdot t + ut_{t+1} - at.
\end{equation}

The equations describing the equilibrium conditions under these expectations were presented in the Appendix A. Second, we take the equilibrium decision rules formed under ambiguity and then characterize the dynamics under the econometrician’s law of motion for productivity described by the probabilistic evolution

\begin{equation}
zt_{t+1} = \rho zt \cdot t + ut_{t+1}
\end{equation}

Let \( w_t \) denote the endogenous variables and \( s_t \) denote the persistent exogenous variables of the model. For notational purposes, split the vector \( s_t \) into the TFP shock \( z_t := \log Z_t \), the ambiguity variable \( a_t \) and the rest of the exogenous variables, \( \tilde{s}_t \), expressed in logs, of size \( n \). We can summarize our procedure for finding the equilibrium dynamics in the following steps:

1. Find the deterministic ‘worst case steady state’. Here we take the steady state values of exogenous variables \( \tilde{s}_0 \). This vector includes setting \( a_t = \bar{a}, \tilde{s}_t = \tilde{s}_0 \) and finding the steady state TFP level of the process in (B1). The latter is

\[ z^0 = -\frac{\bar{a}}{1 - \rho_z} \]

Using \( \tilde{s}_0 := (\tilde{s}_0^{0, n \times 1}, \bar{a}) \) one can compute the ‘worst case steady state’ of the endogenous variables. This can be done by analytically solving the equilibrium conditions, presented in section A, evaluated at the deterministic steady state \( \tilde{s}_0 \). Denote the steady state values of these endogenous variables as a vector \( \tilde{w}_0^{0, n \times 1} \).

2. Linearize the model around the ‘worst case steady state’. Denote the deviation from the worst case steady state by \( \tilde{w}_t^0 := wt_0 - \tilde{w}_0^0 \) and \( \tilde{s}_t^0 := st_0 - s^0 \). Then we can solve the model using standard techniques for forward-looking general equilibrium rational expectations models. In particular we describe here an approach based on Sims (2002).\(^{11}\) Using the linearized equilibrium conditions, we can write the canonical form

\begin{equation}
\Gamma_0 \tilde{y}_t^0 = \Gamma_1 \tilde{y}_{t-1}^0 + \Psi \omega_t + \Upsilon \eta_t
\end{equation}

\(^{11}\)Since at this stage we solve for the decision rules as a model of expected utility under the worst-case belief, any of the usual methods for solving linear rational expectations could be employed (such as for example those presented in Uhlig (1999), Klein (2000), Christiano (2002) and Sims (2002)). Readily available packages such as Dynare also produce such a linear solution.
where $\tilde{y}_t^0$ is a column vector of size $k$, and contains all the endogenous variables, thus including $\hat{w}_t^0$, time $t$ conditional expectations of $\hat{y}_{t+1}^0$ and $\bar{s}_t^0$. Here $\tilde{y}_t^0 = y_t - \bar{y}^0$ denotes deviations from the worst-case steady state and $\eta_t$ are expectational errors, defined as $\eta_t = \tilde{y}_t^0 - E_{t-1}^* \tilde{y}_t^0$, such that $E_{t-1}^* \eta_t = 0$. Notice that to specify the matrices $\Gamma_0$ and $\Gamma_1$ we use the linear evolution of the exogenous variables under the worst-case belief:

$$\hat{s}_t^0 := \begin{bmatrix} \hat{z}_t \\ \hat{a}_t \\ \hat{\tilde{s}}_t \\ \hat{\tilde{z}}_{t-1} \\ \hat{\tilde{a}}_{t-1} \end{bmatrix} = P \begin{bmatrix} \hat{s}_{t-1} \\ \hat{z}_{t-1} \\ \hat{a}_{t-1} \\ \tilde{z}_{t-1} \\ \tilde{a}_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ u_t \\ \varepsilon_{a,t} \end{bmatrix}$$

Thus, $\omega_t$ contains the exogenous random disturbances $\omega_t = [\varepsilon_t \ u_t \ \varepsilon_{a,t}]'$ with $\omega_t \sim i.i.d. N(0, \Sigma)$. To reflect the time $t$ worst case belief about $\tilde{z}_{t+1}$, recall (B1) so the matrix $P$ is given by:

$$(B4) \quad P = \begin{bmatrix} \rho_{n \times n} & 0 & 0 \\ 0 & \rho_z & -1 \\ 0 & 0 & \rho_a \end{bmatrix}$$

where $\rho$ is a matrix reflecting the autocorrelation structure of the elements in $\tilde{s}_t$.

