Center for Research in Econometric Analysis of Time Series

CREATES Research Paper 2007-15

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Jumps and Betas: A New Framework for Disentangling and Estimating Systematic Risks*

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July 25, 2007

Abstract

We provide a new theoretical framework for disentangling and estimating sensitivity towards systematic diffusive and jump risks in the context of factor pricing models. Our estimates of the sensitivities towards systematic risks, or betas, are based on the notion of increasingly finer sampled returns over fixed time intervals. In addition to establishing consistency of our estimators, we also derive Central Limit Theorems characterizing their asymptotic distributions. In an empirical application of the new procedures using high-frequency data for forty individual stocks and an aggregate market portfolio, we find the estimated diffusive and jump betas with respect to the market to be quite different for many of the stocks. Our findings have direct and important implications for empirical asset pricing finance and practical portfolio and risk management decisions.

Keywords: Factor models, systematic risk, common jumps, high-frequency data, realized variation.

JEL classification: C13, C14, G10, G12.

*Bollerslev's work was supported by a grant from the NSF to the NBER and CREATES funded by the Danish National Research Foundation. We thank George Tauchen for many discussions on closely related ideas. We would also like to thank Tzuo Hann Law for help with extracting and cleaning the high-frequency data used in the empirical analysis.

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1 Introduction

Linear discrete-time factor models permeate academic asset pricing finance and also form the basis for a wide range of practical portfolio and risk management decisions. Importantly, within this modeling framework equilibrium considerations imply that only non-diversifiable risk, as measured by the factor loading(s) or the sensitivity to the systematic risk factor(s), should be priced, or carry a risk premium. Conversely, so-called neutral strategies that immunize the impact of the systematic risk factor(s) should earn the risk free rate.

Specifically, consider the one-factor representation,

$$r_i \;=\; \alpha_i \;+\; \beta_i r_0 \;+\; \epsilon_i, \qquad i = 1, ..., N, \tag{1}$$

where r_i and r_0 denote the returns on the i^{th} asset and the systematic risk factor, respectively, and the idiosyncratic risk, ϵ_i, is assumed to be uncorrelated with r_0. Then, provided sufficiently weak cross-asset dependencies in the idiosyncratic risks (see Ross (1976) and Chamberlain and Rothschild (1983)), the absence of arbitrage implies that $\mathbb{E}(r_i) \;=\; r_f \;+\; \lambda_0 \beta_i$, so that the differences in expected returns across assets are solely determined by the cross-sectional variation in the betas. This generic one-factor setup obviously encompasses the popular market model and CAPM implications in which the betas are proportional to the covariation of the assets with respect to the aggregate market portfolio. However, the use of other benchmark portfolios in place of r_0, or more general multi-factor representations, attach the same key import to the corresponding betas.

The beta(s) of an asset is(are), of course, not directly observable. The traditional way of circumventing this problem and estimating betas rely on rolling linear regression, typically based on five years of monthly data, see, e.g., the classical studies by Fama and MacBeth (1973) and Fama and French (1992).[1] Meanwhile, the recent advent of readily-available high-frequency financial prices have spurred a renewed interest into alternative ways for more accurately estimating betas. In particular, Andersen et al. (2005), Andersen et al. (2006), Bollerslev and Zhang (2003), Barndorff-Nielsen and Shephard (2004a) and Hooper et al. (2006) among others, have all explored new procedures for measuring and forecasting period-by-period betas based on so-called realized variation measures constructed from the summation of squares and cross-products of higher frequency within period returns. These studies generally confirm that the use of high-frequency data results in statistically far superior beta estimates relative to the traditional regression based procedures.

Meanwhile, another strand of the burgeon recent empirical literature concerned with the analysis of high-frequency intraday financial data have argued that it is important to allow for the possibility of price dis-continuities, or jumps, in satisfactorily describing financial asset prices; see, e.g., Andersen et al. (2007) Barndorff-Nielsen and Shephard (2006), Huang and Tauchen (2005) and Lee and Mykland (2007). This is further corroborated by the mounting empirical evidence from options markets that the variation in returns associated with sharp price dis-continuities seem to carry a separate risk premium from the one associated with continuous price moves; see, e.g., Bates (2000), Eraker (2004), Pan (2002) and Todorov (2007). Similarly, the results in Wright and Zhou (2007) suggest that bond prices contain a separate premium for jump risk.

[1] For additional references on the estimation of time-varying betas based on more sophisticated data-driven filters and explicit parametric models see, e.g., Ghysels and Jacquier (2006).

Combining these recent ideas and empirical results naturally suggests decomposing the return on the benchmark portfolio(s) within the linear factor model framework into the returns associated with continuous and dis-continuous price moves (r_0^c and r_0^d respectively). In particular, decomposing the return on the benchmark portfolio in the one-factor model in equation (1), the model is naturally extended as,

$$r_i \;=\; \alpha_i \;+\; \beta_i^c r_0^c \;+\; \beta_i^d r_0^d \;+\; \epsilon_i, \qquad i = 1, ..., N, \tag{2}$$

where by definition $r_0 = r_0^c + r_0^d$, and the two separate betas represent the systematic risks attributable to each of the two return components. Of course, for $\beta_i^c = \beta_i^d$ the model trivially reduces to the standard one-factor model in equation (1). However, there is no apriori theoretical reason to restrict, let alone expect, the two betas to be the same. Indeed, the classical paper by Merton (1976) hypothesized that in the context of the market model, jump risks for individual stocks are likely to be non-systematic, so that effectively $\beta_i^d = 0$. On the other hand, the evidence for larger cross-asset correlations for extreme returns documented in Ang and Chen (2002) among others, indirectly suggests non-zero jump sensitivities. Despite the obvious importance both from a theoretical asset pricing as well as a practical portfolio management perspective, direct empirical assessment of this issue have hitherto been hampered by the lack of formal statistical procedures for actually estimating different types of betas.

The present paper fills this void by developing a general theoretical framework for disentangling and separately estimating sensitivity towards systematic continuous and systematic jump risks. The asymptotic theory underlying our results rely on the notion of increasingly finer sampled returns over a fixed time-interval. Our estimation and inference procedures thus extend the results in Barndorff-Nielsen and Shephard (2004a) on realized covariation measures for continuous sample path diffusions. For simplicity and ease of notation we will focus on the one-factor representation in equation (2). However, the same ideas and estimation procedures extend to more general multi-factor representations. The derivation of the results directly builds on and extends the work of Jacod (2006) on power variation for general semimartingales (containing jumps) and the recent work of Ait-Sahalia and Jacod (2006) and Jacod and Todorov (2007) on testing for jumps in discretely sampled univariate and multivariate processes. Related ideas have also recently been explored by Mancini (2006) and Gobbi and Mancini (2007). We also utilize the procedures of Barndorff-Nielsen and Shephard (2004b) and Barndorff-Nielsen et al. (2005) for measuring the continuous sample path variation in the construction of feasible estimates for the asymptotic variances of the betas.

To illustrate the practical usefulness of the new procedures, we estimate separate continuous and jump betas with respect to an aggregate market portfolio for a sample of forty individual stocks. Consistent with the aforementioned previous studies on high-frequency beta estimates which implicitly restrict the two kind of betas to be the same, we find overwhelming empirical evidence that both kinds of systematic risks in the stocks vary non-trivially from month-to-month. Our findings of systematically positive jump betas for all of the stocks directly contradict the notion that jump risk is diversifiable. More importantly, our results also show that for some of the stocks the two types of betas can be quite different. In particular, the estimated jump betas are often *larger* and generally *less* persistent than their continuous counterparts. Although the calendar time span of high-frequency data available for the empirical analysis is too short to allow for the construction of meaningful tests for whether the separate betas truly reflect differences in priced systematic risks, the differences in the estimates are large enough to conjecture that they could make an important difference in terms of pricing and allow for more informed portfolio and risk management decisions.

The rest of the paper proceeds as follows. Section 2 details our theoretical setup and assumptions, along with the intuition for how to calculate continuous and jump betas in the unrealistic situation when continuous price records are available. Our new procedures for actually estimating separate betas based on discretely sampled high-frequency observations and the corresponding asymptotic distributions allowing for formal statistical inference are presented in Section 3. Our empirical application entailing estimates of the betas for the extended market model for the forty individual stocks is discussed in Section 3. Section 4 concludes. All of the proofs are relegated to a technical Appendix.

2 The Continuous Record Case and Assumptions

Discrete-time models and procedures along the lines of the simple one-factor model in equation (1) are commonly used in finance for describing returns over annual, quarterly, monthly or even daily horizons. Our goal here is to make inference for the separate betas in the extended one-factor model in (2) under minimal assumptions about the processes that govern the returns within the discrete time intervals. To this end, assume that within some fixed interval $[0, T]$ the log-price process p_i is generated by the following general process (defined on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$),

$$\begin{aligned} dp_{it} &= \alpha_{it}dt + \beta_i^c\sigma_{0t}dW_{0t} + \sigma_{it}dW_{it} + \beta_i^d\int_{E^0}\kappa(\delta_0(t,x))\tilde{\mu}_0(dt,d\mathbf{x}) + \beta_i^d\int_{E^0}\kappa'(\delta_0(t,x))\mu_0(dt,d\mathbf{x}) \\ &\quad + \int_{E_i}\kappa(\delta_i(t,x))\tilde{\mu}_i(dt,d\mathbf{x}) + \int_{E_i}\kappa'(\delta_i(t,x))\mu_i(dt,d\mathbf{x}), \qquad i = 1, ..., N, \end{aligned} \tag{3}$$

where $(W_0, W_1, ..., W_N)$ denotes a $(N+1)\times 1$ standard Brownian motion with independent elements; μ_0 is a Poisson random measure on $[0,\infty)\times E_0$ with $(E_0, \mathcal{E}_0)$ an auxiliary measurable space, with the compensator of μ_0 denoted $\nu_0(ds, dz) = ds\otimes\lambda_0(dz)$ for some σ-finite measure λ_0 on $(E_0, \mathcal{E}_0)$; μ_i is a Poisson random measure on $[0,\infty)\times E_i$ with $(E_i, \mathcal{E}_i)$ an auxiliary measurable space, with the compensator of μ_i denoted $\nu_i(ds, dz) = ds\otimes\lambda_i(dz)$ for some σ-finite measure λ_i on $(E_i, \mathcal{E}_i)$; the two measures μ_0 and μ_i are independent from each other; $\kappa(x)$ is a continuous function on $\mathbb{R}$ into itself with compact support such that $\kappa(x) = x$ around 0 and $\kappa'(x) = x - \kappa(x)$.[2]

This very general theoretical framework essentially encompasses all discrete-time one-factor models described by the benchmark representation in equation (1). The systematic diffusive risk is captured by $\sigma_{0t}dW_{0t}$, explicitly allowing for time-varying stochastic volatility. The systematic jump risk is determined by the Poisson measure μ_0 and the jump size

[2]Note that $\kappa(x) + \kappa'(x) = x$, and therefore we integrate the big jumps with respect to the jump measure and the small jumps with respect to the compensated measure. This is so because, the big jumps are almost surely finite and thus integration for them can be done in the usual sense, while the small jumps can have a lot of variation, which requires defining them in a stochastic sense (with respect to a martingale measure (see Jacod and Shiryaev (2003))).

function $\delta_0(\cdot,\cdot)$, which allow for both time-varying jump intensities and jump sizes. Consistent with the extended discrete-time model in (2), the continuous-time representation in (3) also explicitly allows for different (but constant over $[0,T]$) sensitivities to the systematic diffusive and jump risks, captured by β_i^c and β_i^d, respectively. Now, suppose that continuous records over the $[0,T]$ time-interval were available for all of the price processes. Is it possible to separately infer the β_i^c and β_i^d, $i = 1, ..., N$ coefficients, without making any additional parametric assumptions about the underlying process? The answer to this question is a qualified 'yes'.

In particular, it follows by standard arguments that for $i \neq j$,

$$[p_i^c, p_j^c]_{(0,T]} = \beta_i \beta_j \int_0^T \sigma_{0s}^2 ds \quad \text{and} \quad \sum_{s \leq T} (\Delta p_{is})^2 (\Delta p_{js})^2 = (\beta_i^d \beta_j^d)^2 \int_{E_0} \delta_0(t,x)^4 \mu_0(dt,dx), \quad (4)$$

where $[p_i^c, p_j^c]_{(0,T]}$ is the quadratic covariation between the continuous parts of p_i and p_j over $[0,T]$. Hence, ratios of the separate betas are readily obtained as,

$$\frac{\beta_i^c}{\beta_j^c} = \frac{[p_i^c, p_k^c]_{(0,T]}}{[p_j^c, p_k^c]_{(0,T]}} \quad \text{and} \quad \left| \frac{\beta_i^d}{\beta_j^d} \right| = \sqrt{\frac{\sum_{s \leq T} (\Delta p_{is})^2 (\Delta p_{ks})^2}{\sum_{s \leq T} (\Delta p_{js})^2 (\Delta p_{ks})^2}} \quad , \quad (5)$$

for some $k = 1, ..., N$, such that $k \neq i$, and $k \neq j$[3]. Note, since the square destroys the sign, the second ratio only gives the absolute value of the relative sensitivity to the systematic jump risk, and not the ratio itself. This is unlikely of little practical concern, as the sign of β_i^d will generally be obvious from the context. Note also that the ratios in equation (5) provide an assessment of the relative magnitude of the sensitivity towards systematic risks for the different assets without the use of any observations on the systematic risk factor itself.

Meanwhile, most practical uses of factor models in finance, and one-factor models in particular, associate the source of the systematic risks with specific assets, or benchmark portfolios. Specifically, suppose that observations are available on some reference asset 0 that is only exposed to the systematic risks, i.e.,

$$dp_{0t} = \alpha_{0t} dt + \sigma_{0t} dW_{0t} + \int_{E^0} \kappa(\delta_0(t,x)) \tilde{\mu}_0(dt, d\mathbf{x}) + \int_{E^0} \kappa'(\delta_0(t,x)) \mu_0(dt, d\mathbf{x}). \quad (6)$$

Standardizing by this benchmark asset in equation (5), it follows that for $i = 1, ..., N$,

$$\beta_i^c = \frac{[p_i^c, p_0^c]_{(0,T]}}{[p_0^c, p_0^c]_{(0,T]}} \quad \text{and} \quad |\beta_i^d| = \sqrt{\frac{\sum_{s \leq T} (\Delta p_{is})^2 (\Delta p_{0s})^2}{\sum_{s \leq T} (\Delta p_{0s})^4}}, \quad (7)$$

so that the actual values of the betas (again subject to sign for the jump betas), and not just their ratios, may be uncovered from the continuous price records. Even if the one-factor structure in equation (3) does not hold, the β_i^c and β_i^d in (7) still provide meaningful

[3] An alternative estimator for the ratio of the jump betas that does not involve a reference asset k may be constructed as

$$\left| \frac{\beta_i^d}{\beta_j^d} \right| = \left(\frac{\sum_{s \leq T} (\Delta p_{is})^{2+\alpha} (\Delta p_{js})^2}{\sum_{s \leq T} (\Delta p_{is})^2 (\Delta p_{js})^{2+\alpha}} \right)^{1/\alpha}, \quad \alpha > 0.$$

measures of the (average over $[0,T]$) sensitivity of asset i to the diffusive and jump moves in the reference asset 0.

The expressions for the betas given above form the basis for all of our estimators and inference procedures discussed below. However, for the results reported on below, we need the following, mostly, technical conditions on the underlying process in (3).

Assumption A1

(a) *The functions α_{it}, σ_{0t}, σ_{it}, $\delta_0(\omega,t,x)$ and $\delta_i(\omega,t,x)$ are left-continuous and have right limits with respect to t for all $i = 1,...,N$.*

(b) *$|\delta_0(\omega,t,x)| \leq \gamma_k(x)$ for $t \leq T_k(\omega)$, where $\gamma_k(x)$ is a deterministic function such that $\int_{E_0} |\gamma_k(x)|^2 \wedge 1\lambda_0(dx) < \infty$, and T_k is a sequence of stopping times increasing to $+\infty$. A similar condition holds for $\delta_i(\omega,t,x)$ for all $i = 1,...,N$.*

(c) *$\int_0^t |\sigma_{0s}|ds > 0$ a.s. for every $t > 0$.*

(d) *σ_0 and σ_i for $i = 1,...,N$, are Itô semimartingales, with coefficients satisfying the conditions in (a) and (b).*

Assumption A1 is identical to the assumptions made in Jacod (2006). It is a rather weak set of assumptions, and with the possible exception of part (c) which rules out pure-jump specifications, virtually all parametric models employed in finance satisfy these conditions.

