Disaster Begets Crisis: The Role of Contagion in Financial Markets*

(Job Market Paper)

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Abstract

Severe economic downturns like the great depression of the 1930s take place over extended periods of time. I model an economy where rare economic disasters increase the likelihood of subsequent near term disasters. The mechanism generates more clustering of disasters than existing models. Serial correlation in disasters has important implications for asset prices. For example, it generates a larger equity premium and a lower risk free rate than similarly calibrated models without this feature. The calibrated model developed here replicates the temporal structure of severe economic downturns in OECD countries. It quantitatively explains the equity premium, the risk-free rate, excess volatility and return predictability. It also generates the implied volatility smile observed in equity index options.

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1 Introduction

To explain what has come to be known as the equity premium puzzle[1], financial economists have pointed to the risk of rare economic disasters. An example of such a disaster occurred from 1930 to 1932, when the U.S. economy contracted by more than 10% each year. A sequence of large annual declines in economic activity like this is difficult to reconcile with existing disaster models developed in Rietz (1988), Barro (2006), Gabaix (2010), and Wachter (2010), which view each disaster as an isolated rare event. The great depression experience suggests that these events are not independent: Once an economic decline is experienced, it is likely to repeat, causing the economic crisis to unfold over an extended period of time.

This paper models the general equilibrium dynamics of asset prices - including stocks, bonds, and options - when large scale declines in consumption tend to cluster. If economic agents rationally anticipate such clustering, their consumption and investment plans will dramatically differ from those generated in the extant literature on rare events. Moreover, they will revise their consumption and investment decisions in response to an initial adverse consumption shock which could mark the beginning of a crisis. The desire to form precautionary savings will then be more pronounced. This will increase demand for riskless investments, resulting in a lower risk-free rate. Furthermore, increased uncertainty about future economic health brought about by the arrival of a rare disaster makes agents reluctant to hold risky assets. Fear of further economic downturns will require these assets to increase their expected returns. Hence, valuation multiples like the price to dividend ratio will drop and price declines will exceed the decline in cash flows. In most instances the economy will prove resilient; economic downturns will be short-lived, and, in hindsight, economic participants will appear to have been overly prudent in their investment decisions. This mechanism results in an ex-post excessive reaction of economic quantities to realized risk, which can account for observed pricing phenomena. These include time-varying expected returns and risk-premia that can be predicted by the price-dividend ratio as well as excess stock market volatility.

The recent financial crisis of 2007-2009 illustrates one such episode where an initial disaster occurred in one part of the financial sector that subsequently caused contagion with widespread consequences throughout the financial system. Fear of this downturn to develop into another great depression led households to slash consumption and investors to shy away from risky assets.[2]

[1] See Mehra and Prescott (1985)

[2] This notion was echoed by respected economics:
Paul Krugman, New York Times, January 4, 2009: "This looks an awful lot like the beginning of a second Great Depression."

During the fourth quarter of 2008, nondurable consumption expenditures declined by 7.66%, which translates into an annualized reduction of 27.29%. This represents the single largest drop in over half a century and constitutes an event more than six standard deviations from its historical mean.[3] During this period, S&P 500 option implied volatility, measured by the Volatility Index (VIX) sharply increased from 18.81% on August 22nd to 80.86% on November 20th and then reverted to a lower level in the months following these events.[4] Explaining such a drastic jump in implied volatility within the context of a consumption based asset pricing model requires a sudden change in one of the state variables characterizing economic uncertainty, like the conditional probability of further economic disasters. Existing models in the disaster risk literature assume the disaster intensity to be either constant over time as in Barro (2006), predicting option-implied volatility to be constant over time as well, or to be subject to time variation that is unrelated to disaster arrival e.g. Wachter (2010). Under either set of assumptions, option-implied volatility does not respond to the occurrence of a disaster itself. The implied volatility pattern experienced during the recent financial crisis can be explained in a model where the arrival of a disaster magnifies the probability of disasters in the near future.

Moreover, during the fourth quarter of 2008, the S&P 500 index declined from 1282.83 on August 29th to 903.25 on December 31st, a 29.59% drop, which far exceeds the contemporaneous decline in personal consumption of 7.66%. This excessive stock market reaction is a recurring phenomenon summarized in Paul Samuelson's remark that "Wall Street indexes [had] predicted nine out of the last five recessions". Indeed, this stylized fact is not unique to U.S. capital markets. Barro and Ursúa (2009) document that only 28% of all stock market declines exceeding 25% are accompanied by a macroeconomic contraction exceeding 10% in OECD countries. I contend that this apparent puzzle can be accounted for by the model developed in this paper. The consumption drop in the fall of 2008 was a rare event that caused a surge in the probability of further disasters. While no similar consumption shock has happened to this day, the fear of another such disaster remains above its normal level as evidenced by high dividend-yields, a low risk-free rate, and a VIX that is above average.

The economic consequences of the possibility that a disaster can beget a crisis are analyzed in the context of a representative agent endowment economy. The model is set in continuous-time where the agent's preferences are expressed using stochastic differential

Robert Barro, Wall Street Journal, March 4, 2009: "... there is ample reason to worry about slipping into a depression. There is a roughly one-in-five chance that U.S. GDP and consumption will fall by 10% or more, something not seen since the early 1930s."

Quoted from Gourio (2010).

[3] BEA, National Income and Product Accounts, Table 1.1.5.

[4] See figure 1.

utility. The departure from standard time-separable expected utility separates risk aversion from the willingness to substitute consumption over time and has been proven successful in addressing the equity premium and risk-free rate puzzles in the long-run risk literature initiated by Bansal and Yaron (2004). Aggregate consumption is subject to both diffusive risk and infrequent jumps, which account for the possibility of rare but potentially disastrous events.

In order to capture the idea of serial correlation in disasters, I introduce a self-exciting jump-diffusion into the consumption-based asset pricing framework. Self-exciting processes, developed in Hawkes (1971b), have recently been brought to bear in the field of credit risk modeling to explain the phenomenon of default clustering.[5] In the context of the model presented here, these processes allow the occurrence of a disaster to affect the intensity of future disaster arrival. The conditional likelihood of further significant economic downturns increases in response to a substantial adverse shock to consumption, leading to the possibility of self-perpetuating economic disasters. Self-exciting disaster risk in conjunction with a preference for early resolution of uncertainty gives rise to an additional channel by which disasters affect risk-premia, interests rates, and asset prices. Expected utility does not exhibit a preference for the timing of resolution of uncertainty, which stochastic differential utility does.

With a preference for the timing of resolution of uncertainty, the pricing kernel involves continuation utility as well as instantaneous consumption growth. This dependence of the stochastic discount factor on continuation utility makes makes it necessary to solve for the representative agent's value function in order to obtain equilibrium asset prices. Since the partial differential equation characterizing indirect utility is nonlinear, standard PDE techniques such as Fourier and Laplace transforms, which are essentially linear operations, are of no avail in this situation. Instead, I resort to a log-linear approximation of the non-linear term in the PDE around the unconditional mean consumption-to-wealth ratio, which is endogenous to the model, in order to address this issue.[6] This approximation is exact if the representative agent has unit elasticity of intertemporal substitution and yields closed form solutions for the pricing kernel, the risk-free rate, and risk-premia. Valuation ratios of claims to aggregate consumption and corporate dividends are then exponentially affine functions of the state variables. This affine structure gives rise to equity option prices in quasi closed-form that can be efficiently determined by Fourier inversion techniques along the lines of

[5]See Azizpour, Giesecke, and Schwenkler (2010)

[6]This approximation has been suggested by Campbell, Chacko, Rodriguez, and Viceira (2004) in the context of a portfolio choice problem and since been employed by Drechsler (2010) in an asset pricing application. Alternative approximations has been proposed by Benzoni, Collin-Dufresne, and Goldstein (2007) and Eraker and Shaliastovich (2008).

Duffie, Pan, and Singleton (2000).

In order to illustrate the model's quantitative implications for stocks, government bonds, and equity options, I calibrate the parameters to historical data on U.S. aggregate consumption and corporate dividends from the 20th century as well as historical macroeconomic crises in OECD countries documented by Barro and Ursúa (2009). Under realistic assumptions on the representative agent's preferences, the model is able to match the magnitude of the observed historical equity premium and the risk free rate. Furthermore, both the model-implied treasury bill rate and the price-dividend ratio decline during times of crisis. The equity premium and the VIX shoot up in response to a disaster giving rise to countercyclical variation of these quantities. Monte Carlo simulations demonstrate the model's ability to account for excess-return predictability. Finally, the mechanism generates implied volatility patterns close to the smile observed for S&P 500 equity index options.

Rare disasters have been proposed as a solution to the equity premium puzzle.[7] Barro (2006) provides international evidence of macroeconomic contractions that can account for the equity premium and the risk-free rate with reasonable levels of risk aversion. Gabaix (2010) models time variation in the disaster magnitude to explain stylized facts including time variation in equity premia and return predictability. Wachter (2010) shows that time variation in the disaster intensity can account for aggregate stock market volatility. In contrast to these models, self-exciting disasters explain extended economic crises, jumps in option implied volatilities during economic downturns, and large stock price reactions in response to moderate declines in consumption.

The theory of self- exciting Hawkes-processes has recently been applied to problems in finance.[8] Azizpour, Giesecke, and Schwenkler (2010) demonstrate the ability of self-exciting processes to account for the observed phenomenon of default clustering in credit markets. Ait-Sahalia, Cacho-Diaz, and Laeven (2010) study portfolio choice when stock returns in different markets are subject to self- and mutually-exciting jumps and use GMM to estimate a model of financial contagion across countries. My paper uses self-exciting processes to study general equilibrium asset pricing.

The paper is organized as follows. Section 2 introduces the model and derives general equilibrium results for claims to corporate dividends and aggregate consumption, government debt, as well as equity index options in this endowment economy. Section 3 presents a calibration to historical U.S. data and empirically investigates the asset pricing implications of the economic channel proposed in this paper. Section 4 concludes the analysis of this

[7] See Rietz (1988) and Mehra and Prescott (1988).

[8] See Hawkes (1971a) and Hawkes (1971b) for the original development. For a generalization to affine point processes see Errais, Giesecke, and Goldberg (2010). Methods for efficient simulation of these generalized Hawkes processes are provided in Blanchet, Giesecke, Glynn, and Zhang (2009).

paper. The appendix presents a generalized version of the model that features long-run risk as well as stochastic volatility in consumption and accounts for the possibility that rare disasters are generated by a finite number of interrelated mutually-exciting jump processes. Technical proofs following in the appendix are provided for the general model.

2 Model

2.1 Endowment Process

I model a representative agent endowment economy. Consumption growth is subject to diffusive risk as well as rare disasters. The conditional probability of a disaster λ_t is governed by a self-exciting process. The dynamics are as follows:

$$\frac{dC_t}{C_{t-}} = \mu_C dt + \sigma_C dB_{C,t} + \left(e^{Y_t^C} - 1\right) dN_t \quad (1)$$

$$d\lambda_t = \kappa_\lambda(\bar{\lambda} - \lambda_t)dt + \sigma_\lambda \sqrt{\lambda_t} dB_{\lambda,t} + Y_t^\lambda dN_t \quad (2)$$

N_t is a counting process with stochastic intensity λ_t. The Brownian motions driving consumption growth C_t and the jump intensity B_λ are assumed to be mutually independent. For succinctness of notation, I will let B_t denote the multidimensional Brownian motion $(B_{C,t}, B_{\lambda,t})^T$. The counting process N_t triggers jumps of random size in consumption growth as well as its conditional intensity λ_t. The distribution of the jump size in consumption, Y_t^C, is allowed to have support on the entire real line. In order to ensure that the disaster intensity remains positive, the jump size in λ_t is the restricted to upwards jumps, i.e. $Y_t^\lambda > 0$. Let Y_t denote the vector $(Y_t^C, Y_t^\lambda)^T$ and assume that Y_t is independent of both B_t and N_t. The joint distribution of the disaster jump size and the upwards revision of the conditional disaster intensity can be characterized by the moment generating function $\Phi^Y(u) = \mathbb{E}\left[\exp\left(u^T \cdot Y_t\right)\right]$.

This setup nests both a continuous-time version of Barro's constant disaster risk model as well as the time-varying rare disaster model by Wachter (2010). The novel feature of the approach presented here is the use of a self-exciting jump process to capture the idea that the occurrence of a rare disaster also results in a sharp increase of the conditional probability of another disaster.

2.2 Preferences

The representative agent has recursive preferences which have been developed by Kreps and Porteus (1978), Epstein and Zin (1989), and Weil (1989) in discrete time. This paper employs its continuous-time counterpart, stochastic differential utility (SDU), introduced by Duffie and Epstein (1992a,b). Continuation utility of the representative agent, J_t, is defined by the recursion

$$J_t = \mathbb{E}_t \left[\int_t^\infty f(C_s, J_s) ds \right], \tag{3}$$

where $f(C, J)$ is the normalized Porteus-Kreps aggregator

$$f(C,J) = \frac{\beta}{1-\frac{1}{\psi}} J(1-\gamma) \left(\frac{C^{1-\frac{1}{\psi}}}{((1-\gamma)J)^{\frac{1}{\theta}}} - 1 \right) \qquad \text{for } \psi \neq 1 \tag{4}$$

$$f(C,J) = \beta(1-\gamma)J \left(\log C - \frac{1}{1-\gamma} \log\left((1-\gamma)J\right) \right) \qquad \text{for } \psi = 1. \tag{5}$$

In this definition, β assumes the role of a subjective time-discount factor, γ is the coefficient of relative risk aversion, ψ denotes the elasticity of intertemporal substitution (EIS), and the constant θ is defined as $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$. In the special case where $\gamma = \frac{1}{\psi}$, the preference structure coincides with power utility. If $\gamma > \frac{1}{\psi}$ the representative agent prefers early over late resolution of uncertainty.

2.3 Solution

The ultimate purpose in solving this asset pricing problem is in obtaining equilibrium characterizations for such economic quantities as the wealth-consumption ratio, the risk free rate, the equity premium, the dividend-yield, and the price of European options. We are furthermore interested in the effect of an upwards revision in the disaster intensity on these quantities in response to the arrival of such an event. The equilibrium stochastic discount factor that prices assets in equilibrium depends on consumption growth as well as continuation utility. It is therefore required to first solve for the equilibrium value function of representative agent in order to study asset pricing problems in this economy.

2.3.1 The Value Function

The value function is obtained by rewriting the recursive definition (3) in terms of a partial differential equation (PDE). This is possible since consumption and the disaster intensity are Markov processes. Hence, continuation utility at time t is a function of consumption and the disaster intensity at time t only. That means. J_t can be written as $J(C_t, \lambda_t)$.[9] Applying

[9]Note that the Markov property implies

$$J_t = \mathbb{E}_t \left[\int_t^\infty f(C_s, J_s) ds \right] = \mathbb{E} \left[\int_t^\infty f(C_s, J_s) ds \,\middle|\, C_t, \lambda_t \right] = J(C_t, \lambda_t).$$

a Feynman-Kac type argument to the recursive definition of continuation utility given in equation (3) results in the PDE

$$\mathcal{D}J_t + f(C_t, J_t) = 0,$$

where $\mathcal{D}J_t$ denotes the infinitesimal generator associated with the dynamics of C_t and λ_t.[10][11] Substituting the expression for the infinitesimal generator derived in the appendix yields the PDE [12]

$$J_C\mu_C C_t + J_\lambda \kappa_\lambda(\bar{\lambda} - \lambda_t) + \frac{1}{2}J_{CC}\sigma_C^2 C_t^2 + \frac{1}{2}J_{\lambda\lambda}\sigma_\lambda^2\lambda_t + \mathbb{E}_{t-}\left[\Delta J_t\right]\lambda_t + f(C_t, J_t) = 0, \quad (6)$$

where ΔJ_t denotes the jump size in the value function conditional on a jump of N.[13] Since the value function is homogeneous of degree $1-\gamma$ in consumption, one can write[14]

$$J(C, \lambda) = \frac{C^{1-\gamma}}{1-\gamma} I(\lambda),$$

where $I(\lambda)$ solves the differential equation

$$(1-\gamma)\mu_C - \frac{1}{2}\gamma(1-\gamma)\sigma_C^2 + \frac{I_\lambda}{I}\kappa_\lambda(\bar{\lambda} - \lambda_t) + \frac{1}{2}\frac{I_{\lambda\lambda}}{I}\sigma_\lambda^2\lambda_t + \mathbb{E}_{t-}\left[e^{Y_t^C}\frac{I(\lambda_{t-} + Y_t^\lambda)}{I(\lambda_t)} - 1\right]\lambda_t + f(C_t, J_t) = 0. \quad (7)$$

This differential equation involves the aggregator $f(C, J)$ which depends on whether the elasticity of intertemporal substitution is equal to or different from one. The following proposition provides a closed form solution for the case where the representative agent has unit elasticity of intertemporal substitution.

Proposition 1 (Equilibrium Value Function for $\psi = 1$)**.** *If the representative agent has unit*

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[10] See the appendix for a heuristic derivation.

[11] The infinitesimal generator of J associated with (C, λ) is defined as

$$\mathcal{D}J_t = \lim_{h \to 0} \frac{\mathbb{E}_{t-}\left[J(C_{t+h}, \lambda_{t+h}) - J(C_{t-}, \lambda_{t-})\right]}{h}.$$

[12] I use the notation g_X to denote the partial derivative of a function g with respect to X, except for g_t, which denotes the value of that function at time t.

[13] That means $\Delta J_t = J(C_t, \lambda_t) - J(C_{t-}, \lambda_{t-}) = J(C_{t-} \cdot e^{Y_t^C}, \lambda_{t-} + Y_t^\lambda) - J(C_{t-}, \lambda_{t-})$. In general, I use ΔX_t to denote the jump size in the process X_t conditional on a jump of N_t at time t.

[14] Homogeneity of the value function is established in lemma B.1.

elasticity of intertemporal substitution $\psi = 1$, then the value function solving (6) *is given by*

$$J(C, \lambda) = \frac{C^{1-\gamma}}{1-\gamma} \cdot \exp\left(A_0 + A_\lambda \lambda\right).$$

where the coefficients A_0 and A_λ satisfy the system of equations

$$\begin{aligned} 0 &= (1-\gamma)\mu_C - \frac{1}{2}\gamma(1-\gamma)\sigma_C^2 + A_\lambda \kappa_\lambda \bar{\lambda} - \beta A_0 \\ 0 &= -A_\lambda(\beta + \kappa_\lambda) + \frac{1}{2}A_\lambda^2 \sigma_\lambda^2 + \left(\Phi^Y\left(\hat{\eta}\right) - 1\right), \end{aligned}$$

with $\hat{\eta} = (1-\gamma, A_\lambda)^T$.

Proof. See appendix. □

The constants A_0 and A_λ are determined by a non-linear system of equations and need to be solved for numerically.

In the case where the EIS is different from one, the differential equation (7) is non-linear and does not have a closed form solution. A method introduced by Campbell, Chacko, Rodriguez, and Viceira (2004), however, admits a log-linearization of the nonlinear term in this equation around the mean consumption-wealth ratio. The approximate solution for $I(\lambda)$ then takes the same functional form as in the case of a unit EIS. The following proposition states the solution for the value function that results from this approximation.

Proposition 2 (Equilibrium Value Function for $\psi \neq 1$)**.** *If the representative agent has elasticity of intertemporal substitution ψ that is different from one, then the value function solving a Campbell, Chacko, Rodriguez, and Viceira (2004) approximation of* (6) *is given by*

$$J(C, \lambda) = \frac{C^{1-\gamma}}{1-\gamma} \cdot \exp\left(A_0 + A_\lambda \lambda\right).$$

The coefficients A_0 and A_λ satisfy the system of equations

$$\begin{aligned} 0 &= (1-\gamma)\mu_C - \frac{1}{2}\gamma(1-\gamma)\sigma_C^2 + A_\lambda \kappa_\lambda \bar{\lambda} + \theta i_0 + \theta i_1 \log\beta - \beta\theta - i_1 A_0 \\ 0 &= -(\kappa_\lambda + i_1)A_\lambda + \frac{1}{2}A_\lambda^2 \sigma_\lambda^2 + \left(\Phi^Y\left(\hat{\eta}\right) - 1\right), \end{aligned}$$

with $\hat{\eta} = (1-\gamma, A_\lambda)^T$. The linearization constants are given by

$$i_1 = \beta \exp\left(-\frac{A_0}{\theta} - \frac{A_\lambda}{\theta}\mathbb{E}\left[\lambda_t\right]\right) \text{ and } i_0 = i_1(1 - \log(i_1)).$$

Proof. See appendix. □

The coefficients A_0 and A_λ must again be determined numerically. In the calibration below, A_λ is positive. Since $\gamma > 1$, the representative agent dislikes an increase in the disaster intensity at the empirically relevant parameter values with a preference for early resolution of uncertainty.

2.3.2 Asset Prices, Risk Premia, and the Risk Free Rate

In the absence of arbitrage, the price $P_{i,t}$ of an asset paying dividend $D_{i,s}$ at time $s \geq t$ solves

$$\pi_t P_{i,t} = \mathbb{E}_t \left[\int_t^\infty \pi_s D_{i,s} ds \right], \tag{8}$$

where π_t denotes the pricing kernel. The following proposition provides a differential characterization for asset prices.

Proposition 3 (No-Arbitrage Pricing PDE)**.** *The no-arbitrage price $P_{i,t}$ of a claim that yields dividends $D_{i,s}$ at time $s \geq t$ satisfies the PDE*

$$\frac{\mathcal{D}(\pi_t \cdot P_{i,t})}{\pi_{t-} \cdot P_{i,t-}} + \frac{D_{i,t-}}{P_{i,t-}} = 0,$$

which can be decomposed as[15]

$$\frac{\mathcal{D}\pi_t^c}{\pi_{t-}} + \frac{\mathcal{D}P_{i,t}^c}{P_{i,t-}} + \frac{d\left[\pi^c, P_i^c\right]_t}{\pi_{t-} \cdot P_{i,t-} \cdot dt} + \frac{\mathbb{E}_{t-}\left[\Delta\left(\pi \cdot P_i\right)_t\right] \lambda_{t-}}{\pi_{t-} \cdot P_{i,t-}} + \frac{D_{i,t-}}{P_{i,t-}} = 0. \tag{9}$$

Proof. The proof of the first equation proceeds along the same line as the derivation of the PDE for the value function. Equation (9) then follows from an application of Ito's rule for jump-diffusions. □

This is an extension of the cash flow pricing equation in continuous-time to a jump-diffusion setting.[16]

A risk free asset pays dividends at a rate $r_{f,t}$ and has a constant price $P_{r_f,t}$. By applying equation (9) one obtains the following characterization for the risk-free rate.

Proposition 4 (Risk Free Rate)**.** *The instantaneous risk free rate is given by*

$$r_{f,t-} = -\frac{\mathcal{D}\pi_t^C}{\pi_t-} - \frac{\mathbb{E}_{t-}\left[\Delta\pi_t\right]\lambda_{t-}}{\pi_{t-}} = -\frac{\mathcal{D}\pi_t}{\pi_{t-}}. \tag{10}$$

[15] The superscript c denotes the continuous part of a process. See Shreve (2004, chap. 11) for details.

[16] See Cochrane (2001, page 32) for the case of purely diffusive risk.

Proof. A risk-free asset with instantaneous dividend yield $\frac{D_t}{P_{D,t}} = r_{f,t}$ has constant price $P_{r_f,t} = P_{r_f}$. Since $dP_{r_f,t} = 0$, we have $\mathcal{D}P_{r_f,t} = 0$, $d\left[\pi^C, P_D^C\right]_t = 0$ and hence $\mathcal{D}\left(\pi^c \cdot P_D^c\right) = 0$, and $\Delta(\pi \cdot P_{r_f})_t = \Delta\pi_t$. Upon substitution into (9), one obtains (10). □

The instantaneous expected return of a risky asset paying dividend stream $\{D_{i,s}\}_{s\geq t}$ is the sum of the expected appreciation of the continuous part, the expected return of the jump component, and the instantaneous dividend yield, that is

$$\mathbb{E}_{t-}[r_{i,t}] = \frac{\mathcal{D}P_{i,t}^c}{P_{i,t-}} + \frac{\mathbb{E}_{t-}\left[\Delta P_{i,t}\right]\lambda_{t-}}{P_{i,t-}} + \frac{D_{i,t-}}{P_{i,t-}}.$$

Combining propositions 3 and 4 and using the definition of the expected return given above one obtains the risk premium of a dividend paying asset, which is stated in the following proposition.

Proposition 5 (Risk Premium)**.** *The instantaneous risk premium of a claim to dividends* $\{D_{i,s}\}_{s\geq t}$ *is given by*

$$\begin{aligned}\mathbb{E}_{t-}\left[r_{i,t}\right] - r_{f,t-} &= \frac{\mathcal{D}P_{i,t}^c}{P_{i,t-}} + \frac{\mathbb{E}_{t-}\left[\Delta P_{i,t}\right]\lambda_{t-}}{P_{i,t-}} + \frac{D_{i,t-}}{P_{i,t-}} - r_{f,t-} \\ &= -\frac{d\left[\pi^c, P_i^c\right]_t}{\pi_{t-}\cdot P_{i,t-}\cdot dt} - \frac{\mathbb{E}_{t-}\left[\Delta\pi_t \cdot \Delta P_{i,t}\right]\lambda_{t-}}{\pi_{t-}\cdot P_{i,t-}}\end{aligned} \tag{11}$$

Proof. See appendix. □

The risk premium consists of two components. The first term represents compensation for diffusive risk, whereas the second component designates the premium arising from the exposure to disaster risk.

2.3.3 The Pricing Kernel

Duffie and Epstein (1992a) and Duffie and Skiadas (1994) show that the process π_t given by

$$\pi_t = \exp\left(\int_0^t f_J(C_s, J_s)ds\right) f_C(C_t, J_t) \tag{12}$$

can serve as a pricing kernel in a representative agent economy with stochastic differential utility. The following proposition provides a characterization of the dynamics of the pricing kernel as well as the equilibrium risk free rate.

Proposition 6 (Pricing Kernel and Risk Free Rate). *The dynamics of the pricing kernel are governed by*

$$\frac{d\pi_t}{\pi_{t-}} = -r_{f,t}dt + \eta^T \sigma_t dB_t + \left(\exp\left(\eta^T Y_t\right) - 1\right) dN_t - \left(\Phi^Y\left(\eta\right) - 1\right) \lambda_t dt,$$

with $\pi_0 = 1$, where η denotes the market price of risk vector

$$\eta = \left(-\gamma, \left(1 - \frac{1}{\theta}\right) A_\lambda\right)^T$$

and σ_t denotes the matrix of diffusion coefficients

$$\sigma_t = \begin{pmatrix} \sigma_C & 0 \\ 0 & \sigma_\lambda \sqrt{\lambda_t} \end{pmatrix}.$$

The equilibrium risk free rate is

$$r_{f,t} = \beta + \frac{1}{\psi}\mu_C - \left[\frac{1}{2}\gamma\left(1 + \frac{1}{\psi}\right)\right]\sigma_C^2 + \frac{1}{2}\frac{1}{\theta}\left(1 - \frac{1}{\theta}\right) A_\lambda^2 \sigma_\lambda^2 \lambda_t + \Lambda\lambda_t,$$

where $\Lambda = \left(1 - \frac{1}{\theta}\right)\left(\Phi^Y\left(\hat{\eta}\right) - 1\right) - \left(\Phi^Y\left(\eta\right) - 1\right)$.

Proof. See appendix. □

The risk-free rate is a linear function of the disaster intensity. The first two terms represent the subjective time preference and the desire to smooth a consumption stream that is expected to grow over time. Both terms make a positive contribution to the risk-free rate. The second effect is declining in the willingness of the representative agent to substitute consumption over time. The third term arises from a motive to form precautionary savings as insurance against diffusive shocks. It is linear in both the degree of risk aversion and the variance of diffusive risk and lowers the risk-free rate. The last term on the right hand side captures the total effect of disasters on the risk-free rate. The term $1/(2\theta)(1 - 1/\theta)A_\lambda^2\sigma_\lambda^2\lambda_t$ accounts for the effect of an increase in the diffusive volatility of the jump intensity $\sigma_\lambda\sqrt{\lambda_t}$ on continuation utility brought about by an increase in the disaster intensity. The desire to insure against shocks to the disaster intensity stemming from diffusive risk is more pronounced if the disaster intensity is high and lowers the risk free rate if both relative risk aversion and the EIS exceed one and the representative agent prefers early resolution of uncertainty, i.e. $\gamma > 1/\psi$.[17] The remaining term $\Lambda\lambda_t$ captures both precautionary savings

[17]To see this, first note that $\left(1 - \frac{1}{\theta}\right) = \frac{\frac{1}{\psi} - \gamma}{1 - \gamma} > 0$ if $\gamma > 1$ and the representative agent prefers early

to compensate for lost consumption in the event of a disaster and hedging demand against the upwards revision of the disaster intensity following disaster arrival. It can be shown that these effects have a negative impact on the risk-free rate if the representative agent has a unit EIS and relative risk aversion is greater than one.[18] in the empirical calibration, which has $\psi > 1$, this term is negative as well. This means that the unconditional risk free rate is lower in equilibrium than in a model without disasters. Furthermore, the fact that self-excitation increases disaster risk λ_t in response to a disaster implies that the risk-free rate drops during times of crisis.

2.4 Defaultable Short-Term Government Debt

Historically, economic crises have often been accompanied by at least a partial default of the government on its liabilities. To account for this possibility, I follow Barro and Wachter in assuming that whenever a rare disaster occurs, the government defaults with probability q. The fraction of notional that is lost in the event of default is identical to the reduction in consumption due to the disaster.

Let $r_{L,t}$ denote the promised interest rate on short term government debt. Given continuous reinvestment of interest payments, the value of government debt $P_{L,t}$ evolves according to

$$\frac{dP_{L,t}}{P_{L,t-}} = r_{L,t}dt + \left(e^{Y_t^L} - 1\right) dN_{k,t},$$

where $Y_t^L = Y_t^C$ with probability q and $Y_t^L = 0$ with probability $1 - q$. Since this investment strategy yields no dividends, $P_{L,t}$ satisfies the pricing equation

$$\frac{\mathcal{D}\pi_t^c}{\pi_{t-}} + \frac{\mathcal{D}P_{L,t}^c}{P_{L,t-}} + \frac{d\left[\pi^c, P_L^c\right]_t}{\pi_{t-}P_{L,t-}dt} + \frac{\mathbb{E}_{t-}\left[\Delta\left(\pi \cdot P_L\right)_t\right]\lambda_{t-}}{\pi_{t-}P_{L,t-}} = 0.$$

The equilibrium rate promised by the government on short-term debt can then be characterized as follows.

Proposition 7 (Equilibrium Interest Rate on Short-Term Government Debt)**.** *The instantaneous interest rate on defaultable short term debt promised by the government in equilibrium*

---

resolution of uncertainty, i.e. $\gamma > 1/\psi$. Furthermore, $\frac{1}{\theta} = \frac{1-\frac{1}{\psi}}{1-\gamma} < 0$ if $\psi > 1$ and $\gamma > 1/\psi$. Hence $1/(2\theta)(1 - 1/\theta)A_\lambda^2\sigma_\lambda^2\lambda_t > 0$

[18] Note that $\psi = 1$ implies $\frac{1}{\theta} = 0$. Hence the expression $\left(1 - \frac{1}{\theta}\right)\left(\Phi^Y(\hat{\eta}) - 1\right) - \left(\Phi^Y(\eta) - 1\right)$ simplifies to $\mathbb{E}\left[\exp\left(-\gamma Y_t^C + A_\lambda Y_t^\lambda\right)\left(\exp\left(Y_t^C\right) - 1\right)\right]$. Since $Y_C^t < 0$, the factor $\exp\left(Y_C^t\right) - 1$ is negative. Additionally, $\exp\left(-\gamma Y_t^C + A_\lambda Y_t^\lambda\right)$ is positive for $A_\lambda > 0$. Hence their product is negative.

is

$$r_{L,t} = r_{f,t} + \left[\Phi^Y(\eta) - \Phi^Y(\tilde{\eta})\right] q\lambda_t,$$

where $\tilde{\eta} = \left(1-\gamma, \left(1-\frac{1}{\theta}\right) A_\lambda\right)^T$.

Proof. See appendix. □

The default risk premium on government debt $\left[\Phi^Y(\eta) - \Phi^Y(\tilde{\eta})\right] q\lambda_t$ is positive and increasing in both the disaster intensity λ_t and the probability of default conditional on a disaster q.[19]

The expected return on government bills $r_{b,t} = \frac{\mathcal{D}P_{L,t}}{P_{L,t-}}$ incorporates an adjustment of the promised rate $r_{L,t}$ for the unconditional expected loss in the event of government default. It is given by[20]

$$r_{b,t} = r_{L,t} + \left(\Phi^Y(e_1) - 1\right) q\lambda_t.$$

This adjustment is the product of the disaster intensity λ_t, the probability of government default conditional on a disaster q, and the expected loss given default $\Phi^Y(e_1) - 1 = \mathbb{E}\left[\exp\left(Y_t^C\right) - 1\right] < 0$.

The risk premium of an asset with respect to the return on government debt is related to the risk premium with respect to the risk-free rate by

$$\mathbb{E}_t\left[r_{i,t} - r_{b,t}\right] = \mathbb{E}_t\left[r_{i,t} - r_{f,t}\right] + \left[\Phi^Y(\tilde{\eta}) - \Phi^Y(\eta) - \left(\Phi^Y(e_1) - 1\right)\right] q\lambda_t.$$

The risk premium measured with respect to the return on government bills is below that with respect to the risk-free rate, which accounts for the exposure to systematic default risk of government debt.[21]

[19] Note that

$$\Phi^Y(\eta) - \Phi^Y(\tilde{\eta}) = \mathbb{E}\left[\exp\left(-\gamma Y_t^C + \left(1-\frac{1}{\theta}\right) A_\lambda Y_t^\lambda\right) \cdot \left(1 - \exp\left(Y_t^C\right)\right)\right] > 0$$

if $A_\lambda > 0$, $\gamma > 1$, and $\gamma > \frac{1}{\psi}$, i.e. investors prefer early resolution of uncertainty, since $Y_t^C < 0$ and $Y_t^\lambda > 0$.

[20] e_i denotes a vector whose ith element is 1 and all remaining elements are 0.

[21] Note that the term

$$\Phi^Y(\tilde{\eta}) - \Phi^Y(\eta) - \left(\Phi^Y(e_1) - 1\right) = \mathbb{E}\left[\left(\exp\left(-\gamma Y_t^C + \left(1-\frac{1}{\theta}\right) A_\lambda Y_t^\lambda\right) - 1\right)\left(\exp\left(Y_t^C\right) - 1\right)\right]$$

is negative, since $\exp\left(-\gamma Y_t^C + \left(1-\frac{1}{\theta}\right) A_\lambda Y_t^\lambda\right) > 1$ and $\exp\left(Y_t^C\right) < 1$ if $\gamma > 1$, $A_\lambda > 0$, and the representative agent prefers early resolution of uncertainty.

2.5 The Price of a Consumption Claim

The representative agent's wealth in this economy is the present value of all future consumption, that is the price of an asset that pays consumption as its dividend given by

$$P_{C,t} = \mathbb{E}_t \left[\int_t^\infty \frac{\pi_s}{\pi_t} C_s \right].$$

The valuation ratio of the consumption claim, i.e. the wealth-consumption ratio, is denoted by $H_t = \frac{P_{C,t}}{C_t}$. The following two propositions give the wealth-consumption ratio and the consumption risk premium for the cases $\psi = 1$ and $\psi \neq 1$ respectively. If the representative agent has unit elasticity of intertemporal substitution, the solution is exact and the wealth-consumption ratio is a constant β^{-1}.

Proposition 8 (Wealth-Consumption Ratio and Consumption Risk Premium for $\psi = 1$)**.** *If the representative agent has unit elasticity of intertemporal substitution $\psi = 1$, then the wealth-consumption ratio is $H_t = \beta^{-1}$. The instantaneous risk premium of a claim to the consumption stream is given by*

$$\mathbb{E}_{t-} \left[r_{C,t} - r_{f,t-} \right] = \gamma \sigma_C^2 + \left[\Phi^Y(\eta) - \Phi^Y(\hat{\eta}) + \Phi^Y(e_1) - 1 \right] \lambda_{t-}$$

Proof. See appendix. □

In the case where ψ is different from one, an approximate solution exists, which log-linearizes the consumption-wealth ratio around its unconditional mean applying the same technique as in proposition 2.

Proposition 9 (Wealth-Consumption Ratio and Consumption Risk Premium for $\psi \neq 1$)**.** *If the representative agent has elasticity of intertemporal substitution different from one, then the wealth-consumption ratio is given by*

$$H(\lambda) = \exp\left(A_0^C + A_\lambda^C \lambda \right),$$

where the $A_0^C = -log\beta + \frac{A_0}{\theta}$ and $A_\lambda^C = \frac{A_\lambda}{\theta}$. The instantaneous risk premium on a claim to consumption with respect to the risk free rate is

$$\mathbb{E}_{t-} \left[r_{C,t} - r_{f,t-} \right] = \gamma \sigma_C^2 + \left[\Phi^Y(\eta) - \Phi^Y(\hat{\eta}) + \Phi^Y\left(\eta^C\right) - 1 \right] \lambda_{t-},$$

with $\eta^C = \left(1, A_\lambda^C\right)^T$.

