Risk and Ambiguity in Models of Business Cycles

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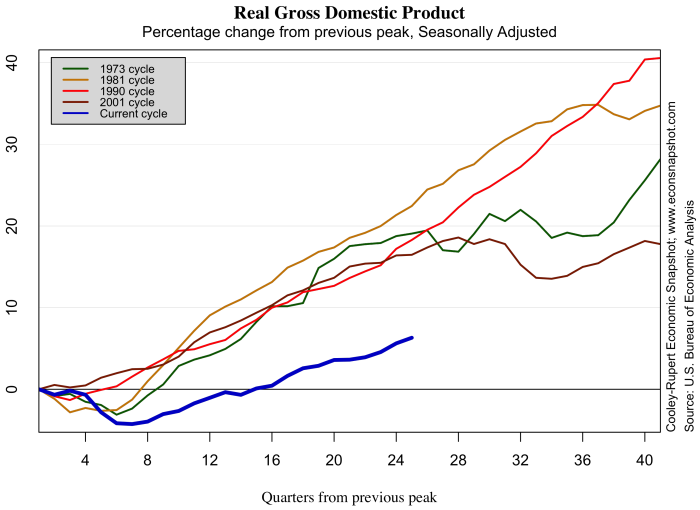
NYU STERN

The "Great Recession" and its aftermath



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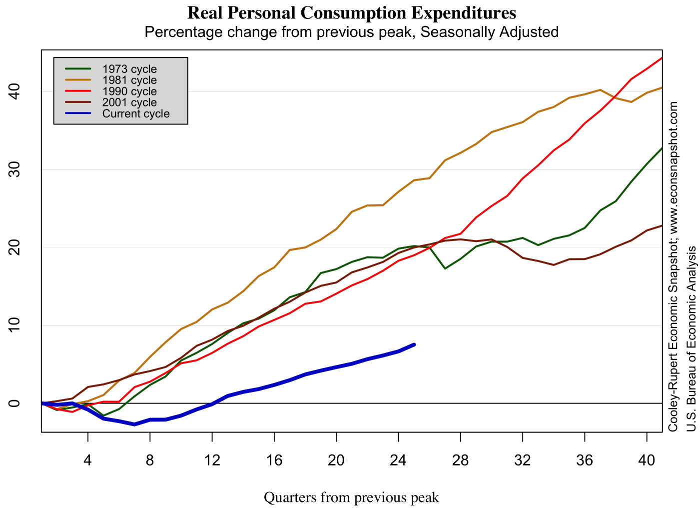

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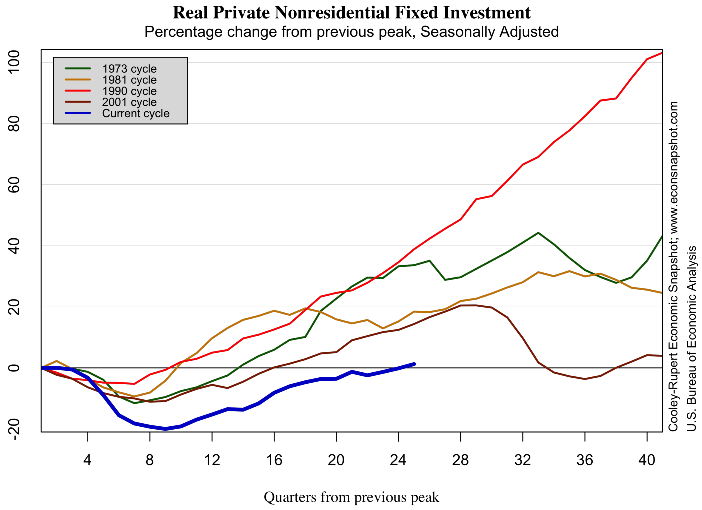

What happened?

- What we see
  - Magnitude: deeper recession than usual
  - Persistence: longer recovery — maybe slower, too
- Like Kydland-Prescott with productivity shocks?
  - Relative magnitudes look right
  - Comovements look right, too
  - But... measured productivity didn't fall very much

More

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- What's missing?

