Disappointment Aversion, Long-Run Risks and Aggregate Asset Prices*

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Abstract

We assess the aggregate asset pricing implications of generalized disappointment aversion (GDA) in the long-run risks model of Bansal and Yaron (2004). Using analytical formulas for asset valuation ratios and several moment and predictive regression statistics we compare thoroughly several recursive utility models with long-run risks. While persistence of expected consumption growth is fundamental for the moment matching ability of Kreps-Porteus preferences, GDA relies mostly on the persistence of consumption volatility. The long-run growth risk, when coupled with Kreps-Porteus preferences, has the undesirable side-effect of generating the wrong predictability pattern: dividend yields forecast consumption growth but not excess returns. With GDA preferences, the persistent volatility of consumption growth capturing economic uncertainty is enough to generate realistic moments and the observed patterns of predictability. These results are robust to an intertemporal elasticity of substitution lower or greater than one.

Keywords: Equilibrium Asset Pricing, Disappointment Aversion, Long-run Risks, Equity Premium, Risk-free Rate Puzzle, Predictability of returns

JEL Classification: G1, G12, G11, C1, C5

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1 Introduction

The Consumption-based Capital Asset Pricing Model (CCAPM) has recently been revived by models of long-run risks (LRR)[1]. Bansal and Yaron (2004) (BY) explain several asset market stylized facts by a model with a small long-run predictable component driving consumption and dividend growth and fluctuating economic uncertainty measured by consumption volatility, together with Epstein and Zin's (1989) preferences, that separate risk aversion from intertemporal substitution. These preferences play an important role in the long-run risks model. The representative agent must have an elasticity of intertemporal substitution greater than one for adverse movements in the long-run growth and volatility risks to lower asset prices. This model stands in contrast with Campbell and Cochrane (1999), which features another model that explains a number of stylized facts but relies on a state-dependent risk aversion varying with the business cycle. Consumption growth is an iid process and the pricing kernel is driven by a heteroskedastic consumption surplus that is an accumulation of past consumption shocks.

To derive model implications for asset prices, both models rely on parameter calibration for consumption and dividend processes, as well as preferences. Moreover, they solve for asset valuation ratios using loglinear approximations and reproduce stylized facts either by numerical or simulation techniques. This means that evidence in support of the models are almost invariably based on a given set of parameters that reproduce the stylized facts. The cost of producing results limits the potential for sensitivity analysis and model assessment is based on the plausibility of the chosen parameters.

Of course, the choice of parameters is a source of lively debate. Take the value of the elasticity of intertemporal substitution. Bansal and Yaron (2004) report empirical evidence in favor of a value greater than 1[2] but mention that Hall (1988) and Campbell (1999) estimate an IES below 1. They also argue that in the presence of time-varying volatility, there is a severe downward bias in the point estimates of the IES. While the argument is correct in principle, Beeler and Campbell (2009) simulate the BY model and report no bias if the riskless interest rate is used as an instrument[3].

A similar empirical debate applies to the consumption growth process. If a very persistent predictable component exists in consumption growth, as proposed by BY, it is certainly

[1]The extensive literature about the equity premium puzzle and other puzzling features of asset markets are reviewed in a collection of essays in Mehra (2008). See also Campbell (2000, 2003), Cochrane and Hansen (1992), Kocherlakota (1996), and Mehra and Prescott (2003).

[2]They cite Hansen and Singleton (1982) and Attanasio and Weber (1989), among others.

[3]They confirm the presence of a bias (negative estimate of the IES) when the equity return is used and attribute it to a weak instrument problem.

hard to detect it as consumption appears very much as a random walk in the data[4].

A third debate relates to the predictability of returns by the dividend yield. Econometric and economic arguments fuel the controversy about the empirical estimates of R^2 in predictive regressions of returns or excess returns over several horizons on the current dividend yield. Some claim that the apparent predictability is a feature of biases inherent to such regressions with persistent regressors, others that it is not spurious since if returns were not predictable, dividend growth should, by accounting necessity, be predictable, which is not the case in the data[5]. Therefore, evidence that a consumption-based asset pricing model is able to reproduce these predictability patterns based on data would certainly clarify the debate.

Given these empirical debates that condition very important economic messages regarding the relationship between economic uncertainty and asset prices, an asset pricing model that shows some robustness in both preference and fundamentals dimensions in reproducing stylized facts appears desirable. In this regard, we would like to be able to solve these asset pricing models easily so thorough sensitivity analysis and comparison between models can be conducted. This paper analyzes the LRR model of BY with the class of generalized disappointment aversion (GDA) preferences, which is a class of recursive utility preferences that admit the Kreps-Porteus (1978) specification as a particular case. Given these generalized preferences, we will be able to explore the sensitivity of the LRR model to preference parameter values, in particular the IES. To conduct this analysis, we map the LRR model into a Markov Switching model to derive analytical formulas for asset valuation ratios, moments of returns and predictive regression R^2 and coefficients. With GDA preferences, log-linear approximations are no longer possible, since the disappointment threshold is endogenous making numerical solutions much more complex.

Disappointment aversion preferences were introduced by Gul (1991) to be consistent with the Allais Paradox. They differ from expected utility by introducing an additional weight to outcomes that are below the certainty equivalent. Routledge and Zin (2004) (RZ) generalized these preferences by allowing the disappointment threshold to be placed at an arbitrary proportion of the certainty equivalent. Disappointment averse preferences are endogenously state-dependent through the certainty equivalent threshold and, therefore, are apt to produce counter-cyclical risk aversion. Investors may become more averse in

[4]Bansal (2007) cites several studies that provide empirical support for the existence of a long-run component in consumption. Bansal, Gallant and Tauchen (2007) and Bansal, Kiku and Yaron (2007) test the LRR model using the efficient and generalized method of moments, respectively. Hansen, Heaton and Li (2008) present evidence for a long-run component in consumption growth suing multivariate analysis.

[5]See in particular Valkanov (2003), Stambaugh (1999), Cochrane (2008) and the special issue of the Review of Financial Studies about the topic of predictability of returns.

recessions if the probability of disappointing outcomes is higher than in booms.

In order to do a thorough comparison of the GDA and KP models and a comprehensive sensitivity analysis, we generate population statistics for a large set of parameter values. We first obtain formulas for the price-consumption, the price-dividend and the risk-free bond for GDA preferences, and a fortiori for KP preferences. Once the equilibrium prices are determined, we can produce the first and second moments of the equity premium and of the risk-free rate, the mean of and the volatility of the price-dividend ratio, the predictability of returns and excess returns by the dividend-price ratio, the predictability of consumption volatility by the dividend-price ratio, as well as the autocorrelation of returns and excess returns at long horizons.

The first step of our investigation is to compare the performance of the Kreps-Porteus preferences specified by BY and a very simple version of GDA, where disappointment aversion is the only source of risk aversion. This simple GDA model is able not only to match returns and price-dividend moments, but also to reproduce the predictability patterns and magnitudes observed in the data. GDA when coupled with LRR generates excess returns that are predictable by the dividend yield. This is not the case for KP. Moreover, consumption and dividend growth are not predicted by the dividend yield with GDA preferences, which is consistent with the data, but the LRR model with KP preferences generates too much predictability for these growth rates in fundamentals[6]. The matching moment ability of the GDA specification is robust to reasonable modifications of the endowment process parameters, while the model with KP preferences is not. Figure 1 illustrates this in a dramatic fashion with the persistence parameter of consumption growth. The graphs of the expected equity premium, risk-free rate and price-dividend ratio show that moving away a small bit from the benchmark value of 0.975 chosen by BY has a big impact on the moments. The equity premium falls quickly as we reduce the persistence, while the risk-free becomes negative as we move towards 1. The most spectacular effect is the behavior of the mean price-dividend ratio that jumps to values greater than 100. In contrast, the LRR-GDA model is more robust, even though it shows some fluctuations between 0.8 and 1.

While persistence of expected consumption growth appears fundamental for the moment matching ability of the LRR model with KP preferences, disappointment aversion relies mostly on the persistence of consumption volatility. When preferences are disappointment averse, the main driver of the asset pricing matching ability of the model is the persistent consumption growth volatility. The GDA model performs well even when coupled with

[6]These counterfactual predictability properties of the LRR model with KP preferences have been confirmed recently by Beeler and Campbell (2009) in long simulations of 1.2 million months of the BY model.

a random walk model for consumption and dividend growth, provided that its volatility is persistent. When volatility is not persistent, there is not so much variation in the probability of disappointment. As a consequence the price-dividend ratio has too little variation, leading to too small excess return variance and predictability.

Models with exogenous reference levels, such as Campbell and Cochrane (1999) and Barberis, Huang ans Santos (2001), generate counter-cyclical risk aversion and link it to return predictability. Investors will be willing to pay a lower price in bad states of the world, implying higher future returns. In Lettau and Van Nieuwerburgh (2008), predictability empirical patterns can be explained by changes in the steady-state mean of the financial ratios. These changes can be rationalized by a LRR model with GDA preferences.

Bernartzi and Thaler (1995) are also using asymmetric preferences over good and bad outcomes to match the equity premium, but instead of using an intertemporal asset pricing framework with preferences defined over consumption streams, they start from preferences defined over one-period returns based on Kahneman and Tversky (1979)'s prospect theory of choice. By defining preferences in this way directly over returns, they avoid the challenge of reconciling the behavior of asset returns with aggregate consumption.

Following the seminal paper by Hamilton (1989), Markov switching models have been used in the consumption-based asset pricing literature to capture the dynamics of the endowment process. While Cechetti, Lam and Mark (1990) and Bonomo and Garcia (1994) estimate univariate models for either consumption or dividend growth, Cechetti, Lam and Mark (1993) estimate a homoscedastic bivariate process for consumption and dividend growth rates, and Bonomo and Garcia (1993, 1994) a heteroscedastic one. Recently, Lettau, Ludvigson and Wachter (2008), and Bhamra, Kuehn, and Strebulaev (2009) have also estimated such processes. Calvet and Fisher (2007) estimate multifractal processes with Markov switching in a large number of states setting in a consumption-based asset pricing model. Apart from capturing changes in regimes, another distinct advantage of Markov switching models is to provide a flexible statistical tool to match other stochastic processes such as autoregressive processes as in Tauchen (1986). In this paper we match the heteroscedastic autoregressive models for consumption and dividend growth rates in Bansal and Yaron (2004), based on the parameter configuration in Bansal, Kiku and Yaron (2007), with a four-state Markov switching model. Recently, Chen (2008) has approximated the dynamics of consumption growth process of the BY LRR model using a discrete-time Markov and the quadrature method of Tauchen and Hussey (1991) in a model to explain credit spreads.

This paper extends considerably the closed-form pricing formulas provided in Bonomo and Garcia (1994) and Cecchetti, Lam and Mark (1990) for the Lucas (1978) and Breeden

(1979) CCAPM model. Bonomo and Garcia (1993) have studied disappointment aversion in a bivariate Markov switching model for consumption and dividend growth rates and solved numerically the Euler equations for the asset valuation ratios. For recursive preferences, solutions to the Euler equations have been mostly found either numerically or after a log linear approximate transformation. However, Chen (2008) and Bhamra, Kuehn, and Strebulaev (2009) use a Markov chain structure for consumption growth to solve analytically for equity and corporate debt prices in an equilibrium setting with Kreps-Porteus preferences, while Calvet and Fisher (2007) focused on the equity premium[7].

Recently, some papers have also proposed to develop analytical formulas for asset pricing models. Abel (1992, 2008) calculate exact expressions for risk premia, term premia, and the premium on levered equity in a framework that includes habit formation and consumption externalities (keeping up or catching up with the Joneses). The formulas are derived under lognormality and an i.i.d. assumption for the growth rates of consumption and dividends. We also assume log-normality but after conditioning on a number of states and our state variable captures the dynamics of the growth rates. Eraker (2008) produces analytic pricing formulas for stocks and bonds in an equilibrium consumption CAPM with Epstein-Zin preferences, under the assumption that consumption and dividend growth rates follow affine processes. However, he uses the Campbell and Shiller (1988) approximation to maintain a tractable analytical form of the pricing kernel. Quite recently, Gabaix (2008) proposed a class of linearity-generating processes that ensures closed-form solutions for the prices of stocks and bonds. This solution strategy is based on reverse-engineering of the processes for the stochastic discount factors and the asset payoffs.

The rest of the paper is organized as follows. Section 2 sets up the preferences and endowment processes. Generalized disappointment averse preferences, the BY long risks model for consumption and dividend growth and the approximating Markov Switching process endowment process model are presented. In section 3 we solve for asset prices and derive formulas for predictive regressions. Section 4 explores empirical implications of a simple version of our GDA asset pricing model, comparing with BY model and illustrating the mechanism at work. Section 5 looks at implications for fully specified GDA preferences, conducts robustness analysis for preference and endowment parameters, and finally explores the implications of an heteroscedastic random walk endowment process in population and in finite samples. Section 6 concludes.

[7]These papers have been developed contemporaneously and independently from the first version of the current paper titled "An Analytical Framework for Assessing Asset pricing Models and Predictability", presented in May 2006 at the CIREQ and CIRANO Conference in Financial Econometrics in Montreal and discussed by Motohiro Yogo.

2 An Asset Pricing Model with GDA Preferences and LRR Fundamentals

Our primary goal in this section is to formulate a model that includes both a long-run risk specification for consumption and dividends and recursive preferences. In BY, the recursive preferences have a Kreps-Porteus (KP) certainty equivalent that disentangles risk aversion from intertemporal substitution. In this paper we want to extend the certainty equivalent to a generalized disappointment aversion structure (GDA). These preferences weight outcomes differently above and below a threshold determined as a fraction of the certainty equivalent. Two more parameters are included with respect to the KP certainty equivalent, one that sets up the kink at the threshold, and another to determine the percentage of the certainty equivalent that determines the threshold. While GDA admits KP as a particular case, it is also the case that we can set up the parameters so that disappointment aversion is the only source of risk aversion. We will examine these specific preferences to build some intuition about the stochastic discount factor that is obtained in equilibrium with a LRR specification for fundamentals.

The LRR model with KP preferences cannot be solved analytically. BY use Campbell and Shiller (1988) approximations to obtain analytical expressions that are useful for understanding the main mechanisms at work, but when it comes to generate numerical results they appeal to numerical simulations of the original model. A second type of approximation, proposed by Hansen, Heaton and Li (2008), is done around a unitary value for the elasticity of intertemporal substitution ψ[8]. However, since the GDA utility is non-differentiable at the kink where disappointment sets in, one cannot rely on the same approximation techniques to obtain analytical solutions of the model. In this paper we propose a methodology that provides an analytical solution to the LRR model with GDA preferences and a fortiori with KP preferences. In other words, we solve for the asset valuation ratios in equilibrium. The key to this analytical solution is to use a Markov Switching process for consumption and dividends that matches the LRR specification. In addition, we report analytical formulas for the population moments of equity premia as well as for the coefficients and R^2 of predictability regressions that have been used to assess the ability of asset pricing models to reproduce stylized facts.

We first discuss GDA preferences, and then the LRR model and its matching Markov switching process.

[8]A previous version of this paper (SSRN working paper 1109080) derived the approximate solutions for the recursive utility model using the Campbell-Shiller and the Hansen-Heaton-Li approximations and analyzed their respective accuracy for various sets of parameter values.

2.1 Generalized Disappointment Aversion

RZ generalized Gul's (1991) disappointment aversion preferences and embedded them in the recursive utility framework of Epstein and Zin (1989). Formally, let V_t be the recursive intertemporal utility functional:

$$V_t = \left\{ (1-\delta) C_t^{1-\frac{1}{\psi}} + \delta \left[\mathcal{R}_t (V_{t+1}) \right]^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \quad \psi > 0, \quad 0 < \delta < 1, \tag{2.1}$$

where C_t is the current consumption, δ is the time preference discount factor, ψ is the elasticity of intertemporal substitution and $\mathcal{R}_t (V_{t+1})$ is the certainty equivalent of the random future utility conditional on time t information. In GDA preferences the certainty equivalent function $\mathcal{R}(.)$ is implicity defined by:

$$\frac{\mathcal{R}^{1-\gamma}}{1-\gamma} = \int_{(-\infty,\infty)} \frac{V^{1-\gamma}}{1-\gamma} dF(V) - \left(\alpha^{-1} - 1\right) \int_{(-\infty,\kappa\mathcal{R})} \left(\frac{(\kappa\mathcal{R})^{1-\gamma}}{1-\gamma} - \frac{V^{1-\gamma}}{1-\gamma} \right) dF(V) \quad \kappa \leq 1. \tag{2.2}$$

Several particular cases are worth mentioning. When α is equal to one, $\mathcal{R}$ becomes the certainty equivalent corresponding to expected utility while V_t represents the Kreps-Porteus preferences. When $\alpha < 1$, outcomes lower than $\kappa\mathcal{R}$ receive an extra weight $(\alpha^{-1} - 1)$, decreasing the certainty equivalent. Thus, α is interpreted as a measure of disappointment aversion, while the parameter κ is the percentage of the certainty equivalent $\mathcal{R}$ such that outcomes below it are considered disappointing[9].