In addition to the minimal Assumption A1, for some of our results we will need an additional slightly stronger assumption.

Assumption A2: *$|\delta_0(\omega,t,x)| \leq \gamma_k(x)$ for $t \leq T_k(\omega)$, where $\gamma_k(x)$ is deterministic function such that $\int_{E_0} |\gamma_k(x)|^s \wedge 1\lambda_0(dx) < \infty$ for some $s \in [0,2]$, and T_k denotes a sequence of stopping times increasing to $+\infty$. A similar condition holds for $\delta_i(\omega,t,x)$ for all $i = 1,...,N$.*

Assumption A2 is again adapted from the general setup in Jacod (2006). It strengthens part (b) of Assumption A1 (when s in A2 is strictly less than 2), in restricting the activity of the jumps in the prices. In the case of time-homogeneous jumps it amounts to requiring the so-called Blumenthal-Getoor index (Blumenthal and Getoor (1961)) of the jumps to be no larger than s. Intuitively, if the small jumps are too frequent, they become statistically indistinguishable from the diffusive part of the process.

3 Estimation of Systematic Diffusive and Jump Risks

The discussion and formula in the preceding section were predication on the notion of continuous price records. In practice, of course, we do not have access to continuously recorded prices over the $[0,T]$ time-interval, but instead we only observe the prices over some discrete time grid, say $i\Delta_n$ for $i = 0, 1, \cdots, [T/\Delta_n]$. Using such discretely observed price data we next discuss how to actually implement the ideas in the previous section in estimating the sensitivity parameters of interest, β_i^c and β_i^d. We also present Central Limit Theorems for the resulting estimators based on the conceptual idea of increasingly finely sampled prices, or $\Delta_n \to 0$. We begin with the estimation of the sensitivity towards systematic jump risk.

3.1 Inference about β_i^d

Our estimator for the sensitivity towards systematic jump risk is constructed by consistently estimating the numerator and denominator in the infeasible ratio in equation (5). In so doing, we build directly on many of the results in Jacod and Todorov (2007). To this end, let $\mathbf{p} = (p_0, p_1, ..., p_N)$, and denote the corresponding vector of discrete price increments,

$$\Delta_i^n \mathbf{p} \;=\; \mathbf{p}_{i\Delta_n} - \mathbf{p}_{(i-1)\Delta_n}. \tag{8}$$

Also, define

$$V(f, \Delta_n)_t \;=\; \sum_{i=1}^{[t/\Delta_n]} f(\Delta_i^n \mathbf{p}), \quad 0 \le t \le T, \tag{9}$$

where the $\mathbb{R}^{N+1}$ measurable function f is given by,

$$f_{ij}(\mathbf{p}) \;=\; (p_i p_j)^2, \tag{10}$$

for $i, j = 0, 1, ..., N$. Our estimator for the ratio of the jump betas between assets i and j may then be compactly expressed as

$$\widehat{\left(\frac{\beta_i^d}{\beta_j^d}\right)} = \sqrt{\frac{V(f_{ki}, \Delta_n)_T}{V(f_{kj}, \Delta_n)_T}}, \tag{11}$$

for some $k = 0, 1, ..., N$. To avoid trivial (and uninteresting) cases we further restrict $i \neq j$, $i \neq k$, and $j = k$ if and only if $j = k = 0$. Note that for $k = j = 0$, the ratio provides a direct estimate of $|\beta_i^d|$.

The feasible estimator in equation (11) directly mirrors the expression in (5) based on continuously recorded prices. Since the price increments only enters in powers of two, the contribution from the continuous part of prices in $f_{ij}(\Delta_i^n \mathbf{p})$ will be negligible (asymptotically). Intuitively, higher powers (higher than two) serve to compress the contribution from the continuous price moves, while at the same time inflating the contribution coming from jumps, in effect making the jumps "visible".[4]

In order to characterize the distribution of the estimator, we will consider an auxiliary space $(\Omega', \mathcal{F}', \mathbb{P}')$, which is an extension of the original one and supports two sequences (U_q) and (U_q') of $N+1$-dimensional standard normals, as well as the sequence (κ_q) of uniform random variables on $[0, 1]$, all of which are mutually independent. We further denote by $(S_q)_{q\ge1}$ the sequence of stopping times that exhausts the "jumps" in the measures μ_0 and μ_i; i.e., for each ω we have $S_p(\omega) \neq S_q(\omega)$ if $p \neq q$, while $\mu_0(\omega, \{t\} \times E_0) = 1$ or $\mu_i(\omega, \{t\} \times E) = 1$ if and only if $t \equiv S_q(\omega)$ for some q. Finally, following Jacod and Todorov (2007) we define the following subsets of Ω,

$$\Omega_T^{(ij)} = \{\omega : \text{ on } [0, T] \text{ the process } \Delta p_{is} \Delta p_{js} \text{ is not identically } 0\}, \tag{12}$$

[4] For simplicity we will here focus on the function in (10) and the accompanying estimator in (11). However, as discussed in more detail in Jacod and Todorov (2007), the same logic applies for arbitrary twice-continuously differentiable functions on $\mathbb{R}^2$, in which the second partial derivatives go to 0 around the origin.

for $i, j = 1, ...N$ and $i \neq j$. The set $\Omega_T^{(ij)}$ represents the events for which there is at least one common jump in p_i and p_j over the $[0,T]$ time-interval. Because of the assumed one-factor structure, these sets are equivalent to the set $\Omega_T^{(0)}$ with at least one systematic jump on $[0,T]$. Note that even if the model allows for systematic jump risk in the assets, it still might be the case that the observed realization of the prices is not in the set $\Omega_T^{(0)}$. This can happen with a positive probability for example if the systematic jumps are compound Poisson. The following theorem provides the distribution of our estimator on all non-empty sets, $\Omega_T^{(0)}$.

Theorem 1 *Assume that p_i and p_0 are governed by equations (3) and (6), respectively, and that $\beta_i \neq 0$. Further assume that Assumption A1 holds. Then for $\Delta_n \to 0$:*[5]

(a)

$$\widehat{\left(\frac{\beta_i^d}{\beta_j^d}\right)} \xrightarrow{\mathbb{P}} \left|\frac{\beta_i^d}{\beta_j^d}\right| \qquad on\ \ \Omega_T^{(0)}, \tag{13}$$

(b)

$$\frac{1}{\sqrt{\Delta_n}}\left(\widehat{\left(\frac{\beta_i^d}{\beta_j^d}\right)} - \left|\frac{\beta_i^d}{\beta_j^d}\right|\right) \xrightarrow{\mathcal{L}-(s)} L_T^d \qquad on\ \ \Omega_T^{(0)}, \tag{14}$$

$$L_T^d = \left|\frac{\beta_i^d}{\beta_j^d}\right| \frac{\sum_{q:S_q\leq T}\left((\Delta p_{0S_q})^3\left[\sqrt{\kappa_q}\overline{R}_q^1 + \sqrt{1-\kappa_q}\overline{R}_q^2\right]\right)}{\sum_{s\leq T}(\Delta p_{0s})^4}, \tag{15}$$

$$\overline{R}_q^1 = \frac{1}{\beta_i^d}\sigma_{iS_q-}U_q^i - 1_{\{j\neq 0\}}\frac{1}{\beta_j^d}\sigma_{jS_q-}U_q^j \quad and \quad \overline{R}_q^2 = \frac{1}{\beta_i^d}\sigma_{iS_q}U_q^{'i} - 1_{\{j\neq 0\}}\frac{1}{\beta_j^d}\sigma_{jS_q}U_q^{'j}. \tag{16}$$

Conditional on $\mathcal{F}_T$, L_T^d has mean 0 and variance

$$V_T = \frac{\sum_{q:S_q\leq T}\left[(\Delta p_{0S_q})^6\left(\left(\sigma_{iS_q-}^2 + \sigma_{iS_q}^2\right) + 1_{\{j\neq 0\}}\left(\frac{\beta_i^d}{\beta_j^d}\right)^2\left(\sigma_{jS_q-}^2 + \sigma_{jS_q}^2\right)\right)\right]}{2(\beta_j^d)^2(\sum_{s\leq T}(\Delta p_{0s})^4)^2}. \tag{17}$$

If in addition $\Delta p_{0S_q}\Delta\sigma_{iS_q} \equiv 0$ for all $S_q \leq T$, then conditional on $\mathcal{F}_T$, L_T^d is normal.

(c) The results in parts (a) and (b) continue to hold for

$$\widetilde{\left(\frac{\beta_i^d}{\beta_j^d}\right)} = \sqrt{\frac{\sum_{i=1}^{[T/\Delta_n]}(\Delta_i^n p_i)^2(\Delta_i^n p_k)^2 1_{\{|\Delta_i^n p_i|>\alpha\Delta_n^{\varpi}\ \cup\ |\Delta_i^n p_k|>\alpha\Delta_n^{\varpi}\}}}{\sum_{i=1}^{[T/\Delta_n]}(\Delta_i^n p_j)^2(\Delta_i^n p_k)^2 1_{\{|\Delta_i^n p_j|>\alpha\Delta_n^{\varpi}\ \cup\ |\Delta_i^n p_k|>\alpha\Delta_n^{\varpi}\}}}}, \tag{18}$$

for arbitrary values of $\alpha > 0$ and $\varpi > \frac{1}{3}$.

[5] The notation $\xrightarrow{\mathcal{L}-(s)}$ means convergence stable in law. This convergence is stronger than the usual convergence in law. Its importance for us comes from the fact that it implies joint convergence of the converging sequence with any random variable defined on the original probability space (see Jacod and Shiryaev (2003) for further details).

(d) *If in addition Assumption A2 holds for some $s < 2$, then the variance V_T for $j = k = 0$ may be consistently estimated by*

$$\widehat{V}_T = \frac{\sum_{i=k_n+2}^{[T/\Delta_n - k_n - 1]} (\Delta_i^n p_0)^6 \left(\widehat{c}(n,-)_j + \widehat{c}(n,+)_j\right) 1_{\{|\Delta_i^n p_0| \geq \alpha \Delta_n^{\varpi}\}}}{2\left(\sum_{i=1}^{[T/\Delta_n]} (\Delta_i^n p_0)^4 1_{\{|\Delta_i^n p_0| \geq \alpha \Delta_n^{\varpi}\}}\right)^2}, \tag{19}$$

$$\widehat{c}(n,\pm)_i = \frac{1}{k_n \Delta_n} \frac{\pi}{2} \sum_{j \in I_{n,\pm}(i)} |\Delta_j^n \widehat{p}_{i0}^{cn}||\Delta_{j-1}^n \widehat{p}_{i0}^{cn}|, \quad \widehat{p}_{i0}^{cn} := p_i - \widehat{\beta}_i^c p_j,$$

where $I_{n,-}(i) = \{i - k_n, i - k_n + 1, \cdots, i-1\}$ for $i > k_n + 1$, $I_{n,+}(i) = \{i+2, \cdots, i+k_n\}$ for $i < [T/\Delta_n] - k_n$; $\alpha > 0$, $\varpi \in (0, \frac{1}{2})$; $k_n \to \infty$ and $k_n \Delta_n \to 0$; and $\widehat{\beta}_i^c$ denotes some consistent estimator for β_i^c.

Proof: See Appendix.

Part (a) of the theorem shows that the proposed estimator does indeed converge to the (absolute value of the) ratio of the sensitivities toward systematic jump risk. Importantly, this convergence is restricted to the set $\Omega_T^{(0)}$.[6] This is, of course, quite natural as it isn't possible to infer any quantities/parameters related to co-jumping when there is no common arrival of jumps in the asset prices. As such, the estimator in equation (11) should only be used in situations when there were actually systematic jumps present.

We note that the convergence in probability and the Central Limit Theorem stated in part (a) and part (b) of the theorem hold under very general conditions and in particular no restriction on the jump activity: finite or infinite activity, finite or infinite variation jumps are all allowed. Several observations regarding the asymptotic limit in (15) are in order. First, the larger the systematic jumps, the lower the asymptotic variance and the more accurate the estimates for the sensitivities to systematic jump risk. Intuitively, smaller common jumps are generally harder to separate from continuous co-movements, and in turn result in less precise estimates of β_i^d. Second, the longer the $[0, T]$ time-interval, the more realizations of systematic jumps on average, and hence the more accurate the estimates. Of course, this assumes that the same one-factor structure with identical jump sensitivities in (3) hold true over the entire time-interval. We will return to this issue in the empirical section below. Third, the less the idiosyncratic risks, the more precise the estimates. In particular, if observations on the common (systematic) factor p_0 are available, the use of these will result in the most precise estimates.

[6]For the events in $(\Omega_T^{(0)})^c$ corresponding to only idiosyncratic jumps in i, j or k, the limiting value of the estimator in (11) is a random quantity conditional on the observed prices. When neither systematic nor idiosyncratic jumps are present on $[0, T]$, the limit equals

$$\sqrt{\frac{3(\beta_i^c \beta_k^c)^2 \int_0^T \sigma_{0s}^4 ds + 1_{\{k \neq 0\}} (\beta_i^c)^2 \int_0^T \sigma_{ks}^2 \sigma_{0s}^2 ds + (\beta_k^c)^2 \int_0^T \sigma_{is}^2 \sigma_{0s}^2 ds + 1_{\{k \neq 0\}} \int_0^T \sigma_{is}^2 \sigma_{ks}^2 ds}{3(\beta_j^c \beta_k^c)^2 \int_0^T \sigma_{0s}^4 ds + 1_{\{k \neq 0\}} (\beta_j^c)^2 \int_0^T \sigma_{ks}^2 \sigma_{0s}^2 ds + 1_{\{j \neq 0\}} (\beta_k^c)^2 \int_0^T \sigma_{js}^2 \sigma_{0s}^2 ds + 1_{\{j \neq 0, k \neq 0\}} \int_0^T \sigma_{js}^2 \sigma_{ks}^2 ds}},$$

which for $j = k = 0$ is strictly greater than the sensitivity towards the (absolute of the) sensitivity towards diffusive systematic risk.

As noted in part (b) of the theorem, the absence of any common jumps between the price levels and the stochastic volatility for the continuous price process implies that the distribution of L_T^d will be normal. In the empirical results reported on below we simply proceed under this maintained assumption. The results reported in Jacod and Todorov (2007) suggest that even if this assumption is violated, the use of the right approximating limit for L_T^d, obtained by substituting the jumps in L_T^d with the price increments and the stochastic volatilities with the square root of the $\widehat{c}$'s, would not give rise to materially different distributions and test statistics.

Part (c) of the theorem formally shows that the asymptotic results in parts (a) and (b) remain true if we drop the terms in $V(f_{ki}, \Delta_n)_T$ for which both price increments are smaller than some pre-specified threshold level. Intuitively, these terms will capture continuous moves and their impact will therefore be negligible asymptotically. In finite samples, however, it might be desirable to use the truncated estimator in equation (18). Of course, for very high values of ϖ the two estimators will be numerically the same. We will discuss reasonable choices for α and ϖ in the empirical section below.

The final part (d) of the theorem provides a consistent estimator for V_T in the case of $j = k = 0$. This is the estimator that we will actually rely on in the empirical section.[7] In addition to the previous Assumption 1, the $\widehat{V}_T$ estimator requires that Assumption 2 holds for some $s < 2$. This is a very weak regularity type assumption. Jumps for which $s = 2$ are extremely active and for practical purposes impossible to separate from the continuous price movements. Otherwise the estimator for V_T is essentially based on a portfolio consisting of assets p_i and p_0, which eliminates the systematic diffusive risk, along with an estimate of the local stochastic variance of the continuous part of this portfolio, $\widehat{c}(n, \pm)_i$. The truncation employed in the estimator is asymptotically immaterial. Just like the truncated estimator itself defined in part (c), the price increments only enter the variance estimator in powers higher than two so that the contribution from the continuous part is asymptotically negligible.

We next turn to a discussion of our estimates for the sensitivities to systematic diffusive risks.

