Ex Ante Skewness and Expected Stock Returns*

Jennifer Conrad† **Robert F. Dittmar**‡ **Eric Ghysels**§

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Abstract

We use a sample of option prices, and the method of Bakshi, Kapadia and Madan (2003), to estimate the *ex ante* higher moments of the underlying individual securities' risk-neutral returns distribution. We find that individual securities' volatility, skewness, and kurtosis are strongly related to subsequent returns. Specifically, we find a negative relation between volatility and returns in the cross-section. We also find a significant relation between skewness and returns, with more negatively (positively) skewed returns associated with subsequent higher (lower) returns, while kurtosis is positively related to subsequent returns. We analyze the extent to which these returns relations represent compensation for risk. We find evidence that, even after controlling for differences in comoments, individual securities' skewness matters. As an application, we examine whether idiosyncratic skewness in technology stocks might explain bubble pricing in Internet stocks. However, when we combine information in the risk-neutral distribution and a stochastic discount factor to estimate the implied physical distribution of industry returns, we find little evidence that the distribution of technology stocks was positively skewed during the bubble period – in fact, these stocks have the lowest skew, and the highest estimated Sharpe ratio, of all stocks in our sample.

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†Department of Finance, Kenan-Flagler Business School, University of North Carolina at Chapel Hill

‡Department of Finance, Stephen Ross School of Business, University of Michigan, Ann Arbor, MI 48109

§Department of Finance, Kenan-Flagler Business School, and Department of Economics, University of North Carolina at Chapel Hill

1 Introduction

Models suggesting that investors consider higher moments in returns have a long history in the literature. Researchers such as Rubinstein (1973) and Kraus and Litzenberger (1976, 1983) develop models of expected returns which incorporate skewness. More recent empirical work provides evidence that higher moments of the return distribution are important in pricing securities. For example, Harvey and Siddique (2000) test whether skewness is priced, and Dittmar (2002) tests whether a security's skewness and kurtosis might influence investors' expected returns.[1] In these papers, although additional restrictions are imposed on investors' utility functions (e.g., that of decreasing absolute prudence), investors are still maximizing expected utility, and evaluating risk in the context of optimal portfolios. Consequently, the higher moments which are relevant for individual securities in these models are co-moments with the aggregate market portfolio; the tests in these papers ask whether a security's co-skewness or co-kurtosis with the market is priced, and use historical returns data to measure these co-moments.

Other recent papers have suggested that additional features of individual securities' payoff distribution may be relevant for understanding differences in assets' returns. For example, Ang, Hodrick, Xing, and Zhang (2006a, 2006b) document that firms' idiosyncratic return volatility contains important information about future returns. The work of Barberis and Huang (2004), Brunnermeier, Gollier and Parker (2007), and the empirical evidence presented in Mitton and Vorkink (2007) and Boyer, Mitton and Vorkink (2008) suggest that the skewness of individual securities may also influence investors' portfolio decisions. Additionally, Xing, Zhang, and Zhao (2007) find that portfolios formed by sorting individual securities on a measure which is related to idiosyncratic skewness generate cross-sectional differences in returns.

In this paper, we examine the importance of higher moments using a different approach. We exploit the fact that if option and stock prices reflect the same information, then it is possible to use options market data to extract estimates of the higher moments of the securities' (risk-neutral) probability density function. Our method has several advantages. First, option prices are a market-based estimate of investors' expectations. Authors such as Bates (1991), Rubinstein (1985, 1994) and Jackwerth and Rubinstein (1996) have argued that option market prices appear to efficiently capture the information of market participants. Second, the use of option prices eliminates the need of a long time series of returns to estimate the moments of the return distribution; this is especially helpful when trying to forecast the payoff

[1]More recently, Chabi-Yo et al. (2006) present a general framework to disentangle the effects of heterogeneous beliefs and preferences on asset prices and find empirically that both skewness and heterogeneous beliefs are priced factors.

distribution of relatively new firms or during periods where expectations, at least for some firms, may change relatively quickly. Third, options reflect a true *ex ante* measure of expectations; they do not give us, as Battalio and Schultz (2006) note, the "unfair advantage of hindsight." As Jackwerth and Rubinstein (1996) state, "not only can the nonparametric method reflect the possibly complex logic used by market participants to consider the significance of extreme events, but it also implicitly brings a much larger set of information ... to bear on the formulation of probability distributions."

We begin with a sample of options on individual stocks, and test whether cross-sectional differences in estimates of the higher moments of an individual security's payoff extracted from options are related to subsequent returns. Consistent with Ang, Hodrick, Xing, and Zhang (2006a, 2006b) findings for physical volatility, we find a negative relation between risk-neutral volatility and subsequent returns.[2] We also document a significant negative relation between firms' risk-neutral skewness and subsequent returns-that is, more negatively skewed securities have higher subsequent returns. In addition, we find a significant positive relation between firms' risk-neutral kurtosis and subsequent returns. These relations persist after controlling for firm characteristics, such as beta, size, and book-to-market ratios, and adjustment for the Fama and French (1993) risk factors.

We examine the extent to which these relations between idiosyncratic higher moments and subsequent returns may be a function of differences in co-moments with the aggregate portfolio. Using several different methods and various proxies for the benchmark portfolio, we find that the relation between idiosyncratic higher moments, particularly skewness, and risk-adjusted returns persists, even after controlling for differences in co-skewness and co-kurtosis.

Our results are consistent with models such as Brunnermeier, Gollier and Parker (2007), and Barberis and Huang (2004) which suggest that investors will trade off the benefits of diversification and skewness, holding more concentrated positions in skewed securities, and result in a negative relation between idiosyncratic skewness and expected returns. These results are also consistent with the empirical evidence in Mitton and Vorkink (2007), who examine the choices of investors in a sample of discount brokerage accounts and find that investors appear to hold relatively undiversified portfolios and accept lower Sharpe ratios for positively skewed portfolios and securities.

Our paper contains several methodological innovations as well. Sorting stocks on the basis of risk neutral pricing is novel to the literature and has other potential applications as well.

[2]Spiegel and Wang (2006) also find a significant relation between idiosyncratic volatility and subsequent returns, although the sign of the relation is reversed from the Ang et al. results.

First, the fact that we sort on option-implied moments means that we sort on market-derived measures of *ex ante* volatility, (co)skewness and (co)kurtosis estimates of individual stocks. Second, we use estimates of the stochastic discount factor (SDF) and estimates of risk neutral densities to obtain *implied physical densities*, or subjective probability distributions. The idea is similar to a procedure in Bliss and Panigirtzoglou (2004), who use the SDF implied by CRRA utility and risk neutral distributions to estimate the subjective probability distribution of two broad-based indices and investigate temporal patterns in risk aversion. However, our procedure differs from theirs both in the characterization of densities and the SDF. Third, the analysis of implied physical densities, and particularly the higher moments of such densities, is of interest, since recent models that consider the effects of skewness and fat tails in individual securities' distributions on expected returns deal with investors' estimates of the physical distribution.

Evidence that other features of an individual security's return distribution are relevant for stock prices seems particularly intriguing in the context of anomalies which affect narrow sectors of the economy, such as the Internet bubble. Given that the rest of the market appeared relatively unaffected during this period–in fact, Siegel (2006) argues that if one removes technology and telecommunication stocks from the S&P 500, the remaining stocks had depressed prices in early 2000 – it seems unlikely that the pricing of *"bubble"* stocks is due to cross-sectional differences in the co-moments of their return distributions with the aggregate portfolio; the dispersion in the co-moments, and the risk premium associated with the exposure, would have to be implausibly high. However, if the characteristics of individual securities' payoff distribution are important, then the concentration of the 'bubble' in particular segments of the market, which have large idiosyncratic differences in return distributions, may be less puzzling.

To examine this possibility, we combine the information about risk-neutral distributions contained in *sector* option prices, and estimates of the stochastic discount factor, to construct the implied physical distributions of industry portfolios. Surprisingly, we find little evidence that investors viewed technology stocks as having markedly higher probabilities of extreme positive payoffs; in fact, technology stocks have the lowest skew, and the highest Sharpe ratios, of any industry in our sample. Overall, while we find substantial evidence that individual securities' skewness affects prices in general, we find no evidence that skewness can explain the bubble in particular.

The remainder of the paper is organized as follows. In section 2, we detail the method we employ for recovering measures of volatility, skewness, and kurtosis, following Bakshi, Kapadia, and Madan (2003) and we discuss the data (filters) used in our analysis. In Section 3 we focus on testing whether estimates of the *ex ante* higher moments of the payoff distribution obtained from options data are related to the subsequent returns of the underlying security. In Section 4, we analyze the extent to which the relations between option-based *ex ante* higher moment sorts and subsequent returns are due to investors seeking compensation for higher co-moment risk, rather than idiosyncratic moments. In Section 5, we discuss the estimation of implied physical distributions for different industries, and present these estimates for various sub-periods in our sample. We conclude in Section 6.

2 Data and Computing Ex Ante Risk-Neutral Moments

We wish to examine the relation, if any, between features of the risk-neutral density function and the pricing of stocks. In this section we describe the data and the methods used to compute *ex ante* estimates of volatility, skewness, and kurtosis.

Our data on option prices are from Optionmetrics (provided through Wharton Research Data Services). We begin with daily option price data for all out-of-the-money calls and puts for all stocks from 1996-2005.[3] Closing prices are constructed as midpoint averages of the closing bid and ask prices. In subsequent tests, we augment these data with complementary data for options on aggregate indices (S&P 500 and Nasdaq 100) and 14 industry index options. We discuss these index option data in greater detail in Sections 4 and 5.

Data on stock returns are obtained from the Center for Research in Security Prices (again provided through Wharton Research Data Services). We employ daily and monthly returns from 1996-2005 for all individual securities covered by CRSP with common shares outstanding. Risk free rates are the yield on secondary market three month Treasury Bills taken from the Federal Reserve Report H.15. Daily returns on the aggregate and industry indices are obtained from Datastream. Finally, we obtain balance sheet data for the computation of book-to-market ratios from Compustat and compute these ratios following the procedure in Davis, Fama, and French (2000).

To estimate the higher moments of the (risk-neutral) density function of individual securities, we use the results in Bakshi and Madan (2000) and Bakshi, Kapadia, and Madan (2003). Bakshi and Madan (2000) show that any payoff to a security can be constructed and priced using a set of option prices with different strike prices on that security. Bakshi, Kapadia, and Madan (2003) demonstrate how to express the risk-neutral density moments in terms of

[3]We do not adjust for early exercise premia in our option prices. As Bakshi, Kapadia and Madan (2003) note, the magnitude of such premia in OTM calls and puts is very small, and the implicit weight that options receive in our estimation of higher moments declines as they get closer to at-the-money. Using the same method, BKM show in their empirical work that, for their sample of OTM options, the implied volatilities from the Black-Scholes model and a model of American option prices have negligible differences.

quadratic, cubic, and quartic payoffs. In particular, Bakshi, Kapadia, and Madan (2003) show that one can express the τ-maturity price of a security that pays the quadratic, cubic, and quartic return on the base security i as

$$V_{i,t}(\tau) = \int_{S_{i,t}}^{\infty} \frac{2(1 - ln(K_i/S_{i,t}))}{K_i^2} C_{i,t}(\tau; K_i) dK_i + \int_{0}^{S_{i,t}} \frac{2(1 + ln(K_i/S_{i,t}))}{K_i^2} P_{i,t}(\tau; K_i) dK_i \tag{1}$$

$$W_{i,t}(\tau) = \int_{S_{i,t}}^{\infty} \frac{6ln(K_i/S_{i,t}) - 3(ln(K_i/S_{i,t}))^2)}{K_i^2} C_{i,t}(\tau; K_i) dK_i + \int_{0}^{S_{i,t}} \frac{6ln(K_i/S_{i,t}) + 3(ln(K_i/S_{i,t}))^2}{K_i^2} P_{i,t}(\tau; K_i) dK_i \tag{2}$$

$$X_{i,t}(\tau) = \int_{S_{i,t}}^{\infty} \frac{12(ln(K_i/S_{i,t}))^2 - 4(ln(K_i/S_{i,t}))^3)}{K_i^2} C_{i,t}(\tau; K_i) dK_i + \int_{0}^{S_{i,t}} \frac{12(ln(K_i/S_{i,t}))^2 + 4(ln(K_i/S_{i,t}))^3}{K_i^2} P_{i,t}(\tau; K_i) dK_i \tag{3}$$

where $V_{i,t}(\tau)$, $W_{i,t}(\tau)$, and $X_{i,t}(\tau)$ are the time t prices of τ-maturity quadratic, cubic, and quartic contracts, respectively. $C_{i,t}(\tau; K)$ and $P_{i,t}(\tau; K)$ are the time t prices of European calls and puts written on the underlying stock with strike price K and expiration τ periods from time t. As equations (1), (2) and (3) show, the procedure involves using a weighted sum of (out-of-the-money) options across varying strike prices to construct the prices of payoffs related to the second, third and fourth moments of returns.

Using the prices of these contracts, standard moment definitions suggest that the risk-neutral moments can be calculated as

$$VAR_{i,t}^{Q}(\tau) = e^{r\tau} V_{i,t}(\tau) - \mu_{i,t}(\tau)^2 \tag{4}$$

$$SKEW_{i,t}^{Q}(\tau) = \frac{e^{r\tau} W_{i,t}(\tau) - 3\mu_{i,t}(\tau) e^{r\tau} V_{i,t}(\tau) + 2\mu_{i,t}(\tau)^3}{[e^{r\tau} V_{i,t}(\tau) - \mu_{i,t}(\tau)^2]^{3/2}} \tag{5}$$

$$KURT_{i,t}^{Q}(\tau) = \frac{e^{r\tau} X_{i,t}(\tau) - 4\mu_{i,t}(\tau) W_{i,t}(\tau) + 6 e^{r\tau} \mu_{i,t}(\tau)^2 V_{i,t}(\tau) - \mu_{i,t}(\tau)^4}{[e^{r\tau} V_{i,t}(\tau) - \mu_{i,t}(\tau)^2]^2} \tag{6}$$

where

$$\mu_{i,t}(\tau) = e^{r\tau} - 1 - e^{r\tau} V_{i,t}(\tau)/2 - e^{r\tau} W_{i,t}(\tau)/6 - e^{r\tau} X_{i,t}(\tau)/24 \tag{7}$$

and r represents the risk-free rate. We follow Dennis and Mayhew (2002), and use a trapezoidal approximation to estimate the integrals in expressions (1)-(3) above using discrete data.[4]

In estimating equations (1) - (3), we use equal numbers of out-of-the-money (OTM) calls and puts for each stock for each day. Thus, if there are n OTM puts with closing prices available on day t we require n OTM call prices. If there are $N > n$ OTM call prices available on day t, we use the n OTM calls which have the most similar distance from stock to strike as the OTM puts for which we have data. We require a minimum n of 2.[5] We also eliminate options with prices less than \$0.50 in order to remove especially thinly traded options. In unreported results, we examine the sensitivity of our results to changing the requirement of options available and both increasing and decreasing the price filter. The results are qualitatively unchanged. As an additional robustness check we investigate the effect of adding a filter on option volume; these results are similar and are discussed in the appendix. The resulting set of data consists of 3,722,700 daily observations across firms and maturities over the 1996-2005 sample period.

In Table 1, we present descriptive statistics for the sample estimates of volatility, skewness, and kurtosis. We report medians, 5th and 95th percentiles across time and securities for each year during the sample period. There are clear patterns in the time series of these moments through the sample period, as well as evidence of interactions between them. Volatility peaks in 2000, during the height of the *"bubble"* period, then declines through 2005. The median risk-neutral skewness is negative, indicating that the distribution is left-skewed; the median value stays relatively flat through 2000 after which it declines sharply, while the median kurtosis estimate increases during that same period, more than doubling from 2000 through 2005.

3 Ex Ante Higher Moments and the Cross-section of Returns

Our focus in this section is on testing whether estimates of the *ex ante* higher moments of the payoff distribution obtained from options data are related to the subsequent returns of the underlying security.

[4] We are grateful to Patrick Dennis for providing us with his code to perform the estimation.

[5] Dennis and Mayhew (2006) examine and estimate the magnitude of the bias induced in Bakshi-Kapadia-Madan estimates of skewness which is due to discreteness in strike prices. For \$5 (\$2.50) differences in strike prices, they estimate the bias in skewness is approximately -0.07 (0.05). Since most stocks have the same differences across strike prices, however, the relative bias should be approximately the same across securities, and should not affect either the ranking of securities into portfolios based on skewness, or the nature of the cross-sectional relation between skewness and returns which we examine. In our empirical implementation, the moneyness of the options in our sample ranged roughly from .8 to 1.2 with on average 5 equally spaced contracts.

3.1 Arbitrage Issues

Under the assumption that no-arbitrage rules hold between the options market and the underlying security prices, the information set contained in both cash and derivatives markets should be the same. Several authors have shown that information in option prices can provide valuable forecasts of features of the payoff distributions in the underlying market. For example, Bates (1991) examines option prices (on futures contracts) prior to the market crash of 1987 and concludes that the market anticipated a crash in the year, but not the two months, prior to the October market decline. He also shows that fears of a crash increased immediately after the crash itself.