Proof. See appendix. □

2.6 Valuation of a Claim to Corporate Dividends

Following Abel (1999), Campbell (2003), and Wachter (2010), I model corporate dividends D_t as a levered claim to consumption by letting $D_t = C_t^{\phi}$. Dividend growth is then governed by

$$\frac{dD_t}{D_{t-}} = \left(\phi\mu_C + \frac{1}{2}\phi(\phi-1)\sigma_C^2\right)dt + \phi\sigma_C dB_{c,t} + \left(e^{\phi Y_t^C} - 1\right)dN_t. \tag{13}$$

I will denote the time t price-dividend ratio of a claim to the dividend stream $\{D_s\}_{s\geq t}$ by $G_t = G(\lambda_t)$. The price of equity $P_{D,t} = D_t \cdot G_t$ follows

$$\begin{aligned}\frac{dP_{D,t}}{P_{D,t-}} = &\left(\phi\mu_C + \frac{1}{2}\phi(\phi-1)\sigma_C^2\right)dt + \frac{G_\lambda}{G_t}\kappa_\lambda(\bar{\lambda}-\lambda_t)dt + \frac{1}{2}\frac{G_{\lambda\lambda}}{G_t}\sigma_\lambda^2\lambda_t dt \\ &+ \phi\sigma_C dB_{c,t} + \sigma_\lambda\frac{G_\lambda}{G_t}\sqrt{\lambda_t}dB_{\lambda,t} + \frac{\Delta(D\cdot G)_t}{D_{t-}G_{t-}}dN_t.\end{aligned}$$

Substitution of the dynamics for the equity price and the stochastic discount factor into equation (9) yields a differential equation for $G(\lambda)$. Log-linearization of the dividend-yield around its unconditional mean gives rise to an approximation whose solution is exponentially affine in the the disaster intensity. The following proposition summarizes this solution and provides and expression of the equity premium with respect to the risk-free rate.

Proposition 10 (Price Dividend Ratio)**.** *The equilibrium price-dividend ratio of a claim to corporate dividends is given by*

$$G(\lambda) = \exp\left(A_0^D + A_\lambda^D\lambda\right),$$

where the coefficients A_0^D and A_λ^D satisfy the system of equations

$$\begin{aligned}0 =& -\beta + \left(\phi - \frac{1}{\psi}\right)\mu_C + \left(\frac{1}{2}\gamma\left(1+\frac{1}{\psi}\right) - \gamma\phi + \frac{1}{2}\phi(\phi-1)\right)\sigma_C^2 + A_\lambda^D\kappa_\lambda\bar{\lambda} + g_0 - g_1 A_0^D \\ 0 =& \frac{1}{2}\left(A_\lambda^D\right)^2\sigma_\lambda^2 - (\kappa_\lambda + g_1)A_\lambda^D + \left(1-\frac{1}{\theta}\right)A_\lambda\sigma_\lambda^2 A_\lambda^D - \frac{1}{2}\frac{1}{\theta}\left(1-\frac{1}{\theta}\right)A_\lambda^2\sigma_\lambda^2 \\ &-\left(1-\frac{1}{\theta}\right)\left(\Phi^Y(\hat{\eta})-1\right) + \left(\Phi^Y\left(\eta+\eta^D\right)-1\right),\end{aligned}$$

with $\eta^D = \left(\phi, A_\lambda^D\right)^T$. The instantaneous equity risk premium with respect to the risk-free rate is given by

$$\mathbb{E}_{t-}\left[r_{D,t} - r_{f,t-}\right] = \gamma\phi\sigma_C^2 - \left(1-\frac{1}{\theta}\right)A_\lambda\sigma_\lambda^2 A_\lambda^D\lambda_{t-} + \Lambda^D\lambda_{t-},$$

where $\Lambda^D = \left[\Phi^Y(\eta) + \Phi^Y\left(\eta^D\right) - \Phi^Y\left(\eta+\eta^D\right) - 1\right]$

Proof. See appendix. □

The equity premium consists of three components. The first term $\gamma\phi\sigma_C^2$ is present in a standard Lucas (1978) endowment economy in the absence of disasters. It denotes compensation for shocks to consumption due to the Brownian motion. The term $-\left(1-\frac{1}{\theta}\right)A_\lambda\sigma_\lambda^2A_\lambda^D\lambda_t$ arises from the representative agent's objective to hedge against diffusive shocks to the disaster intensity. In the calibration presented below, this term, which has been studied by Wachter (2010), contributes positively to the equity risk premium and is increasing in the disaster intensity.[22] The remaining term $\Lambda^D\lambda_t$ captures the effect of both disasters themselves and the resulting increase of the disaster intensity due to their arrival. These effects raise the equity premium.[23] The assessment of the relative magnitude of these two effects is a quantitative exercise that is carried out as part of the calibration presented below. It turns out that at the empirically relevant parameters, the second effect accounts for the major part of the equity premium. Hence, the representative agent is less anxious about the loss in consumption due a disaster itself than the possibility that this event could lead to a sequence of disasters.

2.7 The Risk-Neutral Measure

In order to determine equilibrium prices of financial derivatives, it is convenient to work under an equivalent risk-neutral measure $\tilde{\mathbb{P}}$. To construct this probability measure, define the exponential martingale $Z_t = \exp\left(\int_0^t r_u du\right)\pi_t$. Since Z_t satisfies $Z_t > 0$ a.s. and $\mathbb{E}\left[Z_t\right] = 1$, it is a Radon-Nikodym derivative. Fix a positive time T and define the equivalent probability measure $\tilde{\mathbb{P}}$ by

$$\tilde{\mathbb{P}}(A) = \int_{\omega\in A} Z_T(\omega)d\mathbb{P}(\omega) \text{ for all } A \in \mathcal{F}_T.$$

Under this measure, the price at time t of an asset that provides a dividend stream $\{D_{i,s}\}_{s=t}^{\infty}$ is given by

$$P_{i,t} = \tilde{\mathbb{E}}\left[\int_t^\infty e^{-\int_t^s r_u du} D_{i,s} ds \middle| \mathcal{F}_t\right].$$

[22] Note that $1-\frac{1}{\theta} = \frac{\frac{1}{\psi}-\gamma}{1-\gamma}$ is positive for $\gamma > 1$ and $\gamma > \frac{1}{\psi}$, i.e if the representative agent prefers early resolution of uncertainty. Moreover, $A_\lambda > 0$ and $A_\lambda^D < 0$, which ensures that $-\left(1-\frac{1}{\theta}\right)A_\lambda\sigma_\lambda^2A_\lambda^D\lambda_t$ is positive.

[23] To see this, first note that $1-\frac{1}{\theta} = \frac{\frac{1}{\theta}-1}{1-\gamma} > 0$ if $\gamma > 1$ and the representative agent has a preference for early resolution of uncertainty. Therefore

$$\Phi^Y(\eta)+\Phi^Y\left(\eta^D\right)-\Phi^Y\left(\eta+\eta^D\right)-1 = \mathbb{E}\left[\left(\exp\left(-\gamma Y_t^C+\left(1-\frac{1}{\theta}\right)A_\lambda Y_t^\lambda\right)-1\right)\left(1-\exp\left(\phi Y_t^C + A_\lambda^D Y_t^\lambda\right)\right)\right] > 0,$$

since $\exp\left(-\gamma Y_t^C + \left(1-\frac{1}{\theta}\right)A_\lambda Y_t^\lambda\right) > 1$ and $\exp\left(\phi Y_t^C + A_\lambda^D Y_t^\lambda\right) < 1$ with $A_\lambda > 0$ and $A_\lambda^D < 0$.

The Radon-Nikodym derivative process Z_t in this model is governed by

$$\frac{dZ_t}{Z_{t-}} = \eta^T \sigma_t dB_t + \left(\exp\left(\eta^T Y_t\right) - 1\right) dN_t - \left(\Phi^Y(\eta) - 1\right) \lambda_t dt,$$

with initial condition $Z_0 = 1$. The dynamics of consumption, dividends, and disaster risk which obtain under $\tilde{\mathbb{P}}$ are summarized by the following proposition.

Proposition 11 (Dynamics Under the Risk-Neutral Measure)**.** *The dynamics for consumption growth, dividend growth, and the state variables under the risk-neutral measure $\tilde{\mathbb{P}}$ are given by*

$$\begin{aligned}
d\ln C_t &= \left[\mu_C - \left(\gamma + \frac{1}{2}\right)\sigma_C^2\right] dt + \sigma_C d\tilde{B}_{C,t} + Y_t^C d\tilde{N}_t \\
d\ln D_t &= \phi\left[\mu_C - \left(\gamma + \frac{1}{2}\right)\sigma_C^2\right] dt + \phi\sigma_C d\tilde{B}_{C,t} + \phi Y_t^C d\tilde{N}_t \\
d\lambda_t &= \left[\kappa_\lambda(\bar{\lambda} - \lambda_t) + \left(1 - \frac{1}{\theta}\right) A_\lambda \sigma_\lambda^2 \lambda_t\right] dt + \sigma_\lambda \sqrt{\lambda_t} d\tilde{B}_t + Y_t^\lambda d\tilde{N}_t,
\end{aligned}$$

where $\tilde{B}_C$ and $\tilde{B}_\lambda$ are independent Brownian motions under $\tilde{\mathbb{P}}$. Furthermore, the intensity of the counting process $\tilde{N}$ is

$$\tilde{\lambda}_t = \Phi^Y(\eta)\lambda_t$$

under the martingale measure $\tilde{\mathbb{P}}$. The moment-generating function of Y_t under $\tilde{\mathbb{P}}$ is given by

$$\tilde{\Phi}^Y(u) = \frac{\Phi^Y(u+\eta)}{\Phi^Y(\eta)}.$$

Proof. See appendix. □

Under the risk-neutral measure, the drift of both log consumption and log dividend growth is reduced by a component which increases in risk aversion, volatility of diffusive shocks to consumption, and corporate leverage. Furthermore, the intensity of disasters under the equivalent martingale measure is higher than under the physical measure.[24]

The dynamics of the logarithm of the price of equity under the risk neutral measure follow immediately from the proposition above.

Corollary 1 (Dynamics of the Price of Equity under $\tilde{\mathbb{P}}$)**.** *The log-Price of equity* $\ln P_{D,t}$ *is*

[24] Note that $\Phi^Y(\eta) = \mathbb{E}\left[\exp\left(-\gamma Y_t^C + A_\lambda Y_t^\lambda\right)\right] > 1$ since $A_\lambda > 0$.

governed by the process

$$
\begin{aligned}
d\ln P_{D,t} = & \left[\phi\left(\mu_C - \left(\gamma + \frac{1}{2}\right)\sigma_C^2\right) + A_\lambda^D\left(\kappa_\lambda\left(\bar{\lambda} - \lambda_t\right) + \left(1 - \frac{1}{\theta}\right)A_\lambda \sigma_\lambda^2 \lambda_t\right)\right]dt \\
& + \phi\sigma_C d\tilde{B}_{C,t} + A_\lambda^D \sigma_\lambda \sqrt{\lambda_t} d\tilde{B}_{\lambda,t} + \left(\phi Y_t^C + A_\lambda^D Y_t^\lambda\right) d\tilde{N}_t
\end{aligned}
$$

under the risk-neutral measure $\tilde{\mathbb{P}}$.

Proof. See appendix. □

2.8 Equilibrium Prices of European Derivatives

Equilibrium Prices of European options that arise in this endowment economy can be computed in quasi-closed form using transform techniques developed by Carr and Madan (1999) and Duffie, Pan, and Singleton (2000). The following proposition provides the Fourier transform of the price of a general state contingent claim on equity with European exercise that pays $f(\ln P_{D,t})$ at maturity date T.

Proposition 12 (Prices of European Derivatives)**.** *Let $x_t = \ln P_{D,t}$ and consider a state contingent claim which yields a cash flow of $f(x_T)$ at expiration T. Denote by $P_{f,t} = \tilde{\mathbb{E}}\left[\exp\left(-\int_t^T r_{f,s}ds\right) f(x_T)\middle| \mathcal{F}_t\right]$ the equilibrium price of the derivative at time $t \leq T$. The Fourier transforms $FP_{f,t}(u) = \int_{-\infty}^{\infty} e^{i\cdot u\cdot x_t} P_{f,t}(x_t)dx_t$ is given by*

$$
FP_{f,t}(u) = \exp(i \cdot u \cdot x_t) \cdot Ff(u) \cdot \tilde{\Psi}^x(t, x, -u, T),
$$

where $\tilde{\Psi}_{t,T}^x(t, x, u, T) = \tilde{\mathbb{E}}\left[\exp\left(-\int_t^T r_{f,s}ds\right)\exp\left(iux_T\right)\middle| \mathcal{F}_t\right]$ denotes the discounted characteristic function of the log price of equity under the risk-neutral measure.

Proof. See appendix. □

This result is an application of the method developed in Lewis (2000) to a representative agent economy with self-exciting disaster risk. The derivative price at time t can then be determined by Fourier inversion, which entails numerical integration along a strip parallel to the real axis, i.e. an evaluation of the integral

$$
P_{f,t} = \frac{1}{2\pi}\int_{i\omega-\infty}^{i\omega+\infty} \exp\left(-iux_t\right) FP_{f,t}(u)du, \tag{14}
$$

for appropriate choice of $\omega \in \mathbb{R}$.

The applicability of this result in practice depends on the availability of simple expressions for the discounted characteristic function of the log price of equity as well as the Fourier transform of the derivative payoff. The appendix provides the joint discounted characteristic function of the log equity price and the disaster intensity arising from this consumption based model up to the solution of a system of ordinary differential equations, which are solved numerically. The Fourier transform of the payoff at expiration of a European option with log strike price k is given by

$$Ff(u) = \frac{\exp\left((iu+1)k\right)}{iu(iu+1)}.$$

For the valuation of call options, the integral (14) is computed with $\omega > 1$ whereas put option prices are obtained by choosing $\omega < 0$.

3 Calibration and Simulation

In order to illustrate the economic implications of self-exciting disaster risk quantitatively, the model is calibrated according to a dataset on economic crises documented in Barro and Ursúa (2009). The ability of self-perpetuating economic disasters to account for stylized asset pricing facts is then investigated based on a simulation of the calibrated model.

3.1 Calibration

The dynamics of consumption growth and the disaster intensity of the model to be calibrated are given by

$$\begin{aligned} dC_t &= \mu_C C_t dt + \sigma_C C_t dB_{C,t} + C_{t-}\left(\exp\left(Y_t^C\right) - 1\right) dN_t \\ d\lambda_t &= \kappa_\lambda\left(\bar{\lambda} - \lambda_t\right) dt + \sigma_\lambda \sqrt{\lambda_t} dB_{\lambda,t} + Y_t^\lambda dN_t, \end{aligned}$$

where $B_{C,t}$ and $B_{\lambda,t}$ are independent Brownian motions and N_t is a counting process with intensity λ_t. In order to ensure that disasters reduce consumption, the jump size Y_t^C has a negative Gamma distribution with shape parameter a_J and scale parameter b_J, i.e. $-Y_t^C \overset{i.i.d.}{\sim} GA(a_J, b_J)$. For simplicity, I assume that whenever a disaster occurs, the conditional disaster intensity λ_t jumps up by a constant Y^λ. A calibration hence involves picking values for the 8 parameters $\mu_C, \sigma_C, \kappa_\lambda, \bar{\lambda}, \sigma_\lambda, a_J, b_J$, and Y^λ.

The consumption dynamics during non-crisis times are identical to existing models in the disaster risk literature such as Barro (2006) and Wachter (2010). In order to facilitate comparison with these models, I adopt their parameter choices for the mean and the standard deviation of consumption growth during normal times and set $\mu_C = 0.0252$ and $\sigma_C = 0.02$. Wachter considers time variation in the disaster intensity that is unrelated to disaster arrival.

To clearly distinguish the ability of self-perpetuating economic disasters to explain stylized asset pricing facts, I shut down the channel investigated in her paper by setting $\sigma_\lambda = 0$.

The remaining parameters of the model, which control both the magnitude and the temporal behavior of economic crises, are calibrated to match salient features of a dataset on macroeconomic crises around the world presented in Barro and Ursúa (2008). The major characteristics of this dataset are summarized in Barro and Ursúa (2009), which also matches economic crises with contemporaneous declines in a stock market index. The study documents 100 severe economic downturns across 30 countries during which the macroeconomy contracted by more than 10%. In this paper, I am trying to explain stylized asset pricing facts for the U.S. I am restricting the sample to crises in 20 OECD countries since their economies and capital markets exhibit a higher degree of similarity with the those of the U.S.

Both the duration and the magnitude of each of the 57 crises remaining after this selection procedure are provided in Barro and Ursúa (2009) and are summarized in table 2 of the present paper. The probability of the economy moving into a recession that results in a contraction of at least 10% is 3.8%. Such a crisis lasts for 5.33 years on average. The mean and standard deviation of the decline in consumption experienced over the course of the crisis are 23.66% and 13.91% respectively. Raising the threshold of the loss in consumption during a crisis from a 10% to 15% reduces the probability of observing such an event in a given year to 2.2% and increases the average duration to 5.64% years. The conditional mean and standard deviation of the decline in consumption are 32.33% and 15.60% respectively. Further raising the bar for the macroeconomic contraction to 25% decreases the odds of observing such a crisis to 1.07% and raises the average crisis length to 6.19 years. The expected macroeconomic contraction associated with this event is 45.00%, while its standard deviation is 13.18% in the data.

The parameters to be calibrated are the speed of mean reversion κ_λ, the mean reversion target $\bar{\lambda}$, the increase in the disaster intensity in response to disaster arrival Y^λ, as well as the shape and scale parameters of the gamma distribution determining the jump size in log consumption, a_J and b_J. In order to find a calibration that replicates the moments summarized above, $5,000$ independent paths are simulated from the model, each with a length of 100 years. The jump intensity of each simulated path is initialized at its unconditional mean $\mathbb{E}[\lambda_t] = \kappa_\lambda \bar{\lambda}/(\kappa_\lambda - Y^\lambda)$. In the first period, a disaster is simulated that results in an increase in the intensity from $\mathbb{E}[\lambda_t]$ to $\mathbb{E}[\lambda_t] + Y^\lambda$ and simultaneously causes a downwards jump in log consumption according to the distribution of Y_t^C. This disaster marks the potential beginning of a crisis. In each simulated path, the end of this crisis is declared when the disaster intensity falls below a crisis threshold for the first time. I define this threshold to be reached once the crisis intensity has reverted halfway back to its unconditional mean from the first jump, i.e. the critical intensity for declaring a crisis is $\mathbb{E}[\lambda_t] + Y^\lambda/2$.

The parameter values that are chosen by this procedure are summarized in table 1. A comparison of the model implied characteristics of a crisis with their sample counterparts is detailed in table 2. The calibration is conservative in that both the probabilities of observing macroeconomic contractions that result in a consumption loss of at least 10%, 15%, and 25% respectively and their associated expected reductions conditional on these thresholds are smaller than in the data. Historically, both small and medium crises have lasted slightly longer than implied by the calibration. The average length of a crisis that results in a consumption reduction of 10% or more is 4.27 years versus 5.33 years in the data. The mean duration of a crisis that destroys at least 15% of consumption is 5.13 years compared with a historical duration of 5.64 years. On the other hand, large crises that result in a consumption loss exceeding 25% last longer in the model than they do in the data (7.19 years in the model versus 6.19 years in the data). Not every disaster in the model turns into a crisis that results a consumption decline of more than 10%. The calibrated model implies a moderate amount of crises that fall below the 10% threshold. The unconditional probability of observing a crisis that results in a macroeconomic contraction between 0% and 10% is 3.92% per year, which translates to roughly one such event every 25 years. The cumulative consumption loss during such an episode, which takes about 2 years, is 4.21% on average, or about 2.12% during each year of the crisis. The distribution of the disaster intensity arising from this calibration is depicted in figure 5. The unconditional mean of the disaster intensity is 10.06%. The intensity remains below 22% with a probability of 90%. In 95% and 99% of all cases the disaster intensity is below 31% and 53% respectively. Despite the simple dynamics of consumption and disaster risk, the calibration does well at explaining the frequency, magnitude, and duration of crises exceeding of 10% without introducing an unreasonable number of small crises.

The leverage of corporate dividends is set to $\phi = 3$, the value chosen by Bansal and Yaron (2004). The probability of partial default by the government in the event of a disaster q is set to the same value as in Wachter's calibration, wherein $q = 0.4$.

Preference parameters are chosen to match the equity premium and the expected return on short term government debt in U.S. data from 1890 to 2004. Relative risk aversion γ assumes a value of 2.65, the elasticity of intertemporal substitution (EIS) ψ, is set to 1.5, and the calibration uses a subjective discount factor of $\beta = 0.02$. The degree of risk aversion is significantly below 10, which is considered an upper bound in the literature for this parameter. There is some disagreement on the magnitude of the EIS between the macroeconomics literature on real business cycles and the long-run risk literature. Models of real business cycles typically require the EIS to be close to but smaller than one in order to match moments of macroeconomic data.[25] In contrast to that, the long-run risk literature requires the EIS to significantly exceed unity. I follow Bansal and Yaron (2004) and use a value of 1.5.

3.2 Simulation Results

Two paths from the calibrated model are simulated for $25,000$ years at a monthly frequency and then aggregated to an annual horizon. Annual moments from the simulation are reported in table 3 along with their sample counterparts in U.S. data. Historical moments on returns, consumption, and dividends spanning the period from 1890 to 2004 are from Robert Shiller's dataset reported in Wachter (2010).[26] Throughout the empirical analysis I use returns on short term government debt in the model as a proxy for returns on U.S. treasury bills.

3.2.1 The Equity Premium and the Return on Government Bills

The model with self-excitation is able to explain an equity premium of 6.08% along with a low expected return on government bills of 2.15%, both with a reasonable level of risk aversion. Furthermore, the model can address the excess volatility puzzle documented by Shiller (1981). Equity return volatility generated by the model is 18.96% per year which compares to 18.48% in U.S. data. The mechanism achieves this result without predicting counterfactually high volatility in the return on treasury bills, which is 2.79% in the simulation and 5.91% historically. The unconditional Sharpe ratio of 0.32 exactly matches its counterpart in the data. The volatility of both log consumption and log dividend growth is slightly above its historical value.

The dependence of the risk-free rate and the yield on government bills on the disaster intensity is depicted in figure 2. Since government debt is subject to default and investors are risk averse, the risk-free rate is below the expected return on government bills, which in turn is below the promised yield of these investments. All of these rates of return are decreasing in the disaster intensity. The risk-free rate is driven by an objective to smooth consumption over time and to form precautionary savings. Both of these motives become more pronounced in response to an upwards revision of the disaster intensity. If the probability of a disaster increases, expected consumption growth declines and the consumption smoothing motive increases the investor's desire to transfer wealth into the future. In addition to that, the precautionary savings objective intensifies during periods in which agents face higher odds

[25] See for e.g. Kydland and Prescott (1982), Lucas (1990), and Jones, Manuelli, and Siu (2000).

[26] The data is available on Shiller's website at http://www.econ.yale.edu/ shiller/data.htm. See Shiller (1989) for details on the construction of this dataset.

of a disaster. Both of these effects contribute towards a negative relationship between the risk-free rate and the disaster probability.

In contrast to a truly risk-free asset, government debt is itself subject to default in the event of a disaster. This systematic default risk causes risk averse investors to demand a premium that is increasing in the disaster probability. This latter effect partially counteracts the consumption smoothing and precautionary savings motives and decreases the sensitivity of the expected return on government debt with respect to the disaster intensity. In turn, the promised yield on these assets must be even higher than the expected yield to provide compensation for the possibility of default during economic disasters. The relationship between the risk-free rate as well as the promised and expected return on government bills are depicted in figure 2. These latter rates are less sensitive to changes in the disaster intensity compared to the risk-free rate. In times of crisis, the expected real return on government bills can be negative. This happens whenever the disaster intensity rises above 23%, which occurs in 9 out of 100 years in the simulation. In contrast to that, the realized real return on treasury bills in U.S. data has been negative in 25 out of 100 years.

The equity premium compensates investors for two sources of risk. The compensation for diffusive risk in prices and consumption considered by Lucas (1978) is too small to account for the observed equity premium in the U.S. Its contribution towards the total equity premium of 6.07% in the calibration presented here amounts to just 0.318%. The major part of the risk premium stems from compensation for the exposure of equity investments to rare economic disasters. In the context of recursive preferences, agents care about disasters for two reasons. The occurrence of a disaster leads to a sharp contemporaneous decline in both corporate dividends and consumption which requires equity to offer a premium for this exposure to systematic risk. In addition to that, the arrival of a disaster causes an upwards revision in the conditional disaster intensity by virtue of self-excitation. This causes both a decline in expected future consumption growth and an increase in the uncertainty about its future realization. A shock to the persistent disaster intensity, which increases uncertainty about future consumption growth affects indirect utility from future consumption. If investors exhibit a preference for the timing of resolution of uncertainty, i.e. if $\psi \neq 1/\gamma$, indirect utility enters into the pricing kernel. In particular, if agents prefer early resolution of uncertainty, which is the case that obtains in this calibration, they dislike an increase in the uncertainty about future consumption growth and hence require additional compensation for the possibility of a positive shock to the intensity of disaster arrival in the future.

Figure 3 illustrates how these individual components determine the equity premium in relation to the disaster intensity. The amount of diffusive risk equity investments are exposed to is constant with respect to the disaster intensity and its contribution to the equity premium is small in magnitude. The compensation for the instantaneous effect of a disaster on consumption is depicted by the dashed line and represents that part of the equity premium which arises from diffusive risk as well as the direct impact of a disaster on consumption. It is the risk premium that arises in a model with power utility such as Barro (2006) if disasters take place over extended periods rather than being concentrated at a single instant. The indirect effect of disasters on the equity premium works through the forward looking component in the pricing kernel that obtains in the case of stochastic differential utility with a preference for early resolution of uncertainty. It is compensation for the upwards revision of the jump intensity that is brought about by the arrival of a disaster.

3.2.2 Stock Market Crashes and Macroeconomic Contractions

Historically, stock market crashes have been observed even in the absence of significant declines in macroeconomic activity. Indeed, Paul Samuelson once famously remarked that "Wall Street indexes [had] predicted nine out of the last five recessions" (Samuelson (1966). In fact, an analysis of the crisis dataset in Barro and Ursúa (2009) indicates that in 28% of all cases, periods of negative stock market returns exceeding 25% were associated with a contemporaneous decline in macroeconomic activity of more than 10% in OECD countries. Likewise, the odds of a macroeconomic contraction exceeding 25% given a stock market decline of 25% were only 11% in this historical dataset.

A standard disaster risk model, in which the entire reduction in consumption is realized instantaneously is unable to account for this phenomenon since the only cause of a significant stock market decline is an economic disaster. The possibility of a self-perpetuating economic crisis can provide an explanation for this apparent puzzle. In the event of a disaster, agents rationally anticipate the possibility of a more severe economic crisis which leads to an increase in the compensation required for holding risky assets. This effect on the equity premium and hence the discount rate makes stock valuations decline by more than the reduction in dividends that is due to the disaster. In most cases, a more severe economic contraction does not materialize and the downturn in financial markets appears to have been excessive ex-post.

A simulation of the calibrated model demonstrates that self-excitation is consistent with this empirical fact. Table 6 illustrates the probability of observing a macroeconomic downturn in excess of a 10% and 25% threshold conditional on a stock market crash of at least 25%. In the simulation, a stock market crash exceeding 25% goes along with a macroeconomic contraction of at least 10% in roughly 4 out of 10 cases, which is slightly higher than in the historical dataset. The calibration exactly hits the mark for crises of more than 25%, which have a conditional probability of 11% both in the model and in the data.

3.2.3 Return Predictability

Valuation ratios on the aggregate stock market negatively predict future excess returns at various horizons in U.S. data. Panel C of table 4 reports slope coefficients from a regression of excess returns in the U.S. stock market over 1, 2, 4, 6, 8, and 10 years on the log price-dividend ratio at the beginning of the period. Low valuations relative to dividends predict higher than average returns and vice versa. Due to persistence in the price-dividend ratio, its predictive power for excess returns is increasing with the horizon. This predictability pattern arises naturally in the context of a model with a self-exciting disaster intensity. The disaster intensity λ completely determines the price-dividend ratio. An increase in λ due to the occurrence of a disaster raises the conditional equity premium and leads to a decrease of the price-dividend ratio. In most cases, another disaster does not occur and a low price-dividend ratio is followed by a period of higher than average returns due to mean reversion of the disaster intensity, which leads to an increase in the valuation ratio.

Table 4, panel A illustrates the model's ability to generate return predictability in the calibration presented here. Slope coefficients from a regression of excess returns on the log price-dividend ratio are negative throughout and decrease with the horizon. The model manages to reproduce the predictability pattern observed at medium horizons of 4 and 6 years. The degree to which excess-returns are predictable by the price-dividend ratio implied by the models is higher than in data at short horizons of 1 and 2 years and lower at long horizons of 8 and 10 years. Furthermore, the R^2's from these regressions are significantly below their counterparts in historical data.

3.2.4 Equity Options

The model succeeds at generating the implied volatility smirk pattern for equity index options qualitatively. Figure 6 and 7 depict the implied volatility of put options with one month and one year to expiration respectively. Option prices are computed at the unconditional mean disaster intensity. High implied volatilities of out of the money put options reflect relatively high prices for these options, which are compensation for insurance against economic disasters provided by these instruments. The smirk generated by the model is more pronounced than its empirical counterpart in put options on the S&P 500 index.

Figure 8 graphs the model-generated VIX computed from both call and put options with 30 days to expiration as a function of the disaster intensity.[27] The model-implied VIX is

[27] The calculation implements the methodology used by the Chicago Board Options Exchange outlined in the document "The CBOE Volatility Index-VIX" available from the CBOE website (www.cboe.com/micro/vix/vixwhite.pdf). The computation is based on model-implied prices for calls and puts with moneyness between 80% and 120%.

positively related to the conditional likelihood of a disaster, which can help in explaining the sharp increase in the VIX during the fall of 2008. A VIX of 16.23 on June 29, 2007 just before the beginning of the crisis corresponds to a disaster intensity of 4.5% per year in the calibration, which is the mean reversion target. The implied volatility of 18.81 on August 22 of 2008 is consistent with a disaster intensity of about 6% per annum. Furthermore, the subsequent surge of the VIX to 80.86 on November 20, 2008 implies that market participants expected disaster arrival to occur at an instantaneous rate of 1.36 events per year. This increase of 130% in the disaster intensity requires more than 6 consumption disasters if the upward revisions in the intensity is 20% as in the calibration. In contrast to that, there was only one such disaster during the fourth quarter of 2008. The decline of the VIX to around 16 by mid April 2010 can be attributed to mean reversion of the disaster intensity in the absence of further large negative consumption declines. This highlights the model's ability to qualitatively account for the observed VIX dynamics but also demonstrates its numerical shortfalls in matching the exact magnitude.

4 Conclusion

In this paper, I have demonstrated the importance of self-perpetuating economic disasters for understanding stylized asset pricing facts. A calibration of a simple model can explain the equity premium and risk free rate with a reasonable degree of risk aversion. Excess volatility and return predictability by valuation multiples arise naturally with this model. It is the first disaster risk model which can account for the empirical observation that severe economic downturns develop over extended periods of time. This feature allows predictions for the behavior of equilibrium quantities during a crisis and can account for an increase in the VIX and the dividend-yield as well as a decline in the risk-free rate in response to the occurrence of a consumption disaster.

Previous disaster risk models have been criticized for their assumption that the entire extent of an economic contraction be realized in a single instant.[28] This paper demonstrates the ability of self-exciting disasters to address this point which gives further justification to this important class of models.

While the model is able to account for the implied volatility smile in equity index options, it fails to capture its magnitude. Implied volatilities of out of the money put options generated by the model exceed the corresponding observations in S&P 500 option markets. This shortcoming might in part be owed to the simple model of corporate cash flows assumed in this paper. In reality, a stock market index is not just a levered claim to aggregate

[28]See for instance Constantinides (2008).

consumption. A more sophisticated model that properly accounts for the corporate fraction in this economy, like the one proposed by Longstaff and Piazzesi (2004), might contribute to resolve this issue.

Promising avenues for future research include the empirical analysis of the dynamics of equity premia and bond yields during severe economic downturns in developed countries as well as an investigation of the implications of self-exciting disasters for the cross-section of expected returns.

A Affine Jump Diffusions

Fix a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual hypothesis, on which are defined a d-dimensional Brownian motion W_t, K counting processes with independent arrival $N_{k,t}$, $k \in \{1, 2, ..., K\}$, and associated with each process N_k, a sequence of d-dimensional i.i.d. random vectors $\{Y_{k,t}\}_{t\geq 0}$.[29] The distribution of $Y_{k,t}$ can be characterized by its joint moment generating function

$$\Phi_k^Y(u) = \mathbb{E}\left[\exp(u^T \cdot Y_{k,t})\right]$$

for some $u \in \mathbb{R}^d$. Define the process $X_t \in \mathcal{D} \subset \mathbb{R}^d$ by

$$dX_t = \mu(X_t)dt + \sigma(X_t)dW_t + \sum_{k=1}^{K} Y_{k,t} dN_{k,t} \tag{A.1}$$

with initial condition $X_0 = x$. The counting processes $\{N_k\}_{k=1,2,...,K}$ have stochastic arrival intensity $\lambda_k(X_t)$. Assume that the coefficients of (A.1) satisfy the restrictions

$$\begin{aligned}
\mu(X) &= K_0 + K_1 \cdot X && \text{, with } K_0 \in \mathbb{R}^d, K_1 \in \mathbb{R}^{d\times d} \\
\sigma(X)\cdot\sigma(X)^T &= H_0 + \sum_{i=1}^{d} H_{1,i} X_i && \text{, with } H_0, H_1 \in \mathbb{R}^{d\times d} \\
\lambda_k(X) &= \lambda_{0,k} + \lambda_{1,k}^T X && \text{, with } \lambda_{0,k} \in \mathbb{R}, \lambda_{1,k} \in \mathbb{R}^d \\
r_{f,t}(X) &= \rho_0 + \rho_1 X && \text{, with } \rho_0 \in \mathbb{R}, \lambda_1 \in \mathbb{R}^d,
\end{aligned} \tag{A.2}$$

where $r_{f,t}$ denotes the risk-free rate. Under these conditions X_t is an affine jump diffusion (AJD) and the following results hold as special cases to the theory developed in Duffie, Pan, and Singleton (2000). The following lemma gives the characteristic function of an affine jump diffusion.

Lemma A.1 (Discounted Characteristic Function of Affine Jump Diffusion)**.** *For some $u \in \mathbb{C}^d$, $T \geq t$, and $X_t \in \mathcal{D}$, let*

$$\Psi^X(t, x, T, u) = \mathbb{E}\left[\exp\left(-\int_t^T r_{f,s} ds\right) \exp(iu^T X_T) \middle| X_t = x\right]$$

denote the characteristic function of the X_T conditional on $X_t = x$. If the jump diffusion (A.1) *satisfies the affine coefficient restrictions* (A.2)*, then its characteristic function is given*

[29] See e.g. Protter (2005, page 3) for a definition of the usual hypothesis.

by

$$\Psi^X(t,x,T,u) = \exp(A_0(T-t) + A_X(T-t)^T x)$$

where $A_0(\tau)$ and $A_X(\tau)$ are functions in $\mathbb{C}$- and $\mathbb{C}^d$ respectively, satisfying the system of ordinary differential equations

$$\begin{aligned}
A_0'(\tau) &= A_X(\tau)^T K_0 + \frac{1}{2} A_X(\tau)^T H_0 A_X(\tau) + \sum_{k=1}^{K} \left(\Phi_k^Y(A_X(\tau)) - 1\right) \lambda_{0,k} - \rho_0 \\
A_{X,i}'(\tau) &= \left[A_X(\tau)^T K_1\right]_i + \frac{1}{2} A_X(\tau)^T H_{1,i} A_X(\tau) + \sum_{k=1}^{K} \left(\Phi_k^Y(A_X(\tau)) - 1\right) \lambda_{1,k,i} - \rho_{1,i}
\end{aligned} \tag{A.3}$$

with initial conditions $A_0(0) = 0$ and $A_X(0) = iu$.

