Disasters Implied by Equity Index Options

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The idea

- Disasters are infrequent $\Rightarrow$ hard to estimate their distribution
- Idea: infer from option prices (market prices of bets on disasters)
- We find:
  - disasters apparent in options data
  - disasters are more modest than what we see in macro data

Outline

- Preliminaries: entropy, Alvarez-Jermann bound, cumulants
- **Disasters in macroeconomic data**
- Risk-neutral probabilities
- **Disasters in options data**
- **Compare the implications of the two approaches**

Alvarez-Jermann bound

- Pricing relation

$$E_t \left(m_{t+1} r^j_{t+1} \right) \quad = \quad 1$$

- Entropy: for any $x > 0$

$$L(x) \quad \equiv \quad \log Ex - E \log x \quad \geq \quad 0$$

- AJ bound

$$L(m) \quad \geq \quad E \left(\log r^j - \log r^1 \right)$$

Cumulants

- Cumulant generating function

$$k(s;x) = \log E e^{sx} = \sum_{j=1}^{\infty} \kappa_j(x) s^j / j!$$

- Cumulants are almost moments

$$\begin{aligned} \text{mean} &= \kappa_1(x) \\ \text{variance} &= \kappa_2(x) \\ \text{skewness} &= \kappa_3(x)/\kappa_2^{3/2}(x) \\ \text{(excess) kurtosis} &= \kappa_4(x)/\kappa_2^2(x) \end{aligned}$$

- If x is normal, $\kappa_j(x) = 0$ for $j > 2$

Entropy and cumulants

Entropy and cumulants

- Entropy of pricing kernel

$$L(m) \quad = \quad \log E e^{\log m} - E \log m \quad = \quad k(1, \log m) - \kappa_1(\log m)$$

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$$L(m) \;=\; \log E e^{\log m} - E \log m \;=\; k(1, \log m) - \kappa_1(\log m)$$

- Zin's "never a dull moment" conjecture

$$L(m) \;=\; \underbrace{\kappa_2(\log m)/2!}_{\text{(log)normal term}} + \underbrace{\kappa_3(\log m)/3! + \kappa_4(\log m)/4! + \cdots}_{\text{high-order cumulants (incl disasters)}}$$

Disasters based on macro fundamentals

Macro disasters: Model

- Consumption growth iid

$$
\begin{aligned}
\log g_{t+1} &= w_{t+1} + z_{t+1} \\
w_{t+1} &\sim \mathcal{N}(\mu, \sigma^2) \\
z_{t+1} | j &\sim \mathcal{N}(j\theta, j\delta^2) \\
j &\geq 0 \text{ has probability } e^{-\omega}\omega^j / j!
\end{aligned}
$$

- Parameter values
  - Match mean and variance of log consumption growth
  - Average number of disasters ($\omega = 0.01$), mean ($\theta = -0.3$) and variance ($\delta^2 = 0.15^2$)
  - Similar to Barro, Nakamura, Steinsson, and Ursua (2009)

Macro disasters: Deviations from normality

- Pricing kernel

$$\log m_{t+1} = \log \beta - \alpha \log g_{t+1}$$

Macro disasters: Deviations from normality

- Pricing kernel

$$\log m_{t+1} \quad = \quad \log \beta - \alpha \log g_{t+1}$$

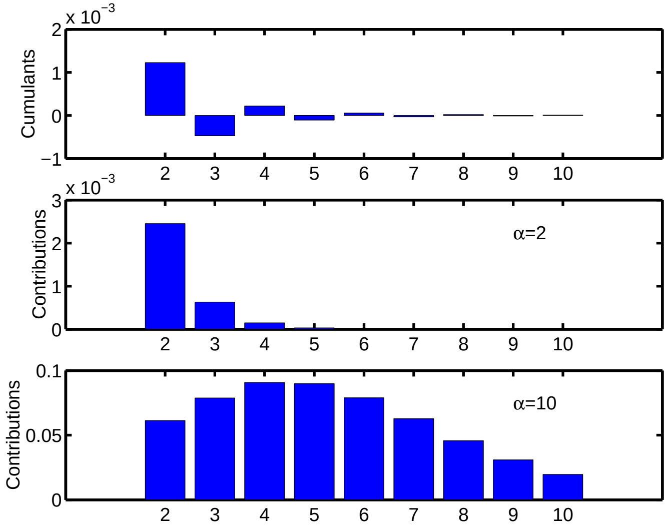
- Yaron's "bazooka"