Our sample period includes the Internet bubble, and some researchers have argued that option prices and equity prices diverged during this period. For example, Ofek and Richardson (2003) propose that the Internet bubble is related to the 'limits to arbitrage' argument of Shleifer and Vishny (1997). This argument requires that investors could not, or did not, use the options market to profit from mis-pricing in the underlying market, and, in fact, they also provide empirical evidence that option prices diverged from the (presumably misvalued) prices of the underlying equity during this period. However, Battalio and Schultz (2006) use a different dataset of option prices than Ofek and Richardson (2003), and conclude that shorting synthetically using the options market was relatively easy and cheap, and that short-sale restrictions are not the cause of persistently high Internet stock prices. A corollary to their results is that option prices and the prices of underlying stocks did not diverge during the 'bubble' period and they argue that Ofek and Richardson's results may be a consequence of misleading or stale option prices in their data set. Note that if option and equity prices do not contain similar information, then our tests should be biased against finding a systematic relation between estimates of higher moments obtained from option prices and subsequent returns in the underlying market.[6] However, motivated by the Battalio and Schultz results, we employ additional filters to try to ensure that our results are not driven by stale or misleading prices. In addition to eliminating option prices below 50 cents and performing robustness checks with additional constraints on option liquidity, as mentioned above, we also remove options with less than one week to maturity, and eliminate days in which closing quotes on put-call pairs violate no-arbitrage restrictions.

[6]Robert Battalio graciously provided us with the OPRA data used in their analysis; unfortunately, these data, provided by a single dealer, do not have a sufficient cross-section of data across calls and puts to allow us to estimate the moments of the risk-neutral density function in which we are interested.

3.2 Portfolio Sorts

We begin by selecting daily observations of prices of out-of-the-money calls and puts on individual securities, which have maturities closest to 1 month, 3 months, 6 months and 12 months, and group these options into separate maturity bins. In each maturity bin, we estimate the moments of the risk-neutral density function for each individual security on a daily basis. We remove observations in the top 1% and bottom 1% of the cross-sectional distribution of volatility, skewness, and kurtosis each day to mitigate the effect of outliers. Finally, we remove firms that have less than 10 trading days of observations in a given calendar month. Following Bakshi, Kapadia and Madan (2003), we average the daily estimates for each stock over time (in our case, the calendar quarter). For each maturity bin, we further sort options into terciles based on the moment estimates (volatility, skewness, or kurtosis); the 'extreme' terciles contain 30% of the sample, while the middle tercile contains 40% of the sample. On the basis of these tercile rankings, we form equally-weighted portfolio returns on a monthly basis over the subsequent calendar quarter.

In Table 2, we report results for portfolios sorted on the basis of estimated volatility (Panel A), estimated skewness (Panel B), and estimated kurtosis (Panel C). Specifically, we report the subsequent raw returns of the equally-weighted moment-ranked portfolios over the next month in the column with label 'Mean'. In the next column, we report the characteristic-adjusted return over that same month. To calculate the characteristic-adjusted return, we perform a calculation similar to that in Daniel et al. (1997). For each individual firm, we assess to which of the 25 Fama-French size- and book-to-market ranked portfolios the security belongs. We subtract the return of that Fama-French portfolio from the individual security return and then average the resulting excess or characteristic-adjusted 'abnormal' return across firms in the moment-ranked portfolio. In the next three columns, for Panels A though C of Table 2, we report the average risk-neutral volatility, skewness and kurtosis estimates for each of the ranked portfolios. Finally, we report average betas, average (log) market value and average book-to-market equity ratios of the securities in the portfolio.

Summary statistics in Panel A of Table 2 suggest a strong negative relation between volatility and subsequent raw returns; for example, in the shortest maturity options (maturity bin 1), the returns differential between high volatility (Portfolio 3) and low volatility (Portfolio 1) securities is -34 basis points per month; longer maturities have differentials between 47 and 55 basis points per month. The columns of data which report the average characteristics of securities in the portfolio show sharp differences in beta and size and more modest differences in book-to-market equity ratios across these volatility-ranked portfolios. Low (high) volatility portfolios tend to contain low (high) beta firms and larger (smaller) firms, while differences in book-to-market equity ratios across portfolios are relatively small and differ across maturity bins. We adjust for these differences in size and book-to-market equity ratio in the characteristic-adjusted return column. After adjusting for the differences in size and book-to-market equity observed across the volatility portfolios, the return differentials are somewhat attenuated in all four maturities. However, although the differential is reduced, it remains significant, with lowest volatility portfolios earning between 12 and 24 basis points per month more than the highest volatility portfolios in all four maturity bins.

Panel A also indicates that there is a weak positive relation between volatility and skewness; in all maturity bins, skewness has a tendency to increase as volatility increases, although the effect is not monotonic.[7] The relation between volatility and kurtosis in Panel A is much stronger: as average volatility increases in the portfolio, kurtosis declines in all four maturity bins. Thus, the relation between volatility and returns may be confounded by the effect, if any, of other moments on returns; we examine this possibility in later sections of the paper. Finally, the average number of securities in each portfolio indicates that the portfolios should be relatively well-diversified. The top and bottom tercile portfolios average 273 firms, whereas the middle tercile portfolio averages 365 firms. Combined with the fact that we are sampling securities which have publicly traded options, this breadth should reduce the effect of outlier firms on our results.

Panel B of Table 2 sorts securities into portfolios on the basis of estimated skewness. Interestingly, we see significant differences in returns across skewness-ranked portfolios. The raw returns differential is negative for all four maturities, at 25, 39, 49 and 45 basis points per month, respectively. That is, on average, in each maturity bin the securities with lower skewness earn higher returns in the next month, while securities with less negative, or positive, skewness earn lower returns. The differentials in raw returns are of the same order of magnitude or larger than that observed in the volatility-ranked portfolios in Panel A. Compared to the volatility-ranked portfolios, the skewness-ranked portfolios show relatively little difference in their betas, and smaller differences in their market value, although differences in book-to-market equity ratios remain. When we adjust for the size- and book-to-market characteristics of securities, the characteristic-adjusted returns are reduced only slightly, and average 22, 35, 38 and 38 basis points per month, respectively, across the maturity bins.[8]

[7]The moments reported in the paper are averages of the estimated risk neutral moments, not portfolio moments or physical moments. In unreported results, we compare rolling physical skewnesses of industry index portfolios to their lagged risk neutral skewness. We find that in both the time series and the cross-section, the physical skewness is positively and significantly related to *ex ante* risk neutral skewness.

[8]In a different application, Xing, Zhang and Zhao (2007) find a positive relation between a skewness metric taken from option prices and the next month's returns. Their measure of skewness is the absolute value of the difference in implied volatilities in out-of-the-money call option contracts, where the strike price is constrained to be within the range of 0.8S to S, and preferably in the range of 0.95S to S. Thus, their skewness measure is related to the slope of the volatility smile over a smaller range of strike prices. We conduct a Monte Carlo exercise

In addition to the differences in returns, the table indicates that there is a negative relation between skewness and kurtosis. That is, kurtosis declines as we move across skewness-ranked portfolios. As in Panel A, interactions between other moments and returns could be masking or exacerbating the relation between skewness and returns. Consequently, in later tests, we control for the relation of other higher moments to returns in estimating their effect.

Panel C of Table 2 reports the results when securities are sorted on the basis of estimated kurtosis. Generally, we see a positive relation between kurtosis and subsequent raw returns; the return differential is economically significant, at 14, 36, 34 and 41 basis points per month across the four maturities. As with the other moment-ranked portfolios, the effect is reduced after adjusting for book-to-market and market capitalization differences, but the differences are very slight and the effect remains highly significant, at 20, 38, 33 and 39 basis points per month across maturity bins. We also observe patterns in the other estimated moments, with both volatility and skewness decreasing as kurtosis increases. Again, this emphasizes the need to control for the relation of all higher moments to returns.

The results in Table 2, Panels A-C, suggest that, on average, higher moments in the distribution of securities' payoffs are related to subsequent returns. Consistent with the evidence in Ang, Hodrick, Xing and Zhang (2006a), we see that securities with higher volatility have lower subsequent returns. We also find that securities with higher skewness have lower subsequent returns, while higher kurtosis is related to higher subsequent returns. In the next section, we examine whether the returns are related to risk premia associated with characteristic-based factors as in Fama and French (1993).

3.3 Factor-Adjusted Returns

In Table 2, we adjust for the differences in characteristics across portfolios, following Daniel et al. (1997), by subtracting the return of the specific Fama-French portfolio to which an individual firm is assigned. However, Fama and French (1993) interpret the relation between characteristics and returns as evidence of risk factors. Consequently, we also adjust for differences in characteristics across our moment-sorted portfolios by estimating a time series regression of the 'factor-mimicking' portfolio returns on the three factors proposed in Fama and French (1993). The dependent variable in these regressions is the monthly return from portfolios re-formed each month (as in Table 2), where the portfolios consist of a long position in the portfolio of securities with the highest estimated moments, and a short position in the

using a Heston model with plausible parameter values, to compare the performance of our skewness metric to theirs in a setting where skewness is known. In this controlled environment we find that the slope estimate of skewness used by Xing et al. is extremely noisy (using a Mean Squared Error metric) compared to the Bakshi et al. approach we use. Simulation details are available upon request.

portfolio of securities with the lowest estimated moments. The three factors used as independent variables in the regressions are the return on the value-weighted market portfolio in excess of the risk-free rate ($r_{MRP,t}$), the return on a portfolio of small capitalization stocks in excess of the return on a portfolio of large capitalization stocks ($r_{SMB,t}$), and the return on a portfolio of firms with high book-to-market equity in excess of the return on a portfolio of firms with low book-to-market equity ($r_{HML,t}$). As in Table 2, firms are grouped by maturity and sorted into portfolios on the basis of estimated moments (volatility, skewness and kurtosis). We report intercepts, slope coefficients for the three factors, and adjusted R-squareds. Standard errors for the coefficients are presented in parentheses, and are adjusted for serial correlation and heteroskedasticity using the Newey and West (1987) procedure.

Panels A-D of Table 3 present results for options closest to one, three, six, and twelve months to maturity, respectively. The first row of each panel contains the results for the long-short portfolio constructed from volatility-sorted portfolios. Consistent with the results in Panel A of Table 2 for characteristic-adjusted returns, we observe negative alphas in our "high-low" portfolio in all four maturity bins. Risk adjustment does not appear to have a material impact on the returns to these portfolios. The alphas range from -41 basis points (with a standard error of 36 bp) for the twelve-month maturity portfolio to -59 basis points (and a standard error of 33 bp) for the one-month maturity portfolio. These alphas are consistent with those of Ang, Hodrick, Xing and Zhang (2006), who show that firms with high idiosyncratic volatility relative to the Fama-French model earn "abysmally low" returns.

The patterns in the intercepts for skewness-sorted portfolios (row 2 of Panels A-D of Table 3) are also consistent with those observed in Panel B of Table 2. Alphas are negative in all four maturities, significant at the 10% level for the three month maturity and at the 5% level for the six- and twelve-month maturities. The alphas remain roughly constant in magnitude as we move from short-maturity options to long-maturity options, at -58, -65, -64 and -60 basis points per month, respectively. The negative alphas still suggest a 'low skewness' premium; that is, securities with more negative skewness earn, on average, higher returns in the subsequent months, while securities with less negative, or positive skewness, earn lower returns in subsequent months.

The evidence that skewness in individual securities is negatively related to subsequent returns is consistent with the models of Barberis and Huang (2004), and Brunnermeier, Gollier and Parker (2005). In their papers, they note that investors who prefer positively skewed distributions may hold concentrated positions in (positively skewed) securities–that is, investors may trade off skewness against diversification, since adding securities to a portfolio will increase diversification, but at the cost of reducing skewness. The preference for skewness will increase the demand for, and consequently the price of, securities with higher skewness and consequently reduce their expected returns. This evidence is also consistent with the empirical results in Boyer, Mitton and Vorkink (2008), who generate a cross-sectional model of expected skewness for individual securities and find that portfolios sorted on expected skew generate a return differential of approximately 67 basis points per month.

In the third rows of Panels A-D of Table 3, we report the results for kurtosis-sorted portfolios. Consistent with the results in Table 2, we see positive intercepts in portfolios that are long kurtosis. Alphas are positive and both economically and statistically significant, at 56, 67, 59 and 67 basis points per month, respectively, across the four maturities. Similar to the characteristic-adjusted returns in Table 3, there is no discernible trend in these intercepts across maturity bins. The magnitude of the alphas with respect to kurtosis is comparable to that observed in the skewness and volatility sorted portfolios.

There is one other noteworthy feature of Table 3. The explanatory power of the Fama-French three factors is, on average, lower for the kurtosis-sorted High-Low portfolios, and much lower for the skewness-sorted portfolios, than the volatility-sorted portfolios. Some of this difference is likely due to the fact that, as Table 2 shows, skewness and kurtosis-sorted portfolios exhibit much smaller differences in size and beta than do the volatility-sorted portfolios. However, it is also possible that there are features of the returns on moment-sorted portfolios that are not captured well by the usual firm characteristics. This evidence suggests that there is potentially important variation in the returns of higher moment sorted portfolios that is not captured by the Fama and French (1993) risk adjustment framework.

As discussed above, one of our concerns following the findings of Battalio and Schultz (2006) is that results might be driven by stale or misleading prices. As discussed above, we employ a number of filters to attempt to mitigate these concerns. We further investigate these issues by requiring additional, more stringent restrictions on the data. Additionally, we consider risk adjustment relative to an aggregate liquidity factor, as in Pastor and Stambaugh (2004). The results of this section are robust to these additional requirements, and are discussed in more detail in the Appendix.

Overall, both the characteristic-adjusted returns in Table 2 and the regression results in Table 3 provide evidence that higher moments in the returns distribution are associated with differences in subsequent returns, and that not all of the return differential observed can be explained by differences in the size, book-to-market, beta or liquidity differentials of the moment-sorted portfolios. That is, on average, we see some relation between the higher moments of risk-neutral returns distributions of individual securities and subsequent returns on these stocks in the underlying market. In the next section, we allow the risk adjustment for subsequent returns to incorporate higher co-moments as well.

4 Higher Moment Returns: Systematic and Idiosyncratic Components

In the previous section, we presented evidence that higher moments of individual securities are related to subsequent returns, and that while firm characteristics and characteristic-based risk factors do a relatively good job of explaining the cross-sectional differences in volatility-sorted portfolios, they perform substantially less well in explaining the returns of skew- and kurtosis-sorted portfolios. In this section, we analyze the extent to which these relations are due to investors seeking compensation for higher co-moment risk, rather than idiosyncratic moments. We perform a series of analyses. In each subsequent test, we decrease the restrictions placed on the stochastic discount factor and test whether the relation between higher moments and subsequent risk-adjusted returns persists.

4.1 Co-skewness, co-kurtosis and a single factor model

In addition to central higher moments, Bakshi, Kapadia, and Madan (2003) also suggest a procedure for computing the co-skewness of an asset with a factor. Assuming a single factor data generating process,

$$R_{i,t} = a_i + b_i R_{m,t} + e_{i,t}, \tag{8}$$

the authors note that co-skewness, as defined by Harvey and Siddique (2000) can be calculated as

$$COSKEW_t^Q\left(R_{i,t+\tau}, R_{m,t+\tau}\right) = \frac{E_t^Q\left[\left(R_{i,t+\tau} - E_t^Q\left[R_{i,t+\tau}\right]\right)\left(R_{m,t+\tau} - E_t^Q\left[R_{m,t+\tau}\right]\right)^2\right]}{\sqrt{VAR_t^Q\left(R_{i,t+\tau}\right)VAR_t^Q\left(R_{m,t+\tau}\right)}} \tag{9}$$

$$= b_i SKEW_{m,t}^Q\left(\tau\right)\frac{VAR_{i,t}^Q\left(\tau\right)}{\sqrt{VAR_{m,t}^Q\left(,\tau\right)}} \tag{10}$$

In these expressions, $R_{i,t+\tau}$ is the τ-period return on the underlying security, $SKEW^Q$ is the risk-neutral skewness, and $COSKEW^Q$ is the risk-neutral co-skewness with the single factor

m. A similar argument can be invoked to derive co-kurtosis,

$$COKURT_t^Q\left(R_{i,t+\tau}, R_{m,t+\tau}\right) = \frac{E_t^Q\left[\left(R_{i,t+\tau} - E_t^Q\left[R_{i,t+\tau}\right]\right)\left(R_{m,t+\tau} - E_t^Q\left[R_{m,t+\tau}\right]\right)^3\right]}{VAR_t^Q\left(R_{i,t+\tau}\right)VAR_t^Q\left(R_{m,t+\tau}\right)} \quad (11)$$

$$= b_i \frac{KURT_{m,t}^Q(\tau)}{VAR_{i,t}^Q(\tau)\,VAR_{m,t}^Q(,\tau)} \quad (12)$$

Bakshi, Kapadia, and Madan note that the parameter b_i is a risk-neutral parameter; in principle, this parameter is the risk-neutral projection coefficient of the return on asset i on the traded asset m.

We calculate the parameter b_i following the procedure used in Coval and Shumway (2001). The authors compute the beta of a call option under the Black-Scholes model as

$$b_i = \frac{S_{i,t}}{C_{i,t}}\mathcal{N}\left(\frac{\ln\left(S_{i,t}/K_i\right) + \left(r - \delta + 0.5\sigma^2\right)\tau}{\sigma\sqrt{\tau}}\right)\beta_i \quad (13)$$

where $\mathcal{N}$ represents the normal distribution, δ is the stock's dividend yield, σ^2 is the volatility of the underlying stock return, and β_i is the slope coefficient from a projection of underlying stock returns on the single factor. The authors report that their estimates following this procedure are very similar to those calculated by directly regressing option returns on the market portfolio. We follow their lead, and use the average ratio of $S_{i,t}/C_{i,t}$ across calls and the risk neutral variance calculated for each security i to compute our estimates of this parameter.[9]

While co-skewness and co-kurtosis can be characterized for any two assets, in general these co-moments are measured relative to some aggregate equity portfolio return. The S&P 500 is a natural choice as a broad index of stocks that also has traded options to permit the calculation of its risk neutral moments. Consequently, we estimate the slope coefficient in the regression, β_i, using one year of daily data on the returns to the individual asset and the S&P 500, ending on the date of the option price data.[10] We compute co-skewness and co-kurtosis using equations 13, 10 and 12, and form portfolios for these co-moments in a manner analogous to the total moment portfolio sorts in Section 3.