RZ emphasize that when GDA preferences are embedded in a dynamic asset-pricing economy, effective risk aversion can be counter-cyclical - a feature that helps to explain the equity-premium puzzle.

GDA preferences imply a stochastic discount factor given by:

$$M_{t,t+1} = z_{t+1}^{1-\gamma} \left(R_{t+1}^m\right)^{-1} \frac{\left[1 + (\alpha^{-1} - 1) I (z_{t+1} < \kappa)\right]}{\left[1 + \kappa^{1-\gamma} (\alpha^{-1} - 1) E_t I (z_{t+1} < \kappa)\right]}, \tag{2.3}$$

where:

$$z_{t+1} = \delta^{\frac{1}{1-\frac{1}{\psi}}} \left(\frac{C_{t+1}}{C_t} \right)^{\frac{1}{1-\psi}} \left(R_{t+1}^m\right)^{\frac{1}{1-\frac{1}{\psi}}}.$$

R_{t+1}^m is the return on an asset that yields aggregate consumption as payoff, which we call the market portfolio. It is clear that when there is no disappointment aversion ($\alpha = 1$), the expression above reduces to the familiar Kreps-Porteus pricing kernel, which was used in BY:

[9] Notice that the certainty equivalent, besides being decreasing in γ, is also increasing in α and decreasing in κ (for $\kappa \leq 1$). Thus α and κ are also measures of risk aversion, but of a different type than γ.

$$
\begin{aligned}
M_{t,t+1} &= z_{t+1}^{1-\gamma}\left(R_{t+1}^{m}\right)^{-1}, & (2.4)\\
&= \delta^{\frac{1-\gamma}{1-\frac{1}{\psi}}}\left(\frac{C_{t+1}}{C_t}\right)^{\frac{1-\gamma}{1-\psi}}\left(R_{t+1}^{m}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}}. & (2.5)
\end{aligned}
$$

In order to clarify the role of disappointment aversion preferences, we will sometimes generate results with an otherwise linear utility function: $\gamma = 0$ and $\psi = \infty$. In this case, the stochastic discount factor becomes:

$$
M_{t,t+1} = \delta \frac{\left[1 + \left(\alpha^{-1} - 1\right) I\left(R_{t+1}^{m} < \frac{\kappa}{\delta}\right)\right]}{\left[1 + \kappa\left(\alpha^{-1} - 1\right) E_t I\left(R_{t+1}^{m} < \frac{\kappa}{\delta}\right)\right]}.
$$

Notice that in this case, the only source of risk aversion is disappointment aversion. For each state of the economy in t, the stochastic discount factor has only two values. The SDF for disappointing outcomes is α^{-1} times the SDF for non-disappointing outcomes. The probability of disappointment occurring is given by the likelihood that the return on the market portfolio is less than the ratio between κ and the time discount factor δ. Suppose for simplicity that κ is equal to δ. Then, disappointment occurs when the gross return is less than one, which means when a negative net return occurs. In that case, the variability of the SDF will depend on the distance between the two outcomes, determined by α, as on the respective likelihoods of positive and negative returns[10]. These likelihoods are conditional on the state at time t and therefore produce state-dependent risk aversion.

Suppose further for illustration purposes that these likelihoods are equal and identical over every possible state. Then, for say a value of α equal to 0.2, the value of the SDF will be five times higher in the negative-return states than in the positive-return ones. This will create a sizable negative covariance between the pricing kernel and the return on a risky asset, making the risk premium sizable. In contrast, if the likelihood of disappointing outcomes is negligible, the covariance will be very small and the risk premia close to zero.

In the next sections we will provide evidence that this GDA pricing kernel has also the potential to generate return predictability by the dividend-price ratio. If states are persistent, as it is the case in the LRR case, then the stochastic discount factor distribution will change gradually, implying persistent and predictable conditional expected returns. As argued by Fama and French (1988), this type of process for expected returns generate mean reversion in asset prices. Therefore, the price-dividend ratio today should be a good predictor of returns over several future periods.

[10]Note that the return on the market portfolio also depends on the pricing kernel. Thus, the probability of a disappointing return on it depends also on α.

2.2 Long-run risks and Markov-switching endowment processes

To model the consumption and dividend growth dynamics, we adopt the long-run risk process of BY:

$$\Delta c_{t+1} = x_t + \sigma_t \epsilon_{c,t+1} \tag{2.6}$$

$$\Delta d_{t+1} = (1 - \phi_d)\, \mu_x + \phi_d x_t + \nu_d \sigma_t \epsilon_{d,t+1} \tag{2.7}$$

$$x_{t+1} = (1 - \phi_x)\, \mu_x + \phi_x x_t + \nu_x \sigma_t \epsilon_{x,t+1} \tag{2.8}$$

$$\sigma^2_{t+1} = (1 - \phi_\sigma)\, \mu_\sigma + \phi_\sigma \sigma^2_t + \nu_\sigma \epsilon_{\sigma,t+1}, \tag{2.9}$$

where c_t is the logarithm of real consumption and d_t is the logarithm of real dividends. In this characterization, x_t, the conditional expectation of the consumption growth, is modeled as a slowly reverting AR(1) process (ϕ_x smaller but close to one). Notice that $\phi_d x_t$ also governs the conditional expectation of the dividend growth, and ϕ_d is assumed to be greater than one - the leverage ratio on consumption growth. The volatility of consumption growth σ_t represents the time-varying economic uncertainty, which is also assumed to be a very persistent process (ϕ_σ smaller but close to one) with unconditional mean μ_σ. We depart from BY by allowing a correlation ρ between innovations in consumption growth and in dividend growth, as in Bansal, Kiku and Yaron (2007). However, we maintain the BY assumption of independence between the innovations in the expected growth processes and in the volatility process.

When we combine this fundamentals' dynamics with GDA preferences, it is not possible to rely on the usual solution techniques based on log linearization used in BY. The kink in the SDF introduces a nonlinearity that is not amenable to the linearization technique. Therefore, we propose to match the fundamentals process (2.6 - 2.9) with a Markov switching process. The main intuition for the matching to work is that a two-state Markov process is similar to an AR(1) process. Given this matching we will be able to solve analytically a complex dynamic asset pricing model.

We postulate that the growth rates of consumption and dividends follow a process where conditional means and variances change according to a Markov process. In the technical appendix, we show that with a four-state Markov switching process - two states governing the conditional mean of both processes and the two other states shifting the conditional variance - it is possible to match several moments of the original BY process. Let s_t be the Markov state at time t. By combining the two states - high and low - in mean and in volatility we obtain four states, $s_t \in \{\mu_L \sigma_L, \mu_L \sigma_H, \mu_H \sigma_L, \mu_H \sigma_H\}$.

$$\Delta c_{t+1} = \mu_c(s_t) \;+\; (\omega_c(s_t))^{1/2}\,\varepsilon_{c,t+1} \tag{2.10}$$

$$\Delta d_{t+1} = \mu_d(s_t) \;+\; (\omega_d(s_t))^{1/2}\varepsilon_{d,t+1}, \tag{2.11}$$

where $\varepsilon_{c,t+1}$ and $\varepsilon_{d,t+1}$ follow a bivariate normal process with mean zero and correlation ρ. The states evolve according to a 4 by 4 transition probability matrix P.

Bonomo and Garcia (1996) proposed and estimated the specification (2.10, 2.11) for the joint consumption-dividends process with a three-state Markov switching process to investigate if an equilibrium asset pricing model with different types of preferences can reproduce various features of the real and excess return series.[11] Here we calibrate the parameters of a four-state process in order to match moments generated by the variant of the BY process proposed by Bansal, Kiku and Yaron (2007)[12]. It should be noted that only moments are necessary for computing asset pricing and predictability statistics as we will see in the formulas derived in the next section. The calibrated parameters of the four-state Markov-switching process are reported in Panel A of Table 1.

While we match some unconditional moments of the original process in Bansal, Kiku and Yaron (2007), it will be important to know whether the fit of the Markov-switching model is adequate in finite samples. To assess the fit, we simulate 10,000 samples of the size of the original data for both the original consumption and dividend processes and the matching MS process, and compute empirical quantiles of several moments of the consumption and dividend processes[13]. The percentile values are very close between the two processes except for the volatilities. As a matter of fact, the mean and median volatilities for consumption and dividend growth produced by the MS model are closer to the volatility values computed with the original data.

[11] Cecchetti, Lam, and Mark (1990) use a two-state homoscedastic specification for a univariate process of the endowment process, and Bonomo and Garcia (1994) a heteroscedastic specification in order to investigate if an equilibrium model could reproduce the mean reversion in asset prices. Cecchetti, Lam, and Mark (1993) use a homoscedastic consumption-dividend process in order to try to match the first and second moments of asset returns. The authors use two models, one with a leverage economy, another with a pure exchange economy without bonds. In both instances, they are unable to replicate the first and second moments taken together. All the articles above use expected utility function.

[12] The details of the matching procedure are given in a technical appendix to this paper available upon request from the authors. The main idea of the matching procedure is that the expected means and conditional variances of the consumption and dividend growth rates are written as linear functions of two two-state Markov chains given that a two-state Markov chain is an AR(1) process.

[13] For space consideration, the results are reported in a technical appendix.

3 Solving for Asset Prices and Return Predictability

Given the specification (2.2), the risk-adjustment $\mathcal{R}_t(V_{t+1})$ to the date $t+1$ continuation value of a consumption plan is implicitly given by:

$$\mathcal{R}_t(V_{t+1}) = \left(E\left[\frac{I_{\alpha,1}\left(\frac{V_{t+1}}{\kappa \mathcal{R}_t(V_{t+1})}\right)}{E\left[I_{\alpha,\kappa}\left(\frac{V_{t+1}}{\kappa \mathcal{R}_t(V_{t+1})}\right) \mid J_t \right]} V_{t+1}^{1-\gamma} \mid J_t \right] \right)^{\frac{1}{1-\gamma}}, \tag{3.12}$$

where

$$I_{\alpha,y}(x) = 1 + \left(\frac{1}{\alpha} - 1\right) y^{1-\gamma} \mathbf{1}_{(x<1)}.$$

The stochastic discount factor (2.3) can equivalently be written in terms of the continuation value as in Hansen, Heaton, Roussanov, and Lee (2007), as:

$$M_{t,t+1} = \delta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left(\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})}\right)^{\frac{1}{\psi}-\gamma} \frac{I_{\alpha,1}\left(\frac{V_{t+1}}{\kappa \mathcal{R}_t(V_{t+1})}\right)}{E\left[I_{\alpha,\kappa}\left(\frac{V_{t+1}}{\kappa \mathcal{R}_t(V_{t+1})}\right) \mid J_t \right]}. \tag{3.13}$$

In general, the Markov state s_t in (2.10) and (2.11) will arbitrarily have N possible values, say $s_t \in \{1, 2, .., N\}$, although 4 values as described in the previous section are sufficient to provide a good approximation of the BY long run risk model. Let $\zeta_t \in \mathbb{R}^N$ be the vector Markov chain equivalent to s_t and such that:

$$\zeta_t = \begin{cases} e_1 = (1, 0, 0, .., 0)^\top & \text{if } s_t = 1 \\ e_2 = (0, 1, 0, .., 0)^\top & \text{if } s_t = 2 \\ ... & \\ e_N = (0, 0, .., 0, 1)^\top & \text{if } s_t = N, \end{cases}$$

where e_i is the $N \times 1$ column vector with zeroes everywhere except in the i^{th} position which has the value one, and $\top$ denotes the transpose operator for vectors and matrices.

The Markov chain s_t evolves according to a transition probability matrix P defined as:

$$P^\top = [p_{ij}]_{1 \le i,j \le N}, \quad p_{ij} = \Pr(s_{t+1} = j \mid s_t = i), \tag{3.14}$$

and is stationary with ergodic distribution and second moments given by:

$$E[\zeta_t] = \Pi \in \mathbb{R}_+^N, \quad E[\zeta_t \zeta_t^\top] = Diag(\Pi_1, .., \Pi_N) \text{ and } Var[\zeta_t] = Diag(\Pi_1, .., \Pi_N) - \Pi\Pi^\top, \tag{3.15}$$

where $Diag(u_1, u_2, .., u_N)$ is the diagonal matrix whose diagonal elements are u_1, u_2,..,u_N.

The dynamics (2.10) and (2.11) of endowments can therefore be written as follows:

$$\Delta c_{t+1} = \mu_c^\top \zeta_t \; + \; (\omega_c^\top \zeta_t)^{1/2} \, \varepsilon_{c,t+1} \tag{3.16}$$

$$\Delta d_{t+1} = \mu_d^\top \zeta_t \; + \; (\omega_d^\top \zeta_t)^{1/2} \, \varepsilon_{d,t+1}, \tag{3.17}$$

where

$$\left(\begin{array}{c}\varepsilon_{c,t+1}\\ \varepsilon_{d,t+1}\end{array}\right) \mid \langle \varepsilon_{c,\tau}, \varepsilon_{d,\tau}, \tau \leq t; \zeta_m, m \in \mathbf{Z}\rangle \sim \mathcal{N}\left(\left(\begin{array}{c}0\\ 0\end{array}\right), \left[\begin{array}{cc}1 & \rho^{\top}\zeta_t\\ \rho^{\top}\zeta_t & 1\end{array}\right]\right). \tag{3.18}$$

3.1 Asset Valuation Ratios

The main objective of this section is to characterize the price-consumption ratio $P_{c,t}/C_t$ (where $P_{c,t}$ is the price of the unobservable portfolio that pays off consumption), the price-dividend ratio $P_{d,t}/D_t$ (where $P_{d,t}$ is the price of an asset that pays off the aggregate dividend), and finally the price $P_{f,t}/1$ of a single-period risk-free bond that pays for sure one unit of consumption.

The Markov property of the model is crucial for deriving our analytical formulas. We will show that the variables $\mathcal{R}_t\left(V_{t+1}\right)/C_t$, V_t/C_t, $P_{d,t}/D_t$, $P_{c,t}/C_t$ and $P_{f,t}/1$ are (non-linear) functions of the state variable ζ_t. On the other hand, the state ζ_t takes a finite number of values. Consequently, any real non-linear function $g(\cdot)$ of ζ_t is a linear function of ζ_t. This property will allow us to characterize analytically the price-payoff ratios while other data generating processes need either linear approximations or numerical methods to solve the model. The structure of the endowment process implies that there will be one such payoff-price ratio per regime and this will help in computing closed-form analytical formulas. For these valuation ratios, we adopt the following notations:

$$\frac{\mathcal{R}_t\left(V_{t+1}\right)}{C_t} = \lambda_{1z}^{\top}\zeta_t, \quad \frac{V_t}{C_t} = \lambda_{1v}^{\top}\zeta_t, \quad \frac{P_{d,t}}{D_t} = \lambda_{1d}^{\top}\zeta_t, \quad \frac{P_{c,t}}{C_t} = \lambda_{1c}^{\top}\zeta_t, \text{ and } \frac{P_{f,t}}{1} = \lambda_{1f}^{\top}\zeta_t. \tag{3.19}$$

Solving the GDA model amounts to characterize the vectors λ_{1d}, λ_{1c} and λ_{2f} as functions of the parameters of the consumption and dividend growth dynamics and of the recursive utility function defined above. We start our analysis by characterizing the vectors λ_{1z} and λ_{1v} defined in (3.19) that represent the ratio of the certainty equivalent of future lifetime utility to current consumption and the ratio of lifetime utility to consumption. The characterization of these vectors is the main difference between Epstein-Zin and CCAPM models. We will show below that when one has the vectors λ_{1z} and λ_{1v}, one gets the price-consumption ratio (i.e. the vector λ_{1c}), the price-dividend ratio (i.e. the vector λ_{1d}) and the risk-free rate (i.e. the vector λ_{1f}) as for the CCAPM. The following proposition characterizes the vectors λ_{1z} and λ_{1v}.