What we do

- Take a streamlined business cycle model
- Ask: How does **uncertainty** affect the **dynamics** of output, consumption, and investment?
  - Magnitude: Does uncertainty magnify fluctuations?
  - Persistence: Can it reduce the speed of recovery?
- Compute solutions with
  - Transparent loglinear approximation
  - Acurate numerical method

Modeling ingredients

- Streamlined **business cycle model**
  - Recursive preferences
  - Unit root in productivity
  - Fixed labor supply
- With fluctuations in **uncertainty**
  - *Risk* (stochastic volatility)
  - *Ambiguity* (unobservable long-term growth)

What we find

Fluctuations in uncertainty have **little impact**

- Persistence
  - Separation property: internal **dynamics independent of risk and risk aversion**
  - Persistence must be in the shock
- Magnitude
  - Impact typically small, but magnified by **risk aversion**

Business cycle properties governed by IES

Risk

- Recursive references

$$
\begin{aligned}
U_t &= V[c_t, \mu_t(U_{t+1})] \\
&= [(1-\beta)c_t^\rho + \beta\mu_t(U_{t+1})^\rho]^{1/\rho} \\
\mu_t(U_{t+1}) &= [E_t(U_{t+1}^\alpha)]^{1/\alpha}
\end{aligned}
$$

V, μ_t homogeneous of degree one, $RA = 1 - \alpha$, $IES \equiv \sigma = 1/(1-\rho)$

- Productivity a_t

$$
\begin{aligned}
\log g_t &= log(a_t/a_{t-1}) = \log g + e^\top x_t \\
x_{t+1} &= Ax_t + v_t^{1/2} Bw_{1t+1} \;\; (\text{"news"}) \\
v_{t+1} &= (1-\varphi_v)v + \varphi_v v_t + \tau w_{2t+1} \;\; (\text{"risk"}) \\
(w_{1t}, w_{2t}) &= \text{iid standard normals}
\end{aligned}
$$

Scaling

- Bellman equation

$$\begin{aligned} J(k_t, x_t, v_t, a_t) &= \max_{c_t} V\left\{c_t, \mu_t[J(k_{t+1}, x_{t+1}, v_{t+1}, a_{t+1})]\right\} \\ \text{s.t.} \qquad k_{t+1} &= f(k_t, a_t n) - c_t \end{aligned}$$

f $hd1$: eg, $f(k, an) = k^{\omega}(an)^{1-\omega} + (1-\delta)k$

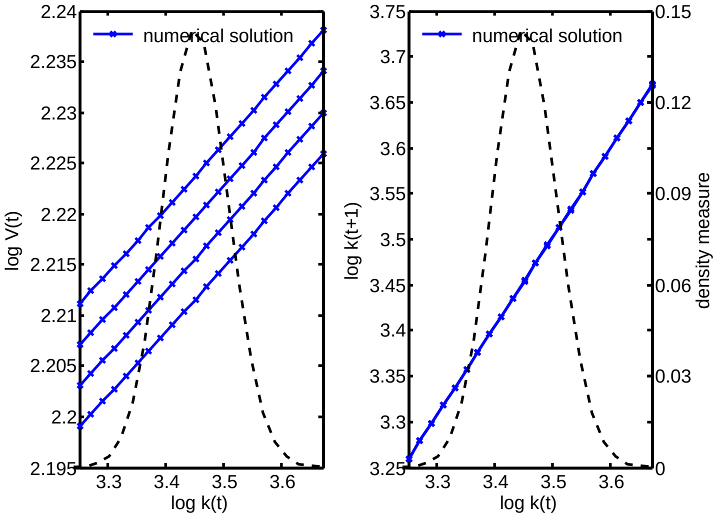
- **Rescaled** Bellman equation $[\tilde{k}_t = k_t/a_t,\ \tilde{c}_t = c_t/a_t]$

$$\begin{aligned} J(\tilde{k}_t, x_t, v_t) &= \max_{\tilde{c}_t} V\left\{\tilde{c}_t, \mu_t[g_{t+1} J(\tilde{k}_{t+1}, x_{t+1}, v_{t+1})]\right\} \\ \text{s.t.} \qquad g_{t+1}\tilde{k}_{t+1} &= f(\tilde{k}_t, n) - \tilde{c}_t \end{aligned}$$

Parameter values

Parameter	Value	Comment
Preferences		
ρ	-1	intertemporal substitution $= \sigma = 1/(1-\rho) = 1/2$
α	-9	risk aversion $= 1 - \alpha = 10$
β	—	chosen to hit $k/y = 10$ (quarterly)
Technology		
ω	$1/3$	Kydland and Prescott (1982, Table I), rounded off
δ	0.025	Kydland and Prescott (1982, Table I)
Productivity growth		
$\log g$	0.004	Tallarini (2000, Table 4)
e	1	normalization
A	0	no predictable component ("news")
B	1	normalization
$v^{1/2}$	0.015	Tallarini (2000, Table 4), rounded off
φ_v	0.95	arbitrary
τ	0.74×10^{-5}	makes v three standard deviations from zero

Model is essentially loglinear



Loglinearization I

- Goal: loglinear decision rule for capital

$$\log \tilde{k}_{t+1} = h_k \log \tilde{k}_t + h_x^\top x_t + h_v v_t - \log g_{t+1}$$

- Dynamic programming version of Campbell (JME, 1994)
- Loglinearization around the **stochastic** steady-state

Loglinearization II

- Loglinearize **capital's marginal product** and **law of motion**

$$\begin{aligned} \log f_{kt} &= \lambda_r \log \tilde{k}_t + \lambda_0 \\ \log \tilde{k}_{t+1} &= \lambda_k \log \tilde{k}_t - \lambda_c \log \tilde{c}_t + \lambda_1 - \log g_{t+1} \end{aligned}$$

where $(\lambda_k, \lambda_c, \lambda_r)$ are steady-state objects.

