Monotonicity of the stochastic discount factor and expected option returns

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Abstract

This paper examines the expected returns of a family of option trading strategies on individual stocks to test whether the (projected) stochastic discount factor (SDF) is monotonic in the terminal stock price. We characterize a class of option trading strategies whose expected returns are increasing in the strike price under a monotonic SDF. Call and put options are special cases, but the set also includes butterfly spreads, bullish call spreads, and binary options. Based on our empirical results, we find that expected returns are increasing in the strike price for all the option trading strategies considered in this paper which is consistent with a monotonic SDF. The framework outlined in this paper can be used to test classes of asset pricing models like the CAPM, representative agent models with expected utility and the Black-Scholes model.

JEL classification: G12, G13, C8, C50

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1 Introduction

This paper examines the expected returns of a family of option trading strategies to test the monotonicity of the (projected) stochastic discount factor (SDF) in the terminal stock price. We characterize the entire class of option trading strategies whose expected returns are increasing in the strike under a monotonic SDF. We then examine average returns of some common option trading strategies that are special cases of this class. Using data on options on individual stocks, we find that the average returns of the option strategies are increasing in strike which is consistent with a monotonically decreasing SDF in the terminal stock price.

The relation between the stochastic discount factor and security payoffs is key to many fundamental results in asset pricing theory. Technicalities aside, the existence of a SDF is equivalent to the law of one price while the absence of arbitrage corresponds to the existence of a strictly positive SDF. In the absence of arbitrage, all asset prices can be expressed as the expected value of the product of the SDF and the security payoff. In a single period setting, the SDF, or pricing kernel is a random variable m_{t+1} which satisfies the equation

$$E_t\left[m_{t+1}x_{t+1}\right] = p_t$$

for every security with payoff x_{t+1} at time $(t+1)$ and price p_t at time t, where E_t denotes the time-t conditional expectation operator. The SDF, when used with a probability model for the states, gives a complete description of asset prices, expected returns, and risk premia.

Non-redundant asset pricing requires some equilibrium model for the SDF. For example, a principal implication of the CAPM is that the SDF is linear in a single factor, the portfolio of aggregate wealth. In a representative agent model with time-additive uitility, the SDF is the intertemporal marginal rate of substitution (IMRS) of the consumer. In such a setting, the pricing equation can be represented as the consumer's intertemporal Euler equation, which is a necessary condition for an individual consumer's optimization problem.

A negative correlation between returns and the SDF is equivalent to a positive risk premium on the security. In the case where the SDF is the aggregate IMRS, a security will earn a positive risk premium if it is positively correlated with aggregate consumption, and therefore negatively correlated with marginal utility of aggregate consumption. The intuition is that such a security is risky as it fails to deliver wealth precisely when it is most valuable to the investor. The investor therefore demands a large risk premium to hold the security. Although the relation between the risk premium and the SDF is easy to understand from the intertemporal choice problem, it can be derived simply from the absence of arbitrage, without assuming that investors maximize well-behaved utility functions.

A stronger condition than a negative correlation is monotonicity: the SDF projected on the terminal stock price is a decreasing function. We show that the monotonicity of the SDF is equivalent to negative correlation between the stock return and the SDF conditioning on the stock price falling within any interval. Alternatively, strict monotonicity of the SDF is essentially equivalent to the positivity of all "conditional" risk premium, defined as the difference in returns between the stock and unit payouts, the payouts being made conditional on the terminal stock price falling within some specified interval. In the Black-Scholes model, for example, the SDF is always monotonic, and is decreasing if and only if the risk premium is positive. Monotonicity also hold in a larger class of option pricing models.

Although testing SDF monotonicity is equivalent to testing that conditional risk premia are positive, it is difficult to conduct such a test as we would need a continuum of binary cash-or-nothing and binary asset-or-nothing contracts. Instead, we provide a much cleaner and practical way to test the monotonicity of the SDF using commonly traded option strategies. We characterize the enitre class of option strategies whose expected returns are increasing in the strike price under a monotonic SDF. The class is characterized by the property that the log-payout functions of the strategies are concave over the set of ending security prices where the payouts are positive. Call and put options are special cases, as shown in Coval and Shumway (2001), but the class also includes butterfly spreads, bullish call spreads, and binary options. One easy way to understand this characterization is that an increase in the strike price for these strategies, shifts the payoff to lower utility states, requiring a higher return to compensate. More precisely, an increase in the strike shifts the probability weighted payoffs to lower values of the SDF, resulting in lower prices and higher expected returns.

A violation of monotonicity (in the strike) of expected returns of any of the strategies implies a violation of the monotonicity of the SDF, but the converse is not true. For example, the conditional covariance conditions in Coval and Shumway (2001) guarantees that the expected returns for call and put options are increasing in strike, but they do not imply that the SDF is a decreasing function of the terminal stock price. Hence we provide a set of weaker SDF monotonicity conditions for some of the option trading strategies which implies monotonicity of expected returns in the strike. Based on our empirical results on options on individual stocks, we find that expected returns are increasing in the strike price for all the option trading strategies considered in this paper, which is consistent with monotonicity of the SDF.

Our paper contributes to the literature in the following ways. We extend the work in Coval and Shumway (2001) by providing necessary and sufficient characterization of the entire class of option strategy payoffs whose expected returns are increasing in strike when the SDF is monotonic in the terminal stock price. Tests of monotonicity of the SDF also provide a test of a family of option pricing models. The Black-Scholes model is a special case, as are models with independent stochastic volatility. Our characterization of monotonicity of the SDF can be applied to options on a proxy for the market portfolio (say the S&P 500) to test classes of models such as the CAPM (where the SDF is linear) or representative agent models with expected utility (where the SDF is strictly monotonic under concave utility). Our paper also contributes to the existing literature on viable price processes. Bick

(1990) outlines conditions under which the state price density is a decreasing function of the terminal stock price under the assumption that the state price density is a function of only the terminal stock price. This assumption is very restrictive and is not generally true (The Black-Scholes model with deterministic but time varying parameters being a simple example). Our characterization of SDF monotonicity is much more general than what is presented in Bick (1990).

Several researchers have attempted to estimate the shape of the SDF from index prices and aggregate consumption data. However, such a study is difficult to conduct using individual stocks as we would have to estimate the shape of the projected SDF on each individual stock. Moreover, the usefulness of options data is limited as the number of available strikes is much fewer for individual stocks compared to a index like the S&P 500. Our paper contributes to the literature by providing a simple framework to test properties of the projected SDF without explicitly estimating its shape.

Our paper also contributes to the empirical literature on option returns. This is one of the few papers (Ni (2007), being one of the exceptions) that analyzes expected returns of option strategies on individual stocks. The strategies that we consider in this paper include traded strategies like calls, puts and butterfly spreads, as well as non-traded strategies like binary calls[1] and bullish call spreads. To test the hypothesis that average strategy returns are increasing in the strike price, we also use a nonparametric test that we borrow from the behavioral sciences. The Page test for ordered alternatives tests the null hypothesis that groups (or measures) are same versus the alternate hypothesis that the groups (or measures) are ordered in a specific sequence. An advantage of the Page test is that it allows us to test the monotonicity of expected returns by calculating a single test statistic. We extend the original Page test statistic to allow for the fact that different underlying stocks have options with varying number of strikes. The procedure outlined for implementing the Page

[1] CBOE has introduced binary options on SPX and VIX indices starting from July 2008

test is quite general and can be applied to any finance problem where a variable of interest is expected to be increasing across multiple, but related, samples.

Finally, this paper uses a very simple method for calculating skewness-adjusted t-statistics and standard errors. The distribution of option returns are extremely skewed as out of the money expirations generate a return of -100%. Central limit approximations are problematic because of the irregular nature of option return distributions and usual standard errors and t-statistics cannot be relied upon for assessing the significance of option returns. The skewness-adjusted t-statistic was developed in Chen (1995) and has shown to be quite accurate for small sample sizes and for distributions as asymmetric as the exponential distribution.

The remainder of the paper is organized as follows. In Section 2, we summarize the prior relevant literature. In Section 3, we derive theoretical results regarding the SDF and expected returns. In Section 4, we discuss our dataset, In Section 5, we test the implications developed in Section 3. In Section 6 we provide some robustness tests. Section 7 concludes the paper.

2 Previous Research

Academic research in options and other forms of derivative assets has grown exponentially in the last few decades. A part of this interest in derivative research can be attributed to the gradual growth in the derivatives market. A significant part, however, can be attributed to the seminal work on options by Black-Scholes and Merton. Hence the Black-Scholes model would be an appropriate starting point when relating our work to the existing literature.

In the constant-parameter setting of Black-Scholes option pricing framework where asset prices follow a Geometric Brownian Motion (GBM), the SDF (projected on the terminal stock price) is a strictly decreasing function of the stock price if and only if the risk premium of the stock is positive. In such a setting, the instantaneous excess expected rate of return (over the risk free rate) of a option is equal to the product of the option's elasticity with respect to the underlying price times the instantaneous excess expected return of the underlying asset. In this framework, it is straightforward to see that expected call and put returns should be increasing in the strike price provided the risk premium of the stock is positive. Morever, expected call returns should never be lower than the return on the underlying stock and expected put returns should never be higher than the risk free rate. The above results can easily be extended to any general one factor model. In particular the results would hold if the volatility of the underlying stock is a function of the price of the underlying stock.

Coval and Shumway (2001) show that the above result hold under much weaker conditions than implied by the Black-Scholes model. They show that under no arbitrage conditions as long as the covariance of the SDF and the terminal stock price, conditional on the option being in the money, is negative for all strikes, expected option returns would be increasing in strike. Empirically, they show that the above restrictions are satisfied by options on the S&P 500 and S&P 100 indices. On the other hand, Ni (2007) studies option returns on individual stocks and shows that deep out-of-the-money (OTM) calls earn significantly lower average returns than deep in-the-money (ITM) calls. She sorts call option contracts on each buying date into portfolios based on the option's moneyness and shows that the portfolio with OTM options earns significantly lower returns than the portfolio with ITM option contracts. She concludes that, unlike index options, individual stock options do not conform to the monotonicity restrictions on call returns. She argues that investors are sometimes risk-seeking and a preference for idiosyncratic skewness leads to a premium for deep OTM options and is a possible explanation for the puzzling call returns.