The relation between co-skewness and subsequent returns is reported in Table 4, Panel A; this table is identical in structure to Table 2. Similar to the results for skewness, sorting

[9]We experimented with using the most and least out-of-the-money calls in our estimates and found little cross-sectional sensitivity to the choice of call options.

[10]Of course, for the purposes of pricing assets, any co-skewness should be related to factors that are relevant for characterizing the stochastic discount factor. We discuss this issue and investigate its empirical importance in greater detail later in Section 4.

on co-skewness generates a negative spread in returns, ranging from 10 basis points for the six-month maturity to 29 basis points for the one-month maturity. In contrast to the results in Table 2, co-skewness seems considerably more related to firm characteristics. Indeed, as noted in the table, characteristic adjustment shrinks the magnitude of the average return difference between high and low co-skew firms, and in fact generates a *positive* relation between characteristic-adjusted returns and co-skewness for all but the one-month maturity portfolio. Co-skewness is positively related to volatility and negatively related to co-kurtosis, but appears to have virtually no relation to total skewness or kurtosis. These results are broadly consistent with Harvey and Siddique (2000), who also document a co-skewness premium and link co-skewness to characteristics.

Similar summary statistics for co-kurtosis are presented in Panel B of Table 4. Co-kurtosis generates a small positive return premium, ranging from nine basis points for the six-month maturity portfolio to 19 basis points for the twelve-month maturity portfolio. Again, characteristic adjustment has a substantial impact on the returns of these portfolios; in all four cases, the difference in high and low characteristic-adjusted co-kurtosis returns becomes negative. Co-kurtosis is negatively related to total volatility and co-skewness, and exhibits some positive relation with total kurtosis. There is little evidence of a relation between co-kurtosis and total skewness.

We test whether the returns related to the total moments presented in the previous section can be traced to co-moments. Specifically, we regress the returns of total moment portfolios on the returns of these co-moment portfolios. We estimate:

$$r_{it}(\tau) = \alpha_i + \beta_{i,mrp} r_{mrp,t} + \beta_{i,cs} r_{cs,t}(\tau) + \beta_{i,ck} r_{ck,t}(\tau) + \epsilon_{i,t} \qquad (14)$$

where $r_{it}(\tau)$ is the 'factor-mimicking' moment portfolio, constructed by taking the time t return of the τ-maturity option highest tercile moment portfolio in excess of the lowest tercile moment portfolio, $r_{mrp,t}$ is the return on the S&P 500 index in excess of the return on a one-month Treasury Bill, $r_{cs,t}(\tau)$ is the return of the τ-maturity option highest tercile co-skewness portfolio in excess of the lowest tercile return, and $r_{ck,t}(\tau)$ is the return of the τ-maturity option highest tercile co-kurtosis portfolio in excess of the lowest tercile return. Results of these regressions are shown in Table 5.

As shown in Table 5, the index and co-moment portfolios explain much of the time-series variation in the returns on volatility-sorted portfolios. The R^2's from the regressions exceed 90% for all four maturities, and the slope coefficients are all precisely estimated. The results suggest that the volatility returns load positively on the S&P 500 index, but negatively on co-skewness and co-kurtosis. However, the portfolios retain substantial returns in excess of that explained by the co-moments. The intercepts are economically large, ranging from -26 basis points to -55 basis points, and are statistically significant in all but the 12-month maturity case. Thus, the table suggests that while co-moment adjustment can explain much of the time series variation in the return on volatility-sorted portfolios, it fails to capture the average return associated with these portfolios.

Similar to the Fama-French three-factor regressions in Table 3, the co-moment factors are much less successful in capturing variation in the returns on skewness and kurtosis-sorted portfolios. The coefficients are estimated with substantially less precision than those for the volatility-sorted portfolios, and the explanatory power of the regressions is much weaker. As a result of the low explanatory power, for both kurtosis-sorted and skewness-sorted portfolios, the intercepts remain economically large, but are not statistically significant. In the case of skewness, these intercepts range from -15 basis points for the one-month-maturity returns to -54 basis points for the six-month-maturity returns. Only the latter is statistically significant at conventional levels, despite the fact that the magnitude of the intercepts is similar to that of the volatility-sorted portfolios. Intercepts for the kurtosis-sorted portfolios range from 9 basis points for the one-month-maturity returns to 41 basis points for the twelve-month-maturity returns. Again, the magnitude is large, but only the six and twelve month maturity returns are marginally statistically significant.

Overall, we note that while risk-neutral co-moments, constructed from a single factor model, do have some association with returns, portfolios sorted on total moments bear premia that do not appear to be related to these co-moment returns. Of course, this may be due to the way in which we measure sources of co-moment risk. In subsequent subsections of the paper, we analyze progressively less restrictive measures of co-moment risk to ask whether these total moments are in fact attributable to co-movement with some source of aggregate risk.

4.2 Parametric stochastic discount factors with higher moments

In the previous section, we attempt to form portfolios that capture time series variation in co-moment risk to isolate sources of total moment risk from co-moment risk. In this section, we follow an approach that similarly assumes that risk premia arise due to exposure to a common discount factor. However, we relax the functional form of this relationship and the nature of the risk premia. Specifically, we start from the observation that, under the law of one price, there exists a stochastic discount factor, $M_t(\tau)$ that satisfies the Euler equation

$$E_t\left[M_t(\tau)\, r_{i,t}(\tau)\right] = 0 \qquad (15)$$

where $r_{i,t}$ is an excess return for asset i. Under a correctly specified stochastic discount factor, this relation will hold exactly, implying that the payoff to asset i is determined by the covariance of the payoff with the stochastic discount factor. In contrast, if this condition does not hold, the implication is that payoffs to the asset cannot be described by covariance with the stochastic discount factor; in our context, where assets are moment-sorted portfolios, the failure of equation (15) suggests that idiosyncratic moments are associated with returns, even after controlling for co-moments with the SDF.

Of course, inferences about the importance of idiosyncratic moments are relative to a particular specification of the stochastic discount factor. Failure of the Euler equation condition to hold may represent the importance of idiosyncratic risk or mis-specification of the stochastic discount factors. In the next three subsections, we use several methods to estimate stochastic discount factors that allow for higher co-moments to influence required returns. These methods differ in the details of specific factor proxies, the number of higher co-moments allowed, and the construction of the stochastic discount factor. However, the goal in each case is to estimate the relation between idiosyncratic moments and residual returns, after adjusting for risk.

We begin by considering a parametric stochastic discount factor (SDF) that incorporates information about higher moments of the SDF, and consequently adjusts for co-moment risk with the SDF. In particular, Harvey and Siddique (2000) and Dittmar (2002) examine polynomial stochastic discount factors that account for co-skewness and co-kurtosis risk, respectively. These stochastic discount factors are nested in the polynomial specification

$$M_t(\tau) = d_0 + d_1 \left(R_t^*(\tau)\right) + d_2 \left(R_t^*(\tau)\right)^2 + d_3 \left(R_t^*(\tau)\right)^3 \tag{16}$$

where $R_t^*(\tau)$ is the τ-period return on a traded portfolio that captures the relevant risks in the SDF. We discuss various approaches to this formulation of the SDF in subsections 4.2.1 and 4.2.2; in section 4.3, we consider a nonparametric approach.

4.2.1 The S&P 500 Index

Similar to our analysis of co-skewness and co-kurtosis above, our first test uses the S&P 500 as the tangency portfolio in estimating M_t using equation (16). While numerous studies have documented violations of the CAPM, evidence in support of higher co-moment CAPMs is stronger. For example, Harvey and Siddique (2000) investigate an SDF that is quadratic in the return on the market portfolio, consistent with a three-moment CAPM. Dittmar (2002) investigates an SDF that is cubic in the return on the market, consistent with a four-moment

CAPM. Both studies document empirical evidence suggesting that higher-moment CAPMs improve upon the standard two-moment CAPM.[11]

The parameters are estimated in equation (16) via GMM using the sample moment restrictions

$$\hat{\alpha} = \frac{1}{T}\sum_{t=1}^{T}\left(R_{i,t}\left(\tau\right)M_t - 1_N\right) = 0 \quad \textbf{(17)}$$

where $R_{i,t}(\tau)$ is a 10×1 vector of gross returns comprising 3 portfolios sorted on τ-maturity risk-neutral volatility, 3 portfolios sorted on τ-maturity risk-neutral skewness, 3 portfolios sorted on τ-maturity risk-neutral kurtosis, and a Treasury Bill. We include the risk-free return since Dahlquist and Söderlind (1999) show that failing to do so can result in an SDF that implies a downward-sloping capital market line. We examine three versions of the polynomial SDF, M_t. The first is linear ($d_2 = d_3 = 0$), accounting for covariance with the tangency portfolio, the second is quadratic ($d_3 = 0$), accounting for coskewness, and the unrestricted version accounts for cokurtosis.

In Table 6, Panels A through D, we report the parameter estimates, Hansen's (1982) J-statistic of overidentifying restrictions, and point estimates of the excess returns (pricing errors) implied by the SDF for the three high minus low moment portfolio returns. In addition, we present Newey and West (1987) standard errors or p-values for the J-statistic in parentheses. Panel A presents results for the moment-sorted portfolios based on one-month maturity options; Panels B-D present complementary results for options based on three, six, and twelve months to maturity. In all cases, we use data over the period 4/30/1996 through 1/31/2005 for 106 monthly observations.[12] The results in Panels A and B suggest that at shorter maturities, the candidate models cannot be rejected at conventional significance levels. However, examination of the standard errors of the parameter estimates suggest that this failure to reject is more likely attributable to lack of power than fit of the model. With the exception of the intercept term, few of the parameter estimates are statistically different than zero at conventional levels.[13] Further, at the longer-horizon maturities shown in Panels C and D, the specifications are formally rejected at the 10% significance level. One positive result is that the point estimates correspond with economic arguments about co-moment preference; negative signs on the coefficients d_1 and d_3 suggest aversion to covariance and co-kurtosis,

[11]The S&P 500 is not a perfect proxy for the value-weighted market portfolio. However, in order to facilitate comparison with later results, we use this index rather than a broader index such as the CRSP value-weighted index.

[12]In later tests, we use overlapping annual returns. For comparison purposes, we truncate the data at 1/31/2005.

[13]Statistical inference in these tests would, of course, be affected by the relatively small sample and the collinearity between the polynomial terms in the SDF that we employ. As a consequence, the magnitude of the intercepts may be a more useful guide in inferring the economic importance of higher moments unrelated to the SDF.

whereas the positive sign on d_2 suggests preference for co-skewness.

More importantly, the point estimates of the excess returns on high minus low volatility, skewness, and kurtosis-sorted portfolios are large in magnitude. Excess returns average -101, -42, and 32 basis points per month across specifications and maturities for the volatility-sorted, skewness-sorted, and kurtosis-sorted high minus low portfolios, respectively. Pricing errors for the skewness-sorted portfolios tend to be somewhat more precisely estimated than those of the volatility-sorted portfolios; pricing errors range from 2 basis points (SE = 0.38) to -72 basis points (SE = 0.28) for the skewness-sorted portfolios, with five out of twelve specifications significant at conventional levels, compared to only three specifications significant for volatility-sorted portfolios. However, the magnitude of the alphas for volatility-sorted portfolios is economically quite large. In contrast, the evidence for significant excess returns in the pricing errors for the kurtosis-sorted portfolios is both economically and statistically weaker; the alphas range from -13 basis points (SE = 0.28) to 65 basis points (SE = 0.29), with only one out of twelve specifications significant at conventional levels.

In summary, the evidence suggests that the payoffs to higher moment-sorted portfolios, particularly skewness-sorted portfolios, cannot be traced to higher co-moments with respect to a value-weighted market proxy. While the statistical magnitude of the pricing errors is not consistent across all specifications, the economic magnitude of the pricing errors is large. Relative to the risks associated with returns on an S&P 500 tangency portfolio, the returns to the moment-sorted high minus low portfolios appear to be idiosyncratic.[14]

4.2.2 Industry Tangency Portfolio

Our second investigation of the systematic and idiosyncratic components of the payoffs to higher-moment sorted portfolios estimates the parameters of an SDF polynomial in the returns on the tangency portfolio constructed by a set of basis assets. Our choice of this proxy is motivated by several considerations. First, we focus on a tangency portfolio as it correctly prices the assets included in its formation by construction. As discussed in Hansen and Jagannathan (1991), there is a one-to-one correspondence under the law of one price between this tangency portfolio and the minimum variance SDF that correctly prices assets. Second, as mentioned above, although the CAPM suggests that the value-weighted market is the tan-

[14]One potential issue in our analysis so far may be that the candidate tangency portfolios examined may be dominated by larger, older firms with less skewness and kurtosis in their payoffs. Hence, measurement of co-skewness may be obscured by the lack of skewness in the index payoff. We therefore examine the Nasdaq 100 index as a proxy for the tangency portfolio as it generally comprises smaller, younger firms that may be more likely to exhibit skewed and leptokurtic payoffs. These results are very similar to those obtained using the S&P 500 as the market proxy; they are discussed in more detail in the Appendix.

gency portfolio, a large body of empirical evidence suggests that this hypothesis is violated. King (1966) and Ahn, Conrad, and Dittmar (2007) suggest that industry portfolios represent a reasonable basis for asset pricing, as sorting on industries tends to maximize within-portfolio covariation and minimize across-portfolio covariation. Consequently, we use a set of 14 industry portfolios to form our tangency portfolio. Descriptions of the industry indices and the tangency portfolio are presented in Table A1.

Table 6, Panels E-H contains the results from estimating the polynomial equation (13) using the industry tangency portfolio to estimate M_t via GMM. As shown in the table, the results are qualitatively unchanged from those estimated using the S&P 500 index. There is a slightly larger tendency to reject the overidentifying restrictions of the model, as indicated by the relatively smaller p-values of the tests compared to those in Panels A-D. However, as in the previous table, any failure to reject seems likely to be due to lack of power, as suggested by the large standard errors of the point estimates of the parameters. We cannot reject the null hypothesis that the parameters are significantly different than zero at conventional levels for any of the specifications.

It should finally be noted that the point estimates of pricing errors in Panels E through H remain large. The average excess return on the high minus low volatility portfolio varies from -40 to -183 basis points per month depending on maturity, comparable to that estimated using the value-weighted market portfolio. Similar results for skewness portfolios indicate average excess returns varying between 5 and -66 basis points, whereas average excess returns for kurtosis-sorted portfolios range from -16 to 67 basis points. Several of these estimates are statistically different from zero at the 10% level. Thus, similar to our conclusion for the value-weighted market portfolio, we conclude that relative to the risks present in the industry tangency portfolio, the returns to moment-sorted extremum portfolios appear to be idiosyncratic.

4.3 Non-Parametric Stochastic Discount Factors with Higher Moments

In the preceding sections, we estimated the parameters of polynomial stochastic discount factors using different proxies for the tangency portfolios, and examined whether these discount factors could explain the returns on moment-sorted portfolios. The evidence suggests that they cannot, indicating that the returns related to these moments appear to be idiosyncratic to the risks embodied in the returns employed in the SDFs. In this section, we pursue a more nonparametric approach for investigating the SDF using the relation between the risk-neutral and physical densities of a candidate asset.

The no-arbitrage condition in asset pricing suggests that the risk-neutral and physical probability measures are related by the equation

$$M_t(s,\tau)P_t(s) = \exp(r\tau)\,Q_t(s) \tag{18}$$

where $M_t(s,\tau)$ is the τ-period SDF at time t, contingent on state s, $P_t(s)$ is the physical probability of state s occurring at time t, and $Q_t(s)$ is the risk neutral probability of state s occurring at time t. Given an estimate of the physical and risk neutral probabilities, this equation implies

$$M_t(s,\tau) = \exp(r\tau)\,Q_t(s)/P_t(s). \tag{19}$$

Researchers have used this relation in several ways. It is possible to use restrictions on M, combined with estimates of the risk-neutral distribution Q and generate an estimate of the physical distribution P. For example, Bliss and Panagirtzoglou (2004) assume that investors have either power or exponential utility functions and estimate the risk-neutral distribution of the FTSE100 and S&P500 using options data in order to generate an estimate of the subjective probability distribution of the underlying indexes. They provide evidence that these subjective distributions are better forecasters of the underlying index returns. Alternatively, it is possible to combine estimates of the physical distribution generated from a time-series of returns, with estimates of the risk-neutral distribution inferred from option prices, and use equation (20) to infer something about the stochastic discount factor M. For example, Jackwerth (2001) and Ait-Sahalia and Lo (2000) employ this approach to estimate empirical risk-aversion functions.

We take a slightly different approach. Specifically, we follow Eriksson, Ghysels and Wang (2009) and use a Normal Inverse Gaussian (NIG) approximation to generate an estimate of both the subjective and risk-neutral probability distributions. The particular appeal to this approach is that the density is characterized entirely by the first four moments of the distribution. Hence, given estimates of the mean, variance, skewness, and kurtosis, we can characterize assets' conditional densities. Importantly, the authors show that this method is particularly well-suited when the distribution exhibits skewness and fat tails, as it does in the returns distributions which we examine in this application.

Since the results in the preceding section were little affected by our choice of benchmark portfolio, for convenience we focus on the stochastic discount factor implied by the S&P 500. This choice allows us to easily compute the risk-neutral moments of the benchmark: options on this index are heavily traded, and we can compute these moments analogously to the procedure employed in Section 3 for individual assets. This contrasts with alternative stochastic discount factors, such as those implied by the industry index tangency portfolio or the Fama-

French factors, for which options are not traded on the combination of the assets that generate the tangency portfolio.