3.2 Inference about β_i^c

Analogous to the estimator for the sensitivity towards jump risk discussed above, our estimator for the sensitivity towards continuous systematic risk is based on the first infeasible ratio in equation (7), replacing the numerator and denominator by feasible estimates. To this end, we need some additional notation. In particular, let X denote a generic N-dimensional semimartingale. The following multidimensional realized truncated variation

$$V_n''(X, \alpha, \varpi)_t = \begin{pmatrix} \sum_{i=1}^{[t/\Delta_n]} (\Delta_i^n X^1)^2 1_{\{||X|| \le \alpha \Delta_n^{\varpi}\}} \\ \vdots \\ \sum_{i=1}^{[t/\Delta_n]} (\Delta_i^n X^d)^2 1_{\{||X|| \le \alpha \Delta_n^{\varpi}\}} \end{pmatrix}, \tag{20}$$

[7] It is possible to construct an estimator for V_T in the general case of $k \neq 0$ by estimating the limiting variance in the multivariate CLT stated as part of the proof of the theorem. However, this estimator is considerably more complicated and less intuitive than $\widehat{V}_T$, and since we do not use it in the empirical analysis, we leave out the details.

then represents a natural extension of the univariate truncated realized variation measures analyzed in Mancini (2001, 2006) and Jacod (2006). Also, define the following vector constructed from the original prices,

$$X_k^{ij} = \begin{pmatrix} p_i + p_k \\ p_i - p_k \\ p_j + p_k \\ p_j - p_k \end{pmatrix}, \tag{21}$$

for $i = 1, ..., N$ and $j, k = 0, 1, ..., N$. Our estimator for the ratio of the continuous betas is then defined as,

$$\widehat{\left(\frac{\beta_i^c}{\beta_j^c}\right)} = \frac{V_n''^1(X_k^{ij}, \alpha, \varpi)_T - V_n''^2(X_k^{ij}, \alpha, \varpi)_T}{V_n''^3(X_k^{ij}, \alpha, \varpi)_T - V_n''^4(X_k^{ij}, \alpha, \varpi)_T}. \tag{22}$$

The following theorem characterizes the behavior of the estimator. As in the previous subsection, to avoid uninteresting cases we restrict $i \neq j$, $i \neq k$, and $j = k$ if and only if $j = k = 0$.

Theorem 2 *Assume that p_i and p_0 are governed by equations (3) and (6), respectively. Further assume that Assumption A1 holds, and let $\alpha > 0$ and $\varpi \in (0, \frac{1}{2})$. Then for $\Delta_n \to 0$:*

(a)

$$\widehat{\left(\frac{\beta_i^c}{\beta_j^c}\right)} \xrightarrow{\mathbb{P}} \frac{\beta_i^c}{\beta_j^c}. \tag{23}$$

(b) If in addition Assumption A2 holds for some $s \leq \frac{4\varpi - 1}{2\varpi}$,

$$\frac{1}{\sqrt{\Delta_n}}\left(\widehat{\left(\frac{\beta_i^c}{\beta_j^c}\right)} - \frac{\beta_i^c}{\beta_j^c}\right) \xrightarrow{\mathcal{L}-(s)} L_T^c := K_T \times U, \tag{24}$$

where $U \sim N(0,1)$ is defined on an extension of the original probability space and is independent of the filtration $\mathcal{F}$, and

$$K_T = \frac{\sqrt{\int_0^T \left((\beta_k^c)^2 \sigma_{0u}^2 + 1_{\{k \neq 0\}} \sigma_{ku}^2\right)\left(\sigma_{iu}^2 + 1_{\{j \neq 0\}} \left(\frac{\beta_i^c}{\beta_j^c}\right)^2 \sigma_{ju}^2\right) du}}{\beta_j^c \beta_k^c \int_0^T \sigma_{0u}^2 du}.$$

(c) The variance K_T may be consistently estimated by

$$\widehat{K}_T = \frac{\sqrt{\widehat{K}_T^1}}{\widehat{K}_T^2},$$

where,

$$\widehat{K}_T^1 = \frac{\pi^2}{4\Delta_n} \sum_{i=1}^{[T/\Delta_n]-3} \left| \Delta_i^n p_k \Delta_{i+1}^n \widehat{p}_{ij}^{cn} \Delta_{i+2}^n p_k \Delta_{i+3}^n \widehat{p}_{ij}^{cn} \right|,$$

$$\widehat{K}_T^2 = \frac{\pi}{8} \sum_{i=1}^{[T/\Delta_n]-1} (|\Delta_i^n(p_j + p_k)\Delta_{i+1}^n(p_j + p_k)| - |\Delta_i^n(p_j - p_k)\Delta_{i+1}^n(p_j - p_k)|),$$

and

$$\widehat{p}_{ij}^{cn} := p_i - \widehat{\left(\frac{\beta_i^c}{\beta_j^c}\right)} p_j.$$

Proof: See Appendix.

Part (a) of the theorem shows that the use of the truncated variation measures afford a consistent estimator for the quantity of interest. This consistency holds true for any values of $\alpha > 0$ and $\varpi \in (0, \frac{1}{2})$. Of course, as discussed further in the empirical section below, the actual numerical value of the estimator for a given Δ_n will depend upon the specific choice of these tuning parameters. Assumption A1, part (c) guarantees non-vanishing systematic diffusive risk, so that in contrast to the estimator for the sensitivity towards systematic jump risk in Theorem 2, which only converges on $\Omega_T^{(0)}$, the estimator for the sensitivity to systematic diffusive risk converges on the whole set Ω.

Unlike the CLT for the jump beta in Theorem 1, which holds quite generally, the CLT for the continuous beta in part (b) of Theorem 2 involves a non-trivial restriction related to the activity of the jumps. In practical applications it is natural to choose ϖ to be close to 0.5, so that in the case of time-homogenous jumps the restriction in part (b) essentially excludes jumps of infinite variation. Given the maintained assumption of non-vanishing continuous price components, we do not believe this to be restrictive. Importantly, the limiting distribution of L_T^c is always normal. In parallel to the estimates for the jump beta, the expression for the asymptotic variance of L_T^c indicates that the precision of the continuous beta estimates increase with the use of longer $[0, T]$ time-periods and assets with less idiosyncratic risk.

The consistent estimator for the asymptotic variance of L_T^c in part (c) is based on multipower variation measures. Analogous to the construction in part (c) in Theorem 1, the estimate for $\widehat{K}_T^1$ involves a linear combination of assets i and j that eliminates the systematic diffusive risk, $\widehat{p}_{ij}^{cn}$. The particular ordering of p_k and $\widehat{p}_{ij}^c$ used in defining $\widehat{K}_T^1$ is, of course, arbitrary.[8]

We next turn to a practical empirical illustration of the new estimators and distributional results in Theorems 1 and 2.

4 Empirical Illustration

Our empirical illustration is based on high-frequency transaction prices for forty large capitalization stocks over the January 1, 2001 to December 31, 2005 sample period, for a total of 1,241 active trading days. The data were obtained from the Trade and Quote Database (TAQ). The name and ticker symbols for each of the individual stocks are given in the tables below. The same data has previously been analyzed by Bollerslev et al. (2007) from a very

[8] An alternative, and somewhat more complicated, estimator for K_T could be constructed from appropriately defined truncated power variation measures.

different perspective, and we refer to the discussion therein for further details concerning the methods and filters employed in cleaning the raw price data.

The theoretical results derived in the preceding section is based on the notion of increasingly finer sample prices, or $\Delta_n \to 0$. Meanwhile, a host of practical market microstructure complications, including bid-ask spreads, price discreteness and non-synchronous trading effects, prevent us from sampling too frequently, while maintaining the fundamental semimartingale assumption underlying our results. Ways in which to best deal with the market microstructure "noise" in the implementation of univariate realized variation measures is currently a very active area of research; see, e.g., Ait-Sahalia et al. (2005), Hansen and Lunde (2006) and Barndorff-Nielsen et al. (2007) and the references therein. However, these procedures do not easily generalize to a multivariate context, where the issues are further confound by non-synchronous recording of prices across assets, and little work has yet been done in regards to the practical estimation of multivariate power variation measures in the presence of "noise". Hence, we simply follow most of the literature in the use of a not-too-fine sampling frequency as a way to strike a reasonable balance between the desire for as finely sampled prices as possible on the one hand and the desire not to overwhelm the measures by market microstructure effects on the other. While the magnitude and the impact of the "noise" obviously differs across stocks and across time, the analysis in Bollerslev et al. (2007) suggests that a conservative sampling frequency of 22.5 minutes mitigates the effect of the "noise" for all of the forty stocks in the sample.[9]

The one-factor market model most often employed in practice identifies the systematic risk factor with the return on the aggregate market portfolio. Hence, rather than estimating the relative factor sensitivities across the forty stocks, we treat the market as asset 0 and focus on the sensitivities with respect to that benchmark as defined in equation (7). These direct beta estimates are obviously somewhat easier to interpret than the more generally valid sensitivity ratios.[10] We use the S&P500 index as our measure for the aggregate market, with the corresponding high-frequency returns constructed from the prices for the SPY Exchange Traded Fund (ETF).

Our model-free approach only permits the estimation of discontinuous betas over periods in which there were actually jumps in the reference asset 0, as formally defined by the set $\Omega_T^{(0)}$. We therefore begin our empirical analysis with testing for systematic jumps in the SPY contract. To do so we use the non-parametric test in Barndorff-Nielsen and Shephard (2006) and Huang and Tauchen (2005) based on the difference in the logarithmic daily realized variance and bipower variation measures. Since the SPY is less susceptible to market microstructure "noise" than many of the forty stocks in the sample, we rely on a finer 5-minute sampling frequency in the implementation of the tests. Also, to avoid falsely classifying no-jump days as jump days, we use a fairly conservative critical value of 3.09 for the normally distributed test statistic, corresponding to a 0.2% significance level. The resulting tests indicate that the market jumped on 106 of the $1,241$ days in sample. At the monthly level 50 out of the 60 months in the sample contained at least one significant jump

[9]For simplicity we decided to maintain the identical sampling frequency for all of of the stocks throughout the sample. However,we also experimented with the use of other sampling frequencies, resulting in the same basic findings as the ones reported below.

[10]Importantly, as noted above the distributional results in Theorems 1 and 2 also imply that the explicit use of the right 0 benchmark asset will give rise to most accurate sensitivity estimates.

day, while all of the 20 quarters contained significant jumps. In the following we restrict our calculation of jump betas to only those significant time periods; i.e., 106 days, 50 months, and 20 quarters.

In calculating the betas, we focus on the estimators for β_i^d and β_i^c defined in equations (18) and (22) for $j = k = 0$, respectively. Both estimators involve truncation of the price increments, necessitating a choice of α and ϖ. As previously noted, choosing $\varpi = 0.49 < 0.5$ essentially excludes jumps of infinite variation, which are (perhaps) hard to differentiate from continuous price moves with discretely sample observations. For $\widehat{\beta_i^d}$ we set $\alpha = 2\sqrt{BV_{(0,T)}}$, where $BV_{(0,T)}$ denotes the bi-power variation of the relevant price process. This choice of α explicitly recognizes that σ_i^2 (and/or σ_0^2) is likely changing over time. Intuitively, over short time-periods the continuous part of the price process is approximately normal, so that our choice of α used in estimating the jump betas discard only those price increments which are within two standard deviations of 0, and thus most likely to be associated with continuous price movements. On the other hand, for $\widehat{\beta_i^c}$ we set $\alpha = 3\sqrt{BV_{(0,T)}}$, discarding only those price increments which are more than three standard deviations away from 0, and thus unlikely to be associated with continuous price moves. These two different values of α arguably reflect a conservative choice in classifying (and consequently discarding) a price increment as being either continuous or one that contains jump(s). Of course, asymptotically the values of α and ϖ do not matter.[11]

Turning to the actual empirical results, Figures 1-4 plot the time series of quarterly, monthly and daily continuous and jump beta estimates for two representative firms, IBM and Genentech. The daily beta estimates are obviously somewhat noisy and difficult to interpret. Meanwhile, the estimates for the monthly betas appear much more stable, while still showing interesting and clearly discernable patterns over time. Even though the same longer run dynamic dependencies are visible in the quarterly betas, they afford a much less detailed picture, and some of the more subtle dependencies appear to have been lost at the quarterly horizon. Hence, in the following we will concentrate our discussion and analysis on the monthly beta estimates.[12]

In order to compare more directly the monthly beta estimates, Figure 5 combines the separate betas for each of the two stocks in the same graph. The plot in the top panel shows that the betas for IBM tend to be close. However, the plot for Genentech in the bottom panel reveals some rather marked differences in the estimates. In particular, for the months in which there were systematic jumps $\widehat{\beta_i^d}$ is almost always greater than $\widehat{\beta_i^c}$, and sometimes by a considerable amount. Before starting to speculate on the economic significance and importance of these findings, it is naturally to ask whether these apparent differences in the betas are actually statistically significant.

The asymptotic distributional results in Theorems 1 and 2 afford a direct way of assessing the accuracy of the beta estimates, and in turn allow for the calculation of period-by-period confidence intervals. Looking at the corresponding 95-percent confidence intervals in Figures 6-7, it is clear that the intervals for the monthly Genentech betas often do not have any points in common suggesting that the betas are indeed different, while the intervals for IBM

[11] We also experimented with other values for these tuning parameters, resulting in very similar beta estimates to the ones reported below. Further details concerning these results are available upon request.

[12] This also mirrors the ubiquitous monthly return regressions in the empirical finance literature.

generally involve some overlap making it impossible to statistically tell the two betas apart. Note that the width of the confidence intervals for the jump betas vary much more than the width of the intervals for the continuous betas. As discussed in connection with Theorem 1 above, this is to be expected. Intuitively, it is much easier to estimate the sensitivity to systematic jump risk in months where the market experienced a few large jumps than it is in months involving more moderate sized jumps.

To illustrate the results on a broader basis, we report in Table 1 the average monthly continuous and jump beta estimates for each of the forty stocks in the sample. We also include (in square brackets) the corresponding 95-percent confidence intervals for the averages, constructed from the sum of the asymptotic variances in Theorems 1 and 2. Consistent with the visual impression from the figures, the average betas for IBM are very close, 0.981 versus 0.990, with overlapping confidence intervals, while those for Genentech are very different, 0.992 versus 1.406, with non-overlapping confidence intervals. In fact, looking across all of the forty stocks for only five of the stocks do the confidence intervals for the average betas overlap, indicating that on average most of the stocks do respond differently to continuous and discontinuous market moves. Moreover, with a few exceptions the average jump betas are greater than the continuous betas, suggesting that for the large capitalization stocks analyzed here, larger (jump) market moves tend to be associate with proportionally larger systematic price reactions than smaller more common (continuous) market moves. Also, while Genentech exhibits the largest numerical difference of 0.414, the differences in the two betas for many of the other stocks are clearly non-trivial and economically important.[13]

In addition to allowing for the estimation of separate betas, one of the main attractive features of the high-frequency based estimation approach developed here is the ability to reliably estimate the betas over relatively short time spans, such as a month. Indeed, as noted above in connection with our discussion of the representative time series plots for IBM and Genentech, the monthly beta estimates for both of the stocks do seem to vary in an orderly and reliable fashion from one month to the next.[14] Of course, the averages reported in Table 1 in effect obscures any variation in the betas. Thus, to complement these results and more directly highlight this important feature of our new procedures, we present a series of tests for constancy of the betas. In particular, let $\widehat{\beta}^c_{i,t}$ denote the estimate for β^c_i for month

[13]We also calculated the proportion of the total diffusive and jump variation for each of the stocks due to idiosyncratic variation, as formally defined by

$$R^c_i = \frac{\int_0^T \sigma^2_{is} ds}{\int_0^T \sigma^2_{is} ds + (\beta^c_i)^2 \int_0^T \sigma^2_{0s} ds} \quad \text{and} \quad R^d_i = \frac{\int_0^T \int_{E_i} \delta^2_i(t,x) \mu_i(dt,dx)}{\int_0^T \int_{E_i} \delta^2_i(t,x) \mu_i(dt,dx) + (\beta^d_i)^2 \int_0^T \int_{E_0} \delta^2_0(t,x) \mu_0(dt,dx)}.$$

The average values of the two measures averaged across the forty stocks and sixty (resp. fifty) months in the sample were close and equal to 0.688 and 0.686, respectively. The average measures for the two types of idiosyncratic risks generally also differed very little for each of the individual stocks, with a maximum difference of only 0.071 for Texas Instruments. Further details of these results are available upon request.

[14]This obviously suggests that temporal variations in the betas might be predictable. We will not pursue the issue of modeling and forecasting the betas here, instead referring to Andersen et al. (2006) where reduced-form time series models for simpler realized monthly betas based on standard realized variation measures are presented.