The assumption that the SDF is negatively correlated with ending security prices is a fundamental concept in any asset pricing theory, the intuition being that the SDF should be high in bad states of the world and low in good states of the world. A sufficient condition for this covariance to be negative is that the projection of the SDF on the ending level of the security price is a monotonically decreasing function of the security price. Several researchers have attempted to recover the stochastic discount factor from security prices (e.g. Hansen and Singleton (1982), Hansen and Singleton (1983) and Chapman (1997)). Jackwerth and Rubinstein (1996), Jackwerth (1997), Brown and Jackwerth (2004) and Ait-Sahalia and Lo (2000) provide empirical procedures for estimating a stochastic discount factor(SDF) from option prices when a finite number of options exist instead of a dense set. Option data is particularly helpful in this context because it is possible to develop a model-free option based estimator that does not depend on a particular specification of consumer preferences or asset price stochastics.

Brown and Jackwerth (2004) estimate the SDF by recovering the risk neutral probability distribution from option prices on the S&P 500 index and the subjective distribution from actual index returns. They estimate the SDF as a function of the ending level of the S&P 500 index and show that overall, the SDF is a decreasing function of the ending index level. However, for index levels approximately ranging from 0.97 to 1.03 (a 3% deviation from the current level), the SDF is increasing. They argue that such a shape of the SDF function is inconsistent with traditional asset pricing framework as this would suggest that the representative agent is locally risk seeking and the utility function is non-concave or locally convex. They attempt to resolve this puzzle by introducing additional state variables. Rosenberg and Engle (2002) analyze the empirical characteristic of investor risk aversion over equity return states by estimating a time-varying SDF. They argue that the state prices and probabilities in Jackwerth (1997) are averaged over time, so their estimates can be intepreted as a measure of the average SDF over the sample period. Thus assets are correctly priced only when risk aversion and state probabilities are at their average level.

Bick (1990)provides necessary and sufficient conditions for viability of diffusion price processes. The paper identifies conditions under which the associated utility function to any diffusion price process is consistent with an equilibrium. In particular, assuming that the state price density (SPD) is only a function of the terminal stock price, the paper identifies conditions under which the SPD is monotonic in the terminal stock price. As discussed in the previous section, this assumption turns out to be quite restrictive for many simple asset pricing models.

Our work contributes to the strands of literature discussed above. Our paper extends the work in Coval and Shumway (2001) by characterizing the entire class of option trading strategies whose expected returns are increasing in the strike price under a monotonic SDF. On the other hand, our paper is related to the huge literature that attempts to estimate the shape of the SDF from option prices. We do not try to estimate the shape of the SDF directly, but instead use expected returns of a set of option strategies to test for violations of the monotonicity of the SDF in the terminal stock price. Our paper also extends the work in Bick (1990) as it provides characterizations of SDF monotonicity under much general conditions. The next section characterizes the class of option trading strategies that we use to test the monotonicity of the SDF and also derives necessary and sufficient conditions for the monotonicity of the SDF.

3 Theoretical Results

3.1 Monotonicity of the SDF

Lemma 1 *Let $\sigma(S_T)$ denote the information set generated by the strictly positive terminal stock price S_T. For any $A \in \sigma(S_T)$ satisfying $P(A) > 0$, let P_A^1 and P_A^S denote the prices of payouts 1 and S_T, respectively, in the set A:*

$$P_A^1 = E(m1_A), \quad P_A^S = E(mS_T 1_A);$$

and let $R^1 = 1/P_A^1$ and $R^S = S_T/P_A^S$ denote the returns on set A. Furthermore, let m denote the stochastic discount factor, and $g(S_T) = E(m|S_T)$ its projection on S_T.

a) If g is continuous and strictly decreasing then $E\big(R^S - R^1_A \big| A\big) > 0$ for all $A \in \sigma(S_T)$ satisfying $P(A) > 0$.

b) If g is continuously differentiable, and the distribution function of F is strictly increasing on $(0, \infty)$, then g is strictly decreasing if and only if $E\big(R^S - R^1_A \big| S_T \in [\alpha, \beta]\big) > 0$ for all $0 \leq \alpha < \beta$.

Proof. See appendix ■

Lemma 1 states that the strict monotonicity of the SDF is essentially equivalent to the positivity of all "conditional" risk premia, defined as the difference in returns between the stock and unit payouts, the payouts being made conditional on the terminal stock price falling within some specified interval. Alternatively, a strictly decreasing SDF would imply that the risk premium of the stock is positive for all possible return distributions. Lemma 1 provides a good understanding of the monotonicity of the SDF with respect to the risk premium, but testing the conditional risk premium empirically is difficult with traded securities.

We instead test the monotoncity of the SDF in a much cleaner way by characterizing a class of option strategies whose expected returns are increasing in the strike price under a monotonic SDF. A violation in the monotonicity of expected returns in the strike price implies a violation of the monotonicity of the SDF, but the converse is not true. Hence we provide weaker conditions for some of the option strategies which implies monotonicity of expected returns in the strike. The theoretical results are presented in the next subsection.

3.2 Theoretical Expected Returns

Let S_T denote the stock price at expiration, and interpret the constant (shift parameter) $K \geq 0$ as the strike price. We characterize the class of functions $G()$ such that the expected return corresponding the payout $G(S_T - K)$ is increasing in the strike for all stock-price distribution functions and all monotonically decreasing stochastic discount factors (that is, for all stochastic discount factors whose projection on S_T are monotonically decreasing functions of S_T) We then consider some common option trading strategies that are special cases, which we use to test weaker forms of monotonicity.

Without loss of generality, we assume throughout the following regularity condition of G:

Condition 1 *Assume $G(x) > 0$ for $x \in (x_1, x_2)$, where $-\infty \leq x_1 < x_2 \leq \infty$, and $G(x) = 0$ for $x \notin [x_1, x_2]$. Furthermore, $G(x)$ is continuous for $x \in (x_1, x_2)$ and $G'(x)$ is piecewise continuous for $x \in (x_1, x_2)$.*

We let m denote a stricly positive stochastic discount factor for pricing time-T payouts, and let $m(s) = E(m|\, S_T = s)$, $s \in [0, \infty)$ denote the projection of m on the terminal stock price.

Proposition 1 *The expected return*

$$R(K) = \frac{E\{G(S_T - K)\}}{E\{m(S_T)\,G(S_T - K)\}}$$

is increasing in K for all distribution functions F (satisfying $E\{G(S_T - K)\} \neq 0$) and any monotonically decreasing $m()$ if and only if

$$\frac{G'(x)}{G(x)} \text{ is decreasing in } x \text{ for all } x \in (x_1, x_2)\,. \tag{1}$$

Proof. See Appendix ■

Proposition 1 says that if the projected stochastic discount factor is monotonically decreasing in the terminal stock price, then concavity of the logarithm of the payoff function is equivalent to monotonicity of the expected return in the strike for all return distribution functions. The concavity of the log-payoff condition implies that increases in the strike shifts (in the first-order dominance sense) the probability-weighted payoffs to higher stock prices; the lower values of the SDF corresponding to higher stock prices implies a lower price (per unit probability-weighted payoff) and therefore a higher expected return.

The following three examples present exchange-traded strategies with payoffs that satisfy the condition in equation (1). Monotonicity of $m()$ therefore implies monotonicity of expected returns in the strike price. Conversely, monotonicity of expected returns in the strike price in each case implies a weaker form of monotonicity.

Example 1 (Call options) *The payoff function is $G(x) = x^{+}$. Coval and Shumway (2001) show that the derivative of the expected return with respect to the strike is proportional to the conditional covariance between the stock price and the SDF on the set where the option expires in the money:*

$$R'(K) = -\left(\frac{P(S_T > K)}{E\left\{m\cdot (S_T - K)^{+}\right\}}\right)^2 Cov\left(S_T, m \mid S_T > K\right).$$

Therefore monotonicity of expected returns in the strike implies negative instantaneous conditional covariances.

Example 2 (Put options) *The payoff function $G(x) = (-x).^{+}$ Analogous to the call case, we get from Coval and Shumway (2001)*

$$R'(K) = -\left(\frac{P(S_T < K)}{E\left\{m\cdot (K - S_T)^{+}\right\}}\right)^2 Cov\left(S_T, m \mid S_T < K\right).$$

Example 3 (Butterfly spread) *A strategy of long 1 call each at strike prices $K - \Delta K$ and $K + \Delta K$, and short two calls at strike K has the payoff function $G(x) = (\Delta K - |x|)^{+}$.*

The expected return is

$$R(K) = \frac{E\{(\Delta K - |x|)^+\}}{E\{m(S_T)(\Delta K - |x|)^+\}}$$

The inverse of the expected return is a weighted average of the stochastic discount factor, with the weights proportional to the product of the density and payoff functions and the result follows directly from Proposition 1

The following examples also satisfy the concavity condition in equation (1), but are not exchange-traded.

Example 4 (Binary cash-or-nothing option) *The payoff function for the call version is* $G(x) = 1_{\{x \geq 0\}}$ *The expected return is*

$$R(K) = \frac{P(S_T \geq K)}{E\left\{m1_{\{S_T \geq K\}}\right\}} = \{E(m \mid S_T \geq K)\}^{-1},$$

and is therefore increasing in K *if and only if* $E(m \mid S_T > K)$ *is decreasing in* K*, which is equivalent to*

$$m(K) > E(m \mid S_T \geq K) \text{ for all } K \geq 0.$$

The payoff function of the put version is $G(x) = 1_{\{x \leq 0\}}$*, and the expected return is increasing in* K *if and only if*

$$m(K) < E(m \mid S_T \leq K) \text{ for all } K \geq 0.$$

If the expected returns of vanilla call options, and the expected returns of binary cash-or-nothing call options are both increasing in the strike, then

$$m(K) > E(m \mid S_T \geq K) > \frac{E(m \cdot (S_T - K) \mid S_T \geq K)}{E(S_T - K \mid S_T \geq K)} \text{ all } K.$$

The call-option-return test puts much more weight on the right tail of the distribution than the binary test. For example, if $m(s)$ is not monotonic, but declines sharply as s becomes large, then call option returns may be increasing in the strike, but the binary calls may not.