- Guess **loglinear value function and derivative**

$$\begin{aligned} \log J_t &= p_k \log \tilde{k}_t + p_x^\top x_t + p_v v_t + p_0 \\ \log J_t^{\rho-1} J_{kt} &= q_k \log \tilde{k}_t + q_x^\top x_t + q_v v_t + q_0 \end{aligned}$$

More

Separation property

Claim (Tallarini)

Consider the loglinear approximation of capital's law of motion,

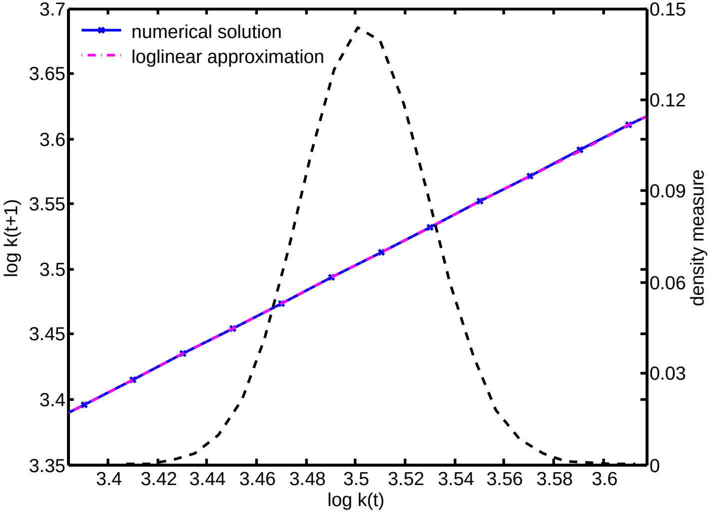
$$\log \tilde{k}_{t+1} = h_0 + h_k \log \tilde{k}_t + h_x^\top x_t + h_v v_t - \log g_{t+1}$$

If we hold constant the stochastic steady state:

1. h_k *is independent of* **properties of all shocks** *and* **risk aversion**
2. h_x *is independent of* **properties of uncertainty shocks** *and* **risk aversion**

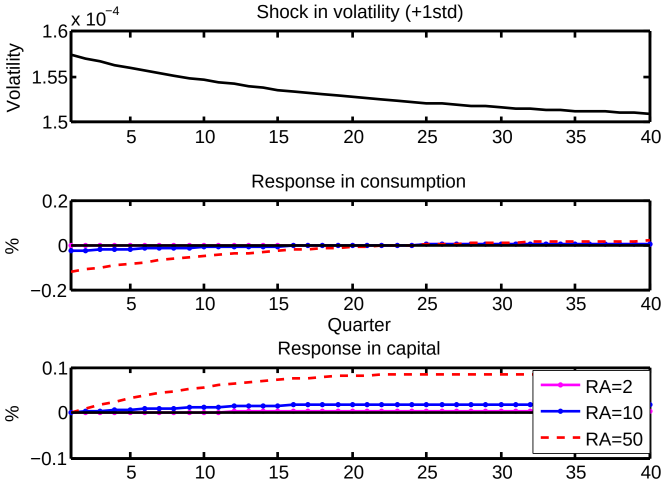
$$\begin{aligned} h_k &= \lambda_k + \sigma\lambda_c(q_k - \lambda_r), \quad h_x^\top = \sigma\lambda_c q_x^\top \\ q_k &= q_k\left[\lambda_k + \sigma\lambda_c(q_k - \lambda_r)\right] + \lambda_r \\ q_x &= -(\sigma^{-1} + q_k)e^T A\left[(1 - \sigma q_k\lambda_c)I - A\right]^{-1} \end{aligned}$$

Loglinearization III



Risk aversion magnifies uncertainty

More



Business cycles and risk aversion

	US Data	Model w/ RA =			Cst. vol.
Risk Aversion		**2**	**10**	**50**	10
Standard deviations (%)					
Output growth	*1.04*	0.82	0.82	0.82	0.82
Consumption growth	*0.55*	0.75	0.75	0.76	0.75
Investment growth	*2.79*	1.03	1.04	1.06	1.02
Correlations with output growth					
Consumption growth	*0.52*	0.99	0.99	0.97	0.99
Investment growth	*0.65*	0.98	0.97	0.93	0.98