The Bakshi, Kapadia, and Madan (2003) procedure provides a straightforward approach for the computation of risk neutral moments; computation of conditional physical moments is somewhat more problematic. While procedures exist for estimating conditional variance, econometric work surrounding the estimation of conditional skewness and kurtosis is lacking. We follow Jackwerth (2000) and use four years of daily data through the first date of our option sample period to estimate sample variance, skewness, and kurtosis. In their empirical work, Bakshi, Kapadia and Madan (2003) note that skew and kurtosis may be underestimated using short windows. Consequently, in the Appendix, we examine the robustness of our approach for measuring physical higher moments to different sample periods, the use of rolling and autoregressive estimates of the moments, as well as longer windows.

Finally, to estimate the conditional physical mean of the market μ_t we follow Jackwerth (2000) and add a risk premium of 8% to the risk-free rate observed at time t.[15] Once physical and risk-neutral distributions are estimated using the NIG method, the τ-period SDF, $M_t(\tau)$ is computed as in equation (19) by taking the risk-free discounted ratio of the risk-neutral to physical distribution.

The time series average of stochastic discount factor functions is depicted in Figure 1. In addition to the SDF obtained using the NIG approximation to the density, we also present averages of SDFs obtained by fitting linear, quadratic, and cubic functions of the S&P 500 return support to the NIG approximation each period. The Figure shows that the linear and quadratic stochastic discount factors are downward sloping throughout their range, consistent with decreasing risk aversion over all levels of wealth. In contrast, the NIG SDF and, to a lesser extent the cubic SDF, are upward-sloping over some portion of the support. In particular, the NIG stochastic discount factor has a segment in the mid-range of the graph which is increasing, consistent with the evidence in Jackwerth (2000) and Brown and Jackwerth (2001).[16]

While the NIG class is versatile (e.g., as Eriksson, Forsberg and Ghysels (2004) note, its domain is much wider than Gram-Charlier or Edgeworth expansions), there are some re-

[15]We examine the sensitivity of our results to this assumption, considering risk premia of 4% and 12% per annum as well. While the shape of the SDF is affected by this assumption, our results are qualitatively unchanged.

[16]Both Jackwerth (2000) and Brown and Jackwerth (2001) examine possible reasons for this pattern, which suggests that the representative agent may be risk-seeking over the upward-sloping range. Although not the focus of our paper, it is interesting to note that we obtain similar results despite using a sample period that does not overlap with Jackwerth (2001) or Brown and Jackwerth (2001) and an entirely different methodology. Golubev et al. (2008) also report a similar pattern of the pricing kernel using German DAX index data, and propose a statistical test for monotonicity. Using their test they find statistically significant against monotonicity; hence, their results also provide support for the presence of upward sloping segments.

strictions on its use. In particular, the parameters of the NIG approximation may become imaginary and so the distribution cannot be computed. This constraint does not arise in the case of 3- and 12-month to maturity options, and arises in only one month for the 6-month maturity options. However, this condition is frequently violated in the case of 1-month to maturity options. As a result, we compute stochastic discount factors using only 3-, 6-, and 12-month maturity options.

In Table 7, we report estimates of alphas (pricing errors) of the moment-sorted portfolios implied by the Euler equation calculated from each of the stochastic discount factors estimated above, using options closest to 3, 6, and 12 months to maturity. In general, across all specifications, precision of the estimates is quite poor; despite this, the results suggest that regardless of the specification of the stochastic discount factor, the sign and the economic magnitudes of the alphas across volatility-, skewness- and kurtosis-sorted portfolios after risk-adjustment remain similar to those observed in Table 2.

In all, the results of this section appear to corroborate the findings from the preceding sections. There is little evidence to suggest that the payoffs of moment-sorted portfolios are related to systematic exposure to a stochastic discount factor. It is important, however, to note that our results do not necessarily imply that the alpha, or residual return, is an arbitrage profit. Related to the possibility of a mis-specified stochastic discount factor, the estimates of the stochastic discount factor used to construct α control only for non-diversifiable risk (including the risk of higher co-moments) in the context of a well-diversified portfolio. For example, if investors have a preference for individual securities' skewness, they may, as in Brunnermeier et al., hold concentrated portfolios and push up the price of securities which are perceived to have a higher probability of an extremely good outcome. As a consequence, the lower subsequent returns of high-skew stocks may represent an equilibrium result.

In the next section of the paper, we address two final issues related to whether perception of highly skewed payoffs may affect investors' valuations. We examine the *ex ante* physical probability measures for different industries implied by the stochastic discount factors examined in this section. More specifically, we ask whether industries that were suspected of having particularly high valuations over our sample period are associated with higher *ex ante* skewness. We also explore the sensitivity of our risk-neutral measure results to measurement under the physical probability distribution.

5 Implied Physical Probability Distributions

To this point, we have focused on the estimation of risk-neutral moments, and the relation of these moments to returns. However, the models that consider the effects of skewness and fat tails in individual securities' distributions on expected returns deal with investors' estimates of the physical distribution. The no arbitrage restriction, equation (18), suggests that, given a stochastic discount factor and a risk neutral distribution, one can retrieve the physical distribution. In this section, we use this procedure to estimate the physical distributions of a set of sector portfolios.

The approach we adopt in this section has similarities with the Bliss and Panigirtzoglou (2004) paper, but is different in several important ways. First, for risk-neutral density approximation, we continue to use the NIG density approximations constructed from risk-neutral moments, as discussed in Section 4.3. Second, we use a different specification of the SDF; Bliss and Panigirtzoglou (2004) use an SDF implied by CRRA utility, whereas ours is constructed to satisfy the law of one price as discussed below. Finally, instead of a single benchmark index, we examine the distributions of a set of sector options spanning several industries. In particular, we use the same underlying indices that were used in Section 4 to create a tangency portfolio. As stated above, industry descriptions are provided in Table A1. The industries cover a number of different sectors of the economy, incorporating financials in the Bank (BKX) and Broker/Dealer (XBD) indices, mainline firms in the Consumer (CMR) and Cyclical (CYC) indices, high-tech firms in the High-Tech (MSH) and Computer Technology (XCI) industries, more traditional growth firms in the Biotech (BTK) and Pharmaceutical (DRG) indices, and commodities in the Gold and Silver (XAU) and Oil (XOI) indices. Components of these indices include Goldman Sachs, Johnson & Johnson, Citigroup, Ford Motor, Apple, Cisco Systems, Merck, and Exxon-Mobil. The broad cross-section of firms contained in these indices should allow us to capture important features of the stochastic discount factor.

One potential limitation of this approach is that the relation linking the physical and risk neutral measures, equation (18), are based on an SDF that satisfies no arbitrage. As our results in the preceding section suggest, this condition is unlikely to be satisfied for any of the SDFs that we have used thus far. However, we can construct an SDF that satisfies the law of one price unconditionally in sample. Specifically, as noted by Hansen and Jagannathan (1991), an SDF that is linear in the base assets under consideration satisfies the Law of One Price. Specifically, an SDF constructed as

$$M_t(s) = d_0 + d_1 r^T(s) \tag{20}$$

where

$$d_1 = -\frac{\bar{r}^T - r_f}{(s^T)^2(1+r_f)} \tag{21}$$

$$d_0 = \frac{1}{1+r_f} - d_1\bar{r}^T. \tag{22}$$

and $r^T(s)$ represents the return on the sample tangency portfolio in state s, which by construction satisfies the Law of One Price.

We use the returns on the 14 industry index portfolios and the returns on the skewness-sorted portfolios as base asset returns. Our rationale for doing so is that we are examining moments of the industry index portfolios, and thus want to ensure that the Law of One Price is satisfied for these assets. Further, we include the skewness sorted portfolios in order to incorporate the information in these portfolios that the results of the previous sections suggest are important for understanding the investment opportunity set. We calculate the weights on the tangency portfolio and the coefficients d_0 and d_1 using sample moments of the base assets. The stochastic discount factor function in each period is then formed by varying $r^T(s)$ over its support.[17]

5.1 Implied Moments of Physical Probability Distributions

We are interested in the moments of the physical probability distributions of the industry index portfolios. To estimate the physical distributions, we start with calculating risk neutral moments of the industries using sector options, and then apply the NIG approximation to obtain the risk neutral density for each sector. Finally, we use (18) with the linear SDF appearing in (20) to obtain the physical distribution. We integrate numerically over the distribution to obtain the first four central moments for each industry every month. We average these estimates over four subperiods. The first subperiod, Q2.1996 - Q2.1998, captures the behavior of the industries prior to the Asian currency crisis, arguably the period prior to the inflation of the so-called Internet Bubble. The second subperiod, Q3.1998 - Q1.2000, captures the expansion of the bubble. The third subperiod, Q2.2000 - Q4.2002, roughly captures the bursting of the bubble, the ensuing recession, and the September 11, 2001 event. The final period, Q1.2003 - Q4.2004, represents the beginning of the economic recovery.

Average moments are presented in Table 8. For brevity, we present results only for options

[17]In unreported results, we have also used the stochastic discount factors investigated in the preceding sections as candidates for the SDF in this section. The qualitative conclusions of this section are not affected by the use of these alternative SDFs.

that have closest to 12 months remaining to maturity.[18] We present results for Subperiod 1 in Panel A and Subperiods 2-4 in Panels B-D. We observe substantial cross-sectional variation in estimated moments across industries. For the first subperiod, the results suggest that the highest expected return is in the Morgan-Stanley High Tech Industry Index (MSH), and the lowest in the Utilities (UTY) index. Interestingly, the MSH index also exhibits high volatility, the lowest skewness of the group, and relatively low kurtosis. In fact, expected returns in this subperiod are strongly correlated with volatility (at 0.82), strongly negatively correlated with skewness (at -0.51), and strongly negatively correlated with kurtosis (at 0.76). However, a cross-sectional regression of expected returns on the moments in this subperiod (not reported) suggests a positive and statistically significant relation between expected returns and volatility, with the remaining moments statistically insignificant.

In the Bubble subperiod, the high-tech index, MSH, is again the index with the highest expected return, although Banking (BKX) and Computer Technology (XCI) also display relatively high returns. Again, these high returns are associated with high *ex ante* volatility; the correlation between expected returns and volatility is 70%. In this subperiod, skewness is only minimally correlated with expected returns (at 0.10), suggesting little economic relation, and kurtosis is negatively correlated with returns, at -83%. In unreported cross-sectional regressions, only the relation with kurtosis is marginally statistically significant–that is, in the period most closely aligned with the bubble, differences in expected returns do not appear to be particularly sensitive to investors' estimates of the likelihood of extreme outcomes. In fact, in this period industries associated with the bubble and growth do not exhibit particularly high skewness. Biotech (BTK), High-Tech (MSH), and Computer Equipment (XCI) all exhibit negative skewness, while Consumer (CMR) and Oil (XOI) are more positively skewed.

The moments in the third subperiod, spanning the collapse of the bubble and the ensuing recession, are the most correlated with mean returns. Volatility is very strongly positively correlated with expected returns (at 0.95), skewness is strongly negatively correlated with expected returns (at -0.83), and kurtosis is very strongly negatively correlated with expected returns, at -0.95. Nearly all of the industry indices exhibit negative skewness; the only ones that do not are Consumer (CMR) and Oil (XOI). Technology-oriented industries, such as MSH and TXX are among the most negatively skewed.

Finally, in the post-recession period, we again observe a positive correlation between expected returns and volatility, a mild negative relation with skewness, and a moderate negative relation with kurtosis. Interestingly, in this subperiod, all fourteen industry indices have negative skewness. The most negatively skewed is Airlines, which may correspond to

[18]These data are also the least likely to violate the conditions for the NIG distribution above, and hence lead to the greatest number of available observations.

expectations about the impact of fuel costs on this industry at that time as well as several bankruptcies. Again, the growth indices, BTK and MSH, are among the most negatively skewed.

We find it striking that the technology-oriented industry indices display little evidence of positive skewness, in contrast to hypotheses that suggest that stocks in these indices might exhibit 'lottery-like' features. Rather, considering the implied means and standard deviations, investors appeared to view technology stocks as relatively good Sharpe ratio bets. In both the first and second subperiods, leading up to the internet bubble, the ratio of expected returns to volatility are highest for the high-tech industry, MSH. That is, according to our estimates, investors viewed these stocks as "good deals" rather than as lotteries.

Two cautions are worth emphasizing. First, inclusion in an index may prevent us from sampling the youngest, most highly skewed firms, particularly in those industries whose composition is changing the most rapidly. More importantly, the use of options data to infer higher moments limits us not just in the cross-section, but in the horizon over which we can estimate investors' expectations. If investors' expectations of moments over horizons longer than one year (the longest horizon for which we have data) are both relevant for prices, and significantly different from the shorter-horizon moments that we have estimated, then our results may be incomplete and/or our inferences may be incorrect. For example, if investors believed that technology stocks' extreme payoffs would occur over, say, five year intervals, then differences in five-year skewness may potentially explain high valuations.[19] While we cannot rule this out, it is worth pointing out that the relation between *shorter* horizon skewness and Sharpe ratios in Table 7 is remarkably stable across all four intervals we examine. Since these intervals include the 'pre-bubble' and 'post-bubble' periods, any separate effect of longer horizon skewness would suggest a very marked term premium in the skew which differs dramatically across these intervals.

5.2 Sorts on Implied Physical Moments

In our final investigation, we note that the theories to which we appeal about the importance of skewness in determining asset valuations pertain to the physical, rather than the risk neutral moments as analyzed in Sections 3 and 4. In principle, one could use the procedure of the previous subsection to retrieve physical moments for every individual security. Unfortunately, practical limitations, including estimating a tangency portfolio across a broad class of assets and computational concerns in integrating densities for a large number of assets, makes doing so somewhat daunting. However, we provide some insight into this question by

[19]We are grateful to Paul Pfleiderer for an analysis of a setting in which this situation could arise.

sorting the industry portfolios into groups on the basis of the physical moments calculated in the preceding section, and analyzing whether these sorts are related to subsequent returns.

Specifically, we form sets of three portfolios each month on the basis of estimates of their expected returns, volatility, skewness, and kurtosis implied by the physical densities calculated in the preceding section. The high-moment portfolio is the equally weighted average of the three industries with the highest moment (mean, volatility, skewness, or kurtosis) in the preceding month and the low-moment portfolio is the equally weighted average return on the three industries with the lowest moments. Since we form our densities based on twelve-month-maturity options, we calculate returns over the subsequent year. We present (overlapping) averages in Table 9 and convert the averages to monthly returns for comparison with earlier results.

For our purposes, the main result of the table is that estimated physical moments predict average returns with the same sign as the risk neutral moments. That is, high physical volatility indices predict lower returns than low volatility indices, high skewness indices predict lower average returns than low skewness indices, and high kurtosis indices predict higher average returns than low kurtosis indices. Further, the magnitude of these average return differentials is economically significant; -52 basis points for volatility, 14 basis points for skewness and 47 basis points for kurtosis. Interestingly, the implied expected return also appears to forecast returns positively; the highest implied expected return firm outperforms the lowest by an average of 48 basis points per month.

6 Conclusions

We explore the possibility that higher moments of the returns distribution are important in explaining security returns. Using a sample of option prices from 1996-2005, we estimate the moments of the risk-neutral density function for individual securities using the methodology of Bakshi, Kapadia and Madan (2003). We analyze the relation between volatility, skewness and kurtosis and subsequent returns.

We find a strong relation between these moments and returns. Specifically, we find that high (low) volatility firms are associated with lower (higher) returns over the next month. This result is consistent with the results of Ang, Hodrick, Xing and Zhang (2006). We also find that skewness has a strong negative relation with subsequent returns; firms with lower negative skewness, or positive skewness, earn lower returns. That is, investors seem to prefer positive skewness, and the returns differential associated with skewness is both economically and statistically significant. We also find a positive relation between kurtosis and returns.

These relations are robust to controls for differences in firm characteristics, such as firm size, book-to-market ratios and betas, as well as liquidity and momentum.

We use several different methods to estimate stochastic discount factors to control for differences in higher co-moments, and their related compensation for risk. We use these stochastic discount factors to calculate risk-adjusted returns to portfolios sorted on the basis of volatility, skewness and kurtosis, where the risk-adjustment explicitly takes higher co-moments into account. After controlling for higher co-moments, we continue to find evidence that idiosyncratic moments matter.

We use estimates of the stochastic discount factor, and the risk-neutral distributions calculated for sector options, to estimate implied physical distributions across different industries. We find several interesting results. First, our results suggest that implied physical distributions are much more stable than those constructed using historical data. Second, in implied physical distributions, we find evidence of a trade-off between skewness in industry portfolios and *ex ante* estimates of the Sharpe ratios for the industry. That is, our results suggest a trade-off between expected returns and higher moments, with higher (lower) traditional risk-reward measures associated with lower (higher) skewness. However, we also find that the portfolio containing technology firms has low *ex ante* physical skew and kurtosis, and a high Sharpe ratio. Consequently, while we find *both* that higher moments matter, and that investors' expectations of higher moments change through time, our results do not appear to be an explanation of bubble pricing in the Internet period.

Finally, we examine whether portfolios sorted on the moments of implied physical moments have predictive power for subsequent returns. We find that the explanatory power of risk-neutral moments are preserved in physical moments–returns differentials for objective volatility, skewness and kurtosis are economically significant and identical in sign to those observed by sorting on risk-neutral measures.

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Table 1: Descriptive Statistics: Risk Neutral Moments

Entries to the table are the 5th percentile, median, and 95th percentiles of risk neutral volatility, skewness, and kurtosis across securities by year. We calculate the risk neutral moments following the procedure in Bakshi, Kapadia, and Madan (2003) using data on out of the money (OTM) puts and calls. We require at least two OTM puts and two OTM calls to calculate the moments. Further, we restrict attention to options with prices in excess of $0.50 for which we have at least 10 quotes per month and are not expiring within one week. Finally, we eliminate any options that violate put-call parity restrictions and lie in the extreme 1% of the distribution of the risk neutral moments. The sample consists of 3,722,700 option-day combinations over the time period January 1996 through December 2005.