To solve for the linear stochastic first-order difference equations in (B3) we use the method developed in Sims (2002) that produces the solution:

$$(B5) \quad \hat{y}_t^0 = T \hat{y}_{t-1}^0 + R \omega_t$$

where $T$ and $R$ are $k \times k$ and $k \times (n+2)$ matrices, respectively.

3. Consider now the dynamics of the model from the perspective of the econometrician. Agents’ response to ambiguity leads to actions and hence equilibrium outcomes given by (B5). At the same time, the exogenous state $\bar{z}_t$ moves according to the equation (B2) so the steady state of $\bar{z}$ equals $\bar{z} = 0$. Thus, we have to correct for the fact that, from the perspective of the agent’s worst case beliefs at $t - 1$, the average innovation of the TFP shock at time $t$ is not equal to 0. Comparing (B2) and (B1), the average innovation is then equal to $a_{t-1}$:

$$(B6) \quad \bar{z}_t = E_{t-1}^* \bar{z}_t + u_t + a_{t-1}.$$

This means that the equilibrium law of motion under the econometrician’s data generating process (DGP) is given by

$$(B7) \quad \tilde{y}_t^0 = T \tilde{y}_{t-1}^0 + R \omega_t + R \begin{bmatrix} 0_{n \times 1} & a_{t-1} & 0 \end{bmatrix}'$$

3.a) Find the ‘zero risk steady state’. Take the law of motion in (B7) and evaluate it at steady state. The zero risk steady state is then the fixed point $\bar{y}$.
that solves

\( \bar{y} - \bar{y}^0 = T (\bar{y} - \bar{y}^0) + R \left[ \begin{array}{cc} -0.1 & 0 \\ 0 & 0 \end{array} \right] \)

3.b) Dynamics around the ‘zero risk steady state’.

Denote by \( \hat{y}_t := y_t - \bar{y} \) the deviations from the zero risk steady state. Combining (B7) and (B8), those deviations follow the law of motion \n
\( \hat{y}_t = T \hat{y}_{t-1} + R\omega_t + R \left[ \begin{array}{c} \hat{a}_{t-1} \\ 0 \end{array} \right] \)

Equation (B9) then characterizes the dynamics of our economy around the zero risk steady state. In the online appendix we present details on how to extend the solution method to allow for higher order approximations of the equilibrium dynamics. The general principle still applies: first, we obtain the higher-order dynamics under the worst-case belief, which can be done using standard perturbation techniques. Second, we feed in the econometrician’s data generating process, which requires to shock the economy with an average innovation that reflects the difference in the conditional mean of an exogenous process.

**Appendix C: Data sources**

The data used to construct the observables are:

1) Real Gross Domestic Product, BEA, NIPA table 1.1.6, line 1, billions of USD, in 2005 chained dollars.

2) Gross Domestic Product, BEA, NIPA table 1.1.5, line 1, billions of USD, seasonally adjusted at annual rates.

3) Personal consumption expenditures on nondurable goods, BEA, NIPA table 1.1.5, line 5, billions of USD, seasonally adjusted at annual rates.

4) Personal consumption expenditures on services, BEA, NIPA table 1.1.5, line 6, billions of USD, seasonally adjusted at annual rates.

5) Gross private domestic investment, fixed investment, nonresidential and residential, BEA, NIPA table 1.1.5, line 8, billions of USD, seasonally adjusted at annual rates.

6) Personal consumption expenditures on durable goods, BEA, NIPA table 1.1.5, line 4, billions of USD, seasonally adjusted at annual rates.

7) Nonfarm business hours worked, BLS PRS85006033, seasonally adjusted at annual rates, index 1992=100.

8) Civilian noninstitutional population over 16, BLS LNU00000000Q.
9) Effective Federal Funds Rate. Source: Board of Governors of the Federal Reserve System.

10) Dispersion, Interdecile range of Survey of Professional Forecasters on one quarter ahead projections for Q/Q real GDP growth.

We then perform the following transformations of the above data to get the observables:

11) GDP deflator: $\frac{(2)}{(1)}$

12) Real per capita consumption: $\frac{[(3)+(4)]}{[(8)*(11)]}$

13) Real per capita investment: $\frac{[(5)+(6)]}{[(8)*(11)]}$

14) Per capital hours: $\frac{(7)}{(8)}$