$t = 1, ..., 60$ in the sample. The following three test statistics,

$$T^c_{i,m} = \sum_{t=1}^{30} \frac{\left(\widehat{\beta}^c_{i,2t} - \widehat{\beta}^c_{i,2t-1}\right)^2}{\widehat{\text{Avar}}(\widehat{\beta}^c_{i,2t}) + \widehat{\text{Avar}}(\widehat{\beta}^c_{i,2t-1})} \xrightarrow{\mathcal{L}} \chi^2_{30}, \tag{25}$$

$$T^c_{i,q} = \sum_{t=1}^{7} \frac{\left(\sum_{j=1}^{3}\left(\widehat{\beta}^c_{i,6(t-1)+3+j} - \widehat{\beta}^c_{i,6(t-1)+j}\right)\right)^2}{\sum_{j=1}^{3}\left(\widehat{\text{Avar}}\left(\widehat{\beta}^c_{i,6(t-1)+3+j}\right) + \widehat{\text{Avar}}\left(\widehat{\beta}^c_{i,6(t-1)+j}\right)\right)} \xrightarrow{\mathcal{L}} \chi^2_{10}, \tag{26}$$

$$T^c_{i,y} = \sum_{t=1}^{2} \frac{\left(\sum_{j=1}^{12}\left(\widehat{\beta}^c_{i,24(t-1)+12+j} - \widehat{\beta}^c_{i,24(t-1)+j}\right)\right)^2}{\sum_{j=1}^{12}\left(\widehat{\text{Avar}}\left(\widehat{\beta}^c_{i,24(t-1)+12+j}\right) + \widehat{\text{Avar}}\left(\widehat{\beta}^c_{i,24(t-1)+j}\right)\right)} \xrightarrow{\mathcal{L}} \chi^2_{2}, \tag{27}$$

then provide direct tests for equality of adjacent monthly betas, quarterly-averaged monthly betas, and annual-averaged monthly betas, respectively. Similarly, we define the test statistics $T^d_{i,m}$, $T^d_{i,q}$ and $T^d_{i,y}$ based on the monthly $\widehat{\beta}^d_{i,t}$ estimates, to test for equality of the jump betas over monthly, quarterly, and annual horizons. Since, there were no systematic jumps in 10 of the 60 months in the sample, $T^d_{i,m}$ has a limiting χ^2_{25} distribution under the null of constant monthly jump betas. The limiting distributions of $T^d_{i,q}$ and $T^d_{i,y}$ are χ^2_{10} and χ^2_2, respectively. The actual results of the tests reported in Table 1 strongly reject that the monthly and quarterly betas stayed the same over the sample. This is true for both types of betas. Meanwhile, for a few of the stocks we are not able to reject the hypothesis that the annual averages are constant.

In a sum, the empirical results for the forty stocks reported in the two tables show that not only did the monthly continuous and jump betas differ significantly for most of stocks in the sample, the betas also changed significantly through time over the five-year sample period. As such the results clearly highlight the benefits and insights afforded by our new procedures vis-a-vis the more traditional regressions based procedures for estimating betas restricting the continuous and jump betas to be the same and implicitly treating the monthly betas to be constant over long five-year periods.

5 Conclusion

Discrete-time factor models are used extensively in asset pricing finance. We provide a new theoretical framework for separately identifying and estimating sensitivities towards continuous and discontinuous systematic risks, or betas, within this popular model setup. Our estimates and distributional results are based on the idea of increasingly finer sampled returns over fixed time-intervals. Using high-frequency data for a large cross-section of individual stocks and a benchmark portfolio mimicking the aggregate market, we find that allowing for separate continuous and jump betas can result in materially very different estimates from the ones restricting the two types of betas to be the same. These results raise a number of new interesting questions.

As discussed in the introduction, several recent studies have argued that the risk premia associated with discontinuous, or jump, risks often appear to be quite different from the premia associated with continuous risks. The relatively limited time-span of high-frequency data available for the empirical analysis here invariable limits the scope of such investigations. Nonetheless, it would be interesting to somehow test whether the two types of betas carry separate risk premia.

Along these lines, our findings of different sensitivities to systematic jump risks also has important implications for practical portfolio and risk management. In particular, our results suggest that portfolios designed to hedge the largest market moves, or systematic price jumps, might have to be constructed differently from portfolios intended to neutralize, or immunize, the more common systematic day-to-day market movements.

At a more fundamental level, the ability to accurately estimate separate betas over relatively short time-spans also raise the possibility of investigating the economic determinants behind the different types of risks. In particular, is it possibly to explain the differences and temporal variation in the betas by underlying economic variables?

In spite of the continued dominance of the market model in practical applications, more complicated multi-factor representations have often been shown to provide more accurate descriptions of the cross-sectional variation in expected returns. It would be interesting to formally extend the theoretical results for the one-factor model presented here to a multi-factor setting allowing for the estimation of different continuous and jump betas with respect to specific factor representing portfolios, including the popular Fama-French book-to-market and size sorted portfolios and momentum based portfolios.

We leave further investigation of all of these issues for future research.

Technical Appendix

A Proof of Theorem 1

Part (a). Part (a) of the Theorem follows directly from Lemma 8.2 in Jacod and Todorov (2007).
Part (b). To prove part (b) we first introduce some additional notation. We define $\overline{\sigma}_t$, a random $(N+1)\times(N+1)$ matrix as follows

$$\overline{\sigma}_t = \begin{pmatrix} \sigma_{0t} & 0 & \dots & \dots & \dots & 0 \\ \beta_1\sigma_{0t} & \sigma_{1t} & 0 & \dots & \dots & 0 \\ \vdots & 0 & \ddots & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & & \ddots & \ddots & 0 \\ \beta_N\sigma_{0t} & 0 & \dots & \dots & 0 & \sigma_{Nt} \end{pmatrix}. \tag{A.1}$$

Note that $\overline{\sigma}_t\overline{\sigma}_t' = c_t$, where $C_t = \int_0^t c_s ds$ is the second characteristic of the Itô semimartingale $\mathbf{p}$ (see Jacod and Shiryaev (2003) for a definition of the characteristics of semimartingales). Using $\overline{\sigma}_t$ we define the $N+1$-dimensional variable

$$R_q = \sqrt{\kappa_q}\ \overline{\sigma}_{S_q-}U_q + \sqrt{1-\kappa_q}\ \overline{\sigma}_{S_q}U_q'. \tag{A.2}$$

The proof of Theorem 1, part (b) is based on the following Lemma.

Lemma 1 *For the Itô semimartingale* $\mathbf{p}$ *satisfying the conditions in Theorem 1 and the functions* $f_{ij}(\cdot)$ *defined in (10) we have*

$$\frac{1}{\sqrt{\Delta_n}}\begin{pmatrix} V(f_{k0},\Delta_n) - \sum_{s\le T} f_{k0}(\Delta\mathbf{p}) \\ V(f_{k1},\Delta_n) - \sum_{s\le T} f_{k1}(\Delta\mathbf{p}) \\ \vdots \\ V(f_{kN},\Delta_n) - \sum_{s\le T} f_{kN}(\Delta\mathbf{p}) \end{pmatrix} \xrightarrow{\mathcal{L}-(s)} \begin{pmatrix} Z_T^{k0} \\ Z_T^{k1} \\ \vdots \\ Z_T^{kd} \end{pmatrix}, \quad k=0,1,...,N, \tag{A.3}$$

where for arbitrary i *we define* $Z_t^{ki} = \sum_{q:S_q\le t} 2(\Delta p_{kS_q})(\Delta p_{iS_q})^2 R_q^k + 2(\Delta p_{kS_q})^2(\Delta p_{iS_q})R_q^i$.

Proof of Lemma 1: First note that the elements Z_t^{ki} are well defined using Lemma 8.1 in Jacod and Todorov (2007). The proof of the stable convergence result in (A.3) follows from Theorem 8.3 in Jacod and Todorov (2007) and Theorem 2.12(i) in Jacod (2006). □

Using the CLT result in equation (A.3) and the Delta method, it follows that $\frac{1}{\sqrt{\Delta_n}}\left(\widehat{\left(\frac{\beta_i^d}{\beta_j^d}\right)} - \left|\frac{\beta_i^d}{\beta_j^d}\right|\right)$ converges stably in law on $\Omega_T^{(0)}$ to the random variable

$$\frac{1}{2}\frac{\left(\sum_{s\le T}(\Delta p_{ks}\Delta p_{is})^2\right)^{-\frac{1}{2}}}{\left(\sum_{s\le T}(\Delta p_{ks}\Delta p_{js})^2\right)^{\frac{1}{2}}}Z_T^{ki} - \frac{1}{2}\frac{\left(\sum_{s\le T}(\Delta p_{ks}\Delta p_{is})^2\right)^{\frac{1}{2}}}{\left(\sum_{s\le T}(\Delta p_{ks}\Delta p_{js})^2\right)^{\frac{3}{2}}}Z_T^{kj}. \tag{A.4}$$

Using equations (3) and (6) we have

$$\frac{\left(\sum_{s\le T}(\Delta p_{ks}\Delta p_{is})^2\right)^{-\frac{1}{2}}}{\left(\sum_{s\le T}(\Delta p_{ks}\Delta p_{js})^2\right)^{\frac{1}{2}}} = \left|\frac{\beta_i^d}{\beta_j^d}\right|\frac{1}{\left(\beta_k^d\beta_i^d\right)^2}\frac{1}{\sum_{s\le T}(\Delta p_{0s})^4}, \tag{A.5}$$

$$\frac{\left(\sum_{s\le T}(\Delta p_{ks}\Delta p_{is})^2\right)^{\frac{1}{2}}}{\left(\sum_{s\le T}(\Delta p_{ks}\Delta p_{js})^2\right)^{\frac{3}{2}}} = \left|\frac{\beta_i^d}{\beta_j^d}\right|\frac{1}{\left(\beta_k^d\beta_j^d\right)^2}\frac{1}{\sum_{s\le T}(\Delta p_{0s})^4}, \tag{A.6}$$

$$Z_t^{ki} = \sum_{q:S_q \le t} 2(\Delta p_{0S_q})^3((\beta_i^d)^2 \beta_k^d R_q^k + \beta_i^d (\beta_k^d)^2 R_q^i), \tag{A.7}$$

for arbitrary i. Plugging the last three expressions into equation (A.4) we get (15). To show (17) we use (A.1) and the definition of R_q^i to write

$$\begin{aligned} R_q^0 &= \sqrt{\kappa_q}\sigma_{0S_q-}U_q^0 + \sqrt{1-\kappa_q}\sigma_{0S_q}U_q^{0'}, \\ R_q^i &= \beta_i^d R_q^0 + \sqrt{\kappa_q}\sigma_{iS_q-}U_q^i + \sqrt{1-\kappa_q}\sigma_{iS_q}U_q^{i'}, \end{aligned} \tag{A.8}$$

for arbitrary $i \neq 0$. Equation (17) now follows trivially. We are left with showing parts (c) and (d) of the Theorem. To do so we will make use of the following generic one-dimensional Itô semimartingale:

$$X_t = X_0 + \int_0^t b_u du + \int_0^t \sigma_u dW_u + \int_0^t \int_E \kappa(\delta(t,x))\tilde{\mu}(du,dx) + \int_0^t \int_E \kappa'(\delta(t,x))\mu(du,dx), \tag{A.9}$$

where W is a Brownian motion and μ is a Poisson measure with compensator $ds \otimes \lambda(dx)$ and all other quantities associated with the process are defined similar to the corresponding price process in (3).

Part(c). The result in part (c) follows if we can show that for arbitrary processes X_1 and X_2 with the same dynamics as the generic process X in (A.9) (but with possibly different coefficients of course) it holds that

$$\frac{1}{\sqrt{\Delta_n}} \sum_{i=1}^{[T/\Delta_n]} (\Delta_i^n X_1)^2 (\Delta_i^n X_2)^2 1_{\{|\Delta_i^n X_1| < \alpha\Delta_n^{\varpi}, |\Delta_i^n X_1| < \alpha\Delta_n^{\varpi}\}} \xrightarrow{\mathbb{P}} 0. \tag{A.10}$$

First, it is convenient to introduce the following two functions $g(x_1,x_2) = x_1^2 x_2^2$ and $g_n(x_1,x_2) = x_1^2 x_2^2 1_{\{|x_1|<\alpha\Delta_n^{\varpi}, |x_2|<\alpha\Delta_n^{\varpi}\}}$. Second, for the generic semimartingale X we set $X_t^c = X_0 + \int_0^t b_u du + \int_0^t \sigma_u dW_u$ and $X^d = X - X^c$, with the same quantities defined similarly for X_1 and X_2. It follows therefore

$$\begin{aligned} &\frac{1}{\sqrt{\Delta_n}} \sum_{i=1}^{[T/\Delta_n]} (\Delta_i^n X_1)^2 (\Delta_i^n X_2)^2 1_{\{|\Delta_i^n X_1| < \alpha\Delta_n^{\varpi}, |\Delta_i^n X_2| < \alpha\Delta_n^{\varpi}\}} \\ &= \frac{1}{\sqrt{\Delta_n}} \sum_{i=1}^{T/\Delta_n} g_n(\Delta_i^n X_1^c, \Delta_i^n X_2^c) + \frac{1}{\sqrt{\Delta_n}} \sum_{i=1}^{T/\Delta_n} \left(g_n(\Delta_i^n X_1, \Delta_i^n X_2) - g_n(\Delta_i^n X_1^c, \Delta_i^n X_2^c)\right). \end{aligned} \tag{A.11}$$

For the first term on the right side of the above equation using the results in Barndorff-Nielsen et al. (2005) we have that

$$0 \le \frac{1}{\sqrt{\Delta_n}} \sum_{i=1}^{T/\Delta_n} g_n(\Delta_i^n X_1^c, \Delta_i^n X_2^c) \le \frac{1}{\sqrt{\Delta_n}} \sum_{i=1}^{T/\Delta_n} g(\Delta_i^n X_1^c, \Delta_i^n X_2^c) \xrightarrow{\mathbb{P}} 0. \tag{A.12}$$

Thus, we are left with showing

$$\frac{1}{\sqrt{\Delta_n}} \sum_{i=1}^{T/\Delta_n} \left(g_n(\Delta_i^n X_1, \Delta_i^n X_2) - g_n(\Delta_i^n X_1^c, \Delta_i^n X_2^c)\right) \xrightarrow{\mathbb{P}} 0. \tag{A.13}$$

We prove this via several inequalities. First, it is easy to show the following algebraic inequality

$$|g_n(x_1+y_1, x_2+y_2) - g_n(x_1,x_2)| \le K(\alpha\Delta_n^{\varpi})^3(|y_1| \wedge (\alpha\Delta_n^{\varpi}) + |x_1| \wedge (\alpha\Delta_n^{\varpi})), \tag{A.14}$$

for arbitrary x_1, x_2, y_1, y_2 and K being some constant. Further, using the Burkholder-Davis-Gundy inequality (see e.g. Protter (2004)) we have

$$\mathbb{E}_{i-1}^n |\Delta_i^n X^d| \le K\sqrt{\Delta_n}, \tag{A.15}$$

where we use the abbreviation $\mathbb{E}_{i-1}^n = \mathbb{E}\left(\cdot|\mathcal{F}_{(i-1)\Delta_n}\right)$[15]. Therefore using (A.14)

$$\frac{1}{\sqrt{\Delta_n}} \mathbb{E}_{i-1}^n |g_n(\Delta_i^n X_1, \Delta_i^n X_2) - g_n(\Delta_i^n X_1^c, \Delta_i^n X_2^c)| \le K\Delta_n^{3\varpi}, \tag{A.16}$$

[15] For this to hold we need boundedness of the jumps as well as $\int_0^t \int_E \kappa^2(\delta(s,x))ds\lambda(dx) < \infty$. We can use a localization argument as in Jacod (2006) and then the local boundedness conditions in A1 to extend the results to general semimartingales satisfying only the weaker conditions of A1.

and since $\varpi > \frac{1}{3}$, the result in (A.13) follows. This completes the proof of part(c).
Part (d). For the proof of part (d) we first state and proof a result of independent interest.

Lemma 2 *For the process X in (A.9) assume that Assumption A1 and Assumption A2 for some $s < 2$ are both satisfied. Then for some $l > 2$ we have*

$$\sum_{i=k_n+2}^{[T/\Delta_n - k_n - 1]} (\Delta_i^n X)^l \left(\widehat{c}(n,-)_j + \widehat{c}(n,+)_j\right) \xrightarrow{\mathbb{P}} \sum_{q:S_q \le T} (\Delta X_{S_q})^l \left(\sigma_{S_q-}^2 + \sigma_{S_q}^2\right), \tag{A.17}$$

where

$$\widehat{c}(n,\pm)_i = \frac{1}{k_n \Delta_n} \frac{\pi}{2} \sum_{j \in I_{n,\pm}(i)} |\Delta_j^n X||\Delta_{j-1}^n X|,$$

and $I_{n,\pm}(i)$ and k_n are defined in Theorem 1.

Proof of Lemma 2: The proof parallels the proof of Theorem 4, part (b) in Ait-Sahalia and Jacod (2006), and we follow the main steps therein. In parallel to that proof, we will prove Lemma 2 under the stronger condition that the drift, the stochastic volatility and the jumps of the process X are bounded. The result after this can be extended to the general case using a localization procedure as in Jacod (2006). Our proof consists of two steps.