Proposition 3.1 ***Characterization of the ratios of utility to consumption.*** *Let*

$$\frac{\mathcal{R}_t\left(V_{t+1}\right)}{C_t} = \lambda_{1z}^{\top}\zeta_t \text{ and } \frac{V_t}{C_t} = \lambda_{1v}^{\top}\zeta_t$$

respectively denote the ratio of the certainty equivalent of future lifetime utility to current consumption and the ratio of lifetime utility to consumption. The components of the vectors λ_{1z} *and* λ_{1v} *are given by:*

$$\lambda_{1z,i} = \exp\left(\mu_{c,i} + \frac{1-\gamma}{2}\omega_{c,i}\right)\left(\sum_{j=1}^{N} p_{ij}^{*}\lambda_{1v,j}^{1-\gamma}\right)^{\frac{1}{1-\gamma}} \tag{3.20}$$

$$\lambda_{1v,i} = \left\{(1-\delta) + \delta\lambda_{1z,i}^{1-\frac{1}{\psi}}\right\}^{\frac{1}{1-\frac{1}{\psi}}} \text{ if } \psi \neq 1 \text{ and } \lambda_{1v,i} = \lambda_{1z,i}^{\delta} \text{ if } \psi = 1, \tag{3.21}$$

where the matrix $P^{*\top} = \left[p_{ij}^{*}\right]_{1\leq i,j\leq N}$ *is defined in the Appendix.*

The equations (3.20) and (3.21) are solved jointly[14].

Given the ratio of the certainty equivalent of future lifetime utility to current consumption and the ratio of lifetime utility to consumption derived in Proposition 3.1, one gets the following expressions for the price-consumption ratio, the equity price-dividend ratio and the single-period risk-free rate.

Proposition 3.2 ***Characterization of asset prices.*** *Let*

$$\frac{P_{d,t}}{D_t} = \lambda_{1d}^{\top}\zeta_t, \quad \frac{P_{c,t}}{C_t} = \lambda_{1c}^{\top}\zeta_t \quad \text{and} \quad \frac{1}{R_{f,t+1}} = \lambda_{1f}^{\top}\zeta_t$$

respectively denote the price-dividend ratio, the price-consumption ratio and the risk-free rate. The components of the vectors λ_{1d}*,* λ_{1c}*, and* λ_{1f} *are given by:*

$$\lambda_{1d,i} = \delta\left(\frac{1}{\lambda_{1z,i}}\right)^{\frac{1}{\psi}-\gamma}\exp\left(\mu_{cd,i} + \frac{\omega_{cd,i}}{2}\right)\left(\lambda_{1v}^{\frac{1}{\psi}-\gamma}\right)^{\top}P^{**}\left(Id - \delta A^{**}\left(\mu_{cd} + \frac{\omega_{cd}}{2}\right)\right)^{-1}e_i \tag{3.22}$$

$$\lambda_{1c,i} = \delta\left(\frac{1}{\lambda_{1z,i}}\right)^{\frac{1}{\psi}-\gamma}\exp\left(\mu_{cc,i} + \frac{\omega_{cc,i}}{2}\right)\left(\lambda_{1v}^{\frac{1}{\psi}-\gamma}\right)^{\top}P^{*}\left(Id - \delta A^{*}\left(\mu_{cc} + \frac{\omega_{cc}}{2}\right)\right)^{-1}e_i \tag{3.23}$$

$$\lambda_{1f,i} = \delta\exp\left(-\gamma\mu_{c,i} + \frac{\gamma^2}{2}\omega_{c,i}\right)\sum_{j=1}^{N}\tilde{p}_{ij}^{*}\left(\frac{\lambda_{1v,j}}{\lambda_{1z,i}}\right)^{\frac{1}{\psi}-\gamma}, \tag{3.24}$$

where the vectors $\mu_{cd} = -\gamma\mu_c + \mu_d$*,* $\omega_{cd} = \omega_c + \omega_d - 2\gamma\rho\odot\omega_c^{1/2}\odot\omega_d^{1/2}$*,* $\mu_{cc} = (1-\gamma)\mu_c$*,* $\omega_{cc} = (1-\gamma)^2\omega_c$*, and the matrices* $P^{**\top} = \left[p_{ij}^{**}\right]_{1\leq i,j\leq N}$ *and* $\tilde{P}^{*\top} = \left[\tilde{p}_{ij}^{*}\right]_{1\leq i,j\leq N}$ *as well as the matrix functions* $A^{**}(u)$ *and* $A^{*}(u)$ *are defined in the Appendix.*

[14] The system is highly nonlinear in terms of the $\lambda_{1z,i}$, $i = 1,..,N$. However, it is easy to solve the system of equations numerically by using numerical algorithms. We did by using the nonlinear equation solver in GAUSS.

3.2 Analytical Formulas for Statistics Reproducing Stylized Facts

Since the seminal paper of Mehra and Prescott (1985), reproducing the equity premium and the risk-free rate has become an acid test for all consumption-based asset pricing models. Follow-up papers added the volatilities of both excess returns and the risk-free rate, as well as predictability regressions where the predictor is most often the price-dividend ratio and the predicted variables are equity returns or excess returns or consumption and dividend growth rates.

Bansal and Yaron (2004) use a number of these stylized to assess the adequacy of their LRR model and Beeler and Campbell (2009) provide a thorough critical analysis of the BY LRR model for a comprehensive set of stylized facts. The methodology used in Beeler and Campbell (2009) to produce population moments from the model rests on solving a loglinear approximate solution to the model and on a single simulation run over 1.2 million months (100,000 years). This simulation has to be run for each configuration of preference parameters considered. Typically, as in most empirical assessments of consumption-based asset pricing models, they consider a limited set of values for preference parameters and fix the parameters of the LRR model at the values chosen by Bansal and Yaron (2004) or Bansal, Kiku and Yaron (2007).

In this section, we provide analytical formulas for the stylized facts used in the literature. We will report below formulas for expected (excess) returns and unconditional moments of (excess) returns, formulas for predictability of (excess) returns and consumption and dividend growth rates by the dividend-price ratio, and formulas for variance ratios of (excess) returns. These analytical formulas will allow us to assess the sensitivity of the results to wide ranges of the parameters of the LRR model and to several sets of preference parameter values.

We will compare these model-produced statistics to the corresponding empirical quantities computed with a data set of quarterly consumption, dividends and returns data for the US economy (1930:1 to 2007:4). The empirical first and second moments of asset prices and the empirical predictability results are reported first in the second column of Table 2 and then repeated for convenience of comparison in all relevant tables.

3.2.1 Expected Returns

In order to study the predictability of the returns and excess returns, we need to connect them to the state variable ζ_t and to the dividend growth. We define the return process,

R_{t+1}, and aggregate returns over h periods, $R_{t+1:t+h}$, as:

$$R_{t+1} = \frac{P_{d,t+1} + D_{t+1}}{P_{d,t}} = \left(\lambda_{2d}^{\top}\zeta_t\right)\left(\lambda_{3d}^{\top}\zeta_{t+1}\right)\exp\left(\Delta d_{t+1}\right) \quad \text{and} \quad R_{t+1:t+h} = \sum_{j=1}^{h} R_{t+j}, \tag{3.25}$$

with $\lambda_{2d} = 1/\lambda_{1d}$ and $\lambda_{3d} = \lambda_{1d} + \iota$ where ι denotes the $N \times 1$ vector with all components equal to one. We also define the excess returns R^e_{t+1} and aggregate excess returns $R^e_{t+1:t+h}$, i.e., $R^e_{t+1} = R_{t+1} - R_{f,t+1}$ and $R^e_{t+1:t+h} = R_{t+1:t+h} - R_{f,t+1:t+h}$. We show that:

$$E\left[R_{t+j} \mid J_t\right] = \psi_d^{\top} P^{j-1}\zeta_t \text{ and } E\left[R^e_{t+j} \mid J_t\right] = \left(\psi_d - \lambda_{2f}\right)^{\top} P^{j-1}\zeta_t, \quad \forall j \geq 2, \tag{3.26}$$

$$E\left[R_{t+1:t+h} \mid J_t\right] = \psi_{h,d}^{\top}\zeta_t \quad \text{and} \quad E\left[R^e_{t+1:t+h} \mid J_t\right] = \left(\psi_{h,d} - \lambda_{h,2f}\right)^{\top}\zeta_t, \tag{3.27}$$

where $\lambda_{2f} = 1/\lambda_{1f}$ and the vectors ψ_d, $\psi_{h,d}$ and $\lambda_{h,2f}$ are given in the Appendix.

3.2.2 Variance of Returns

The variance of returns over h periods is given by:

$$\begin{aligned} Var\left[R_{t+1:t+h}\right] &= h\theta_2^{\top} E\left[\zeta_t\zeta_t^{\top}\right] P^{\top}\theta_3. \\ &+ h\left(\theta_1 \odot \theta_1\right)^{\top} E\left[\zeta_t\zeta_t^{\top}\right] P^{\top}\left(\lambda_{3d} \odot \lambda_{3d}\right) - h^2\left(\theta_1^{\top} E\left[\zeta_t\zeta_t^{\top}\right] P^{\top}\lambda_{3d}\right)^2 \\ &+ 2\sum_{j=2}^{h}\left(h - j + 1\right)\theta_1^{\top} E\left[\zeta_t\zeta_t^{\top}\right] P^{\top}\left(\lambda_{3d} \odot \left(\left(P^{j-2}\right)^{\top}\left(\theta_1 \odot \left(P^{\top}\lambda_{3d}\right)\right)\right)\right), \end{aligned} \tag{3.28}$$

where θ_1, θ_2 and θ_3 are given in the Appendix. Similar formulas obtain for excess returns.

3.2.3 Predictability Regressions

Stylized facts show a strong predictability of (excess) returns by the dividend-price ratio, which increases with the horizon. Although a vast literature discusses whether this predictability is actually present or not because of several statistical issues, we will sidestep the various corrections suggested since we are looking for a model that rationalizes the observed stylized facts.

The predictability is inevitably measured with finite samples of data, but when considering the ability of a model to reproduce some empirical facts it is important to consider population moments[15]. Therefore, we provide below the formulas for population coefficients of determination in regressions of returns aggregated over a number of periods on the current dividend-price ratio, as it is common in the asset pricing literature to run

[15]Papers in the literature on consumption-based asset pricing usually compute by simulation small sample and large sample statistics (see for example Cecchetti, Lam and Mark (1990) and Bonomo and Garcia (1994) in the older literature, Beeler and Campbell (2009) in the most recent one.

such predictive regressions. Similar regressions can be run with cumulative excess returns, consumption growth or dividend growth as the dependent variable.

Typically, when one runs the linear regression of a variable, say $y_{t+1:t+h}$, onto another one, say x_t, and a constant, one gets

$$y_{t+1:t+h} = a(h) + b(h)\, x_t + \eta_{y,1,t+h}(h), \tag{3.29}$$

where

$$b(h) = \frac{Cov\left(y_{t+1:t+h}, x_t\right)}{Var\left[x_t\right]}, \tag{3.30}$$

while the corresponding population coefficient of determination denoted $R^2(h)$ is given by:

$$R^2(h) = \frac{\left(Cov\left(y_{t+1:t+h}, x_t\right)\right)^2}{Var\left[y_{t+1:t+h}\right] Var\left[x_t\right]}. \tag{3.31}$$

In order to use these formulas to characterize the predictive ability of the dividend-price ratio for future expected returns, one needs the variance of payoff-price ratios, covariances of payoff-price ratios with aggregate returns and variance of aggregate returns. We show that:

$$Var\left[\frac{D_t}{P_{d,t}}\right] = \lambda_{2d}^{\top} Var\left[\zeta_t\right] \lambda_{2d} \quad \text{and} \quad Cov\left(R_{t+1:t+h}, \frac{D_t}{P_{d,t}}\right) = \psi_{h,d}^{\top} Var\left[\zeta_t\right] \lambda_{2d}, \tag{3.32}$$

and the variance of aggregate returns is given by (3.28). Similar formulas obtain for excess returns, consumption growth and dividend growth.

4 Asset Pricing Implications of a Simple GDA

In order to illustrate the potential for Generalized Disappointment Aversion to produce realistic asset pricing and predictability implications we start with the simplest GDA preference - one in which intertemporal substitution is perfectly elastic and disappointment aversion is the only source of risk aversion[16]. That is we set $\gamma = 0$ and $\psi = \infty$, and call this specification GDA0. The SDF is given by equation (2.4):

$$M_{t,t+1} = \delta \frac{\left[1 + \left(\alpha^{-1} - 1\right) I\left(R_{t+1}^{m} < (\kappa/\delta)\right)\right]}{\left[1 + \kappa\left(\alpha^{-1} - 1\right) E_t I\left(R_{t+1}^{m} < (\kappa/\delta)\right)\right]}.$$

Notice that if disappointment aversion were not present ($\alpha = 1$) the stochastic discount factor would be equal to the constant time discount factor δ. This simplistic specification of the GDA preferences will allow us to gain intuition about the replication potential of

[16]RZ also examine this case with a simple two-state Markov chain endowment process.

asset pricing and predictability stylized facts by the model, without using the curvature engendered by the other parameters. In the next section we will show the asset pricing implications of more realistic specifications of GDA preferences. We will examine two cases that will differ mainly by a value of the intertemporal elasticity of substitution above and below 1.

4.1 Asset Pricing Implications: KP vs GDA0

In terms of asset pricing implications we will look at a set of moments for returns and price-dividend ratios, namely the expected value and the standard deviation of the equity premium, the risk-free rate and the price-dividend ratio. We should stress that the moments are population moments and are computed with the analytical formulas reported in the previous section. We also consider several predictability regressions by the price-dividend ratio, for excess returns, consumption growth and dividend growth. Again the R^2 and the regression coefficients are computed analytically with the formulas reported in the previous section. There are, therefore, population statistics.

We start by reporting the asset pricing and predictability implications for the benchmark endowment process of BY, as specified in Bansal, Kiku and Yaron (2007). They set the autocorrelation of expected consumption growth ϕ_x to 0.975, the leverage of dividends on consumption ϕ_d at 2.5, and the persistence of volatility ϕ_σ at the extreme value of 0.999[17]. In the original calibration of Bansal and Yaron (2004), the volatility parameter was less persistent, which gave a bigger role to the persistence in consumption growth in determining asset prices. Beeler and Campbell (2009) do a detailed comparative analysis of the two calibrated processes in terms of reproducing stylized facts.

Under the benchmark LRR heading in Table 2, we compare KP preferences used in BY ($\delta = 0.9989$, $\psi = 1.5$, $\gamma = 10$) to the simple GDA0 preferences described above (with $\delta = 0.998$ and the disappointment aversion parameter α set to 0.2, which means that the disappointment threshold is placed at 98% of the certainty equivalent).

4.1.1 Matching the Moments

A first observation is that the values reported for the return and price-dividend ratio moments for KP are close to the values reported in BY. They reproduce the mean and the standard deviation of equity returns and the mean of the risk-free rate. The standard deviation of the latter is small compared to the observed volatility in post-war data. For the price-dividend ratio, the expected value is lower than in the data and the standard

[17] The correlation ρ of consumption and dividend growth innovations is chosen to be 0.4. The other parameters are $\mu_x = 0.0015$, $\nu_x = 0.038$, $\sqrt{\mu_\sigma} = 0,0072$, $\nu_\sigma = 0.28 \times 10^{-5}$.

deviation of the dividend yield is about a third of what is observed. The simple GDA0 specification does a very good job at matching these statistics, except the volatility of the risk-free rate.

The interpretation of the results for KP preferences is known. Given the two economic sources of risk at play - the persistent long-run component in the expected growth of consumption and economic uncertainty represented by a time-varying and very persistent volatility of consumption growth, BY make the point that the reaction of the agent to the first source of risk, that is for constant volatility of consumption, accounts for a large part (around two-thirds) of the risk premium. Indeed, when the ϕ_x parameter is set at 0.9 [18] instead of 0.975, the risk premium produced by the model falls to 2.26 as reported under the corresponding heading in Table 2, which represents roughly one-third of the premium explained by the time-varying volatility in BY.

We also provide a very telling graphical illustration of the sensitivity of the six moments to ϕ_x in Figure 1. We keep all the other parameters fixed and vary ϕ_x over the whole range of values between 1 and 0. It should be noted that each graph is produced with a grid of a thousand values for the parameter ϕ_x. The model is solved analytically for each new value of the parameter. This will represent a huge task if we had to solve the model numerically and compute the statistics by simulation.