Example 5 (Modified bullish call spread) *Consider a portfolio that is long a call with strike K, short a call with strike $K + \Delta K$, where $\Delta K > 0$, and short a cash-or-nothing binary call with payoff $\Delta K \cdot 1_{\{S_T > K + \Delta K\}}$ (all on the same stock and same expiration T). The portfolio payoff is $(S_T - K)\, 1_{\{S_T \in [K, K+\Delta K]\}}$, the expected return of the portfolio is*

$$R(K) = \frac{E\left\{(S_T - K)\, 1_{\{S_T \in [K, K+\Delta K]\}}\right\}}{E\left\{m\,(S_T - K)\, 1_{\{S_T \in [K, K+\Delta K]\}}\right\}},$$

and the derivative with respect to K is

$$R'(K) = -\left(\frac{P\left(S_T \in [K, K+\Delta K]\right)}{E\left\{m\cdot (S_T - K)\, 1_{\{S_T \in [K, K+\Delta K]\}}\right\}}\right)^2 Cov\left(S_T, m\,\left|S_T \in [K, K+\Delta K]\right.\right)$$

As the interval shrinks to zero, $R'(K)$ becomes proportional to the slope of $m()$:

$$\lim_{\Delta k \downarrow 0} \frac{Cov\left(S_T, m|\ S_T \in [K, K+\Delta K]\right)}{Var\left(S_T|\ S_T \in [K, K+\Delta K]\right)} = m'(K)\,.$$

The final example presents a payoff function which results in an expected return invariant to K for all distribution functions F

Example 6 *(Strike-Invariant Expected Return) Suppose $G(x) = \exp(-kx)$, some $k \in \mathbb{R}$. Then the expected return (when it exists) is invariant to K for any distribution function F.*

4 Data

The data on options are from the OptionMetrics Ivy DB database. The dataset contains information on the entire US equity option market from 1996 to 2008 and includes daily volume, open interest, best daily closings bid and ask quotes, option Greeks and implied volatilities. The implied volatilities and Greeks are calculated using a binomial tree model developed by Cox, Ross, and Rubinstein (1979). The data set also includes information on daily prices, returns and distribution of all exchange traded stocks.

We select option contracts that satisfy the following criteria. Since the monotonicity restrictions on options are applicable to European style options, we restrict our analysis to only those option contracts for which the underlying stock do not have an ex-dividend date during the remaining life of the contract.[2] It is well known that it is never optimal to exercise non-dividend paying American style call options early. American put options however might be optimally exercised early irrespective of whether the underlying stock pays dividend or not.

In accordance with the standard practice in empirical option studies, we choose only those call and put option contracts for which the bid price is greater than or equal to $0.125. We also eliminate contracts for which the recorded ask price is lower than the bid price.[3] The arbitrage bound filter that we use requires that option prices, estimated as the bid-ask midpoint, should be greater than $S - Ke^{-r\tau}$ for calls and $Ke^{-r\tau} - S$ for puts,where S is the price of the underlying asset, K is the options strike price, r is the risk free rate and τ is the time to expiration. In an unreported robustness check, our results hold when we also run our tests on option contracts that satisfy the put-call parity bounds. That is, the bid and ask prices should satisfy $C_{bid} - P_{ask} \leq S - Ke^{-r\tau} \leq C_{ask} - P_{bid}$, where C and P are the call and put prices respectively. On each expiration Friday from January, 1996 to June, 2005, we first identitfy option contracts that expire on the next expiration Friday. We then use prices observed on Tuesdays to calculate weekly returns for the option contracts identified in the first step. Our initial sample consists of 2,123,004 call returns and the sample reduces

[2]It is not difficult to adjust the returns for the case when we have an ex-dividend date prior to maturity. We will include them in our next version.

[3]A significant violation is observed in the data.

to 1,643,925 after all the restrictions have been applied. The mean (median) number of unique stocks on each buying date is 1380 (1421) and the mean (median) number of strikes for each stock on each buying date is 3.43 (3.00). Since deep in-the-money (ITM) and deep out-of-the-money (OTM) options are thinly traded we note the possibility of prices being estimated from stale quotes. As a robustness test we repeat our analysis with only those option contracts which have a positive volume on the buying date. The number of calls in our sample that have a positive volume on the buying date is 598,826.

5 Empirical Results

5.1 Average returns

We consider some common option trading strategies that are special cases of the particular class of payoff functions for which the logarithm of the payoff function is concave. We find results in support of the hypothesis that the average return of these option strategies are monotonic with respect to shifts in the payoff of the strategey induced by an increase in strike, except for put options for which the evidence is somewhat weak. The results are reported in Table 1.

A Call options

To test the monotonicity restriction on call returns, we first divide the calls into strike groups based on the difference between their strike prices and the strike price of the call which is closest to being at the money. In particular, for any underlying stock on any buying date we identify the call option contract that is closest to being at the money and assign a strike group value of 3. The next two higher strikes and the two lower strikes are assigned strike group values of 4, 5 and 2, 1 respectively. Note that the option contracts are first divided into strike groups before any filters are applied to the data. Excess returns for any particular strike group are calculated by taking the return differences between the return of an option in a particular strike group and the return in the immediate lower strike group. Returns for strike group 1 are represented in excess of the return on the underlying stock for the same holding period. Since a non-dividend paying stock is essentially equivalent to a zero-strike call option, excess returns for strike group 1 can be thought of as being calculated in excess of the return on a call with the lowest possible strike.

The return differences for each strike group are averaged across different stocks to form a weekly time series of average return differences. The average of the weekly time series and the t statistics are reported in Table 1. We find that the average return differences for call options are positive and statistically significant suggesting that average call returns are monotonically increasing in the strike price.

Table 1 also provides a skewness-adjusted t-statistic for the average return differences. The distribution of call option returns are extremely skewed as OTM expirations generate a return of -100%. Central limit approximations are problematic because of the short sample size and irregular nature of option return distributions. Thus usual standard errors and t statistics cannot be relied upon for assessing the significance of option returns. We use a very simple method for calculating skewness-adjusted standard errors and t-statistics. This statistic for testing the mean of positively skewed distributions was developed by Chen (1995) and is an extension of Johnson (1978) and Sutton (1993). It is derived using Hall's t-type inversion of the Edgeworth expansion and has been shown to be quite accurate for sample sizes as small as 13 and for distributions as asymmetric as the exponential distribution. Further details on this test statistic are provided in the appendix. The skewness-adjusted t statistic would provide estimates closer to the usual t statistics if the distribution is close to normal. As one would expect, the adjusted t statistics estimate for out of the money calls differ significantly from the usual t statistic. This is because the return distribution for OTM calls is much more positively skewed compared to the return distribution for ATM or

ITM calls.

B Put options

The possibility of early exercise complicates analysis of put option returns. For American put options early exercise might be optimal even if the underlying stock pays no dividends. Optimal stopping theory implies that it is optimal to exercise a put option at a given time if and only if the market price and the intrinsic price are identical. Under complete markets and additional regularity conditions, it is suboptimal to not exercise at the first instance when the market price coincides with the intrinsic value of the put options [See Duffie, Liu, and Poteshman (2005)]. In practice, American put option should always be exercised early if it is sufficiently in the money.

Several researches have attempted to estimate this early exercise premium for puts using different valuation approaches. Engstrom and Norden (2000) uses Swedish equity options data and finds a substantial early exercise premium for American put options. The premium increases with moneyness and time left to expiration, while the effect of volatility and interest rates is dependent on the moneyness of the option [4]. In our analysis of average put option returns we believe that early exercise premium should be small because of the short maturity of our option contracts and low interest rates for the period.

Although, the monotonicity test result from Table 1 for put options is somewhat weak, it is still consistent with our theoretical predictions. In-the-money puts earn significantly higher average returns than at-the-money and out-of-the-money puts. Moreover, the Page test and the elasticity test to be discussed later also provide strong evidence in support of the monotonicity hypothesis.

To adjust the expected returns to account for an early exercise premium, we would fisrt have to estimate the critical stock price boundary below which early exericse would be

[4]This is surprising since the early exercise premium should be increasing in the interest rate.

optinal. Using the idea behind the representation in Theorem 1 in Carr, Jarrow, and Myneni (1992), the gross return on an American put option over the interval $(t, t + \Delta t)$, including any interest earned on the strike over any subinterval in which early exercise is optimal is

$$\frac{P(S_{t+\Delta t}, t + \Delta t) + rK \int_t^{t+\Delta t} e^{r(t+\Delta t-u)} 1_{\{S_u \leq B_u\}} du}{P(S_t, t)}$$

where r is the short rate, and B_t is the critical time-t stock price below which early exercise is optimal at t (we assume that the stock price is the only source of uncertainty). A future version of this paper would have some results on put returns adjusted for early exercise using the above method.

C Butterfly spreads

A butterfly spread with call options is a strategy that is created by taking a long position in a call with strike price $(K - \Delta K)$, a long position in a call with strike price $(K + \Delta K)$, and a short position in two calls with strike K. To calculate the return differences for the butterfly spreads, we impose the following restrictions: 1) There should exist at least four valid strikes for any underlying stock on any buying date to estimate at least one return difference. 2) The estimated price of the spread must be greater than \$0.01 to reduce the possibility of unrealistic returns [5] and 3) The value of ΔK should be the same across all butterfly spreads on the same underlying stock and the same buying date. We allow the spreads to have overlapping strikes, that is, the middle and right strike of a spread could be the left and middle strike respectively of the next higher spread. Results are stronger if we restrict the butterfly spreads to have non-overlapping strikes and/or the value of ΔK is increased but this comes at a cost of losing a lot of observations. The total number of valid butterfly returns after the restrictions on the calls and the spreads are imposed is 192,823.

[5] Although convexity in option prices should ensure that the price of a butterfly spread be always positive, a significant violation is observed in the data

D Binary call options

Estimating average return differences for non-exchange traded securities is slightly more involved. Note that the cash-or-nothing binary option pays a fixed amount of cash if the option expires in-the-money and nothing otherwise. The binary options are not exchange traded [6] and so we estimate their prices by using a smooth implied voltility curve estimated using calls on all available strikes on the same underlying stock and same trading day. Let $B(K)$ denote the price of the binary option with strike K that pays a dollar if it expires in the money.

$$B(K) = E\left\{m1_{\{S_T>K\}}\right\}.$$

In general,

$$\frac{d}{dK}C(K) = \frac{d}{dK}E\left\{m\cdot(S_T - K)\,1_{\{S_T>K\}}\right\} = -B(K).$$

If the short rate is deterministic, $B(K)$ is the product of the risk-neutral probability of the call being in the money, $P^{rn}(S_T > K)$, and the discount factor.