Proof. First note that $\exp\left(-\int_0^t r_{f,s} ds\right) \Psi^X(t,x,T,u)$ is a martingale under $\mathbb{P}$, i.e. for any $t_1 \leq t_2 \leq T$, we have

$$\begin{aligned}
&\mathbb{E}\left[\exp\left(-\int_0^{t_2} r_{f,s} ds\right) \Psi^X(t_2, X_{t_2}, T, u) \middle| X_{t_1} = x\right] \\
=&\mathbb{E}\left[\mathbb{E}\left[\exp\left(-\int_0^{T} r_{f,s} ds\right) \exp(iu^T X_T) \middle| X_{t_2} = x\right] \middle| X_{t_1} = x\right] \\
=&\exp\left(-\int_0^{t_1} r_{f,s} ds\right) \mathbb{E}\left[\exp\left(-\int_{t_1}^{T} r_{f,s} ds\right) \exp(iu^T X_T) \middle| X_{t_1} = x\right] \\
=&\exp\left(-\int_0^{t_1} r_{f,s} ds\right) \Psi^X(t_1, X_{t_1}, T, u).
\end{aligned}$$

Let $D_t = \exp\left(-\int_0^t r_{f,s} ds\right)$. The dynamics of $D_t \Psi_t^X = D_t \Psi^X(t, X_t, s, u)$ follow from an application of Ito's formula for jump-diffusions, which yields

$$\begin{aligned}
\frac{d\left(D_t \Psi_t^X\right)}{D_{t-}} =& \left[\frac{\partial \Psi^X}{\partial t} + \frac{\partial \Psi^X}{\partial X^T}\mu(X_t) + \frac{1}{2}\operatorname{tr}\left(\frac{\partial^2 \Psi^X}{\partial X \partial X^T}\sigma(X_t)\sigma(X_t)^T\right) + \sum_{k=1}^{K} \mathbb{E}_{t-}\left[\Delta \Psi_{k,t}^X\right] \lambda_k(X_t) - r_{f,t}\right] dt \\
&+ \frac{\partial \Psi^X}{\partial X^T}\sigma(X_t) dW_t + \sum_{k=1}^{K}\left[\Delta \Psi_{k,t}^X dN_{k,t} - \mathbb{E}_{t-}\left[\Delta \Psi_{k,t}^X\right] \lambda_k(X_{t-}) dt\right].
\end{aligned}$$

The martingale restriction on $\Psi^X(t, X_t, s, u)$ gives rise to the partial differential equation

$$\frac{\partial \Psi^X}{\partial t} + \frac{\partial \Psi^X}{\partial X^T}\mu(X_t) + \frac{1}{2}\operatorname{tr}\left(\frac{\partial^2 \Psi^X}{\partial X \partial X^T}\sigma(X_t)\sigma(X_t)^T\right) + \sum_{k=1}^{K} \mathbb{E}_{t-}\left[\Delta \Psi_{k,t}^X\right] \lambda_k(X_{t-}) - r_{f,t} = 0.$$

Substitution of the conjecture $\Psi^X(t,x,T,u) = \exp(A_0(T-t) + A_X(T-t)^T x)$ yields the differential equation

$$- \left(A_0'(\tau) + A_X'(\tau)^T x\right) + A_x(\tau)^T \left(K_0 + K_1 x\right) + \frac{1}{2} A_X(\tau)^T H_0 A_X(\tau)$$
$$+\frac{1}{2}\sum_{i=1}^{d} A_X(\tau)^T H_{1,i} A_X(\tau) x_i + \sum_{k=1}^{K} \left(\Phi_k^Y(A_X(\tau)) - 1\right)(\lambda_{0,k} + \lambda_{1,k} x) - (\rho_0 + \rho_1 x) = 0.$$

Since the coefficients multiplying each x_i and the constant must be zero individually to make the equation hold for all x, one obtains the system of ordinary differential equations of the proposition. □

The next result gives a characterization of the dynamics of the affine jump diffusion under a change of measure.

Lemma A.2 (Change of Measure for Affine Jump Diffusion)**.** *Let X_t be an affine jump-diffusion and $\eta \in \mathbb{R}^d$. Denote by Z_t the Radon-Nikodym process*

$$\frac{dZ_t}{Z_{t-}} = \eta^T \sigma(X_t) dW_t - \sum_{k=1}^{K} \left[\left(1 - \exp\left(\eta^T Y_{k,t}\right)\right) dN_{k,t} - \left(1 - \Phi_k^Y(\eta)\right) \lambda_k(X_{t-}))dt\right],$$

with $Z_0 = 0$. Fix $T > 0$ and define the probability measure $\tilde{\mathbb{P}}$ by

$$\tilde{\mathbb{P}}(A) = \int_{\omega \in A} Z_T(\omega) d\mathbb{P}(\omega) \text{ for all } A \in \mathcal{F}_T$$

Under the probability measure $\tilde{\mathbb{P}}$ the process X_t is an affine jump diffusion with dynamics

$$dX_t = \tilde{\mu}(X_t)dt + \sigma(X_t) d\tilde{W} + \sum_{k=1}^{K} Y_{k,t} d\tilde{N}_{k,t},$$

where $\tilde{W}_t$ is a d-dimensional Brownian motion and $\tilde{N}_k$, $k \in \{1, 2, ..., K\}$ are K independent counting processes with intensities $\tilde{\lambda}_k(X_t)$ under $\tilde{\mathbb{P}}$. The drift coefficient is given by $\tilde{\mu}(X) = \tilde{K}_0 + \tilde{K}_1 X$, with

$$\tilde{K}_0 = K_0 + H_0^T \eta$$
$$\tilde{K}_1 = K_1 + \left[H_{1,1}^T \eta ... H_{1,d}^T \eta\right].$$

The arrival intensity of the point process N_k is given by $\tilde{\lambda}_k(X) = \tilde{\lambda}_{0,k} + \tilde{\lambda}_{1,k} X$ with

$$\tilde{\lambda}_{0,k} = \lambda_{0,k} \Phi_k^Y(\eta)$$
$$\tilde{\lambda}_{1,k} = \lambda_{1,k} \Phi_k^Y(\eta).$$

Under $\tilde{\mathbb{P}}$, the random variable $Y_{k,t}$ has moment generating function

$$\tilde{\Phi}_k^Y(u) = \frac{\Phi_k^Y(u+\eta)}{\Phi_k^Y(\eta)}.$$

Proof. Since Z_t is an exponential martingale, we have $Z_t > 0$ a.s. and $\mathbb{E}\left[Z_t\right] = 1$ for all $t \geq 0$. Hence Z_t is a Radon-Nikodym derivative and $\tilde{\mathbb{P}}$ is a probability measure. The proof proceeds by showing that the characteristic function of X_t under the measure $\tilde{\mathbb{P}}$ is that of an affine jump diffusion with the proposed parameters. The characteristic function of X_s under $\tilde{\mathbb{P}}$ conditional on $\mathcal{F}_t$ is defined as

$$\tilde{\Psi}^X(t, X_t, s, u) = \tilde{\mathbb{E}}\left[\exp\left(iu^T X_s\right)\middle|\, \mathcal{F}_t\right] = \mathbb{E}\left[Z_s \exp\left(iu^T X_s\right)\middle|\, \mathcal{F}_t\right].$$

The process $Z_t\tilde{\Psi}^X(t, X_t, s, u)$ is a $\mathbb{P}$-martingale, since for any $t_1 \leq t_2 \leq s$, we have

$$\begin{aligned}
\mathbb{E}\left[Z_{t_2}\tilde{\Psi}^X(t_2, X_{t_2}, s, u)\middle|\, \mathbb{F}_{t_1}\right] &= \mathbb{E}\left[Z_{t_2}\tilde{\mathbb{E}}\left[\exp(iu^T X_s)\middle|\, \mathcal{F}_{t_2}\right]\middle|\, \mathcal{F}_{t_1}\right] \\
&= \mathbb{E}\left[Z_{t_2}\mathbb{E}\left[\frac{Z_s}{Z_{t_2}}\exp(iu^T X_s)\middle|\, \mathcal{F}_{t_2}\right]\middle|\, \mathcal{F}_{t_1}\right] \\
&= Z_{t_1}\mathbb{E}\left[\frac{Z_s}{Z_{t_1}}\exp(iu^T X_s)\middle|\, \mathcal{F}_{t_1}\right] \\
&= Z_{t_1}\tilde{\Psi}^X(t_1, X_{t_1}, s, u).
\end{aligned}$$

In the following let $\tilde{\Psi}_t^X$ denote $\tilde{\Psi}^X(t, X_t, s, u)$. By an application of Ito's rule, the dynamics of $Z_t\tilde{\Psi}_t^X$ under $\mathbb{P}$ are given by

$$\begin{aligned}
d\left(Z_t\tilde{\Psi}_t^X\right) =& \left[\frac{\partial\tilde{\Psi}^X}{\partial t} + \frac{1}{2}\operatorname{tr}\left(\frac{\partial^2\tilde{\Psi}^X}{\partial X\partial X^T}\sigma(X_t)\sigma(X_t)^T\right) + \sum_{k=1}^K\left(1 - \Phi_k^Y(\eta)\right)\tilde{\Psi}_t^X\lambda_k(X_t)\right. \\
&\left. + \left(\mu(X_t)^T + \eta^T\sigma(X_t)\sigma(X_t)^T\right)\frac{\partial\tilde{\Psi}^X}{\partial X}\right] Z_t dt \\
&+ \left[\frac{\partial\tilde{\Psi}^X}{\partial X^T}\sigma(X_t) + \eta^T\sigma(X_t)\right] Z_t dW_t + \sum_{k=1}^K \Delta\left(Z\tilde{\Psi}^X\right)_{k,t} dN_{k,t}.
\end{aligned}$$

Since $Z_t\tilde{\Psi}_t^X$ is a martingale under $\mathbb{P}$, the characteristic function of X_t under $\tilde{\mathbb{P}}$, $\tilde{\Psi}^X(t, X_t, s, u)$

satisfies the partial differential equation

$$\frac{\partial \tilde{\Psi}^X}{\partial t} + \frac{1}{2}\operatorname{tr}\left(\frac{\partial^2 \tilde{\Psi}^X}{\partial X \partial X^T}\sigma(X)\sigma(X)^T\right) + \sum_{k=1}^{K}\left(1 - \Phi_k^Y(\eta)\right)\tilde{\Psi}_t^X \lambda_k(X)$$
$$+ \left(\mu(X)^T + \eta^T\sigma(X)\sigma(X)^T\right)\frac{\partial \tilde{\Psi}^X}{\partial X} + \sum_{k=1}^{K}\mathbb{E}_{t-}\left[\Delta\left(Z\tilde{\Psi}^X\right)_{k,t}\right]\lambda_k(X) = 0$$

Substituting the conjecture $\tilde{\Psi}^X(t, X, s, u) = \exp\left(\tilde{A}_0(s-t) + \tilde{A}_X(s-t)^T X\right)$ and letting $\tau = s - t$ gives rise to the ordinary differential equation

$$-\left(\tilde{A}_0'(\tau) + \tilde{A}_X'(\tau)^T X\right) + \frac{1}{2}\tilde{A}_X(\tau)^T H_0 \tilde{A}_X(\tau) + \tilde{A}_X(\tau)^T K_0 + \tilde{A}_X(\tau)^T K_1 X$$
$$+ \frac{1}{2}\sum_{i=1}^{d}\tilde{A}_X(\tau)^T H_{1,i}\tilde{A}_X(\tau) X_i + \sum_{k=1}^{K}\left(1 - \Phi_k^Y(\eta)\right)\lambda_{0,k} + \sum_{k=1}^{K}\left(1 - \Phi_k^Y(\eta)\right)\lambda_{1,k}X$$
$$+ \eta^T H_0 \tilde{A}_X(\tau) + \eta^T\sum_{i=1}^{d} H_{1,i}\tilde{A}_X(\tau) X_i + \sum_{k=1}^{K}\left(\Phi_k^Y\left(\eta + \tilde{A}_X(\tau)\right) - 1\right)\left(\lambda_{0,k} + \lambda_{1,k}X\right) = 0.$$

Since this equation holds for all values of X, the coefficients multiplying each of the components X_i and the constant term must be separately equal to zero. This yields the system of simultaneous differential equations

$$\tilde{A}_0'(\tau) = \left(K_0^T + \eta^T H_0\right)\tilde{A}_X(\tau) + \frac{1}{2}\tilde{A}_X(\tau)^T H_0 \tilde{A}_X(\tau) + \sum_{k=1}^{K}\left(\Phi_k^Y\left(\eta + \tilde{A}_X(\tau)\right) - \Phi_k^Y(\eta)\right)\lambda_{0,k}$$
$$\tilde{A}_{X,i}'(\tau) = \left(\left(K_1^T + \eta^T H_{1,i}\right)\tilde{A}_X(\tau)\right)_i + \frac{1}{2}\tilde{A}_X(\tau)^T H_{1,i}\tilde{A}_X(\tau) + \sum_{k=1}^{K}\left(\Phi_k^Y\left(\eta + \tilde{A}_X(\tau)\right) - \Phi_k^Y(\eta)\right)\lambda_{1,k,i}.$$

The claim then follows by comparing coefficients of the differential equation above with the one for the characteristic function given in lemma A.1. □

The following result provides the conditional expectation of an affine jump-diffusion.

Lemma A.3 (Expectation of Affine Jump-Diffusion)**.** *Let X_t be an affine jump-diffusion. The conditional expectation $\mathbb{E}\left[X_T \middle| \mathcal{F}_t\right]$ is given by*

$$\mathbb{E}\left[X_T \middle| \mathcal{F}_t\right] = \frac{1}{i}\left(\alpha_0(T-t) + \alpha_X(T-t)\cdot X_t\right),$$

where $\alpha_0 : \mathbb{R}_+ \mapsto \mathbb{C}^d$ *and* $\alpha_X : \mathbb{R}_+ \mapsto \mathbb{C}^{d\times d}$ *solve the differential equations*

$$\begin{aligned}
\alpha_0'(\tau) &= \alpha_X(\tau) \cdot K_0 + \sum_{k=1}^{K} \alpha_X(\tau) \nabla \Phi_k^Y(0) \lambda_{0,k} \\
\alpha_{X,i}'(\tau) &= [\alpha_X(\tau) \cdot K_1]_{(1,\ldots,d),i} + \sum_{k=1}^{K} \alpha_X(\tau) \nabla \Phi_k^Y(0) \lambda_{1,k,i}
\end{aligned} \tag{A.4}$$

with initial conditions $\alpha_0(0) = (0, ..., 0)^T$ *and* $\alpha_{X,j}(0) = i \cdot e_j^T$*.* [30]

Proof. Let $\Psi^{X^*}(t, X_t, T, u) = \mathbb{E}\left[\exp(iu^T X_T) \middle| \mathcal{F}_t\right]$, i.e. the characteristic function of X_t, which can be obtained from lemma A.1 by setting $\rho_0 = 0$ and $\rho_1 = 0$. By the properties of characteristic functions, the first moment of X_t, can be computed as

$$\begin{aligned}
\mathbb{E}\left[X_T | \mathcal{F}_t\right] &= \frac{1}{i} \left. \frac{\partial \Psi^{X^*}(t, X_t, T, u)}{\partial u} \right|_{u=0} \\
&= i^{-1}\, \Psi^{X^*}(t, X_t, T, u) \left. \left(\frac{\partial A_0(\tau, u)}{\partial u} + \frac{\partial \big(A_X(\tau, u)^T\big)}{\partial u} \cdot X_t \right) \right|_{u=0} \\
&= i^{-1} \Psi^{X^*}(t, X_t, T, 0) \left(\alpha_0(\tau) + \alpha_X(\tau) \cdot X_t \right),
\end{aligned}$$

where

$$\alpha_0(\tau) = \left. \frac{\partial A_0(\tau, u)}{\partial u} \right|_{u=0} \text{ and } \alpha_X(\tau) = \left. \frac{\partial \big(A_X(\tau, u)^T\big)}{\partial u} \right|_{u=0}.$$

Differentiation of (A.3) with respect to u yields

$$\begin{aligned}
\alpha_0'(\tau) &= \alpha_X(\tau) \cdot K_0 + \alpha_X(\tau) \cdot H_0 \cdot A_X(\tau) + \sum_{k=1}^{K} \alpha_X(\tau) \nabla \Phi_k^Y(A_X(\tau)) \lambda_{0,k} \\
\alpha_{X,i}'(\tau) &= [\alpha_X(\tau) \cdot K_1]_{(1,\ldots,d),i} + \alpha_X(\tau) \cdot H_{1,i} \cdot A_X(\tau) + \sum_{k=1}^{K} \alpha_X(\tau) \nabla \Phi_k^Y(A_X(\tau)) \lambda_{1,k,i}.
\end{aligned}$$

The boundary conditions given in the lemma arise by differentiation of the boundary conditions for the equations characterizing $A_0(\tau)$ and $A_X(\tau)$ with respect to u. Since $u = 0$, $\Psi^{X^*}(t, X_t, T, 0) = \mathbb{E}[\exp(i \cdot 0 \cdot X_T) | X_t] = 1$. Accordingly $A_0(\tau) = 0$ and $A_X(\tau) = (0, ..., 0)^T$.

[30] $\nabla \Phi^Y(u)$ denotes the gradient $\left(\frac{\partial \Phi^Y}{\partial u_1}, \frac{\partial \Phi^Y}{\partial u_2}, ..., \frac{\partial \Phi^Y}{\partial u_d}\right)^T$.

Hence, the differential equations above simplify to

$$\alpha_0'(\tau) = \alpha_X(\tau) \cdot K_0 + \sum_{k=1}^{K} \alpha_X(\tau) \nabla \Phi_k^Y(0) \lambda_{0,k}$$

$$\alpha_{X,i}'(\tau) = [\alpha_X(\tau) \cdot K_1]_{(1,\ldots,d),i} + \sum_{k=1}^{K} \alpha_X(\tau) \nabla \Phi_k^Y(0) \lambda_{1,k,i}.$$

□

B A Generalized Asset Pricing Model

This appendix presents a generalizes asset pricing model, which extends the model considered in this paper towards long-run risk and stochastic volatility. Furthermore, disasters may be driven by a finite number of interrelated self-exciting jump processes.

B.1 Endowment Process

I model a representative agent endowment economy. Consumption growth contains a small persistent component and is subject to both diffusive risk with time-varying uncertainty and occasional rare disasters. The dynamics are as follows:

$$dC_t = (\mu_C + X_t) C_t dt + \sqrt{V_t} C_t dB_{C,t} + C_{t-} \sum_{k=1}^{K} \left(e^{Y_{k,t}^C} - 1 \right) dN_{k,t} \quad \text{(B.1)}$$

$$dX_t = -\kappa_X X_t dt + \sigma_X \sqrt{V_t} dB_{X,t} + \sum_{k=1}^{K} Y_{k,t}^X dN_{k,t} \quad \text{(B.2)}$$

$$dV_t = \kappa_V (\bar{V} - V_t) dt + \sigma_V \sqrt{V_t} dB_{V,t} + \sum_{k=1}^{K} Y_{k,t}^V dN_{k,t} \quad \text{(B.3)}$$

$$d\lambda_{k,t} = \kappa_{\lambda,k} (\bar{\lambda}_k - \lambda_{k,t}) dt + \sigma_{\lambda,k} \sqrt{\lambda_{k,t}} dB_{k,t} + \sum_{j=1}^{K} Y_{j,t}^{\lambda_k} dN_{j,t} \quad \text{(B.4)}$$

The counting processes $N_{1,t}$ through $N_{K,t}$ have independent arrival times with stochastic intensity $\lambda_{k,t}$ respectively. The Brownian motions driving consumption growth C_t, expected consumption growth X_t, economic uncertainty V_t and jump intensities B_{λ_k} are assumed to be mutually independent. For succinctness of notation, I will let B_t denote the multidimensional Brownian motion $(B_{C,t}, B_{X,t}, B_{V,t}, B_{\lambda_1,t}, ..., B_{\lambda_K,t})^T$. Each jump process $N_{k,t}$ triggers jumps of random size in consumption growth and the state variables X_t, V_t, and $\{\lambda_{k,t}\}_{k=1}^K$. The distribution of the jump size to both consumption, $Y^C_{k,t}$, and expected consumption growth $Y^X_{k,t}$ is allowed to have support on the entire real line. In order to ensure that economic uncertainty V_t and the intensities $\lambda_{k,t}$ remain positive, the restrictions $Y^V_{k,t} > 0$ and $Y^{\lambda_j}_{k,t} > 0$ are imposed. Let $Y_{k,t}$ denote the vector $(Y^C_{k,t}, Y^X_{k,t}, Y^V_{k,t}, Y^{\lambda_1}_{k,t}, ..., Y^{\lambda_K}_{k,t})^T$ and assume that it is independent of B_t and $\{N_k\}_{k=1}^K$. The joint distribution of jump sizes can be characterized by the moment generating function $\Phi^Y_k(u) = \mathbb{E}\left[\exp\left(u^T \cdot Y_{k,t}\right)\right]$.

This setup nests both a continuous-time version of the long-run risk model due to Bansal and Yaron (2004) and the time-varying rare disaster model by Wachter (2010). The novel feature of the approach presented here is the use of a self-exciting jump process to capture the idea that the occurrence of a rare disaster also results in a sharp increase of the conditional probability of another disaster.

B.2 Preferences

The representative agent has recursive preferences developed in discrete time by Kreps and Porteus (1978), Epstein and Zin (1989), and Weil (1989). The model employs its continuous-time counterpart, stochastic differential utility (SDU), introduced by Duffie and Epstein (1992a,b). Continuation utility of the representative agent, J_t, is defined by the recursion

$$J_t = \mathbb{E}_t\left[\int_t^\infty f(C_s, J_s)ds\right], \tag{B.5}$$

where $f(C, J)$ is the normalized Porteus-Kreps aggregator

$$f(C,J) = \frac{\beta}{1-\frac{1}{\psi}} J(1-\gamma)\left(\frac{C^{1-\frac{1}{\psi}}}{((1-\gamma)J)^{\frac{1}{\theta}}} - 1\right) \qquad \text{for } \psi \neq 1 \tag{B.6}$$

$$f(C,J) = \beta(1-\gamma)J\left(\log C - \frac{1}{1-\gamma}\log\left((1-\gamma)J\right)\right) \qquad \text{for } \psi = 1. \tag{B.7}$$

In this definition, β assumes the role of a subjective time-discount factor, γ is the coefficient of relative risk aversion, ψ denotes the elasticity of intertemporal substitution, and the constant θ is defined as $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$. In the special case $\gamma = \frac{1}{\psi}$, the preference structure coincides with power utility.

B.3 Solution

We are ultimately interested in the impact of time-variation in the stochastic opportunity set, in particular the effect of a jump in the disaster intensity in response to a rare event, on equilibrium quantities like the wealth-consumption ratio, the risk free rate, the dividend-yield, and equity option prices. This requires an expression for the pricing kernel, which can be directly inferred from the intertemporal marginal rate of substitution (IMRS) of the representative agent and the dynamics of aggregate consumption in models with power utility. In the case of recursive preferences, the IMRS of the representative agent also depends on continuation utility, that is the value function, which is determined endogenously. Hence I begin by deriving the value function for a representative agent in this economy.

B.3.1 The Value Function

In order to make progress on the derivation of the value function J_t, I rewrite the recursion (B.5) in terms of a partial differential equation (PDE) in J. We first however exploit the fact that the dynamics of C_t, X_t, V_t, and $\{\lambda_{k,t}\}_{k=1}^K$ follow a Markov process, which implies that continuation utility is a function of the state at time t only, i.e. J_t can be written as $J(C_t, X_t, V_t, \lambda_{1,t}, ..., \lambda_{K,t})$.[31] Applying a Feynman-Kac type argument to the recursive definition of continuation utility given in equation (B.5) results in the PDE

$$\mathcal{D}J_t + f(C_t, J_t) = 0,$$

where $\mathcal{D}J_t$ denotes the infinitesimal generator associated with the dynamics of C_t, X_t, V_t, and $\{\lambda_{k,t}\}_{k=1}^K$.[32][33] Substituting the expression for the infinitesimal generator derived below yields the PDE [34]

$$\begin{aligned} &J_C(\mu_C + X_t)C_t - J_X\kappa_X X_t + J_V\kappa_V(\bar{V} - V_t) + \sum_{k=1}^K J_{\lambda_k}\kappa_{\lambda_k}(\bar{\lambda}_k - \lambda_{k,t}) + \frac{1}{2}J_{CC}V_tC_t^2 \\ &+\frac{1}{2}J_{XX}\sigma_X^2 V_t + \frac{1}{2}J_{VV}\sigma_V^2 V_t + \frac{1}{2}\sum_{k=1}^K J_{\lambda_k\lambda_k}\sigma_{\lambda_k}^2\lambda_{k,t} + \sum_{k=1}^K \mathbb{E}_{t-}\left[\Delta J_{t,k}\right]\lambda_{k,t-} + f(C_t, J_t) = 0, \end{aligned} \tag{B.8}$$

[31]Note that the Markov property implies

$$J_t = \mathbb{E}_t\left[\int_t^\infty f(C_s, J_s)ds\right] = \mathbb{E}\left[\int_t^\infty f(C_s, J_s)ds \middle| C_t, X_t, V_t, \lambda_{1,t}, ..., \lambda_{K,t}\right] = J(C_t, X_t, V_t, \lambda_{1,t}, ..., \lambda_{K,t}).$$

[32]See the section on derivations and proofs below for a heuristic derivation.

[33]The infinitesimal generator is defined as

$$\mathcal{D}J_t = \lim_{h\to 0}\frac{\mathbb{E}_{t-}\left[J(C_{t+h}, X_{t+h}, V_{t+h}, \lambda_{1,t+h}, ..., \lambda_{K,t+h}) - J(C_{t-}, X_{t-}, V_{t-}, \lambda_{1,t-}, ..., \lambda_{K,t-})\right]}{h}.$$

[34]I use the notation g_X to denote the partial derivative of a function g with respect to X, except for g_t, which denotes the value of that function at time t.

where $\Delta J_{k,t}$ denotes the jump size in the value function conditional on a jump of N_k, i.e.[35]

$$
\begin{aligned}
\Delta J_{k,t} &= J(C_t, X_t, V_t, \lambda_{1,t}, ..., \lambda_{K,t}) - J(C_{t-}, X_{t-}, V_{t-}, \lambda_{1,t-}, ..., \lambda_{K,t-}) \\
&= J(C_{t-} \cdot e^{Y^C_{k,t}}, X_{t-} + Y^X_{k,t}, V_{t-} + Y^V_{k,t}, \lambda_{1,t-} + Y^{\lambda_1}_{k,t}, ..., \lambda_{K,t-} + Y^{\lambda_K}_{k,t}) \\
&- J(C_{t-}, X_{t-}, V_{t-}, \lambda_{1,t-}, ..., \lambda_{K,t-}).
\end{aligned}
$$

The solution for the value function takes the form[36]

$$
J(C, X, V, \lambda_1, \lambda_K) = \frac{C^{1-\gamma}}{1-\gamma} I(X, V, \lambda_1, ..., \lambda_K), \tag{B.9}
$$

where the function $I(\cdot)$ remains to be determined. Upon substituting (B.9) into (B.8) one obtains[37]

$$
\begin{aligned}
&(1-\gamma)(\mu_C + X_t) - \frac{I_X}{I}\kappa_X X_t + \frac{I_V}{I}\kappa_V(\bar{V} - V_t) + \sum_{k=1}^{K} \frac{I_{\lambda_k}}{I}\kappa_{\lambda_k}(\bar{\lambda}_k - \lambda_{k,t}) - \frac{1}{2}\gamma(1-\gamma)V_t \\
&+\frac{1}{2}\frac{I_{XX}}{I}\sigma_X^2 V_t + \frac{1}{2}\frac{I_{VV}}{I}\sigma_V^2 V_t + \frac{1}{2}\sum_{k=1}^{K}\frac{I_{\lambda_k\lambda_k}}{I}\sigma^2_{\lambda_k}\lambda_{k,t} + \sum_{k=1}^{K}\mathbb{E}_{t-}\left[e^{Y^C_{k,t}}\frac{I_{k,t}}{I_{k,t-}} - 1\right]\lambda_{k,t-} + f(C_t, J_t) = 0,
\end{aligned} \tag{B.10}
$$

where $I_{k,t}$ and $I_{k,t-}$ are used to denote $I(X_{t-} + Y^X_{k,t}, V_{t-} + Y^V_{k,t}, \lambda_{1,t-} + Y^{\lambda_1}_{k,t}, ..., \lambda_{K,t-} + Y^{\lambda_K}_{k,t})$ and $I(X_{t-}, V_{t-}, \lambda_{1,t-}, ..., \lambda_{K,t-})$ respectively.

Since the form of the aggregator depends on whether the EIS is different from unity, I will treat the cases $\psi = 1$ and $\psi \neq 1$ separately in the analysis that follows. I will begin by analyzing the case $\psi = 1$ which has an exact closed form solution for $I(\cdot)$ of the form

$$
I(X, V, \lambda_1, ..., \lambda_K) = \exp\left(A_0 + A_X \cdot X + A_V \cdot V + \sum_{k=1}^{K} A_{\lambda_k}\lambda_k\right).
$$

The coefficients A_0, A_X, A_V, and $A_{\lambda_1}, ..., A_{\lambda_K}$ are determined by a nonlinear system of equations given in the following proposition. Details of the computation are presented in the section on derivations and proofs below.

Proposition B.1 (Equilibrium Value Function for $\psi = 1$)**.** *If the representative agent has*

[35] To see that $C_t = C_{t-} \cdot e^{Y^C_{k,t}}$ at jump times of N_k, first note that equation (B.1) implies $\Delta C_{t,k} = C_{t-}\left(\exp\left(Y^C_{k,t}\right) - 1\right)$. It then follows that at jump times of the kth counting process, we have $C_t = C_{t-} + \Delta C_{t,k} = C_{t-} \cdot \exp\left(Y^C_{k,t}\right)$.

[36] This follows from the homogeneity of the value function with respect to consumption. See lemma B.1.

[37] See section on derivations and proofs below for details on the derivation.

unit elasticity of intertemporal substitution $\psi = 1$, *then the value function solving* (B.8) *is given by*

$$J(C, X, V, \lambda_1, ..., \lambda_K) = \frac{C^{1-\gamma}}{1-\gamma} \cdot \exp\left(A_0 + A_X \cdot X + A_V \cdot V + \sum_{k=1}^{K} A_{\lambda_k} \lambda_k\right).$$

where the coefficients A_0, A_X, A_V, *and* $A_{\lambda_1}, ..., A_{\lambda_K}$ *satisfy the system of equations*

$$\begin{aligned}
0 &= (1-\gamma)\mu_C + A_V \kappa_V \bar{V} + \sum_{k=1}^{K} A_{\lambda_k} \kappa_{\lambda_k} \bar{\lambda}_k - \beta A_0 \\
0 &= (1-\gamma) - A_X \kappa_X - \beta A_X \\
0 &= -\beta A_V - A_V \kappa_V - \frac{1}{2}\gamma(1-\gamma) + \frac{1}{2} A_X^2 \sigma_X^2 + \frac{1}{2} A_V^2 \sigma_V^2 \\
0 &= -A_{\lambda_k} \kappa_{\lambda_k} + \frac{1}{2} A_{\lambda_k}^2 \sigma_{\lambda_k}^2 - \beta A_{\lambda_k} + \left(\Phi_k^Y(\hat{\eta}) - 1\right) \ \forall k \in \{1, ..., K\},
\end{aligned}$$

with $\hat{\eta} = (1-\gamma, A_X, A_V, A_{\lambda_1}, ..., A_{\lambda_K})^T$.

Proof. See section on derivations and proofs. □

Having established the value function for the case of a unit elasticity of intertemporal substitution, I now turn to the analysis for $\psi \neq 1$. This situation is slightly more complicated as an exact closed form solution to the ensuing nonlinear partial differential equation does not exist. A method introduced by Campbell, Chacko, Rodriguez, and Viceira (2004), however, admits a log-linearization of the nonlinear term in the PDE around the mean consumption-wealth ratio. The solution of the approximate problem then takes the same functional form as for the case of $\psi = 1$. The following proposition states the solution for the value function resulting from this approximation, with the details once again being relegated to the section on derivations and proofs below.

Proposition B.2 (Equilibrium Value Function for $\psi \neq 1$)**.** *If the representative agent has elasticity of intertemporal substitution* ψ *that is different from one, then the value function solving a log-linear approximation of the PDE* (B.8) *is given by*

$$J(C, X, V, \lambda_1, ..., \lambda_K) = \frac{C^{1-\gamma}}{1-\gamma} \cdot \exp\left(A_0 + A_X \cdot X + A_V \cdot V + \sum_{k=1}^{K} A_{\lambda_k}, \lambda_k\right).$$

where the coefficients $A_0, A_X, A_V,$ and $A_{\lambda_1}, ..., A_{\lambda_K}$ satisfy the system of equations

$$
\begin{aligned}
0 &= (1-\gamma)\mu_C + A_V \kappa_V \bar{V} + \sum_{k=1}^{K} A_{\lambda_k} \kappa_{\lambda_k} \bar{\lambda}_k + \theta i_0 + \theta i_1 \log \beta - \beta\theta - i_1 A_0 \\
0 &= (1-\gamma) - A_X \kappa_X - i_1 A_X \\
0 &= -i_1 A_V - A_V \kappa_V - \frac{1}{2}\gamma(1-\gamma) + \frac{1}{2}A_X^2 \sigma_X^2 + \frac{1}{2}A_V^2 \sigma_V^2 \\
0 &= -(\kappa_{\lambda_k} + i_1) A_{\lambda_k} + \frac{1}{2} A_{\lambda_k}^2 \sigma_{\lambda_k}^2 + \left(\Phi_k^Y(\hat{\eta}) - 1\right) \text{ for all } k \in \{1, ..., K\},
\end{aligned}
$$

with $\hat{\eta} = (1-\gamma, A_X, A_V, A_{\lambda_1}, ..., A_{\lambda_K})^T$.

Proof. See section on derivations and proofs. □

B.3.2 Asset Prices, Risk Premia, and the Risk Free Rate

In the absence of arbitrage, the price $P_{i,t}$ of an asset paying dividend $D_{i,s}$ at time $s \geq t$ solves

$$\pi_t P_{i,t} = \mathbb{E}_t \left[\int_t^\infty \pi_s D_{i,s} ds \right], \tag{B.11}$$

where π_t denotes the pricing kernel. The following proposition provides a differential characterization for asset prices.

Proposition B.3 (No-Arbitrage Pricing PDE)**.** *The no-arbitrage price $P_{i,t}$ of a claim yielding dividends $D_{i,s}$ at time $s \geq t$ satisfies the PDE*

$$\frac{\mathcal{D}(\pi_t \cdot P_{i,t})}{\pi_{t-} \cdot P_{i,t-}} + \frac{D_{i,t-}}{P_{i,t-}} = 0,$$

which can be decomposed as[38]

$$\frac{\mathcal{D}\pi_t^c}{\pi_{t-}} + \frac{\mathcal{D}P_{i,t}^c}{P_{i,t-}} + \frac{d\left[\pi^c, P_i^c\right]_t}{\pi_{t-} \cdot P_{i,t-} \cdot dt} + \frac{\sum_{k=1}^K \mathbb{E}_{t-}\left[\Delta\left(\pi \cdot P_i\right)_{k,t}\right] \lambda_{k,t-}}{\pi_{t-} \cdot P_{i,t-}} + \frac{D_{i,t-}}{P_{i,t-}} = 0. \tag{B.12}$$

Proof. The proof of the first equation proceeds along the same line as the derivation of the PDE for the value function. Equation (B.12) then follows from an application of Ito's rule for jump-diffusions. □

This is an extension of the pricing equation for cash flows in continuous-time discussed in e.g. Cochrane (2001, page 32) to a jump-diffusion setting.

[38]The superscript c denotes the continuous part of a process. See Shreve (2004, chap. 11) for details.

A risk free asset pays dividends at rate $r_{f,t}$ and has a constant price $P_{r_f,t}$. By applying equation (B.12) one obtains the following characterization for the risk-free rate.

Proposition B.4 (Risk Free Rate)**.** *The instantaneous risk free rate is given by*

$$r_{f,t-} = -\frac{\mathcal{D}\pi_t^C}{\pi_t-} - \frac{\sum_{k=1}^{K} \mathbb{E}_{t-}\left[\Delta\pi_{k,t}\right]\lambda_{k,t-}}{\pi_{t-}} = -\frac{\mathcal{D}\pi_t}{\pi_{t-}}. \tag{B.13}$$

Proof. A risk-free asset with instantaneous dividend yield $\frac{D_t}{P_{D,t}} = r_{f,t}$ has constant price $P_{r_f,t} = P_{r_f}$. Since $dP_{r_f,t} = 0$, we have $\mathcal{D}P_{r_f,t} = 0$, $d\left[\pi^C, P_D^C\right]_t = 0$ and hence $\mathcal{D}\left(\pi^c \cdot P_D^c\right) = 0$, and $\Delta(\pi \cdot P_{r_f})_{k,t} = \Delta\pi_{k,t}$. Upon substitution into (B.12), one obtains (B.13). $\square$

The instantaneous expected return of a risky asset paying dividend stream $\{D_{i,s}\}_{s\geq t}$ is the sum of the expected appreciation of the continuous part, the expected return of the jump component, and the instantaneous dividend yield, that is

$$\mathbb{E}_t[r_{i,t-}] = \frac{\mathcal{D}P_{i,t}^c}{P_{i,t-}} + \frac{\sum_{k=1}^{K} \mathbb{E}_{t-}\left[\Delta P_{i,k,t}\right]\lambda_{k,t-}}{P_{i,t-}} + \frac{D_{i,t-}}{P_{i,t-}}.$$

Combining propositions B.3 and B.4 and using the definition of the expected return given above one obtains the risk premium of a dividend paying asset, which is stated in the following proposition.