Step1. We denote $\delta_i^n = \sigma_{(i-1)\Delta_n} \Delta_i^n W$. Then in this first step we show that Lemma 2 will follow if we have proved the following

$$\frac{1}{k_n \Delta_n} \frac{\pi}{2} \sum_{i=k_n+2}^{[T/\Delta_n - k_n - 1]} \sum_{j \in I_n(i)} (\Delta_i^n X)^l |\delta_j^n||\delta_{j-1}^n| \xrightarrow{\mathbb{P}} \sum_{q:S_q \le T} (\Delta X_{S_q})^l \left(\sigma_{S_q-}^2 + \sigma_{S_q}^2\right), \tag{A.18}$$

where $I_n(i) = I_{n,-}(i) \cup I_{n,+}(i)$. We note that this is somewhat similar to the result in Barndorff-Nielsen et al. (2006) regarding the robustness of realized multipower variation estimators with respect to Lévytype jumps. To establish Step 1 we first prove some preliminary results. Recall from the proof of part (c) the abbreviation $\mathbb{E}_{i-1}^n = \mathbb{E}\left(\cdot|\mathcal{F}_{(i-1)\Delta_n}\right)$. Using the boundedness of b_u, σ_u and $\delta(u,x)$, the following three inequalities are straightforward,

$$\mathbb{E}_{i-1}^n \left| \int_{(i-1)\Delta_n}^{i\Delta_n} b_u du \right| \le K\Delta_n, \tag{A.19}$$

$$\mathbb{E}_{i-1}^n \left| \int_{(i-1)\Delta_n}^{i\Delta_n} (\sigma_u - \sigma_{(i-1)\Delta_n}) dW_u \right| \le \sqrt{\mathbb{E}_{i-1}^n \left(\int_{(i-1)\Delta_n}^{i\Delta_n} (\sigma_u - \sigma_{(i-1)\Delta_n})^2 du \right)} \le K\Delta_n^{1/2}, \tag{A.20}$$

$$\mathbb{E}_{i-1}^n \left| \int_{(i-1)\Delta_n}^{i\Delta_n} \int_E \kappa'(\delta(u,x)) \mu(du,dx) \right| \le \mathbb{E}_{i-1}^n \left(\int_{(i-1)\Delta_n}^{i\Delta_n} \int_E |\kappa'(\delta(u,x))| du \lambda(dx) \right) \le K\Delta_n. \tag{A.21}$$

We proceed with bounding the conditional expectation of the increment of X due to the jump martingale. First if $s < 1$, the jump martingale can be split into two integrals (one with respect to μ and the other one with respect to ν) and we can then bound the conditional expectation of the jump martingale as in the above case. Thus, assume that $s > 1$ and choose an arbitrary α such that $s < \alpha < 2$. Then, using Jensen's inequality and the Burkholder-Davis-Gundy inequality we have

$$\begin{aligned} \mathbb{E}_{i-1}^n \left| \int_{(i-1)\Delta_n}^{i\Delta_n} \int_E \kappa(\delta(u,x)) \tilde{\mu}(du,dx) \right| &\le \left(\mathbb{E}_{i-1}^n \left| \int_{(i-1)\Delta_n}^{i\Delta_n} \int_E \kappa(\delta(u,x)) \tilde{\mu}(du,dx) \right|^\alpha \right)^{1/\alpha} \\ &\le \left(\mathbb{E}_{i-1}^n \left(\int_{(i-1)\Delta_n}^{i\Delta_n} \int_E \kappa^2(\delta(u,x)) \mu(ds,dx) \right)^{\alpha/2} \right)^{1/\alpha} \\ &\le \left(\mathbb{E}_{i-1}^n \left(\int_{(i-1)\Delta_n}^{i\Delta_n} \int_E |\kappa(\delta(u,x))|^\alpha \mu(ds,dx) \right) \right)^{1/\alpha} \le K\Delta_n^{1/\alpha}. \end{aligned} \tag{A.22}$$

Further (see e.g. Jacod (2006) for a proof),

$$\mathbb{E}_{i-1}^n\left(|\Delta_i^n X|^q\right) \leq K\Delta^{q/2\wedge 1}, \quad q \geq 1. \tag{A.23}$$

We also have the following basic algebraic inequality

$$\begin{aligned}\Big||\Delta_i^n X||\Delta_{i-1}^n X| - |\delta_i^n||\delta_{i-1}^n|\Big| \quad &\leq \quad |\Delta_i^n X - \delta_i^n||\Delta_{i-1}^n X - \delta_{i-1}^n| \\ &\quad +|\delta_i^n||\Delta_{i-1}^n X - \delta_{i-1}^n| + |\Delta_i^n X - \delta_i^n||\delta_{i-1}^n|.\end{aligned}$$

It follows as an elementary consequence of the Lebesgue convergence theorem (and the boundedness of σ^2) that

$$\mathbb{E}\left(\int_0^T \left(\sigma_u^2 - \sigma_{[u/\Delta_n]\Delta_n}^2\right)^2 du\right) \longrightarrow 0. \tag{A.24}$$

Finally, we make use of the fact that $\Delta_i^n X$ and $\widehat{c}(n,\pm)_i$ in (A.17) are defined over non-overlapping periods and the same holds true for $\Delta_i^n X$, δ_j^n and δ_j^n in (A.18). Together with the above inequalities (A.19)-(A.24) this proves *Step 1.*

Step 2. In the second step we verify that

$$\frac{\pi}{2}\frac{1}{k_n\Delta_n}\sum_{j\in I_{n,q}^-}|\delta_{j-1}^n||\delta_j^n| \xrightarrow{\mathbb{P}} \sigma_{iS_q-}^2, \quad \frac{\pi}{2}\frac{1}{k_n\Delta_n}\sum_{j\in I_{n,q}^+}|\delta_{j-1}^n||\delta_j^n| \xrightarrow{\mathbb{P}} \sigma_{iS_q-}^2, \tag{A.25}$$

where as in Ait-Sahalia and Jacod (2006) we set $i(n,q) = \inf(i : i\Delta_n \geq S_q)$, $I_{n,q}^- = \{j : j \neq i(n,q), |j - i(n,q)| \leq k_n, j < k_n\}$, $I_{n,q}^+ = \{j : j \neq i(n,q), |j - i(n,q)| \leq k_n, j > k_n\}$ and recall that S_q is any sequence of stopping times exhausting the jump times of X. Then similar to Ait-Sahalia and Jacod (2006), we also define

$$U_q^n = \frac{\pi}{2}\frac{1}{k_n\Delta_n}\sum_{j\in I_{n,q}^-}|\Delta_{j-1}^n W||\Delta_j^n W| \qquad t_q^n = \inf_{u\in[S_q-k_n\Delta_n,S_q)}\sigma_u^2, \quad T_q^n = \sup_{u\in[S_q-k_n\Delta_n,S_q)}\sigma_u^2. \tag{A.26}$$

Then by a standard Law of Large Numbers, we have that $U_q^n \xrightarrow{a.s.} 1$, $t_q^n \xrightarrow{a.s.} \sigma_{S_q-}^2$ and $T_q^n \xrightarrow{a.s.} \sigma_{S_q-}^2$. Hence the first part of (A.25) follows. The second part of (A.25) is proved analogously. This establishes *Step 2.*

Combining *Step 1* and *Step 2* along with the proof of Theorem 4, part (b) in Ait-Sahalia and Jacod (2006) (where loosely speaking it is shown that substitution of $|\Delta_i^n X|^l$ with the jumps $|\Delta X|^l$ does not change the estimator) the claim in Lemma 2 follows. □

Using Lemma 2 trivially establishes part (d) of Theorem 1. The difference between $\widehat{V}_T$ and the same estimator with $\widehat{\beta}_i^c$ substituted by β_i^c is a sum of functions of the type $(\widehat{\beta}_i^c - \beta_i^c)^p K_n$, where $K_n \xrightarrow{\mathbb{P}} K$ for some processes K_n and K. Therefore part (d) follows from the consistency of $\widehat{\beta}_i^c$ for β_i^c. □

B Proof of Theorem 2

Part (a). Part (a) follows trivially from the following results, taking into account the restrictions on i, j and k in defining X_k^{ij} in (21)

$$\begin{aligned}
V_n^{''1}(X_k^{ij},\alpha,\varpi)_t \quad &\xrightarrow{\mathbb{P}} \quad (\beta_i^c + \beta_k^c)^2\int_0^t \sigma_{0s}^2 ds + \int_0^t (\sigma_{is}^2 + 1_{\{k\neq 0\}}\sigma_{ks}^2)ds, \\
V_n^{''2}(X_k^{ij},\alpha,\varpi)_t \quad &\xrightarrow{\mathbb{P}} \quad (\beta_i^c - \beta_k^c)^2\int_0^t \sigma_{0s}^2 ds + \int_0^t (\sigma_{is}^2 + 1_{\{k\neq 0\}}\sigma_{ks}^2)ds, \\
V_n^{''3}(X_k^{ij},\alpha,\varpi)_t \quad &\xrightarrow{\mathbb{P}} \quad (\beta_j^c + \beta_k^c)^2\int_0^t \sigma_{0s}^2 ds + \int_0^t (1_{\{j\neq 0\}}\sigma_{js}^2 + 1_{\{k\neq 0\}}\sigma_{ks}^2)ds, \\
V_n^{''4}(X_k^{ij},\alpha,\varpi)_t \quad &\xrightarrow{\mathbb{P}} \quad (\beta_j^c - \beta_k^c)^2\int_0^t \sigma_{0s}^2 ds + \int_0^t (1_{\{j\neq 0\}}\sigma_{js}^2 + 1_{\{k\neq 0\}}\sigma_{ks}^2)ds,
\end{aligned}$$

Part (b). We make use of the following Lemma.

Lemma 3 *Let X be a N-dimensional Itô semimartingale, satisfying the same conditions as price process* $\mathbf{p}$ *in Theorem 2 defined on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Set $C_t = \int_0^t c_u du$ to be the second characteristic of the semimartingale X.*

(a) We have

$$\frac{1}{\sqrt{\Delta_n}}\left(V_n^{''}(X,\alpha,\varpi) - \begin{pmatrix} \int_0^\bullet c_u^{11} du \\ \vdots \\ \int_0^\bullet c_u^{dd} du \end{pmatrix}\right) \xrightarrow{\mathcal{L}-(s)} \sqrt{2}\int_0^\bullet A_u d\overline{W}_u, \tag{B.1}$$

where $\overline{W}$ is a N-dimensional Brownian motion defined on an extension of the original probability space, independent from the filtration $\mathcal{F}$; A_u is a $N \times N$ matrix satisfying $(c_u^{ij})^2 = \sum_{s=1}^N a_u^{is} a_u^{js}$.

(b) A consistent estimator for the asymptotic variance covariance matrix $\int_0^T A_u A_u' du$ is given by $\widehat{D}_T$, defined for $i, j = 1, ..., N$ by,

$$\begin{aligned} \widehat{D}_T^{ij} &= \frac{\pi^2}{32\Delta_n}\sum_{i=1}^{[T/\Delta_n]-3}\Big(|\Delta_i^n(X^i+X^j)\Delta_{i+1}^n(X^i+X^j)\Delta_{i+2}^n(X^i+X^j)\Delta_{i+3}^n(X^i+X^j)| \\ &\quad +|\Delta_i^n(X^i-X^j)\Delta_{i+1}^n(X^i-X^j)\Delta_{i+2}^n(X^i-X^j)\Delta_{i+3}^n(X^i-X^j)| \\ &\quad -2|\Delta_i^n(X^i+X^j)\Delta_{i+1}^n(X^i+X^j)\Delta_{i+2}^n(X^i-X^j)\Delta_{i+3}^n(X^i-X^j)|\Big). \end{aligned} \tag{B.2}$$

Using this Lemma the proof of part (b) is easy. Set

$$\begin{aligned} \overline{V}_n^{''1}(X_k^{ij},\alpha,\varpi) &= \frac{1}{\sqrt{\Delta_n}}\left(V_n^{''1}(X_k^{ij},\alpha,\varpi) - \int_0^T [(\beta_i^c+\beta_k^c)^2\sigma_{0u}^2 + \sigma_{iu}^2 + 1_{\{k\neq 0\}}\sigma_{ks}^2] du\right), \\ \overline{V}_n^{''2}(X_k^{ij},\alpha,\varpi) &= \frac{1}{\sqrt{\Delta_n}}\left(V_n^{''2}(X_k^{ij},\alpha,\varpi) - \int_0^T [(\beta_i^c-\beta_k^c)^2\sigma_{0u}^2 + \sigma_{iu}^2 + 1_{\{k\neq 0\}}\sigma_{ks}^2] du\right), \\ \overline{V}_n^{''3}(X_k^{ij},\alpha,\varpi) &= \frac{1}{\sqrt{\Delta_n}}\left(V_n^{''3}(X_k^{ij},\alpha,\varpi) - \int_0^T [(\beta_j^c+\beta_k^c)^2\sigma_{0u}^2 + 1_{\{j\neq 0\}}\sigma_{ju}^2 + 1_{\{k\neq 0\}}\sigma_{ks}^2] du\right), \\ \overline{V}_n^{''4}(X_k^{ij},\alpha,\varpi) &= \frac{1}{\sqrt{\Delta_n}}\left(V_n^{''4}(X_k^{ij},\alpha,\varpi) - \int_0^T [(\beta_j^c-\beta_k^c)^2\sigma_{0u}^2 + 1_{\{j\neq 0\}}\sigma_{ju}^2 + 1_{\{k\neq 0\}}\sigma_{ks}^2] du\right). \end{aligned}$$

Then a Delta method and the above Lemma implies that

$$\begin{aligned} \frac{1}{\sqrt{\Delta_n}}\left(\widehat{\left(\frac{\beta_i^c}{\beta_j^c}\right)} - \frac{\beta_i^c}{\beta_j^c}\right) &= \frac{1}{4\beta_j^c\beta_k^c}\frac{1}{\int_0^T \sigma_{0u}^2 du}\Big(\overline{V}_n^{''1}(X_k^{ij},\alpha,\varpi) - \overline{V}_n^{''2}(X_k^{ij},\alpha,\varpi) \\ &\quad - \frac{\beta_i^c}{\beta_j^c}\overline{V}_n^{''3}(X_k^{ij},\alpha,\varpi) + \frac{\beta_i^c}{\beta_j^c}\overline{V}_n^{''4}(X_k^{ij},\alpha,\varpi)\Big) + o_p(1). \end{aligned} \tag{B.3}$$

Applying Lemma 3 to the process X_k^{ij} we have

$$\text{Avar}\left(\overline{V}_n^{''1}(X_k^{ij},\alpha,\varpi)\right) = 2\int_0^T [(\beta_i^c+\beta_k^c)^2\sigma_{0u}^2 + \sigma_{iu}^2 + 1_{\{k\neq 0\}}\sigma_{ks}^2]^2 du,$$

$$\text{Avar}\left(\overline{V}_n^{''2}(X_k^{ij},\alpha,\varpi)\right) = 2\int_0^T [(\beta_i^c-\beta_k^c)^2\sigma_{0u}^2 + \sigma_{iu}^2 + 1_{\{k\neq 0\}}\sigma_{ks}^2]^2 du,$$

$$\text{Avar}\left(\overline{V}_n^{''3}(X_k^{ij},\alpha,\varpi)\right) = 2\int_0^T [(\beta_j^c+\beta_k^c)^2\sigma_{0u}^2 + 1_{\{j\neq 0\}}\sigma_{ju}^2 + 1_{\{k\neq 0\}}\sigma_{ks}^2]^2 du,$$

$$\text{Avar}\left(\overline{V}_n''^4(X_k^{ij},\alpha,\varpi)\right) = 2\int_0^T [(\beta_j^c-\beta_k^c)^2\sigma_{0u}^2 + 1_{\{j\neq 0\}}\sigma_{ju}^2 + 1_{\{k\neq 0\}}\sigma_{ks}^2]^2 du,$$

$$\text{Acov}\left(\overline{V}_n''^1(X_k^{ij},\alpha,\varpi),\overline{V}_n''^2(X_k^{ij},\alpha,\varpi)\right) = 2\int_0^T [((\beta_i^c)^2-(\beta_k^c)^2)\sigma_{0u}^2 + \sigma_{iu}^2 - 1_{\{k\neq 0\}}\sigma_{ku}^2]^2 du,$$

$$\text{Acov}\left(\overline{V}_n''^1(X_k^{ij},\alpha,\varpi),\overline{V}_n''^3(X_k^{ij},\alpha,\varpi)\right) = 2\int_0^T [(\beta_i^c+\beta_k^c)(\beta_j^c+\beta_k^c)\sigma_{0u}^2 + 1_{\{k\neq 0\}}\sigma_{ku}^2]^2 du,$$

$$\text{Acov}\left(\overline{V}_n''^1(X_k^{ij},\alpha,\varpi),\overline{V}_n''^4(X_k^{ij},\alpha,\varpi)\right) = 2\int_0^T [(\beta_i^c+\beta_k^c)(\beta_j^c-\beta_k^c)\sigma_{0u}^2 - 1_{\{k\neq 0\}}\sigma_{ku}^2]^2 du,$$

$$\text{Acov}\left(\overline{V}_n''^2(X_k^{ij},\alpha,\varpi),\overline{V}_n''^3(X_k^{ij},\alpha,\varpi)\right) = 2\int_0^T [(\beta_i^c-\beta_k^c)(\beta_j^c+\beta_k^c)\sigma_{0u}^2 - 1_{\{k\neq 0\}}\sigma_{ku}^2]^2 du,$$

$$\text{Acov}\left(\overline{V}_n''^2(X_k^{ij},\alpha,\varpi),\overline{V}_n''^4(X_k^{ij},\alpha,\varpi)\right) = 2\int_0^T [(\beta_i^c-\beta_k^c)(\beta_j^c-\beta_k^c)\sigma_{0u}^2 + 1_{\{k\neq 0\}}\sigma_{ku}^2]^2 du,$$

$$\text{Acov}\left(\overline{V}_n''^3(X_k^{ij},\alpha,\varpi),\overline{V}_n''^4(X_k^{ij},\alpha,\varpi)\right) = 2\int_0^T [((\beta_j^c)^2-(\beta_k^c)^2)\sigma_{0u}^2 + 1_{\{j\neq 0\}}\sigma_{ju}^2 - 1_{\{k\neq 0\}}\sigma_{ku}^2]^2 du.$$

Combining everything yields the result in (24).