Because strikes are not dense enough, a better way to compute $\frac{d}{dK}C(K)$ (rather than differencing premiums at different strikes and scaling) is to estimate a smooth Black-Scholes implied volatility curve $\sigma(K)$ and then estimate $B(K)$ from the B-S model in the following way. Assuming a deterministic short rate, and letting r denote the annualized continuously compounded zero yield from 0 to T (on a discount bond), then (assuming a smooth B-S implied volatility curve $\sigma()$):

$$B(K) = -\frac{d}{dK}C^{BS}(S, K, \sigma(K)) = -C_K^{BS}(S, K, \sigma(K)) - \frac{d\sigma(K)}{dK}C_\sigma^{BS}(S, K, \sigma(K))$$

[6] CBOE has introduced binary options on SPX and VIX indices from July 2008

Substituting, we get

$$B(K) = e^{-rT} \left\{ N(d_2) - KN'(d_2)\sqrt{T}\sigma'(K) \right\}. \tag{2}$$

The no-arbitrage bound on the binary call options are calculated as follows: If there is no arbitrage (and a continuum of strikes), the European calls must satisfy

$$\frac{dC}{dK} \in \left(-e^{-rT}, 0\right).$$

implying that the prices of the binary calls should lie in the interval $\left(0, e^{-rT}\right)$. The second derivative satisfies

$$\frac{d^2C}{dK^2} = e^{-rT} \left\{ N'(d_2) - KN''(d_2)\sqrt{T}\sigma'(K) \right\} \left\{ \frac{1}{K\sigma(K)\sqrt{T}} + d_1 \frac{\sigma'(K)}{\sigma(K)} \right\} + e^{-rT} KN''(d_2)\sqrt{T}\sigma''(K)$$

Substituting $N''(x) = -xN'(x)$ we get

$$\frac{d^2C}{dK^2} = e^{-rT} \frac{N'(d_2)}{\sigma(K)} \left\{ 1 + d_2 K\sqrt{T}\sigma'(K) \right\} \left\{ \frac{1}{K\sqrt{T}} + d_1\sigma'(K) \right\} + e^{-rT} KN'(d_2)\sqrt{T}\sigma''(K) \tag{3}$$

and the no-arbitrage condition (assuming a continuum) is

$$\frac{d^2C}{dK^2} \geq 0.$$

To estimate a smooth B-S implied volatility curve, we first compute the implied volatilities of all the call options in our sample. The implied volatilities corresponding to calls on the same stock and same buying date are then used to fit a smooth curve using cubic spline interpolation. We also compute the first and second derivatives of the estimated implied volatility curve at the traded points. The first derivative is used to compute the price of the binary call option using equation (2) and the second derivative is used to estimate equation (3) to check if the binary call satisfies the no-arbitrage condition. To ensure better estimation of the implied volatility curve, we restrict our sample to only those calls whose underlying stocks have at least four strikes on any trading day. [7] Finally, to reduce the possibility of unrealistic returns, we restrict our sample to only those binary calls for which the estimated price is at least \$0.01 giving us a total of 162,964 valid binary call returns that satisfies all the restrictions. Results indicate that the average returns of the binary calls are monotonically increasing in the strike price.

E Modified bullish call spreads

Once we have the prices of the cash-or-nothing binary call option, computing the prices of the modified bullish call spread is straightforward. Recall that we defined a modified bullish call spread as a portfolio that is long a call with strike K, short a call with strike $K + \Delta K$, where $\Delta K > 0$, and short a cash-or-nothing binary call with payoff $\Delta K \cdot 1_{\{S_T > K + \Delta K\}}$.(all on the same stock and same expiration T). We do not impose any other restriction on the bullish call spreads other than setting the minimum bound on the prices to 0.01 giving us a total of 80,988 bullish call return observations. Average returns are seen to be increasing in K.

5.2 Page test for ordered alternatives

In summary, the results in Table 1 are consistent with monotonicity of expected returns in the strike price. In particular, we show that average return for each strike group is greater than the average return for the immediate lower strike group. However, testing the monotonicity

[7] We get similar results when we require at least three strikes instead of four.

restriction convincingly requires that we design a test that allows us to compare more than two strike groups simultaneously. When $k(> 2)$ samples are to be compared, it is necessary to use a statistical test which will indicate whether there is an overall difference among the k samples before choosing any pair of samples to test the significance of the difference between them. In other words, we need a single statistic for monotonicity of expected returns in the strike price.

Consider our setting where we use a two-sample statistical test to test for differences among five consecutive strike groups. We conduct five two-sample statistical tests in order to test if the average returns are increasing when we move from one strike group to the next higher strike group. Such a procedure could lead to erroneous conclusions because it capitalizes on chance. We are giving ourselves five chances rather than one chance to reject the null hypothesis. Now, with an α of 0.05, we are taking the risk of rejecting the null hypothesis erroneously five percent of the time. But if we make five statistical tests of the same hypothesis, we are increasing the probability to 0.23 that a two sample statistical test will find one or more significant differences.The " actual significance " level in such a procedure becomes $\alpha = 0.23$.[8] It is only when an overall k-sample test allows us to reject the null hypothesis that we are justified in employing a procedure for testing differences between any two of the k samples [See Siegel and Castellan Jr. (1988) for a review of available k-sample tests]. In other words, an appropriate k-sample test would compliment the results in Table 1 and together would be a convincing and rigorous test of the monotonicity restrictions.

The k-sample test that we use is the "Page test for ordered alternatives" and it tests the null hypothesis that the groups (or measures) are the same versus the alternative that the groups (or measures) are ordered in a specific sequence. In particular let θ_j be the population mean for the jth group. Then we may write the null hypothesis that the means are the same

[8] $\alpha = 1 - (0.95)^5 = 0.23$

as

$$H_0 : \theta_1 = \theta_2 = \cdots = \theta_k$$

and the alternative hypothesis may be written as

$$H_A : \theta_1 \leq \theta_2 \leq \cdots \leq \theta_k$$

If the null hypothesis is rejected then at least one of the differences is a strict inequality ($<$). To apply the Page test we first assign the option strategies to strike groups as in Table 1. We then estimate expected returns for the strategy by computing the time series average of the strategy return for each underlying stock and each strike group. Assuming k_i strikes for the ith underlying stock, we rank the average returns of the strike groups for the ith stock on a scale of 1 to k_i. Assuming we had k strikes for each of the m underlying stocks (we will relax this assumption momentarily), the ranks are cast into a two-way table having m rows and k columns. The null hypothesis is that the average rank in each of the columns are the same against the alternative that the average rank increases across groups 1 to k. The Page statistic L_m is defined as

$$L_m = \sum_{j=1}^{k} Y_j \sum_{i=1}^{m} X_{ij} = \sum_{i=1}^{m} \left(\sum_{j=1}^{k} Y_j X_{ij} \right)$$

where i indexes the m observations (unique underlying stocks in our case), j indexes the k strike groups, X_{ij} is the rank of the average return of the ith stock and the jth strike group and Y_j is the a priori ordering of the group which in our case is equal to j. Then under the null hypothesis

$$E \sum_{j=1}^{k} Y_j X_{ij} = \frac{k\,(k+1)^2}{4}.$$

and

$$Var\left(\sum_{j=1}^{k} Y_j X_{ij}\right) = \frac{1}{144} k^2 (k+1)^2 (k-1).$$

Assuming the weighted ranks $\left\{\sum_{j=1}^{k} Y_j X_{ij}\right\}_{i=1,\ldots,m}$ are independent across different stocks, the CLT implies

$$\frac{L_m - mE\sum_{j=1}^{k} Y_j X_{ij}}{\left(mVar\left(\sum_{j=1}^{k} Y_j X_{ij}\right)\right)^{-1/2}} \to N(0,1).$$

From the results above, this is equivalent to

$$z = \frac{12L_m - 3mk(k+1)^2}{(mk^2(k+1)(k^2-1))^{1/2}} \to N(0,1).$$

The expression above is the original Page test statistic[9] (see Page (1963) and Pirie (1985)) and is only valid for samples which have the same k treatments for each observation. To account for the fact that underlying stocks have diffrent number of strikes, we modify the original Page test statistic to allow for varying number of treatments. Let k_i be the number of treatments for the ith observation (e.g. the number of strikes for the ith underlying stock). We then have

$$\begin{aligned} E\sum_{j=1}^{k_i} Y_j X_{ij} &= \frac{1}{4} k_i (k_i+1)^2 \\ Var\left(\sum_{j=1}^{k_i} Y_j X_{ij}\right) &= \frac{1}{144} k_i^2 (k_i+1)^2 (k_i-1) \end{aligned}$$

[9]See appendix for a detailed proof of the original Page statistic

Then, defining

$$L_m = \sum_{i=1}^{m} \left(\sum_{j=1}^{k_i} Y_j X_{ij} \right)$$

the CLT implies

$$z^{high} = \frac{12L_m - 3\sum_{i=1}^{m} k_i (k_i+1)^2}{\left(\sum_{i=1}^{m} k_i^2 (k_i+1)(k_i^2-1) \right)^{1/2}} \rightarrow N(0,1) \tag{4}$$

The statistic z^{high} essentially gives less weight to observations (i's) with fewer treatments(strike groups). Alternatively, we can standardize the weighted rank for each observation, and define

$$\tilde{L}_m = \sum_{i=1}^{m} \left(\frac{12\sum_{j=1}^{k_i} Y_j X_{ij} - 3k_i (k_i+1)^2}{\left(k_i^2 (k_i+1)^2 (k_i-1) \right)^{1/2}} \right)$$

Here we essentially scale up the statistics for observations with fewer strikes. Then

$$z^{low} = \frac{\tilde{L}_m}{\sqrt{m}} \rightarrow N(0,1) \tag{5}$$

Table 2 reports the number of unique underlying stocks and the values of the modified Page test statistics for all the option strategies. Recall that the statistic z^{high} refers to the case where we give less weight to stocks that have fewer number of strikes and z^{low} refers to the case where we scale up the statistics for stocks that have fewer number of strikes. Note that to be consistent with the analysis in Table 1, the maximum number of strike groups that we consider is five. Panel A reports the results for the whole sample and panel B reports the results for subsamples which have to satisfy an additional restriction. To ensure better estimation of the expected returns, we require that the option strategies on each underlying stock and for each strike group have a minimum number of weekly observations. For calls and puts we require at least 150 weekly observations. For the remaining strategies, we require at least 50 weekly observations [10] to be included in the Page test. From Table 2, we see that the Page statistics for all the option strategies are positive and very highly significant suggesting that the average returns of the option strategies are ordered in an increasing sequence across strike groups. As an added robustness test, we repeat the above analysis with only those observations which have at least five strike groups so that the assumptions of the original Page test are satisfied. Results are qualitatively similar to those reported in Panel A.