Proposition B.5 (Risk Premium)**.** *The instantaneous risk premium of a claim to dividends* $\{D_{i,s}\}_{s\geq t}$ *is given by*

$$\begin{aligned} \mathbb{E}_t\left[r_{i,t-}\right] - r_{f,t-} &= \frac{\mathcal{D}P_{i,t}^c}{P_{i,t-}} + \frac{\sum_{k=1}^{K} \mathbb{E}_t\left[\Delta P_{i,k,t}\right]\lambda_{k,t-}}{P_{i,t-}} + \frac{D_{i,t-}}{P_{i,t-}} - r_{f,t-} \\ &= -\frac{d\left[\pi^c, P_i^c\right]_t}{\pi_{t-}\cdot P_{i,t-}\cdot dt} - \frac{\sum_{k=1}^{K} \mathbb{E}_{t-}\left[\Delta\pi_{k,t}\cdot\Delta P_{i,k,t}\right]\lambda_{k,t-}}{\pi_{t-}\cdot P_{i,t-}} \end{aligned} \tag{B.14}$$

Proof. See section on derivations and proofs. $\square$

The risk premium consists of two components. The first term represents compensation for diffusive risk, whereas the second component designates the premium arising from the exposure to disaster risk.

B.3.3 The Pricing Kernel

Duffie and Epstein (1992a) and Duffie and Skiadas (1994) show that the process π_t given by

$$\pi_t = \exp\left(\int_0^t f_J(C_s, J_s)ds\right) f_C(C_t, J_t) \tag{B.15}$$

can serve as a pricing kernel in a representative agent economy with stochastic differential utility. The following proposition provides a characterization of the dynamics of the pricing kernel as well as the equilibrium risk free rate.

Proposition B.6 (Pricing Kernel and Risk Free Rate)**.** *The dynamics of the pricing kernel are governed by*

$$\frac{d\pi_t}{\pi_{t-}} = -r_{f,t}dt + \eta^T \sigma_t dB_t + \sum_{k=1}^{K} \left[\exp\left(\eta^T Y_{k,t}\right) - 1\right] dN_{k,t} - \sum_{k=1}^{K} \left[\Phi_k^Y(\eta) - 1\right] \lambda_{k,t} dt,$$

with $\pi_0 = 1$, where η denotes the market price of risk vector

$$\eta = \left(-\gamma, \left(1 - \frac{1}{\theta}\right) A_X, \left(1 - \frac{1}{\theta}\right) A_V, \left(1 - \frac{1}{\theta}\right) A_{\lambda_1}, ..., \left(1 - \frac{1}{\theta}\right) A_{\lambda_K}\right)^T$$

and σ_t denotes the matrix of diffusion coefficients

$$\sigma_t = \begin{pmatrix} \sqrt{V_t} & 0 & 0 & 0 & \dots & 0 \\ 0 & \sigma_X\sqrt{V_t} & 0 & 0 & \dots & 0 \\ 0 & 0 & \sigma_V\sqrt{V_t} & 0 & \dots & 0 \\ 0 & 0 & 0 & \sigma_{\lambda_1}\sqrt{\lambda_{1,t}} & \dots & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \dots & \sigma_{\lambda_K}\sqrt{\lambda_{K,t}} \end{pmatrix}.$$

The equilibrium risk free rate is

$$\begin{aligned} r_{f,t} =& \beta + \frac{1}{\psi}\mu_C + \frac{1}{\psi}X_t - \left[\frac{1}{2}\gamma\left(1 + \frac{1}{\psi}\right) + \frac{1}{2}\frac{1}{\theta}\left(1 - \frac{1}{\theta}\right)\left(A_X^2\sigma_X^2 + A_V^2\sigma_V^2\right)\right] V_t \\ &+ \sum_{k=1}^{K} \left[\left(1 - \frac{1}{\theta}\right)\left(\left(\Phi_k^Y(\hat{\eta}) - 1\right) + \frac{1}{2}\frac{1}{\theta}A_{\lambda_k}^2\sigma_{\lambda_k}^2\right) - \left(\Phi_k^Y(\eta) - 1\right)\right] \lambda_{k,t}. \end{aligned}$$

Proof. See section on derivations and proofs. □

The expressions simplify with a unit elasticity of intertemporal substitution since $\psi = 1$ implies $\frac{1}{\theta} = 0$. The proposition establishes that the risk-free rate is a linear function of the state variables.

B.4 Defaultable Short-Term Government Debt

Historically, economic crises have often been accompanied by at least a partial default of the government on its liabilities. To account for this possibility, I follow Barro and Wachter in assuming that whenever a rare disaster of type k occurs, the government defaults with probability q_k. Furthermore, the fraction of notional that is lost in the event of default is identical to the reduction in consumption due to the disaster. Let $r_{L,t}$ denote the promised interest rate on short term government debt. Given continuous reinvestment of interest payments, the value of government debt $P_{L,t}$ evolves according to

$$\frac{dP_{L,t}}{P_{L,t-}} = r_{L,t}dt + \sum_{k=1}^{K} \left(e^{Y_{k,t}^L} - 1\right) dN_{k,t},$$

where $Y_{k,t}^L = Y_{k,t}^C$ with probability q_k and $Y_{k,t}^L = 0$ with probability $1 - q_k$. Since this investment strategy yields no dividends, $P_{L,t}$ satisfies the pricing equation

$$\frac{\mathcal{D}\pi_t^c}{\pi_{t-}} + \frac{\mathcal{D}P_{L,t}^c}{P_{L,t-}} + \frac{d\left[\pi^c, P_L^c\right]_t}{\pi_{t-}P_{L,t-}dt} + \frac{\sum_{k=1}^{K} \mathbb{E}_{t-}\left[\Delta\left(\pi \cdot P_L\right)_{k,t}\right]\lambda_{k,t-}}{\pi_{t-}P_{L,t-}} = 0.$$

The equilibrium rate promised by the government on short-term debt can then be characterized as follows.

Proposition B.7 (Equilibrium Interest Rate on Short-Term Government Debt)**.** *The instantaneous interest rate on defaultable short term debt promised by the government in equilibrium is*

$$r_{L,t} = r_{f,t} + \sum_{k=1}^{K} \left[\Phi_k^Y(\eta) - \Phi_k^Y(\tilde{\eta})\right] q_k \lambda_{k,t},$$

where $\tilde{\eta} = \left(1-\gamma, \left(1-\frac{1}{\theta}\right)A_X, \left(1-\frac{1}{\theta}\right)A_V, \left(1-\frac{1}{\theta}\right)A_{\lambda_1}, ..., \left(1-\frac{1}{\theta}\right)A_{\lambda_K}\right)^T$.

Proof. See section on derivations and proofs. □

The expected return on government bill's $r_{b,t} = \frac{\mathcal{D}P_{L,t}}{P_{L,t-}}$, which adjust the promised rate for the possibility of default, is then given by[39]

$$\begin{aligned} r_{b,t} &= r_{L,t} + \sum_{k=1}^{K} \left(q_k\mathbb{E}_t\left[\exp\left(Y_{k,t}^C\right) - 1\right] + (1-q_k)\mathbb{E}_t\left[\exp(0) - 1\right]\right)\lambda_{k,t} \\ &= r_{L,t} + \sum_{k=1}^{K} \left(\Phi_k^Y(e_1) - 1\right) q_k \lambda_{k,t}. \end{aligned}$$

The risk premium of an asset with respect to the return on government debt is related to

[39] e_i denotes a vector whose ith element is 1 and all remaining elements are 0.

the risk premium with respect to the risk-free rate by

$$\mathbb{E}_{t-}\left[r_{i,t} - r_{b,t-}\right] = \mathbb{E}_{t-}\left[r_{i,t} - r_{f,t-}\right] + \sum_{k=1}^{K}\left[\Phi_k^Y\left(\tilde{\eta}\right) - \Phi_k^Y\left(\eta\right) - \left(\Phi_k^Y(e_1) - 1\right)\right] q_k \lambda_{k,t}$$

B.5 The Price of a Consumption Claim

The representative agent's wealth in this economy is the present value of all future consumption, i.e the price of an asset that pays consumption as its dividend, i.e.

$$P_{C,t} = \mathbb{E}_t\left[\int_t^\infty \frac{\pi_s}{\pi_t} C_s\right].$$

The valuation ratio of the consumption claim, i.e. the wealth-consumption ratio, is denoted by $H_t = \frac{P_{C,t}}{C_t}$. The following two propositions give the wealth-consumption ratio and the consumption risk premium for the cases $\psi = 1$ and $\psi \neq 1$ respectively. If the representative agent has unit elasticity of intertemporal substitution, the solution is exact and the wealth-consumption ratio is a constant β^{-1}.

Proposition B.8 (Wealth-Consumption Ratio and Consumption Risk Premium for $\psi = 1$)**.** *If the representative agent has unit elasticity of intertemporal substitution $\psi = 1$, then the wealth-consumption ratio is $H_t = \beta^{-1}$. The risk premium of a claim to the consumption stream is given by*

$$\mathbb{E}_{t-}\left[r_{C,t} - r_{f,t-}\right] = \gamma V_t + \sum_{k=1}^{K}\left[\Phi_k^Y\left(\eta\right) - \Phi_k^Y\left(\hat{\eta}\right) + \Phi_k^Y(e_1) - 1\right] \lambda_{k,t-}$$

Proof. See section on derivations and proofs. □

In the case where ψ is different from one, an approximate solution exists, which log-linearizes the nonlinear term in the pricing PDE employing the same technique as in proposition B.2.

Proposition B.9 (Wealth-Consumption Ratio and Consumption Risk Premium for $\psi \neq 1$)**.** *If the representative agent has elasticity of intertemporal substitution different from one, then the wealth-consumption ratio is given by*

$$H(X, V, \lambda_1, ..., \lambda_K) = \exp\left[A_0^C + A_X^C X + A_V^C V + \sum_{k=1}^{K} A_{\lambda_k}^C \lambda_k\right],$$

where the $A_0^C = -log\beta + \frac{A_0}{\theta}$, $A_X^C = \frac{A_X}{\theta}$, $A_V^C = \frac{A_V}{\theta}$, *and* $A_{\lambda_k}^C = \frac{A_{\lambda_k}}{\theta}$ *for all* $k = 1, ..., K$. *The risk premium on a claim to consumption is*

$$\mathbb{E}_{t-}\left[r_{C,t} - r_{f,t-}\right] = \left[\gamma - \left(1 - \frac{1}{\theta}\right)\frac{1}{\theta}\left(A_X^2\sigma_X^2 + A_V^2\sigma_V^2\right)\right]V_{t-} + \sum_{k=1}^{K}\left[\Phi_k^Y(\eta) - \Phi_k^Y(\hat{\eta}) + \Phi_k^Y\left(\eta^C\right) - 1\right]\lambda_{k,t-},$$

with $\eta^C = \left(1, A_X^C, A_V^C, A_{\lambda_1}^C, ..., A_{\lambda_K}^C\right)^T$.

Proof. See section on derivations and proofs. □

B.6 Valuation of a Claim to Corporate Dividends

Following Abel (1999), Campbell (2003), and Wachter (2010), I model corporate dividends D_t as a levered claim to consumption by letting $D_t = C_t^\phi$. Dividend growth is then governed by

$$\frac{dD_t}{D_{t-}} = \left(\phi(\mu_C + X_t) + \frac{1}{2}\phi(\phi - 1)V_t\right)dt + \phi\sqrt{V_t}dB_{c,t} + \sum_{k=1}^{K}\left(e^{\phi Y_{k,t}^C} - 1\right)dN_{k,t}. \qquad \text{(B.16)}$$

I will denote the time t price-dividend ratio of a claim to the dividend stream $\{D_s\}_{s\geq t}$ by $G_t = G(X_t, V_t, \lambda_{1,t}, ..., \lambda_{K,t})$. By an application of Ito's rule for jump-diffusions, the price of equity $P_{D,t} = D_t \cdot G_t$ follows

$$\begin{aligned}
\frac{dP_{D,t}}{P_{D,t-}} = &\left(\phi(\mu_C + X_t) + \frac{1}{2}\phi(\phi - 1)V_t\right)dt + \phi\sqrt{V_t}dB_{c,t} \\
&+ \frac{G_X}{G_t}\left(-\kappa_X X_t dt + \sigma_X\sqrt{V_t}dB_{X,t}\right) + \frac{G_V}{G_t}\left(\kappa_V(\bar{V} - V_t)dt + \sigma_V\sqrt{V_t}dB_{V,t}\right) \\
&+ \sum_{k=1}^{K}\frac{G_{\lambda_k}}{G_t}\left(\kappa_{\lambda_k}(\bar{\lambda}_k - \lambda_{k,t})dt + \sigma_{\lambda_k}\sqrt{\lambda_{k,t}}dB_{k,t}\right) \\
&+ \frac{1}{2}\frac{G_{XX}}{G_t}\sigma_X^2 V_t dt + \frac{1}{2}\frac{G_{VV}}{G_t}\sigma_V^2 V_t dt + \frac{1}{2}\sum_{k=1}^{K}\frac{G_{\lambda_k\lambda_k}}{G_t}\sigma_{\lambda_k}^2\lambda_{k,t}dt + \sum_{k=1}^{K}\frac{\Delta(D \cdot G)_t}{D_{t-}G_{t-}}dN_{k,t}.
\end{aligned}$$

Substitution of the dynamics for $P_{D,t}$ and the pricing kernel into equation (B.12) yields a PDE, in which the dividend-yield is log-linearized around its unconditional mean using the method developed by Campbell, Chacko, Rodriguez, and Viceira (2004). This yields the approximation

$$\frac{1}{G_t} = \exp\left(\log(D_t) - \log(P_{D,t})\right) \approx g_0 + g_1\left(\log(D_t) - \log(P_{D,t})\right) = g_0 - g_1\log(G_t),$$

with $g_1 = \exp\left(\mathbb{E}\left[\log(D_t) - \log(P_{D,t}]\right), g_0 = g_1(1 - \log(g_1))$. The approximate PDE has a closed-form solution which is exponentially affine in the state variables. The following proposition summarizes this solution and gives the equity premium with respect to the risk-free rate.

Proposition B.10 (Price Dividend Ratio)**.** *The equilibrium price-dividend ratio of a claim to corporate dividends is given by*

$$G(X, V, \lambda_1, ..., \lambda_K) = \exp\left(A_0^D + A_X^D X + A_V^D V + \sum_{k=1}^{K} A_{\lambda_k}^D \lambda_k\right),$$

where the coefficients A_0^D, A_X^D, A_V^D, and $A_{\lambda_1}^D, ..., A_{\lambda_K}^D$ satisfy the system of equations

$$\begin{aligned}
0 =& -\beta + \left(\phi - \frac{1}{\psi}\right)\mu_C + A_V^D \kappa_V \bar{V} + \sum_{k=1}^{K} A_{\lambda_k}^D \kappa_{\lambda_k} \bar{\lambda}_k + g_0 - g_1 A_0^D \\
0 =& \phi - \frac{1}{\psi} - (\kappa_X + g_1) A_X^D \\
0 =& \frac{1}{2}\gamma\left(1 + \frac{1}{\psi}\right) - \gamma\phi + \frac{1}{2}\phi(\phi - 1) - (\kappa_V + g_1) A_V^D + \frac{1}{2}\left(A_X^D\right)^2 \sigma_X^2 + \frac{1}{2}\left(A_V^D\right)^2 \sigma_V^2 \\
&+ \left(1 - \frac{1}{\theta}\right)\left(A_X \sigma_X^2 A_X^D + A_V \sigma_V^2 A_V^D\right) - \frac{1}{2}\frac{1}{\theta}\left(1 - \frac{1}{\theta}\right)\left(A_X^2 \sigma_X^2 + A_V^2 \sigma_V^2\right) \\
0 =& \frac{1}{2}\left(A_{\lambda_k}^D\right)^2 \sigma_{\lambda_k}^2 - (\kappa_{\lambda_k} + g_1) A_{\lambda_k}^D + \left(1 - \frac{1}{\theta}\right) A_{\lambda_k} \sigma_{\lambda_k}^2 A_{\lambda_k}^D - \frac{1}{2}\frac{1}{\theta}\left(1 - \frac{1}{\theta}\right) A_{\lambda_k}^2 \sigma_{\lambda_k}^2 \\
&- \left(1 - \frac{1}{\theta}\right)\left(\Phi_k^Y(\hat{\eta}) - 1\right) + \left(\Phi_k^Y\left(\eta + \eta^D\right) - 1\right),
\end{aligned}$$

with $\eta^D = \left(\phi, A_X^D, A_V^D, A_{\lambda_1}^D, ..., A_{\lambda_K}^D\right)^T$. The instantaneous equity risk premium with respect to the risk-free rate is given by

$$\begin{aligned}
\mathbb{E}_{t-}\left[r_{D,t-} - r_{f,t-}\right] =& \left[\gamma\phi - \left(1 - \frac{1}{\theta}\right)\left(A_X \sigma_X^2 A_X^D + A_V \sigma_V^2 A_V^D\right)\right] V_{t-} \\
&+ \sum_{k=1}^{K}\left[\Phi_k^Y(\eta) + \Phi_k^Y\left(\eta^D\right) - \Phi_k^Y\left(\eta + \eta^D\right) - 1 - \left(1 - \frac{1}{\theta}\right) A_{\lambda_k} \sigma_{\lambda_k}^2 A_{\lambda_k}^D\right] \lambda_{k,t-}.
\end{aligned}$$

Proof. See section on derivations and proofs. □

B.7 The Risk-Neutral Measure

In order to determine equilibrium prices of financial derivatives, it is often convenient to work under an equivalent risk-neutral probability measure $\tilde{\mathbb{P}}$. To obtain this alternative measure, introduce a martingale $Z_t > 0$ a.s. with $\mathbb{E}[Z_t] = 1$ and $\pi_t = \exp\left(-\int_0^t r_u du\right) Z_t$. Fix a positive time T and define the equivalent probability measure $\tilde{\mathbb{P}}$ by

$$\tilde{\mathbb{P}}(A) = \int_{\omega \in A} Z_T(\omega) d\mathbb{P}(\omega) \text{ for all } A \in \mathcal{F}_T.$$

Under this measure, the price at time t of an asset that provides a dividend stream $\{D_{i,s}\}_{s=t}^{\infty}$ is given by

$$P_{i,t} = \tilde{\mathbb{E}}\left[\int_t^\infty e^{-\int_t^s r_u du} D_{i,s} ds \middle| \mathcal{F}_t\right].$$

An appropriate Radon-Nikodym derivative process Z_t satisfying the conditions outlined above is given by

$$\frac{dZ_t}{Z_{t-}} = \eta^T \sigma_t dB_t + \sum_{k=1}^K \left[\exp\left(\eta^T Y_{k,t}\right) - 1\right] dN_{k,t} - \sum_{k=1}^K \left[\Phi_k^Y(\eta) - 1\right] \lambda_{k,t} dt,$$

with initial condition $Z_0 = 1$. The economic dynamics which obtain under $\tilde{\mathbb{P}}$ are summarized in the following proposition.

Proposition B.11 (Dynamics Under the Risk-Neutral Measure)**.** *The dynamics for consumption growth, dividend growth, and the state variables under the risk-neutral measure $\tilde{\mathbb{P}}$ defined above are given by*

$$\begin{aligned}
d\ln C_t &= \left[\mu_C + X_t - \left(\gamma + \frac{1}{2}\right) V_t\right] dt + \sqrt{V_t} d\tilde{B}_{C,t} + \sum_{k=1}^K Y_{k,t}^C d\tilde{N}_{k,t} \\
d\ln D_t &= \phi \left[\mu_C + X_t - \left(\gamma + \frac{1}{2}\right) V_t\right] dt + \phi\sqrt{V_t} d\tilde{B}_{C,t} + \sum_{k=1}^K \phi Y_{k,t}^C d\tilde{N}_{k,t} \\
dX_t &= -\kappa_X X_t dt + \left(1 - \frac{1}{\theta}\right) \sigma_X^2 A_X V_t dt + \sigma_X \sqrt{V_t} d\tilde{B}_{X,t} + \sum_{k=1}^K Y_{k,t}^X d\tilde{N}_{k,t} \\
dV_t &= \left[\kappa_V(\bar{V} - V_t) + \left(1 - \frac{1}{\theta}\right) \sigma_V^2 A_V V_t\right] dt + \sigma_V \sqrt{V_t} d\tilde{B}_{V,t} + \sum_{k=1}^K Y_{k,t}^V d\tilde{N}_{k,t} \\
d\lambda_{k,t} &= \left[\kappa_{\lambda,k}(\bar{\lambda}_k - \lambda_{k,t}) + \left(1 - \frac{1}{\theta}\right) A_{\lambda_k} \sigma_{\lambda_k}^2 \lambda_{k,t}\right] dt + \sigma_{\lambda,k} \sqrt{\lambda_{k,t}} d\tilde{B}_{k,t} + \sum_{j=1}^K Y_{j,t}^{\lambda_k} d\tilde{N}_{j,t},
\end{aligned}$$

where $\tilde{B}_C$, $\tilde{B}_X$, $\tilde{B}_V$, and $\tilde{B}_{\lambda_k}$ $k = 1, ..., K$ are independent Brownian motions under $\tilde{\mathbb{P}}$. Furthermore, the arrival of the processes $\tilde{N}_k$ $k = 1, ..., K$ is independent with intensity

$$\tilde{\lambda}_{k,t} = \Phi_k^Y(\eta)\,\lambda_{k,t}$$

under the martingale measure $\tilde{\mathbb{P}}$. The moment-generating function of jump sizes of $Y_{k,t}$ under $\tilde{\mathbb{P}}$ is given by

$$\tilde{\Phi}_k^Y(u) = \frac{\Phi_k^Y(u+\eta)}{\Phi_k^Y(\eta)}.$$

Proof. See section on derivations and proofs. □

Hence, under the risk-neutral measure, expected consumption and dividend growth are adjusted downwards by an additive component that is increasing in both risk aversion and leverage. This result immediately gives rise to the dynamics of the log price of equity under the risk neutral measure.

Corollary B.1 (Dynamics of the Price of Equity under $\tilde{\mathbb{P}}$). *The log-Price of equity $\ln P_{D,t}$ is governed by the process*

$$\begin{aligned} d\ln P_{D,t} = & \Bigg(\phi\left[\mu_C + X_t - \left(\gamma + \frac{1}{2}\right)V_t\right] - A_X\kappa_X X_t + \left(1 - \frac{1}{\theta}\right)A_X^D A_X \sigma_X^2 V_t \\ & + A_V^D\left[\kappa_V\left(\bar{V} - V_t\right) + \left(1 - \frac{1}{\theta}\right)\sigma_V^2 A_V V_t\right] \\ & + \sum_{k=1}^K A_{\lambda_k}^D\left[\kappa_{\lambda_k}\left(\bar{\lambda}_k - \lambda_{k,t}\right) + \left(1 - \frac{1}{\theta}\right)A_{\lambda_k}\sigma_{\lambda_k}^2\lambda_{k,t}\right]\Bigg)dt \\ & + \phi\sqrt{V_t}d\tilde{B}_{C,t} + A_X^D\sigma_X\sqrt{V_t}d\tilde{B}_{X,t} + A_V^D\sigma_V\sqrt{V_t}d\tilde{B}_{V,t} + \sum_{k=1}^K A_{\lambda_k}^D\sigma_{\lambda_k}\sqrt{\lambda_{k,t}}d\tilde{B}_{k,t} \\ & + \sum_{j=1}^K\left(\phi Y_{j,t}^C + A_X^D Y_{j,t}^X + A_V^D Y_{j,t}^V + \sum_{k=1}^K A_{\lambda_k}^D Y_{j,t}^{\lambda_k}\right)d\tilde{N}_{j,t} \end{aligned}$$

under the risk-neutral measure $\tilde{\mathbb{P}}$.

Proof. See section on derivations and proofs. □

B.8 Equilibrium Prices of European Derivatives

Equilibrium Prices of European options that arise in this endowment economy can be computed in quasi-closed form using transform techniques developed by Carr and Madan (1999) and Duffie, Pan, and Singleton (2000). The following proposition provides the Fourier transform of the price of a general state contingent claim on equity with European exercise that pays $f(\ln P_{D,t})$ at maturity date T.

Proposition B.12 (Prices of European Derivatives)**.** *Let $x_t = \ln P_{D,t}$ and consider a state contingent claim which yields a cash flow of $f(x_T)$ at expiration T. Denote by $P_{f,t} = \tilde{\mathbb{E}}\left[\exp\left(-\int_t^T r_{f,s}ds\right) f(x_T)\middle| \mathcal{F}_t\right]$ the equilibrium price of the derivative at time $t \leq T$. The Fourier transforms $FP_{f,t}(u) = \int_{-\infty}^{\infty} e^{i\cdot u\cdot x_t} P_{f,t}(x_t)dx_t$ is given by*

$$FP_{f,t}(u) = \exp(i \cdot u \cdot x_t) \cdot Ff(u) \cdot \tilde{\Psi}^x(t, x, -u, T),$$

where $\tilde{\Psi}^x_{t,T}(t, x, u, T) = \tilde{\mathbb{E}}\left[\exp\left(-\int_t^T r_{f,s}ds\right) \exp\left(iux_T\right)\middle| \mathcal{F}_t\right]$ denotes the discounted characteristic function of the log price of equity under the risk-neutral measure.

Proof. See section on derivations and proofs. □

This result is an extension of a method developed in Lewis (2000) toward this representative agent economy with self-exciting disasters. The derivative price at time t can then be determined Fourier inversion, which entails numerical integration along a strip parallel to the real axis, i.e. an evaluation of the integral

$$P_{f,t} = \frac{1}{2\pi} \int_{i\omega-\infty}^{i\omega+\infty} \exp\left(-iux_t\right) FP_{f,t}(u)du,$$

for appropriate choice of $\omega \in \mathbb{R}$.

The applicability of this result hinges on the availability of simple expressions for the discounted characteristic function of the log price of equity as well as the Fourier transform of the derivative payoff. The ODEs defining the discounted joint characteristic function of the log price of equity and the state variables arising in this consumption based model is provided in the section on derivations and proofs below. In the case of a European option, the Fourier transform of the payoff at expiration with log strike price k takes on the particularly straightforward form

$$Ff(u) = \frac{\exp\left((iu + 1)k\right)}{iu(iu + 1)}.$$

For the valuation of call options, the integral is computed with $\omega > 1$ whereas put option prices are obtained by choosing $\omega < 0$.

B.9 Derivations and Proofs

B.9.1 Steady state mean and expected mean reversion of self-exciting jump intensity

The jump intensity follows

$$d\lambda_t = \kappa_\lambda(\bar{\lambda} - \lambda_t)dt + \sigma_\lambda\sqrt{\lambda_t}dB_t + Y_t^\lambda dN_t,$$

where B_t is a Brownian motion, N_t is a counting process with conditional intensity λ_t, and Y_t^λ is a random variable that has bounded support with $\bar{Y}^\lambda \equiv \mathbb{E}\left[Y_t^\lambda\right] < \kappa_\lambda$ and is independent of B_t and N_t. Taking expectations on both sides and dividing by dt one obtains

$$\frac{\mathbb{E}\left[d\lambda_t\right]}{dt} = \kappa_\lambda(\bar{\lambda} - \mathbb{E}\left[\lambda_t\right]) + \bar{Y}^\lambda\mathbb{E}\left[\lambda_t\right].$$

Letting $m(t) = \mathbb{E}[\lambda_t]$ one can express the dynamics of $\mathbb{E}[\lambda_t]$ in terms of the ordinary differential equation

$$m'(t) = \left(\bar{Y}^\lambda - \kappa_\lambda\right)m(t) + \kappa_\lambda\bar{\lambda}$$

with initial condition $m(0) = \lambda_0$. The solution to this linear first order equation takes the form

$$m(t) = \frac{\kappa_\lambda\bar{\lambda} - \exp\left(\left(\bar{Y}^\lambda - \kappa_\lambda\right)t\right)\left(\kappa_\lambda(\bar{\lambda} - \lambda_0) + \lambda_0\bar{Y}^\lambda\right)}{\kappa_\lambda - \bar{Y}^\lambda}.$$

The steady state mean $\mathbb{E}\left[\lambda_\infty\right]$ is defined as

$$\mathbb{E}\left[\lambda_\infty\right] \equiv \lim_{t\to\infty} m(t) = \frac{\kappa_\lambda\bar{\lambda}}{\kappa_\lambda - \bar{Y}^\lambda}.$$

The steady state mean of the jump intensity is finite and non-negative if and only if $\kappa_\lambda > \bar{Y}^\lambda$, i.e. the speed of mean-reversion is greater than the expected size of the jump in the intensity. Under this condition, the expectation of the jump intensity approaches the steady state mean as $t \to \infty$. In fact, $m(t)$ is strictly decreasing for $\lambda_0 > \mathbb{E}\left[\lambda_\infty\right]$ and strictly increasing for $\lambda_0 < \mathbb{E}\left[\lambda_\infty\right]$. This can be seen by differentiating $m(t)$ with respect to t, which yields

$$m'(t) = \exp\left(\left(\mathbb{E}\left[Y_t^\lambda\right] - \kappa_\lambda\right)t\right)\left(\kappa_\lambda(\bar{\lambda} - \lambda_0) + \lambda_0\mathbb{E}\left[Y_t^\lambda\right]\right).$$

This expression is strictly negative for $\lambda_0 > \mathbb{E}\left[\lambda_\infty\right]$ and strictly positive for $\lambda_0 < \mathbb{E}\left[\lambda_\infty\right]$.

B.9.2 Infinitesimal generator of $C, X, V, \lambda_1, ..., \lambda_K$

For any compactly supported function $g(C, X, V, \lambda_1, ..., \lambda_K)$ that is twice continuously differentiable in the arguments $C, X, V, \lambda_1, ..., \lambda_K$, define the infinitesimal generator

$$\mathcal{D}g_t = \lim_{h\to 0} \frac{\mathbb{E}_{t-}\left[g(C_{t+h}, X_{t+h}, V_{t+h}, \lambda_{1,t+h}, ..., \lambda_{K,t+h}) - g(C_{t-}, X_{t-}, V_{t-}, \lambda_{1,t-}, ..., \lambda_{K,t-})\right]}{h},$$

where the dynamics of $C_t, X_t, V_t, \lambda_{1,t}, ..., \lambda_{K,t}$ are given in equation (B.1) through (B.4). An application of Ito's rule for jump diffusions yields the dynamics

$$\begin{aligned}
dg_t = {} & g_C dC_t^c + g_X dX_t^c + g_V dV_t^c + \sum_{k=1}^{K} g_{\lambda_k} d\lambda_{k,t}^c + \frac{1}{2} g_{CC} d\left[C^c, C^c\right]_t + \frac{1}{2} g_{XX} d\left[X^c, X^c\right]_t \\
& + \frac{1}{2} g_{VV} d\left[V^c, V^c\right]_t + \frac{1}{2}\sum_{k=1}^{K}\sum_{j=1}^{K} g_{\lambda_k\lambda_j} d\left[\lambda_k^c, \lambda_j^c\right]_t + g_{CX} d\left[C^c, X^c\right]_t + g_{CV} d\left[C^c, V^c\right]_t + g_{XV} d\left[X^c, V^c\right]_t \\
& + \sum_{k=1}^{K} g_{C\lambda_k} d\left[C^c, \lambda_k^c\right]_t + \sum_{k=1}^{K} g_{X\lambda_k} d\left[X^c, \lambda_k^c\right]_t + \sum_{k=1}^{K} g_{V\lambda_k} d\left[V^c, \lambda_k^c\right]_t + \sum_{k=1}^{K} \Delta g_{k,t} dN_{k,t},
\end{aligned}$$

where $\Delta g_{k,t} = g(C_t, X_t, V_t, \lambda_{1,t}, ..., \lambda_{K,t}) - g(C_{t-}, X_{t-}, V_{t-}, \lambda_{1,t-}, ..., \lambda_{K,t-})$ is the jump size of g conditional on a jump in the kth point process N_k at time t, i.e.

$$\begin{aligned}
\Delta g_{k,t} = {} & g(C_{t-} \cdot e^{Y_{k,t}^C}, X_{t-} + Y_{k,t}^X, V_{t-} + Y_{k,t}^V, \lambda_{1,t-} + Y_{k,t}^{\lambda_1}, ..., \lambda_{K,t-} + Y_{k,t}^{\lambda_K}) \\
& - g(C_{t-}, X_{t-}, V_{t-}, \lambda_{1,t-}, ..., \lambda_{K,t-}).
\end{aligned}$$

Upon substituting the dynamics of $C_t, X_t, V_t, \lambda_{1,t}, ..., \lambda_{K,t}$ one obtains

$$\begin{aligned}
dg_t = {} & g_C\left((\mu_C + X_t)C_t dt + \sqrt{V_t} C_t dB_{C,t}\right) + g_X\left(-\kappa_X X_t dt + \sigma_X \sqrt{V_t} dB_{X,t}\right) \\
& + g_V(\kappa_V(\bar{V} - V_t)dt + \sigma_V \sqrt{V_t} dB_{V,t}) + \sum_{k=1}^{K} g_{\lambda_k}\left(\kappa_{\lambda_k}(\bar{\lambda}_k - \lambda_{k,t})dt + \sigma_{\lambda_k}\sqrt{\lambda_{k,t}} dB_{k,t}\right) \\
& + \frac{1}{2} g_{CC} V_t C_t^2 dt + \frac{1}{2} g_{XX} \sigma_X^2 V_t dt + \frac{1}{2} g_{VV} \sigma_V^2 V_t dt + \frac{1}{2}\sum_{k=1}^{K} g_{\lambda_k\lambda_k} \sigma_{\lambda_k}^2 \lambda_{k,t} dt \\
& + \sum_{k=1}^{K} \mathbb{E}_t\left[\Delta g_{k,t}\right] \lambda_{k,t} dt + \sum_{k=1}^{K} \Delta g_{k,t} dN_{k,t} - \sum_{k=1}^{K} \mathbb{E}_{t-}\left[\Delta g_{k,t}\right] \lambda_{k,t} dt.
\end{aligned}$$

Recognizing that $\sum_{k=1}^{K} \Delta g_t dN_{k,t} - \sum_{k=1}^{K} \mathbb{E}_t \left[\Delta g_t\right] \lambda_{k,t}$ is a martingale, the infinitesimal generator can be written as

$$\begin{aligned}\mathcal{D}g_t = {} & g_C(\mu_C + X_t)C_t - g_X \kappa_X X_t + g_V \kappa_V(\bar{V} - V_t) + \sum_{k=1}^{K} g_{\lambda_k}\kappa_{\lambda_k}(\bar{\lambda}_k - \lambda_{k,t}) \\ & + \frac{1}{2} g_{CC} V_t C_t^2 + \frac{1}{2} g_{XX}\sigma_X^2 V_t + \frac{1}{2} g_{VV}\sigma_V^2 V_t + \frac{1}{2}\sum_{k=1}^{K} g_{\lambda_k\lambda_k}\sigma_{\lambda_k}^2 \lambda_{k,t} + \sum_{k=1}^{K} \mathbb{E}_{t-}\left[\Delta g_{k,t}\right]\lambda_{k,t}.\end{aligned}$$

B.9.3 Homegeneity of Indirect Utility with Porteus-Kreps aggregator

The following lemma establishes that the value function for stochastic differential utility with a Porteus-Kreps aggregator is homogeneous in consumption of degree $1-\gamma$.

