Discussion of:
Disasters Implied by Equity Index Options

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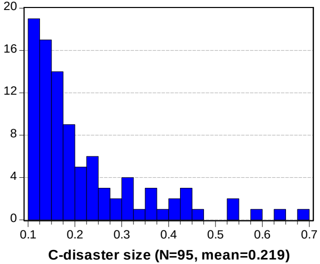
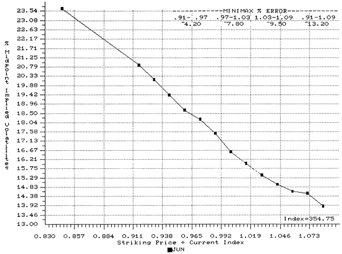
What is the question?

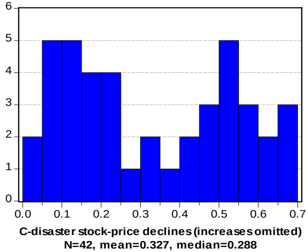
- Is the skew observed in equity index options consistent with Barro-Rietz style consumption disasters required to explain the equity premium in consumption based models with standard preferences and iid consumption, and vice versa?

What is the question? (cont)

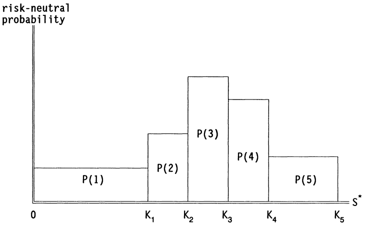




versus



Barro and Ursua (2008)



Rubinstein (1994)

The answer (in my interpretation)

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- Disaster models and options data have some striking commonalities
  - Both imply negatively skewed consumption growth or positively skewed stochastic discount factors (pricing kernels)
  - Both exhibit similar "moderate" tail behavior

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$$\text{Prob}[3 \text{ std jump}] \simeq 1\%$$

- But there are some important differences
  - Disaster models require much larger extreme tail events

$$\text{Prob}_{\text{Disaster}}[5 \text{ std jump}] \simeq 0.8\% \quad \text{versus} \quad \text{Prob}_{\text{Options}}[5 \text{ std jump}] \simeq 0\%$$

  - Options data imply heavily state-dependent preferences to explain large SRs from selling deep out-of-the-money puts

What is new?

- Combining consumption-based models with options data is not new
  - There are lots of papers that try to use consumption-based models to price options and match the implied volatility skew
  - There are also lots of papers that use options data to address macro-finance puzzles, particularly the role of volatility and jump risk in the equity premium
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- Rare events are a focal point of the options literature, so that is not new

What is new? (cont)

- The main contribution is methodological – the paper reverse engineers the higher-order moment properties of the pricing kernel required either to fit the equity premium via the disaster channel or to fit option prices
  - Focusing on the properties of the pricing kernel is nice because it serves as neutral ground between two relatively different modeling paradigms
  - Reminds me of Backus and Zin (1994), which reverse engineers the autocorrelation properties of the pricing kernel implied by term premia

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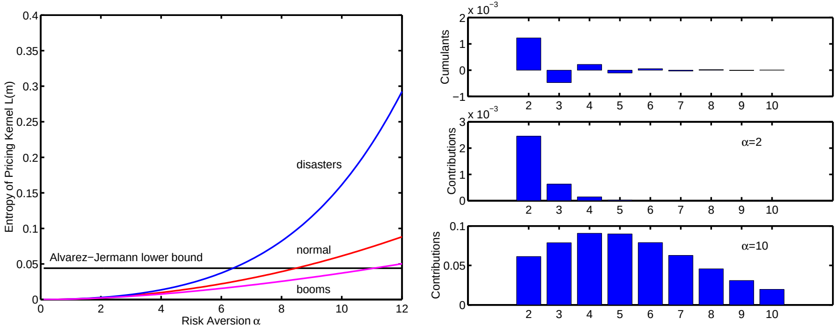
- Alvarez and Jermann (2005) derive a Hansen-Jagannathan style bound on the entropy of pricing kernel m

$$L(m) \geq \mathrm{E}[\ln r - \ln rf]$$

  which shows conceptually that the risk premium puzzle can be solved with low volatility but positive skewness of m (the Barro-Rietz point)