6 Robustness Tests

The previous section provides strong evidence that average returns of the option strategies whose payout belong to a particular class of payout functions are montonic in the strike price of the strategy. This is consistent with the hypothesis that the (projected) SDF is a decreasing function of the terminal stock price. This section of the paper provides several robustness test on call returns. We focus primarily on calls because of the recent empirical evidence suggesting that average call returns violate the monotonicity of expected returns in the strike price. Moreover, all the option strategies considered in this paper are combinations of call options and it is imperative that our results on calls are robust to different testing procedures [11].

A traditional approach to understanding why expected call returns should be increasing in strike is the following: A call option is equivalent to a levered long position in the stock. In terms of rate of return, a levered position is more risky than an unlevered one, and leverage amplifies the risk premium of the option. Since higher strike calls are more levered positions than lower strike calls, expected returns would be larger for higher strike prices.

[10]Raising the minimum number of required observations to 100 or 150 reduces the number of unique stocks significantly.

[11]Note that the empirical tests conducted in this section are also valid for other strategies and results could be provided if necessary

As moneyness increases with strike price of the option, expected option returns should be increasing in the moneyness of the option.

A seemingly intuitive way to test the above hypothesis would be to sort option contracts into portfolios based one their moneyness. Ni (2007) sorts call option contracts on each buying date into five portfolios based on moneyness and shows that the portfolio with OTM option contracts earns significantly lower returns than the portfolio with ITM option contracts. She suggests that investors are sometimes risk-seeking and a preference for idiosyncratic skewness leads to a premium for deep OTM options and is a possible explanation for the puzzling call returns. We will provide evidence that such a testing procedure could lead to erroneous conclusions primarily because the underlying asset of the call options do not remain the same. Consider the example provided in Table 3. The example looks at actual data on option contracts on two underlying stocks on the same buying date with approximately the same moneyness range. However, the Black-Scholes elasticity of the lowest strike call on "AIG" is almost equal to the elasticity of the highest strike call on "USRX". Thus call options on "AIG" on this buying date are more levered positions than call options on "USRX" and should be expected to earn higher returns.

Table 4 reports the Black-Scholes deltas and elasticites of calls sorted into strike groups based on the option's moneyness. The sample and moneyness cutoffs [12] used are from Ni (2007). We see that although the average deltas are monotonically increasing across strike groups, the same cannot be said about the elasticities. The mean and median elasticities of deep out of the money calls are smaller than that of at the money calls. Also note that the standard deviation of the elasticities are enormous suggesting huge variations within strike groups. In the Black-Scholes continuous time framework the elasticity (or leverage) of an option would depend on the moneyness of the option and the volatility of the underlying

[12] The moneyness cutoffs used are as follows: $[K/S \leq 0.85, 0.85 < K/S \leq 0.95, 0.95 < K/S \leq 1.05, 1.05 < K/S \leq 1.15, K/S > 1.15]$

stock. For options on the same underlying stock, elasticities and hence expected returns are monotonically increasing in the moneyness of the option. When the underlying asset is not constant, a higher moneyness would not necessarilly imply a higher elasticity [This is also easy to confirm in a simulation setting]. Note that monotoncity of the elasticity in strike price is true for all the option strategies considered in this paper. Of course, unlike call and put options, the elasticities would not necessarily be of the same sign. For example, the elasticity of a butterfly spread is negative for in the money and positive for out of the money contracts.

Another problem with testing the monotonicity hypothesis by sorting by moneyness is illustrated through the following example: Consider a call option with a strike price of 35 when the underlying stock is worth $30. The moneyness of this option is 1.17 and would be assigned a strike group value of five using the cutoffs used in Ni (2007). In fact, all out of the money calls in this case will be assigned a strike group value of five. Alternatively, when the stock price is high ($175 for example), most of the out of the money calls will be assigned a strike group value of three. In other words, when current stock prices are low or high, the sorting of option contracts into bins depend more on the level of the current stock price than on its leverage or its distance from being at the money. In summary, this procedure neither test whether expected returns are increasing when moving from one strike to the next higher strike nor does it test whether expected returns are increasing in the leverage of the option.

Table 5 reports the average of the monthly and weekly time series of call returns and return differences sorted by leverage. We sort option contracts into leverage groups by sorting option contracts on each buying date into quintiles based on their elasticities with respect to the underlying stock. Panel A reports the one month holding-till-maturity returns [13] and Panel B reports the weekly buy-and-hold returns. Since the underlying stock that does not pay dividend has the same return structure as a call option with zero strike, we calculate

[13]One month holding till maturity returns are calculated following Ni (2007).

all option returns in excess of that of the return on the underlying stocks. [14] Options on individual stocks earn higher returns than the underlying stock and returns are monotonically increasing with leverage. The portfolio with the highest leverage call option contracts earn significantly higher returns than the portfolio with lowest leverage call options. Our results are also robust to the cases when (1) we remove the top and bottom 1% of the elasticities from our sample, and (2) when we consider only those option contracts that have a positive volume on the buying date.

We estimate expected put returns in the same way as we do for call options and the results are reported in Table 6. Since a put option with infinite strike price has an expected return equal to the risk-free rate, expected put returns must always be below the risk-free rate (if the risk premium of the underlying stock is positive). Expected stock returns are typically higher than the risk-free rate; so expected put returns should always be lower than the return on the underlying stock. We find that average put returns are lower than the average return on the underlying stock and the returns are increasing with leverage. We also confirm that the average put returns are never higher than the average risk free rate.

In Table 7 we use regressions to test the monotonicty restriction in call option returns. Note that option payoffs are non-linear functions of the underlying stock and any discrete time linear model that attempts to explain option returns would be inherently wrong. In fact, our regression results confirm the presence of significant non-linearity's in call returns. However, we use regression models as an added robustness test to check if the results are consistent with our theoretical predictions.

In our regressions, the dependent variable is the excess call option return over the risk free rate. Averages of the weekly cross-sectional regression estimates as well as Fama-MacBeth t statistics adjusted for third order serial dependency are reported. Specification (1) of Table 7 regresses the excess option return on dummy variables for the strike groups where

[14] Results are stronger if returns are not calculated in excess of the return on the underlying stock

strike group 1 is considered as the base group for all our regressions. The strike groups are defined in the same way as before. The estimates in specification (1) are consistent with the results in Table 1. The coefficients of the strike group dummy variables are all positive, significant and increasing with strike group. In specification (2) we add the independent variable *excess stock return* which is the excess return of the underlying stock over the risk free rate. Controlling for the risk premium of the underlying stock, we still find that coefficients on the dummy variables are positive and increasing except for the coefficient on *strike* which is the dummy variable for strike group 5. Although, the coefficient on $strike5$ is slightly lower than that on $strike4$, the difference is not statistically significant. This can be confirmed by repeating the above regression with strike group 4 or 5 as the base group.

The effect of leverage in option returns can be seen in specifications (3), (4) and (5). The basic idea is to test whether the increase in expected call option returns for increases in the excess return on the underlying stock is the same for all strike groups. We include interaction terms of the strike group dummy variables with *excess stock return* to allow for a difference in slopes for different strike groups. In specification (3) we find that the coefficients on the interaction terms are all positive, significant and increasing except again for the coefficient on interaction term of $strike5$ and *excess stock return*, which is although slightly lower than the interaction term of strike group 4 but is not significantly different. We find that the intercepts of the different strike groups are all negative. This is not surprising because the intercepts indicate the return differentials when the excess return of the stock is zero. When the excess return on the underlying stock is very low or zero, the expected call option returns should be negative. Specification (3) captures the effect of leverage in call option returns. The results show that for an increase in the excess return of the underlying stock, the increase in the excess return of the call option would be higher for a higher strike price option.

As discussed earlier, option payoffs are nonlinear functions of the underlying stock return and any discrete time linear model trying to explain option returns should be incorrect. We confirm this by including a nonlinear explanatory variable $(excess\ stock\ return * elast)^2$ in specification (4). This variable is defined as the square of the elasticity weighted excess stock return. The coefficient on this variable is very significant suggesting significant nonlinearities in call option returns. The significance of the nonlinear explanatory variable is robust to different choices of the explanatory variables. The other variables that we have used are the square of the excess return of the underlying stock and the excess return of the stock multiplied by an indicator function for a positive excess return. Note that we do not try to interpret the coefficients on these nonlinear explanatory variables. These are just used as proxies to test for nonlinearities in the call returns. Thus we conclude that, although the results from the regression analysis are consistent with our hypotheses, a linear model for call option returns can be rejected.

In specification (5) we introduce other explanatory variables that have been suggested in the literature to have an effect on option returns. Coval and Shumway (2001) and Buraschi and Jackwerth (2001) find evidence that volatility is priced in option markets. We use the change in implied volatility *delivol* as an explanatory variable in call option returns. In an efficient market, any deviation from no arbitrage values should trigger an immediate response from investors leading to the rapid disappearance of the arbitrage opportunity. However, the empirical asset pricing literature suggests the existence of market frictions that impede the arbitrage process. Liquidity has often been argued as an important limit to arbitrage because it makes arbitrage more risky and costly and might lead to persistence in mispricing in assets. We use the call trading volume on the buying date as a proxy for liquidity in our regressions. From specification (5) we find that the coefficients on *delivol* turn out to be statistically significant. However, the value of the coefficient on *volume* is very small and not statistically significant and does not appear to have a significant effect on option returns. The sign of the coefficient on *volume* however turn out to be consistent with theoretical predictions. The nonlinear term remains significant in this specification.

Finally, in specification (6) we regress excess option returns on elasticity weighted excess return on the stock; Vega weighted change in implied volatility, trading volume on the buying date and a nonlinear term. If discrete time option returns were just explained by the CAPM, the coefficient on the elasticity weighted excess return would be one or at least very close to one. A coefficient of 0.87 and significantly different from 1 suggests that discrete time option returns are not fully explained by the CAPM. The other explanatory variables along with the nonlinear term are also significant but the value of the coefficient on *volume* remains very small. Note however, that elasticity weighted excess return of the stock, by itself, explains about 90% of the variation in call returns.

Table 8 presents some more robustness tests on average call returns. To eliminate call prices estimated from stale quotes, Panel A repeats the test for call options reported in Table 1 with the added restrictions that the calls should have a positive volume on the buying date. Although the results are qualitatively similar to those reported in Table 1, we find that the return difference estimates for out of the money calls are lower consistent with the fact that out of the money calls are thinly traded in comparison to at the money calls. Panel B tests the monotonicty restriction where the average returns are calculated in the same way as for the Page test. We first estimate expected returns for the calls by computing the time series average of the call returns for each underlying stock and each strike group. Return differences are calculated for each stock and then averaged across different stocks. Average return differences are all positive and significant consistent with the results in table 1. The results in panel B are robust to the case when we restrict our analysis to the sub-sample where calls on each underlying stock and each strike group have at lease 150 weekly observations. The results are reported in Panel C.