Lemma B.1 (Homogeneity of Indirect Utility with Porteus-Kreps aggregator)**.** *Let J_t denote indirect utility associated with the stochastic sequence of consumption $C = \{C_s\}_{s=t}^{\infty}$. Let $C^\lambda = \{\lambda C_s\}_{s=t}^{\infty}$. Indirect utility J_t^λ associated with C^λ is $J_t^\lambda = \lambda^{1-\gamma} J_t$.*

Proof. We first establish that the aggregator satisfies $f(\lambda C, \lambda^{1-\gamma}) = \lambda^{1-\gamma} f(C, J)$. If $\psi = 1$, we have

$$\begin{aligned} f(\lambda C, \lambda^{1-\gamma} J) &= \beta(1-\gamma)\lambda^{1-\gamma} J \left(\log(\lambda C) - \frac{1}{1-\gamma}\log\left((1-\gamma)\lambda^{1-\gamma} J\right)\right) \\ &= \lambda^{1-\gamma}\beta(1-\gamma) J \left(\log(C) - \frac{1}{1-\gamma}\log\left((1-\gamma)J\right)\right) \\ &= \lambda^{1-\gamma} f(C, J). \end{aligned}$$

Likewise, if $\psi \neq 1$, we can write

$$\begin{aligned} f(\lambda C, \lambda^{1-\gamma} J) &= \frac{\beta}{1-\frac{1}{\psi}} J \lambda^{1-\gamma} \left(\frac{\lambda^{1-\frac{1}{\psi}} C^{1-\frac{1}{\psi}}}{\left((1-\gamma)\lambda^{1-\gamma} J\right)^{\frac{1}{\theta}}} - 1\right) \\ &= \lambda^{1-\gamma}\frac{\beta}{1-\frac{1}{\psi}} J \left(\frac{C^{1-\frac{1}{\psi}}}{\left((1-\gamma)J\right)^{\frac{1}{\theta}}} - 1\right) \\ &= \lambda^{1-\gamma} f(C, J). \end{aligned}$$

Hence

$$
\begin{aligned}
J_t^\lambda &= \mathbb{E}_t\left[\int_t^\infty f(\lambda C_s, J_s^\lambda)ds\right] \\
&= \mathbb{E}_t\left[\int_t^\infty f(\lambda C_s, \lambda^{1-\gamma} J_s)ds\right] \\
&= \lambda^{1-\gamma}\mathbb{E}_t\left[\int_t^\infty f(C_s, J_s)ds\right] \\
&= \lambda^{1-\gamma} J_t.
\end{aligned}
$$

□

B.9.4 Partial derivatives of the normalized Porteus-Kreps aggregator

The normalized Porteus-Kreps aggregator for the stochastic differential utility process $J_t = \mathbb{E}_t\left[\int_t^\infty f(C_s, J_s)ds\right]$ is given by

$$
f(C,J) = \frac{\beta}{1-\frac{1}{\psi}} J(1-\gamma)\left(\frac{C^{1-\frac{1}{\psi}}}{((1-\gamma)J)^{\frac{1}{\theta}}} - 1\right) \text{ for } \psi \neq 1
$$

and

$$
f(C,J) = \beta(1-\gamma)J\left(\log C - \frac{1}{1-\gamma}\log\left((1-\gamma)J\right)\right) \text{ for } \psi = 1,
$$

with $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$. The partial derivatives with respect to C and J are

$$
\begin{aligned}
f_C(C,J) &= \beta C^{-\frac{1}{\psi}}((1-\gamma)J)^{1-\frac{1}{\theta}} && \text{for } \psi \neq 1 \\
f_C(C,J) &= \beta(1-\gamma)\frac{J}{C} && \text{for } \psi = 1
\end{aligned}
$$

and

$$
\begin{aligned}
f_J(C,J) &= \beta\theta\left(\left(1-\frac{1}{\theta}\right)\frac{C^{1-\frac{1}{\psi}}}{((1-\gamma)J)^{\frac{1}{\theta}}} - 1\right) && \text{for } \psi \neq 1 \\
f_J(C,J) &= \beta\left((1-\gamma)\log C - \log((1-\gamma)J) - 1\right) && \text{for } \psi = 1.
\end{aligned}
$$

B.9.5 A Heuristic Derivation of the PDE for the Value Function

The recursion given in equation (B.5) can be written as follows

$$
\begin{aligned}
J_t &= \mathbb{E}_t\left[\int_t^{\infty} f(C_s, J_s)ds\right] = \mathbb{E}_t\left[\int_t^{t+\Delta t} f(C_s, J_s)ds + \int_{t+\Delta t}^{\infty} f(C_s, J_s)ds\right] \\
&= \mathbb{E}_t\left[\int_t^{t+\Delta t} f(C_s, J_s)ds + \mathbb{E}_{t+\Delta t}\left[\int_{t+\Delta t}^{\infty} f(C_s, J_s)ds\right]\right] = \mathbb{E}_t\left[\int_t^{t+\Delta t} f(C_s, J_s)ds + J_{t+\Delta t}\right],
\end{aligned}
$$

where the third equality follows from the law of iterated expectations and the final step applies the definition of the value function to the second term inside the expectation. Subtracting $J_t = J(C_t, X_t, V_t, \lambda_{1,t}, ..., \lambda_{K,t})$ on both sides, dividing by Δt, and taking the limit $\Delta t \to 0$ gives rise to

$$
0 = \lim_{\Delta t \to 0} \frac{\mathbb{E}_t\left[\int_t^{t+\Delta t} f(C_s, J_s)ds\right]}{\Delta t} + \lim_{\Delta t \to 0} \frac{\mathbb{E}_t\left[J_{t+\Delta t} - J_t\right]}{\Delta t}.
$$

Under regularity conditions the first term converges to $f(C_t, J_t)$ and the second term is the infinitesimal generator of the value function, i.e. $\mathcal{D}J_t + f(C_t, J_t) = 0$, which establishes the claim.

B.9.6 Solution of the PDE for the Value function

The solution for the PDE (B.8) takes the form

$$
J(C, X, V, \lambda_1, \lambda_K) = \frac{C^{1-\gamma}}{1-\gamma} I(X, V, \lambda_1, ..., \lambda_K)
$$

for both $\psi = 1$ and $\psi \neq 1$. Substituting this conjecture into the aggregator yields

$$
\begin{aligned}
f(C, J) &= \beta\theta J\left(I^{-\frac{1}{\theta}} - 1\right) && \text{for } \psi \neq 1 \\
f(C, J) &= -\beta J \log(I) && \text{for } \psi = 1.
\end{aligned}
$$

The jump term $\Delta J_{k,t}$ is given by

$$
\begin{aligned}
\Delta J_{k,t} &= \frac{C_{t-}^{1-\gamma} \cdot e^{(1-\gamma)Y_{k,t}^C}}{1-\gamma} \cdot I(X_{t-} + Y_{k,t}^X, V_{t-} + Y_{k,t}^V, \lambda_{1,t-} + Y_{k,t}^{\lambda_k}, ..., \lambda_{K,t-} + Y_{k,t}^{\lambda_K}) \\
&\quad - \frac{C_{t-}^{1-\gamma}}{1-\gamma} \cdot I(X_{t-}, V_{t-}, \lambda_{1,t-}, ..., \lambda_{K,t-}) \\
&= J_{t-} \cdot \left(e^{(1-\gamma)Y_{k,t}^C} \frac{I_{k,t}}{I_{k,t-}} - 1 \right),
\end{aligned}
$$

where $I_{k,t-} = I(X_{t-}, V_{t-}, \lambda_{1,t-}, ..., \lambda_{K,t-})$ and $I_{k,t} = I(X_{t-}+Y_{k,t}^X, V_{t-}+Y_{k,t}^V, \lambda_{1,t-}+Y_{k,t}^{\lambda_1}, ..., \lambda_{K,t-}+ Y_{k,t}^{\lambda_K})$. Substituting the expression for $\Delta J_{k,t}$ and the partial derivatives of the conjectured solution (B.9) into (B.8) yields (B.10).

B.9.6.1 Exact Closed Form Solution for $\psi = 1$ (Proof of Proposition B.1) Substituting the aggregator for the case $\psi = 1$ into (B.10) results in the PDE

$$
\begin{aligned}
&(1-\gamma)(\mu_C + X_t) - \frac{I_X}{I}\kappa_X X_t + \frac{I_V}{I}\kappa_V(\bar{V} - V_t) + \sum_{k=1}^K \frac{I_{\lambda_k}}{I}\kappa_{\lambda_k}(\bar{\lambda}_k - \lambda_{k,t}) - \frac{1}{2}\gamma(1-\gamma)V_t \\
&+\frac{1}{2}\frac{I_{XX}}{I}\sigma_X^2 V_t + \frac{1}{2}\frac{I_{VV}}{I}\sigma_V^2 V_t + \frac{1}{2}\sum_{k=1}^K \frac{I_{\lambda_k\lambda_k}}{I}\sigma_{\lambda_k}^2 \lambda_{k,t} + \sum_{k=1}^K \mathbb{E}_{t-}\left[e^{(1-\gamma)Y_{k,t}^C} \frac{I_{k,t}}{I_{k,t-}} - 1 \right] \lambda_{k,t} - \beta J \log(I) = 0,
\end{aligned}
$$

which has an analytical solution of the form

$$
I(X, V, \lambda_1, ..., \lambda_K) = \exp\left(A_0 + A_X \cdot X + A_V \cdot V + \sum_{k=1}^K A_{\lambda_k}\lambda_k \right).
$$

Given this functional form, the jump term simplifies to

$$
\begin{aligned}
\frac{I_{k,t}}{I_{k,t-}} &= \frac{\exp\left(A_X \left(X_{t-} + Y_{k,t}^X\right) + A_V \left(V_{t-} + Y_{k,t}^V\right) + \sum_{k=1}^K A_{\lambda_k} \left(\lambda_{k,t-} + Y_{k,t}^{\lambda_k}\right)\right)}{\exp\left(A_X X_{t-} + A_V V_{t-} + \sum_{k=1}^K A_{\lambda_k}\lambda_{k,t-}\right)} \\
&= \exp\left(A_X Y_{k,t}^X + A_V Y_{k,t}^V + \sum_{k=1}^K A_{\lambda_k} Y_{k,t}^{\lambda_k} \right).
\end{aligned}
$$

Substituting this solution into the PDE and collecting terms involving $X_t, V_t, \lambda_{1,t}, ..., \lambda_{K,t}$ yields

$$
\begin{aligned}
0 &= (1-\gamma)\mu_C + A_V\kappa_V\bar{V} + \sum_{k=1}^{K} A_{\lambda_k}\kappa_{\lambda_k}\bar{\lambda}_k - \beta A_0 \\
&+ \left[(1-\gamma) - A_X\kappa_X - \beta A_X\right] X_t + \left[-\beta A_V - A_V\kappa_V - \frac{1}{2}\gamma(1-\gamma) + \frac{1}{2}A_X^2\sigma_X^2 + \frac{1}{2}A_V^2\sigma_V^2\right] V_t \\
&+ \sum_{k=1}^{K}\left[-A_{\lambda_k}\kappa_{\lambda_k} + \frac{1}{2}A_{\lambda_k}^2\sigma_{\lambda_k}^2 - \beta A_{\lambda_k} + \left(\Phi_k^Y(\hat{\eta}) - 1\right)\right]\lambda_{k,t}.
\end{aligned}
$$

The terms multiplying the state variables $X_t, V_t, \lambda_{1,t}, ..., \lambda_{K,t}$ have to be separately zero, which implies the following nonlinear system of equations in the coefficients $A_0, A_X, A_V, A_{\lambda_1}, ..., A_{\lambda_K}$:

$$
\begin{aligned}
0 &= (1-\gamma)\mu_C + A_V\kappa_V\bar{V} + \sum_{k=1}^{K} A_{\lambda_k}\kappa_{\lambda_k}\bar{\lambda}_k - \beta A_0 \\
0 &= (1-\gamma) - A_X\kappa_X - \beta A_X \\
0 &= -\beta A_V - A_V\kappa_V - \frac{1}{2}\gamma(1-\gamma) + \frac{1}{2}A_X^2\sigma_X^2 + \frac{1}{2}A_V^2\sigma_V^2 \\
0 &= -A_{\lambda_k}\kappa_{\lambda_k} + \frac{1}{2}A_{\lambda_k}^2\sigma_{\lambda_k}^2 - \beta A_{\lambda_k} + \left(\Phi_k^Y(\hat{\eta}) - 1\right) \ \forall k \in \{1, ..., K\}
\end{aligned}
$$

This concludes the proof to the proposition.

B.9.6.2 Approximate Solution for $\psi \neq 1$ (Proof of Proposition B.2) In the case $\psi \neq 1$ a closed form solution to the PDE is not available. Substituting the aggregator for this case into equation (B.10) yields

$$
\begin{aligned}
&(1-\gamma)(\mu_C + X_t) - \frac{I_X}{I}\kappa_X X_t + \frac{I_V}{I}\kappa_V(\bar{V} - V_t) + \sum_{k=1}^{K}\frac{I_{\lambda_k}}{I}\kappa_{\lambda_k}(\bar{\lambda}_k - \lambda_{k,t}) - \frac{1}{2}\gamma(1-\gamma)V_t \\
&+\frac{1}{2}\frac{I_{XX}}{I}\sigma_X^2 V_t + \frac{1}{2}\frac{I_{VV}}{I}\sigma_V^2 V_t + \frac{1}{2}\sum_{k=1}^{K}\frac{I_{\lambda_k\lambda_k}}{I}\sigma_{\lambda_k}^2\lambda_{k,t} + \sum_{k=1}^{K}\mathbb{E}_{t-}\left[e^{(1-\gamma)Y_{k,t}^C}\frac{I_{k,t}}{I_{k,t-}} - 1\right]\lambda_{k,t} - \beta\theta + \beta\theta I^{-\frac{1}{\theta}} = 0.
\end{aligned}
$$

The last term on the left hand side makes the PDE nonlinear. Recognizing that $\beta I^{-\frac{1}{\theta}}$ is the consumption-wealth ratio, we can apply the approximation introduced by Campbell, Chacko, Rodriguez, and Viceira (2004). This method results in the following log-linearization of the last term appearing in the PDE around the mean consumption-wealth ratio

$$
\frac{C_t}{W_t} = \beta \cdot I^{-\frac{1}{\theta}} = \exp\left(\log(C_t) - \log(W_t)\right) \approx i_0 + i_1\left(\log(C_t) - \log(W_t)\right) = i_0 + i_1\left(\log(\beta) - \frac{1}{\theta}\log(I)\right),
$$

with $i_1 = \exp\left(\mathbb{E}[\log(C_t) - \log(W_t)]\right)$ and $i_0 = i_1(1 - \log(i_1))$.

The solution to this log-linearized PDE can be now be obtained analytically and takes the same form as for $\psi = 1$. Substituting the approximation and the functional form of I into the PDE and collecting terms yields

$$
\begin{aligned}
&(1-\gamma)\mu_C + A_V \kappa_V \bar{V} + \sum_{k=1}^{K} A_{\lambda_k}\kappa_{\lambda_k}\bar{\lambda}_k + \theta i_0 + \theta i_1 \log\beta - \beta\theta - i_1 A_0 \\
&+ \left[(1-\gamma) - A_X\kappa_X - i_1 A_X\right] X_t + \left[-i_1 A_V - A_V\kappa_V - \frac{1}{2}\gamma(1-\gamma) + \frac{1}{2}A_X^2\sigma_X^2 + \frac{1}{2}A_V^2\sigma_V^2\right] V_t \\
&+ \sum_{k=1}^{K}\left[-(\kappa_{\lambda_k} + i_1)A_{\lambda_k} + \frac{1}{2}A_{\lambda_k}^2\sigma_{\lambda_k}^2 + \left(\Phi_k^Y(\hat{\eta}) - 1\right)\right]\lambda_{k,t} = 0.
\end{aligned}
$$

The method of undetermined coefficients once again implies a system of equations. The coefficients A_0, A_X, A_V, and $A_{\lambda_1}, ..., A_{\lambda_K}$ simultaneously satisfy

$$
\begin{aligned}
0 &= (1-\gamma)\mu_C + A_V\kappa_V\bar{V} + \sum_{k=1}^{K} A_{\lambda_k}\kappa_{\lambda_k}\bar{\lambda}_k + \theta i_0 + \theta i_1\log\beta - \beta\theta - i_1 A_0 \\
0 &= (1-\gamma) - A_X\kappa_X - i_1 A_X \\
0 &= -i_1 A_V - A_V\kappa_V - \frac{1}{2}\gamma(1-\gamma) + \frac{1}{2}A_X^2\sigma_X^2 + \frac{1}{2}A_V^2\sigma_V^2 \\
0 &= -(\kappa_{\lambda_k} + i_1)A_{\lambda_k} + \frac{1}{2}A_{\lambda_k}^2\sigma_{\lambda_k}^2 + \left(\Phi_k^Y(\hat{\eta}) - 1\right) \text{ for all } k \in \{1, ..., K\},
\end{aligned}
$$

which concludes the proof of the proposition.

B.9.7 Derivation of the Risk Premium (Proof of Proposition B.5)

Substituting equation (B.13) into the pricing PDE (B.12) yields

$$
\begin{aligned}
&\frac{\mathcal{D}P_{i,t}^c}{P_{i,t-}} + \frac{\sum_{k=1}^{K}\mathbb{E}_{t-}\left[\Delta P_{i,k,t}\right]\lambda_{k,t-}}{P_{i,t-}} + \frac{D_{i,t-}}{P_{i,t-}} - r_{f,t-} \\
&= -\frac{d\left[\pi^c, P_i^c\right]_t}{\pi_{t-}\cdot P_{i,t-}\cdot dt} - \left(\frac{\sum_{k=1}^{K}\mathbb{E}_{t-}\left[\Delta\left(\pi\cdot P_i\right)_{k,t}\right]\lambda_{k,t-}}{\pi_{t-}\cdot P_{i,t-}} - \frac{\sum_{k=1}^{K}\mathbb{E}_{t-}\left[\Delta\pi_{k,t}\right]\lambda_{k,t-}}{\pi_{t-}} - \frac{\sum_{k=1}^{K}\mathbb{E}_{t-}\left[\Delta P_{i,k,t}\right]\lambda_{k,t-}}{P_{i,t-}}\right)
\end{aligned}
$$

Using the definitions $\Delta\left(\pi\cdot P_i\right)_{k,t} = \pi_{k,t}P_{i,k,t} - \pi_{t-}P_{i,t-}$, $\Delta\pi_{k,t} = \pi_{k,t} - \pi_{t-}$, and $\Delta P_{i,k,t} = P_{i,k,t} - P_{i,t-}$, one can rewrite the sums inside the parentheses on the right hand side of the previous equation as

$$\sum_{k=1}^{K} \mathbb{E}_{t-}\left[\frac{\pi_{k,t}P_{i,k,t} - \pi_{t-}P_{i,t-}}{\pi_{t-}\cdot P_{i,t-}} - \frac{\pi_{k,t} - \pi_{t-}}{\pi_{t-}} - \frac{P_{i,k,t} - P_{i,t-}}{P_{i,t-}}\right]\lambda_{k,t-}.$$

Rearranging terms gives rise to

$$\sum_{k=1}^{K} \mathbb{E}_{t-}\left[\frac{(\pi_{k,t} - \pi_{t-})\left(P_{i,k,t} - P_{t-}\right)}{\pi_{t-}\cdot P_{i,t-}}\right]\lambda_{k,t-} = \sum_{k=1}^{K} \mathbb{E}_{t-}\left[\frac{\Delta\pi_{k,t}\cdot\Delta P_{i,k,t}}{\pi_{t-}\cdot P_{i,t-}}\right]\lambda_{k,t-}.$$

Substituting back into the equation above yields the expression for the risk premium (B.14) and concludes the proof of proposition B.5.

B.9.8 Derivation of Pricing Kernel Dynamics (Proof of Proposition B.6)

The general form of the pricing kernel in an endowment economy under stochastic differential utility is given by equation (B.15). An application of Ito's formula then implies the dynamics

$$\frac{d\pi_t}{\pi_{t-}} = f_J(C_t, J_t)dt + \frac{df_C(C_t, J_t)}{f_C(C_t, J_t)}, \tag{B.17}$$

with

$$\begin{aligned} df_{C,t} = {} & f_{CC}dC_t^c + \frac{1}{2}f_{CCC}d\left[C^c, C^c\right]_t + f_{CJ}dJ_t^c \\ & + \frac{1}{2}f_{CJJ}d\left[J^c, J^c\right]_t + f_{CCJ}d\left[C^c, J^c\right]_t + \sum_{k=1}^{K}\Delta f_{C,k,t}dN_{k,t}. \end{aligned} \tag{B.18}$$

The continuous part of the value function evolves according to

$$\begin{aligned} \frac{dJ_t^c}{J_{t-}} = & \Bigg[(1-\gamma)(\mu_C + X_t) - A_X\kappa_X X_t + A_V\kappa_V(\bar{V} - V_t) + \sum_{k=1}^{K} A_{\lambda_k}\kappa_{\lambda_k}(\bar{\lambda}_k - \lambda_{k,t}) \\ & + \frac{1}{2}A_X^2\sigma_X^2 V_t + \frac{1}{2}A_V^2\sigma_V^2 V_t + \frac{1}{2}\sum_{k=1}^{K} A_{\lambda_k}^2\sigma_{\lambda_k}^2\lambda_{k,t} - \frac{1}{2}\gamma(1-\gamma)V_t\Bigg]dt \\ & + (1-\gamma)\sqrt{V_t}dB_{C,t} + A_X\sigma_X\sqrt{V_t}dB_{X,t} + A_V\sigma_V\sqrt{V_t}dB_{V,t} + \sum_{k=1}^{K} A_{\lambda_k}\sigma_{\lambda_k}\sqrt{\lambda_{k,t}}dB_{k,t}. \end{aligned}$$

The remainder of the derivation depends on the functional form of the aggregator which differs for $\psi = 1$ and $\psi \neq 1$. Hence the two cases are treated separately in what follows.

B.9.8.1 Pricing Kernel and Risk Free Rate for $\psi = 1$ To proceed with the calculation of df_C we need to establish higher order partial derivatives of the aggregator in the case of $\psi = 1$. The relevant partial derivatives are given by

$$
\begin{aligned}
f_C(C, J) &= \beta(1-\gamma)\frac{J}{C} \\
f_{CC}(C, J) &= -f_C(C, J)\frac{1}{C} \\
f_{CCC}(C, J) &= 2f_C(C, J)\frac{1}{C^2} \\
f_{CJ}(C, J) &= f_C(C, J)\frac{1}{J} \\
f_{CJJ}(C, J) &= 0 \\
f_{CCJ}(C, J) &= -f_C(C, J)\frac{1}{CJ}.
\end{aligned}
$$

We next derive an expression for the jump term appearing in (B.18). At times t when N_k jumps, we have $C_t = C_{t-}e^{Y^C_{k,t}}$ and

$$
J_t = \frac{C_{t-}^{1-\gamma}e^{(1-\gamma)Y^C_{k,t}}}{1-\gamma}\exp\left(A_X\left(X_{t-} + Y^X_{k,t}\right) + A_V\left(V_{t-} + Y^V_{k,t}\right) + \sum_{j=1}^K A_{\lambda_j}(\lambda_{j,t-} + Y^{\lambda_j}_{k,t})\right).
$$

For the jump term of f_C at times of the kth jump, i.e. $\Delta f_{C,k,t}$, we obtain

$$
\begin{aligned}
&\Delta f_{C,k,t} = f_C(C_t, J_t) - f_C(C_{t-}, J_{t-}) \\
=&\beta C_{t-}^{-\gamma}\exp\left(-\gamma Y^C_{k,t} + A_X\left(X_{t-} + Y^X_{k,t}\right) + A_V\left(V_{t-} + Y^V_{k,t}\right) + \sum_{j=1}^K A_{\lambda_j}(\lambda_{j,t-} + Y^{\lambda_j}_{k,t})\right) \\
-&\beta C_{t-}^{-\gamma}\exp\left(A_X X_{t-} + A_V V_{t-} + \sum_{j=1}^K A_{\lambda_j}\lambda_{j,t-}\right) \\
=&\beta C_{t-}^{-\gamma}\exp\left(A_X X_{t-} + A_V V_{t-} + \sum_{j=1}^K A_{\lambda_j}\lambda_{j,t-}\right)\left[\exp\left(-\gamma Y^C_{k,t} + A_X Y^X_{k,t} + A_V Y^V_{k,t} + \sum_{j=1}^K A_{\lambda_j}Y^{\lambda_j}_{k,t}\right) - 1\right] \\
=&f_C(C_{t-}, J_{t-})\left[\exp\left(-\gamma Y^C_{k,t} + A_X Y^X_{k,t} + A_V Y^V_{k,t} + \sum_{j=1}^K A_{\lambda_j}Y^{\lambda_j}_{k,t}\right) - 1\right].
\end{aligned}
$$

Substituting the equations from proposition B.1 into the value function dynamics above, we obtain

$$\frac{dJ_t^c}{J_t} = \left[\beta A_0 + \beta A_X X_t + \beta A_V V_t + \sum_{k=1}^{K}\left(\beta A_{\lambda_k} - \left(\Phi_k^Y\left(\hat{\eta}\right) - 1\right)\right)\lambda_{k,t}\right] dt$$
$$+(1-\gamma)\sqrt{V_t}dB_{C,t} + A_X\sigma_X\sqrt{V_t}dB_{X,t} + A_V\sigma_V\sqrt{V_t}dB_{V,t} + \sum_{k=1}^{K} A_{\lambda_k}\sigma_{\lambda_k}\sqrt{\lambda_{k,t}}dB_{k,t}.$$

Combining theses results with (B.18), we find

$$\frac{df_C(C_t,J_t)}{f_C(C_{t-},J_{t-})} = \left[-\mu_C + \beta A_0 + \left(\beta A_X - 1\right)X_t + \left(\beta A_V + \gamma\right)V_t + \sum_{k=1}^{K}\left(\beta A_{\lambda_k} - \left(\Phi_k^Y\left(\hat{\eta}\right) - 1\right)\right)\lambda_{k,t}\right] dt$$
$$-\gamma\sqrt{V_t}dB_{C,t} + A_X\sigma_X\sqrt{V_t}dB_{X,t} + A_V\sigma_V\sqrt{V_t}dB_{V,t} + \sum_{k=1}^{K} A_{\lambda_k}\sigma_{\lambda_k}\sqrt{\lambda_{k,t}}dB_{k,t}$$
$$+\sum_{k=1}^{K}\left[\exp\left(\eta^T Y_{k,t}\right) - 1\right] dN_{k,t}.$$

Substituting the solution for J_t into the partial derivative of the aggregator with respect to the value function $f_J(C_t, J_t)$ yields

$$f_J(C_t,J_t) = -\beta\left(A_0 + A_X X_t + A_V V_t + \sum_{k=1}^{K} A_{\lambda_k}\lambda_{k,t}\right) - \beta.$$

Putting this result together with (B.17) gives rise to the following dynamics for the pricing kernel

$$\frac{d\pi_t}{\pi_{t-}} = \left[-\beta - \mu_C - X_t + \gamma V_t - \sum_{k=1}^{K}\left[\left(\Phi_k^Y\left(\hat{\eta}\right) - 1\right)\right]\lambda_{k,t}\right] dt$$
$$-\gamma\sqrt{V_t}dB_{C,t} + A_X\sigma_X\sqrt{V_t}dB_{X,t} + A_V\sigma_V\sqrt{V_t}dB_{V,t} + \sum_{k=1}^{K} A_{\lambda_k}\sigma_{\lambda_k}\sqrt{\lambda_{k,t}}dB_{k,t}$$
$$+\sum_{k=1}^{K}\left[\exp\left(\eta^T Y_{k,t}\right) - 1\right] dN_{k,t}.$$

The risk-free rate can now be computed using proposition B.4, which results in

$$r_{f,t} = \beta + \mu_C + X_t - \gamma V_t + \sum_{k=1}^{K}\left[\Phi_k^Y\left(\hat{\eta}\right) - \Phi_k^Y\left(\eta\right)\right]\lambda_{k,t}.$$

Substituting back into the dynamics of the pricing kernel, we obtain the statement in proposition B.6 for a unit EIS by noting that $\psi = 1$ implies $\frac{1}{\theta} = 0$.

B.9.8.2 Pricing Kernel and Risk Free Rate for $\psi \neq 1$ Once again we first need to establish some higher order partial derivatives of the aggregator. In the case of $\psi \neq 1$ we obtain

$$
\begin{aligned}
f_{CC}(C,J) &= -\frac{1}{\psi} f_C(C,J) \frac{1}{C} \\
f_{CCC}(C,J) &= \frac{1}{\psi}\left(1+\frac{1}{\psi}\right) f_C(C,J) \frac{1}{C^2} \\
f_{CJ}(C,J) &= \left(1-\frac{1}{\theta}\right) f_C(C,J) \frac{1}{J} \\
f_{CJJ}(C,J) &= -\frac{1}{\theta}\left(1-\frac{1}{\theta}\right) f_C(C,J) \frac{1}{J^2} \\
f_{CCJ}(C,J) &= -\frac{1}{\psi}\left(1-\frac{1}{\theta}\right) f_C(C,J) \frac{1}{CJ}.
\end{aligned}
$$

Using the equations from proposition B.2, we obtain the dynamics of the continuous part of the value function

$$
\begin{aligned}
\frac{dJ_t^c}{J_t} = & \Bigg[i_1 A_0 + \theta(i_0 + i_1 \log\beta - \beta) + i_1 A_X X_t + i_1 A_V V_t \\
& + \sum_{k=1}^{K} \left(i_1 A_{\lambda_k} - \left(\Phi_k^Y(\hat{\eta}) - 1 \right) \right) \lambda_{k,t} \Bigg] dt \\
& + (1-\gamma)\sqrt{V_t} dB_{C,t} + A_X \sigma_X \sqrt{V_t} dB_{X,t} + A_V \sigma_V \sqrt{V_t} dB_{V,t} + \sum_{k=1}^{K} A_{\lambda_k} \sigma_{\lambda_k} \sqrt{\lambda_{k,t}} dB_{k,t}.
\end{aligned}
$$

The jump term of $f_{C,t}$ for a jump of N_k can be computed as

$$
\begin{aligned}
\Delta f_{C,k,t} &= f(C_t, J_t) - f(C_{t-}, J_{t-}) \\
&= \beta C_{t-}^{-\frac{1}{\psi}} e^{-\frac{1}{\psi} Y_{k,t}^C} \left(C_{t-}^{1-\gamma} e^{(1-\gamma)Y_{k,t}^C + A_0 + A_X \left(X_{t-} + Y_{k,t}^X\right) + A_V \left(V_{t-} + Y_{k,t}^V\right) + \sum_{j=1}^K A_{\lambda_j} \left(\lambda_{j,t-} + Y_{k,t}^{\lambda_j}\right)} \right)^{1-\frac{1}{\theta}} \\
&- \beta C_{t-}^{-\frac{1}{\psi}} \left(C_{t-}^{1-\gamma} e^{A_0 + A_X X_{t-} + A_V V_{t-} + \sum_{j=1}^K A_{\lambda_j} \lambda_{j,t-}} \right)^{1-\frac{1}{\theta}} \\
&= f_C(C_{t-,J_{t-}}) \left[e^{\left(-\frac{1}{\psi} + 1 - \gamma - 1 + \frac{1}{\psi}\right) Y_{k,t}^C + \left(1 - \frac{1}{\theta}\right) \left(A_X Y_{k,t}^X + A_V Y_{k,t}^V + \sum_{j=1}^K A_{\lambda_j} Y_{k,t}^{\lambda_j} \right)} - 1 \right] \\
&= f_C(C_{t-,J_{t-}}) \left[e^{-\gamma Y_{k,t}^C + \left(1 - \frac{1}{\theta}\right) \left(A_X Y_{k,t}^X + A_V Y_{k,t}^V + \sum_{j=1}^K A_{\lambda_j} Y_{k,t}^{\lambda_j} \right)} - 1 \right].
\end{aligned}
$$

Substitution of these results into (B.18) yields

$$
\begin{aligned}
\frac{df_{C,t}}{f_{C,t-}} =& \Bigg[-\frac{1}{\psi}\mu_C + \left(1 - \frac{1}{\theta}\right) \left(i_1 A_0 - \theta \left(i_0 + i_1 \log\beta - \beta\right)\right) + \left[\left(1 - \frac{1}{\theta}\right) i_1 A_X - \frac{1}{\psi} \right] X_t \\
&+ \left[\frac{1}{2}\gamma \left(1 + \frac{1}{\psi}\right) + \left(1 - \frac{1}{\theta}\right) i_1 A_V - \frac{1}{2}\frac{1}{\theta} \left(1 - \frac{1}{\theta}\right) \left(A_X^2 \sigma_X^2 + A_V^2 \sigma_V^2\right) \right] V_t \\
&+ \left(1 - \frac{1}{\theta}\right) \sum_{k=1}^K \left[i_1 A_{\lambda_k} - \left(\Phi_k^Y\left(\hat{\eta}\right) - 1\right) - \frac{1}{2}\frac{1}{\theta} A_{\lambda_k}^2 \sigma_{\lambda_k}^2 \right] \lambda_{k,t} \Bigg] dt \\
&- \gamma \sqrt{V_t} dB_{C,t} + \left(1 - \frac{1}{\theta}\right) A_X \sigma_X \sqrt{V_t} dB_{X,t} + \left(1 - \frac{1}{\theta}\right) A_V \sigma_V \sqrt{V_t} dB_{V,t} \\
&+ \left(1 - \frac{1}{\theta}\right) \sum_{k=1}^K A_{\lambda_k} \sigma_{\lambda_k} \sqrt{\lambda_{k,t}} dB_{k,t} + \sum_{k=1}^K \left(\exp\left(\eta^T Y_{k,t}\right) - 1\right) dN_{k,t}.
\end{aligned}
$$

Using the approximation around the mean consumption-wealth ratio, the partial derivative of the aggregator with respect to indirect utility evaluated at the solution for J_t is given by

$$
\begin{aligned}
f_J(C_t, J_t) &= \left(1 - \frac{1}{\theta}\right) \beta\theta I_t^{-\frac{1}{\theta}} - \beta\theta \\
&\approx (\theta - 1)\left(i_0 + i_1 \log\beta\right) - \left(1 - \frac{1}{\theta}\right) i_1 \left(A_0 + A_X X_t + A_V V_t + \sum_{k=1}^K A_{\lambda_k} \lambda_{k,t} \right) - \beta\theta.
\end{aligned}
$$

Combining these results with (B.17) and once again using the equations in proposition B.2, gives rise to the pricing kernel dynamics

$$\begin{aligned}\frac{d\pi_t}{\pi_{t-}} =& \Bigg[-\beta - \frac{1}{\psi}\mu_C - \frac{1}{\psi}X_t + \left[\frac{1}{2}\gamma\left(1+\frac{1}{\psi}\right) - \frac{1}{2}\frac{1}{\theta}\left(1-\frac{1}{\theta}\right)\left(A_X^2\sigma_X^2 + A_V^2\sigma_V^2\right)\right]V_t \\ &- \left(1-\frac{1}{\theta}\right)\sum_{k=1}^{K}\left[\left(\Phi_k^Y(\hat{\eta}) - 1\right) + \frac{1}{2}\frac{1}{\theta}A_{\lambda_k}^2\sigma_{\lambda_k}^2\right]\lambda_{k,t}\Bigg]dt \\ &-\gamma\sqrt{V_t}dB_{C,t} + \left(1-\frac{1}{\theta}\right)A_X\sigma_X\sqrt{V_t}dB_{X,t} + \left(1-\frac{1}{\theta}\right)A_V\sigma_V\sqrt{V_t}dB_{V,t} \\ &+ \left(1-\frac{1}{\theta}\right)\sum_{k=1}^{K}A_{\lambda_k}\sigma_{\lambda_k}\sqrt{\lambda_{k,t}}dB_{k,t} + \sum_{k=1}^{K}\left(\exp\left(\eta^T Y_{k,t}\right) - 1\right)dN_{k,t}.\end{aligned}$$

Applying proposition B.4 yields the risk free rate

$$\begin{aligned}r_{f,t} =& \beta + \frac{1}{\psi}\mu_C + \frac{1}{\psi}X_t - \left[\frac{1}{2}\gamma\left(1+\frac{1}{\psi}\right) + \frac{1}{2}\frac{1}{\theta}\left(1-\frac{1}{\theta}\right)\left(A_X^2\sigma_X^2 + A_V^2\sigma_V^2\right)\right]V_t \\ &+ \sum_{k=1}^{K}\left[\left(1-\frac{1}{\theta}\right)\left(\left(\Phi_k^Y(\hat{\eta}) - 1\right) + \frac{1}{2}\frac{1}{\theta}A_{\lambda_k}^2\sigma_{\lambda_k}^2\right) - \left(\Phi_k^Y(\eta) - 1\right)\right]\lambda_{k,t}.\end{aligned}$$

Substitution of the risk-free rate into the pricing kernel dynamics above yields the result of proposition B.6 for an EIS different from unity and concludes the proof.