$$\kappa_j(\log m)/j! \quad = \quad \kappa_j(\log g)(-\alpha)^j/j!$$

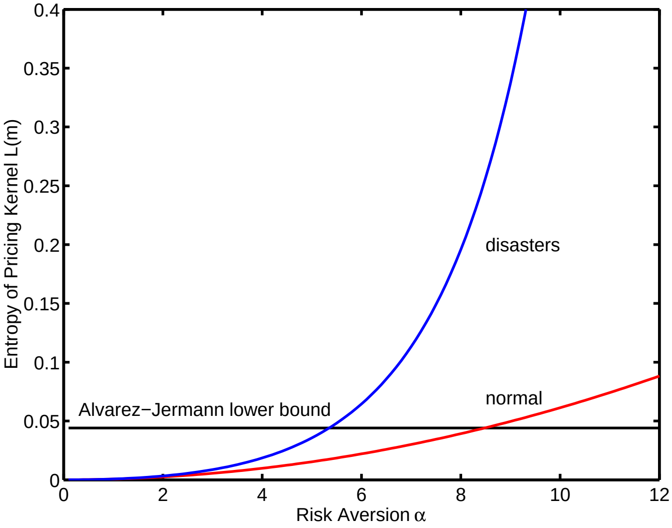
- The contribution of higher-order cumulants peaks at $j = \alpha$

$$\frac{\alpha^j}{j!} = \frac{\alpha}{1} \cdot \frac{\alpha}{2} \cdot \ldots \cdot \frac{\alpha}{j-1} \cdot \frac{\alpha}{j}$$

Macro disasters: Cumulants



Macro disasters: Entropy



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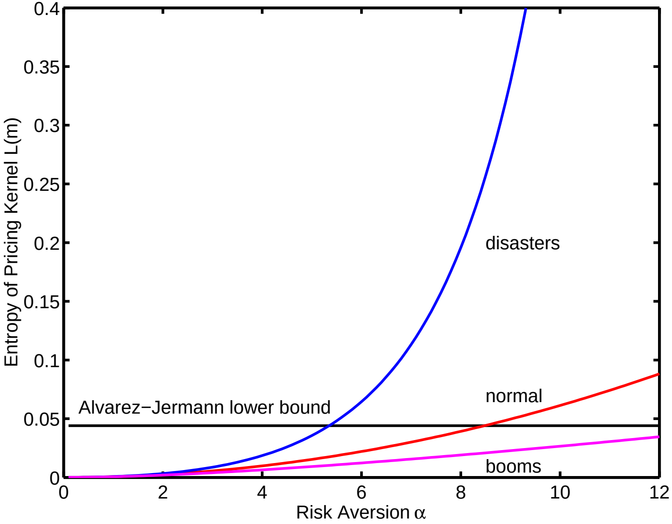

Macro-finance and risk-neutral pricing

Macro-finance and risk-neutral pricing

- Pricing relation

$$q^1 E_t^* \left(r_{t+1}^j \right) = 1,$$

  - where q^1 is a price of a one-period riskless bond

- Translating between preferences and risk-neutral probabilities

$$\begin{aligned} p(x)m(x) &= q^1 p^*(x) \\ p^*(x) &= p(x)m(x)/q^1 \end{aligned}$$

Macro-finance and risk-neutral pricing: Examples

- Normal log consumption growth
  - If $\log g \sim \mathcal{N}(\mu, \sigma^2)$ (true distribution)
  - Then risk-neutral distribution also lognormal with $\mu^* = \mu - \alpha\sigma^2, \sigma^* = \sigma$

- Poisson log consumption growth
  - If disasters have probability ω and distribution $\mathcal{N}(\theta, \delta^2)$
  - Then risk-neutral distribution has same form with $\omega^* = \omega \exp(-\alpha\theta + (\alpha\delta)^2/2), \theta^* = \theta - \alpha\delta^2, \delta^* = \delta$