Part (c). For the case when $\widehat{\frac{\beta_i^c}{\beta_j^c}}$ is replaced with $\frac{\beta_i^c}{\beta_j^c}$ in $\widehat{K}_T$, part (c) of the Theorem follows from general results about realized multipower variation in Barndorff-Nielsen et al. (2005) (the presence of jumps does not affect the limit). As shown exactly in the proof of Theorem 1, part (d), the substitution of a consistent estimator for $\frac{\beta_i^c}{\beta_j^c}$ does not alter the results. □

C Proof of Lemma 3

Lemma 3, part (a) is essentially a multivariate extension of Theorem 2.11 in Jacod (2006), and our proof follows the same steps as in the proof of that Theorem. For ease of reference, we largely preserve the same notation as in Jacod (2006).

First, we have trivially

$$V_1 := \begin{pmatrix} \sum_{i=1}^{[t/\Delta_n]} \overline{g}_n(\Delta_i^n X^1) \\ \vdots \\ \sum_{i=1}^{[t/\Delta_n]} \overline{g}_n(\Delta_i^n X^N) \end{pmatrix} \leq V_n''(X,\alpha,\varpi)_t \leq V_2 := \begin{pmatrix} \sum_{i=1}^{[t/\Delta_n]} g_n(\Delta_i^n X^1) \\ \vdots \\ \sum_{i=1}^{[t/\Delta_n]} g_n(\Delta_i^n X^N) \end{pmatrix}, \tag{C.1}$$

where

$$g_n(x) = x^2\psi(x/\alpha\Delta_n^\varpi) \quad \text{and} \quad \overline{g}_n(x) = x^2\psi(2\sqrt{N}x/\alpha\Delta_n^\varpi), \tag{C.2}$$

for any $\psi(x)$ which is C^∞ and satisfies $1_{\{|x|\leq 1\}} \leq \psi(x) \leq 1_{\{|x|\leq 2\}}$. The inequalities in (C.1) should be interpreted component-by-component. To prove the Lemma, we need to show the result for V_1 and V_2 and also that $\frac{1}{\sqrt{\Delta_n}}||V_1 - V_2|| \xrightarrow{u.c.p.} 0$.

We can make the following decomposition

$$\frac{1}{\sqrt{\Delta_n}}\left(V_2 - \begin{pmatrix} \int_0^\bullet c_u^{11} du \\ \vdots \\ \int_0^\bullet c_u^{NN} du \end{pmatrix}\right) = A^1 + A^2 + A^3, \tag{C.3}$$

$$A_t^1 = \sqrt{\Delta_n} \sum_{i=1}^{[t/\Delta_n]} \begin{pmatrix} (\beta_i^{n,1})^2 - c_{i/n}^{11} \\ \vdots \\ (\beta_i^{n,N})^2 - c_{i/n}^{NN} \end{pmatrix}, \tag{C.4}$$

$$A_t^2 = \sqrt{\Delta_n} \sum_{i=1}^{[t/\Delta_n]} \begin{pmatrix} \mathbb{E}_{i-1}^n g_n(\Delta_i^n X^1/\sqrt{\Delta_n}) \\ \vdots \\ \mathbb{E}_{i-1}^n g_n(\Delta_i^n X^N/\sqrt{\Delta_n}) \end{pmatrix} - \frac{1}{\sqrt{\Delta_n}} \begin{pmatrix} \int_0^t c_u^{11} du \\ \vdots \\ \int_0^t c_u^{NN} du \end{pmatrix}, \tag{C.5}$$

$$A_t^3 = \sum_{i=1}^{[t/\Delta_n]} \left(\zeta_i^n - \mathbb{E}_{i-1}^n(\zeta_i^n)\right), \quad \zeta_i^n = \sqrt{\Delta_n} \begin{pmatrix} g_n(\Delta_i^n X^1) - (\beta_i^{n,1})^2 \\ \vdots \\ g_n(\Delta_i^n X^N) - (\beta_i^{n,N})^2 \end{pmatrix}, \tag{C.6}$$

where $\beta_i^n = \sigma_{(i-1)\Delta_n} \Delta_i^n W/\sqrt{\Delta_n}$; W is the Brownian motion with respect to which the continuous (local) martingale part of X is defined; σ_u is a square root of c_u (and therefore $\sigma_u \sigma_u^{'} = c_u$); and as in the proof of Lemma 2, $\mathbb{E}_{i-1}^n$ is a shorthand for the conditional expectation with respect to the filtration $\mathcal{F}_{(i-1)\Delta_n}$.

Using Theorem 2.3. in Barndorff-Nielsen et al. (2005) we have

$$A^1 \xrightarrow{\mathcal{L}-(s)} \sqrt{2} \int_0^{\bullet} A_u d\overline{W}_u. \tag{C.7}$$

Therefore we are left with showing

$$A^2 \xrightarrow{u.c.p.} 0 \quad A^3 \xrightarrow{u.c.p.} 0. \tag{C.8}$$

However, this result is established in Jacod (2006) componentwise and hence we are done. Finally, the same decomposition as in (C.3) holds true for V_1

$$\frac{1}{\sqrt{\Delta_n}} \left(V_1 - \begin{pmatrix} \int_0^{\bullet} c_u^{11} du \\ \vdots \\ \int_0^{\bullet} c_u^{NN} du \end{pmatrix} \right) = \overline{A}^1 + \overline{A}^2 + \overline{A}^3, \tag{C.9}$$

with $\overline{A}^i$ defined as A^i for $i = 1, 2, 3$ with $g_n(\cdot)$ replaced by $\overline{g}_n(\cdot)$. Thus $\frac{1}{\sqrt{\Delta_n}} ||V_1 - V_2|| = ||A^2 + A^3 - \overline{A}^2 - \overline{A}^3||$, and using (C.8) (and analogous results for $\overline{A}^2$ and $\overline{A}^3$) we have $\frac{1}{\sqrt{\Delta_n}} ||V_1 - V_2|| \xrightarrow{u.c.p.} 0$. $\square$

Table 1: Average Monthly Betas

Stock	$\bar{\beta}_i^c$	$\bar{\beta}_i^d$
Abbott Laboratories (ABT)	0.7643 [0.7460; 0.7826]	0.9097 [0.8887; 0.9306]
AIG (AIG)	0.9337 [0.9166; 0.9508]	1.0172 [1.0008; 1.0335]
American Express (AXP)	1.0216 [1.0040; 1.0392]	1.1887 [1.1712; 1.2060]
Bank of America (BAC)	0.8196 [0.8043; 0.8350]	0.8969 [0.8816; 0.9123]
BellSouth(BLS)	0.9435 [0.9240; 0.9630]	1.1123 [1.0920; 1.1325]
Bristol-Myers (BMY)	0.7992 [0.7790; 0.8194]	0.9743 [0.9525; 0.9961]
Citigroup (C)	1.1106 [1.0954; 1.1257]	1.1719 [1.1570; 1.1867]
Genentech (DNA)	0.9915 [0.9588; 1.0243]	1.4055 [1.3742; 1.4368]
Fannie Mae (FNM)	0.6853 [0.6657; 0.7049]	0.9571 [0.9363; 0.9780]
General Electric (GE)	1.0922 [1.0769; 1.1074]	1.1598 [1.1448; 1.1748]
Goldman Sachs (GS)	1.0996 [1.0806; 1.1187]	1.1377 [1.1179; 1.1576]
The Home Depot (HD)	1.0837 [1.0644; 1.1030]	1.2432 [1.2234; 1.2630]
IBM (IBM)	0.9809 [0.9609; 0.9900]	0.9903 [0.9760; 1.0047]
Johnson & Johnson (JNJ)	0.6354 [0.6212; 0.6495]	0.7959 [0.7823; 0.8096]
JP Morgan Chase (JPM)	1.1690 [1.1490; 1.1889]	1.2267 [1.2072; 1.2462]
Coca Cola (KO)	0.7040 [0.6885; 0.7194]	0.8346 [0.8192; 0.8501]
Eli Lilly (LLY)	0.7620 [0.7436; 0.7804]	0.9528 [0.9336; 0.9719]
Lowe's Companies (LOW)	1.0312 [1.0101; 1.0523]	1.2666 [1.2433; 1.2900]
Mcdonald's (MCD)	0.7993 [0.7786; 0.8200]	1.0112 [0.9902; 1.0322]
Medtronic (MDT)	0.6841 [0.6668; 0.7013]	0.8686 [0.8495; 0.8876]
Merrill Lynch (MER)	1.2344 [1.2127; 1.2562]	1.2457 [1.2237; 1.2677]
3M Co. (MMM)	0.8057 [0.7908; 0.8206]	0.9118 [0.8971; 0.9265]
Altria Group (MO)	0.6326 [0.6128; 0.6524]	0.8505 [0.8321; 0.8690]
Motorola (MOT)	1.4497 [1.4204; 1.4789]	1.5442 [1.5137; 1.5747]
Merck & Co. (MRK)	0.7767 [0.7552; 0.7983]	0.8927 [0.8700; 0.9155]
Nokia (NOK)	1.2119 [1.1878; 1.2359]	1.1983 [1.1770; 1.2195]
Pepsico (PEP)	0.6306 [0.6163; 0.6449]	0.7722 [0.7575; 0.7870]
Pfizer Inc. (PFE)	0.8359 [0.8180; 0.8538]	0.9781 [0.9536; 1.0027]
Procter & Gamble Co. (PG)	0.6113 [0.5966; 0.6261]	0.7107 [0.6952; 0.7261]
Schlumberger Limited (SLB)	0.7012 [0.6758; 0.7267]	1.0561 [1.0312; 1.0810]
Target (TGT)	1.1038 [1.0815; 1.1261]	1.2853 [1.2631; 1.3075]
Texas Instruments (TXN)	1.8551 [1.8252; 1.8851]	1.8538 [1.8230; 1.8847]
Tyco International (TYC)	1.0953 [1.0662; 1.1244]	1.3320 [1.2967; 1.3673]
UPS (UPS)	0.5533 [0.5409; 0.5657]	0.7093 [0.6958; 0.7227]
United Technologies (UTX)	0.8856 [0.8679; 0.9032]	1.0449 [1.0261; 1.0638]
Verizon (VZ)	0.8751 [0.8572; 0.8929]	1.0239 [1.0050; 1.0429]
Wachovia (WB)	0.9017 [0.8857; 0.9176]	1.0451 [1.0289; 1.0613]
Wells Fargo & Co.	0.7402 [0.7268; 0.7536]	0.9022 [0.8885; 0.9160]
WalMart (WMT)	0.9396 [0.9236; 0.9555]	1.0292 [1.0134; 1.0450]
Exxon Mobil (XOM)	0.8173 [0.8010; 0.8336]	0.8988 [0.8832; 0.9145]
25-th quantile	0.7511	0.9005
50-th quantile	0.8803	1.0206
75-th quantile	1.0879	1.1935

Table 2: Tests for Equality of Betas

Stock	$T^c_{i,\mathrm{m}}$	$T^c_{i,\mathrm{q}}$	$T^c_{i,\mathrm{y}}$	$T^d_{i,\mathrm{m}}$	$T^d_{i,\mathrm{q}}$	$T^d_{i,\mathrm{y}}$
Abbott Laboratories (ABT)	101.6	69.8	42.0	2402.3	264.3	20.8
AIG (AIG)	131.9	40.1	257.2	2092.4	583.7	221.7
American Express (AXP)	125.0	82.6	310.5	8545.2	993.0	187.4
Bank of America (BAC)	198.4	133.9	13.1	10474	867.2	11.2
BellSouth(BLS)	108.8	147.3	249.1	7444.2	476.7	111.5
Bristol-Myers (BMY)	116.6	75.1	45.1	1159.9	358.2	128.4
Citigroup (C)	222.4	81.6	106.5	1916.6	710.0	3.5
Genentech (DNA)	131.2	88.1	7.4	2224.7	723.8	96.3
Fannie Mae (FNM)	148.3	37.1	41.8	1166.5	641.1	12.3
General Electric (GE)	116.1	111.3	57.3	6843.7	787.0	22.4
Goldman Sachs (GS)	269.7	146.1	16.7	4295	759.6	182.5
The Home Depot (HD)	213.1	156.6	4.6	1337.9	755.0	14.9
IBM (IBM)	172.3	106.0	22.8	831.9	1079.3	105.3
Johnson & Johnson (JNJ)	170.1	95.3	144.5	1873.0	552.2	69.8
JP Morgan Chase (JPM)	154.3	98.7	156.0	5970.3	333.2	20.0
Coca Cola (KO)	120.5	91.6	16.5	6151.3	570.9	5.1
Eli Lilly (LLY)	135.9	58.8	189.8	1927.9	381.2	45.0
Lowe's Companies (LOW)	260.5	143.1	6.5	4124.4	1238.8	0.2
Mcdonald's (MCD)	135.5	35.0	21.0	1562.5	472.1	17.9
Medtronic (MDT)	92.5	31.1	3.8	2123.6	612.7	17.7
Merrill Lynch (MER)	230.9	63.5	21.1	24846	755.5	30.6
3M Co. (MMM)	127.1	83.6	9.5	2019.2	968.3	190.6
Altria Group (MO)	86.0	122.3	2.6	4901.2	313.5	7.1
Motorola (MOT)	142.0	65.8	38.0	3857.5	769.7	55.6
Merck & Co. (MRK)	108.0	78.4	130.0	7540.3	112.4	54.3
Nokia (NOK)	177.2	150.6	59.4	1168.4	602.8	5.9
Pepsico (PEP)	107.8	166.3	105.4	3125.3	443.4	3.3
Pfizer Inc. (PFE)	180.4	71.9	48.0	1814.3	490.3	97.5
Procter & Gamble Co. (PG)	129.5	60.8	15.9	1238.4	292.4	124.4
Schlumberger Limited (SLB)	115.2	71.3	22.1	3526.7	169.8	70.6
Target (TGT)	199.3	90.4	16.3	1129.7	1426.3	11.9
Texas Instruments (TXN)	190.5	117.2	82.2	1870.6	972.9	17.1
Tyco International (TYC)	184.6	86.3	207.1	11466.0	379.7	179.0
UPS (UPS)	158.2	54.7	144.4	1309.1	733.8	139.9
United Technologies (UTX)	152.0	66.9	19.3	1309.6	779.1	15.8
Verizon (VZ)	137.5	71.6	165.0	8462.9	567.8	149.8
Wachovia (WB)	210.7	71.3	204.5	10531.0	678.2	18.9
Wells Fargo & Co.	139.4	53.5	42.7	1423.7	594.9	119.7
WalMart (WMT)	225.8	126.6	3.2	2561.0	1343.0	48.3
Exxon Mobil (XOM)	163.9	94.5	50.1	2243.7	865.6	32.5
χ^2 95-th quantile	43.8	18.3	6.0	37.7	18.3	6.0