B.9.9 The Interest Rate on Defaultable Short-Term Government Debt

The equilibrium promised rate $r_{L,t}$ is determined by the equation

$$\frac{\mathcal{D}\pi_t^c}{\pi_{t-}} + \frac{\mathcal{D}P_{L,t}^c}{P_{L,t-}} + \frac{d\left[\pi^c, P_L^c\right]_t}{\pi_{t-}P_{L,t-}dt} + \frac{\sum_{k=1}^{K}\mathbb{E}_{t-}\left[\Delta\left(\pi\cdot P_L\right)_{k,t}\right]\lambda_{k,t-}}{\pi_{t-}P_{L,t-}} = 0.$$

We have $\frac{\mathcal{D}P_{L,t}^c}{P_{L,t-}} = r_{L,t}$ and $d\left[\pi^c, P_L^c\right]_t = 0$. Furthermore, at times of the process N_k, we have

$$\frac{P_{L,k,t}}{P_{L,t-}} = \exp\left(Y_{k,t}^Z\right)$$

and

$$\frac{\pi_{k,t}}{\pi_{t-}} = \exp\left(-\gamma Y_{k,t}^C + \left(1-\frac{1}{\theta}\right)\left(A_X Y_{k,t}^X + A_V Y_{k,t}^V + \sum_{j=1}^{K}A_{\lambda_j}Y_{k,t}^{\lambda_j}\right)\right).$$

Hence it is straightforward to compute the expectations

$$\frac{\mathbb{E}_{t-}\left[\Delta\left(\pi\cdot P_L\right)_{k,t}\right]}{\pi_{t-}\cdot P_{L,t-}} = \mathbb{E}_t\left[\exp\left(Y^Z_{k,t}-\gamma Y^C_{k,t}+\left(1-\frac{1}{\theta}\right)\left(A_X Y^X_{k,t}+A_V Y^V_{k,t}+\sum_{j=1}^K A_{\lambda_j}Y^{\lambda_j}_{k,t}\right)\right)-1\right]$$
$$= q_k\Phi^Y_k\left(1-\gamma,\left(1-\frac{1}{\theta}\right)A_X,\left(1-\frac{1}{\theta}\right)A_V,\left(1-\frac{1}{\theta}\right)A_{\lambda_1},...,\left(1-\frac{1}{\theta}\right)A_{\lambda_K}\right)$$
$$+(1-q_k)\Phi^Y_k\left(-\gamma,\left(1-\frac{1}{\theta}\right)A_X,\left(1-\frac{1}{\theta}\right)A_V,\left(1-\frac{1}{\theta}\right)A_{\lambda_1},...,\left(1-\frac{1}{\theta}\right)A_{\lambda_K}\right)$$
$$-1$$
$$= q_k\Phi^Y_k\left(\tilde{\eta}\right)+(1-q_k)\Phi^Y_k\left(\eta\right)-1$$

and

$$\frac{\mathbb{E}_{t-}\left[\Delta\pi_{k,t}\right]}{\pi_{t-}} = \Phi^Y_k\left(-\gamma,\left(1-\frac{1}{\theta}\right)A_X,\left(1-\frac{1}{\theta}\right)A_V,\left(1-\frac{1}{\theta}\right)A_{\lambda_1},...,\left(1-\frac{1}{\theta}\right)A_{\lambda_K}\right)-1$$
$$= \Phi^Y_k\left(\eta\right)-1.$$

Combining these results gives rise to

$$\frac{\mathbb{E}_{t-}\left[\Delta\left(\pi\cdot P_L\right)_{k,t}\right]}{\pi_{t-}\cdot P_{L,t-}} = \frac{\mathbb{E}_{t-}\left[\Delta\pi_{k,t}\right]}{\pi_{t-}}+q_k\left(\Phi^Y_k\left(\tilde{\eta}\right)-\Phi^Y_k\left(\eta\right)\right).$$

Substitution into the pricing equation yields

$$r_{L,t} = -\frac{\mathcal{D}\pi^c_t}{\pi_{t-}}-\frac{\sum_{k=1}^K\mathbb{E}_{t-}\left[\Delta\pi_{k,t}\right]\lambda_{k,t-}}{\pi_{t-}}+\sum_{k=1}^K\left(\Phi^Y_k\left(\eta\right)-\Phi^Y_k\left(\tilde{\eta}\right)\right)q_k\lambda_{k,t-}.$$

Recognizing that the risk-free rate is given by

$$r_{f,t-} = -\frac{\mathcal{D}\pi^C_t}{\pi_{t-}}-\frac{\sum_{k=1}^K\mathbb{E}_{t-}\left[\Delta\pi_{k,t}\right]\lambda_{k,t-}}{\pi_{t-}},$$

one obtains the equation given in the proposition.

B.9.10 The Wealth Consumption Ratio and the Consumption Risk Premium

I am going to show that the wealth-consumption ratio is $\beta^{-1}I^{\frac{1}{\theta}}$ and derive an expression for the risk premium on a claim to aggregate consumption. According to proposition B.3, wealth, which is the price of an asset that delivers consumption C_s as its dividend at time

$s \geq t$, satisfies the PDE

$$\frac{\mathcal{D}\pi_t^c}{\pi_{t-}} + \frac{\mathcal{D}P_{C,t}^c}{P_{C,t-}} + \frac{d\left[\pi^c, P_C^c\right]_t}{\pi_{t-} \cdot P_{C,t-} \cdot dt} + \frac{\sum_{k=1}^K \mathbb{E}_{t-}\left[\Delta\left(\pi \cdot P_C\right)_{k,t}\right]\lambda_{k,t-}}{\pi_{t-} \cdot P_{C,t-}} + \frac{C_{t-}}{P_{C,t-}} = 0. \tag{B.19}$$

In the following, the wealth-consumption ratio, that is the price-dividend ratio of a claim to consumption, will be denoted by $H_t = H(X_t, V_t, \lambda_{1,t}, ..., \lambda_{K,t})$. The dynamics of the wealth-consumption ratio are then given by

$$\begin{aligned} dH_t =& \Bigg[- H_X\kappa_X X_t + H_V\kappa_V(\bar{V} - V_t) + \sum_{k=1}^K H_{\lambda_k}\kappa_{\lambda_k}(\bar{\lambda}_k - \lambda_{k,t}) \\ &+ \frac{1}{2}H_{XX}\sigma_X^2 V_t + \frac{1}{2}H_{VV}\sigma_V^2 V_t + \frac{1}{2}\sum_{k=1}^K H_{\lambda_k\lambda_k}\sigma_{\lambda_k}^2\lambda_{k,t}\Bigg]dt \\ &+ H_X\sigma_X\sqrt{V_t}dB_{X,t} + H_V\sigma_V\sqrt{V_t}dB_{V,t} + \sum_{k=1}^K H_{\lambda_k}\sigma_{\lambda_k}\sqrt{\lambda_{k,t}}dB_{k,t} + \sum_{k=1}^K \Delta H_{k,t}dN_{k,t}. \end{aligned}$$

The price of a claim to consumption can be written as $P_{C,t} = H_t \cdot C_t$. Its continuous part follows the process

$$dP_{C,t}^c = d\left(H_t^c C_t^c\right) = H_t dC_t^c + C_t dH_t^c + d\left[H^c, C^c\right]_t.$$

Substituting the dynamics of consumption, we have

$$\begin{aligned} \frac{dP_{C,t}^c}{P_{C,t}} =& \Bigg[\mu_C + X_t - \frac{H_X}{H_t}\kappa_X X_t + \frac{H_V}{H_t}\kappa_V(\bar{V} - V_t) + \sum_{k=1}^K \frac{H_{\lambda_k}}{H_t}\kappa_{\lambda_k}(\bar{\lambda}_k - \lambda_{k,t}) \\ &+ \frac{1}{2}\frac{H_{XX}}{H_t}\sigma_X^2 V_t + \frac{1}{2}\frac{H_{VV}}{H_t}\sigma_V^2 V_t + \frac{1}{2}\sum_{k=1}^K \frac{H_{\lambda_k\lambda_k}}{H_t}\sigma_{\lambda_k}^2\lambda_{k,t}\Bigg]dt \\ &+ \sqrt{V_t}dB_{C,t} + \frac{H_X}{H_t}\sigma_X\sqrt{V_t}dB_{X,t} + \frac{H_V}{H_t}\sigma_V\sqrt{V_t}dB_{V,t} + \sum_{k=1}^K \frac{H_{\lambda_k}}{H_t}\sigma_{\lambda_k}\sqrt{\lambda_{k,t}}dB_{k,t}. \end{aligned}$$

The infinitesimal generator of the continuous part of the wealth process is hence given by

$$\begin{aligned} \frac{\mathcal{D}P_{C,t}^c}{P_{C,t}} =& \mu_C + X_t - \frac{H_X}{H_t}\kappa_X X_t + \frac{H_V}{H_t}\kappa_V(\bar{V} - V_t) + \sum_{k=1}^K \frac{H_{\lambda_k}}{H_t}\kappa_{\lambda_k}(\bar{\lambda}_k - \lambda_{k,t}) \\ &+ \frac{1}{2}\frac{H_{XX}}{H_t}\sigma_X^2 V_t + \frac{1}{2}\frac{H_{VV}}{H_t}\sigma_V^2 V_t + \frac{1}{2}\sum_{k=1}^K \frac{H_{\lambda_k\lambda_k}}{H_t}\sigma_{\lambda_k}^2\lambda_{k,t}. \end{aligned}$$

The remainder of the derivation depends on the value of the elasticity of intertemporal substitution. The following subsections will treat the cases $\psi = 1$ and $\psi \neq 1$ respectively.

B.9.10.1 Price of a Consumption Claim with $\psi = 1$ (Proof of Proposition B.8)

In the case where $\psi = 1$, the infinitesimal generator of the continuous part of the pricing kernel is given by

$$\frac{\mathcal{D}\pi_t^c}{\pi_t} = -\beta - \mu_C - X_t + \gamma V_t - \sum_{k=1}^{K} \left(\Phi_k^Y\left(\hat{\eta}\right) - 1\right)\lambda_{k,t}.$$

The cross-variation of the continuous parts of the pricing kernel and wealth is

$$\frac{d\left[\pi^c, P_C^c\right]_t}{\pi_t P_{C,t} dt} = \left[-\gamma + A_X \sigma_X^2 \frac{H_X}{H_t} + A_V \sigma_V^2 \frac{H_V}{H_t}\right] V_t + \sum_{k=1}^{K} A_{\lambda_k} \sigma_{\lambda_k}^2 \frac{H_{\lambda_k}}{H_t} \lambda_{k,t}.$$

The wealth consumption ratio is conjectured to be constant at $H_t = \beta^{-1}$. This implies that the jump term is given by

$$\begin{aligned}
\frac{\Delta\left(\pi \cdot P_C\right)_{k,t}}{\pi_{t-} \cdot P_{C,t-}} &= \frac{\pi_{k,t}}{\pi_{t-}} \exp\left(Y_{k,t}^C\right) - 1 \\
&= \exp\left((1-\gamma) Y_{k,t}^C + A_X Y_{k,t}^X + A_V Y_{k,t}^V + \sum_{j=1}^{K} A_{\lambda_j} Y_{k,t}^{\lambda_j}\right) - 1.
\end{aligned}$$

Substitution of these results verifies that the conjecture satisfies the PDE (B.19), i.e. that the wealth consumption ratio is indeed given by β^{-1}. The risk premium of a consumption claim is given by

$$\mathbb{E}_{t-}[r_{C,t} - r_{f,t-}] = -\frac{d\left[\pi^c, P_c^c\right]_t}{\pi_t \cdot P_{C,t} \cdot dt} - \frac{\sum_{k=1}^{K} \mathbb{E}_{t-}\left[\Delta\pi_{k,t} \Delta P_{C,k,t}\right] \lambda_{k,t-}}{\pi_{t-} \cdot P_{C,t-}}.$$

The cross-variation between the continuous parts of the pricing kernel and the price of a consumption claim is

$$\frac{d\left[\pi^c, P_C^c\right]_t}{\pi_t P_{C,t} dt} = -\gamma V_t.$$

Furthermore, the cross-variation of the jump components of the pricing kernel and the price of consumption claim is given by

$$\begin{aligned}
\frac{\sum_{k=1}^{K}\mathbb{E}_t\left[\Delta\pi_{k,t}\Delta P_{C,k,t}\lambda_{k,t-}\right]}{\pi_{t-}\cdot P_{C,t-}} &= \sum_{k=1}^{K}\mathbb{E}_t\left[\exp\left((1-\gamma)Y_{k,t}^{C}+A_X Y_{k,t}^{X}+A_V Y_{k,t}^{V}+\sum_{k=1}^{K}A_{\lambda_j}Y_{k,t}^{\lambda_j}\right)-1\right]\lambda_{k,t-}\\
&-\sum_{k=1}^{K}\mathbb{E}_t\left[\exp\left(-\gamma Y_{k,t}^{C}+A_X Y_{k,t}^{X}+A_V Y_{k,t}^{V}+\sum_{k=1}^{K}A_{\lambda_j}Y_{k,t}^{\lambda_j}\right)-1\right]\lambda_{k,t-}\\
&-\sum_{k=1}^{K}\mathbb{E}_t\left[\exp\left(Y_{k,t}^{C}\right)-1\right]\lambda_{k,t-}\\
&=\sum_{k=1}^{K}\left(\Phi_k^Y\left(\hat{\eta}\right)-\Phi_k^Y\left(\eta\right)-\Phi_k^Y\left(e_1\right)+1\right)\lambda_{k,t-}.
\end{aligned}$$

Combining these results yields the consumption risk premium

$$\mathbb{E}_t[r_{C,t}-r_{f,t-}]=\gamma V_t+\sum_{k=1}^{K}\left(\Phi_k^Y\left(\eta\right)-\Phi_k^Y\left(\hat{\eta}\right)+\Phi_k^Y\left(e_1\right)-1\right)\lambda_{k,t-},$$

as claimed in the proposition.

B.9.10.2 Price of a Consumption Claim with $\psi\neq 1$ (Proof of Proposition B.9)

When the EIS is different from unity, the infinitesimal generator of the continuous part of the pricing kernel is given by

$$\begin{aligned}
\frac{\mathcal{D}\pi_t^c}{\pi_t}=&-\beta-\frac{1}{\psi}\mu_C-\frac{1}{\psi}X_t+\left[\frac{1}{2}\gamma\left(1+\frac{1}{\psi}\right)-\frac{1}{2}\frac{1}{\theta}\left(1-\frac{1}{\theta}\right)\left(A_X^2\sigma_X^2+A_V^2\sigma_V^2\right)\right]V_t\\
&-\left(1-\frac{1}{\theta}\right)\sum_{k=1}^{K}\left[\left(\Phi_k^Y\left(\hat{\eta}\right)-1\right)+\frac{1}{2}\frac{1}{\theta}A_{\lambda_k}^2\sigma_{\lambda_k}^2\right]\lambda_{k,t}.
\end{aligned}$$

The cross-variation between the continuous parts of wealth and the pricing kernel is

$$\begin{aligned}
\frac{d\left[\pi^c,P_C^c\right]_t}{\pi_{t-}\cdot P_{C,t-}\cdot dt}=&\left[-\gamma+\left(1-\frac{1}{\theta}\right)A_X\sigma_X^2\frac{H_X}{H_t}+\left(1-\frac{1}{\theta}\right)A_V\sigma_V^2\frac{H_V}{H_t}\right]V_t\\
&+\left(1-\frac{1}{\theta}\right)\sum_{k=1}^{K}A_{\lambda_k}\sigma_{\lambda_k}^2\frac{H_{\lambda_k}}{H_t}\lambda_{k,t}
\end{aligned}$$

Substituting the expressions for infinitesimal generator of the the wealth and pricing kernel processes along with their cross-variation into (B.19) gives rise to the partial differential equation

$$
\begin{aligned}
0 = & \left(1-\frac{1}{\psi}\right)\mu_C - \beta + \frac{H_V}{H_t}\kappa_V\bar{V} + \sum_{k=1}^{K}\frac{H_{\lambda_k}}{H_t}\kappa_{\lambda_k}\bar{\lambda}_k + \left[\left(1-\frac{1}{\psi}\right) - \frac{H_X}{H_t}\kappa_X\right]X_t \\
& + \left[\frac{1}{2}\frac{H_{XX}}{H_t}\sigma_X^2 + \frac{1}{2}\frac{H_{VV}}{H_t}\sigma_V^2 - \frac{H_V}{H_t}\kappa_V + \left(1-\frac{1}{\theta}\right)\left(A_X\sigma_X^2\frac{H_X}{H_t} + A_V\sigma_V^2\frac{H_V}{H_t}\right)\right. \\
& \quad \left. + \frac{1}{2}\gamma\left(\frac{1}{\psi}-1\right) - \frac{1}{2}\frac{1}{\theta}\left(1-\frac{1}{\theta}\right)\left(A_X^2\sigma_X^2 + A_V^2\sigma_V^2\right)\right]V_t \\
& + \sum_{k=1}^{K}\left[\frac{1}{2}\frac{H_{\lambda_k\lambda_k}}{H_t}\sigma_{\lambda_k}^2 - \frac{1}{2}\frac{1}{\theta}\left(1-\frac{1}{\theta}\right)A_{\lambda_k}^2\sigma_{\lambda_k}^2 + \left(1-\frac{1}{\theta}\right)A_{\lambda_k}\sigma_{\lambda_k}^2\frac{H_{\lambda_k}}{H_t} - \frac{H_{\lambda_k}}{H_t}\kappa_{\lambda_k}\right. \\
& \quad \left. - \left(1-\frac{1}{\theta}\right)\left(\Phi_k^Y(\hat{\eta}) - 1\right) + \frac{\mathbb{E}_t\left[\Delta\left(\pi\cdot P_C\right)_{k,t}\right]\lambda_{k,t-}}{\pi_{t-}\cdot P_{C,t-}}\right]\lambda_{k,t} + H_t^{-1}.
\end{aligned}
$$

At this point we will approximate the last term on the right hand side around the mean consumption-wealth ratio, i.e.

$$
H_t^{-1} = \exp\left(\log C_t - \log P_{C,t}\right) \approx h_0 - h_1\log\left(\log C_t - \log P_{C,t}\right) = h_0 - h_1\log H_t,
$$

with $h_1 = \exp\left(\mathbb{E}\left[\log C_t - \log P_{C,t}\right]\right)$ and $h_0 = h_1\left(1-\log h_1\right)$. The solution to the linearized PDE is of the form

$$
H(X_t, V_t, \lambda_{1,t}, ..., \lambda_{K,t}) = \exp\left(A_0^C + A_X^C X_t + A_V^C V_t + \sum_{k=1}^{K} A_{\lambda_k}^C\lambda_{k,t}\right).
$$

The jump term for process N_k can be computed as

$$
\frac{\Delta\left(\pi P_C\right)_{k,t}}{\pi_{t-}P_{C,t-}} = \frac{\pi_{k,t}P_{C,k,t}}{\pi_{t-}P_{C,t-}} - 1,
$$

where

$$
\frac{P_{C,k,t}}{P_{C,t-}} = \exp\left(Y_{k,t}^C + A_X^C Y_{k,t}^X + A_V^C Y_{k,t}^V + \sum_{j=1}^{K} A_{\lambda_j}^C Y_{k,t}^{\lambda_j}\right)
$$

and

$$
\frac{\pi_{k,t}}{\pi_{t-}} = \exp\left(-\gamma Y_{k,t}^C + \left(1-\frac{1}{\theta}\right)\left(A_X Y_{k,t}^X + A_V Y_{k,t}^V + \sum_{j=1}^{K} A_{\lambda_j} Y_{k,t}^{\lambda_j}\right)\right).
$$

The expression for the jump component in the PDE is then

$$\begin{aligned}
\frac{\sum_{k=1}^{K}\mathbb{E}_t\left[\Delta\left(\pi P_C\right)_{k,t}\right]}{\pi_{t-}P_{C,t-}} &= \sum_{k=1}^{K}\mathbb{E}_t\Bigg[\exp\Bigg((1-\gamma)Y_{k,t}^{C}+\left[\left(1-\frac{1}{\theta}\right)A_X+A_X^C\right]Y_{k,t}^{X} \\
&\qquad +\left[\left(1-\frac{1}{\theta}\right)A_V+A_V^C\right]Y_{k,t}^{V}+\sum_{j=1}^{K}\left[\left(1-\frac{1}{\theta}\right)A_{\lambda_j}+A_{\lambda_j}^C\right]Y_{k,t}^{\lambda_j}\Bigg)-1\Bigg] \\
&= \sum_{k=1}^{K}\Phi_k^Y\left(\eta+\eta^C\right),
\end{aligned}$$

where $\eta^C = \left(1, A_X^C, A_V^C, A_{\lambda_1}^C, ..., A_{\lambda_K}^C\right)^T$. Substituting the trial solution and the expression for the jump terms into the PDE, we obtain

$$\begin{aligned}
0 &= \left(1-\frac{1}{\psi}\right)\mu_C-\beta+A_V^C\kappa_V\bar{V}+\sum_{k=1}^{K}A_{\lambda_k}^C\kappa_{\lambda_k}\bar{\lambda}_k+h_0-h_1A_0^C+\left[\left(1-\frac{1}{\psi}\right)-A_X^C\kappa_X-h_1A_X^C\right]X_t \\
&\quad+\Bigg[\frac{1}{2}\left(A_X^C\right)^2\sigma_X^2+\frac{1}{2}\left(A_V^C\right)^2\sigma_V^2-A_V^C\kappa_V+\frac{1}{2}\gamma\left(\frac{1}{\psi}-1\right) \\
&\qquad+\left(1-\frac{1}{\theta}\right)\left(A_X\sigma_X^2A_X^C+A_V\sigma_V^2A_V^C\right)-\frac{1}{2}\frac{1}{\theta}\left(1-\frac{1}{\theta}\right)\left(A_X^2\sigma_X^2+A_V^2\sigma_V^2\right)-h_1A_V^C\Bigg]V_t \\
&\quad+\sum_{k=1}^{K}\Bigg[\frac{1}{2}\left(A_{\lambda_k}^C\right)^2\sigma_{\lambda_k}^2-\frac{1}{2}\frac{1}{\theta}\left(1-\frac{1}{\theta}\right)A_{\lambda_k}^2\sigma_{\lambda_k}^2+\left(1-\frac{1}{\theta}\right)A_{\lambda_k}\sigma_{\lambda_k}^2A_{\lambda_k}^C-A_{\lambda_k}^C\kappa_{\lambda_k}-h_1A_{\lambda_k}^C \\
&\qquad\quad-\left(1-\frac{1}{\theta}\right)\left(\Phi_k^Y\left(\hat{\eta}\right)-1\right)+\left(\Phi_k^Y\left(\eta+\eta^C\right)-1\right)\Bigg]\lambda_{k,t}.
\end{aligned}$$

Applying a separation argument gives rise to the following system of equations for the coefficients of the wealth-consumption ratio

$$
\begin{aligned}
0 &= \left(1-\frac{1}{\psi}\right)\mu_C - \beta + A_V^C \kappa_V \bar{V} + \sum_{k=1}^{K} A_{\lambda_k}^C \kappa_{\lambda_k} \bar{\lambda}_k + h_0 - h_1 A_0^C \\
0 &= \left(1-\frac{1}{\psi}\right) - A_X^C \kappa_X - h_1 A_X^C \\
0 &= \frac{1}{2}\left(A_X^C\right)^2 \sigma_X^2 + \frac{1}{2}\left(A_V^C\right)^2 \sigma_V^2 - A_V^C \kappa_V + \left(1-\frac{1}{\theta}\right)\left(A_X \sigma_X^2 A_X^C + A_V \sigma_V^2 A_V^C\right) \\
&\quad + \frac{1}{2}\gamma\left(\frac{1}{\psi}-1\right) - \frac{1}{2}\frac{1}{\theta}\left(1-\frac{1}{\theta}\right)\left(A_X^2\sigma_X^2 + A_V^2\sigma_V^2\right) - h_1 A_V^C \\
0 &= \frac{1}{2}\left(A_{\lambda_k}^C\right)^2 \sigma_{\lambda_k}^2 - \frac{1}{2}\frac{1}{\theta}\left(1-\frac{1}{\theta}\right) A_{\lambda_k}^2 \sigma_{\lambda_k}^2 + \left(1-\frac{1}{\theta}\right) A_{\lambda_k}\sigma_{\lambda_k}^2 A_{\lambda_k}^C - A_{\lambda_k}^C \kappa_{\lambda_k} - h_1 A_{\lambda_k}^C \\
&\quad - \left(1-\frac{1}{\theta}\right)\left(\Phi_k^Y(\hat{\eta}) - 1\right) + \left(\Phi_k^Y\left(\eta + \eta^C\right) - 1\right) \text{ for all } k = 1, ..., K.
\end{aligned}
$$

The proposition postulates that the wealth-consumption ratio is $\beta^{-1} I^{\frac{1}{\theta}}$, which implies $A_0^C = -\log\beta + \frac{1}{\theta}A_0$, $A_X^C = \frac{1}{\theta}A_X$, $A_V^C = \frac{1}{\theta}A_V$, and $A_{\lambda_k}^C = \frac{1}{\theta}A_{\lambda_k}$ for all $k = 1, ..., K$. Since then, both the PDE determining $I(\cdot)$ and the PDE for $H(\cdot)$ are linearized around the mean consumption-wealth ratio, the linearization constants must be the same, i.e. $h_0 = i_0$ and $h_1 = i_1$. Making use of this conjecture, the equations simplify to

$$
\begin{aligned}
0 &= (1-\gamma)\mu_C - \theta\beta + A_V \kappa_V \bar{V} + \sum_{k=1}^{K} A_{\lambda_k}\kappa_{\lambda_k}\bar{\lambda}_k + \theta i_0 - i_1 A_0 + \theta\log\beta \\
0 &= (1-\gamma) - (\kappa_X + i_1) A_X \\
0 &= \frac{1}{2}\gamma\left(\frac{1}{\psi}-1\right) - \frac{1}{\theta}(\kappa_V + i_1)A_V \\
&\quad + \left[\frac{1}{2}\frac{1}{\theta^2} + \frac{1}{\theta}\left(1-\frac{1}{\theta}\right) - \frac{1}{2}\frac{1}{\theta}\left(1-\frac{1}{\theta}\right)\right] A_X^2\sigma_X^2 + \left[\frac{1}{2}\frac{1}{\theta^2} + \frac{1}{\theta}\left(1-\frac{1}{\theta}\right) - \frac{1}{2}\frac{1}{\theta}\left(1-\frac{1}{\theta}\right)\right] A_V^2\sigma_V^2 \\
0 &= \frac{1}{2}\frac{1}{\theta^2}\left(A_{\lambda_k}\right)^2\sigma_{\lambda_k}^2 - \frac{1}{2}\frac{1}{\theta}\left(1-\frac{1}{\theta}\right)A_{\lambda_k}^2\sigma_{\lambda_k}^2 + \left(1-\frac{1}{\theta}\right)\frac{1}{\theta}A_{\lambda_k}\sigma_{\lambda_k}^2 A_{\lambda_k} - \frac{1}{\theta}A_{\lambda_k}\kappa_{\lambda_k} - \frac{1}{\theta}i_1 A_{\lambda_k} \\
&\quad + \frac{1}{\theta}\left(\Phi_k^Y(\hat{\eta}) - 1\right) \text{ for all } k = 1, ..., K.
\end{aligned}
$$

Using the equations for the coefficients of the value function given in proposition B.2 it is easily verified that the system of equations above is satisfied as well, which establishes that $H_t = \beta^{-1} I_t^{1/\theta}$. In order to derive the consumption risk premium, we first note that the cross-variation between the continuous parts of the pricing kernel and the price of consumption is

$$\frac{d\left[\pi^c, P_C^c\right]_t}{\pi_{t-}\cdot P_{C,t-}\cdot dt} = \left[-\gamma + \frac{1}{\theta}\left(1-\frac{1}{\theta}\right)\left(A_X\sigma_X^2 A_X + A_V\sigma_V^2 A_V\right)\right]V_t + \frac{1}{\theta}\left(1-\frac{1}{\theta}\right)\sum_{k=1}^{K} A_{\lambda_k}\sigma_{\lambda_k}^2 A_{\lambda_k}\lambda_{k,t}.$$

Since the jump components of the pricing kernel and the price of the claim to consumption are respectively given by

$$\frac{\Delta\pi_{k,t}}{\pi_{t-}} = \exp\left(-\gamma Y_{k,t}^C + \left(1-\frac{1}{\theta}\right)\left(A_X Y_{k,t}^X + A_V Y_{k,t}^V + \sum_{j=1}^{K} A_{\lambda_j} Y_{k,t}^{\lambda_j}\right)\right) - 1$$

and

$$\frac{\Delta P_{C,k,t}}{P_{C,t-}} = \exp\left(Y_{k,t}^C + A_X^C Y_{k,t}^X + A_V^C Y_{k,t}^V + \sum_{j=1}^{K} A_{\lambda_j}^C Y_{k,t}^{\lambda_j}\right) - 1,$$

their cross-variation is

$$\begin{aligned}
&\frac{\sum_{k=1}^{K}\mathbb{E}_t\left[\Delta\pi_{k,t}\Delta P_{C,k,t}\right]\lambda_{k,t-}}{\pi_{t-}\cdot P_{C,t-}} \\
&= \sum_{k=1}^{K}\mathbb{E}_t\left[\exp\left((1-\gamma)Y_{k,t}^C + A_X Y_{k,t}^X + A_V Y_{k,t}^V + \sum_{j=1}^{K} A_{\lambda_j} Y_{k,t}^{\lambda_j}\right) - 1\right]\lambda_{k,t-} \\
&- \sum_{k=1}^{K}\mathbb{E}_t\left[\exp\left(-\gamma Y_{k,t}^C + \left(1-\frac{1}{\theta}\right)\left(A_X Y_{k,t}^X + A_V Y_{k,t}^V + \sum_{j=1}^{K} A_{\lambda_j} Y_{k,t}^{\lambda_j}\right)\right) - 1\right]\lambda_{k,t-} \\
&- \sum_{k=1}^{K}\mathbb{E}_t\left[\exp\left(Y_{k,t}^C + \frac{1}{\theta}\left(A_X Y_{k,t}^X + A_V Y_{k,t}^V + \sum_{j=1}^{K} A_{\lambda_j} Y_{k,t}^{\lambda_j}\right)\right) - 1\right]\lambda_{k,t-} \\
&= \sum_{k=1}^{K}\left(\Phi_k^Y\left(\hat{\eta}\right) - \Phi_k^Y\left(\eta\right) - \Phi_k^Y\left(\eta^C\right) + 1\right)\lambda_{k,t-}
\end{aligned}$$

Combining these results establishes the risk premium of the consumption claim

$$\begin{aligned}
\mathbb{E}_t[r_{C,t} - r_{f,t-}] = &\left[\gamma - \frac{1}{\theta}\left(1-\frac{1}{\theta}\right)\left(A_X\sigma_X^2 A_X + A_V\sigma_V^2 A_V\right)\right]V_{t-} - \frac{1}{\theta}\left(1-\frac{1}{\theta}\right)\sum_{k=1}^{K} A_{\lambda_k}\sigma_{\lambda_k}^2 A_{\lambda_k}\lambda_{k,t-} \\
&+ \sum_{k=1}^{K}\left(\Phi_k^Y\left(\eta\right) - \Phi_k^Y\left(\hat{\eta}\right) + \Phi_k^Y\left(\eta^C\right) - 1\right)\lambda_{k,t-}
\end{aligned}$$

This concludes the proof of proposition B.9.

B.9.11 The Price-Dividend Ratio and the Equity Premium (Proof of Proposition B.10)

I model corporate dividends as a levered claim to consumption letting $D_t = C_t^{\phi}$. An application of Ito's formula gives the dynamics of dividend growth as

$$dD_t = D_t \left(\phi \left(\mu_C + X_t + \frac{1}{2}(\phi - 1)V_t \right) dt + \phi\sqrt{V_t} dB_{C,t} \right) + D_{t-} \sum_{k=1}^{K} \left(\exp\left(\phi Y_{k,t}^C - 1\right)\right) dN_{k,t}.$$

The price dividend-ratio, denoted by G_t, is a function of the state variables

$$\frac{P_{D,t}}{D_t} = G(X_t, V_t, \lambda_{1,t}, ..., \lambda_{K,t}).$$

The dynamics of the price-dividend ratio can be obtained by Ito's rule to yield

$$\begin{aligned} dG_t =& \Bigg[- G_X \kappa_X X_t + G_V \kappa_V (\bar{V} - V_t) + \sum_{k=1}^{K} G_{\lambda_k} \kappa_{\lambda_k} (\bar{\lambda}_k - \lambda_{k,t}) \\ &+ \frac{1}{2} G_{XX} \sigma_X^2 V_t + \frac{1}{2} G_{VV} \sigma_V^2 V_t + \frac{1}{2} \sum_{k=1}^{K} G_{\lambda_k \lambda_k} \sigma_{\lambda_k}^2 \lambda_{k,t} \Bigg] dt \\ &+ G_X \sigma_X \sqrt{V_t} dB_{X,t} + G_V \sigma_V \sqrt{V_t} dB_{V,t} + \sum_{k=1}^{K} G_{\lambda_k} \sigma_{\lambda_k} \sqrt{\lambda_{k,t}} dB_{k,t} + \sum_{k=1}^{K} \Delta G_{k,t} dN_{k,t}. \end{aligned}$$

Another Ito calculation yields the price process

$$\begin{aligned} \frac{dP_{D,t}}{P_{D,t-}} = \frac{d\left(G_t \cdot D_t\right)}{G_{t-} \cdot D_{D-}} =& \Bigg[\phi \mu_C + \frac{G_V}{G_t} \kappa_V \bar{V} + \sum_{k=1}^{K} \frac{G_{\lambda_k}}{G_t} \kappa_{\lambda_k} \bar{\lambda}_k + \left(\phi - \kappa_X \frac{G_X}{G_t} \right) X_t \\ &+ \left(\frac{1}{2} \phi(\phi - 1) - \kappa_V \frac{G_V}{G_t} + \frac{1}{2} \frac{G_{XX}}{G_t} \sigma_X^2 + \frac{1}{2} \frac{G_{VV}}{G_t} \sigma_V^2 \right) V_t \\ &+ \sum_{k=1}^{K} \left(\frac{1}{2} \frac{G_{\lambda_k \lambda_k}}{G_t} \sigma_{\lambda_k}^2 - \kappa_{\lambda_k} \frac{G_{\lambda_k}}{G_t} \right) \lambda_{k,t} \Bigg] dt \\ &+ \phi \sqrt{V_t} dB_{C,t} + \frac{G_X}{G_t} \sigma_X \sqrt{V_t} dB_{X,t} + \frac{G_V}{G_t} \sigma_V \sqrt{V_t} dB_{V,t} + \sum_{k=1}^{K} \frac{G_{\lambda_k}}{G_t} \sigma_{\lambda_k} \sqrt{\lambda_{k,t}} dB_{k,t} \\ &+ \frac{\sum_{k=1}^{K} \Delta (G \cdot D)_{k,t}}{G_{t-} \cdot D_{t-}} dN_{k,t}. \end{aligned}$$

The infinitesimal generator of the continuous part is given by

$$
\begin{aligned}
\frac{\mathcal{D}P_{D,t}^c}{P_{D,t-}} =&\phi\mu_C + \frac{G_V}{G_t}\kappa_V\bar{V} + \sum_{k=1}^{K}\frac{G_{\lambda_k}}{G_t}\kappa_{\lambda_k}\bar{\lambda}_k + \left(\phi - \kappa_X\frac{G_X}{G_t}\right)X_t \\
&+\left(\frac{1}{2}\phi(\phi-1) - \kappa_V\frac{G_V}{G_t} + \frac{1}{2}\frac{G_{XX}}{G_t}\sigma_X^2 + \frac{1}{2}\frac{G_{VV}}{G_t}\sigma_V^2\right)V_t + \sum_{k=1}^{K}\left(\frac{1}{2}\frac{G_{\lambda_k\lambda_k}}{G_t}\sigma_{\lambda_k}^2 - \kappa_{\lambda_k}\frac{G_{\lambda_k}}{G_t}\right)\lambda_{k,t}.
\end{aligned}
$$

In the absence of arbitrage, the valuation ratio G_t satisfies the PDE

$$
\frac{\mathcal{D}\pi_t^c}{\pi_{t-}} + \frac{\mathcal{D}P_{D,t}^c}{P_{D,t-}} + \frac{d\left[\pi^c, P_D^c\right]_t}{\pi_{t-}\cdot P_{D,t-}\cdot dt} + \frac{\sum_{k=1}^{K}\mathbb{E}_{t-}\left[\Delta\left(\pi\cdot P_D\right)_{k,t}\right]\lambda_{k,t-}}{\pi_{t-}\cdot P_{D,t-}} + G_t^{-1} = 0. \tag{B.20}
$$

The remainder of the derivation depends on whether the elasticity of intertemporal substitution is equal to or different from unity. The following sections treat the two cases separately.