In the upper left graph corresponding to the equity premium, one clearly sees the almost vertical fall in the premium produced by the KP model as ϕ_x moves away from 1. The value settles at about 1%, which will be produced by the time-varying volatility. Two other moment statistics also deteriorate when the persistence is reduced to 0.9. The expected value of the dividend-price ratio becomes very high and its volatility goes to zero. For ϕ_x equal to 0.9, we can see in Table 2 that the mean of the price-dividend ratio is equal to 130.18 while the dividend yield volatility falls to 0.03.

For generalized disappointment aversion (GDA0), the interpretation of the results in terms of sources of risk is opposite to KP. For an investor with GDA0 preferences, it is the macroeconomic uncertainty that explains the high equity premium. In the high volatility state, which happens about 20% of the time in the benchmark case, the required premium is much higher than in the low volatility state. To confirm this, we can observe in Table 2 that the moment statistics produced by GDA0 are essentially the same when ϕ_x is set to 0.9. This is also very clear in Figure 1 where the lines associated with all moment statistics show robustness to changes in the ϕ_x value.

The sensitivity to another key parameter ϕ_d, the leverage ratio on expected consumption

[18] We do compensating changes in the parameters for the variance of innovations in consumption and dividends to keep their variance unchanged.

growth, is also worth investigating. In the last column of Table 2, we set the parameter ϕ_d to 1 instead of 2.5 in the benchmark case. Since a lower leverage ϕ_d reduces the volatility of dividends, the equilibrium equity premium decreases for both KP and GDA0 but the effect is much more pronounced for KP. Again the expected value of the price-dividend ratio shoots up (146.91) and the dividend yield volatility goes down dramatically (0.03) for KP. The movements are similar for GDA0 but their magnitude is much smaller. This is well illustrated in Figure 2, where we vary ϕ_d between 0 and 10 while keeping all other parameters constant. The moments react much more dramatically to ϕ_d with KP preferences than with GDA0. In particular, the equity premium goes to zero and the stock price tends to infinity as ϕ_d decreases to one.

Overall, apart from the high sensitivity of the statistics produced by the KP model for small changes of some key parameters, we should retain that this model produces lower values for the level of the price-dividend ratio and the dividend yield volatility than in the data. As we will see in the next section, this will have an important implication for predictability regressions.

4.1.2 Predictability

In Panels B, C and D of Table 2, we report predictability statistics for the one, three and five-year excess returns, consumption growth, and dividend growth when the dividend yield is used to forecast them. More specifically, we compute analytically the population R^2 and the slope of the regressions of the forecasted series on dividend-price ratios. The return regressions are key predictability regressions in empirical asset pricing, while the regressions on the economic fundamentals are there to confirm the low level of predictability of the dividend and consumption growth rates observed in the data.

The population R^2 of excess return regressions for the KP model are very small when compared to the data and do not exhibit an increasing pattern. While the model gives R^2s of 0.06%, 0.06%, and 0.02%, for 1, 3, and 5-year regressions, the actual values are 7%, 14.67%, and 27.26%. The low R^2s can be explained by the low volatility of the dividend yield produced by the model. Moreover, the results deteriorate a lot when we perturb the parameters ϕ_x and ϕ_d, with negative and huge regression coefficients, since the mean level and the volatility of the dividend yield collapse.

The model with GDA0 preferences can match both the magnitude and the pattern of R^2 of excess-return regression in the data: 7.26%, 18.68% and 27.20% for 1, 3, and 5 years, respectively, as well as the magnitudes of the regression coefficients. This is related to the fact that the GDA0 model reproduces well both the volatility (for the R^2s) and the level (for the regression coefficients) of the dividend yield. The kink associated with generalized disappointment aversion creates large differences in dividend-price ratios between the low and high volatility states.

Neither consumption growth nor dividend growth are predictable by the dividend yield in the data. The KP model generates relatively large R^2 in the consumption growth regressions: 16.50%, 21.05% and 18.22% per cent, as compared to practically zero R^2s in the data. The GDA0 preferences produce results much more in line with the data with less than one per cent R^2s at all horizons. For dividend growth, KP produces smaller predictability than for consumption growth, although much larger than in the data. The GDA0 model behaves much better and produces predictability of the same order of magnitude than in the data. Results are also robust to perturbations in the persistence in expected consumption growth and in the leverage ratio.

In summary, GDA0 reproduces rather well both asset pricing moments and predictability stylized facts with the benchmark long-run risks model of BY. For KP, we have shown that the model reproduces well the return moment statistics in the benchmark case, but that its performance, especially in terms of equity premium, deteriorates considerably when the persistence parameter in expected consumption growth and the leverage ratio of consumption for dividends are perturbed. Therefore, the KP-LRR model depends on a specification for the endowment process that works for a very narrow set of parameter values, which is an undesirable feature from a robustness point of view. Moreover we showed that in population the KP-LRR model is not able to reproduce the predictability of excess returns by the dividend yield and that it produces a counterfactual predictability of consumption growth by the dividend yield.

In the next section, we will explain why the GDA0 model reproduces the stylized facts in terms of pricing kernels and assess the level of risk aversion, compared to an expected utility model, that is implied by the level of disappointment aversion in the GDA0 specification.

4.2 GDA0 Stochastic Discount Factor

To better understand why the generalized disappointment model explains well the stylized facts, we have a closer look at the underlying stochastic discount factor. As we showed before in the description of the endowment matching, the MS endowment process we are using has four states: $\{\mu_L\sigma_L, \mu_L\sigma_H, \mu_H\sigma_L, \mu_H\sigma_H\}$. Table 1 reports the transition probability matrix between the states. The process evolves dynamically through changes between states $\mu_L\sigma_L$ and $\mu_H\sigma_L$ or changes between states $\mu_L\sigma_H$ and $\mu_H\sigma_H$. In other words, variance states are very persistent, with changes in the variance happening less often than the changes in mean.

Panel A of Table 3 reports the SDF of the GDA0 basic specification. For each state the SDF is a binary distribution where the disappointing outcome is associated with a realization of the SDF which is 5 times greater than that of non-disappointing outcomes[19]. The states with low variance - $\mu_L\sigma_L$ and $\mu_H\sigma_L$ - have a very high probability of a non-disappointing outcome. Therefore, the SDF in those states is close to a constant, and the risk premium is very low. The states with high variances are the ones with a more variable SDF and with a higher risk premium. The switching between low and high variance states produce state-dependent risk aversion, which is essential for predictability.

We saw in Table 2 that when the coefficient of autocorrelation in expected consumption growth is changed from 0.975 to 0.9 the KP model fails to reproduce the equity premium, while for GDA0 it is slightly increased. We see in Panel B of Table 3 that the distribution of the GDA0 SDF does not change much and that the probability of disappointment becomes a bit higher for the high variance states. This increase in the disappointment probabilities is responsible for the higher risk premium under this specification.

How one compares the risk aversion of KP and GDA0 preferences? If the better results obtained with GDA0 preferences were due to a much higher risk aversion, the results obtained would become less interesting. Figure 3 shows that this is not the case. There we plot indifference curves for a hypothetical gamble with two outcomes with equal probability for our GDA0 preferences ($\alpha = 0.2$ and $\kappa = 0.98$) and expected utility preferences with coefficient of relative risk aversion 5 and 10. While GDA0 preferences exhibit higher risk aversion than both expected utility preferences for small gambles, the same is not true for larger gambles. When the size of the gamble is about 20%, the GDA0 indifference curve crosses the expected utility indifference curve with risk aversion equal to 10, becoming less risk averse for larger gambles. For higher gamble sizes it approaches the expected utility with relative risk aversion equal to 5.

5 Full GDA preferences, endowment specification, and asset pricing implications

We showed that a very simple specification of the generalized disappointment aversion preferences, one in which disappointment aversion is the only source of risk aversion, can reproduce the stylized facts for return moments and predictive regressions of returns, consumption and dividend growth on the dividend yield. On the contrary, predictability results implied by Kreps-Porteus preferences were at odds with the data. We also saw that the asset pricing implications were robust to reasonable modifications of the endowment process for GDA0 but not for KP. While it is a good feature to be able generate realistic asset

[19]Recall that for $\alpha = 0.2$, the disappointing outcomes have a SDF that is α^{-1} times that of the non-disappointing outcomes.

pricing implications with such a simple preference model, the fact that the elasticity of intertemporal substitution is infinity in this specification may be a cause of concern. In fact, the value of the elasticity of intertemporal substitution ψ and its implications for the BY-LRR asset pricing model are a matter of debate. Bansal and Yaron (2004) argue for a value larger than 1 for this parameter since it is critical for reproducing the asset pricing stylized facts. Their main argument is that the presence of fluctuating consumption volatility leads to a serious downward bias in the estimates of the IES using the instrumental variable (IV) regression approach pursued in Hall (1988). Beeler and Campbell (2009) simulate the long-run risks model to see whether the downward bias is important in IV estimates of ψ and conclude that there is no downward bias when the riskless interest rate is used as instrument, but that there is a poor finite-sample performance of IV regressions with stock returns as instrument, reflecting a weak instrument problem. They add that the high volatility of the real interest rate is hard to reconcile with an IES greater than 1. We have seen in Table 2 that it is indeed one dimension over which both KP and GDA0 were not performing well.

Given this debate over the value of ψ, we introduce two specifications of GDA preferences, one that has the same elasticity of intertemporal substitution as the benchmark BY-LRR model ($\psi = 1.5$) and another that has an elasticity of substitution smaller than one ($\psi = 0.75$). Both have a low coefficient of risk aversion (with γ values of 1.25 and 2.5, respectively). For both specifications we maintain the benchmark endowment process of BY. We explore the sensitivity of the results to a wide range of modifications in the endowment process through a graphical analysis. We show that these two specifications of GDA preferences can reproduce as well the asset pricing stylized facts and are robust to changes in the endowment process.

5.1 GDA Preferences and the LRR Model

In Table 4, we report results for the two GDA specifications (GDA1, $\psi = 0.75$, and GDA2, $\psi = 1.5$) and the LRR-BY model. We also report the previous infinite elasticity specification (GDA0) for comparison purposes. All previous results that we reported for GDA0 hold for the two new specifications. Two important results are worth pointing out. First, stylized facts are reproduced for elasticities above and below 1. With KP preferences in BY, the value of 1 was pivotal for the results. If ψ is below 1 the model cannot reproduce the stylized facts. This is quite important because it tells us that we do not need to keep searching for reasons to support an IES greater than 1 to salvage the LRR model. GDA preferences provide the flexibility required to accommodate situations where consumption today and consumption tomorrow can be viewed as substitutes or complements.

Second, we notice that GDA1 produces a higher and a more variable interest rate and tends to accentuate the predictability of excess returns by the dividend price ratio. These interest rate changes are a direct consequence of an IES lower than 1. With ψ less than 1, consumption today and consumption tomorrow along a deterministic path are complements, while they are substitutes for ψ above 1.

It is customary for studies on consumption-based asset pricing models to stop here since we have found parameters that reproduce the stylized facts we selected to evaluate the model. The main reason is that often the model is hard to solve and that very long simulations are necessary to compute population statistics. The recent study of Beeler and Campbell (2009), which looks comprehensively at the LRR model with KP preferences, uses simulations of length of 1.2 million months to compute population return moments and predictability statistics. They report a large set of statistics but for two given set of parameters for fundamentals and preferences, one used in the original study of Bansal and Yaron (2004) and Bansal, Kiku and Yaron (2007). In the next two sections, we will assess thoroughly the sensitivity of the asset pricing and predictability statistics to large variations in the key parameters of the consumption process, namely the persistence of the mean and volatility of consumption growth, and in the preference parameters. Given that we do not estimate any of the parameters, it is crucial to know how much results depend on the particular values chosen. We have already seen that if we lowered a bit the persistence in expected consumption, then the LRR model with KP preferences will not reproduce the average equity premium. We want to scrutinize the ability of the GDA model to price the assets in a way consistent with the data. We will rely on the analytical formulas we developed in the preceding sections to check the robustness of the model in many directions.

5.2 Robustness to Changes in the Endowment Process

We gauge the sensitivity of the statistics through graphs. In each figure about the asset pricing moments, we exhibit 6 graphs, one for each moment. For the figures on predictability, we show 9 graphs, that is three horizons for each quantity to be predicted: excess returns, consumption growth and dividend growth. In each graph, we plot KP and the three specifications of GDA preferences. We start with the robustness of asset pricing moments to changes in the persistence of expected consumption growth (ϕ_x) in Figure 4. All the curves associated with GDA are almost parallel straight lines to the horizontal axis showing that the computed moments are insensitive to the expected growth persistence parameter. For GDA0, the patterns are a bit different for values of ϕ_x close to 1 but settle to straight lines as we reduce ϕ_x. For KP, as already mentioned, the parameter ϕ_x is key. All results obtain for values close to 1, emphasizing the essential role of a very persistent component in expected consumption growth. The pattern of the expected price dividend ratio for KP is particularly striking, increasing steeply from a low value of 20 for the benchmark BY value of 0.975 to values greater than 100 as we just move away from it.

The sensitivity of the asset pricing moments to persistence in consumption volatility is reported in Figure 5. We investigate a very fine grid of values between 1 and 0.9. There is now more variability in the GDA results. While the expected excess return and risk-free rate do not vary much, we can see that their volatility increases when ϕ_σ gets closer to one, starting at about 0.96. A somewhat similar pattern emerges for the expected price-dividend ratio and the dividend yield volatility but the increase takes place for values closer to one. The patterns with GDA0 are similar but a bit more accentuated, while the statistics values are a bit different. For KP, the results are more stable, leading to the conclusion that from the two sources of long-run risk, the risk in expected consumption growth matters more for this type of preference, while consumption volatility is more relevant for GDA preferences.

In Figures 6 and 7 we now explore the implications for predictability of varying the persistence parameters in expected consumption growth and volatility respectively. For ϕ_x, the two GDAs exhibit predictability of excess returns consistent with the predictability observed in the data, while it is not the case for KP. Predictability increases for KP when we reduce the value of the persistence parameter but we know that the moments are no longer matched for these values. For consumption and dividend growth, the benchmark ϕ_x produces too much predictability when it gets close to 1. Otherwise it is flat at zero. Here again, we cannot reproduce the low predictability of the consumption growth and the dividend growth and the moments at the same time. For ϕ_σ, the persistence parameter in consumption volatility, the predictability results for GDA are consistent with the data only for values close to 1. At $\phi_\sigma = 0.9$, there is no predictability of excess returns and predictability in growth rates, contrary to what is observed. This is due to the low volatility of the dividend-price ratio in this case. If the high volatility state is not very persistent, the probability of disappointing outcomes will differ less across states and the prices of the risky asset will be closer together.

We can conclude from this sensitivity analysis that the source of long-run risk, whether in the mean or the volatility of consumption growth, needs to be persistent for the agent's preferences to operate in a way consistent with the observed data. For KP preferences in the Bansal and Yaron (2004) model, we see a strong tension as ϕ_x, the persistence of expected consumption growth, moves away from 1. The ability to reproduce asset pricing moments deteriorates quickly while the predictability statistics improve. For the GDA preferences that we advocate in this paper, the persistence in the volatility of consumption growth ϕ_σ is key for reproducing the predictability stylized facts but we do not find the tension present in KP preferences. The means of the equity and risk-free returns are pretty insensitive to ϕ_σ, while their volatilities decrease but not drastically as ϕ_σ moves away from one.

5.3 Robustness to Changes in Preference Parameters

We now keep the value of the risk aversion parameter γ to 2.5 and vary the disappointment aversion parameter α, the elasticity of intertemporal substitution ψ and the kink parameter κ. In Figure 8, we study the implications of the changes in α in three horizontal panels for expected excess returns, the risk-free rate and the price-dividend ratio respectively, where κ varies between 0.975 and 0.985. We look at three values for ψ, 0.9, 1 and 1.5, which results in three sets of three panels.

The equity premium increases with κ and decreases with α. Increasing α makes the agent less disappointed and therefore prices will be higher and risky returns lower. The parameter κ acts in the opposite direction. When it gets closer to 1, there are more outcomes that makes the investor disappointed. As the elasticity of intertemporal substitution increases, it produces only a small increase in the level of the equity premium.