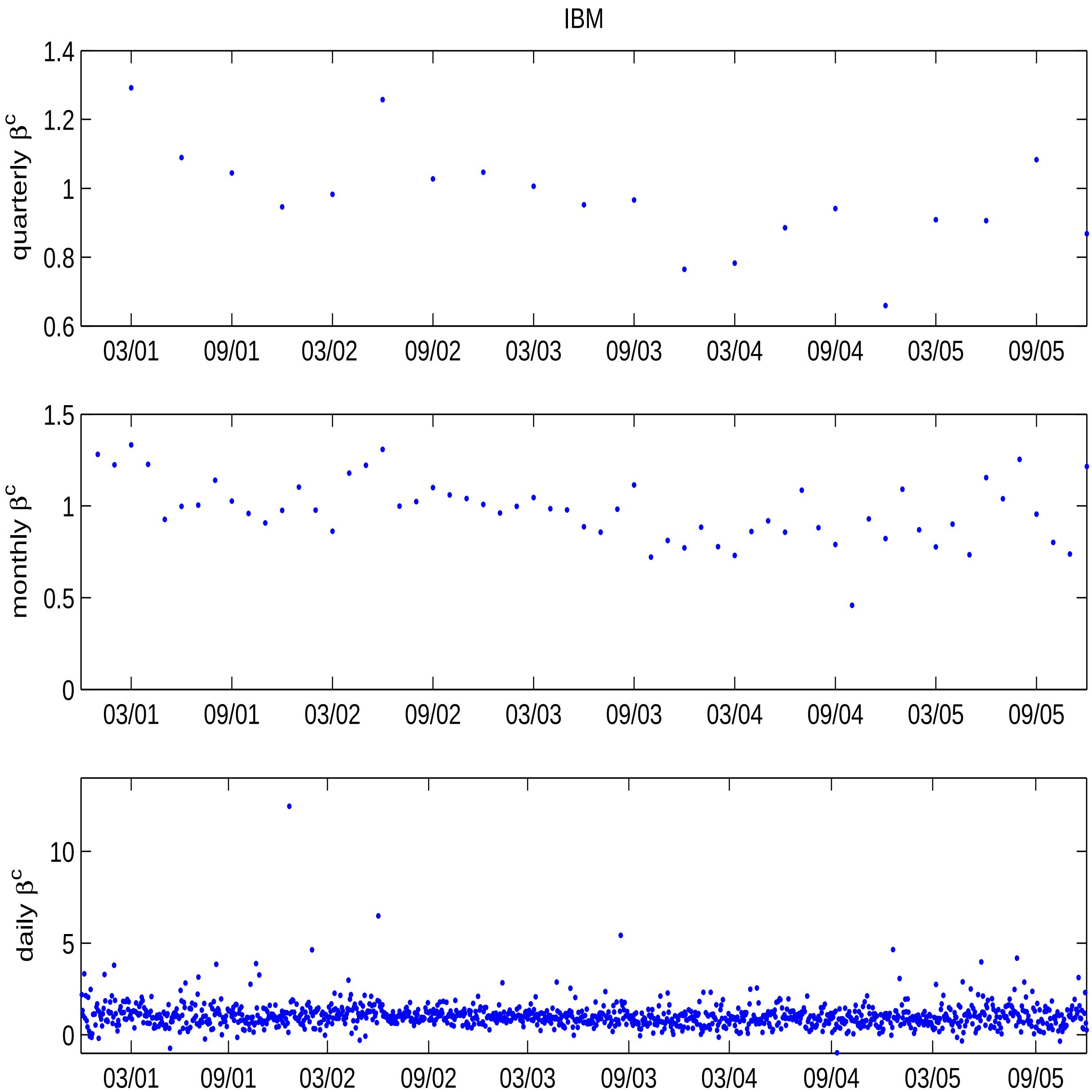

Figure 1: Quarterly, monthly and daily continuous betas for IBM.

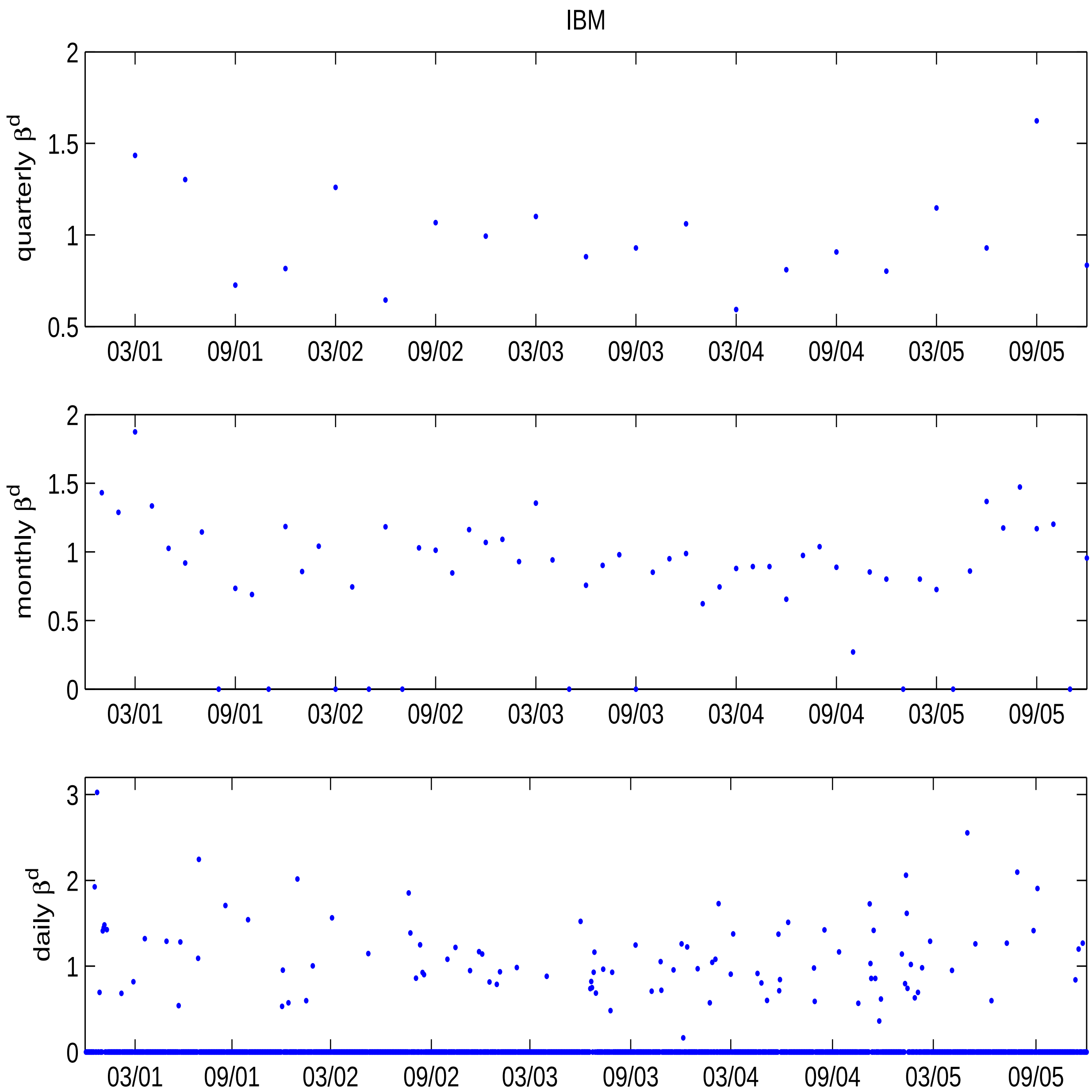

Figure 2: Quarterly, monthly and daily jump betas for IBM.

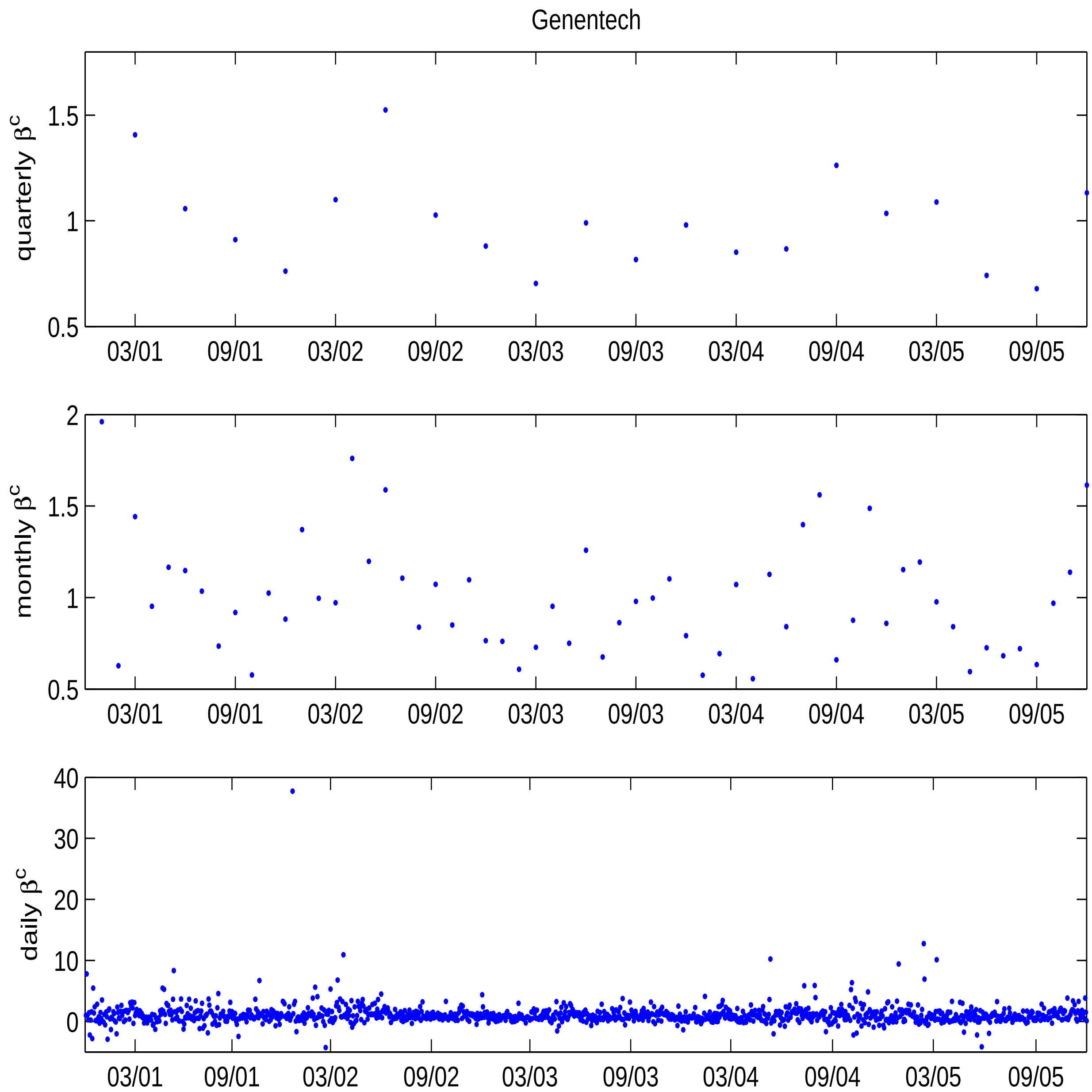

Figure 3: Quarterly, monthly and daily continuous betas for Genentech.

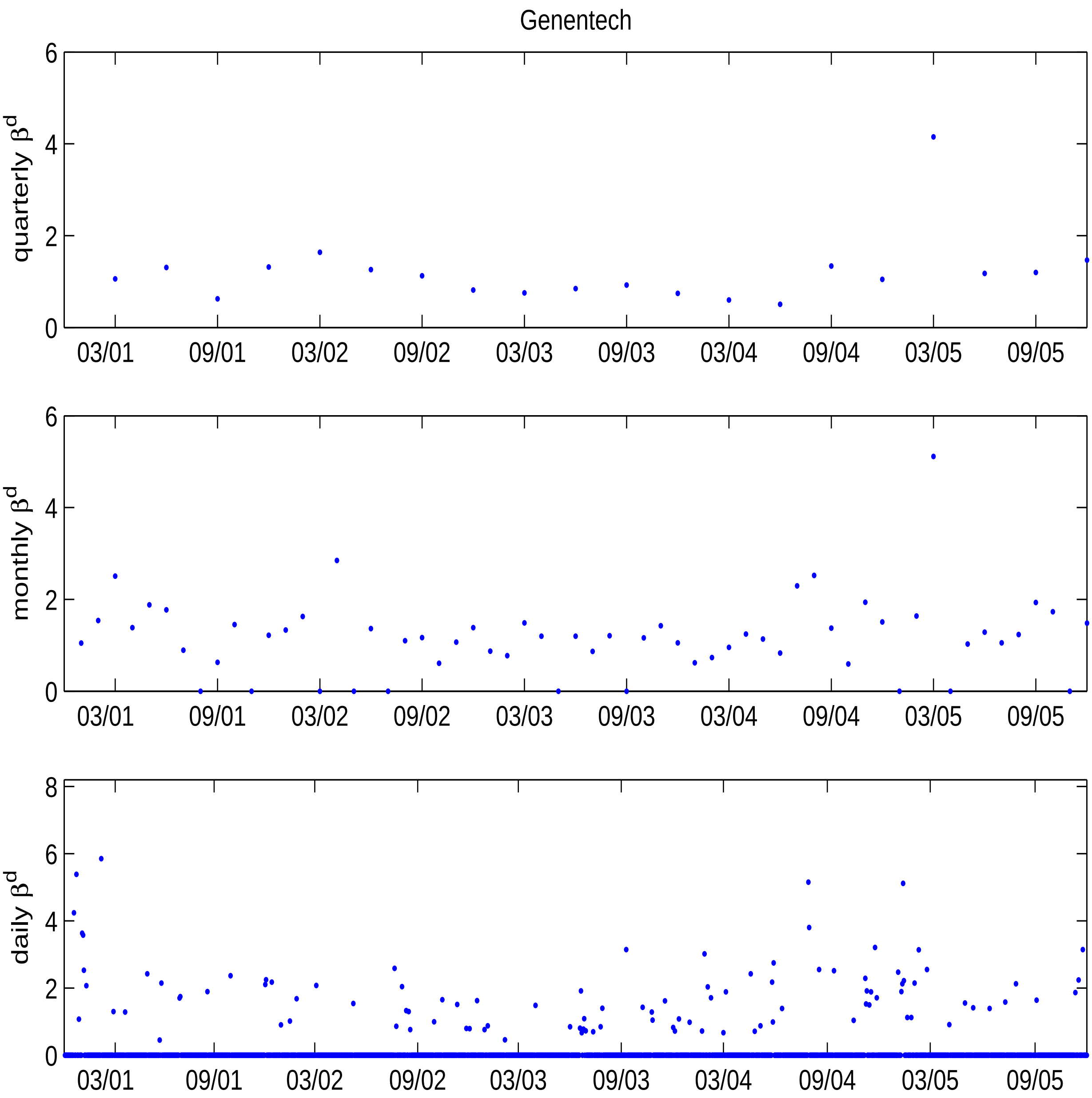

Figure 4: Quarterly, monthly and daily jump betas for Genentech.