	Volatility			Skewness			Kurtosis		
Year	P5	P50	P95	P5	P50	P95	P5	P50	P95
1996	11.09	24.20	43.91	-3.61	-0.30	0.92	1.23	3.75	18.59
1997	11.42	23.89	44.10	-4.04	-0.32	0.88	1.22	3.77	23.05
1998	12.27	24.76	48.01	-3.63	-0.30	1.06	1.24	3.96	21.85
1999	13.31	27.03	55.34	-3.88	-0.35	0.85	1.12	3.67	23.04
2000	15.49	30.55	61.91	-3.47	-0.42	0.86	1.09	3.68	20.55
2001	14.53	30.17	69.55	-3.28	-0.57	0.81	1.30	4.05	19.08
2002	13.81	27.54	69.26	-3.55	-0.63	0.89	1.41	4.50	22.11
2003	12.03	25.57	81.23	-4.46	-1.14	0.63	1.55	5.58	27.95
2004	10.52	23.81	74.79	-4.87	-1.20	0.77	1.71	6.78	33.38
2005	9.39	22.33	55.73	-5.48	-1.34	0.76	1.76	7.70	38.66

Table 2: Descriptive Statistics

Panels A-C present summary statistics for portfolios sorted on measures of firms' risk-neutral moments. Firms are sorted on average risk-neutral volatility, skewness, and kurtosis within each calendar quarter into terciles based on 30^{th} and 70^{th} percentiles. We then form equally-weighted portfolios of these firms, holding the moment ranking constant for the subsequent calendar quarter. Risk-neutral moments are calculated using the procedure in Bakshi, Kapadia, and Madan (2003); the options used are those closest to one, three, six, and twelve months to maturity. The first column of each panel presents mean monthly returns. The second column presents characteristic-adjusted returns, calculated by determining, for each firm, the Fama-French 5X5 size- and book-to-market portfolio to which it belongs and subtracting that return. The next three columns present the average beta, log market value and book-to-market equity ratio of the portfolio, while the next three columns present the average volatility, skewness and kurtosis of the portfolio. Monthly return data cover the period 4/96 through 12/05, for a total of 117 monthly observations.

Panel A: Volatility-Sorted Portfolios

1 Month to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.22	0.27	16.49	-1.52	11.38	0.89	15.71	0.37
2	0.98	0.14	25.78	-1.04	7.50	1.28	14.31	0.39
3	0.87	0.15	44.83	-1.13	5.33	1.78	13.61	0.42
3-1	-0.34	-0.12	28.34	0.38	-6.05	0.89	-2.10	0.05

3 Months to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.21	0.25	17.57	-1.33	9.38	0.84	15.68	0.38
2	1.08	0.21	27.27	-1.00	6.93	1.29	14.30	0.39
3	0.74	0.06	46.74	-1.15	5.31	1.83	13.64	0.40
3-1	-0.47	-0.19	29.17	0.18	-4.08	0.99	-2.04	0.02

6 Months to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.21	0.23	19.23	-0.86	5.56	0.82	15.62	0.40
2	1.13	0.27	29.46	-0.65	4.56	1.29	14.33	0.39
3	0.67	-0.01	48.61	-0.73	3.64	1.86	13.66	0.39
3-1	-0.55	-0.24	29.38	0.14	-1.92	1.05	-1.97	-0.01

12 Months to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.24	0.23	19.84	-0.84	5.39	0.82	15.46	0.40
2	1.06	0.20	30.36	-0.68	4.64	1.29	14.35	0.39
3	0.74	0.09	50.77	-0.80	3.77	1.85	13.81	0.38
3-1	-0.50	-0.14	30.94	0.04	-1.62	1.02	-1.65	-0.02

Table continued on next page...

Panel B: Skewness-Sorted Portfolios

1 Month to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.23	0.38	26.47	-2.65	15.03	1.25	15.37	0.34
2	0.88	0.07	30.37	-1.03	5.78	1.35	14.38	0.40
3	0.99	0.15	28.73	-0.02	3.97	1.27	13.86	0.44
3-1	-0.25	-0.22	2.26	2.63	-11.06	0.02	-1.51	0.10

3 Months to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.27	0.40	29.30	-2.52	13.20	1.24	15.34	0.35
2	0.93	0.12	31.43	-0.97	5.31	1.35	14.41	0.39
3	0.88	0.05	29.47	0.00	3.67	1.29	13.86	0.43
3-1	-0.39	-0.35	0.17	2.52	-9.53	0.05	-1.47	0.08

6 Months to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.37	0.50	32.40	-1.73	8.10	1.22	15.35	0.38
2	0.85	0.00	31.51	-0.59	3.45	1.30	14.48	0.39
3	0.88	0.12	32.72	0.07	2.58	1.37	13.76	0.41
3-1	-0.49	-0.38	0.32	1.80	-5.53	0.15	-1.59	0.03

12 Months to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.35	0.50	34.74	-1.84	8.42	1.21	15.42	0.38
2	0.85	0.00	32.32	-0.58	3.33	1.31	14.43	0.39
3	0.90	0.12	33.27	0.07	2.51	1.38	13.74	0.41
3-1	-0.45	-0.38	-1.47	1.91	-5.91	0.17	-1.68	0.04

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Panel C: Kurtosis-Sorted Portfolios

1 Month to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.01	0.11	34.80	-0.35	2.24	1.35	13.69	0.46
2	0.93	0.15	28.71	-1.00	5.81	1.32	14.37	0.39
3	1.15	0.32	22.62	-2.37	16.70	1.22	15.55	0.32
3-1	0.14	0.20	-12.18	-2.02	14.46	-0.12	1.87	-0.14

3 Months to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	ln MV	BM
1	0.92	0.06	35.36	-0.33	2.18	1.37	13.68	0.45
2	0.89	0.09	30.26	-0.94	5.38	1.33	14.38	0.39
3	1.28	0.44	24.97	-2.23	14.57	1.18	15.54	0.34
3-1	0.36	0.38	-10.40	-1.90	12.39	-0.20	1.87	-0.12

6 Months to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	ln MV	BM
1	0.92	0.10	36.13	-0.18	1.71	1.38	13.70	0.43
2	0.91	0.07	32.45	-0.58	3.46	1.36	14.39	0.39
3	1.26	0.43	27.73	-1.50	8.95	1.14	15.51	0.36
3-1	0.34	0.33	-8.40	-1.32	7.24	-0.24	1.81	-0.07

12 Months to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	ln MV	BM
1	0.90	0.07	36.49	-0.17	1.67	1.40	13.71	0.43
2	0.89	0.08	33.75	-0.57	3.35	1.36	14.36	0.39
3	1.30	0.45	29.61	-1.61	9.22	1.12	15.55	0.36
3-1	0.41	0.39	-6.88	-1.44	7.55	-0.27	1.84	-0.06

Table 3: Time Series Regressions: Fama-French Factor Risk Adjustment

The table presents the results of time series regressions of excess return differentials (Hi-Lo) between portfolios ranked on risk neutral volatility, skewness, and kurtosis on the three Fama and French (1993) factors MRP (the return on the value-weighted market portfolio in excess of a one-month T-Bill), SMB (the difference in returns on a portfolio of small capitalization and large capitalization stocks), and HML (the difference in returns on a portfolio of high and low book equity to market equity stocks). The moment-sorted portfolios are equally-weighted, formed on the basis of terciles and re-formed each quarter. The table presents point estimates of the coefficients and Newey-West standard errors in parentheses. Estimates that are statistically significant at the 10% or higher critical level are presented in boldfaced type. Data cover the period April 1996 through December 2005 for 117 monthly observations.

	Panel A: 1 Month to Maturity					Panel B: 3 Months to Maturity				
	α	β_{MRP}	β_{SMB}	β_{HML}	R^2	α	β_{MRP}	β_{SMB}	β_{HML}	R^2
Vol	**-0.59**	**0.51**	**0.82**	**-0.55**	75.83	-0.54	**0.56**	**0.88**	**-1.02**	83.85
	(0.33)	(0.10)	(0.09)	(0.11)		(0.34)	(0.11)	(0.09)	(0.12)	
Skew	-0.58	**0.18**	-0.02	**0.50**	18.34	**-0.65**	**0.24**	-0.07	0.32	10.52
	(0.37)	(0.09)	(0.16)	(0.14)		(0.35)	(0.08)	(0.17)	(0.15)	
Kurt	**0.56**	**-0.16**	**-0.24**	**-0.53**	28.04	**0.67**	**-0.28**	-0.15	**-0.24**	14.27
	(0.22)	(0.06)	(0.10)	(0.08)		(0.24)	(0.07)	(0.13)	(0.12)	

	Panel C: 6 Months to Maturity					Panel D: 12 Months to Maturity				
	α	β_{MRP}	β_{SMB}	β_{HML}	R^2	α	β_{MRP}	β_{SMB}	β_{HML}	R^2
Vol	-0.52	**0.59**	**0.89**	**-1.22**	85.28	-0.41	**0.54**	**0.81**	**-1.27**	85.16
	(0.35)	(0.12)	(0.09)	(0.13)		(0.35)	(0.11)	(0.08)	(0.12)	
Skew	**-0.64**	**0.23**	0.07	0.00	11.44	**-0.60**	**0.26**	0.09	-0.02	14.76
	(0.28)	(0.07)	(0.15)	(0.14)		(0.28)	(0.08)	(0.17)	(0.14)	
Kurt	**0.59**	**-0.29**	**-0.22**	-0.05	28.47	**0.67**	**-0.32**	**-0.25**	-0.04	33.26
	(0.22)	(0.06)	(0.12)	(0.10)		(0.22)	(0.07)	(0.13)	(0.10)	

Table 4: Descriptive Statistics: Co-Moment Portfolios

Panels A-B present summary statistics for portfolios sorted on measures of firms' risk-neutral co-moments. Firms are sorted on average risk-neutral co-skewness, and co-kurtosis with respect to the S&P 500 index within each calendar quarter into terciles based on 30^{th} and 70^{th} percentiles. We then form equally-weighted portfolios of these firms, holding the moment ranking constant for the subsequent calendar quarter. Risk-neutral moments are calculated using the procedure in Bakshi, Kapadia, and Madan (2003); the options used are those closest to one, three, six, and twelve months to maturity. The first column of each panel presents mean monthly returns. The second column presents characteristic-adjusted returns, calculated by determining, for each firm, the Fama-French 5X5 size- and book-to-market portfolio to which it belongs and subtracting that return. The next three columns present the risk-neutral volatility, skewness, and kurtosis for the portfolios, followed by the beta, risk-neutral co-skewness, and risk-neutral co-kurtosis. The final two columns present the average log market value and book-to-market equity ratio of the portfolio. Monthly return data cover the period 4/96 through 12/05, for a total of 117 monthly observations.

Panel A: Co-Skewness-Sorted Portfolios

1 Month to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	Co-Skew	Co-Kurt	ln MV	BM
1	1.29	0.24	0.21	-1.20	9.63	1.55	-6.46	6.25	15.03	0.38
2	1.16	0.22	0.28	-1.11	9.03	1.43	-3.10	2.00	14.45	0.39
3	1.00	0.21	0.34	-1.24	9.69	0.86	-1.15	0.54	14.37	0.40
3-1	-0.29	-0.03	0.13	-0.04	0.06	0.69	5.31	-5.71	-0.66	0.02

3 Months to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	Co-Skew	Co-Kurt	ln MV	BM
1	1.23	0.18	0.22	-1.18	9.15	1.56	-2.28	11.00	15.06	0.38
2	1.19	0.24	0.29	-1.06	8.25	1.44	-1.09	3.76	14.46	0.39
3	1.03	0.24	0.36	-1.13	8.24	0.85	-0.38	0.89	14.33	0.41
3-1	-0.20	0.06	0.14	0.05	-0.91	-0.71	1.90	-10.11	-0.73	0.03

6 Months to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	Co-Skew	Co-Kurt	ln MV	BM
1	1.19	0.16	0.23	-0.71	5.90	1.59	-2.53	13.72	15.13	0.38
2	1.17	0.22	0.31	-0.66	5.47	1.45	-1.29	4.77	4.47	0.39
3	1.09	0.29	0.37	-0.73	5.41	0.80	-0.47	1.26	14.24	0.41
3-1	-0.10	0.13	0.14	-0.02	-0.49	-0.79	-2.06	-12.46	-0.89	0.03

12 Months to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	Co-Skew	Co-Kurt	ln MV	BM
1	1.24	0.19	0.24	-0.71	5.83	1.61	-2.03	14.96	15.03	0.38
2	1.12	0.18	0.32	-0.71	5.49	1.44	-1.03	4.95	14.49	0.38
3	1.10	0.31	0.39	-0.80	5.50	0.79	-0.36	1.32	14.33	0.41
3-1	-0.14	0.12	0.15	-0.09	-0.33	-0.82	1.67	-13.64	-0.70	0.03

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Panel B: Co-Kurtosis-Sorted Portfolios

1 Month to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	Co-Skew	Co-Kurt	ln MV	BM
1	0.99	0.19	0.39	-1.22	8.31	1.12	-1.25	0.46	14.08	0.42
2	1.25	0.29	0.26	-1.08	9.12	1.42	-3.15	1.91	14.43	0.39
3	1.17	0.17	0.18	-1.25	10.88	1.30	-6.28	6.44	15.35	0.37
3-1	0.18	-0.02	-0.21	-0.03	2.57	0.28	-5.03	5.98	1.27	-0.05

3 Months to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	Co-Skew	Co-Kurt	ln MV	BM
1	1.00	0.19	0.40	-1.17	7.36	1.13	-0.42	0.74	14.06	0.41
2	1.26	0.31	0.28	-1.04	8.41	1.43	-1.11	3.58	14.44	0.39
3	1.16	0.14	0.19	-1.17	9.82	1.29	-2.21	11.38	15.35	0.38
3-1	0.16	-0.05	-0.21	0.00	2.46	0.16	-1.79	10.64	1.29	-0.03

6 Months to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	Co-Skew	Co-Kurt	ln MV	BM
1	1.07	0.25	0.42	-0.77	5.02	1.08	-0.52	1.08	14.03	0.41
2	1.20	0.28	0.30	-0.64	5.56	1.47	1.31	4.59	14.45	0.38
3	1.16	0.12	0.21	-0.70	6.18	1.28	-2.46	14.13	15.38	0.39
3-1	0.09	-0.13	-0.19	0.07	1.16	0.20	-1.98	13.05	1.35	-0.02

12 Months to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	Co-Skew	Co-Kurt	ln MV	BM
1	1.04	0.22	0.44	-0.86	5.13	1.08	-0.40	1.14	14.16	0.41
2	1.16	0.23	0.31	-0.68	5.58	1.46	-1.04	4.78	14.44	0.38
3	1.25	0.21	0.21	-0.69	6.08	1.29	-1.97	15.37	15.25	0.39
3-1	0.19	-0.01	-0.13	0.17	0.95	0.21	-1.57	14.23	1.09	-0.02

Table 5: Time Series Regressions: Co-Moment Risk Adjustment

The table presents the results of time series regressions of excess return differentials (Hi-Lo) between portfolios ranked on risk neutral volatility, skewness, and kurtosis on portfolios representing co-moment risk. Excess portfolio returns are regressed on the excess return on the S&P 500 in excess of the one month T-Bill return (MRP), the excess return on a portfolio long stocks with high risk-neutral co-skewness with the S&P 500 and short low risk-neutral co-skewness (CS), and the excess return on a portfolio long stocks with high risk-neutral co-kurtosis with the S&P 500 and short low risk-neutral co-kurtosis (CK). The moment-sorted portfolios are equally-weighted, formed on the basis of terciles and re-formed each quarter. The table presents point estimates of the coefficients and Newey-West standard errors in parentheses. Point estimates that are statistically significant at the 10% critical level or higher are presented in boldfaced type. Data cover the period April 1996 through December 2005 for 117 monthly observations.