7 Conclusion

This paper examines the expected returns of a family of option trading strategies on individual stocks to test whether the (projected) stochastic discount factor(SDF) is monotonic in the terminal stock price. We show that strict monotonicity of the SDF is essentially equivalent to the positivity of all "conditional" risk premia, defined as the difference in returns between the stock and unit payouts, the payouts being made conditional on the terminal stock price falling within some specified interval. Alternatively, we show that a strictly monotonic SDF in the terminal stock price guarantees a positive risk premium for all possible return distributions of the underlying stock return.

To test the monotonicity of the SDF, we characterize a class of strategies whose expected returns are increasing in the strike price under a monotonic SDF. The class of strategies include all payout functions for which the logarithm of the payout function is concave. The concavity of the log-payout function implies that increases in the strike shifts the probability weighted payoffs to lower values of the SDF and therefore a higher expected return. Call and put options are special cases, but the class also include butterfly spreads, bullish call spreads and binary calls.

A violation in monotonicity of expected returns for any of the strategies would imply a violation in monotonicty of the SDF, but the converse is not true. Hence, we also provide weaker conditions for monotoncity of expected returns in the strike price for some of the option trading strategies. Using data on option contracts on individual stocks, we find that the average weekly returns of the option trading strategies are increasing in the strike price of the strategy which is consistent with a monotonic SDF in the terminal stock price. Our theoretical results characterizes the entire class of option trading strategies whose expected returns are increasing under a monotonic SDF in the terminal stock price. Our characterization of monotonicity of the SDF can be applied, in future work, to test classes of models such as the CAPM or representative agent models with expected utility.

8 Appendix

8.1 Monotoncity of the SDF

Lemma 1 *Let $\sigma(S_T)$ denote the information set generated by the strictly positive terminal stock price S_T. For any $A \in \sigma(S_T)$ satisfying $P(A) > 0$, let P_A^1 and P_A^S denote the prices of payouts 1 and S_T, respectively, in the set A:*

$$P_A^1 = E(m 1_A), \quad P_A^S = E(m S_T 1_A);$$

and let $R^1 = 1/P_A^1$ and $R^S = S_T/P_A^S$ denote the returns on set A. Furthermore, let m denote the stochastic discount factor, and $g(S_T) = E(m|\, S_T)$ its projection on S_T.

a) If g is continuous and strictly decreasing then $E\big(R^S - R_A^1\big|\, A\big) > 0$ for all $A \in \sigma(S_T)$ satisfying $P(A) > 0$.

b) If g is continuously differentiable, and the distribution function of F is strictly increasing on $(0, \infty)$, then g is strictly decreasing if and only if $E\big(R^S - R_A^1\big|\, S_T \in [\alpha, \beta]\big) > 0$ for all $0 \leq \alpha < \beta$.

Proof. From the definitions of P_A^1 and P_A^S we get

$$0 = E\left\{ m\left(R^S - R_A^1\right)\middle|\, A\right\}$$

and therefore

$$\mathrm{E}\left(R^S - R_A^1\middle|\, A\right) = -\frac{\mathrm{Cov}\left(m, R^S\middle|\, A\right)}{\mathrm{E}(m|\, A)}.$$

If $A \in \sigma(S_T)$ (that is, 1_A is a function of S_T) then, letting $g(S_T) =$E$(m|\, S_T)$,

$$\mathrm{E}\left(R^S - R_A^1\middle|\, A\right) = -\frac{\mathrm{Cov}\left(g(S_T), S_T|\, A\right)}{P_A^S \mathrm{E}(m|\, A)} \quad \text{for all } A \in \sigma(S_T).$$

The result follows from Lemma 2 and the strict positivity of m and therefore $\mathrm{E}(m|\, A)$ and P_A^S. ■

Lemma 2 *Suppose Y is a random variable with distribution function F, and let $\sigma(Y)$ denote the information set generated by Y. a) If g is continuous and strictly decreasing then*

$$Cov(Y, g(Y)|\, A) < 0 \quad \text{for all } A \in \sigma(Y) \text{ and } P(A) > 0.$$

b) If g is continuously differentiable, the support of Y is $[a, b]$, where $-\infty \leq a < b \leq \infty$, and F is strictly increasing on $[a, b]$ (no zero probability intervals), then g is strictly decreasing if and only if

$$Cov(Y, g(Y)|\, Y \in [\alpha, \beta]) < 0 \quad \text{for all } a \leq \alpha < \beta \leq b. \tag{1}$$

Proof. Part a) Define

$$h(x; A) = \int^{x} \{E(Y|\, A) - y\} 1_{\{y \in A\}} \frac{dF(y)}{P(A)},$$

which satisfies $h(\alpha) = h(\beta) = 0$ and $h(x) > 0$ for all x such that $P(Y \in A \text{ and } Y \leq x) \in (0, 1)$. Using integration by parts:

$$\begin{aligned}
\mathrm{Cov}(Y, g(Y)|\, A) &= \int (y - E(Y|\, A)) 1_{\{y \in A\}} g(y) \frac{dF(y)}{P(A)} \\
&= -\int g(y)\, dh(y; A) \\
&= \int h(y; A)\, dg(y)
\end{aligned}$$

If g is strictly decreasing then the left hand side must be strictly negative.

Part b) Suppose (1) holds, but g is not strictly decreasing. Then there must exist points $c < e$ such that $g(c) \leq g(e)$, and therefore a point $d \in (c, e)$ such that $g'(d) > 0$ (we can rule out $g'(x) = 0$ for all $x \in (c, e)$, because this contradicts $\mathrm{Cov}\big(Y, g(Y)\, 1_{\{Y \in [c,e]\}}\big) < 0$). Continuity of g' implies $g'(x) > 0$ for x in a neighborhood of d, implying $\mathrm{Cov}\big(Y, g(Y)\, 1_{\{Y \in [d-\varepsilon, d+\varepsilon]\}}\big) > 0$ for some $\varepsilon > 0$, contradicting (1). ■

8.2 Monotoncity in expected returns

Proposition 1 *The expected return*

$$R(K) = \frac{EG(S_T - K)}{E\{m(S_T)\, G(S_T - K)\}}$$

is increasing in K for all distribution functions F (satisfying $EG(S_T - K) \neq 0$) and any monotonically decreasing $m()$ if and only if

$$\frac{G'(x)}{G(x)} \text{ is decreasing in } x \text{ for all } x \in (x_1, x_2). \tag{1}$$

Proof. We assume, for simplicity, that G is right continuous at x_1 and left continuous at x_2 (that is, $G(x_1) = \lim_{s \downarrow x_1} G(s)$ and $G(x_2) = \lim_{s \uparrow x_2} G(s)$).

a) Sufficiency of (1): Let F be some absolutely continuous distribution function F and let $\alpha = \int_0^\infty G(u - K)\, dF(u)$ (which represents the expected payout under F). Define the distribution function W:

$$W(s; K) = \frac{1}{\alpha} \int_0^s G(u - K)\, dF(u) \quad s \in [0, \infty). \tag{2}$$

The inverse of the expected return is

$$\frac{1}{R(K)} = \int_0^\infty m(s)\, dW(s; K). \tag{3}$$

We show that $R(K)$ is strictly increasing in K when $m(s)$ is strictly decreasing in s by showing that $W(s;K)$ is decreasing in K for all $s \in (0,\infty)$ and $K \geq 0$. That is, $W(\cdot;K_2)$ stochastically dominates $W(\cdot;K_1)$ in the first order sense for any $0 \leq K_1 < K_2$.[15] If $s-K < x_1$, then $W(s;k) = 0$ for k in some neighborhood of K and therefore $\frac{d}{dK}W(s;K) = 0$. We therefore consider only $s \geq K + x_1$. Differentiating (2), we have $\frac{d}{dK}W(s;K) \leq 0$ is equivalent to $H(s) \leq 0$ where

$$H(s) = -\alpha \int_0^s F'(u)\, dG(u-K) + \int_0^\infty F'(u)\, dG(u-K) \int_0^s G(u-K)\, dF(u).$$

Substituting $W'(s;K) = \alpha^{-1} G(s-K) F'(s)$, then $\frac{d}{dK}W(s;K) \leq 0$ if and only if

$$\int_0^\infty 1_{u-K\in[x_1,x_2]} W'(u;K) \frac{dG(u-K)}{G(u-K)} \leq \int_0^s 1_{\{u-K\in[x_1,x_2]\}} \frac{W'(u;K)}{W(s;K)} \frac{dG(u-K)}{G(u-K)}$$

which follows for all s because of the monotonicity condition in (1).

a) Necessity of (1): Consider first the case of a jump in $G'()/G()$, and suppose contrary to the hypothesis that

$$\lim_{s\uparrow\hat{s}} \frac{G'(s)}{G(s)} < \lim_{s\downarrow\hat{s}} \frac{G'(s)}{G(s)} \text{ some } \hat{s} \in (x_1, x_2),$$

and define the midpoint

$$m = \frac{1}{2}\left(\lim_{s\uparrow\hat{s}} \frac{G'(s)}{G(s)} + \lim_{s\downarrow\hat{s}} \frac{G'(s)}{G(s)}\right).$$

Using piecewise continuity, choose $\varepsilon > 0$ sufficiently small that $G'(x)/G(x) < m$ for $x \in [\hat{s}-\varepsilon, \hat{s})$ and $G'(x)/G(x) > m$ for $x \in (\hat{s}, \hat{s}+\varepsilon)$, and concentrate F on $[\hat{s}-\varepsilon, \hat{s}+\varepsilon]$ (let $F(\hat{s}-\varepsilon) = 0$, and $F(\hat{s}+\varepsilon) = 1$). Define

$$\Phi(s) = \int_{\hat{s}-\varepsilon}^s \frac{G'(u)}{G(u)} dW(u;0).$$

[15] See Huang and Litzenberger (1988, Ch 2).