The risk-free rate goes down as disappointment increases, that is when α is decreasing and κ is increasing. The effect of κ is much more pronounced since the curves fan out. The effect of ψ on the risk-free rate is important since it affects directly intertemporal trade-offs in terms of consumption. Below the value of 1 the investor sees consumption at two different times as complementary and this results in a higher level of the risk-free rate, while above 1 they are perceived as substitutes and the equilibrium risk-free rate is lower.

Finally, the expected price-dividend ratio decreases with disappointment aversion, with the main factor being κ, since the curves bunch up as κ gets closer to 1. Decreasing ψ lowers the level of the expected price-dividend ratio and makes it less sensitive to changes in α.

In Figure 9, we apply the same sensitivity analysis, with identical changes in the parameters, to the predictability of excess returns at one, three and five-year horizons. The main conclusion is that predictability necessitates a large amount of disappointment aversion. It appears to be consistent with the data for lower values of α and higher values of κ. Changing ψ does not affect much predictability since both the levels and the slopes are identical across graphs.

5.4 GDA Preferences and A Random Walk Model For the Endowment

If a very persistent predictable component exists in consumption growth, as proposed by BY, it is certainly hard to detect it as consumption appears very much as a random walk in the data[20]. In the previous section we have argued that, for agents with disappointment aversion preferences, it is mainly the long-run volatility risk that matters. In this section, we want to investigate if a heteroscedastic random walk model, coupled with disappointment aversion, is able to reproduce the stylized facts that we have put forward for aggregate asset prices.

We restrict the LRR model by assuming a constant expected consumption growth, resulting in the following specification:

$$\begin{aligned}\Delta c_{t+1} &= \mu_x + \sigma_t \varepsilon_{c,t+1} \\ \Delta d_{t+1} &= \mu_x + \nu_d \sigma_t \varepsilon_{d,t+1} \\ \sigma^2_{t+1} &= (1-\phi_\sigma)\mu_\sigma + \phi_\sigma \sigma^2_t + \nu_\sigma \varepsilon_{\sigma,t+1}.\end{aligned} \tag{5.33}$$

We have argued that generalized disappointment aversion was important to reproduce asset pricing and predictability results. In this section we pursue two goals. First we want to see if a simple random walk model can generate these results. Second we want to restrict the parameter κ to be 1 so the agent has disappointment aversion with a kink exactly at the certainty equivalent. This will help us identify the role played by the flexibility in the position of the disappointment threshold brought about by GDA.

The parameters of the random walk process are reported in Panel B of Table 1. The unconditional probability of being in the low volatility state is close to 80% and the parameters of consumption and dividend volatility in the two regimes are close to what we had before in the LRR model. The asset pricing and predictability results for the RW process are reported in Table 5. In Panels A1 and A2, we kept the benchmark preference parameters that we used with the LRR model (see Tables 2 and 4) to assess the marginal effect of changing the endowment process. As we saw before when lowering the persistence parameter in expected consumption growth, the effect on the equity premium is more pronounced with KP preferences. It falls to 1.51% confirming that persistence in expected consumption is the key economic feature in the BY model. Additionally, the expected price-dividend ratio explodes and the regression coefficients in the predictability regressions of excess returns become negative. On the contrary, predictability results remain practically unchanged for

[20]Campbell and Cochrane (1999) use a random walk model for consumption and a heteroscedastic slowly mean-reverting surplus that is dynamically driven by consumption growth innovations that feeds into habit persistent preferences. This random walk model is close in spirit to the model in Calvet and Fisher (2007).

the three GDA specifications. In terms of asset pricing implications, the main statistics that are affected are the level of the equity premium, that is reduced by 1 to 2 percentage points, and the level of the price-dividend ratio, that increases. The volatility of the dividend yield decrease slightly.

We can now keep the RW process but try to modify the preference parameters to best match the stylized facts. This is done in Panels B1 and B2 where the subscript a is added to the preference acronyms. For KP preferences we manage to get much closer to the observed asset pricing statistics by changing the risk aversion and the elasticity of intertemporal substitution, but not as close as in the LRR model. However what is striking is that we now have some predictability of returns, while there was none in the BY model, even though the regression coefficients are much too big compared with the data. For the disappointment preferences, one case is particularly interesting. It is the so-called GDA0a, which is in fact not generalized disappointment aversion but just disappointment aversion since κ is set to one. In this case the risk-free rate is fixed so $\sigma\left[R_f\right]$ is equal to 0. The other asset pricing moments are quite close to the data and we also obtain reasonable predictability patterns.

In Figures 10 and 11, we plot the sensitivity of the asset pricing statistics and predictability statistics, respectively, to variations in the persistence parameter of consumption volatility ϕ_σ. In Figure 10 we observe that all asset pricing statistics for KP preferences, while out of line with the data, remain roughly insensitive to variations of ϕ_σ from 0.9 to 1. For GDA, the patterns are similar across the three specifications. The biggest changes occur in the volatility of the dividend yield that goes towards zero as we approach 0.9. Otherwise, the other statistics remain pretty much the same as we vary ϕ_σ from 0.9 to 1. In Figure 11, the patterns in R^2 for all preference specifications are similar. Their values decrease steeply as ϕ_σ approaches 0.9. As we mentioned before, KP preferences show some predictability but the values of the slopes become unrealistic (they do not show in the graphs for 3 and 5 years). One can see that the magnitude of predictability for the GDA specifications depends very much on the value of ϕ_σ, but that some predictability remains for a sizable range of values.

Our endowment process can be somewhat related to Campbell and Cochrane (1999) model, which was also successful in matching asset pricing moments and generating the right predictability patterns. While our preferences are also state-dependent, the action here is engendered by consumption volatility, while in their paper the main driving process is the surplus, which is more related to the business cycle. Insofar as the volatility of consumption is related to the business cycle, these two models are close to each other in spirit.

Optimizing the preference parameter values for GDA1a and GDA2a does not change the results much compared to the benchmark preferences in Table 5, with GDA2a producing somewhat stronger predictability of excess returns than observed in the data. From these results we can conclude that a random walk model with persistent consumption growth volatility and GDA preferences with an IES less than 1 appears as a rather good model for pricing assets.

5.5 Robustness to sample size: small sample results

The results in the above sections were based on population statistics. In order to assess whether our main conclusions still hold in finite samples, we simulate the random walk model with the set of alternative preferences used in the previous subsection. For each specification we simulate the model for 936 months, repeating it one thousand times[21]. The results are reported in Table 6.

Table 6 displays the mean and median small sample statistics for the moment and predictability regressions of the four preference sets KPa, GDA0a, GDA1a and GDA2a. The main patterns obtained for population statistics are maintained.

The equity premium for Kreps-Porteus preferences is still too low, with a median of 2.87%. The median values of the excess returns distributions generated under the alternative generalized disappointment aversion specifications are much closer to the data value: 5.44 for GDA0a, 5.97 for GDA1a, and 5.82 for GDA2a. It continues to be a challenge to reproduce the standard deviation of the risk-free rate. For GDA1a, with an IES less than 1, the finite sample mean and median standard deviations of the risk-free rate (2.33 and 2.74 respectively) are less than the population mean value of 3.36. As well, the standard deviation of the dividend yield is lower than for the population statistics for GDA preferences. It is close to zero, as in population, for KP.

The median of the small sample distribution of regression coefficients for the excess return regression on the dividend-price ratio is much lower than the observed R^2 for KP preferences, but the mean is higher than in population. The median is closer to the population values. The median of the R^2 distribution is closer to the observed statistics for the GDA1 specification, which has an elasticity of substitution parameter below 1.

When consumption growth is regressed on the dividend price ratio, we find some predictability in average for all preference configurations. It is less pronounced at the median of the distribution but one cannot differentiate between the preferences in finite samples

[21]In order to attenuate the effect of the discreteness of the Markov switching model we weighted the statistics for each state by its ex-post (filtered) probability. We also reduced very slightly the persistence of the volatility process ϕ_σ to 0.994.

based on these regressions. Of course, consumption growth being a random walk, no predictability should be observed but one will need longer samples to arrive at this conclusion. This emphasizes the value of analytical formulas for population statistics to assess the ability of models to reproduce stylized facts.

6 Conclusion

We examined how long-run risks of the type advocated by Bansal and Yaron (2004) interact with Kreps-Porteus and generalized disappointment aversion preferences in shaping asset prices. While persistence of expected consumption growth is fundamental for the moment matching ability of the Kreps-Porteus model, disappointment averse preferences rely mostly on the persistence of consumption volatility. The slow mean reverting process for expected consumption growth when coupled with Kreps-Porteus preferences has the undesirable side-effect of generating the wrong predictability pattern: dividend yields forecast consumption growth but not excess returns.

When preferences are disappointment averse, this source of long-run risk ceases to have this counterfactual effect. The persistent volatility of consumption growth strongly interacts with disappointment aversion to generate realistic moments and predictability patterns, notwithstanding the presence of the Bansal and Yaron's (2004) main source of risk. In fact, disappointment aversion alone could reproduce the patterns present in both asset pricing moments and predictive regressions in a random walk model for consumption.

Appendix: Collection of Expressions Referred to in the Text

The components of the matrix $P^{*\top} = \left[p^*_{ij}\right]_{1\leq i,j\leq N}$ in (3.20) and (3.23), and the matrix function $A^*(u)$ also in (3.23) are defined by:

$$p^*_{ij} = p_{ij}\frac{1 + \left(\alpha^{-1} - 1\right)\Phi\left(\frac{\ln\left(\kappa\frac{\lambda_{1z,i}}{\lambda_{1v,j}}\right) - \mu_{c,i}}{\omega^{1/2}_{c,i}} - (1-\gamma)\,\omega^{1/2}_{c,i}\right)}{1 + \left(\alpha^{-1} - 1\right)\kappa^{1-\gamma}\sum_{j=1}^{N} p_{ij}\Phi\left(\frac{\ln\left(\kappa\frac{\lambda_{1z,i}}{\lambda_{1v,j}}\right) - \mu_{c,i}}{\omega^{1/2}_{c,i}}\right)}, \tag{A.1}$$

$$A^*(u) = Diag\left(\left(\frac{\lambda_{1v,1}}{\lambda_{1z,1}}\right)^{\frac{1}{\psi}-\gamma}\exp(u_1), ..., \left(\frac{\lambda_{1v,N}}{\lambda_{1z,N}}\right)^{\frac{1}{\psi}-\gamma}\exp(u_N)\right)P^*. \tag{A.2}$$

The components of the matrix $P^{**\top} = \left[p^{**}_{ij}\right]_{1\leq i,j\leq N}$ in (3.22), and the matrix function $A^{**}(u)$ also in (3.22) are defined by:

$$p^{**}_{ij} = p_{ij}\frac{1 + \left(\alpha^{-1} - 1\right)\Phi\left(\frac{\ln\left(\kappa\frac{\lambda_{1z,i}}{\lambda_{1v,j}}\right) - \mu_{c,i}}{\omega^{1/2}_{c,i}} - \left(\rho_i\omega^{1/2}_{d,i} - \gamma\omega^{1/2}_{c,i}\right)\right)}{1 + \left(\alpha^{-1} - 1\right)\kappa^{1-\gamma}\sum_{j=1}^{N} p_{ij}\Phi\left(\frac{\ln\left(\kappa\frac{\lambda_{1z,i}}{\lambda_{1v,j}}\right) - \mu_{c,i}}{\omega^{1/2}_{c,i}}\right)}, \tag{A.3}$$

$$A^{**}(u) = Diag\left(\left(\frac{\lambda_{1v,1}}{\lambda_{1z,1}}\right)^{\frac{1}{\psi}-\gamma}\exp(u_1), ..., \left(\frac{\lambda_{1v,N}}{\lambda_{1z,N}}\right)^{\frac{1}{\psi}-\gamma}\exp(u_N)\right)P^{**}. \tag{A.4}$$

The components of the matrix $\tilde{P}^{*\top} = \left[\tilde{p}^*_{ij}\right]_{1\leq i,j\leq N}$ in (3.24) are defined by:

$$\tilde{p}^*_{ij} = p_{ij}\frac{1 + \left(\alpha^{-1} - 1\right)\Phi\left(\frac{\ln\left(\kappa\frac{\lambda_{1z,i}}{\lambda_{1v,j}}\right) - \mu_{c,i}}{\omega^{1/2}_{c,i}} + \gamma\omega^{1/2}_{c,i}\right)}{1 + \left(\alpha^{-1} - 1\right)\kappa^{1-\gamma}\sum_{j=1}^{N} p_{ij}\Phi\left(\frac{\ln\left(\kappa\frac{\lambda_{1z,i}}{\lambda_{1v,j}}\right) - \mu_{c,i}}{\omega^{1/2}_{c,i}}\right)}. \tag{A.5}$$

The vectors θ_1, θ_2 and θ_3 appearing in equation (3.28) are given by:

$$\theta_1 = \lambda_{2d} \odot (\exp(\mu_{d,1} + \omega_{d,1}/2), ..., \exp(\mu_{d,N} + \omega_{d,N}/2))^\top, \tag{A.6}$$

$$\theta_2 = (\theta_1 \odot \theta_1 \odot (\exp(\omega_{d,1}), ..., \exp(\omega_{d,N}))^\top) - (\theta_1 \odot \theta_1), \tag{A.7}$$

$$\theta_3 = \lambda_{3d} \odot \lambda_{3d}. \tag{A.8}$$

The vectors ψ_d, $\psi_{h,d}$ and $\lambda_{h,2f}$ appearing in (3.26) and (3.27) are given by:

$$\psi_{d,i} = \lambda_{2d,i}\exp(\mu_{d,i} + \omega_{d,i}/2)\lambda^\top_{3d}Pe_i, \quad i = 1, ..., N, \tag{A.9}$$

$$\psi_{h,d} = \left(\sum_{j=1}^{h} P^{j-1}\right)^\top \psi_d \quad \text{and} \quad \lambda_{h,2f} = \left(\sum_{j=1}^{h} P^{j-1}\right)^\top \lambda_{2f}. \tag{A.10}$$

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Table 1: **(LRR) Parameters of the Long-Run Risk and Random Walk Markov-Switching Models.**
In Panel A, we report the parameters of the four-state monthly Markov-switching model of the form (2.10,2.11) that matches the long-run risk model of Bansal and Yaron (2004). The long-run risk model is calibrated as in Bansal, Kiku and Yaron (2007), with $\mu_x = 0.0015$, $\phi_d = 2.5$, $\nu_d = 6.5$, $\phi_x = 0.975$, $\nu_x = 0.038$, $\sqrt{\mu_h} = 0.0072$, $\phi_h = 0.999$, $\nu_h = 0.28 \times 10^{-5}$ and $\rho_1 = 0.39985$. In Panel B, we report the parameters of the two-state monthly Markov-switching model of the form (2.10,2.11) such that $\mu_{c,1} = \mu_{c,2}$ and $\mu_{d,1} = \mu_{d,2}$. From the long run risk model, we set $\phi_x = 0$ and $\nu_x = 0$ to obtain a random walk model, and we adjust the other parameters when necessary such that consumption and dividend growth means, variances and covariance remain unchanged from the original model. The random walk model is then calibrated with $\mu_x = 0.0015$, $\nu_d = 6.42322$, $\sqrt{\mu_h} = 0.0073$, $\phi_h = 0.999$, $\nu_h = 0.28 \times 10^{-5}$ and $\rho_1 = 0.40434$. In both panels, μ_c and μ_d are conditional means of consumption and dividend growths, ω_c and ω_d are conditional variances of consumption and dividend growths and ρ is the conditional correlation between consumption and dividend growths. $P^\top$ is the transition matrix across different regimes and Π is the vector of unconditional probabilities of regimes. Means and standard deviations are in percent.