	Panel A: 1 Month to Maturity					Panel B: 3 Months to Maturity				
	α	β_{MRP}	β_{CS}	β_{CK}	R^2	α	β_{MRP}	β_{CS}	β_{CK}	R^2
Vol	**-0.44**	**0.16**	**-1.55**	**-2.36**	92.95	**-0.45**	**0.17**	**-1.64**	**-2.54**	95.07
	(0.20)	(0.06)	(0.08)	(0.07)		(0.20)	(0.05)	(0.12)	(0.08)	
Skew	-0.15	0.15	**0.76**	0.43	15.43	-0.43	**0.31**	**0.71**	0.40	20.63
	(0.34)	(0.15)	(0.38)	(0.36)		(0.33)	(0.14)	(0.31)	(0.29)	
Kurt	0.09	0.04	-0.07	0.08	3.83	0.39	-0.18	-0.08	0.14	16.75
	(0.26)	(0.12)	(0.32)	(0.28)		(0.24)	(0.12)	(0.26)	(0.25)	

	Panel C: 6 Months to Maturity					Panel D: 12 Months to Maturity				
	α	β_{MRP}	β_{CS}	β_{CK}	R^2	α	β_{MRP}	β_{CS}	β_{CK}	R^2
Vol	**-0.55**	**0.18**	**-1.63**	**-2.47**	96.28	-0.26	**0.13**	**-1.70**	**-2.52**	96.45
	(0.18)	(0.05)	(0.09)	(0.07)		(0.17)	(0.04)	(0.08)	(0.07)	
Skew	**-0.5**4	0.18	-0.05	-0.21	16.67	-0.47	0.17	-0.18	-0.30	19.43
	(0.27)	(0.12)	(0.25)	(0.25)		(0.29)	(0.13)	(0.27)	(0.27)	
Kurt	**0.39**	**-0.16**	0.21	**0.35**	31.94	**0.41**	-0.15	**0.32**	**0.45**	37.88
	(0.22)	(0.09)	(0.17)	(0.16)		(0.22)	(0.10)	(0.15)	(0.16)	

Table 6: Parametric Stochastic Discount Factor Risk Adjustments

The table presents point estimates of the parameters of a stochastic discount factor polynomial in the returns of two candidate portfolios. Results using the S&P 500 index as the candidate portfolio are presented in Panels A-D and results using the tangency portfolio return implied by 14 industry index portfolios are presented in Panels E-H. The stochastic discount factor is specified as

$$m_t = d_0 + d_1 r_{T,t} + d_2 r_{T,t}^2 + d_3 r_{T,t}^3$$

where $r_{T,t}$ is either the return on the S&P 500 index (Panels A-D) or the return on the industry index tangency portfolio (Panels E-H). The parameters are estimated via GMM using the sample moment restrictions

$$\hat{\alpha} = \frac{1}{T} \sum_{t=1}^{T} \left((1 + r_t)\, m_t - 1_N \right) = 0$$

where r_t is a 10×1 vector of returns comprising 3 portfolios sorted on risk-neutral volatility, 3 portfolios sorted on risk-neutral skewness, 3 portfolios sorted on risk-neutral kurtosis, and a Treasury Bill. The column titled 'J' presents the test statistic for the overidentifying restrictions. In addition to point estimates, we present the pricing errors associated with high-low factor mimicking portfolios formed on volatility, skewness, and kurtosis in the columns α_{vol}, α_{skew}, and α_{kurt}, respectively. We examine three versions of the model above. The first restricts $d_2 = d_3 = 0$, representing a linear specification, the second restricts $d_3 = 0$, representing a quadratic specification, and the final, representing a cubic specification, is unrestricted. Panel A and E presents results for returns formed on the basis of options with one month to maturity; Panels B-D and F-H present complementary results for options based on three, six, and twelve months to maturity. The 14 industries used in defining the tangency portfolio are described in Table A1. Newey-West standard errors are presented in parentheses below the point estimates and p-values for the J-statistic are presented in parentheses below the statistic. Point estimates of coefficients that are statistically significant at the 10% level or greater are presented in boldfaced type. The data cover the period 4/30/1996 through 1/31/2005 for 106 monthly observations.

Panel A: $r_{T,t}$ S&P 500 index and 1 Month to Maturity

d_0	d_1	d_2	d_3	J	α_{vol}	α_{skew}	α_{kurt}
1.00	-3.24			9.35	-0.79	0.02	-0.13
(0.03)	(2.04)			(0.31)	(0.70)	(0.38)	(0.28)
0.99	-1.57	0.58		9.61	-0.54	**-0.21**	0.08
(0.01)	(5.84)	(5.82)		(0.21)	(0.44)	(0.09)	(0.23)
1.00	0.44	4.15	-7.14	8.85	**-1.16**	-0.12	0.05
(0.04)	(5.95)	(7.55)	(9.90)	(0.18)	(0.55)	(0.22)	(0.15)

Panel B: $r_{T,t}$ S&P 500 index and 3 Months to Maturity

d_0	d_1	d_2	d_3	J	α_{vol}	α_{skew}	α_{kurt}
1.00	-3.28			12.20	-1.23	-0.22	0.24
(0.03)	(2.02)			(0.14)	(0.87)	(0.35)	(0.34)
0.99	-0.73	1.09		8.69	-0.54	**-0.50**	0.43
(0.01)	(5.54)	(5.74)		(0.28)	(0.32)	(0.20)	(0.31)
1.00	-3.10	4.18	-3.28	8.52	**-1.33**	-0.40	0.36
(0.04)	(4.71)	(6.54)	(7.57)	(0.20)	(0.77)	(0.25)	(0.27)

Panel C: $r_{T,t}$ S&P 500 index and 6 Months to Maturity

d_0	d_1	d_2	d_3	J	α_{vol}	α_{skew}	α_{kurt}
1.00	**-3.51**			19.30	-1.39	-0.48	0.32
(0.03)	(1.95)			(0.01)	(0.92)	(0.35)	(0.32)
0.99	-0.17	1.43		17.65	**-0.43**	**-0.65**	0.47
(0.01)	(5.22)	(6.49)		(0.01)	(0.22)	(0.31)	(0.34)
1.00	-4.34	5.40	-2.66	15.79	-1.43	**-0.70**	**0.49**
(0.05)	(6.11)	(8.77)	(7.72)	(0.02)	(0.88)	(0.25)	(0.26)

Panel D: $r_{T,t}$ S&P 500 index and 12 Months to Maturity

d_0	d_1	d_2	d_3	J	α_{vol}	α_{skew}	α_{kurt}
1.00	**-3.57**			15.12	-1.36	-0.44	0.41
(0.03)	(2.00)			(0.06)	(0.90)	(0.37)	(0.34)
0.99	-0.26	1.38		12.68	**-0.40**	**-0.59**	0.51
(0.01)	(5.30)	(6.70)		(0.08)	(0.20)	(0.31)	(0.33)
1.00	-2.88	7.43	-5.78	11.73	**-1.52**	**-0.72**	0.65
(0.06)	(6.64)	(9.76)	(9.00)	(0.07)	(0.89)	(0.28)	(0.29)

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Panel E: $r_{T,t}$ Industry Tangency and 1 Month to Maturity

$\dot{d}_0$	d_1	d_2	d_3	J	α_{vol}	α_{skew}	α_{kurt}
1.00	**-3.60**			7.73	-0.84	0.05	-0.16
(0.03)	(2.08)			(0.46)	(0.70)	(0.38)	(0.28)
0.99	-1.57	0.58		8.48	-0.54	**-0.21**	-0.08
(0.01)	(5.84)	(5.81)		(0.29)	(0.44)	(0.09)	(0.23)
1.00	0.41	3.24	-6.90	7.22	**-1.20**	-0.02	-0.02
(0.04)	(4.92)	(7.18)	(8.55)	(0.30)	(0.62)	(0.21)	(0.15)

Panel F: $r_{T,t}$ Industry Tangency and 3 Months to Maturity

d_0	d_1	d_2	d_3	J	α_{vol}	α_{skew}	α_{kurt}
1.00	-3.36			12.83	-1.22	-0.19	0.23
(0.03)	(2.02)			(0.12)	(0.85)	(0.37)	(0.31)
0.99	-0.73	1.09		13.27	-0.54	**-0.50**	-0.43
(0.01)	(5.54)	(5.57)		(0.07)	(0.32)	(0.20)	(0.31)
1.00	-3.03	0.73	-1.03	12.84	**-1.29**	-0.21	0.25
(0.03)	(4.81)	(5.53)	(7.47)	(0.05)	(0.76)	(0.26)	(0.26)

Panel G: $r_{T,t}$ Industry Tangency and 6 Months to Maturity

$\dot{d}_0$	d_1	d_2	d_3	J	α_{vol}	α_{skew}	α_{kurt}
1.00	**-3.71**			16.95	-1.44	-0.47	0.31
(0.03)	(1.96)			(0.03)	(0.93)	(0.35)	(0.32)
0.99	-0.17	1.43		15.81	**-0.43**	**-0.65**	0.47
(0.01)	(5.21)	(6.49)		(0.03)	(0.22)	(0.31)	(0.34)
1.00	0.30	5.41	-8.46	12.31	**-1.83**	**-0.59**	0.48
(0.05)	(6.40)	(9.71)	(8.04)	(0.06)	(0.90)	(0.26)	(0.29)

Panel H: $r_{T,t}$ Industry Tangency and 12 Months to Maturity

d_0	d_1	d_2	d_3	J	α_{vol}	α_{skew}	α_{kurt}
1.00	**-3.60**			14.38	-1.36	-0.44	0.41
(0.03)	(2.00)			(0.07)	(0.90)	(0.37)	(0.34)
0.99	-0.27	1.38		12.12	**-0.40**	**-0.59**	0.51
(0.01)	(5.30)	(6.69)		(0.10)	(0.20)	(0.31)	(0.33)
1.00	0.48	7.52	-9.65	10.37	**-1.71**	**-0.66**	**0.67**
(0.06)	(8.54)	(10.09)	(9.25)	(0.11)	(0.84)	(0.29)	(0.31)

Table 7: Parametric versus Non-Parametric Stochastic Discount Factor Risk Adjustments

The table presents risk adjustments for the volatility, skewness, and kurtosis factor mimicking portfolios using stochastic discount factors implied by the S&P 500 risk neutral and physical densities. The stochastic discount factor is formed as a risk-free scaled ratio of the risk-neutral to physical probability measure

$$m_t(x, s, \tau) = e^{-r_t^f(\tau)} \frac{f_t^Q(x, s, \tau)}{f_t^P(x, s, \tau)}$$

where $f_t^Q(\cdot)$ is the risk-neutral probability measure at time t, $f_t^P(\cdot)$ is the physical probability measure at time t, and τ is the horizon. We approximate the risk-neutral and physical probability distributions using the Normal Inverse Gaussian (NIG) distribution. The risk neutral measure is approximated using the risk neutral moments calculated in the paper and the physical measure is calculated using returns data on the S&P 500 over the 1000 days prior to 3/31/1996. The table presents excess returns implied by discounting the factor mimicking portfolios by the stochastic discount factor,

$$\hat{\alpha} = \frac{1}{T} \sum_{t=1}^{T} r_t(\tau)\, m_t(x_t, \tau)$$

where $r_t(\tau)$ is the τ-period return on the factor-mimicking portfolio at time t, and $m_t(x_t, \tau)$ is the stochastic discount factor evaluated at the observed τ-period realization of the S&P 500 at time t. The column labeled "NIG" represents the discount factor implied by the NIG approximations to the densities. Columns "Linear," "Quad," and "Cubic" represent discount factors obtained by projecting the density-implied discount factor onto a linear, quadratic, and cubic polynomial, respectively. Panel A presents results for the volatility-sorted factor mimicking portfolio with rows representing portfolios formed on volatility estimated using options with one, three, six, and twelve-months to maturity. Panels B and C present complementary results for skewness- and kurtosis-sorted factor mimicking portfolios. We separately examine stochastic discount factors based on options and returns with three, six, and 12 month horizons. Data for the three, six, and twelve month horizons begin in January, 1997, July, 1996, and April,1996, respectively. All three horizons extend through December, 2005 for 106 (overlapping) observations. Point estimates are scaled to the monthly frequency, and Newey-West standard errors are presented in parentheses below the point estimates. Point estimates that are significantly different than zero at the 10% or higher significance level are presented in boldfaced type.

Panel A: Volatility

	Three Month				Six Month				Twelve Month			
	NIG	Linear	Quad	Cubic	NIG	Linear	Quad	Cubic	NIG	Linear	Quad	Cubic
1 Month	-0.17	0.15	0.17	0.16	-0.55	-0.24	-0.22	-0.26	**-1.03**	-0.17	-0.21	-0.27
	(0.37)	(0.37)	(0.37)	(0.37)	(0.79)	(0.69)	(0.71)	(0.70)	(0.61)	(0.64)	(0.65)	(0.65)
3 Month	-0.19	0.17	0.18	0.16	-0.72	-0.48	-0.45	-0.50	-1.51	-0.31	-0.40	-0.46
	(0.51)	(0.50)	(0.50)	(0.50)	(1.12)	(0.95)	(0.96)	(0.96)	(0.92)	(0.90)	(0.91)	(0.91)
6 Month	-0.19	0.18	0.17	0.15	-0.79	-0.58	-0.56	-0.60	-1.73	-0.37	-0.47	-0.55
	(0.58)	(0.57)	(0.58)	(0.57)	(1.29)	(1.08)	(1.10)	(1.10)	(1.07)	(1.03)	(1.04)	(1.04)
12 Month	-0.16	0.19	0.18	0.16	-0.74	-0.60	-0.58	-0.62	-1.70	-0.35	-0.46	-0.53
	(0.58)	(0.57)	(0.57)	(0.57)	(1.29)	(1.06)	(1.07)	(1.07)	(1.07)	(1.03)	(1.04)	(1.04)

Table continued on next page ...

Panel B: Skewness

	Three Month				Six Month				Twelve Month			
	NIG	Linear	Quad	Cubic	NIG	Linear	Quad	Cubic	NIG	Linear	Quad	Cubic
1 Month	-0.21	-0.22	-0.20	-0.20	-0.52	-0.09	-0.03	-0.05	0.08	-0.43	-0.34	-0.32
	(0.26)	(0.31)	(0.32)	(0.31)	(0.60)	(0.53)	(0.55)	(0.55)	(0.45)	(0.54)	(0.55)	(0.54)
3 Month	-0.29	-0.27	-0.27	-0.27	-0.70	-0.28	-0.24	-0.25	-0.27	-0.58	-0.52	-0.51
	(0.21)	(0.25)	(0.26)	(0.25)	(0.44)	(0.39)	(0.40)	(0.40)	(0.27)	(0.42)	(0.42)	(0.241)
6 Month	**0.30**	**-0.27**	**-0.27**	**-0.27**	**-0.79**	**-0.49**	-0.44	-0.46	**-0.60**	**-0.69**	**-0.65**	**-0.65**
	(0.13)	(0.16)	(0.16)	(0.16)	(0.28)	(0.28)	(0.29)	(0.28)	(0.24)	(0.31)	(0.30)	(0.30)
12 Month	**-0.28**	-0.21	-0.21	-0.21	**-0.70**	-0.39	-0.34	-0.36	**-0.55**	**-0.58**	**-0.54**	**-0.55**
	(0.12)	(0.16)	(0.16)	(0.16)	(0.27)	(0.28)	(0.28)	(0.28)	(0.23)	(0.28)	(0.27)	(0.27)

Panel C: Kurtosis

	Three Month				Six Month				Twelve Month			
	NIG	Linear	Quad	Cubic	NIG	Linear	Quad	Cubic	NIG	Linear	Quad	Cubic
1 Month	0.15	0.07	0.05	0.05	0.33	-0.06	-0.11	-0.09	-0.05	0.20	0.15	0.14
	(0.19)	(0.23)	(0.23)	(0.23)	(0.44)	(0.41)	(0.41)	(0.41)	(0.37)	(0.38)	(0.38)	(0.38)
3 Month	**0.26**	0.16	0.15	0.16	**0.69**	0.28	0.24	0.26	**0.53**	**0.52**	**0.49**	**0.50**
	(0.12)	(0.17)	(0.17)	(0.17)	(0.28)	(0.30)	(0.31)	(0.30)	(0.22)	(0.30)	(0.30)	(0.29)
6 Month	**0.23**	0.13	0.12	0.13	**0.63**	0.30	0.27	0.29	**0.60**	0.48	0.47	**0.49**
	(0.12)	(0.15)	(0.15)	(0.15)	(0.27)	(0.32)	(0.33)	(0.32)	(0.28)	(0.30)	(0.30)	(0.29)
12 Month	**0.27**	0.16	0.15	0.16	**0.69**	0.36	0.33	0.35	**0.69**	**0.52**	**0.52**	**0.53**
	(0.13)	(0.14)	(0.15)	(0.15)	(0.28)	(0.33)	(0.33)	(0.33)	(0.30)	(0.28)	(0.29)	(0.28)

Table 8: Implied Physical Moments

The table presents the moments of the implied physical distributions of 14 industry index portfolios. Distributions are imputed by letting the physical distribution, $f^P(x, s, \tau)$ be related to the risk neutral distribution, $f^Q(x, s, \tau)$ by

$$f^P(x, s, \tau) = e^{-r_f \tau} \frac{f^Q(x, s, \tau)}{m(x, s, \tau)}$$

where $m(x, s, \tau)$ is the linear stochastic discount factor implied by the tangency portfolio pricing the 14 industry portfolios plus the two extreme skewness portfolios. The risk neutral distribution is the NIG approximation to the industry risk neutral portfolio using risk neutral moments computed from industry index portfolio option data via the Bakshi, Kapadia, and Madan (2003) methodology. The 14 industries used in defining the tangency portfolio are described in Table A1. Average moments are computed for four subperiods: 1996 Q2 - 1998 Q2, 1998 Q3 - 2000 Q1, 2000 Q2 - 2002 Q4, and 2003 Q1 - 2005 Q4 in Panels A, B, C, and D, respectively. In all three cases, we utilize the stochastic discount factor with a horizon of 12 months.