Then $H(\hat{s}) \le 0$ is equivalent to $\Phi(\hat{s}+\varepsilon) \le \Phi(\hat{s})/W(\hat{s};0)$ But $\Phi(\hat{s})/W(\hat{s};0) < m$ and

$$\begin{aligned}\Phi(\hat{s}+\varepsilon) &= W(\hat{s};0)\left(\frac{\Phi(\hat{s})}{W(\hat{s};0)}\right) + \{1-W(\hat{s};0)\}\left(\frac{\Phi(\hat{s}+\varepsilon)-\Phi(\hat{s})}{1-W(\hat{s};0)}\right) \\ &> W(\hat{s};0)\left(\frac{\Phi(\hat{s})}{W(\hat{s};0)}\right) + \{1-W(\hat{s};0)\}\, m\end{aligned}$$

Together with

$$\frac{\Phi(\hat{s}+\varepsilon)-\Phi(\hat{s})}{1-W(\hat{s};0)} > m > \frac{\Phi(\hat{s})}{W(\hat{s};0)}$$

we get $\Phi(\hat{s}+\varepsilon) > \Phi(\hat{s})/W(\hat{s};0)$, a contradiction. Therefore $G'()/G()$ cannot increase at a jump.

Now consider the possibility of an increase in $G'()/G()$ over any interval. Suppose

$$\frac{G'(s_2)}{G(s_2)} > \frac{G'(s_1)}{G(s_1)} \tag{4}$$

for some $s_1 < s_2$, where $s_1, s_2 \in (x_1, x_2)$. Let $s^* \in \arg\inf\{G'(s)/G(s); s \in [s_1, s_2]\}$. Concentrating F on $[s^*, s_2]$ (let $F(s^*) = 0$, and $F(s_2) = 1$), then, for any $s \in (s^*, s_2)$, $H(s) \le 0$ is equivalent to

$$\int_{s^*}^{s_2} \frac{G'(u)}{G(u)} dW(u;0) \le \int_{s^*}^{s} \frac{G'(u)}{G(u)} \frac{dW(u;0)}{W(s;0)},$$

Letting $s \downarrow s^*$,

$$\int_{s_1}^{s_2} \frac{G'(u)}{G(u)} dW(u;0) \le \lim_{s \downarrow s^*} \frac{G'(s)}{G(s)},$$

which contradicts (4). ■

8.3 Skewness-adjusted t statistics

The proposed statistic for testing the mean of positively skewed distributions is taken from Chen (1995) and is derived using the Edgeworth expansion as follows:

$$P\left\{\frac{\sqrt{n}(\bar{X}-\mu)}{S} \leq x - \frac{1}{6\sqrt{n}}\hat{\beta}_1(1+2x^2)\right\} = \Phi(x) + o(n^{-1/2}), \tag{1}$$

where $\Phi(x)$ is the standard normal distribution, $S^2 = \frac{\sum(X_i-\bar{X})^2}{(n-1)}$, and $\hat{\beta}_1 = \frac{n^{-1}\sum(X_i-\bar{X})^3}{S^3}$ (For more details on the Edgeworth expansion, see Hall (1983)). To test the hypothesis $H_0 : \mu_x = \mu_0$ against $H_0 : \mu_x > \mu_0$ is to reject H_0 when

$$\frac{\sqrt{n}(\bar{X}-\mu_0)}{S} > z_\alpha - \frac{1}{6\sqrt{n}}\hat{\beta}_1(1+2z^2) \tag{2}$$

where z_α satisfies the equation $1-\Phi(z_\alpha) = \alpha$. Chen (1995) argues that the critical point of the above hypothesis test depends on the skewness of each sample. thus to find a more accurate and powerful test with a common critical point for a given significance level α, we use the following approach. We first solve $\frac{\sqrt{n}(\bar{X}-\mu_0)}{S} > x - \frac{1}{6\sqrt{n}}\hat{\beta}_1(1+2x^2)$ for x. Let $\alpha = \frac{\hat{\beta}_1}{6\sqrt{n}}$ and $t = \frac{\sqrt{n}(\bar{X}-\mu)}{S}$. Thus, when n is large such that $1-8a(t+a) \geq 0$, we have from (1),

$$P\left\{\frac{1-\sqrt{1-8a(t+a)}}{4a} > z_\alpha \text{ or } \frac{1+\sqrt{1-8a(t+a)}}{4a} < z_\alpha\right\} = \alpha + o(n^{-1/2})$$

Thus we have

$$P\left\{\frac{1-\sqrt{1-8a(t+a)}}{4a} > z_\alpha\right\} \leq \alpha$$

Using Taylor series expansion we then have

$$\frac{1-\sqrt{1-8a(t+a)}}{4a} = t + a + 2at^2 + 4a^2(t+2t^3) + o(n^{-1/2}) \tag{3}$$

Equation (3) is the skewness adjusted t statistic that we have used in this paper. An advantage of this test statistic is that it can be used for sample sizes as small as 13 and for distributions as asymmetric as the exponential distribution. Expressing (3) in terms of the original variables, we get the following expression:

$$t_{skew} = \frac{\sqrt{n}(\bar{X}-\mu_0)}{S} + \frac{1}{6\sqrt{n}}\hat{\beta}_1\left[1+2\left(\frac{\sqrt{n}(\bar{X}-\mu_0)}{S}\right)^2\right] + \frac{1}{9n}\hat{\beta}_1^2\left[\frac{\sqrt{n}(\bar{X}-\mu_0)}{S}+2\left(\frac{\sqrt{n}(\bar{X}-\mu_0)}{S}\right)^2\right] \quad (4)$$

8.4 Proof of the Page test

The Page statistic L_m is defined as

$$L_m = \sum_{j=1}^{k} Y_j \sum_{i=1}^{m} X_{ij} = \sum_{i=1}^{m}\left(\sum_{j=1}^{k} Y_j X_{ij}\right)$$

where i indexes the m observations (or "levels" or "ranking") and j indexes the k treatments. Let $\tilde{p}(i)$ denote the ith position of a random permutation of $\{1,\ldots,k\}$ (each permutation is equally likely). Then

$$E\sum_{j=1}^{k} Y_j X_{ij} = E\sum_{j=1}^{k} j\tilde{p}(j) = \sum_{j=1}^{k} jE\tilde{p}(j) = \left(\frac{k+1}{2}\right)\sum_{j=1}^{k} j = \frac{k(k+1)^2}{4}. \quad (1)$$

The variance is derived next.

Lemma 1

$$Var\left(\sum_{j=1}^{k} Y_j X_{ij}\right) = \frac{1}{144}k^2(k+1)^2(k-1) \quad (2)$$

Proof. The variance of any index value is

$$Var\left(\tilde{p}\left(i\right)\right)=\frac{1}{k}\sum_{j=1}^{k}\left(j-\frac{k+1}{2}\right)^{2}=\frac{1}{12}\left(k+1\right)\left(k-1\right),\quad \text{for } i\in\left\{1,\ldots,k\right\},$$

where we have used

$$\sum_{j=1}^{k}j^{2}=\frac{k\left(k+1\right)\left(2k+1\right)}{6}.$$

We next show (recall that $\tilde{p}\left(j\right)$ and $\tilde{p}\left(i\right)$ are the values ith and jth positions of a given random permutation $\tilde{p}$)

$$Cov\left(\tilde{p}\left(j\right),\tilde{p}\left(i\right)\right)=-\frac{1}{k-1}Var\left(\tilde{p}\left(j\right)\right)\quad i\neq j,\ i,j\in\left\{1,\ldots,k\right\}$$

This is done by defining the random variable

$$x=\begin{cases}\tilde{p}\left(i\right) & \text{with probability} \quad \left(k-1\right)/k\\ \tilde{p}\left(j\right) & \text{with probability} \quad 1/k\end{cases}$$

Note that x is uniformly distributed on $\left\{1,\ldots,k\right\}$ and is independent of $\tilde{p}\left(j\right)$. Therefore

$$0=Cov\left(\tilde{p}\left(j\right),x\right)=\frac{k-1}{k}Cov\left(\tilde{p}\left(j\right),\tilde{p}\left(i\right)\right)+\frac{1}{k}Var\left(\tilde{p}\left(j\right)\right).$$

Finally

$$
\begin{aligned}
Var\left(\sum_{j=1}^{k} Y_j X_{ij}\right) &= \sum_{j=1}^{k}\sum_{i=1}^{k} ijCov\left(\tilde{p}\left(j\right),\tilde{p}\left(i\right)\right) \\
&= Var\left(\tilde{p}\left(1\right)\right)\left\{\sum_{j=1}^{k} j^2 - \frac{1}{k-1}\sum_{j=1}^{k}\sum_{i\neq j} ij\right\} \\
&= Var\left(\tilde{p}\left(1\right)\right)\left\{\sum_{j=1}^{k} j^2 - \frac{1}{k-1}\left(\left[\sum_{j=1}^{k} j\right]^2 - \sum_{j} j^2\right)\right\} \\
&= Var\left(\tilde{p}\left(1\right)\right)\frac{k^2\left(k+1\right)}{12}
\end{aligned}
$$

Substituting gives the result. ■

Assuming the weighted ranks $\left\{\sum_{j=1}^{k} Y_j X_{ij}\right\}_{i=1,\ldots,m}$ are independent, the CLT implies

$$
\frac{L_m - mE\sum_{j=1}^{k} Y_j X_{ij}}{\left(mVar\left(\sum_{j=1}^{k} Y_j X_{ij}\right)\right)^{-1/2}} \rightarrow N\left(0,1\right).
$$

From the results above, this equivalent to

$$
\frac{12L_m - 3mn\left(k+1\right)^2}{\left(mn^2\left(k+1\right)\left(k^2-1\right)\right)^{1/2}} \rightarrow N\left(0,1\right). \tag{3}
$$

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Table 1

Average Return Differences

This table reports the weekly buy and hold return differences for some common option trading strategies that are special cases of the class of payout functions for which the growth rate is monotonic. Strike groups are defined using the following algorithm: For each underlying stock and buying date we find the option contract which has a strike price is closest to the price of the underlying stock and assign that to strike group 3. The next two higher strikes are assigned to groups 4 and 5 respectively. Similarly, the previous two lower strikes are assigned to groups 1 and 2 respectively. The options have to satisfy the following conditions to be included in the sample: (1) The bid price is strictly larger than $0.125, (2) the ask price is greater than the bid price, (3) the underlying stock does not have an ex-dividend date prior to maturity and (4) the option prices satisfies a no-arbitrage restriction.