Panel A	$\mu_L\sigma_L$	$\mu_L\sigma_H$	$\mu_H\sigma_L$	$\mu_H\sigma_H$
$\mu_c^\top$	-0.20197	-0.20197	0.19307	0.19307
$\mu_d^\top$	-0.72991	-0.72991	0.25769	0.25769
$\left(\omega_c^\top\right)^{1/2}$	0.44071	1.31462	0.44071	1.31462
$\left(\omega_d^\top\right)^{1/2}$	2.86569	8.54824	2.86569	8.54824
$\rho^\top$	0.39985	0.39985	0.39985	0.39985
		$P^\top$		
$\mu_L\sigma_L$	0.97752	0.00021	0.02227	0.00000
$\mu_L\sigma_H$	0.00077	0.97695	0.00002	0.02226
$\mu_H\sigma_L$	0.00273	0.00000	0.99706	0.00021
$\mu_H\sigma_H$	0.00000	0.00272	0.00079	0.99649
$\Pi^\top$	0.08600	0.02304	0.70268	0.18828

Panel B	σ_L	σ_H
$\mu_c^\top$	0.15	0.15
$\mu_d^\top$	0.15	0.15
$\left(\omega_c^\top\right)^{1/2}$	0.46	1.32
$\left(\omega_d^\top\right)^{1/2}$	2.94	8.48
$\rho^\top$	0.40434	0.40434
	$P^\top$	
σ_L	0.99979	0.00021
σ_H	0.00079	0.99921
$\Pi^\top$	0.78868	0.21132

Table 2: **(LRR) Asset Prices and Predictability: KP and GDA0**

The entries of Panel A are model population values of asset prices. The expressions $E[R - R_f]$, $E[R_f] - 1$ and $E[P_d/D]$ are respectively the annualized equity premium, mean risk-free rate and mean price-dividend ratio. The expressions $\sigma[R]$, $\sigma[R_f]$ and $\sigma[D/P_d]$ are respectively the annualized standard deviations of market return, risk-free rate and dividend-price ratio. Panels B, C and D show the R^2 and the slope of the regression $y_{t+1:t+12h} = a(h) + b(h)\left(\frac{D}{P_d}\right)_{t-11:t} + \eta_{t+12h}(h)$, where y stands for excess returns, consumption growth and dividend growth respectively.

	Data	Benchmark LRR		$\phi_x = 0.90$		$\phi_d = 1$	
		KP	GDA0	KP	GDA0	KP	GDA0
δ		0.9989	0.998	0.9989	0.998	0.9989	0.998
γ		10	0	10	0	10	0
ψ		1.5	∞	1.5	∞	1.5	∞
α		1	0.2	1	0.2	1	0.2
κ		1	0.98	1	0.98	1	0.98
Panel A. Asset Pricing Implications							
$E[R - R_f]$	7.25	6.87	6.01	2.26	6.63	2.74	4.82
$\sigma[R]$	19.52	18.11	18.99	16.86	18.36	16.45	19.43
$E[R_f] - 1$	1.21	1.09	1.34	1.72	0.72	1.09	1.34
$\sigma[R_f]$	4.1	1.01	1.58	1.00	1.53	1.01	1.58
$E[P_d/D]$	30.57	22.28	29.64	130.18	28.92	146.91	45.79
$\sigma[D/P_d]$	1.52	0.49	1.72	0.03	1.82	0.03	1.59
Panel B. Predictability of Excess Returns							
$R^2(1)$	7	0.06	7.26	0.48	8.40	0.97	7.88
$b(1)$	3.12	0.92	3.07	-45.38	3.04	-63.13	3.53
$R^2(3)$	14.67	0.06	18.68	1.70	21.52	2.80	20.52
$b(3)$	7.05	1.57	9.10	-150.18	9.01	-188.59	10.45
$R^2(5)$	27.26	0.02	27.20	2.88	31.25	4.47	30.14
$b(5)$	12.34	1.29	14.97	-255.67	14.84	-312.25	17.20
Panel C. Predictability of Consumption Growth							
$R^2(1)$	0.06	16.50	0.78	2.69	0.01	5.49	0.11
$b(1)$	-0.02	-2.40	-0.15	-17.75	-0.02	-25.94	-0.06
$R^2(3)$	0.09	21.05	0.99	1.46	0.01	7.00	0.14
$b(3)$	-0.05	-5.47	-0.33	-24.18	-0.02	-59.22	-0.13
$R^2(5)$	0.24	18.22	0.86	0.88	0.00	6.06	0.12
$b(5)$	-0.11	-7.15	-0.44	-24.69	-0.02	-77.34	-0.17
Panel D. Predictability of Dividend Growth							
$R^2(1)$	0	3.07	0.14	0.48	0.00	0.17	0.00
$b(1)$	0.04	-5.99	-0.37	-44.37	-0.04	-25.94	-0.06
$R^2(3)$	0.2	5.00	0.23	0.29	0.00	0.29	0.01
$b(3)$	-0.48	-13.68	-0.83	-60.45	-0.06	-59.22	-0.13
$R^2(5)$	0.08	4.90	0.23	0.18	0.00	0.30	0.01
$b(5)$	-0.37	-17.87	-1.09	-61.73	-0.06	-77.34	-0.17

Table 3: **(LRR) Conditional Probability Distribution Function of the Stochastic Discount Factor: GDA0.**

The entries of the table are the probability density function of the stochastic discount factor conditional in each state of the economy, namely $g(M \mid J_t)$, when the representative investor has time separable preferences with $\gamma = 1/\psi = 0$. The distribution is concentrated on two points, $a_{i\cdot}$ and $a_{i\cdot}/\alpha$ where i is the state of the economy, and the table shows each point with the associated weight. Panel A shows the distribution for the benchmark case where the long-run risk model is calibrated as in Bansal, Kiku and Yaron (2007), with $\mu_x = 0.0015$, $\phi_d = 2.5$, $\nu_d = 6.5$, $\phi_x = 0.975$, $\nu_x = 0.038$, $\sqrt{\mu_h} = 0.0072$, $\phi_h = 0.999$, $\nu_h = 0.28 \times 10^{-5}$ and $\rho_1 = 0.4$. In Panel B, the persistence of long-run risks is changed to $\phi_x = 0.90$, and ν_x is adjusted to maintain the same variance for expected consumption growth as in the benchmark case.

i	$a_{i\cdot}$	$Prob\,(a_{i\cdot})$	$a_{i\cdot}/\alpha$	$Prob\,(a_{i\cdot}/\alpha)$
Panel A. Benchmark LRR				
$\mu_L\sigma_L$	0.997	0.99976	4.985	0.00024
$\mu_L\sigma_H$	0.766	0.92256	3.828	0.07744
$\mu_H\sigma_L$	0.987	0.99706	4.933	0.00294
$\mu_H\sigma_H$	0.839	0.95181	4.197	0.04819
Panel B. $\phi_x = 0.90$				
$\mu_L\sigma_L$	0.997	0.99975	4.985	0.00025
$\mu_L\sigma_H$	0.766	0.92259	3.828	0.07741
$\mu_H\sigma_L$	0.957	0.98899	4.784	0.01101
$\mu_H\sigma_H$	0.816	0.94310	4.080	0.05690

Table 4: **(LRR) Asset Prices and Predictability: GDA**

The entries of Panel A are model population values of asset prices. The expressions $E[R-R_f]$, $E[R_f]-1$ and $E[P_d/D]$ are respectively the annualized equity premium, mean risk-free rate and mean price-dividend ratio. The expressions $\sigma[R]$, $\sigma[R_f]$ and $\sigma[D/P_d]$ are respectively the annualized standard deviations of market return, risk-free rate and dividend-price ratio. Panels B, C and D show the R^2 and the slope of the regression $y_{t+1:t+12h} = a(h) + b(h)\left(\frac{D}{P_d}\right)_{t-11:t} + \eta_{t+12h}(h)$, where y stands for excess returns, consumption growth and dividend growth respectively.

	Data	Benchmark LRR		
		GDA0	GDA1	GDA2
δ		0.998	0.9989	0.9989
γ		0	1.25	2.5
ψ		∞	0.75	1.5
α		0.2	0.25	0.33
κ		0.98	0.987	0.986
Panel A. Asset Pricing Implications				
$E[R-R_f]$	7.25	6.01	5.48	6.27
$\sigma[R]$	19.52	18.99	18.40	18.61
$E[R_f]-1$	1.21	1.34	2.04	1.22
$\sigma[R_f]$	4.1	1.58	3.77	2.11
$E[P_d/D]$	30.57	29.64	27.91	27.90
$\sigma[D/P_d]$	1.52	1.72	1.96	1.69
Panel B. Predictability of Excess Returns				
$R^2(1)$	7	7.26	14.01	8.40
$b(1)$	3.12	3.07	3.75	3.30
$R^2(3)$	14.67	18.68	32.96	21.39
$b(3)$	7.05	9.10	11.10	9.74
$R^2(5)$	27.26	27.20	44.91	30.85
$b(5)$	12.34	14.97	18.25	15.99
Panel C. Predictability of Consumption Growth				
$R^2(1)$	0.06	0.78	0.20	0.70
$b(1)$	-0.02	-0.15	-0.07	-0.14
$R^2(3)$	0.09	0.99	0.26	0.90
$b(3)$	-0.05	-0.33	-0.15	-0.32
$R^2(5)$	0.24	0.86	0.22	0.78
$b(5)$	-0.11	-0.44	-0.20	-0.42
Panel D. Predictability of Dividend Growth				
$R^2(1)$	0	0.14	0.04	0.13
$b(1)$	0.04	-0.37	-0.16	-0.36
$R^2(3)$	0.2	0.23	0.06	0.21
$b(3)$	-0.48	-0.83	-0.37	-0.81
$R^2(5)$	0.08	0.23	0.06	0.21
$b(5)$	-0.37	-1.09	-0.49	-1.06

Table 5: **(RW) Asset Prices and Predictability: KP and GDA**

The entries of Panel A are model population values of asset prices. The expressions $E[R-R_f]$, $E[R_f]-1$ and $E[P_d/D]$ are respectively the annualized equity premium, mean risk-free rate and mean price-dividend ratio. The expressions $\sigma[R]$, $\sigma[R_f]$ and $\sigma[D/P_d]$ are respectively the annualized standard deviations of market return, risk-free rate and dividend-price ratio. Panels B, C and D show the R^2 and the slope of the regression $y_{t+1:t+12h} = a(h) + b(h)\left(\frac{D}{P_d}\right)_{t-11:t} + \eta_{t+12h}(h)$, where y stands for excess returns, consumption growth and dividend growth respectively.

	Data	Benchmark RW			
		KP	GDA0	GDA1	GDA2
Panel A1. Asset Pricing Implications					
$E[R-R_f]$	7.25	1.51	3.88	4.76	4.96
$\sigma[R]$	19.52	16.35	22.09	19.45	19.15
$E[R_f]-1$	1.21	1.76	1.46	1.89	1.21
$\sigma[R_f]$	4.1	0.49	1.79	3.36	1.94
$E[P_d/D]$	30.57	692.21	81.62	38.98	44.82
$\sigma[D/P_d]$	1.52	0.00	1.33	1.95	1.61
Panel A2. Predictability of Excess Returns					
$R^2(1)$	7	1.48	6.92	14.03	8.97
$[b(1)]$	3.12	-809.76	4.47	3.97	3.70
$R^2(3)$	14.67	4.20	18.54	33.40	23.05
$[b(3)]$	7.05	-2400.40	13.25	11.75	10.96
$R^2(5)$	27.26	6.64	27.82	45.90	33.49
$[b(5)]$	12.34	-3953.31	21.82	19.36	18.05
		KPa	GDA0a	GDA1a	GDA2a
Panel B1. Asset Pricing Implications					
$E[R-R_f]$	7.25	3.53	5.62	6.23	6.23
$\sigma[R]$	19.52	16.39	17.08	17.79	18.03
$E[R_f]-1$	1.21	1.49	1.32	1.25	1.09
$\sigma[R_f]$	4.1	0.77	0.00	3.33	2.23
$E[P_d/D]$	30.57	53.14	28.88	27.09	28.72
$\sigma[D/P_d]$	1.52	0.08	1.15	1.81	1.86
Panel B2. Predictability of Excess Returns					
$R^2(1)$	7	2.17	3.21	14.50	10.96
$[b(1)]$	3.12	30.02	2.68	4.00	3.36
$R^2(3)$	14.67	6.09	8.96	33.91	27.12
$[b(3)]$	7.05	88.99	7.95	11.85	9.95
$R^2(5)$	27.26	9.54	13.92	46.05	38.29
$[b(5)]$	12.34	146.57	13.10	19.51	16.39

Table 6: **(RW) Asset Prices and Predictability: KPa and GDAa, Small Sample Results**

The entries of Panel A are model finite sample values of asset prices. The expressions $E[R-R_f]$, $E[R_f]-1$ and $E[P_d/D]$ are respectively the annualized equity premium, mean risk-free rate and mean price-dividend ratio. The expressions $\sigma[R]$, $\sigma[R_f]$ and $\sigma[D/P_d]$ are respectively the annualized standard deviations of market return, risk-free rate and dividend-price ratio. Panels B and C show finite sample R^2 and slope of the regression $y_{t+1:t+4h} = a(h) + b(h)\left(\frac{D}{P_d}\right)_{t-3:t} + \eta_{t+4h}(h)$, where y stands for excess returns and consumption growth respectively. Finite sample distributions are based on a series of 1000 simulated samples with equivalent length to the data (936 months). The persistence of the volatility process is set to $\phi_h = 0.994$.

	Data	Benchmark RW							
		KPa		GDA0a		GDA1a		GDA2a	
		Mean	50%	Mean	50%	Mean	50%	Mean	50%
Panel A. Asset Pricing Implications									
$E[R-R_f]$	7.25	3.19	2.87	5.72	5.44	6.81	5.97	6.52	5.82
$\sigma[R]$	19.52	15.78	14.70	16.50	15.72	17.04	16.38	17.41	16.87
$E[R_f]-1$	1.21	1.74	1.88	1.32	1.32	1.24	1.76	1.25	1.61
$\sigma[R_f]$	4.10	0.63	0.75	0.00	0.00	2.33	2.74	1.64	1.94
$E[P_d/D]$	30.57	54.52	54.77	26.52	27.01	21.54	22.05	23.46	24.09
$\sigma[D/P_d]$	1.52	0.04	0.05	0.39	0.46	0.65	0.77	0.71	0.85
Panel B. Predictability of Excess Returns									
$R^2(1)$	7.00	3.51	1.43	4.45	1.86	10.71	8.14	9.60	6.59
$[b(1)]$	3.12	24.15	39.79	8.32	6.19	9.98	8.10	9.59	6.99
$R^2(3)$	14.67	7.54	2.71	9.60	3.90	21.43	18.15	19.74	15.08
$[b(3)]$	7.05	90.49	95.26	18.59	15.79	21.50	20.76	19.85	18.15
$R^2(5)$	27.26	9.79	3.49	12.51	4.94	25.75	23.25	24.30	19.86
$[b(5)]$	12.34	157.44	139.92	27.31	23.27	31.71	30.13	28.65	26.34
Panel C. Predictability of Consumption Growth									
$R^2(1)$	0.06	2.32	0.87	2.32	0.86	2.33	0.86	2.33	0.86
$[b(1)]$	-0.02	-7.34	-0.37	-0.96	-0.04	-0.63	-0.02	-0.61	-0.02
$R^2(3)$	0.09	5.02	1.64	5.02	1.62	5.02	1.63	5.02	1.64
$[b(3)]$	-0.05	9.44	0.34	1.25	0.04	0.82	0.02	0.80	0.02
$R^2(5)$	0.24	6.82	2.03	6.82	2.03	6.82	2.02	6.82	2.02
$[b(5)]$	-0.11	3.28	0.39	0.39	0.04	0.25	0.03	0.24	0.03

Figure 1: **(LRR) Sensitivity of Asset Prices to the Persistence of Expected Consumption Growth: GDA0 and KP.**
The figure displays population values of asset prices as functions of the persistence of expected consumption growth. The expressions $E[R - R_f]$ and $E[P_d/D]$ are respectively the annualized equity premium and mean price-dividend ratio. The expressions $\sigma[R - R_f]$ and $\sigma[D/P_d]$ are respectively the annualized standard deviations of the equity excess return and the equity dividend-price ratio.