Panel A: Subperiod I: 1996 Q2 - 1998 Q2

	BKX	BTK	CMR	CYC	DRG	MSH	TXX	UTY	XAL	XAU	XBD	XCI	XNG	XOI
Mean	4.50	4.77	4.26	4.02	4.65	5.34	4.33	2.91	3.89	3.78	5.28	5.28	3.62	3.36
Std. Dev.	11.01	12.73	10.61	10.13	11.20	12.58	11.26	8.47	9.41	12.02	11.66	12.24	10.25	9.41
Skew.	0.21	0.01	0.34	0.32	0.25	-0.12	0.13	0.29	0.37	-0.21	0.03	0.20	0.37	0.56
Kurt.	24.09	13.98	21.10	21.68	19.46	16.61	21.88	28.89	25.09	16.30	18.58	14.25	22.11	26.39

Panel B: Subperiod II: 1998 Q3 - 2000 Q1

	BKX	BTK	CMR	CYC	DRG	MSH	TXX	UTY	XAL	XAU	XBD	XCI	XNG	XOI
Mean	5.62	4.46	5.36	5.16	5.02	5.66	4.71	3.52	3.71	4.23	4.64	5.00	3.49	4.16
Std. Dev.	13.48	12.52	12.27	12.41	12.73	12.68	11.58	10.35	10.24	13.82	13.12	12.49	10.68	11.16
Skew.	0.02	-0.14	0.50	0.32	0.18	-0.10	0.00	0.34	-0.20	-0.51	-0.19	-0.12	0.04	0.40
Kurt.	11.13	16.78	10.59	11.98	12.08	14.46	20.01	21.89	19.36	14.12	14.26	13.10	22.09	16.46

Panel C: Subperiod III: 2000 Q2 - 2002 Q4

	BKX	BTK	CMR	CYC	DRG	MSH	TXX	UTY	XAL	XAU	XBD	XCI	XNG	XOI
Mean	5.15	5.38	3.36	4.28	3.96	5.25	4.84	3.73	4.69	5.41	4.81	5.02	4.06	3.52
Std. Dev.	12.88	14.25	10.39	11.83	11.92	13.76	13.65	11.32	13.48	14.47	13.17	12.76	12.00	10.79
Skew.	-0.27	-0.61	0.09	-0.03	-0.19	-0.45	-0.51	-0.14	-0.58	-0.50	-0.39	-0.38	-0.36	0.14
Kurt.	16.36	14.84	25.49	19.27	19.03	15.26	15.55	21.86	17.14	12.78	16.25	14.62	20.80	21.08

Panel D: Subperiod IV: 2003 Q1 - 2005 Q4

	BKX	BTK	CMR	CYC	DRG	MSH	TXX	UTY	XAL	XAU	XBD	XCI	XNG	XOI
Mean	2.24	3.32	1.67	2.93	2.33	3.30	3.22	1.95	1.28	4.62	3.46	3.00	1.97	1.69
Std. Dev.	8.46	11.20	7.79	10.64	9.27	11.22	11.24	7.74	3.79	12.88	11.22	10.86	8.61	7.88
Skew.	-0.41	-0.59	-0.24	-0.37	-0.40	-0.58	-0.45	-0.30	-1.21	-0.77	-0.47	-0.47	-0.41	-0.23
Kurt.	49.33	29.17	61.09	28.38	39.52	28.06	26.01	74.75	15.99	16.84	27.48	31.50	47.72	58.63

Table 9: Sorts on Implied P-measure Moments

The table presents average returns on portfolios of industry indices sorted on P-measure moments. P-measure probability distributions are computed by assuming that the stochastic discount factor, $M(x, s, \tau)$ is given by

$$M(R^T, s, \tau) = d_0 + d_1 R^T(s, \tau)$$

where d_0 and d_1 are coefficients implied by moments of the tangency portfolio, R^T. The tangency portfolio is defined by the fourteen index returns plus the extreme skewness-sorted portfolio returns. The physical measure is constructed as

$$P(i, R^T, s, \tau) = \exp(r\tau)\frac{Q(i, R^T, s, \tau)}{M(R^T, s, \tau)}$$

where $Q(i, R^T, s, \tau)$ is the risk-neutral distribution implied by the NIG approximation calculated from the industry index' τ-period risk neutral moments. Each month, we integrate over the physical probability measure to obtain expectations, standard deviations, skewnesses, and kurtoses of the 14 industry indices and rank industry indices on the basis of these physical moments. We form equally-weighted portfolios on the basis of these rankings and hold the portfolio for the subsequent month. Portfolio P1 represents the bottom three industries, P2 the middle eight, and P3 the top three. The table presents average monthly returns over the period 4/1997-12/2005.

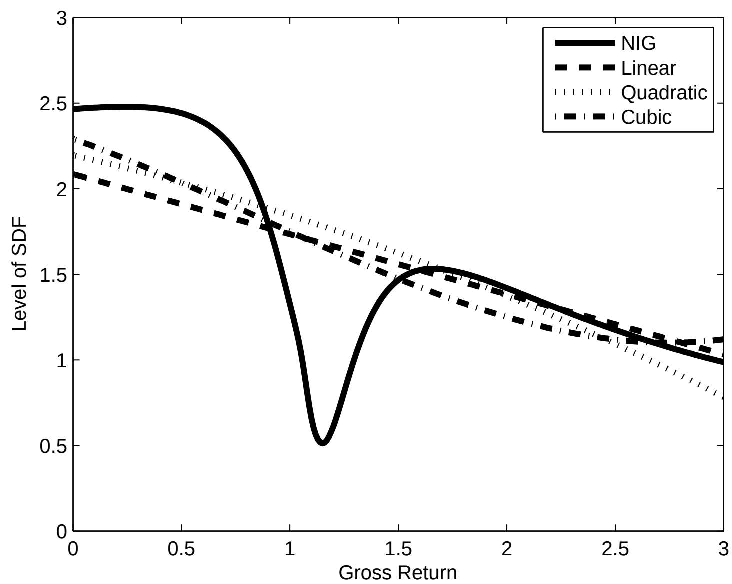
	P1	P2	P3
Expectation	0.95	1.02	1.44
Std. Dev.	1.14	1.25	0.63
Skewness	1.19	1.08	1.05
Kurtosis	1.15	0.88	1.62

Figure 1: Stochastic Discount Factors

The plots depict stochastic discount factors formed using risk neutral moments of S&P 500 index options at the 12-month maturity. The plot labeled 'NIG' represents stochastic discount factors, $M(x, s, \tau)$, formed as

$$M(x, s, \tau) = e^{-r_f \tau} \frac{f^Q(x, s, \tau)}{f^P(x, s, \tau)}$$

where $f(\cdot)$ is the NIG probability density function, Q denotes the risk-neutral probability measure, and P denotes the physical measure. The risk neutral measure is calculated using risk neutral moments retrieved from option prices and the physical measure using the historical moments of the S&P 500 index from 1992 through 1995. 'Linear,' 'Quadratic,' and 'Cubic' represent linear, quadratic, and cubic polynomial fits to the NIG kernel.



A1: Robustness Checks

We perform several additional robustness checks on our results to examine the possibility that return differentials are driven by liquidity issues, either in the underlying equity returns or by stale or illiquid option prices. To examine the latter possibility, we add an additional filter to our sample, and eliminate the observation if there is no trading in any of the out-of-the-money options on a particular day. These results are presented in Appendix Table A2. The principal impact of this restriction is to substantially reduce our sample. As discussed above, on average there are 911 firms per month in our original sample (273/365/273 by tercile). Imposing the trading restriction reduces the average number of firms to 307. However, as shown in the table, with the exception of short-maturity kurtosis-sorted portfolios, the magnitude of return differentials across portfolios remains stable, or actually increases. Thus, we continue to find that returns are negatively related to volatility and skewness, and positively related to kurtosis.[20]

Second, we add the liquidity factor of Pastor and Stambaugh (2003) to our time series regression and re-estimate the factor-adjusted returns. These results are presented in Appendix Table A3. Data on the liquidity factor are taken from Wharton Research Data Services, and cover the sample period only through December, 2004. The basic results change very little. The intercepts retain negative signs for volatility and skewness and positive signs for kurtosis across all three maturity bins. Statistical significance declines slightly; the alpha for the volatility portfolio loses its statistical significance for all maturities. The alpha for the skewness portfolio also loses significance in the shortest maturity option sort. Finally, the liquidity factor does add explanatory power, particularly in the case of the skewness-sorted portfolios, with regression R^2's nearing or exceeding 40%. However, the overall conclusions are similar: high volatility and high skewness stocks earn negative excess returns, and high kurtosis stocks earn positive excess returns.

A third robustness check involves the stochastic discount factor of Section 4.2. As noted therein, our choice of the S&P 500 and industry indices as the index returns in the stochastic discount factor may bias our results toward larger, more established firms that exhibit less skewness in returns. Consequently, we replicate our analysis using the Nasdaq 100 index as an alternative. Results are presented in Table A4. As shown in the table, the use of the Nasdaq 100 does not qualitatively change our conclusions. The majority of moment-sorted portfolio alphas remain large in economic magnitude, but inference remains problematic as the standard errors are large. These results mirror those using the S&P 500 and the industry

[20]For brevity, we report only the average and characteristic-adjusted average returns to these portfolios. The remaining characteristics exhibit similar patterns to those depicted in Table 2. These results are available from the authors upon request.

tangency portfolio as the arguments of the stochastic discount factor.

The implied physical distributions in Sections 4.3 and 5 are constructed with a combination of forward-looking data from option prices, and an estimate of the market's physical distribution estimated from historical market returns data. Since this use of historical market returns is the only instance where *ex post* data are used, we explore the sensitivity of our results in Section 4.3 to different choices of the historical record to estimate the market's underlying returns distribution. First, we maintain the length of the 1000-day window, but allow the window to roll forward with the corresponding risk-neutral distribution obtained from option prices. Results of this estimation are presented in Table A5. As shown in the Table, results for the skewness and kurtosis-sorted portfolios are materially unchanged; skewness-sorted returns maintain negative and frequently statistically significant pricing errors, while kurtosis-sorted returns retain positive pricing errors. The main difference in these results is that, using the three month and six month stochastic discount factor, the volatility-sorted portfolio pricing errors are no longer statistically or economically significant.

Second, to ensure that the window includes relatively rare 'extreme' events, but to maintain the rolling nature of our estimates, we estimate an AR(1) for each of the moments over the period January, 1962 through March, 1996. At the end of each month, we calculate the moment over the past 1000 days as above. We then estimate an AR(1) on each moment, θ_t^n,

$$\theta_t^n = \theta_0^n + \rho_{\theta^n}\theta_{t-1}^n + \eta_t^n \tag{A1}$$

where $n = 2, 3, 4$ are the moments. Each month from April, 1996, through December, 2005, we calculate the predicted moment from these parameters,

$$\hat{\theta}_t^n = \hat{\theta}_0^n + \hat{\rho}_{\theta^n}\theta_{t-1}^n \tag{A2}$$

where θ_{t-1}^n is the rolling sample moment. Results using these estimates of physical moments are presented in Table A6. As shown in the table, these results are quite similar to those reported previously.

Table A1: Industry Definitions

Ticker	Description
BKX	KBW Bank Index
BTK	AMEX Biotechnology Index
CMR	Morgan Stanley Consumer Index
CYC	Morgan Stanley Cyclical Index
DRG	AMEX Pharmaceutcial Index
MSH	Morgan Stanley High-Technology Index
TXX	CBOE Technology Index
UTY	PHLX Utility Sector Index
XAL	AMEX Airline Index
XAU	PHLX Gold and Silver Sector Index
XBD	AMEX Securities Broker/Dealer Index
XCI	AMEX Computer Technology Index
XNG	AMEX Natural Gas Index
XOI	AMEX Oil Index

Table A2: : Summary Statistics: Moment-Sorted Portfolios with Volume Screens

Panels A-C present summary statistics for portfolios sorted on measures of firms' risk-neutral moments. Firms are sorted on average risk-neutral volatility, skewness, and kurtosis within each calendar quarter into terciles based on 30^{th} and 70^{th} percentiles. We then form equally-weighted portfolios of these firms, holding the moment ranking constant for the subsequent calendar quarter. Risk-neutral moments are calculated using the procedure in Bakshi, Kapadia, and Madan (2003); the options used are those closest to one, three, six, and twelve months to maturity. We eliminate firms that do not have trading volume in at least one OTM put and OTM call in a calendar month. The first column of each panel presents mean monthly returns. The second column presents characteristic-adjusted returns, calculated by determining, for each firm, the Fama-French 5X5 size- and book-to-market portfolio to which it belongs and subtracting that return. Monthly return data cover the period 4/96 through 12/05, for a total of 117 monthly observations.

Panel A: Volatility-Ranked Portfolios

1 Month Maturity			3 Month Maturity			6 Month Maturity			12 Month Maturity		
Rank	Mean	Char Adj	Rank	Mean	Char Adj	Rank	Mean	Char Adj	Rank	Mean	Char Adj
1	1.29	0.35	1	1.28	0.37	1	1.34	0.44	1	1.35	0.44
2	0.89	0.52	2	0.99	0.10	2	1.01	0.14	2	0.99	0.09
3	1.00	0.22	3	0.86	0.13	3	0.77	0.00	3	0.79	0.08
3-1	-0.29	-0.13	3-1	-0.42	-0.24	3-1	-0.57	-0.44	3-1	-0.56	-0.36

Panel B: Skewness-Ranked Portfolios

1 Month Maturity			3 Month Maturity			6 Month Maturity			12 Month Maturity		
Rank	Mean	Char Adj	Rank	Mean	Char Adj	Rank	Mean	Char Adj	Rank	Mean	Char Adj
1	1.28	0.38	1	1.36	0.46	1	1.45	0.55	1	1.45	0.57
2	0.97	0.16	2	0.99	0.20	2	0.98	0.15	2	1.04	0.20
3	0.89	0.04	3	0.79	-0.10	3	0.71	-0.12	3	0.63	-0.22
3-1	-0.39	-0.34	3-1	-0.57	-0.56	3-1	-0.74	-0.67	3-1	-0.82	-0.79

Panel C: Kurtosis-Ranked Portfolios

1 Month Maturity			3 Month Maturity			6 Month Maturity			12 Month Maturity		
Rank	Mean	Char Adj	Rank	Mean	Char Adj	Rank	Mean	Char Adj	Rank	Mean	Char Adj
1	0.95	0.329	1	0.74	-0.08	1	0.62	-0.22	1	0.63	-0.25
2	0.97	0.103	2	1.06	0.23	2	1.13	0.30	2	1.11	0.31
3	1.22	0.196	3	1.30	0.42	3	1.34	0.46	3	1.35	0.46
3-1	0.27	-0.133	3-1	0.56	0.50	3-1	0.72	0.68	3-1	0.72	0.71

Table A3: Time Series Regressions

The table presents the results of time series regressions of excess return differentials (Hi-Lo) between portfolios ranked on risk neutral volatility, skewness, and kurtosis on the three Fama and French (1993) factors MRP (the return on the value-weighted market portfolio in excess of a one-month T-Bill), SMB (the difference in returns on a portfolio of small capitalization and large capitalization stocks), and HML (the difference in returns on a portfolio of high and low book equity to market equity stocks), and LIQ, the liquidity factor from Pastor and Stambaugh (2001). The moment-sorted portfolios are equally-weighted, formed on the basis of terciles and re-formed each quarter. The table presents point estimates of the coefficients and standard errors in parentheses. Point estimates that are statistically significant at the 10% level or greater are in boldfaced type. Data cover the period April 1996 through December 2005 for 117 monthly observations.

	Panel A: 1 Month to Maturity						Panel B: 3 Months to Maturity					
	α	β_{MRP}	β_{SMB}	β_{HML}	β_{LIQ}	R^2	α	β_{MRP}	β_{SMB}	β_{HML}	β_{LIQ}	R^2
Vol	-0.58	**0.52**	**1.01**	**-0.61**	**-0.28**	80.89	-0.60	**0.57**	**1.00**	**-1.06**	**-0.17**	85.47
	(0.36)	(0.11)	(0.10)	(0.12)	(0.06)		(0.39)	(0.13)	(0.11)	(0.13)	(0.07)	
Skew	-0.48	**0.16**	**0.23**	**0.41**	**-0.40**	51.71	**-0.57**	**0.22**	0.16	**0.23**	**-0.37**	41.15
	(0.32)	(0.06)	(0.10)	(0.12)	(0.07)		(0.33)	(0.07)	(0.11)	(0.14)	(0.08)	
Kurt	**0.51**	**-0.15**	**-0.38**	**-0.48**	**0.23**	49.37	**0.64**	**-0.27**	**-0.28**	-0.19	**0.22**	29.58
	(0.22)	(0.05)	(0.07)	(0.08)	(0.04)		(0.26)	(0.07)	(0.10)	(0.11)	(0.06)	

	Panel C: 6 Months to Maturity						Panel D: 12 Months to Maturity					
	α	β_{MRP}	β_{SMB}	β_{HML}	β_{LIQ}	R^2	α	β_{MRP}	β_{SMB}	β_{HML}	β_{LIQ}	R^2
Vol	-0.58	**0.59**	**1.00**	**-1.26**	**-0.15**	86.62	-0.45	**0.55**	**0.90**	**-1.31**	**-0.13**	86.28
	(0.40)	(0.13)	(0.11)	(0.14)	(0.07)		(0.40)	(0.13)	(0.11)	(0.13)	(0.07)	
Skew	**-0.63**	**0.22**	**0.24**	-0.07	**-0.28**	32.80	**-0.58**	**0.25**	**0.28**	-0.11	**-0.30**	36.89
	(0.28)	(0.07)	(0.11)	(0.13)	(0.07)		(0.29)	(0.07)	(0.11)	(0.13)	(0.08)	
Kurt	**0.58**	**-0.27**	**-0.33**	0.01	**0.20**	40.70	**0.65**	**-0.30**	**-0.37**	0.02	**0.22**	46.67
	(0.24)	(0.07)	(0.09)	(0.09)	(0.05)		(0.24)	(0.08)	(0.09)	(0.09)	(0.05)	

Table A4: Parametric Stochastic Discount Factor Risk Adjustments: Nasdaq 100

The table presents point estimates of the parameters of a stochastic discount factor polynomial in the returns of two candidate portfolios. Results using the Nasdaq 100 index as the candidate portfolio are presented in Panels A-D. The stochastic discount factor is specified as

$$m_t = d_0 + d_1 r_{T,t} + d_2 r_{T,t}^2 + d_3 r_{T,t}^3$$

where $r_{T,t}$ is either the return on the S&P 500 index (Panels A-D) or the return on the industry index tangency portfolio (Panels E-H). The parameters are estimated via GMM using the sample moment restrictions

$$\hat{\alpha} = \frac{1}{T} \sum_{t=1}^{T} \left((1 + r_t) \, m_t - 1_N \right) = 0$$

where r_t is a 10×1 vector of returns comprising 3 portfolios sorted on risk-neutral volatility, 3 portfolios sorted on risk-neutral skewness, 3 portfolios sorted on risk-neutral kurtosis, and a Treasury Bill. The column titled 'J' presents the test statistic for the overidentifying restrictions. In addition to point estimates, we present the pricing errors associated with high-low factor mimicking portfolios formed on volatility, skewness, and kurtosis in the columns α_{vol}, α_{skew}, and α_{kurt}, respectively. We examine three versions of the model above. The first restricts $d_2 = d_3 = 0$, representing a linear specification, the second restricts $d_3 = 0$, representing a quadratic specification, and the final, representing a cubic specification, is unrestricted. Panel A presents results for returns formed on the basis of options with one month to maturity; Panels B-D present complementary results for options based on three, six, and twelve months to maturity. Newey-West standard errors are presented in parentheses below the point estimates and p-values for the J-statistic are presented in parentheses below the statistic. Point estimates that are statistically significant at the 10% level or greater are in boldfaced type. The data cover the period 4/30/1996 through 1/31/2005 for 106 monthly observations.