Table 1 (Average return differences by strike group)					
Strike Groups	1	2'-1'	3'-2'	4'-3'	5'-4'
call returns	0.010	0.006	0.014	0.102	0.070
t statistics	2.33	2.25	2.23	8.81	3.82
Skewness adjusted t statistics	2.30	2.25	2.28	15.07	6.27
put returns	-0.009	0.002	-0.004	0.013	0.012
t statistics	-0.16	0.07	-0.36	0.97	1.73
Skewness adjusted t statistics	-0.11	0.17	-0.39	0.85	1.63
butterfly spread returns	0.108	-0.009	0.079	0.193	0.333
t statistics	6.48	-0.53	3.95	4.94	3.08
Skewness adjusted t statistics	7.76	-0.55	4.31	6.74	5.21
binary call returns	0.050	0.000	0.006	0.059	0.225
t statistics	8.32	0.04	0.54	3.30	3.04
Skewness adjusted t statistics	7.80	0.04	0.63	3.70	5.63
modified bullish call returns	0.174	0.062	0.092	0.334	0.351
t statistics	9.28	3.53	4.98	6.67	6.06
Skewness adjusted t statistics	15.63	3.42	5.35	36.82	9.62

Table 2

Page Test for Ordered Alternatives

Panel A reports the results of the "Page Test for Ordered Alternatives" for option strategies on individual stocks. Option contracts are first divided into strike groups using the algorithm described in Table 1. Expected strategy returns for each underlying stock and strike group are estimated by taking the average of the available weekly returns for the same underlying stock and strike group. Average strategy returns for each underlying stock are ranked in an increasing sequence across strike groups giving us a set of rankings for each underlying stock. These sets of ranking are then used to calculate the statistics for the Page test. Option contracts satisfy the same restrictions outlined in Table 1. Panel B repeats the analysis in Panel A with the added restriction that there should be minimum number of weekly observations available to estimate the expected returns. We require at least 150 weekly observations for the call and put options and a minimum of 50 observations for the remaining strategies

Table 2 : Page Test

Panel A			
	No. of stocks	z_{high}	z_{low}
call options	3531	41.23	40.73
put options	3532	17.39	16.60
butterfly spreads	2600	26.30	22.54
binary call options	1683	28.24	21.23
modified bullish call spreads	1771	26.52	25.24
Panel B			
	No. of stocks	z_{high}	z_{low}
call options	1222	35.84	36.36
put options	927	9.17	8.69
butterfly spreads	360	8.97	5.09
binary call options	69	4.72	3.53
modified bullish call spreads	75	2.90	2.44

Table 3

Appropriate measure of leverage

This table illustrates that sorting by moneyness might not be equivalent to sorting by leverage by using an example from the actual data

Table 3:

date	ticker	option id	stock price	strike	moneyness	elasticity
19-Jan-96	AIG	10415728	94.25	80.00	0.85	6.24
19-Jan-96	AIG	10394678	94.25	85.00	0.90	9.19
19-Jan-96	AIG	11008635	94.25	90.00	0.95	14.86
19-Jan-96	AIG	11132774	94.25	95.00	1.01	24.34
19-Jan-96	AIG	10105028	94.25	100.00	1.06	33.24
date	ticker	option id	stock price	strike	moneyness	elasticity
19-Jan-96	USRX	11566685	89.50	75.00	0.84	4.27
19-Jan-96	USRX	10755696	89.50	80.00	0.89	4.54
19-Jan-96	USRX	11569192	89.50	85.00	0.95	5.29
19-Jan-96	USRX	10055356	89.50	90.00	1.01	5.95
19-Jan-96	USRX	11499707	89.50	95.00	1.06	6.90

Table 4

Average Deltas and Elasticities by strike group

This table reports the mean, median and standard deviation of the elasticities and deltas of call options on each buying date sorted by moneyness. Strike Groups are estimating using the Ni (2007) moneyness cutoffs. They are as follows: $[K/S \leq 0.085, 0.85 < K/S \leq 0.95, 0.95 < K/S \leq 1.05, 1.05 < K/S \leq 1.15, K/S > 1.15]$. Call options are selected on each option expiration date that mature on the next expiration date if the following conditions are satisfied: (1) The bid price is strictly larger than $0.125, (2) the ask price is greater than the bid price, (3) the underlying stock does not have an ex-dividend date prior to maturity and (4) the call and put prices satisfies a no-arbitrage restriction.

Table 4:

Panel A: call deltas sorted by moneyness					
Strike Group	1	2	3	4	5
mean	0.92	0.81	0.54	0.30	0.20
median	0.94	0.81	0.55	0.30	0.19
std. deviation	0.05	0.09	0.12	0.09	0.09
Panel B: call elasticities sorted by moneyness					
mean	3.82	7.20	12.87	13.60	9.51
median	3.79	6.83	11.13	12.37	8.80
std. deviation	1.05	2.18	7.52	6.49	3.59

Table 5

Average call returns sorted by elasticity

This Table reports average returns and return differences of call options sorted by elasticity. Panel A reports the one-month holding till maturity (HTM) returns and Panel B reports the weekly returns. HTM returns are calculated following Ni (2007). Usual restrictions on call options apply. HTM returns are calculated following Ni (2007).

Table 5

Panel A : average call option returns sorted by elasticity (holding till maturity)								
Leverage Group	1	2	3	4	5	5'-1'	4'-2'	5'-3'
Avg. Return	0.004	0.012	0.035	0.059	0.083	0.080	0.047	0.049
t stats.	0.16	0.39	0.97	1.41	1.48	2.04	2.70	1.52
t stats. (skewness adjusted)	0.16	0.40	0.98	1.43	1.53	2.16	2.79	1.61
Panel B : average call option returns sorted by elasticity (weekly buy-and-hold)								
Leverage Group	1	2	3	4	5	5'-1'	4'-2'	5'-3'
Avg. Return	0.010	0.028	0.060	0.103	0.123	0.113	0.075	0.063
t stats.	0.90	1.79	3.03	4.43	4.78	5.98	6.72	4.49
t stats. (skewness adjusted)	0.95	1.92	3.43	5.44	5.91	8.28	12.79	5.31

Table 6

Average put returns sorted by elasticity

This Table reports average returns and return differences of put options sorted by elasticity. Panel A reports the one-month holding till maturity (HTM) returns and Panel B reports the weekly returns. Usual restrictions on put options apply.

Table 6

Panel A : average put option returns sorted by elasticity (holding till maturity)								
Leverage Group	1	2	3	4	5	5'-1'	4'-2'	5'-3'
Avg. Return	-0.210	-0.170	-0.143	-0.103	-0.063	0.146	0.067	0.080
t stats.	-2.20	-2.34	-2.03	-1.65	-1.27	1.97	2.14	2.38
Panel B : average put option returns sorted by elasticity (weekly buy-and-hold)								
Leverage Group	1	2	3	4	5	5'-1'	4'-2'	5'-3'
Avg. Return	-0.618	-0.507	-0.442	-0.338	-0.170	0.448	0.169	0.272
t stats.	-24.87	-21.63	-19.37	-15.14	-9.25	22.52	14.58	26.56

Table 7

Regression Analysis

In this table we regress excess option returns on a number of factors. "*strike*(i)" is a dummy variable that takes values 1 if the option contract is assigned to strike group "i". "*excess stock ret*" is the excess return of the underlying stock over the risk free rate, "*volume*" is the trading volume on each buying date, "*delivol*" is the change in implied volatility of the option, "*vega*" is the option Vega, and "*elast*" is the elasticity of the option. Averages of the cross-sectional estimates as well as Fama-MacBeth t statistics adjusted for third order serial dependency are reported.

Table 7						
Dependent variable is excess call return						
	(1)	(2)	(3)	(4)	(5)	(6)
constant	0.014	-0.051	0.000	-0.020	-0.140	-0.218
	(2.14)	(-8.23)	(0.27)	(-10.66)	(-28.84)	(-53.08)
*strike*2	0.014	-0.051	0.000	-0.020	-0.140	-0.21754
	(2.42)	(3.32)	(-1.5)	(-13.93)	(7.82)	
*strike*3	0.020	0.024	-0.015	-0.070	-0.004	
	(2.65)	(3.08)	(-5.65)	(-29.78)	(-0.75)	
*strike*4	0.084	0.066	-0.078	-0.237	-0.178	
	(4.25)	(3.68)	(-11.22)	(-43.82)	(-26.84)	
*strike*5	0.107	0.056	-0.177	-0.425	-0.398	
	(3.4)	(2.26)	(-11.17)	(-31.69)	(-29.58)	
excess stock ret		7.244	3.162	3.833	3.448	
		(54.3)	(89.09)	(74.82)	(22.2)	
$excess\ stock\ ret * strike2$			1.065	0.970	1.562	
			(48.97)	(27.88)	(11.49)	
$excess\ stock\ ret * strike3$			4.088	3.528	4.924	
			(32.38)	(29.51)	(24.37)	
$excess\ stock\ ret * strike4$			9.590	7.026	7.695	
			(20.52)	(23.99)	(24.22)	
$excess\ stock\ ret * strike5$			8.581	5.991	5.885	
			(16.75)	(18.63)	(14.91)	
volume					0.000	0.000
					(-0.63)	(-11.48)
delivol					0.612	
					(29.85)	
$excess\ stock\ ret * elast$						0.870
						(201.42)
$delivol * vega$						0.359
						(22.4)
$(excess\ stock\ ret * elast)^2$				0.278	0.343	0.287
				(38.66)	(40.83)	(68.09)
Average R-Squared	0.027	0.525	0.629	0.814	0.846	0.91

Table 8

Robsutness Tests

This table presents some more robustness tests on call returns. Panel A repeats the analyis for call options reported in Table 1 with the added restriction that calls should have a positive volume on the buying date. Panel B reports average return differences calculated as follows: Option contracts are first divided into strike groups using the algorithm described in Table 1. Expected strategy returns for each underlying stock and strike group are estimated by taking the average of the available weekly returns for the same underlying stock and strike group. Return differences are calculated between consequent strike groups and then averaged across different stocks. Panel C repeats the analysis in Panel B with the added restriction that there should be at least 150 weekly observations available to estimate the expected returns.

Table 8					
Panel A					
Strike Groups	1	2'-1'	3'-2'	4'-3'	5'-4'
Average call returns	0.011	0.005	0.015	0.072	0.055
t statistics	1.60	1.34	2.13	5.07	3.25
Panel B					
Strike Groups	1	2'-1'	3'-2'	4'-3'	5'-4'
Avg. call returns	0.001	0.008	0.016	0.124	0.040
t statistics.	1.11	8.27	13.59	25.74	3.95
Panel C					
Strike Groups	1	2'-1'	3'-2'	4'-3'	5'-4'
Avg. call returns	0.013	0.010	0.019	0.094	0.033
t statistics.	16.52	16.95	15.83	33.83	3.82