Disasters, risk aversion, and entropy



Option prices and entropy

Model	Entropy	Variance/2	High-Order Cumulants Odd	High-Order Cumulants Even
Normal consumption growth				
$\alpha = 2$	0.0025	0.0025	0	0
$\alpha = 5$	0.0153	0.0153	0	0
$\alpha = 10$	0.0613	0.0613	0	0
$\alpha = 10.52^*$	0.0678	0.0678	0	0
Bernoulli consumption growth				
$\alpha = 2$	0.0029	0.0025	0.0004	0.0000
$\alpha = 5$	0.0234	0.0153	0.0060	0.0021
$\alpha = 10$	0.1614	0.0613	0.0621	0.0380
$\alpha = 10, \theta = +0.3$ (boom)	0.0372	0.0613	–0.0621	0.0380
$\alpha = 10, \theta = -0.15, \omega = 0.02$	0.0765	0.0613	0.0115	0.0038
$\alpha = 6.59^*$	0.0478	0.0266	0.0147	0.0065
Poisson consumption growth				
$\alpha = 2$	0.0033	0.0025	0.0007	0.0002
$\alpha = 5$	0.0356	0.0153	0.0132	0.0071
$\alpha = 10$	0.5837	0.0613	0.2786	0.2439
$\alpha = 5.38^*$	0.0449	0.0177	0.0173	0.0099
Models fit to option prices				
Merton equity returns	0.7647	0.4699	0.1130	0.1819
Implied consumption growth	0.0650	0.0621	0.0023	0.0006

A closer look at the models

Parameter	Normal Cons Gr (1)	Bernoulli Cons Gr (2)	Poisson Cons Gr (3)	Merton Returns (4)	Implied Cons Gr (5)
Preferences					
α	10.52	6.59	5.38	—	10.07
True distribution					
μ	0.0200	0.0230	0.0230	0.0832	0.0283
σ	0.0350	0.0183	0.0100	0.1377	0.0212
ω	—	0.0100	0.0100	1.5120	1.3864
θ	—	–0.3000	–0.3000	–0.0259	–0.0060
δ	—	—	0.1500	0.0407	0.0229
Risk-neutral distribution					
μ^*	0.0071	0.0208	0.0225	0.0547	0.0238
ω^*	—	0.0680	0.0695	1.5120	1.5120
θ^*	—	–0.4210	–0.4210	–0.0482	–0.0112
δ^*	—	—	0.1500	0.0981	0.0229
Properties of distributions					
γ_1 (true)	0	–6.11	–11.02	–0.07	–0.31
γ_2 (true)	0	50.26	145.06	0.05	0.87
γ_1^* (risk-neutral)	0	–3.15	–4.33	–0.32	–0.53
γ_2^* (risk-neutral)	0	8.72	20.20	0.46	0.91
γ_1 ($\log m$)	0	6.11	11.02	–0.08	0.31
γ_2 ($\log m$)	0	50.26	145.06	2.16	0.87
Tail prob (≤ -3 st dev)	0.0013	0.0100	0.0090	0.0040	0.0086
Tail prob (≤ -5 st dev)	0.0000	0.0100	0.0079	0.0000	0.0002
Entropy					
$L(m) = L(p^*/p)$	0.0678	0.0478	0.0449	0.7647	0.0650

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- The options model has huge entropy (0.75 versus 0.05) but less severe tails
- Explanation
  - High entropy is required to explain the high SR of selling out-of-the-money puts – the Alvarez-Jermann bound holds within the model

$$L(m) \geq \mathrm{E}[\ln r_{\text{puts}} - \ln rf]$$

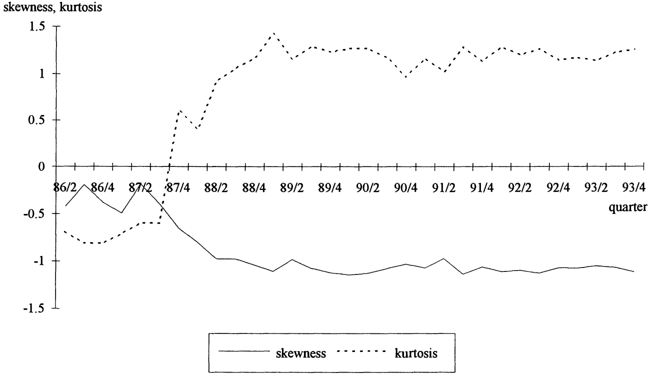
  - High entropy is generated by a large price of left-tail risk
  - Such large price of tail risk, in turn, requires state-dependent preferences, not CRRA type preferences, consistent with Bates (2008)

What's going on with the options models? (cont)

- Back to the objective of the paper – do we call this a success or failure in reconciling the the disaster models with option prices?
  - Yes, option prices do imply (too) high entropy (success!) but the economic mechanism by which option pricing models achieve high entropy is very different from the disaster models (failure?)