Intertemporal elasticity of substitution: 0.5

Business cycles and IES

	US Data	Model	
IES		**0.5**	**1.5**
Standard deviations (%)			
Output growth	*1.04*	0.82	0.82
Consumption growth	*0.55*	0.75	0.39
Investment growth	*2.79*	1.04	1.92
Correlations with output growth			
Consumption growth	*0.52*	0.99	0.98
Investment growth	*0.65*	0.97	0.93

Risk aversion: 10

Risk and ambiguity

- Divide state in two: $s_t = (s_{1t}, s_{2t})$ *(ask about Stan's story)*
- **Smooth ambiguity**

$$\begin{aligned} \text{risk} &= p_{1t}(s_{1t+1}|s_{2t+1}, \mathcal{I}_t) \\ \text{ambiguity} &= p_{2t}(s_{2t+1}|\mathcal{I}_t) \end{aligned}$$

- Two-part certainty equivalent

$$\begin{aligned} \mu_{1t}(U_{t+1}) &= \left[E_{1t}(U_{t+1}^{\alpha})\right]^{1/\alpha} \quad (\text{“risk”}) \\ \mu_{2t}[\mu_{1t}(U_{t+1})] &= \left\{E_{2t}[\mu_{1t}(U_{t+1})]^{\gamma})\right\}^{1/\gamma} \quad (\text{“ambiguity”}) \end{aligned}$$

α controls risk aversion, $\gamma < \alpha$ controls ambiguity aversion

Ambiguity about what?

- Rule of thumb
  - Risk about observables
  - Ambiguity about unobservables
- Example: observe productivity growth g_t but not its mean x_t

$$\begin{aligned} \text{Risk:} \quad & \log g_{t+1} | x_{t+1} \ \sim \ \mathcal{N}(\log g + x_{t+1}, b) \\ \text{Ambiguity:} \quad & x_{t+1} \ \sim \ \text{AR}(1) \end{aligned}$$

- Filtering gives us (say)

$$x_{t+1} | \mathcal{I}_t \ \sim \ \mathcal{N}(\widehat{x}_{t+1}, h_{t+1}), \quad \mathcal{I}_t = g^t$$

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- But: **none of this has much impact**

Summary

- Uncertainty fluctuations have intuitive appeal
- But they add little to standard business cycle model
  - Magnitude: impact is small with common parameter values
  - Persistence: they add nothing to internal dynamics, just the persistence of the shocks themselves

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- Uncertainty fluctuations have intuitive appeal
- But they add little to standard business cycle model
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  - Persistence: they add nothing to internal dynamics, just the persistence of the shocks themselves
- Where next?
  - Uncertainty about parameters?
  - Endogenous uncertainty? (Veldkamp, Schaal)
  - Micro uncertainty with financial frictions? (Arellano, Bai, & Kehoe)
  - Cause or effect? (Alessandria, Choi, Kaboski, & Midrigan)

Related work (some of it)

- Recursive business cycles
  - Campanale, Castro, & Clementi; Tallarini
- Approximation methods
  - Anderson, Hansen, McGrattan, & Sargent; Campbell; Kaltenbrunner and Lochstoer; Malkhozov
- Risk and business cycles
  - Basu & Bundick; Caldara, Fernandez-Villaverde, Rubio-Ramirez, & Wen; Justiniano & Primiceri; Liu & Miao
- Ambiguity and business cycles
  - Klibanoff, Marinacci, & Mukerji; Ju & Miao; Ilut & Schneider; Jahan-Parvar & Miao

Intetemporal substitution and uncertainty

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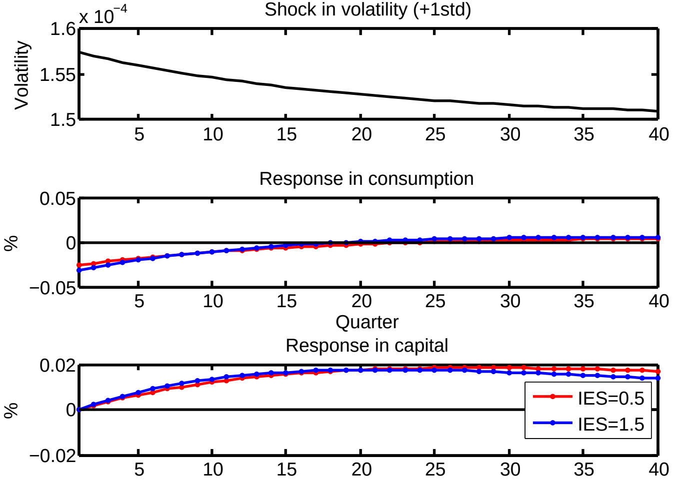

Productivity