Panel A: $r_{T,t}$ 1 Month to Maturity

d_0	d_1	d_2	d_3	J	α_{vol}	α_{skew}	α_{kurt}
1.00	-1.70			10.41	**-1.05**	-0.17	0.22
(0.02)	(2.08)			(0.24)	(0.48)	(0.39)	(0.30)
0.99	-0.28	1.01		7.61	**-0.50**	-0.22	0.12
(0.01)	(1.92)	(5.80)		(0.37)	(0.22)	(0.31)	(0.17)
1.00	-0.61	0.97	-1.43	9.93	**-0.98**	-0.16	0.19
(0.02)	(5.93)	(4.50)	(8.18)	(0.13)	(0.42)	(0.38)	(0.24)

Panel B: $r_{T,t}$ 3 Months to Maturity

d_0	d_1	d_2	d_3	J	α_{vol}	α_{skew}	α_{kurt}
1.00	-0.09			16.76	-0.59	-0.42	0.41
(0.00)	(1.42)			(0.03)	(0.62)	(0.40)	(0.28)
1.00	1.33	16.84		8.18	-0.33	-0.33	0.04
(0.17)	(4.39)	(13.44)		(0.32)	(0.22)	(0.78)	(0.70)
1.00	1.64	9.54	-2.96	12.05	-0.91	-0.35	0.35
(0.08)	(9.74)	(7.75)	(15.83)	(0.06)	(0.71)	(0.53)	(0.33)

Panel C: $r_{T,t}$ 6 Months to Maturity

d_0	d_1	d_2	d_3	J	α_{vol}	α_{skew}	α_{kurt}
1.00	-0.29			21.24	-0.77	**-0.58**	**0.45**
(0.00)	(1.50)			(0.01)	(0.63)	(0.35)	(0.24)
0.99	1.56	**16.33**		9.33	-0.26	-0.70	0.06
(0.01)	(5.21)	(6.49)		(0.23)	(0.22)	(0.49)	(0.64)
1.00	19.73	16.97	-29.33	9.62	-0.26	-0.55	-0.40
(0.17)	(12.88)	(10.99)	(21.25)	(0.14)	(1.07)	(0.65)	(0.49)

Panel D: $r_{T,t}$ 12 Months to Maturity

d_0	d_1	d_2	d_3	J	α_{vol}	α_{skew}	α_{kurt}
0.96	-0.62			17.85	-0.88	-0.55	0.56
(0.01)	(1.52)			(0.02)	(0.63)	(0.36)	(0.25)
0.99	1.36	14.43		11.42	-0.24	-0.71	0.15
(0.14)	(4.03)	(15.32)		(0.12)	(0.19)	(0.41)	(0.57)
1.00	40.85	21.23	-62.85	6.59	-0.41	-0.47	0.47
(0.33)	(38.25)	(22.92)	(63.19)	(0.36)	(1.67)	(1.06)	(0.76)

Table A5: Parametric versus Non-Parametric Stochastic Discount Factor Risk Adjustments: Rolling Physical Moments

The table presents risk adjustments for the volatility, skewness, and kurtosis factor mimicking portfolios using stochastic discount factors implied by the S&P 500 risk neutral and physical densities. The stochastic discount factor is formed as a risk-free scaled ratio of the risk-neutral to physical probability measure

$$m_t(x, s, \tau) = e^{-r_t^f(\tau)} \frac{f_t^Q(x, s, \tau)}{f_t^P(x, s, \tau)}$$

where $f_t^Q(\cdot)$ is the risk-neutral probability measure at time t, $f_t^P(\cdot)$ is the physical probability measure at time t, and τ is the horizon. We approximate the risk-neutral and physical probability distributions using the Normal Inverse Gaussian (NIG) distribution. The risk neutral measure is approximated using the risk neutral moments calculated in the paper. Physical moments are estimated as sample moments calculated over the past 1000 days, updated each month. The table presents excess returns implied by discounting the factor mimicking portfolios by the stochastic discount factor,

$$\hat{\alpha} = \frac{1}{T} \sum_{t=1}^{T} r_t(\tau)\, m_t(x_t, \tau)$$

where $r_t(\tau)$ is the τ-period return on the factor-mimicking portfolio at time t, and $m_t(x_t, \tau)$ is the stochastic discount factor evaluated at the observed τ-period realization of the S&P 500 at time t. The column labeled "NIG" represents the discount factor implied by the NIG approximations to the densities. Columns "Linear," "Quad," and "Cubic" represent discount factors obtained by projecting the density-implied discount factor onto a linear, quadratic, and cubic polynomial, respectively. Panel A presents results for the volatility-sorted factor mimicking portfolio with rows representing portfolios formed on volatility estimated using options with one, three, six, and twelve-months to maturity. Panels B and C present complementary results for skewness- and kurtosis-sorted factor mimicking portfolios. We separately examine stochastic discount factors based on options and returns with three, six, and 12 month horizons. Data for the three, six, and twelve month horizons begin in January, 1997, July, 1996, and April,1996, respectively. All three horizons extend through December, 2005 for 106 (overlapping) observations. Point estimates are scaled to the monthly frequency, and Newey-West standard errors are presented in parentheses below the point estimates. Point estimates that are statistically significant at the 10% level or greater are in boldfaced type.

Panel A: Volatility

Three Month				Six Month				Twelve Month			
NIG	Linear	Quad	Cubic	NIG	Linear	Quad	Cubic	NIG	Linear	Quad	Cubic
-0.04	-0.02	0.02	0.01	-0.07	-0.48	-0.42	-0.42	-0.65	-0.34	-0.30	-0.34
(0.31)	(0.37)	(0.38)	(0.37)	(0.85)	(0.62)	(0.65)	(0.64)	(0.54)	(0.59)	(0.64)	(0.63)
-0.02	-0.01	0.04	0.03	-0.01	-0.69	-0.61	-0.61	-0.93	-0.47	-0.45	-0.49
(0.41)	(0.49)	(0.52)	(0.51)	(1.17)	(0.85)	(0.91)	(0.90)	(0.77)	(0.83)	(0.90)	(0.89)
-0.01	0.00	0.05	0.03	0.04	-0.78	-0.69	-0.70	-1.06	-0.53	-0.52	-0.56
(0.46)	(0.55)	(0.59)	(0.58)	(1.36)	(0.97)	(1.04)	(1.03)	(0.89)	(0.94)	(1.03)	(1.01)
0.00	0.03	0.08	0.06	0.08	-0.75	-0.67	-0.67	-1.01	-0.48	-0.47	-0.51
(0.45)	(0.55)	(0.59)	(0.58)	(1.35)	(0.96)	(1.04)	(1.02)	(0.89)	(0.94)	(1.04)	(1.02)

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Panel B: Skewness

	Three Month				Six Month				Twelve Month			
	NIG	Linear	Quad	Cubic	NIG	Linear	Quad	Cubic	NIG	Linear	Quad	Cubic
1 Month	-0.20	-0.27	-0.31	-0.30	-0.73	-0.23	-0.26	-0.26	-0.20	-0.51	-0.50	-0.48
	(0.23)	(0.29)	(0.32)	(0.31)	(0.65)	(0.49)	(0.54)	(0.53)	(0.43)	(0.50)	(0.56)	(0.55)
3 Month	-0.25	-0.31	-0.36	-0.35	-0.75	-0.39	-0.43	-0.43	-0.43	-0.65	-0.65	-0.64
	(0.19)	(0.24)	(0.26)	(0.25)	(0.51)	(0.36)	(0.40)	(0.39)	(0.30)	(0.39)	(0.43)	(0.42)
6 Month	**-0.23**	**-0.32**	**-0.34**	**-0.34**	**-0.70**	**-0.60**	**-0.61**	**-0.61**	**-0.60**	**-0.74**	**-0.77**	**-0.75**
	(0.14)	(0.15)	(0.16)	(0.16)	(0.29)	(0.27)	(0.28)	(0.28)	(0.23)	(0.28)	(0.31)	(0.30)
12 Month	-0.20	**-0.27**	**-0.30**	**-0.29**	**-0.59**	**-0.51**	**-0.52**	**-0.52**	**-0.55**	**-0.65**	**-0.67**	**-0.65**
	(0.13)	(0.15)	(0.16)	(0.16)	(0.27)	(0.26)	(0.27)	(0.27)	(0.22)	(0.25)	(0.27)	(0.27)

Panel C: Kurtosis

	Three Month				Six Month				Twelve Month			
	NIG	Linear	Quad	Cubic	NIG	Linear	Quad	Cubic	NIG	Linear	Quad	Cubic
1 Month	0.13	0.14	0.15	0.14	0.42	0.09	0.10	0.10	0.12	0.30	0.27	0.26
	(0.17)	(0.20)	(0.22)	(0.21)	(0.43)	(0.36)	(0.39)	(0.38)	(0.32)	(0.34)	(0.38)	(0.37)
3 Month	0.19	0.21	0.23	0.23	**0.57**	0.38	0.40	0.40	**0.51**	**0.57**	**0.58**	**0.57**
	(0.13)	(0.15)	(0.15)	(0.15)	(0.31)	(0.25)	(0.26)	(0.26)	(0.21)	(0.26)	(0.28)	(0.27)
6 Month	0.14	0.18	0.20	0.19	**0.45**	0.40	0.42	0.42	**0.51**	**0.53**	**0.56**	**0.56**
	(0.14)	(0.13)	(0.13)	(0.13)	(0.24)	(0.26)	(0.27)	(0.27)	(0.24)	(0.25)	(0.27)	(0.26)
12 Month	0.16	**0.22**	**0.23**	**0.23**	**0.48**	**0.46**	**0.47**	**0.47**	**0.57**	**0.57**	**0.60**	**0.60**
	(0.13)	(0.13)	(0.12)	(0.13)	(0.23)	(0.27)	(0.27)	(0.26)	(0.23)	(0.23)	(0.25)	(0.24)

Table A6: Parametric versus Non-Parametric Stochastic Discount Factor Risk Adjustments: Autoregressive Rolling Physical Moments

The table presents risk adjustments for the volatility, skewness, and kurtosis factor mimicking portfolios using stochastic discount factors implied by the S&P 500 risk neutral and physical densities. The stochastic discount factor is formed as a risk-free scaled ratio of the risk-neutral to physical probability measure

$$m_t(x,s,\tau) = e^{-r_t^f(\tau)} \frac{f_t^Q(x,s,\tau)}{f_t^P(x,s,\tau)}$$

where $f_t^Q(\cdot)$ is the risk-neutral probability measure at time t, $f_t^P(\cdot)$ is the physical probability measure at time t, and τ is the horizon. We approximate the risk-neutral and physical probability distributions using the Normal Inverse Gaussian (NIG) distribution. The risk neutral measure is approximated using the risk neutral moments calculated in the paper. Physical moments are estimated as predicted moments;

$$\hat{\theta}_t = \hat{\theta}_0 + \hat{\rho}_\theta \theta_{t-1}$$

where $\hat{\theta}_0$ and $\hat{\rho}$ are AR(1) estimates for the moments over the period January, 1962 through March, 1996, and θ_t represents the volatility, skewness, or kurtosis of returns on the S&P 500 index over the past 1000 days, sampled at the monthly frequency. The table presents excess returns implied by discounting the factor mimicking portfolios by the stochastic discount factor,

$$\hat{\alpha} = \frac{1}{T} \sum_{t=1}^{T} r_t(\tau)\, m_t(x_t,\tau)$$

where $r_t(\tau)$ is the τ-period return on the factor-mimicking portfolio at time t, and $m_t(x_t,\tau)$ is the stochastic discount factor evaluated at the observed τ-period realization of the S&P 500 at time t. The column labeled "NIG" represents the discount factor implied by the NIG approximations to the densities. Columns "Linear," "Quad," and "Cubic" represent discount factors obtained by projecting the density-implied discount factor onto a linear, quadratic, and cubic polynomial, respectively. Panel A presents results for the volatility-sorted factor mimicking portfolio with rows representing portfolios formed on volatility estimated using options with one, three, six, and twelve-months to maturity. Panels B and C present complementary results for skewness- and kurtosis-sorted factor mimicking portfolios. We separately examine stochastic discount factors based on options and returns with three, six, and 12 month horizons. Data for the three, six, and twelve month horizons begin in January, 1997, July, 1996, and April,1996, respectively. All three horizons extend through December, 2005 for 106 (overlapping) observations. Point estimates are scaled to the monthly frequency, and Newey-West standard errors are presented in parentheses below the point estimates. Point estimates that are statistically significant at the 10% level or greater are in boldfaced type.

Panel A: Volatility

	Three Month				Six Month				Twelve Month			
	NIG	Linear	Quad	Cubic	NIG	Linear	Quad	Cubic	NIG	Linear	Quad	Cubic
1 Month	-0.20	0.05	0.02	0.00	-0.17	-0.35	-0.40	-0.41	-0.48	-0.23	-0.29	-0.33
	(0.30)	(0.37)	(0.36)	(0.35)	(0.84)	(0.65)	(0.65)	(0.64)	(0.51)	(0.63)	(0.63)	(0.61)
3 Month	-0.20	0.06	0.00	-0.02	-0.07	-0.54	-0.60	-0.60	-0.55	-0.34	-0.44	-0.47
	(0.36)	(0.49)	(0.50)	(0.48)	(1.12)	(0.90)	(0.91)	(0.89)	(0.65)	(0.87)	(0.88)	(0.84)
6 Month	-0.20	0.07	-0.01	-0.03	0.01	-0.63	-0.69	-0.69	-0.61	-0.39	-0.50	-0.54
	(0.39)	(0.56)	(0.57)	(0.55)	(1.29)	(1.02)	(1.04)	(1.02)	(0.73)	(0.99)	(1.00)	(0.96)
12 Month	-0.18	0.09	0.02	-0.01	0.07	-0.61	-0.67	-0.67	-0.54	-0.35	-0.46	-0.49
	(0.37)	(0.56)	(0.56)	(0.55)	(1.28)	(1.01)	(1.03)	(1.01)	(0.72)	(1.00)	(1.01)	(0.96)

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Panel B: Skewness

	Three Month				Six Month				Twelve Month			
	NIG	Linear	Quad	Cubic	NIG	Linear	Quad	Cubic	NIG	Linear	Quad	Cubic
1 Month	-0.18	-0.25	-0.24	-0.22	-0.73	-0.24	-0.24	-0.25	-0.39	-0.51	-0.45	-0.43
	(0.18)	(0.30)	(0.31)	(0.29)	(0.61)	(0.52)	(0.53)	(0.52)	(0.38)	(0.54)	(0.54)	(0.52)
3 Month	-0.21	-0.30	-0.30	-0.29	-0.72	-0.40	-0.41	-0.41	-0.51	-0.63	-0.59	-0.57
	(0.15)	(0.24)	(0.24)	(0.23)	(0.48)	(0.38)	(0.39)	(0.38)	(0.30)	(0.41)	(0.41)	(0.39)
6 Month	**-0.20**	**-0.30**	**-0.31**	**-0.30**	**-0.66**	**-0.58**	**-0.58**	**-0.58**	**-0.50**	**-0.72**	**-0.70**	**-0.68**
	(0.12)	(0.16)	(0.16)	(0.15)	(0.28)	(0.27)	(0.27)	(0.27)	(0.20)	(0.29)	(0.28)	(0.27)
12 Month	-0.19	**-0.25**	**-0.26**	**-0.25**	**-0.57**	**-0.49**	**-0.50**	**-0.50**	**-0.47**	**-0.62**	**-0.60**	**-0.59**
	(0.12)	(0.15)	(0.15)	(0.15)	(0.25)	(0.27)	(0.26)	(0.26)	(0.18)	(0.26)	(0.25)	(0.24)

Panel C: Kurtosis

	Three Month				Six Month				Twelve Month			
	NIG	Linear	Quad	Cubic	NIG	Linear	Quad	Cubic	NIG	Linear	Quad	Cubic
1 Month	0.16	0.11	0.10	0.09	0.46	0.07	0.08	0.09	0.29	0.28	0.24	0.23
	(0.15)	(0.21)	(0.22)	(0.21)	(0.39)	(0.39)	(0.39)	(0.38)	(0.27)	(0.37)	(0.37)	(0.36)
3 Month	0.20	0.19	0.21	0.20	**0.55**	0.37	0.39	0.39	**0.49**	**0.54**	**0.54**	**0.52**
	(0.13)	(0.16)	(0.15)	(0.15)	(0.30)	(0.27)	(0.27)	(0.26)	(0.20)	(0.28)	(0.27)	(0.26)
6 Month	0.16	0.17	0.18	0.18	**0.44**	0.38	0.41	0.41	**0.41**	**0.51**	**0.52**	**0.50**
	(0.13)	(0.14)	(0.14)	(0.14)	(0.22)	(0.28)	(0.27)	(0.27)	(0.19)	(0.27)	(0.26)	(0.25)
12 Month	0.18	0.19	0.21	0.20	**0.48**	0.43	**0.46**	**0.46**	**0.47**	**0.54**	**0.56**	**0.54**
	(0.13)	(0.13)	(0.13)	(0.13)	(0.21)	(0.28)	(0.27)	(0.27)	(0.18)	(0.25)	(0.24)	(0.24)