Some comments. . .

. . . on cumulants and entropy and so on

Let M be an SDF and let $\psi(\theta) \equiv \log \mathbb{E} e^{\theta \cdot \log M}$ be the CGF of $\log M$. We know some things about ψ:

- it's convex
- it passes through the origin: $\psi(0) = 0$
- the riskless rate tells us $\psi(1)$: $\psi(1) = -\log R_f$
- the entropy, $J(M)$, equals $\psi(1) - \psi'(0)$. . .
- . . . so it can be visualized geometrically: draw the tangent to ψ at 0. [It really helps to draw a graph here!] The gap between $\psi(1)$ and this line, at "$x = 1$", is $J(M)$—in other words, entropy is a measure of the curvature of the CGF *not only at the origin*—curvature there captures variance—*but also away from the origin*. Another way of looking at this is that the gap is capturing the effect of $\psi''(0)$ but also the higher cumulants $\psi'''(0)$, $\psi''''(0)$, etc...
- what you describe in your notes as the asset with "highest expected return" is the growth-optimal return R^* that attains the upper bound in the Jensen's inequality expression $\mathbb{E} \log R \leq \mathbb{E} \log(1/M)$, ie the return $R^* = 1/M$. (I'm assuming that $1/M$ is a tradable return. . . Incidentally, this makes it easy to see that setting $\lambda = \alpha$ gives the growth-optimal asset in the power utility case.)
- the expected return on this growth-optimal asset is $\mathbb{E}R^* = \mathbb{E}1/M = e^{\psi(-1)}$, or more conveniently, $\log \mathbb{E}R^* = \psi(-1)$ so knowledge of realized growth-optimal returns gives us information about $\psi(-1)$ (but thinking about it, it's not obvious how to estimate the growth-optimal return empirically in a reasonable

way. . . perhaps use some simple forecasting model to identify a sensible guess at the growth-optimal portfolio one step ahead of time?)

- the Hansen-Jagannathan bound is monotonic in $\psi(2) - 2\psi(1)$:

$$HJB = \sqrt{\exp(\psi(2) - 2\psi(1)) - 1}.$$

So high Sharpe ratios tell us something about $\psi(2)$, too

- if we're working with a simple power utility consumption-based model, the price-dividend ratio of "next-period consumption" will be simply expressible in terms of $\psi(\cdot)$, too (can work with the consumption strip, ie aggregate wealth, if the world is i.i.d.)

- in conclusion, I can see how to get info about $\psi(-1)$, $\psi(0)$, $\psi(1)$ and $\psi(2)$, and maybe more if there's a reasonable proxy for the price-dividend ratio in the previous bullet point. Then the convexity of ψ gives us a bunch of inequalities that relate the required sizes of these quantities to each other (as I did in my higher cumulants paper, though I didn't talk about growth-optimal stuff in that)

. . . on options and the empirical side

- as well as the smile, there are other dimensions on which in principle we should be able to extract info about disasters etc. Notably. . .

- . . . very low expected returns on ATM options (Coval and Shumway, JF 2000, I think?) seems to be consistent with disasters in a Naik-Lee world — this is true whether or not disasters happen in sample. I think this is intriguing

- . . . what's the term structure of ATM (say) vol? Again, Naik-Lee type model that I was playing with generates an upward-sloping term structure of vol. Not sure what the facts are here—I had a data set from about 2001-2006 which had this feature, but that's an unusual time period in which short dated vol was very low... could just be a mean-reversion of vol story...

- from options, we get P^*; from the time series, we get P; but if we can't accurately measure P in the states of the world for which P^* is really large, relative to P—ie disasters—I think we're still not going to be able to measure entropy (of P/P^* or equivalently M) accurately?

. . . on other stuff

- p. 10, time intervals. I discuss this a bit in my higher cumulants paper, in a section on the asymptotic lognormality of consumption growth. Although departures from lognormality are smaller with longer time intervals, people care about them more over long periods

- if disaster probabilities are going to be allowed to move around over time, this may have strong implications for the riskless rate if we use power utility—tiny increases in disaster frequency or size can drive the riskless rate down a LOT! this effect would be lessened with Epstein-Zin prefs...