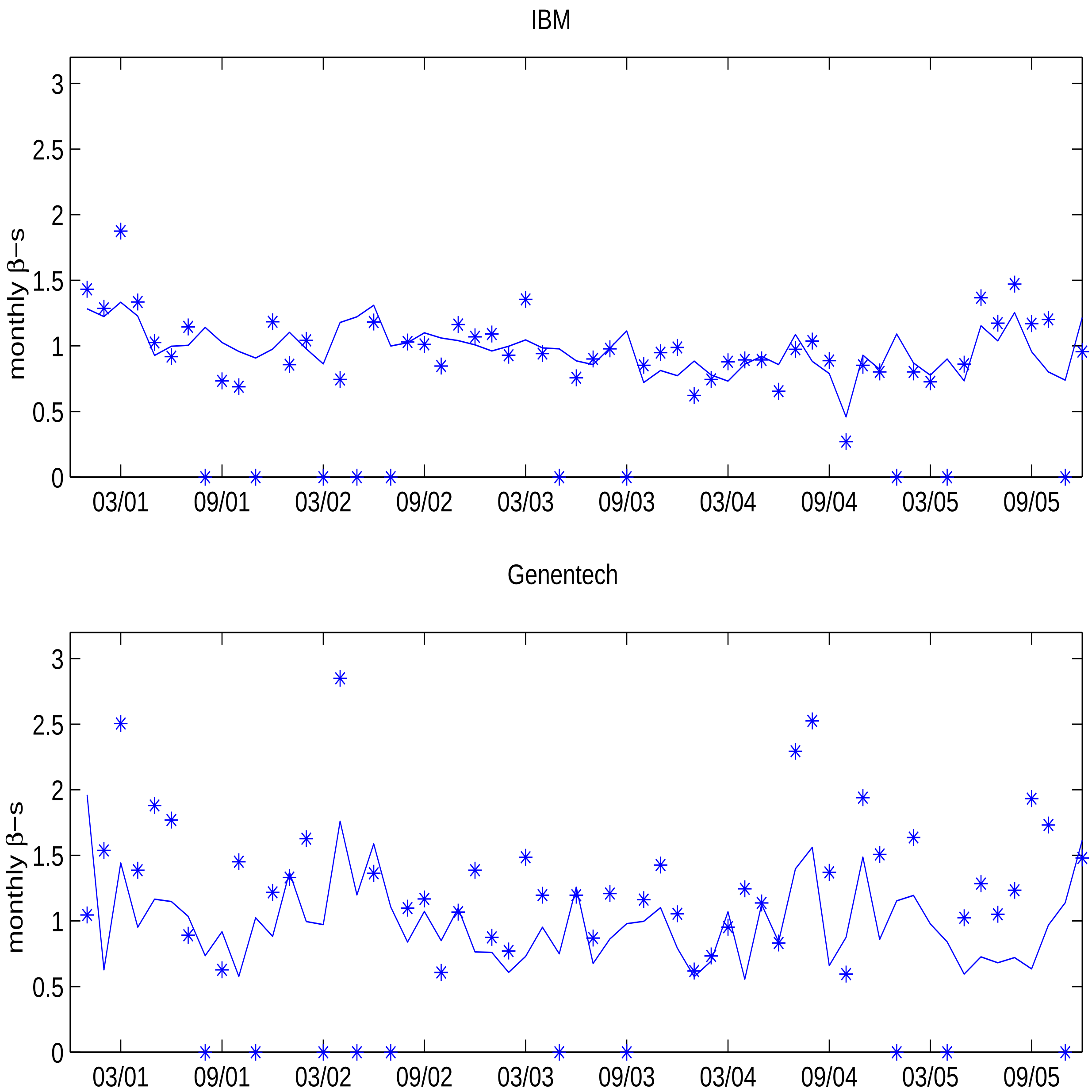

Figure 5: Monthly continuous and jump betas for IBM and Genentech.

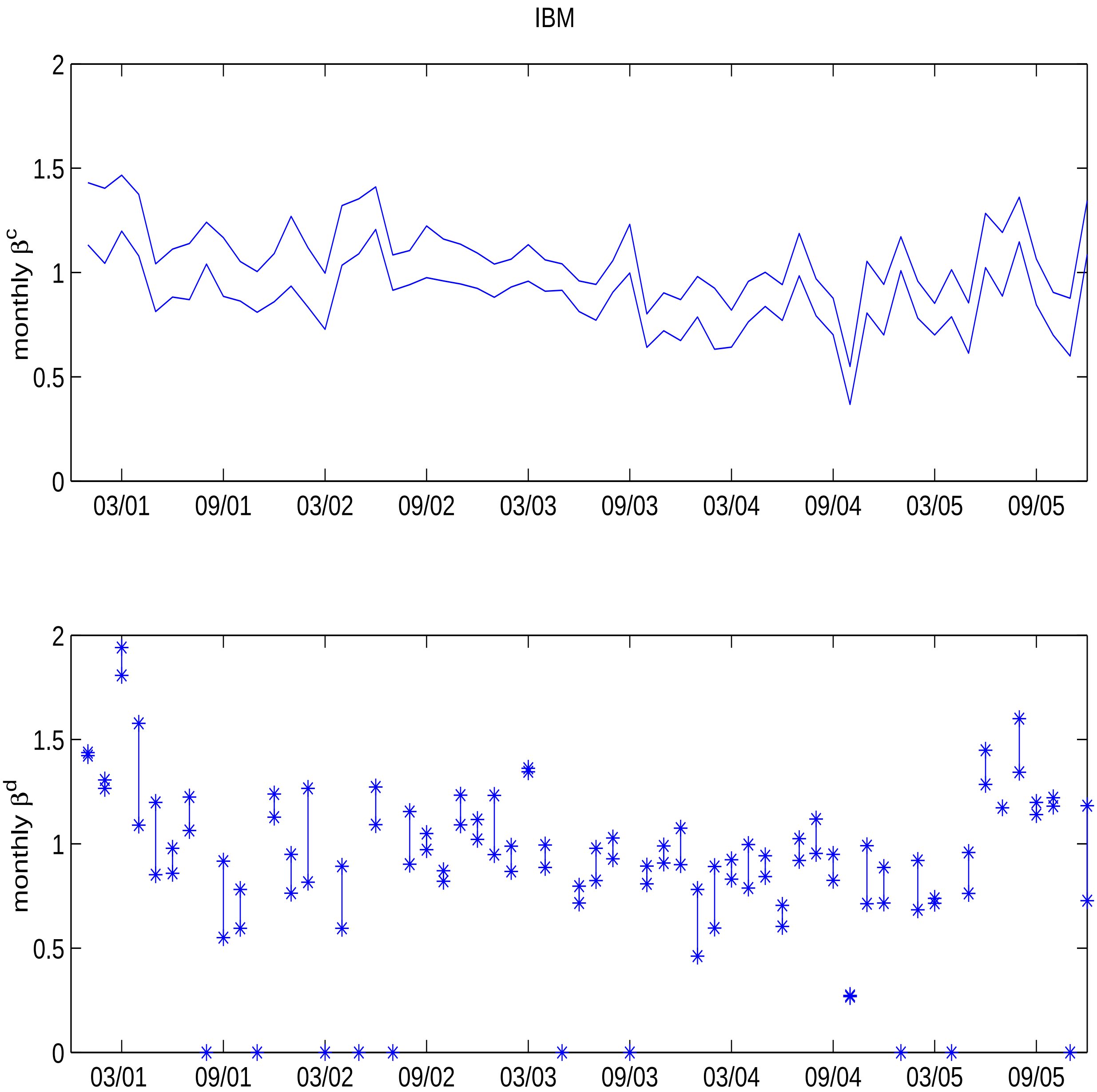

Figure 6: 95% confidence intervals for monthly IBM betas.

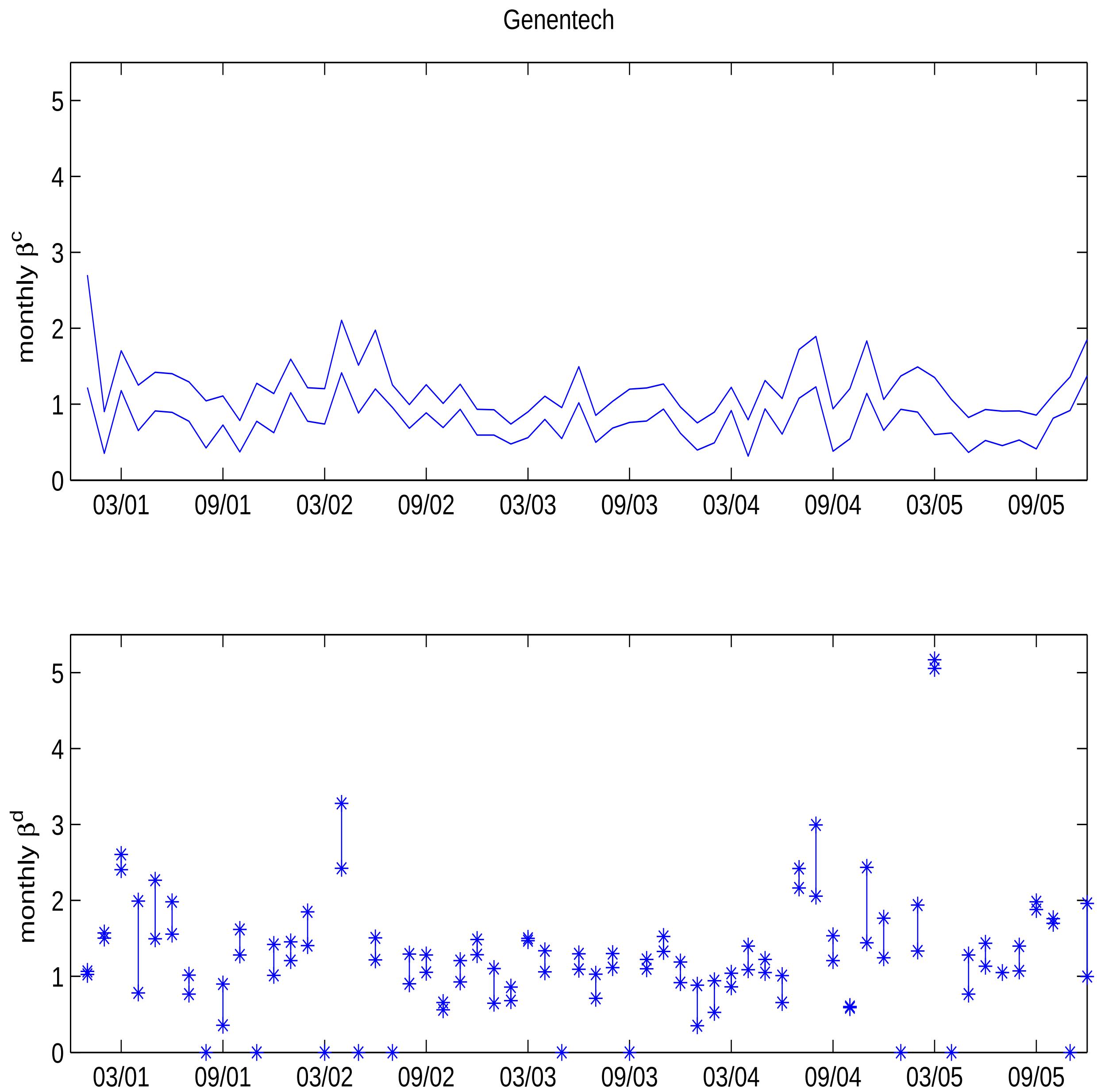

Figure 7: 95% confidence intervals for monthly Genentech betas.

References

Ait-Sahalia, Y. and J. Jacod (2006). Testing for Jumps in a Discretely Observed Process. Working paper, Princeton University and Université de Paris-6.

Ait-Sahalia, Y., P. Mykland, and L. Zhang (2005). How Often to Sample a Continuous Time Process in the Presence of Market Microstructure Noise. *Review of Financial Studies 18*, 351–416.

Andersen, T., T. Bollerslev, and F. Diebold (2007). Roughing It Up: Including Jump Components in the Measurement, Modeling, and Forecasting of Return Volatility. *Review of Economics and Statistcis 89*, forthcoming.

Andersen, T., T. Bollerslev, F. Diebold, and G. Wu (2005). A Framework for Exploring the Macroeconomic Determinants of Systematic Risk. *American Economic Review 95*, 398–404.

Andersen, T., T. Bollerslev, F. Diebold, and G. Wu (2006). Realized Beta: Persistence and Predictability. In T. Fomby and D. Terrell (Eds.), *Advances in Econometrics: Econometric Analysis of Economic and Financial Time Series*, Volume 20. Elsevier Science.

Ang, A. and J. Chen (2002). Asymmetric Correlations of Equity Portfolios. *Journal of Financial Economics 63*, 443–494.

Barndorff-Nielsen, O., S. Graversen, J. Jacod, M. Podolskij, and N. Shephard (2005). A Central Limit Theorem for Realised Power and Bipower Variations of Continuous Semimartingales. In Y. Kabanov and R. Lipster (Eds.), *From Stochastic Analysis to Mathematical Finance, Festschrift for Albert Shiryaev.* Springer.

Barndorff-Nielsen, O., P. Hansen, A. Lunde, and N. Shephard (2007). Designing Realised Kernels to Measure the ex-post Variation of Equity Prices in the Presence of Noise. Working paper, Oxford University.

Barndorff-Nielsen, O. and N. Shephard (2004a). Econometric Analysis of Realized Covariation: High- Frequency Covariance, Regression and Correlation in Financial Economics. *Econometrica 72*, 885–925.

Barndorff-Nielsen, O. and N. Shephard (2004b). Power and Bipower Variation with Stochastic Volatility and Jumps. *Journal of Financial Econometrics 2*, 1–37.

Barndorff-Nielsen, O. and N. Shephard (2006). Econometrics of Testing for Jumps in Financial Economics using Bipower Variation. *Journal of Financial Econometrics 4*, 1–30.

Barndorff-Nielsen, O., N. Shephard, and M. Winkel (2006). Limit Theorems for Multipower Variation in the Presence of Jumps in Financial Econometrics. *Stochastic Processes and Their Applications 116*, 796–806.

Bates, D. (2000). Post-'87 Crash Fears in S&P 500 Futures Options. *Journal of Econometrics 94*, 181–238.

Blumenthal, R. and R. Getoor (1961). Sample Functions of Stochastic Processes with Independent Increments. *Journal of Math. Mech. 10*, 493–516.

Bollerslev, T., T. Law, and G. Tauchen (2007). Risk, Jumps, and Diversification. Working paper, Duke University.

Bollerslev, T. and B. Zhang (2003). Measuring and Modeling Systematic Risk in Factor Pricing Models using High-Frequency Data. *Journal of Empirical Finance 10*, 533–558.

Chamberlain, G. and M. Rothschild (1983). Arbitrage, Factor Structure, and Mean-Variance Analysis on Large Asset Markets. *Econometrica 51*, 1281–1304.

Eraker, B. (2004). Do Stock Prices and Volatility Jump? Reconciling Evidence from Spot and Option Prices. *Journal of Finance 59*, 1367–1403.

Fama, E. and K. French (1992). The Cross-Section of Expected Stock Returns. *Journal of Finance 47*, 427–465.

Fama, E. and J. MacBeth (1973). Risk Return and Equilibrium Empirical Tests. *Journal of Political Economy 81*, 607–636.

Ghysels, E. and E. Jacquier (2006). Market Beta Dynamics and Portfolio Choice. Working paper, University of North Carolina and HEC Montréal.

Gobbi, C. and C. Mancini (2007). Diffusion Covariation and Co-Jumps in Bidimensional Asset Price Processes with Stochastic Volatility and Infinite Activity Lévy Jumps. Working paper, University of Firenze.

Hansen, P. and A. Lunde (2006). Realized Variance and Market Microstructure Noise. *Journal of Business and Economic Statistics 24*, 127–161.

Hooper, E., K. Ng, and J. Reeves (2006). Monthly Forecasts of Systematic Risk: An Evaluation. Working paper, University of New South Wales.

Huang, X. and G. Tauchen (2005). The Relative Contributions of Jumps to Total Variance. *Journal of Financial Econometrics 3*, 456–499.

Jacod, J. (2006). Asymptotic Properties of Power Variations and Associated Functionals of Semimartingales. *Stochastic Processes and their Applications, forthcoming*.

Jacod, J. and A. N. Shiryaev (2003). *Limit Theorems For Stochastic Processes* (2nd ed.). Berlin: Springer-Verlag.

Jacod, J. and V. Todorov (2007). Testing for Common Arrivals of Jumps for Discretely Observed Multidimensional Processes. Working paper, Duke University and Université de Paris-6.

Lee, S. and P. Mykland (2007). Jumps in Financial Markets: A New Nonparametric Test and Jump Dynamics. *Review of Financial Studies 20*, forthcoming.

Mancini, C. (2001). Disentangling the Jumps of the Diffusion in a Geometric Jumping Brownian Motion. *Giornale dell'Instituto Italiano degli Attuari LXIV*, 19–47.

Mancini, C. (2006). Estimating the Integrated Volatility in Stochastic Volatility Models with Lévy-type Jumps. Working paper, University of Firenze.

Merton, R. (1976). Option Pricing when Underlying Asset Returns are Discontinuous. *Journal of Financial Economics 3*, 125–144.

Pan, J. (2002). The Jump Risk Premium Implicit in Options: Evidence from an Integrated Time-Series Study. *Journal of Financial Economics 53*, 3–50.

Protter, P. (2004). *Stochastic Integration and Differential Equations* (2nd ed.). Berlin: Springer-Verlag.

Ross, S. (1976). Arbitrage Theory of Capital Asset Pricing. *Journal of Economic Theory 13*, 341–360.

Todorov, V. (2007). Variance Risk Premium Dynamics. Working paper, Duke University.

Wright, J. and H. Zhou (2007). Bond Risk Premia and Realized Jump Risk. Working paper, Federal Reserve Board.

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