B.9.11.1 Price-Dividend Ratio and Equity Premium with $\psi = 1$ The cross-variation of the continuous parts of the pricing kernel and the price-dividend ratio is

$$
\frac{d\left[\pi^c, P_D^c\right]_t}{\pi_{t-}\cdot P_{D,t-}\cdot dt} = \left[-\gamma\phi + \left(A_X\sigma_X^2\frac{G_X}{G_t} + A_V\sigma_V^2\frac{G_V}{G_t}\right)\right]V_t + \sum_{k=1}^{K}A_{\lambda_k}\sigma_{\lambda_k}^2\frac{G_{\lambda_k}}{G_t}\lambda_{k,t}.
$$

The price of equity satisfies the PDE

$$
\begin{aligned}
&-\beta + (\phi-1)\mu_C + \frac{G_V}{G_t}\kappa_V\bar{V} + \sum_{k=1}^{K}\frac{G_{\lambda_k}}{G_t}\kappa_{\lambda_k}\bar{\lambda}_k + \left[\phi - 1 - \kappa_X\frac{G_X}{G_t}\right]X_t \\
&+\left[(1-\phi)\gamma + \frac{1}{2}\phi(\phi-1) - \kappa_V\frac{G_V}{G_t} + \frac{1}{2}\frac{G_{XX}}{G_t}\sigma_X^2 + \frac{1}{2}\frac{G_{VV}}{G_t}\sigma_V^2 + \left(A_X\sigma_X^2\frac{G_X}{G_t} + A_V\sigma_V^2\frac{G_V}{G_t}\right)\right]V_t \\
&+\sum_{k=1}^{K}\left[\frac{\mathbb{E}_t\left[\Delta\left(\pi\cdot P_D\right)_{k,t}\right]}{\pi_{t-}\cdot P_{D,t-}} - \left(\Phi_k^Y(\hat{\eta}) - 1\right) + \frac{1}{2}\frac{G_{\lambda_k\lambda_k}}{G_t}\sigma_{\lambda_k}^2 - \kappa_{\lambda_k}\frac{G_{\lambda_k}}{G_t} + A_{\lambda_k}\sigma_{\lambda_k}^2\frac{G_{\lambda_k}}{G_t}\right]\lambda_{k,t} + G_t^{-1} = 0.
\end{aligned}
$$

Using the same linear approximation as before, i.e. $G_t^{-1} \approx g_0 - g_1\log G_t$ with $g_1 = \exp\left(\mathbb{E}\left[\log P_{D,t} - \log D_t\right]\right)$ and $g_0 = g_1(1-\log g_1)$, the solution to the PDE takes the exponentially affine form

$$
G(X, V, \lambda_1, ..., \lambda_K) = \exp\left(A_0^D + A_X^D X + A_V^D V + \sum_{k=1}^{K}A_{\lambda_k}^D\lambda_k\right).
$$

Computing the jump term gives rise to

$$\frac{\pi_{k,t}\cdot G_{k,t}\cdot D_{k,t}}{\pi_{t-}\cdot G_{t-}\cdot D_{t-}}=\exp\biggl((\phi-\gamma)Y^C_{k,t}+\left[A_X+A^D_X\right]Y^X_{k,t}+\left[A_V+A^D_V\right]Y^V_{k,t}+\sum_{j=1}^{K}\left[A_{\lambda_j}+A^D_{\lambda_j}\right]Y^{\lambda_j}_{k,t}\biggr).$$

Substituting the functional form of the solution into the linearized PDE, one obtains

$$\begin{aligned}
&-\beta+(\phi-1)\,\mu_C+A^D_V\kappa_V\bar{V}+\sum_{k=1}^{K}A^D_{\lambda_k}\kappa_{\lambda_k}\bar{\lambda}_k+g_0-g_1A^D_0+\left[\phi-1-(\kappa_X+g_1)A^D_X\right]X_t\\
&+\left[(1-\phi)\gamma+\frac{1}{2}\phi(\phi-1)-(\kappa_V+g_1)A^D_V+\frac{1}{2}\left(A^D_X\right)^2\sigma_X^2+\frac{1}{2}\left(A^D_V\right)^2\sigma_V^2+\Bigl(A_X\sigma_X^2A^D_X+A_V\sigma_V^2A^D_V\Bigr)\right]V_t\\
&+\sum_{k=1}^{K}\left[+\frac{1}{2}\left(A^D_{\lambda_k}\right)^2\sigma^2_{\lambda_k}-(\kappa_{\lambda_k}+g_1)A^D_{\lambda_k}+A_{\lambda_k}\sigma^2_{\lambda_k}A^D_{\lambda_k}+\Phi^Y_k\left(\eta+\eta^D\right)-\Phi^Y_k\left(\hat{\eta}\right)\right]\lambda_{k,t}=0.
\end{aligned}$$

The constant term and the coefficients multiplying each of the state variables must be independently zero, whiche implies that A^D_0, A^D_X, A^D_V, and $A^D_{\lambda_1}$, ..., $A^D_{\lambda_K}$ satisfy the system of equations

$$\begin{aligned}
0=&-\beta+(\phi-1)\,\mu_C+A^D_V\kappa_V\bar{V}+\sum_{k=1}^{K}A^D_{\lambda_k}\kappa_{\lambda_k}\bar{\lambda}_k+g_0-g_1A^D_0\\
0=&\phi-1-(\kappa_X+g_1)A^D_X\\
0=&(1-\phi)\gamma+\frac{1}{2}\phi(\phi-1)-(\kappa_V+g_1)A^D_V+\frac{1}{2}\left(A^D_X\right)^2\sigma_X^2+\frac{1}{2}\left(A^D_V\right)^2\sigma_V^2+\Bigl(A_X\sigma_X^2A^D_X+A_V\sigma_V^2A^D_V\Bigr)\\
0=&\frac{1}{2}\left(A^D_{\lambda_k}\right)^2\sigma^2_{\lambda_k}-(\kappa_{\lambda_k}+g_1)A^D_{\lambda_k}+A_{\lambda_k}\sigma^2_{\lambda_k}A^D_{\lambda_k}+\Phi^Y_k\left(\eta+\eta^D\right)-\Phi^Y_k\left(\hat{\eta}\right).
\end{aligned}$$

The equity risk premium is

$$\mathbb{E}_{t-}\left[r_{i,t-}-r_{f,t-}\right]=-\frac{d\left[\pi^c,P^c_D\right]_t}{\pi_{t-}\cdot P_{D,t-}\cdot dt}-\frac{\sum_{k=1}^{K}\mathbb{E}_{t-}\left[\Delta\pi_{k,t}\cdot\Delta P_{D,k,t}\right]\lambda_{k,t-}}{\pi_{t-}\cdot P_{D,t-}}.$$

Specializing the cross-variation of the continuous parts to the proposed functional form results in

$$\frac{d\left[\pi^c,P^c_D\right]_t}{\pi_{t-}\cdot P_{D,t-}\cdot dt}=\left[-\gamma\phi+A_X\sigma_X^2A^D_X+A_V\sigma_V^2A^D_V\right]V_t+\sum_{k=1}^{K}A_{\lambda_k}\sigma^2_{\lambda_k}A^D_{\lambda_k}\lambda_{k,t}.$$

The jump component is

$$
\begin{aligned}
\frac{\Delta\pi_{k,t}\cdot\Delta P_{D,k,t}}{\pi_{t-}\cdot P_{D,t-}} =& \left(\exp\left(-\gamma Y^C_{k,t}+A_X Y^X_{k,t}+A_V Y^V_{k,t}+\sum_{j=1}^K A_{\lambda_j}Y^{\lambda_j}_{k,t}\right)-1\right)\\
&\cdot\left(\exp\left(\phi Y^C_{k,t}+A^D_X Y^X_{k,t}+A^D_V Y^V_{k,t}+\sum_{j=1}^K A^D_{\lambda_j}Y^{\lambda_j}_{k,t}\right)-1\right)\\
=& \exp\left((\phi-\gamma)Y^C_{k,t}+\left[A_X+A^D_X\right]Y^X_{k,t}+\left[A_V+A^D_V\right]Y^V_{k,t}+\sum_{j=1}^K\left[A_{\lambda_j}+A^D_{\lambda_j}\right]Y^{\lambda_j}_{k,t}\right)\\
&-\exp\left(-\gamma Y^C_{k,t}+\left(A_X Y^X_{k,t}+A_V Y^V_{k,t}+\sum_{j=1}^K A_{\lambda_j}Y^{\lambda_j}_{k,t}\right)\right)\\
&-\exp\left(\phi Y^C_{k,t}+A^D_X Y^X_{k,t}+A^D_V Y^V_{k,t}+\sum_{j=1}^K A^D_{\lambda_j}Y^{\lambda_j}_{k,t}\right)+1.
\end{aligned}
$$

Combining these two terms gives rise to the equity premium

$$
\begin{aligned}
\mathbb{E}_{t-}\left[r_{i,t-}-r_{f,t-}\right] =& \left[\gamma\phi-\left(A_X\sigma^2_X A^D_X+A_V\sigma^2_V A^D_V\right)\right]V_t\\
&+\sum_{k=1}^K\left[\Phi^Y_k(\eta)+\Phi^Y_k\left(\eta^D\right)-\Phi^Y_k\left(\eta+\eta^D\right)-1-A_{\lambda_k}\sigma^2_{\lambda_k}A^D_{\lambda_k}\right]\lambda_{k,t}
\end{aligned}
$$

B.9.11.2 Price-Dividend Ratio and Equity Premium with $\psi\neq 1$ The cross-variation of the continuous parts of the pricing kernel and the price-dividend ratio is

$$
\frac{d\left[\pi^c,P^c_D\right]_t}{\pi_{t-}\cdot P_{D,t-}\cdot dt}=\left[-\gamma\phi+\left(1-\frac{1}{\theta}\right)\left(A_X\sigma^2_X\frac{G_X}{G_t}+A_V\sigma^2_V\frac{G_V}{G_t}\right)\right]V_t+\left(1-\frac{1}{\theta}\right)\sum_{k=1}^K A_{\lambda_k}\sigma^2_{\lambda_k}\frac{G_{\lambda_k}}{G_t}\lambda_{k,t}.
$$

The PDE satisfied by the price of equity is hence given by

$$
\begin{aligned}
&-\beta+\left(\phi-\frac{1}{\psi}\right)\mu_C+\frac{G_V}{G_t}\kappa_V\bar{V}+\sum_{k=1}^{K}\frac{G_{\lambda_k}}{G_t}\kappa_{\lambda_k}\bar{\lambda}_k+\left[\phi-\frac{1}{\psi}-\kappa_X\frac{G_X}{G_t}\right]X_t\\
&+\left[\frac{1}{2}\gamma\left(1+\frac{1}{\psi}\right)-\frac{1}{2}\frac{1}{\theta}\left(1-\frac{1}{\theta}\right)\left(A_X^2\sigma_X^2+A_V^2\sigma_V^2\right)-\gamma\phi+\frac{1}{2}\phi(\phi-1)\right.\\
&\quad\left.-\kappa_V\frac{G_V}{G_t}+\frac{1}{2}\frac{G_{XX}}{G_t}\sigma_X^2+\frac{1}{2}\frac{G_{VV}}{G_t}\sigma_V^2+\left(1-\frac{1}{\theta}\right)\left(A_X\sigma_X^2\frac{G_X}{G_t}+A_V\sigma_V^2\frac{G_V}{G_t}\right)\right]V_t\\
&+\sum_{k=1}^{K}\left[-\left(1-\frac{1}{\theta}\right)\left(\Phi_k^Y\left(\hat{\eta}\right)-1\right)+\frac{\mathbb{E}_{t-}\left[\Delta\left(\pi\cdot P_D\right)_{k,t}\right]}{\pi_{t-}\cdot P_{D,t-}}\right.\\
&\qquad\left.+\frac{1}{2}\frac{G_{\lambda_k\lambda_k}}{G_t}\sigma_{\lambda_k}^2-\kappa_{\lambda_k}\frac{G_{\lambda_k}}{G_t}+\left(1-\frac{1}{\theta}\right)A_{\lambda_k}\sigma_{\lambda_k}^2\frac{G_{\lambda_k}}{G_t}-\frac{1}{2}\frac{1}{\theta}\left(1-\frac{1}{\theta}\right)A_{\lambda_k}^2\sigma_{\lambda_k}^2\right]\lambda_{k,t}+G_t^{-1}=0.
\end{aligned}
$$

Using the same linear approximation as above, i.e. $G_t^{-1}\approx g_0-g_1\log G_t$ with

$$
g_1=\exp\left(\mathbb{E}\left[\log P_{D,t}-\log D_t\right]\right)\ \text{ and }\ g_0=g_1(1-\log g_1),
$$

the solution to the PDE takes the exponentially affine form

$$
G(X,V,\lambda_1,...,\lambda_K)=\exp\left(A_0^D+A_X^D X+A_V^D V+\sum_{k=1}^{K}A_{\lambda_k}^D\lambda_k\right).
$$

Computing the jump term gives rise to

$$
\begin{aligned}
\frac{\pi_{k,t}\cdot G_{k,t}\cdot D_{k,t}}{\pi_{t-}\cdot G_{t-}\cdot D_{t-}}=\exp\Bigg(&(\phi-\gamma)Y_{k,t}^C+\left[\left(1-\frac{1}{\theta}\right)A_X+A_X^D\right]Y_{k,t}^X+\left[\left(1-\frac{1}{\theta}\right)A_V+A_V^D\right]Y_{k,t}^V\\
&+\sum_{j=1}^{K}\left[\left(1-\frac{1}{\theta}\right)A_{\lambda_j}+A_{\lambda_j}^D\right]Y_{k,t}^{\lambda_j}\Bigg).
\end{aligned}
$$

Substituting the functional form of the solution into the linearized PDE one obtains

$$
\begin{aligned}
&-\beta+\left(\phi-\frac{1}{\psi}\right)\mu_C+A_V^D\kappa_V\bar{V}+\sum_{k=1}^{K}A_{\lambda_k}^D\kappa_{\lambda_k}\bar{\lambda}_k+g_0-g_1A_0^D+\left[\phi-\frac{1}{\psi}-(\kappa_X+g_1)A_X^D\right]X_t\\
&+\Bigg[\frac{1}{2}\gamma\left(1+\frac{1}{\psi}\right)-\frac{1}{2}\frac{1}{\theta}\left(1-\frac{1}{\theta}\right)\left(A_X^2\sigma_X^2+A_V^2\sigma_V^2\right)+\frac{1}{2}\phi(\phi-1)-\gamma\phi\\
&\quad-(\kappa_V+g_1)A_V^D+\frac{1}{2}\left(A_X^D\right)^2\sigma_X^2+\frac{1}{2}\left(A_V^D\right)^2\sigma_V^2+\left(1-\frac{1}{\theta}\right)\Big(A_X\sigma_X^2A_X^D+A_V\sigma_V^2A_V^D\Big)\Bigg]V_t\\
&+\sum_{k=1}^{K}\Bigg[+\frac{1}{2}\left(A_{\lambda_k}^D\right)^2\sigma_{\lambda_k}^2-(\kappa_{\lambda_k}+g_1)A_{\lambda_k}^D+\left(1-\frac{1}{\theta}\right)A_{\lambda_k}\sigma_{\lambda_k}^2A_{\lambda_k}^D-\frac{1}{2}\frac{1}{\theta}\left(1-\frac{1}{\theta}\right)A_{\lambda_k}^2\sigma_{\lambda_k}^2\\
&\qquad-\left(1-\frac{1}{\theta}\right)\left(\Phi_k^Y\left(\hat{\eta}\right)-1\right)+\left(\Phi_k^Y\left(\eta+\eta^D\right)-1\right)\Bigg]\lambda_{k,t}=0.
\end{aligned}
$$

The coefficients A_0^D, A_X^D, A_V^D, and $A_{\lambda_1}^D$, ..., $A_{\lambda_K}^D$ must hence satisfy the system of equations

$$
\begin{aligned}
0=&-\beta+\left(\phi-\frac{1}{\psi}\right)\mu_C+A_V^D\kappa_V\bar{V}+\sum_{k=1}^{K}A_{\lambda_k}^D\kappa_{\lambda_k}\bar{\lambda}_k+g_0-g_1A_0^D\\
0=&\phi-\frac{1}{\psi}-(\kappa_X+g_1)A_X^D\\
0=&\frac{1}{2}\gamma\left(1+\frac{1}{\psi}\right)-\frac{1}{2}\frac{1}{\theta}\left(1-\frac{1}{\theta}\right)\left(A_X^2\sigma_X^2+A_V^2\sigma_V^2\right)-\gamma\phi+\frac{1}{2}\phi(\phi-1)\\
&-(\kappa_V+g_1)A_V^D+\frac{1}{2}\left(A_X^D\right)^2\sigma_X^2+\frac{1}{2}\left(A_V^D\right)^2\sigma_V^2+\left(1-\frac{1}{\theta}\right)\Big(A_X\sigma_X^2A_X^D+A_V\sigma_V^2A_V^D\Big)\\
0=&\frac{1}{2}\left(A_{\lambda_k}^D\right)^2\sigma_{\lambda_k}^2-(\kappa_{\lambda_k}+g_1)A_{\lambda_k}^D+\left(1-\frac{1}{\theta}\right)A_{\lambda_k}\sigma_{\lambda_k}^2A_{\lambda_k}^D-\frac{1}{2}\frac{1}{\theta}\left(1-\frac{1}{\theta}\right)A_{\lambda_k}^2\sigma_{\lambda_k}^2\\
&-\left(1-\frac{1}{\theta}\right)\left(\Phi_k^Y\left(\hat{\eta}\right)-1\right)+\left(\Phi_k^Y\left(\eta+\eta^D\right)-1\right).
\end{aligned}
$$

The risk premium of a dividend claim is

$$
\mathbb{E}_{t-}\left[r_{i,t-}-r_{f,t-}\right]=-\frac{d\left[\pi^c,P_D^c\right]_t}{\pi_{t-}\cdot P_{D,t-}\cdot dt}-\frac{\sum_{k=1}^{K}\mathbb{E}_{t-}\left[\Delta\pi_{k,t}\cdot\Delta P_{D,k,t}\right]\lambda_{k,t-}}{\pi_{t-}\cdot P_{D,t-}}.
$$

The cross-variation of the continuous parts has been computed above for general G and in this particular instance of an exponantially affine function takes the form

$$
\frac{d\left[\pi^c,P_D^c\right]_t}{\pi_{t-}\cdot P_{D,t-}\cdot dt}=\left[-\gamma\phi+\left(1-\frac{1}{\theta}\right)\Big(A_X\sigma_X^2A_X^D+A_V\sigma_V^2A_V^D\Big)\right]V_t+\left(1-\frac{1}{\theta}\right)\sum_{k=1}^{K}A_{\lambda_k}\sigma_{\lambda_k}^2A_{\lambda_k}^D\lambda_{k,t}.
$$

The jump component is

$$
\begin{aligned}
\frac{\Delta \pi_{k,t} \cdot \Delta P_{D,k,t}}{\pi_{t-} \cdot P_{D,t-}} =& \left(\exp \left(-\gamma Y_{k,t}^C + \left(1 - \frac{1}{\theta}\right) \left(A_X Y_{k,t}^X + A_V Y_{k,t}^V + \sum_{j=1}^K A_{\lambda_j} Y_{k,t}^{\lambda_j} \right) \right) - 1 \right) \\
& \cdot \left(\exp \left(\phi Y_{k,t}^C + A_X^D Y_{k,t}^X + A_V^D Y_{k,t}^V + \sum_{j=1}^K A_{\lambda_j}^D Y_{k,t}^{\lambda_j} \right) - 1 \right) \\
=& \exp\Bigg((\phi - \gamma) Y_{k,t}^C + \left[\left(1 - \frac{1}{\theta}\right) A_X + A_X^D \right] Y_{k,t}^X + \left[\left(1 - \frac{1}{\theta}\right) A_V + A_V^D \right] Y_{k,t}^V \\
& + \sum_{j=1}^K \left[\left(1 - \frac{1}{\theta}\right) A_{\lambda_j} + A_{\lambda_j}^D \right] Y_{k,t}^{\lambda_j} \Bigg) \\
& - \exp \left(-\gamma Y_{k,t}^C + \left(1 - \frac{1}{\theta}\right) \left(A_X Y_{k,t}^X + A_V Y_{k,t}^V + \sum_{j=1}^K A_{\lambda_j} Y_{k,t}^{\lambda_j} \right) \right) \\
& - \exp \left(\phi Y_{k,t}^C + A_X^D Y_{k,t}^X + A_V^D Y_{k,t}^V + \sum_{j=1}^K A_{\lambda_j}^D Y_{k,t}^{\lambda_j} \right) + 1.
\end{aligned}
$$

Combining these two terms, one finds the equity risk premium when $\psi \neq 1$ to be

$$
\begin{aligned}
\mathbb{E}_{t-} \left[r_{i,t-} - r_{f,t-} \right] =& \left[\gamma \phi - \left(1 - \frac{1}{\theta}\right) \left(A_X \sigma_X^2 A_X^D + A_V \sigma_V^2 A_V^D \right) \right] V_{t-} \\
& + \sum_{k=1}^K \left[\Phi_k^Y(\eta) + \Phi_k^Y\left(\eta^D\right) - \Phi_k^Y\left(\eta + \eta^D\right) - 1 - \left(1 - \frac{1}{\theta}\right) A_{\lambda_k} \sigma_{\lambda_k}^2 A_{\lambda_k}^D \right] \lambda_{k,t-}.
\end{aligned}
$$

B.9.12 Dynamics under the Risk-Neutral Measure (Proof of Proposition B.11)

Dynamics under the risk-neutral measure are obtained by exploiting the fact that the process under the physical measure belongs to the class of affine-jump diffusion under and invoking lemma A.2 which gives a characterization under an equivalent probability measure. The dynamics of $X_t = (\ln C_t, X_t, V_t, \lambda_{1,t}, ..., \lambda_{K,t})^T$ are governed by an affine jump diffusion. The drift coefficients are given by

$$
K_0 = \begin{pmatrix} \mu_C \\ 0 \\ \kappa_V \bar{V} \\ \kappa_{\lambda_1} \bar{\lambda}_1 \\ \vdots \\ \kappa_{\lambda_K} \bar{\lambda}_k \end{pmatrix}, K_1 = \begin{pmatrix} 0 & 1 & -\frac{1}{2} & 0 & \dots & 0 \\ 0 & -\kappa_X & 0 & 0 & \dots & 0 \\ 0 & 0 & -\kappa_V & 0 & \dots & 0 \\ 0 & 0 & 0 & -\kappa_{\lambda_1} & \dots & 0 \\ \vdots & \vdots & \vdots & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \dots & -\kappa_{\lambda_K} \end{pmatrix}.
$$

The diffusion coefficient is

$$\sigma(X_t) = \begin{pmatrix} \sqrt{V_t} & 0 & 0 & 0 & \dots & 0 \\ 0 & \sigma_X\sqrt{V_t} & 0 & 0 & \dots & 0 \\ 0 & 0 & \sigma_V\sqrt{V_t} & 0 & \dots & 0 \\ 0 & 0 & 0 & \sigma_{\lambda_1}\sqrt{\lambda_{1,t}} & \dots & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \dots & \sigma_{\lambda_K}\sqrt{\lambda_{K,t}} \end{pmatrix}.$$

Since the class of affine jump diffusion imposes the restriction $\sigma(X_t)\sigma(X_t)^T = H_0 + \sum_{i=1}^d H_{1,i}X_{i,t}$, we have $H_0 = 0, H_{1,1} = 0, H_{1,2} = 0$, and

$$H_{1,3} = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & \sigma_X^2 & 0 & 0 & \dots & 0 \\ 0 & 0 & \sigma_V^2 & 0 & \dots & 0 \\ 0 & 0 & 0 & \sigma_{\lambda_1}^2 & \dots & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \dots & \sigma_{\lambda_K}^2 \end{pmatrix}.$$

For $i \in \{4, \dots, 3+K\}$, $H_{1,i}$ is $\sigma_{\lambda_k}^2$ at element (3 + k, 3 + k) and zero everywhere else.

The coefficients of the jump intensity are $\lambda_{0,k} = 0$ for all $k \in 1, \dots, K$ and $\lambda_{1,k}$ is zero, except at position $3+k$, where it is one. The change of measure is characterized by the Radon-Nikodym derivative process

$$\frac{dZ_t}{Z_{t-}} = \eta^T\sigma(X_t)dB_t - \sum_{k=1}^{K}\left[\left(1 - \exp(\eta^T Y_{k,t})\right) - \left(1 - \Phi_k^Y(\eta)\right)(\lambda_{k,0} + \lambda_{k,1}X_t)\,dt\right],$$

where

$$\eta^T = \begin{pmatrix} -\gamma \\ \left(1 - \frac{1}{\theta}\right)A_X \\ \left(1 - \frac{1}{\theta}\right)A_V \\ \left(1 - \frac{1}{\theta}\right)A_{\lambda_1} \\ \vdots \\ \left(1 - \frac{1}{\theta}\right)A_{\lambda_K} \end{pmatrix}.$$

Under the equivalent measure $\tilde{\mathbb{P}}$, by lemma A.2, the constant component in the drift coefficient is $\tilde{K}_0 = K_0 + H_0^T\eta = K_0$. The coefficient multiplying the state X_t is given by

$\tilde{K}_1 = K_1 + [H_{1,1}\eta \dots H_{1,K}\eta]$. We have $H_{1,1}\eta = 0$, $H_{1,2}\eta = 0$, and

$$H_{1,3}\eta = \begin{pmatrix} -\gamma \\ \left(1-\frac{1}{\theta}\right) A_X \sigma_X^2 \\ \left(1-\frac{1}{\theta}\right) A_V \sigma_V^2 \\ 0 \\ \vdots \\ 0 \end{pmatrix}.$$

Furthermore, for $i \in \{4, \dots, 3+K\}$, the term $H_{1,i}\eta$ has element $-\left(1-\frac{1}{\theta}\right) A_{\lambda_k}\sigma_{\lambda_k}^2$ at position $k = i-3$ and is zero elsewhere. Accordingly, we have

$$\tilde{K}_1 = \begin{pmatrix} 0 & 1 & -\gamma - \frac{1}{2} & 0 & \dots & 0 \\ 0 & -\kappa_X & \left(1-\frac{1}{\theta}\right) A_X \sigma_X^2 & 0 & \dots & 0 \\ 0 & 0 & -\kappa_V + \left(1-\frac{1}{\theta}\right) A_V \sigma_V^2 & 0 & \dots & 0 \\ 0 & 0 & 0 & -\kappa_{\lambda_1} + \left(1-\frac{1}{\theta}\right) A_{\lambda_1}\sigma_{\lambda_1}^2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & -\kappa_{\lambda_K} + \left(1-\frac{1}{\theta}\right) A_{\lambda_K}\sigma_{\lambda_K}^2 \end{pmatrix}.$$

Both, the distribution of the jump size and the jump intensity under the equivalent measure are stated in lemma A.2. The dynamics for X_t then follow from the fact that the process remains within the class of affine jump diffusions under $\tilde{\mathbb{P}}$. The diffusion coefficient is unaffected by the change of measure. The process for log-dividend growth easily follows by noting that $D_t = C_t^{\phi}$ implies $\ln D_t = \phi \ln C_t$ and hence $d \ln D_t = \phi d \ln C_t$, which yields

$$d \ln D_t = \phi \left[\mu_C + X_t - \left(\gamma + \frac{1}{2}\right) V_t \right] dt + \phi \sqrt{V_t} d\tilde{B}_{C,t} + \sum_{k=1}^{K} \phi Y_{k,t}^C d\tilde{N}_{k,t}.$$

This concludes the proof of the proposition.

B.9.13 Dynamics of the Equity Price under the Risk-Neutral Measure (Proof of Corollary B.1)

This section derives the log price process of claim to the dividend stream $\{D_s\}_{s\geq t}$. Since the log price of equity is given by

$$\begin{aligned}\ln P_{D,t} &= \ln D_t + \ln G(X_t, V_t, \lambda_{1,t}, ..., \lambda_{K,t}) \\ &= \ln D_t + A_0^D + A_X^D X_t + A_V^D V_t + \sum_{k=1}^{K} A_{\lambda_k}^D \lambda_{k,t},\end{aligned}$$

the price process satisfies

$$d\ln P_{D,t} = d\ln D_t + A_X^D dX_t + A_V^D dV_t + \sum_{k=1}^{K} A_{\lambda_k}^D d\lambda_{k,t}.$$

Substituting the dynamics of log dividends and state variables under the risk-neutral measure one obtains

$$\begin{aligned}d\ln P_{D,t} =& \Bigg(\phi\left[\mu_C + X_t - \left(\gamma + \frac{1}{2}\right)V_t\right] - A_X \kappa_X X_t + \left(1 - \frac{1}{\theta}\right) A_X^D A_X \sigma_X^2 V_t \\ &+ A_V^D \left[\kappa_V\left(\bar{V} - V_t\right) + \left(1 - \frac{1}{\theta}\right)\sigma_V^2 A_V V_t\right] \\ &+ \sum_{k=1}^{K} A_{\lambda_k}^D \left[\kappa_{\lambda_k}\left(\bar{\lambda}_k - \lambda_{k,t}\right) + \left(1 - \frac{1}{\theta}\right) A_{\lambda_k}\sigma_{\lambda_k}^2 \lambda_{k,t}\right]\Bigg) dt \\ &+ \phi\sqrt{V_t}d\tilde{B}_{C,t} + A_X^D \sigma_X \sqrt{V_t} d\tilde{B}_{X,t} + A_V^D \sigma_V \sqrt{V_t} d\tilde{B}_{V,t} + \sum_{k=1}^{K} A_{\lambda_k}^D \sigma_{\lambda_k} \sqrt{\lambda_{k,t}} d\tilde{B}_{k,t} \\ &+ \sum_{j=1}^{K}\left(\phi Y_{j,t}^C + A_X^D Y_{j,t}^X + A_V^D Y_{j,t}^V + \sum_{k=1}^{K} A_{\lambda_k}^D Y_{j,t}^{\lambda_k}\right) d\tilde{N}_{j,t}.\end{aligned}$$

The joint process for the log equity price and the state variables $X_t = (\ln P_{D,t}, X_t, V_t, \lambda_{1,t}, ..., \lambda_{K,t})^T$ is an affine jump diffusion under the risk-neutral measure $\tilde{\mathbb{P}}$. Accordingly, the drift coefficient satisfies $\mu(X_t) = K_0 + K_1 X_t$ with

$$K_0 = \begin{pmatrix} \phi\mu_C + A_V^D \kappa_V \bar{V} + \sum_{k=1}^K A_{\lambda_k}^D \kappa_{\lambda_k} \bar{\lambda_k} \\ 0 \\ \kappa_V \bar{V} \\ \kappa_{\lambda_1} \bar{\lambda}_1 \\ \vdots \\ \kappa_{\lambda_K} \bar{\lambda}_k \end{pmatrix}, K_1 = \begin{pmatrix} 0 & K_{1,X} & K_{1,V} & K_{1,\lambda_1} & \dots & K_{1,\lambda_K} \\ 0 & -\kappa_X & 0 & 0 & \dots & 0 \\ 0 & 0 & K_{1,V,V} & 0 & \dots & 0 \\ 0 & 0 & 0 & K_{1,\lambda_1,\lambda_1} & \dots & 0 \\ \vdots & \vdots & \vdots & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \dots & K_{1,\lambda_K,\lambda_K} \end{pmatrix},$$

with

$$\begin{aligned} K_{1,X} &= \phi - A_X \kappa_X \\ K_{1,V} &= -\phi\left(\gamma + \frac{1}{2}\right) + \left(1 - \frac{1}{\theta}\right) A_X^D A_X \sigma_X^2 - A_V^D \kappa_V + \left(1 - \frac{1}{\theta}\right) A_V A_V^D \sigma_V^2 \\ K_{1,\lambda_k} &= -A_{\lambda_k}^D \kappa_{\lambda_k} + \left(1 - \frac{1}{\theta}\right) A_\lambda^D A_{\lambda_k} \sigma_{\lambda_k}^2 \\ K_{1,V,V} &= -\kappa_V + \left(1 - \frac{1}{\theta}\right) A_V \sigma_V^2 \\ K_{1,\lambda_k,\lambda_k} &= -\kappa_{\lambda_k} + \left(1 - \frac{1}{\theta}\right) A_{\lambda_k} \sigma_{\lambda_k}^2 \end{aligned}$$

The diffusion coefficient is

$$\sigma(X_t) = \begin{pmatrix} \phi\sqrt{V_t} & A_X^D \sigma_X \sqrt{V_t} & A_V^D \sigma_V \sqrt{V_t} & A_{\lambda_1}^D \sigma_{\lambda_1} \sqrt{\lambda_{1,t}} & \dots & A_{\lambda_K}^D \sigma_{\lambda_K} \sqrt{\lambda_{K,t}} \\ 0 & \sigma_X \sqrt{V_t} & 0 & 0 & \dots & 0 \\ 0 & 0 & \sigma_V \sqrt{V_t} & 0 & \dots & 0 \\ 0 & 0 & 0 & \sigma_{\lambda_1} \sqrt{\lambda_{1,t}} & \dots & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \dots & \sigma_{\lambda_K} \sqrt{\lambda_{K,t}} \end{pmatrix}.$$

The class of affine models requires $\sigma(X_t)\sigma(X_t)^T$ to be a linear function of the state variables. Here, we have $\sigma(X_t)\sigma(X_t)^T$

$$\sigma(X_t)\sigma(X_t)^T = \begin{pmatrix} \left[\sigma(X_t)\sigma(X_t)^T\right]_{1,1} & A_X^D\sigma_X^2 V_t & A_V^D\sigma_V^2 V_t & A_{\lambda_1}^D\sigma_{\lambda_1}^2\lambda_{1,t} & \dots & A_{\lambda_K}^D\sigma_{\lambda_K}^2\lambda_{K,t} \\ A_X^D\sigma_X^2 V_t & \sigma_X^2 V_t & 0 & 0 & \dots & 0 \\ A_V^D\sigma_V^2 V_t & 0 & \sigma_V^2 V_t & 0 & \dots & 0 \\ A_{\lambda_1}^D\sigma_{\lambda_1}^2\lambda_{1,t} & 0 & 0 & \sigma_{\lambda_1}^2\lambda_{1,t} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{\lambda_K}^D\sigma_{\lambda_K}^2\lambda_{K,t} & 0 & 0 & 0 & \dots & \sigma_{\lambda_K}^2\lambda_{K,t} \end{pmatrix},$$

where $\left[\sigma(X_t)\sigma(X_t)^T\right]_{1,1} = \phi^2 V_t + \left(A_X^D\right)^2\sigma_X^2 V_t + \left(A_V^D\right)^2\sigma_V^2 V_t + \sum_{k=1}^K \left(A_{\lambda_k}^D\right)^2\sigma_{\lambda_k}^2\lambda_{k,t}$. Accordingly, we have $H_{1,1} = 0$, $H_{1,2} = 0$, and the matrices $H_{1,3}$ and $H_{1,i}$, with $i = k+3, k = 1, ..., K$ are respectively

$$H_{1,3} = \begin{pmatrix} [H_{1,3}]_{1,1} & A_X^D\sigma_X^2 & A_V^D\sigma_V^2 & 0 & \dots & 0 \\ A_X^D\sigma_X^2 & \sigma_X^2 & 0 & 0 & \dots & 0 \\ A_V^D\sigma_V^2 & 0 & \sigma_V^2 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 0 \end{pmatrix},$$

with $[H_{1,3}]_{1,1} = \phi^2 + \left(A_X^D\right)^2\sigma_X^2 + \left(A_V^D\right)^2\sigma_V^2$, and

$$H_{1,i} = \begin{pmatrix} \left(A_{\lambda_k}^D\right)^2\sigma_{\lambda_k}^2 & 0 & 0 & \dots & A_{\lambda_k}^D\sigma_{\lambda_k}^2 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ A_{\lambda_k}^D\sigma_{\lambda_k}^2 & 0 & 0 & \dots & \sigma_{\lambda_k}^2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \dots & \dots & 0 \end{pmatrix}.$$

The ordinary differential equations that determine the characteristic function of the joint transition density of the log price of equity and the state variable can now be obtained from lemma A.1.

B.9.14 Equilibrium Prices of European Derivatives (Proof of Proposition B.12)

In the absence of arbitrage, the price of a derivative security with European exercise that provides $f(x_T)$ at expiration T is given by the following expectation under the risk-neutral measure:

$$P_{f,t}(x_t) = \tilde{\mathbb{E}}\left[\exp\left(-\int_t^T r_{f,s}ds\right) f(x_T)\middle| \mathcal{F}_t\right].$$

Closed form solutions to this expectations are not available in many cases of interest. However, the Fourier transform with respect to x_t can be computed up to the solution of a system of ordinary differential equations for the class of affine jump-diffusions. The Fourier transform is defined as

$$FP_{f,t}(u) = \int_{-\infty}^{\infty} \exp\left(iux_t\right) P_{f,t}(x_t)dx_t.$$

Substituting the definition and applying the properties of the Fourier transform, one obtains

$$\begin{aligned}
FP_{f,t}(u) &= \int_{-\infty}^{\infty} \exp\left(iux_t\right) \tilde{\mathbb{E}}\left[\exp\left(-\int_t^T r_{f,s}ds\right) f(x_T)\middle| \mathcal{F}_t\right] dx_t \\
&= \tilde{\mathbb{E}}\left[\int_{-\infty}^{\infty} \exp\left(iux_t\right) \exp\left(-\int_t^T r_{f,s}ds\right) f(x_T)dx_t\middle| \mathcal{F}_t\right] \\
&= \tilde{\mathbb{E}}\left[\exp\left(-\int_t^T r_{f,s}ds\right) \int_{-\infty}^{\infty} \exp\left(iux_t\right) f(x_t + \Delta x_{t,T})dx_t\middle| \mathcal{F}_t\right] \\
&= \tilde{\mathbb{E}}\left[\exp\left(-\int_t^T r_{f,s}ds\right) \exp\left(-iu\Delta x_{t,T}\right) \int_{-\infty}^{\infty} \exp\left(iux_t\right) f(x_t)dx_t\middle| \mathcal{F}_t\right] \\
&= \exp\left(iux_t\right) \tilde{\mathbb{E}}\left[\exp\left(-\int_t^T r_{f,s}ds\right) \exp\left(-iux_T\right)\middle| \mathcal{F}_t\right] Ff(u) \\
&= \exp\left(iux_t\right) \Psi^x(t, x, -u, T) Ff(u),
\end{aligned}$$

which confirms the proposition.