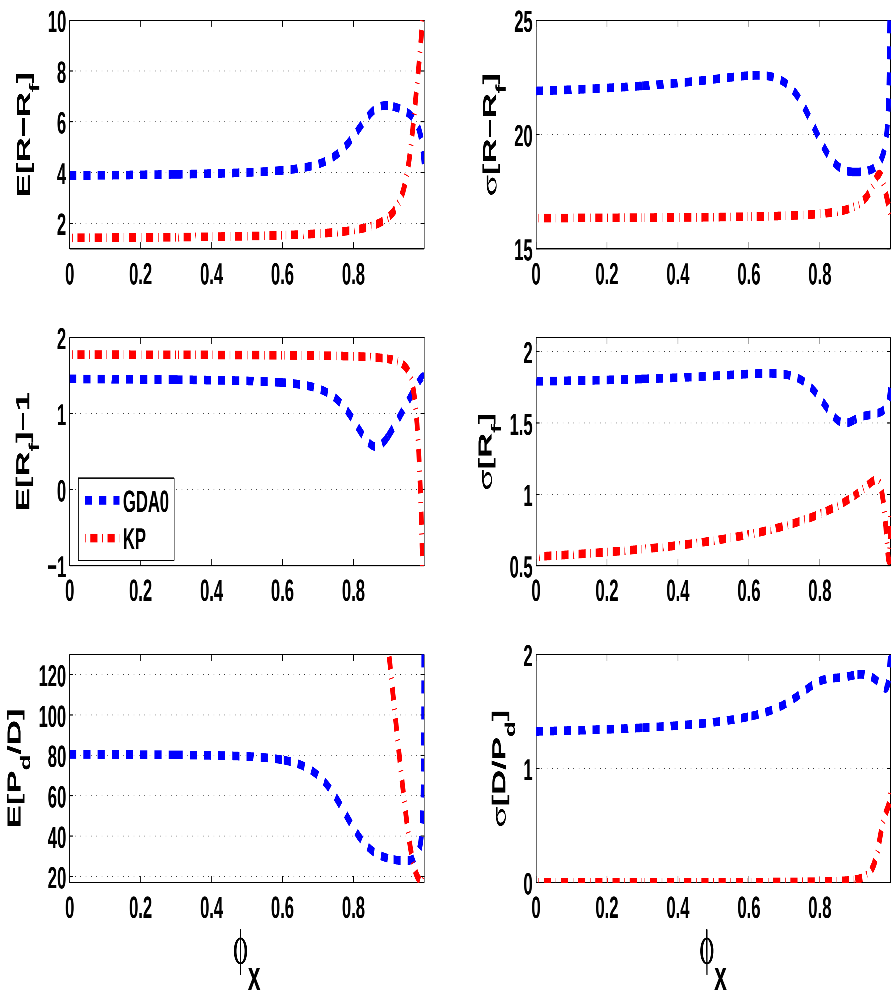

Figure 2: **(LRR) Sensitivity of Asset Prices to the Leverage of Dividends on Consumption: GDA0 and KP.**
The figure displays population values of asset prices as functions of the leverage of dividends on consumption. The expressions $E[R - R_f]$ and $E[P_d/D]$ are respectively the annualized equity premium and mean price-dividend ratio. The expressions $\sigma[R - R_f]$ and $\sigma[D/P_d]$ are respectively the annualized standard deviations of the equity excess return and the equity dividend-price ratio.

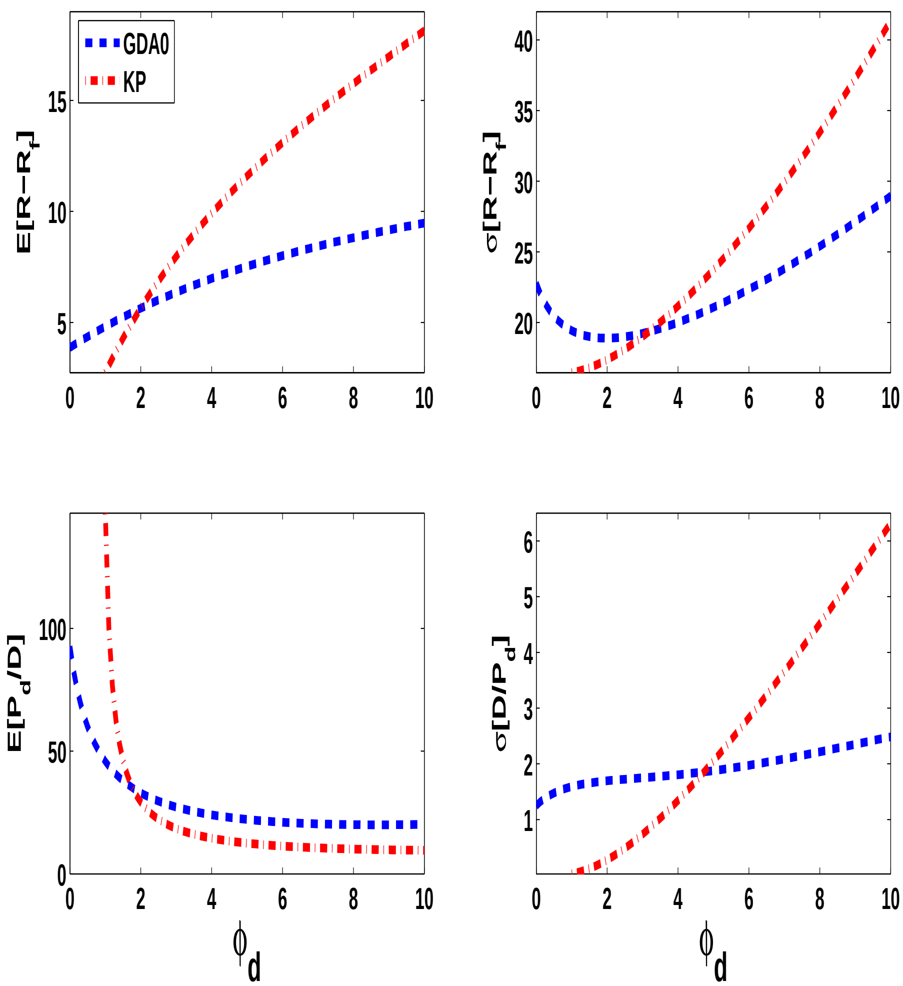

Figure 3: **Indifference Curves for GDA Preferences**
Indifference curves over two outcomes x and y with the fixed probability $p = Prob\,(x) = 1/2$.

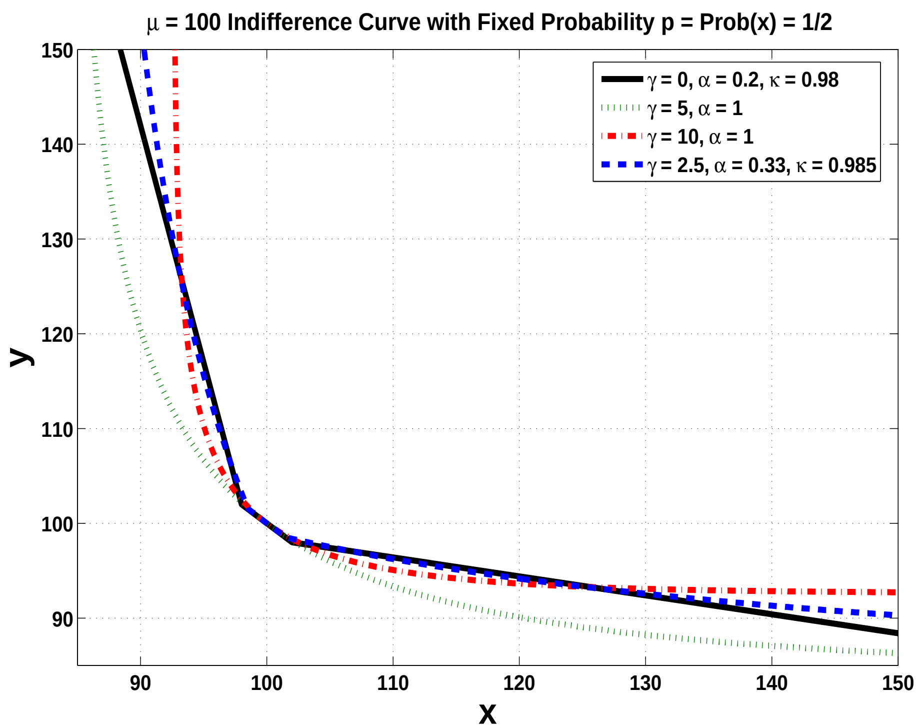

Figure 4: **(LRR) Sensitivity of Asset prices to the Persistence of Expected Consumption Growth: KP and GDA.**
The figure shows the population R^2 of the monthly regression $y_{t+1:t+h} = a\,(h) + b\,(h)\,\frac{D_t}{P_{d,t}} + \eta_{t+h}\,(h)$ for horizons corresponding to one year ($h = 12$), three years ($h = 36$) and five years ($h = 60$). The variable y stands for excess returns $R - R_f$, consumption growth Δc and dividend growth Δd. The R^2 is plotted as a function of the persistence of expected consumption growth.

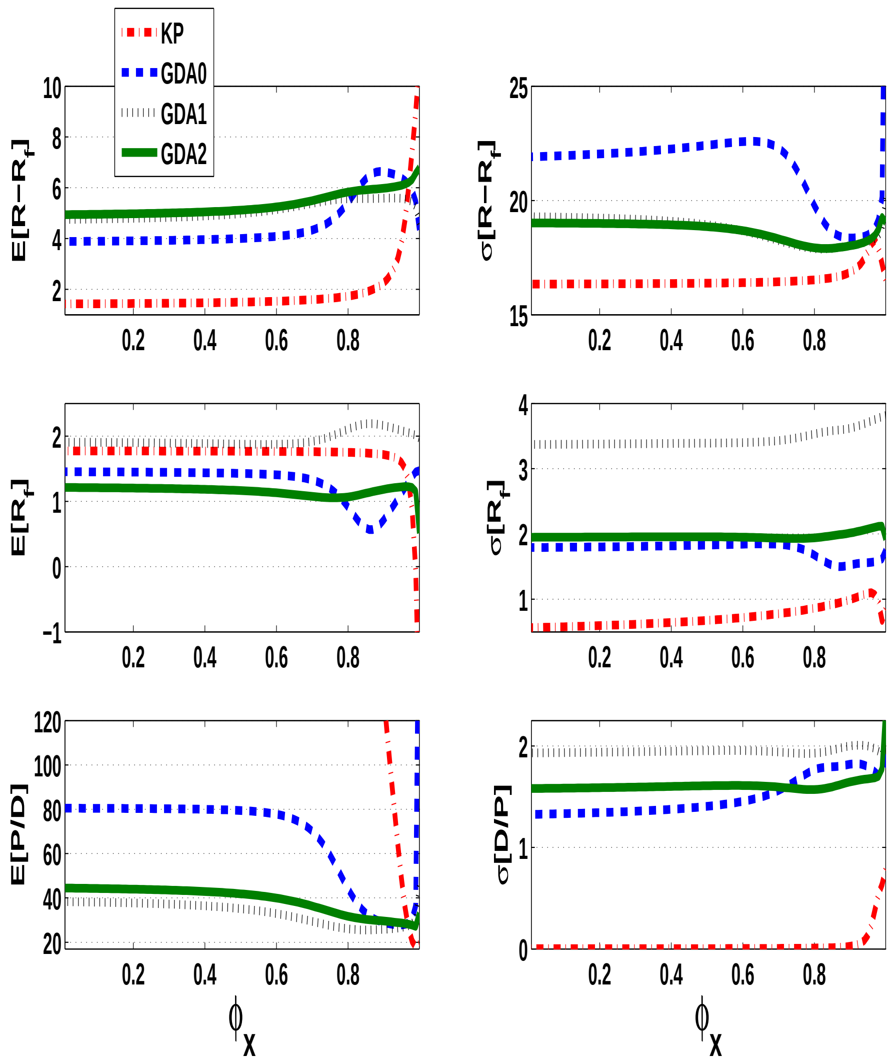

Figure 5: **(LRR) Sensitivity of Asset Prices to the Persistence of Consumption Volatility: KP and GDA.**
The figure displays population values of asset prices as functions of the persistence of consumption volatility. The expressions $E[R - R_f]$ and $E[P_d/D]$ are respectively the annualized equity premium and mean price-dividend ratio. The expressions $\sigma[R - R_f]$ and $\sigma[D/P_d]$ are respectively the annualized standard deviations of the equity excess return and the equity dividend-price ratio.

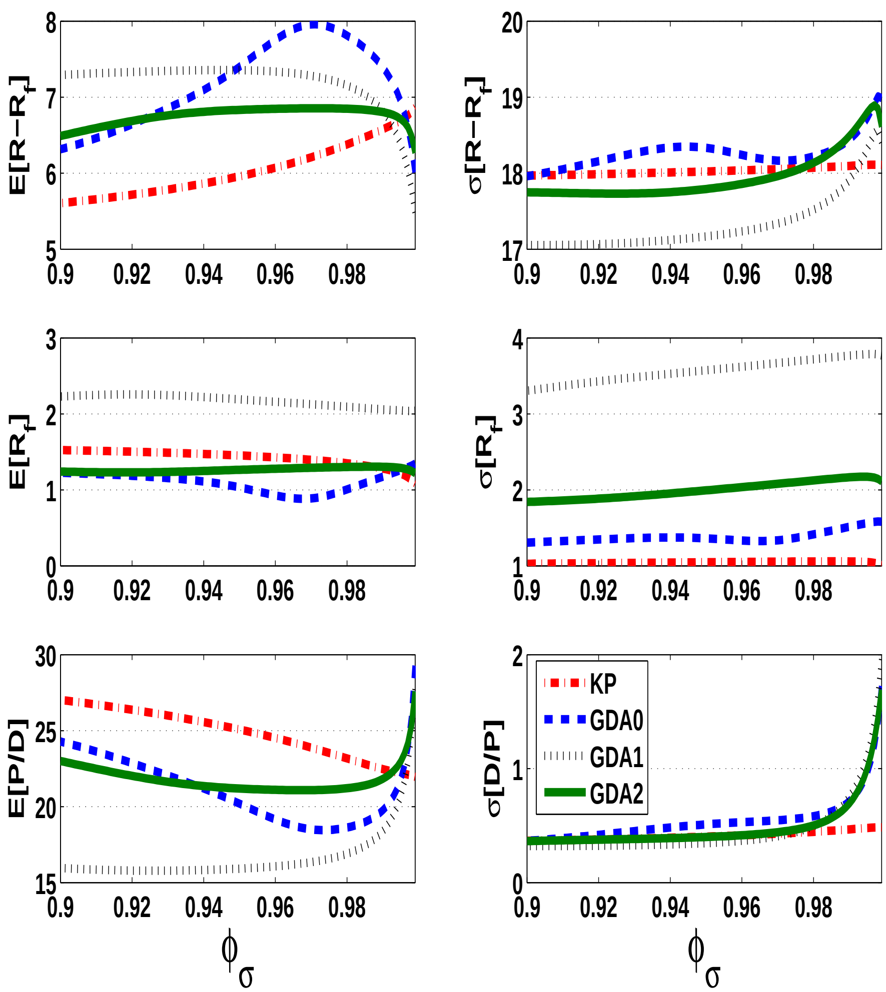

Figure 6: **(LRR) Sensitivity of Excess Return and Growth Rates Predictability to the Persistence of Expected Consumption Growth: KP and GDA.** The figure shows the population R^2 of the monthly regression $y_{t+1:t+h} = a\,(h) + b\,(h)\,\frac{D_t}{P_{d,t}} + \eta_{t+h}\,(h)$ for horizons corresponding to one year ($h = 12$), three years ($h = 36$) and five years ($h = 60$). The variable y stands for excess returns $R - R_f$ and consumption growth Δc. The R^2 is plotted as a function of the persistence of expected consumption growth.