Taking a few steps back

- I am skeptical that option prices are consistent with Barro-Rietz



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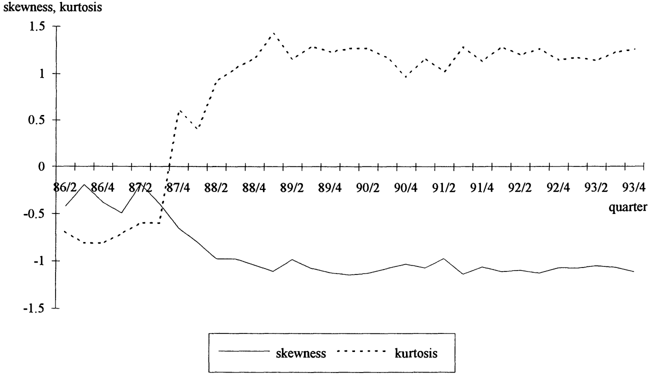
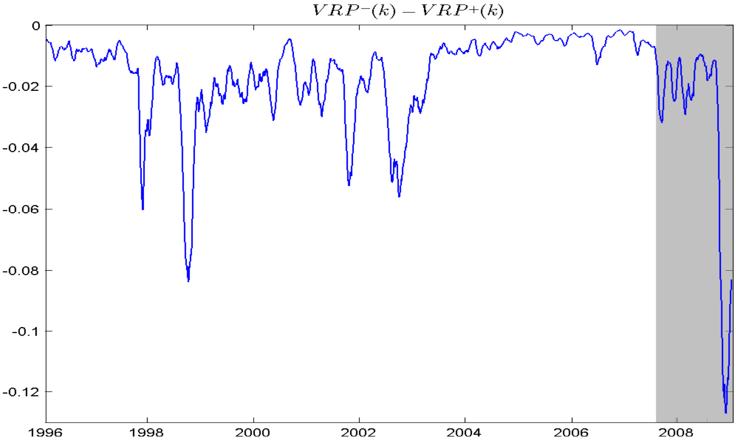


Jackwerth and Rubinstein (1996)

- Option markets learned about tail risk (or their aversion to it) in 1987
- The equity risk premium, in contrast, was just as high, if not higher, pre 1987
  - E.g., Mehra and Prescott (1985)

State dependent preferences



Bollerslev and Todorov (2009)

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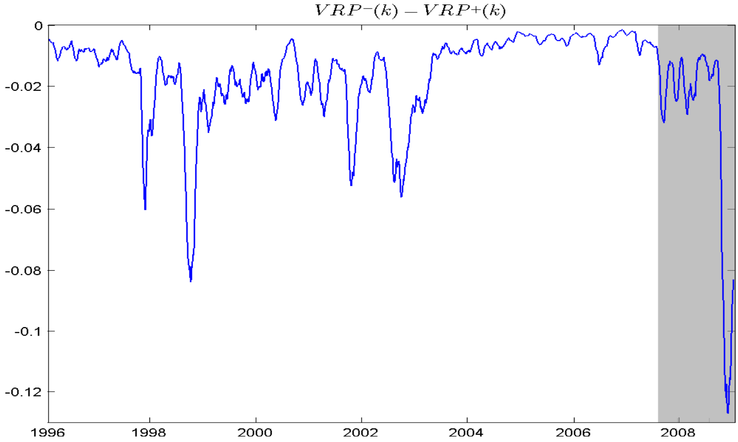

Bollerslev and Todorov (2009)

- Question: Once we have state dependent preferences, do we really still need consumption disasters to explain the equity risk premium?

Revising the HJ/AJ bounds?

- Question: With more consumption-based papers looking at options data, is it not time to change the hurtle for these models from explaining the equity risk premium (SR $\simeq$ 0.5) to explaining the risk premium earned in the option market (SR $\simeq$ 1.5?) $\Rightarrow$ pricing kernels are three times as volatile!