C Tables

Table 1: Calibrated parameters

Parameters of consumption and dividend dynamics	
Drift coefficient of consumption growth μ_C	0.0252
Diffusion coefficient of consumption growth σ_C	0.02
Mean reversion coefficient of jump intensity process κ_λ	0.368
Mean reversion target of disaster intensity process $\bar{\lambda}$	0.045
Diffusion coefficient of disaster intensity process σ_λ	0.00
Shape parameter of disaster size in log-consumption a_J	0.83
Scale parameter of disaster size in log-consumption b_J	0.10
Upwards revision of disaster intensity in response to disaster arrival Y^λ	0.20
Leverage ϕ	3.00
Probability of government default in the event of disaster q	0.4
Preference parameters	
Time discount factor β	0.02
Relative risk aversion γ	2.65
Elasticity of intertemporal substitution ψ	1.5

Parameters are from an annual calibration of the model

$$\begin{aligned} dC_t &= \mu_C C_t dt + \sigma_C C_t dB_{C,t} + C_{t-}\left(\exp\left(Y_t^C\right) - 1\right) dN_t \\ d\lambda_t &= \kappa_\lambda \left(\bar{\lambda} - \lambda_t\right) dt + \sigma_\lambda \sqrt{\lambda_t} dB_{\lambda,t} + Y^\lambda dN_t, \end{aligned}$$

where $B_{C,t}$ and $B_{\lambda,t}$ are independent Brownian motions and N_t is a counting process with intensity λ_t. The jump size in log consumption is distributed according to $-Y_t^C \overset{i.i.d.}{\sim} GA(a_J, b_J)$. Dividends are a levered claim to consumption, i.e. $D_t = C_t^\phi$. The representative agent has recursive preferences with a normalized Porteus-Kreps aggregator

$$J_t = \mathbb{E}_t\left[\int_t^\infty f(C_s, J_s) ds\right], f(C,J) = \frac{\beta}{1-\frac{1}{\psi}} J(1-\gamma)\left(\frac{C^{1-\frac{1}{\psi}}}{((1-\gamma)J)^{\frac{1}{\theta}}} - 1\right).$$

Table 2: Distribution of crisis in the model and in the data

	all crises	consumption loss						
		$\in [0, 10]$ %	$> 10\%$		$> 15\%$		$> 25\%$	
			model	data	model	data	model	data
Crisis probability (percent)	7.08	3.96	3.12	3.8	2.10	2.20	1.01	1.07
Expected crisis duration (years)	2.97	1.94	4.27	5.33	5.13	5.64	7.19	6.19
Expected number of disasters during crisis	2.10	1.32	3.09	-	3.77	-	5.54	-
Expected consumption loss during crisis (percent)	12.78	4.21	23.66	24.04	29.21	32.33	39.82	45.00
Std dev. of consumption loss during crisis (percent)	13.52	2.82	13.91	15.38	13.89	15.60	13.26	13.18

Monte carlo results are for $5,000$ paths of 100 years simulated at a monthly frequency. At the initial point of each path a disaster is simulated that causes consumption to drop and the jump intensity to increase from $\mathbb{E}[\lambda_t]$ to $\mathbb{E}[\lambda_t] + Y^{\lambda}$. The duration of the crisis is computed as the first time the disaster intensity λ_t falls below the crisis threshold.

Table 3: Simulated moments from the model and sample moments in the data

		Model	U.S. Data
Expected return on government debt	$\mathbb{E}\left[r_{B,t}\right]$	2.15%	2.01%
Std. dev. of return on government debt	$\sigma\left[r_{B,t}\right]$	2.79%	5.91%
Equity risk premium	$\mathbb{E}\left[r_{D,t} - r_{B,t}\right]$	6.08%	5.97%
Std. dev. of return on equity	$\sigma\left[r_{D,t}\right]$	18.96%	18.48%
Sharpe Ratio	$\mathbb{E}\left[r_{D,t} - r_{B,t}\right]/\sigma\left[r_{D,t}\right]$	0.32	0.32
Std. dev. of log consumption growth	$\sigma\left[\Delta\ln(C_t)\right]$	4.61%	3.56%
Std. dev. of log dividend growth	$\sigma\left[\Delta\ln(D_t)\right]$	13.83%	11.51%

Moments for the model are from 2 paths of a monthly simulation of $25,000$ years aggregated to an annual frequency. Each path is initialized at $\lambda_0 = \mathbb{E}[\lambda_t]$. Sample moments are from U.S. data spanning the period from 1890 to 2004 as reported in Wachter (2010).

Table 4: Long-horizon predictive regression of excess returns on the price-dividend ratio

	Horizon in years					
	1	2	4	6	8	10
	Panel A: Model (unconditional)					
β_1	-0.13	-0.24	-0.40	-0.50	-0.61	-0.67
R^2	0.02	0.04	0.06	0.07	0.08	0.08
	Panel B: Model (conditional)					
β_1	-0.25	-0.44	-0.69	-0.85	-0.92	-1.00
R^2	0.08	0.14	0.21	0.24	0.24	0.24
	Panel C: U.S. data					
β_1	-0.09	-0.14	-0.36	-0.48	-0.73	-0.98
R^2	0.04	0.04	0.12	0.12	0.17	0.23

Results are from a regression of excess log returns on equity over the horizon $t, t+h$ on the log price-dividend ratio at t. The regression is specified as

$$\sum_{i=1}^{h} \log r_{D,t+i} - \log r_{B,t+i} = \beta_0 + \beta_1 \left(\log P_{D,t} - \log D_t\right) + \epsilon_t$$

The first and second sets of results is from a Monte carlo simulation of the model detailed in the notes accompanying table 3. The first set includes all simulated periods, while the second set drops periods that include a macro contraction exceeding 10%. The third set of results is from U.S. data from 1890 to 2004 as reported in Wachter (2010).

Table 5: Long-horizon predictive regression of log consumption growth on the price-dividend ratio

	Horizon in years					
	1	2	4	6	8	10
	Panel A: Model (unconditional)					
β_1	0.031	0.057	0.099	0.131	0.145	0.162
R^2	0.0294	0.0456	0.0615	0.0679	0.0560	0.0518
	Panel B: Model (conditional)					
β_1	0.009	0.016	0.030	0.034	0.046	0.051
R^2	0.0037	0.0063	0.0104	0.0088	0.0123	0.0111
	Panel C: U.S. data					
β_1	0.003	-0.001	-0.007	0.002	0.006	0.040
R^2	0.0011	0.0000	0.0013	0.0001	0.0004	0.0129

Results are from a regression of log consumption growth over the horizon $t, t+h$ on the log price-dividend ratio at t. The regression is specified as

$$\sum_{i=1}^{h} \Delta \log C_{t+i} = \beta_0 + \beta_1 \left(\log P_{D,t} - \log D_t\right) + \epsilon_t$$

The first and second sets of results is from a Monte carlo simulation of the model detailed in the notes accompanying table 3. The first set includes all simulated periods, while the second set drops periods that included a macro contraction exceeding 10%. The third set of results is from U.S. data from 1890 to 2004 as reported in Wachter (2010).

Table 6: Macroeconomic Contractions and Stock Market Crashes

Probability of macroeconomic contraction conditional on stock market decline $\geq 25\%$	Model	Data
Macroeconomic contraction $\geq 10\%$	41.25%	28%
Macroeconomic contraction $\geq 25\%$	11.38%	11%

D Figures

Figure 1: CBOE S&P 500 Implied Volatility Index (VIX)

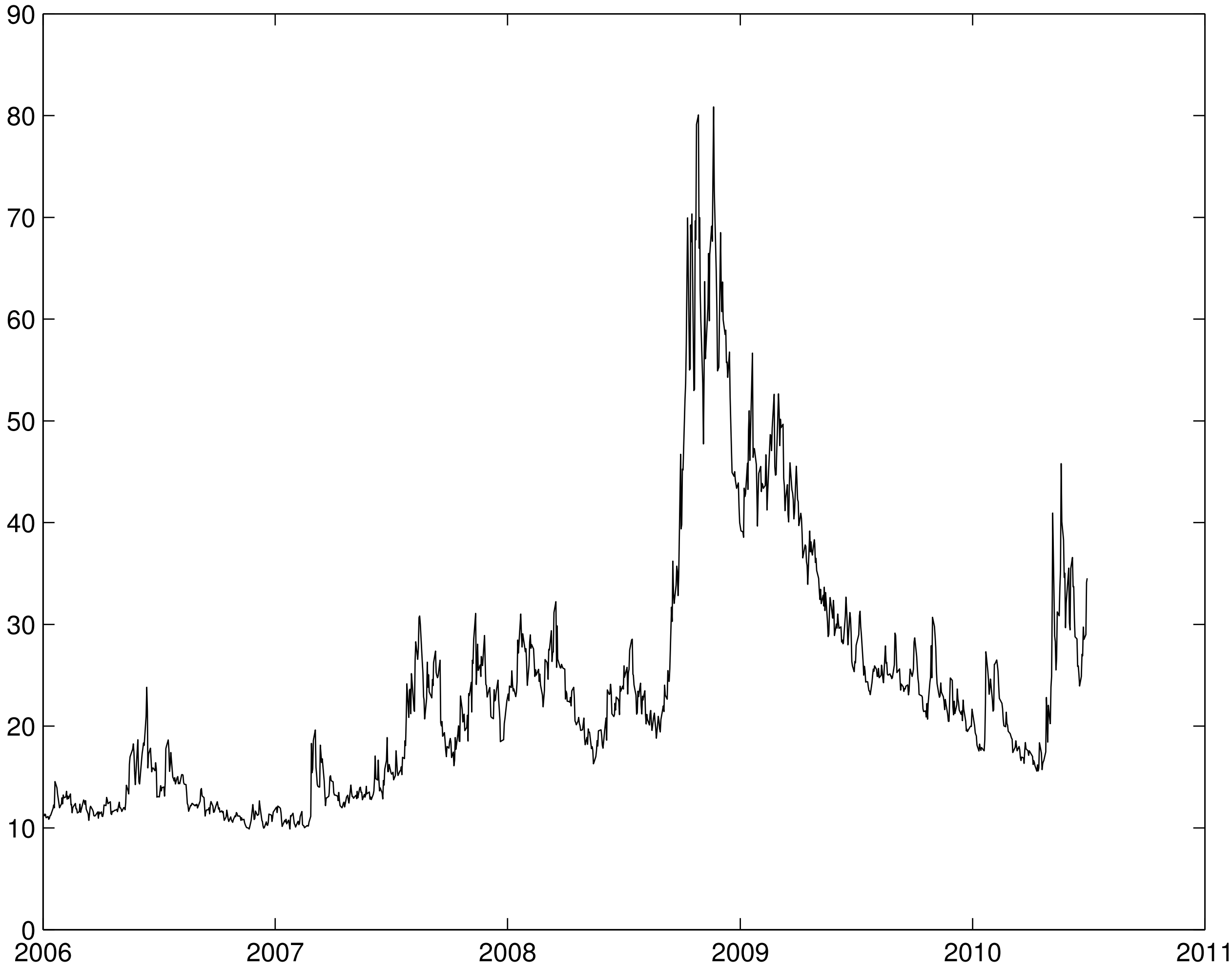
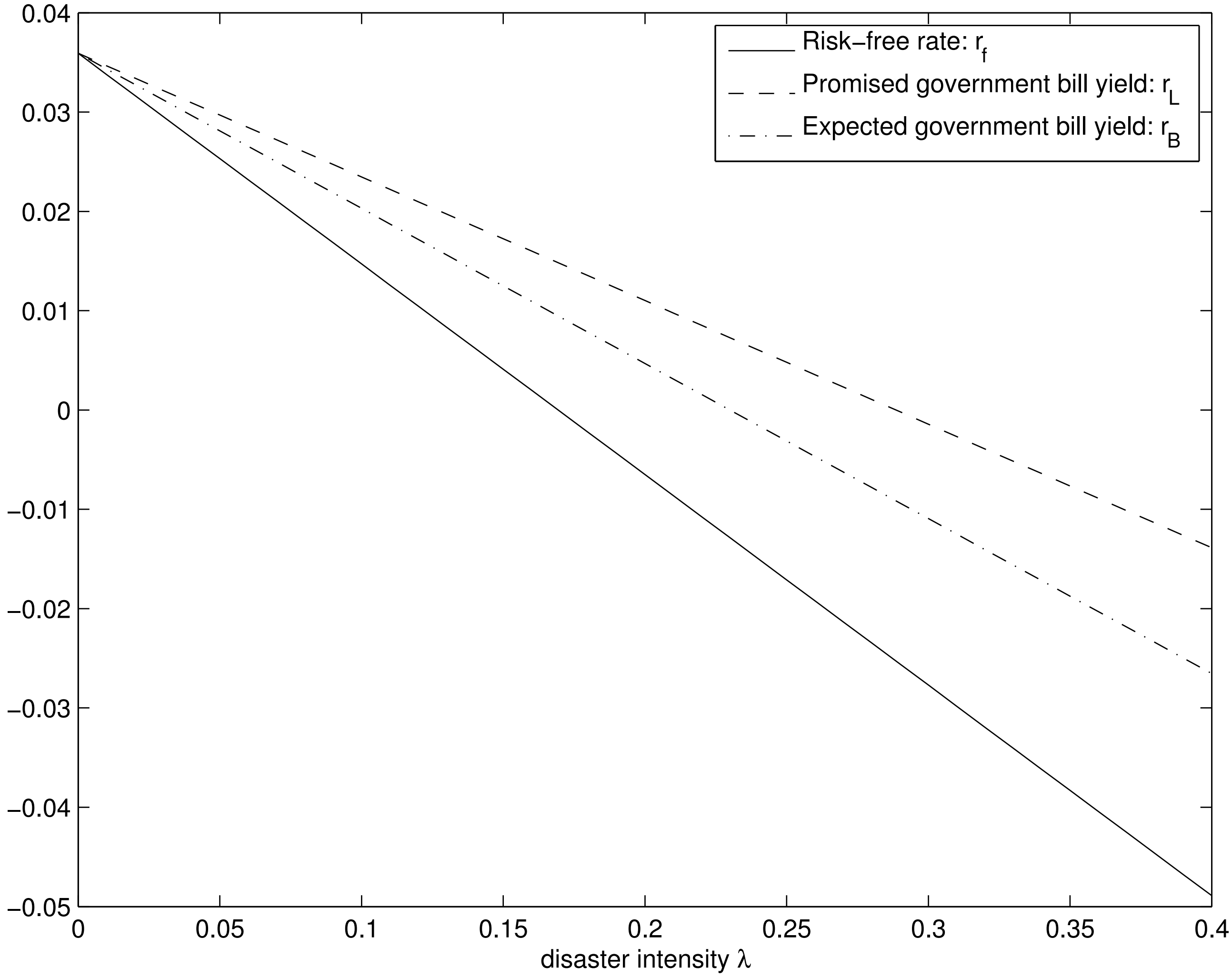
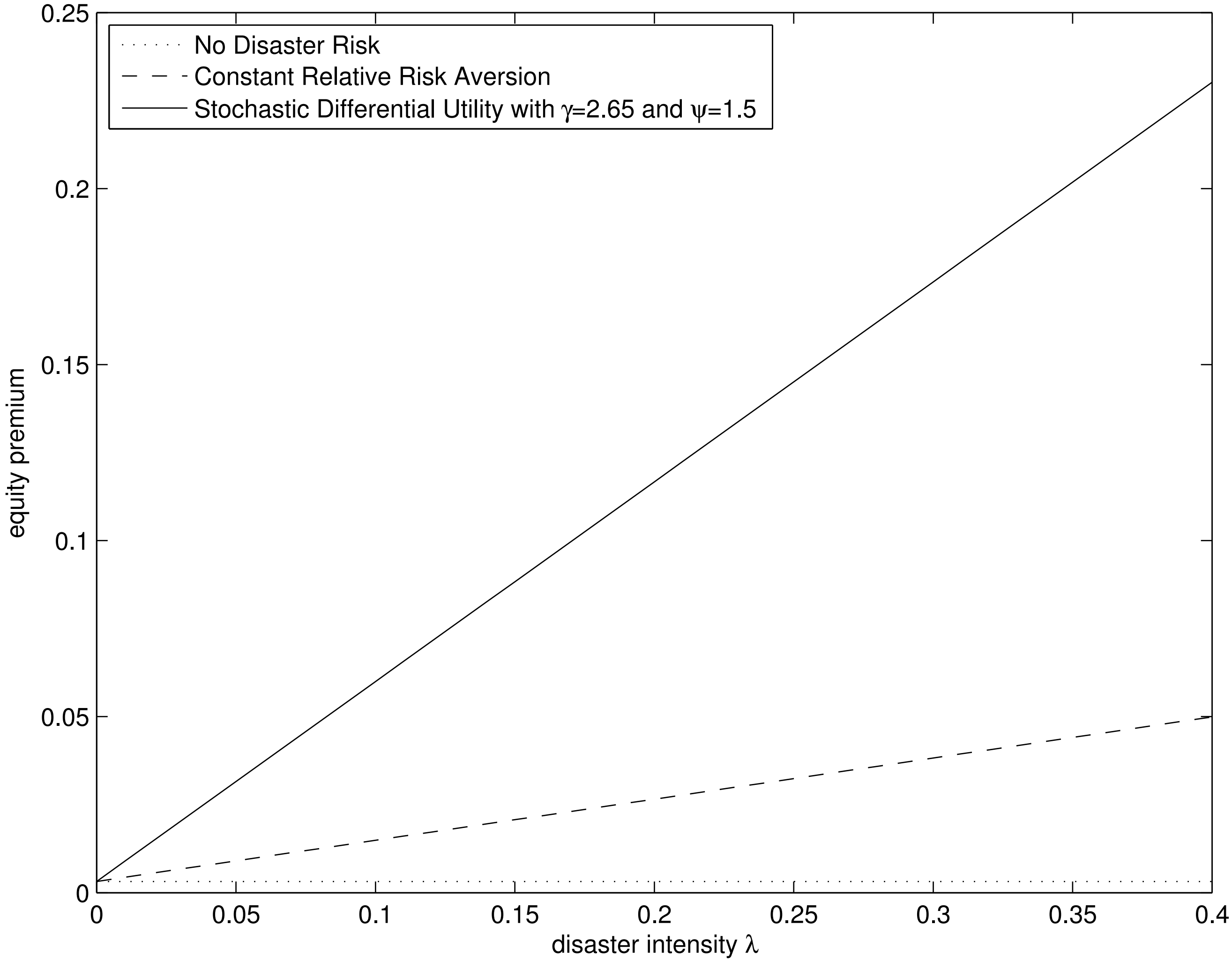


Figure 2: Relationship between short term rates and the disaster intensity



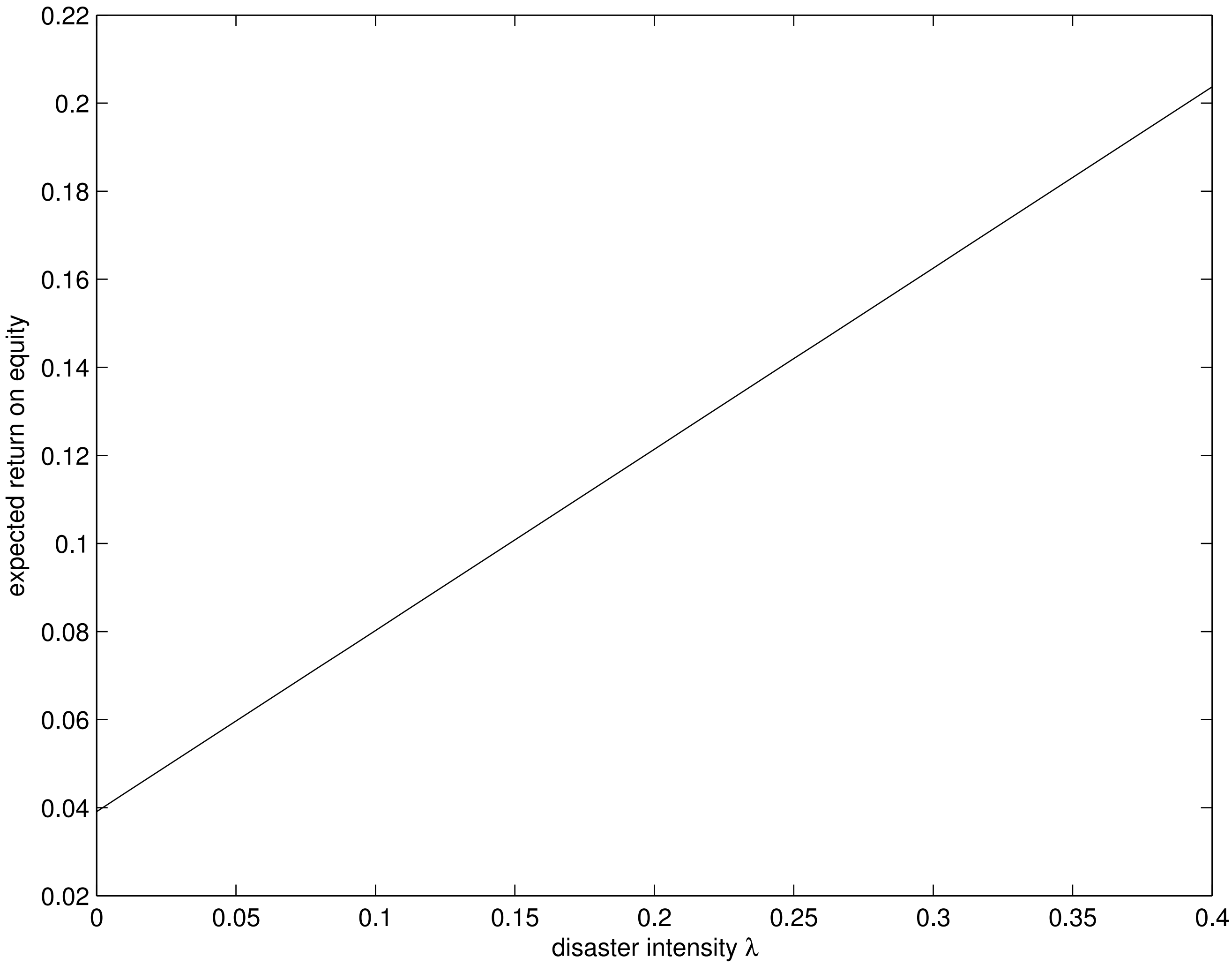
This graph depicts the dependence between the risk-free rate r_f, the promised yield on government debt r_L, and the expected return on government bills r_B on the intensity λ implied by a calibration of the self-exciting disaster risk model.

Figure 3: Relationship between the equity premium and the disaster intensity



This figure graphs the relationship of equity premium with respect to the disaster intensity. The dotted line represents the case without disaster risk. The dashed line illustrates the equity premium with expected utility. In this case investors require no compensation for the increase in disaster intensity in response to the arrival of a disaster. With stochastic differential utility with agents exhibiting a preference for early resolution of uncertainty the equity premium is given by the solid line.

Figure 4: Relationship between the expected return on equity and the disaster intensity



This figure graphs the relationship of the expected return on equity with respect to the disaster intensity.

Figure 5: Probability density of disaster intensity

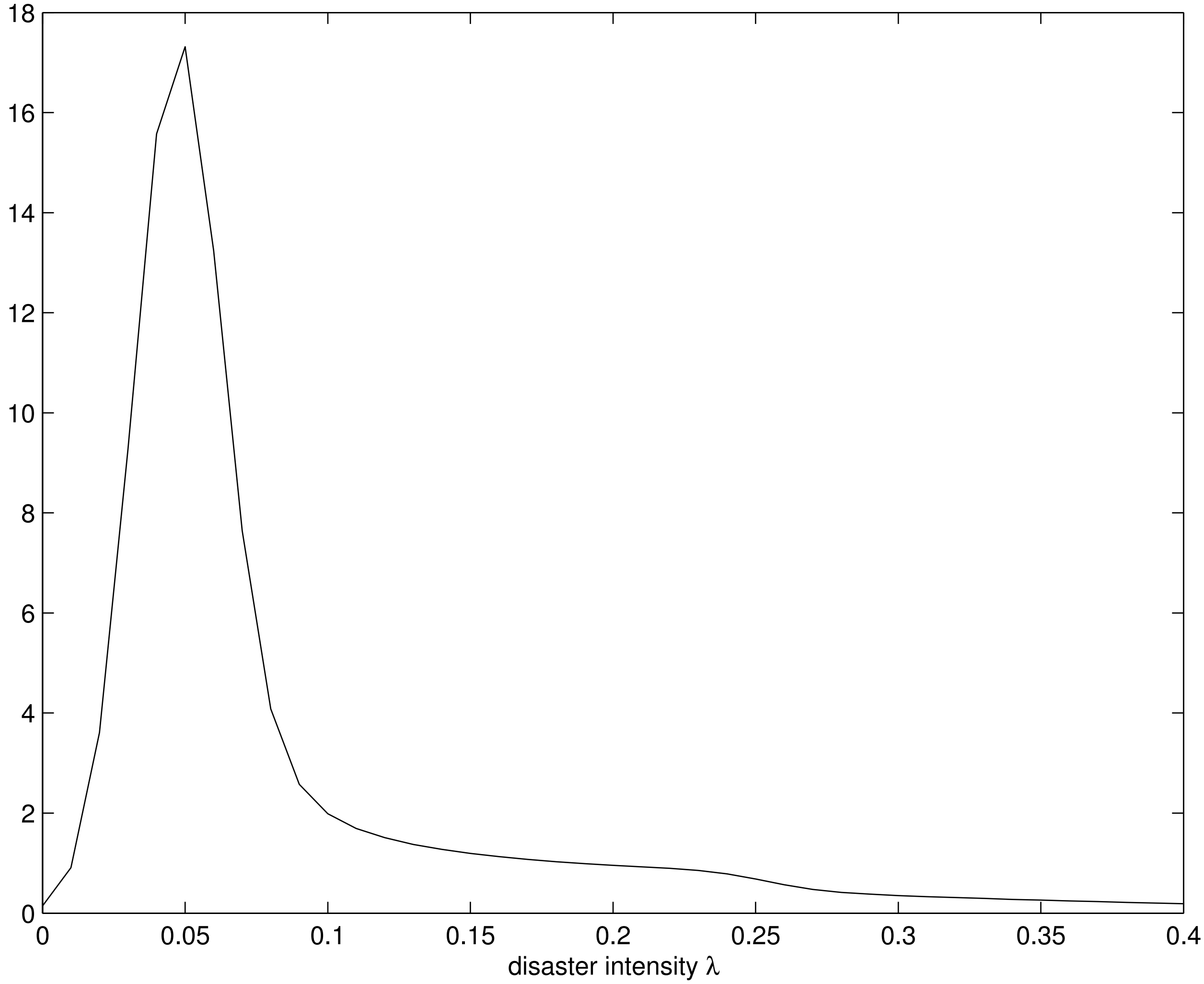


Figure 6: Implied volatility of equity index put options with 1 year until expiration

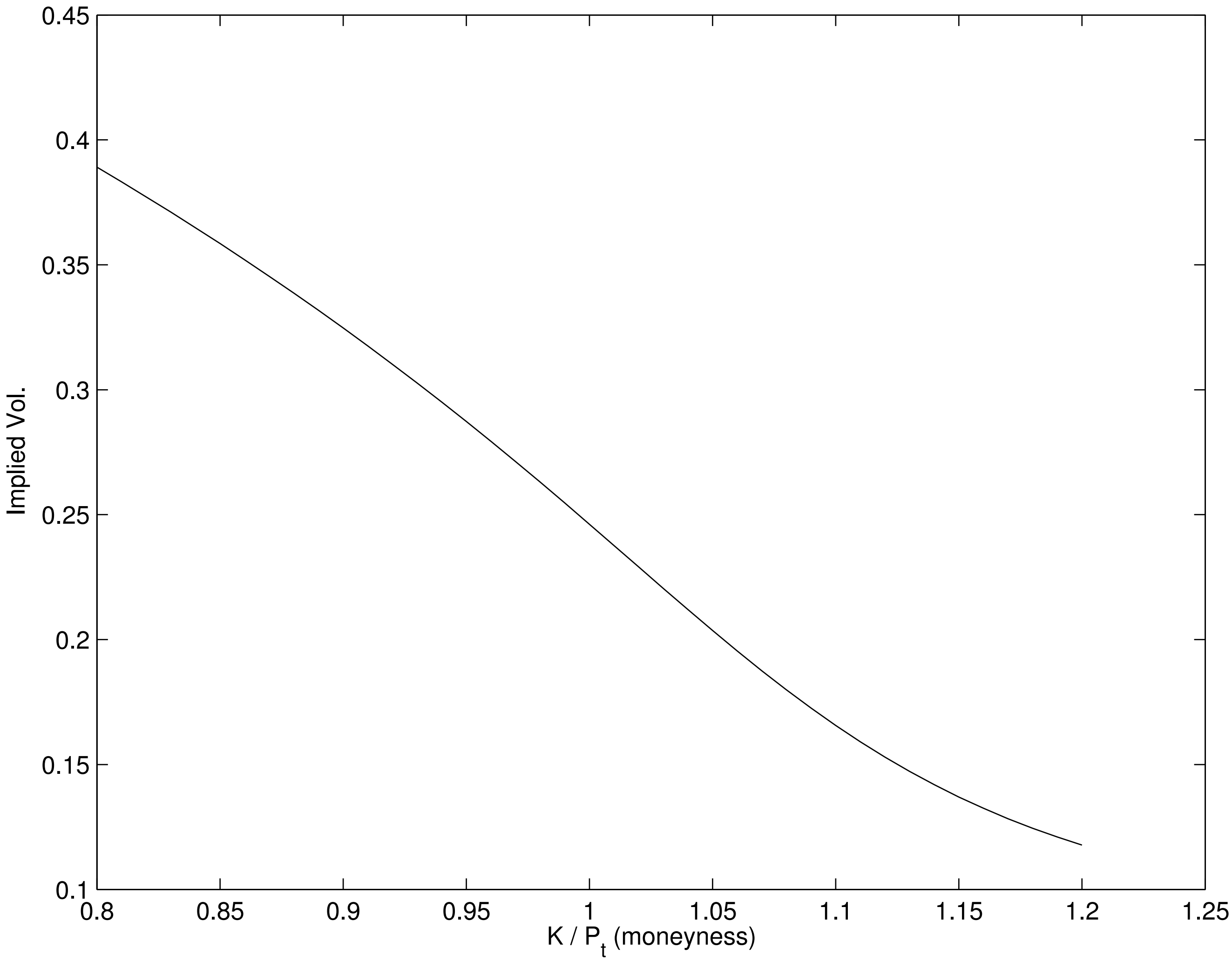


Figure 7: Implied volatility of equity index put options with 1 month until expiration

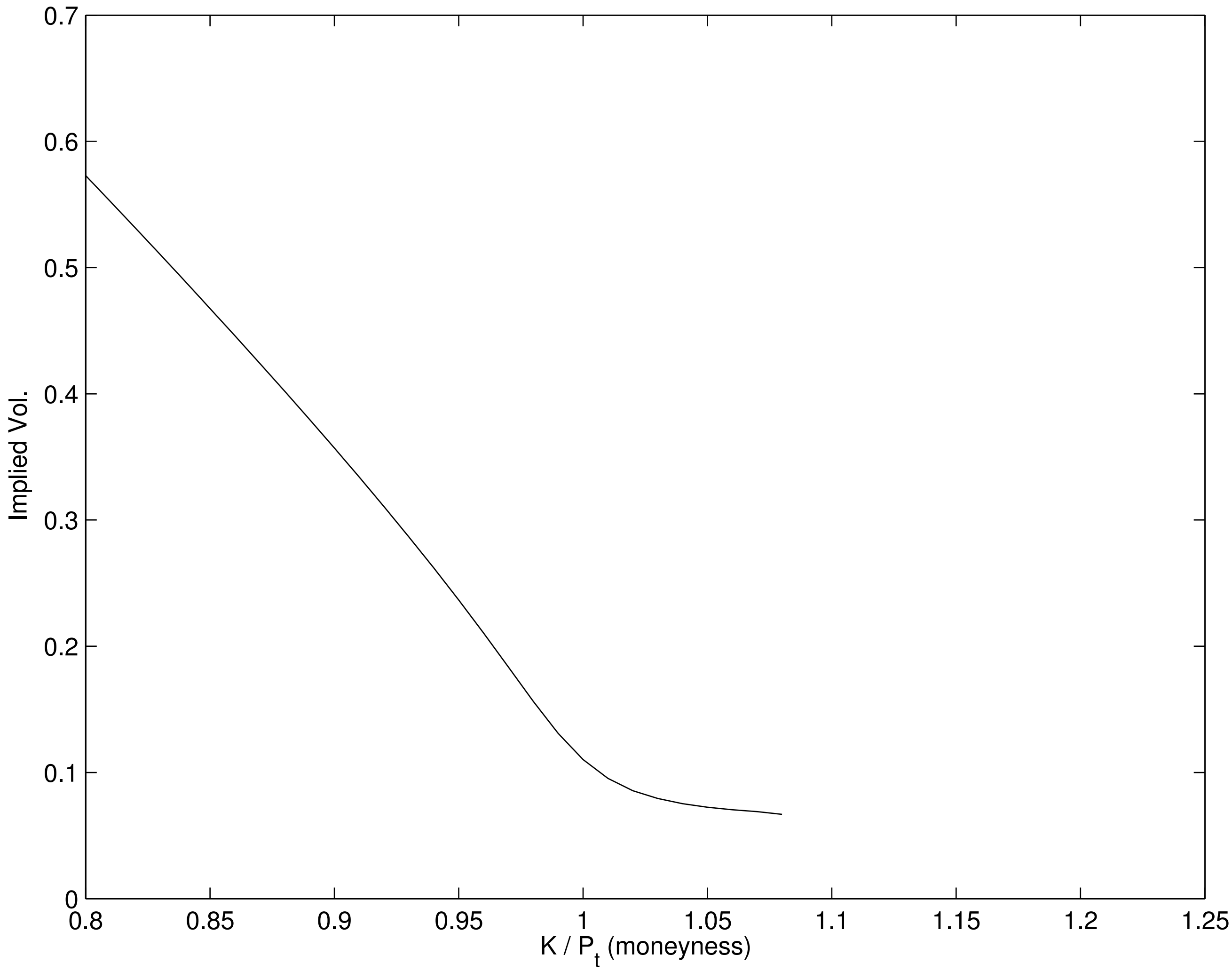
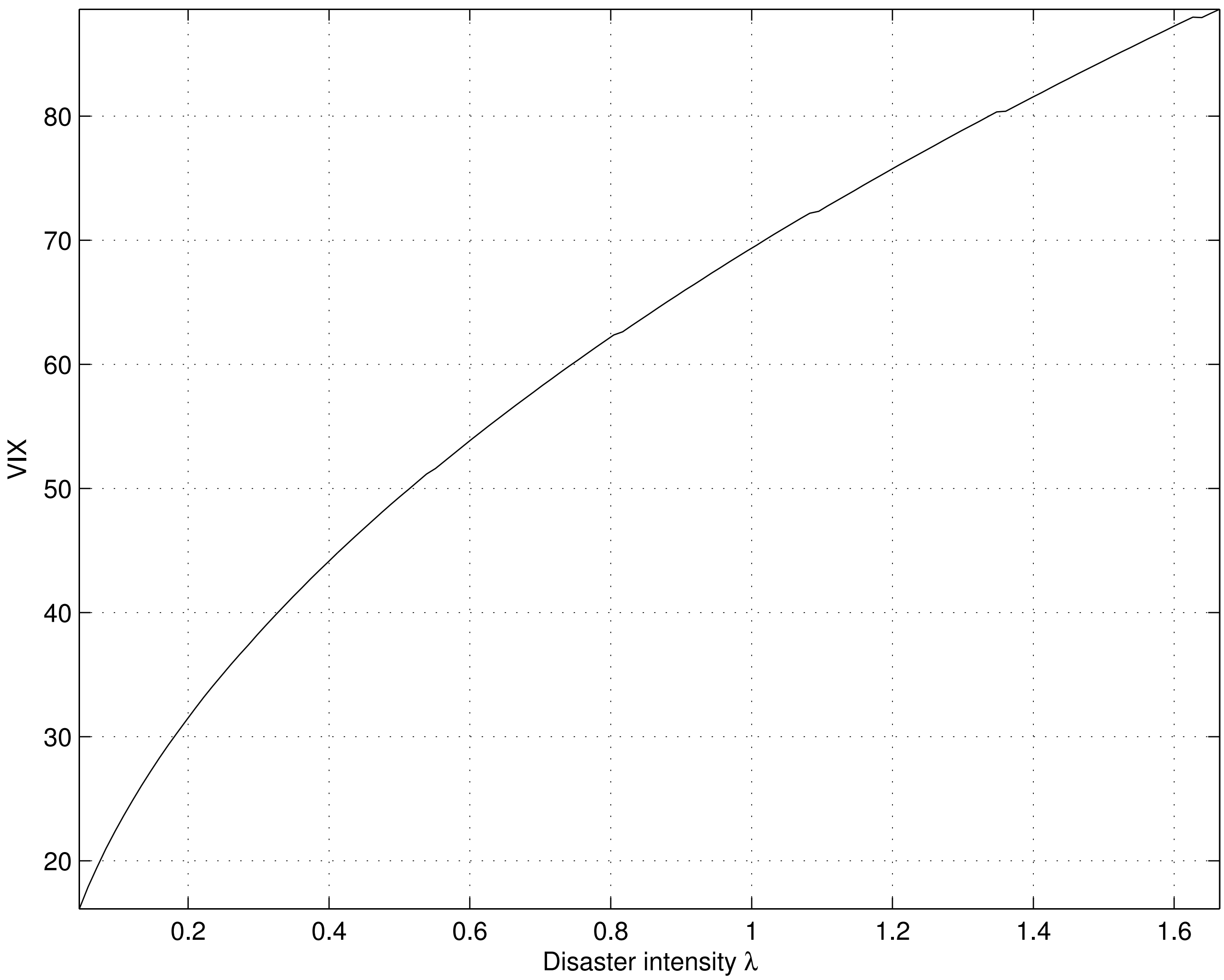


Figure 8: Model-implied VIX computed from 30 days OTM call and put option prices as a function of the disaster intensity



This graph depicts the relationship between the conditional disaster intensity and the model-implied VIX computed from 30 days OTM call and put prices.

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