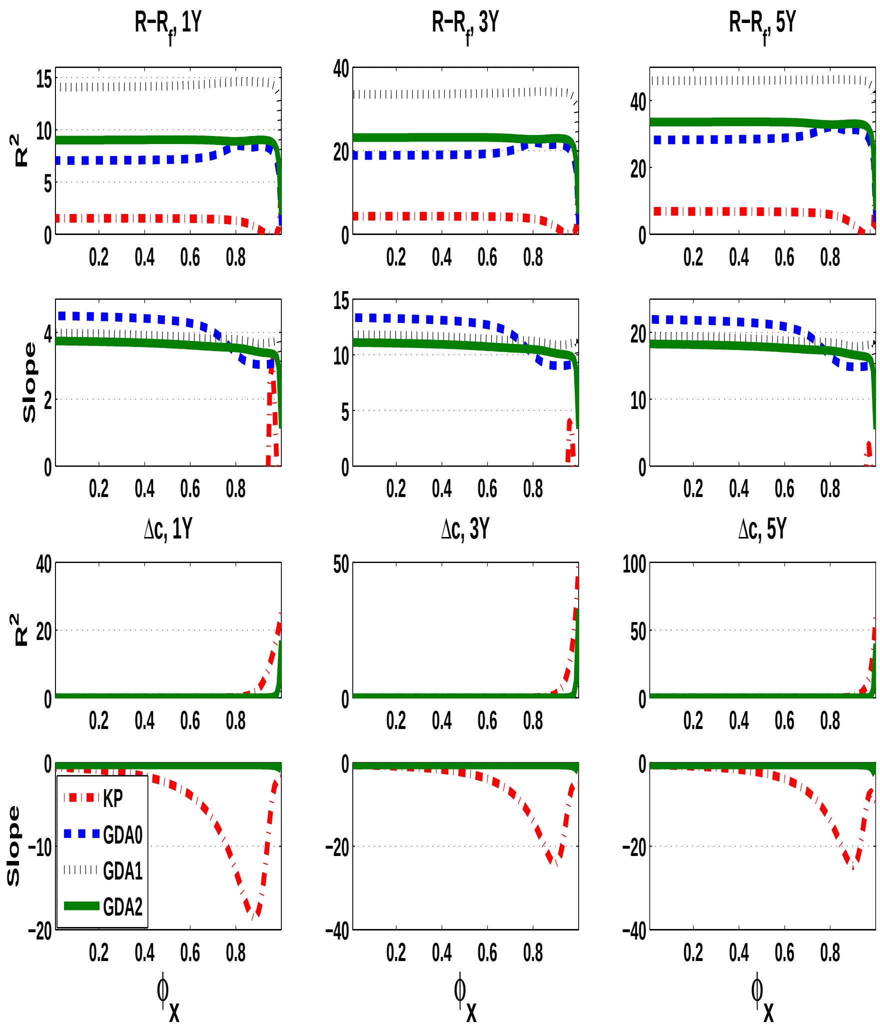

Figure 7: **(LRR) Sensitivity of Excess Return and Growth Rates Predictability to the Persistence of Consumption Volatility: KP and GDA.**
The figure shows the population R^2 of the monthly regression $y_{t+1:t+h} = a(h) + b(h)\frac{D_t}{P_{d,t}} + \eta_{t+h}(h)$ for horizons corresponding to one year ($h = 12$), three years ($h = 36$) and five years ($h = 60$). The variable y stands for excess returns $R - R_f$ and consumption growth Δc. The R^2 is plotted as a function of the persistence of consumption volatility.

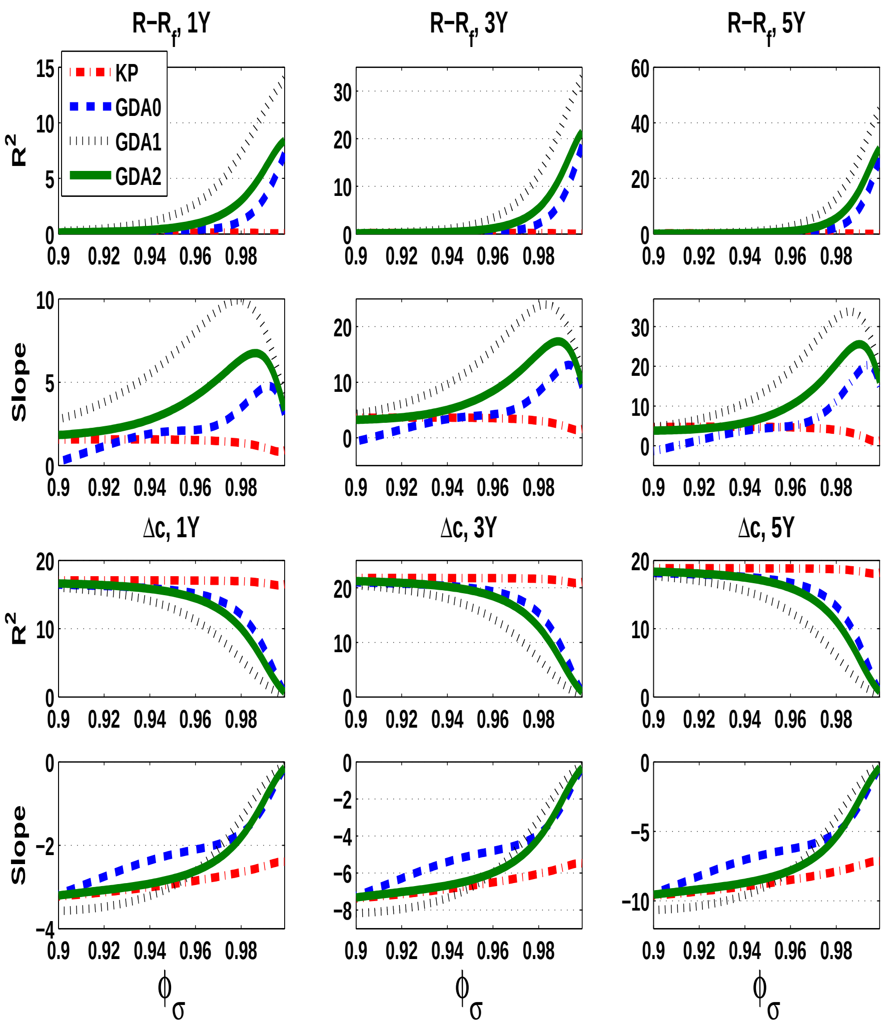

Figure 8: **(LRR) Equity Premium, Risk-Free Rate and Valuation Ratio,** $\gamma = 2.5$
The figure displays population values of asset prices. The expressions $E[R^e]$, $E[R_f] - 1$ and $E[P_d/D]$ are respectively the annualized equity premium, mean risk-free rate and mean price-dividend ratio.

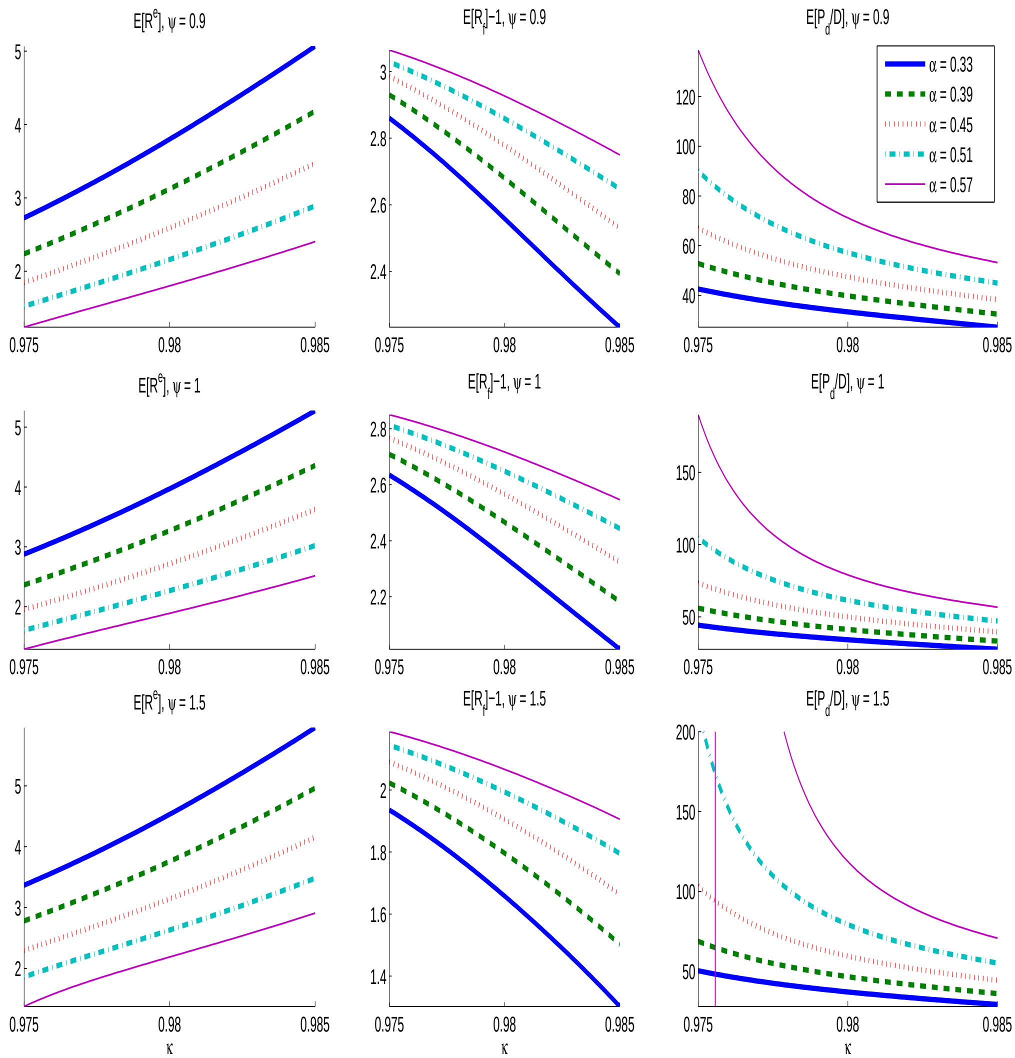

Figure 9: **(LRR) Predictability of Excess Returns (R^2),** $\gamma = 2.5$
The figure shows the population R^2 of the monthly regression $R^e_{t+1:t+h} = a\,(h) + b\,(h)\,\frac{D_t}{P_{d,t}} + \eta_{t+h}\,(h)$ for horizons corresponding to one year ($h = 12$), three years ($h = 36$) and five years ($h = 60$).

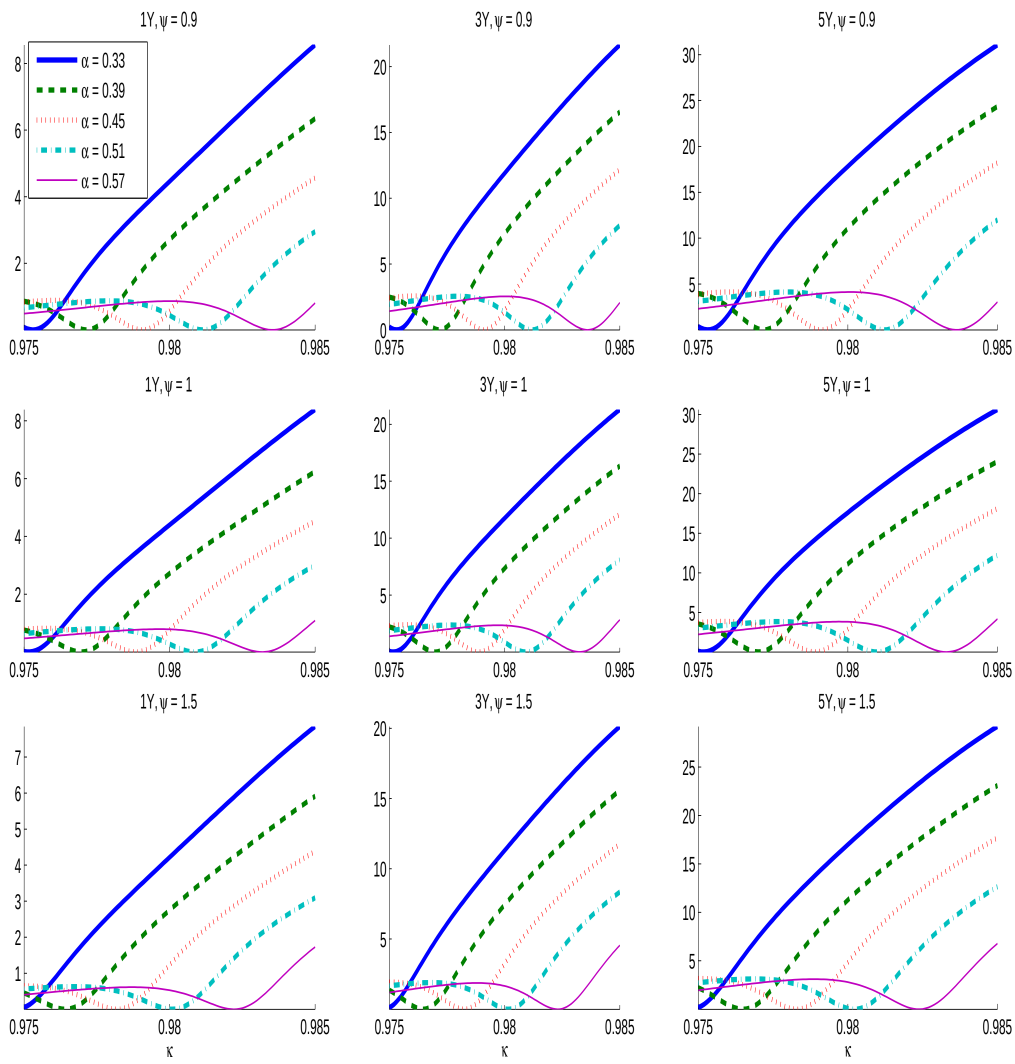

Figure 10: **(RW) Sensitivity of Asset Prices to the Persistence of Consumption Volatility: KP and GDA.**
The figure displays population values of asset prices as functions of the persistence of consumption volatility. The expressions $E[R - R_f]$ and $E[P_d/D]$ are respectively the annualized equity premium and mean price-dividend ratio. The expressions $\sigma[R - R_f]$ and $\sigma[D/P_d]$ are respectively the annualized standard deviations of the equity excess return and the equity dividend-price ratio.

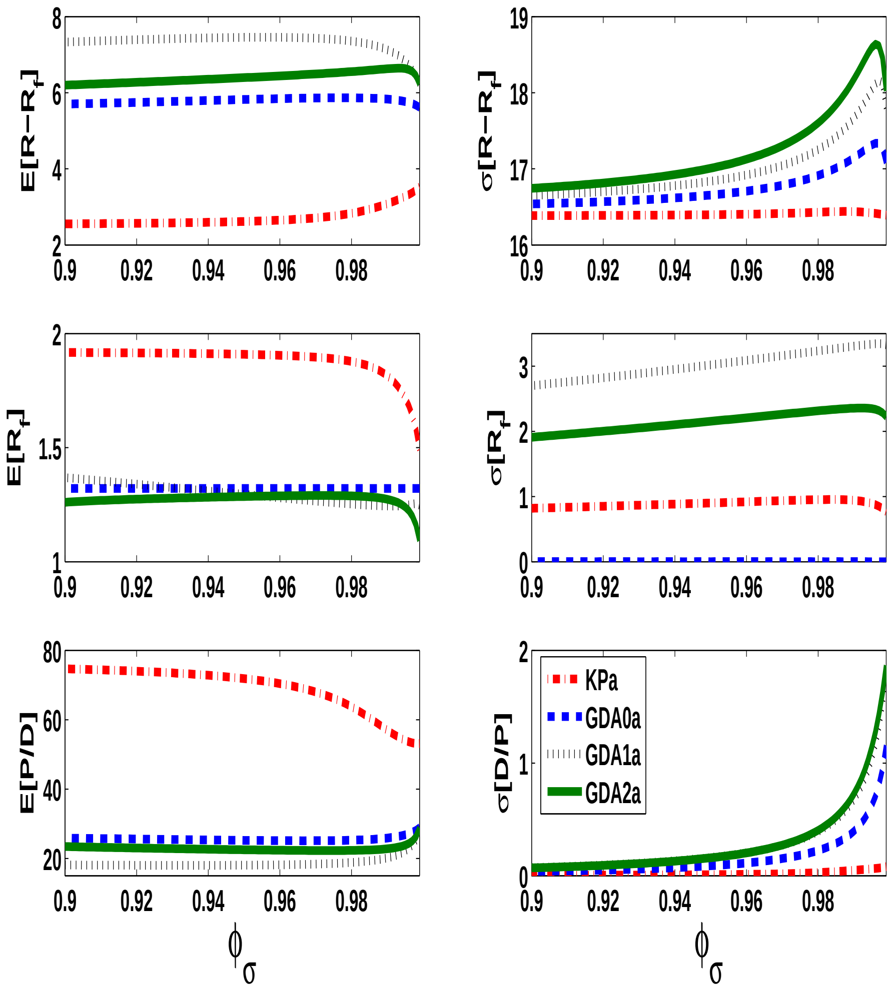

Figure 11: **(RW) Sensitivity of Excess Return Predictability to the Persistence of Consumption Volatility.**

The figure shows the population R^2 of the monthly regression $y_{t+1:t+h} = a(h) + b(h) \frac{D_t}{P_{d,t}} + \eta_{t+h}(h)$ for horizons corresponding to one year ($h = 12$), three years ($h = 36$) and five years ($h = 60$). The variable y stands for excess returns $R - R_f$. The R^2 is plotted as a function of the persistence of consumption volatility.