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- Entropy

$$L(m) \;=\; L(p^*/p) \;=\; -E \log(p^*/p)$$

Disasters in options

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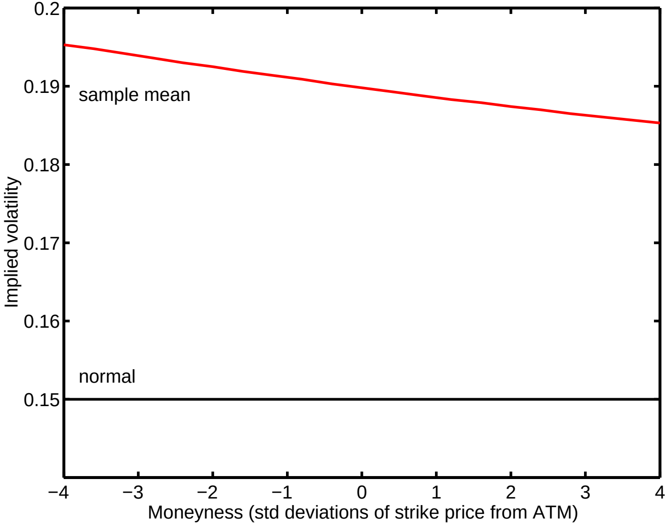
- Put option (bet on low returns)

$$q_t^p \quad = \quad q^1 E_t^*(b - r_{t+1}^e)^+$$

- Estimate p^* by varying strike price b (cross section)

- Black-Scholes-Merton benchmark
  - Quote prices as implied volatilities [high price $\Leftrightarrow$ high vol]
  - Horizontal line if (log)normal
  - “Skew” suggests disasters

Disasters in options: Data vs normal benchmark



Disasters in options: Merton model

- Equity returns iid

$$\begin{aligned} \log r^e_{t+1} - \log r^1 &= w_{t+1} + z_{t+1} \\ w_{t+1} &\sim \mathcal{N}(\mu, \sigma^2) \\ z_{t+1}|j &\sim \mathcal{N}(j\theta, j\delta^2) \\ j &\geq 0 \text{ has probability } e^{-\omega}\omega^j/j! \end{aligned}$$

- Risk-neutral distribution: the same with *s
- Parameter values
  - Choose risk-neutral parameters to match option prices
  - Average number of disasters ($\omega^* = 1.5$), mean ($\theta^* = -0.05$) and variance ($\delta^{*2} = 0.01^2$)
  - Calibration is based on Broadie, Chernov, and Johannes (2007)

Comparing macro- and option-based models

- Entropy and cumulants of pricing kernel
    - Result: option-based entropy is large

- Consumption growth implied by option prices
    - Option-based $p^* +$ power utility $\Rightarrow p$
    - Result: more modest skewness and kurtosis, tail probabilities

- Option prices implied by consumption growth
    - Macro-based $p +$ power utility $\Rightarrow p^*$
    - Compute option prices
    - Result: steeper volatility smile

Comparing models: components of entropy

			High-Order Cumulants	
Model	Entropy	Variance/2	Odd	Even
Macro ($\alpha = 5.38$)	0.0449	0.0177	0.0173	0.0099
		39%	39%	22%
Options	0.7647	0.4699	0.1130	0.1819
		61%	15%	24%

Comparing models: consumption implied by options

	Calibration	Implied
α	5.38	10.07
ω	0.0100	1.3864
θ	–0.3000	–0.0060
δ	0.1500	0.0229

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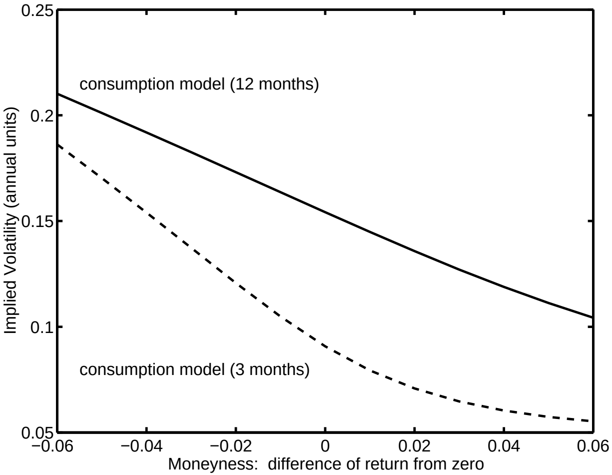
Tail prob (≤ -5 st dev)	0.0079	0.0002

Comparing models: options implied by macro model



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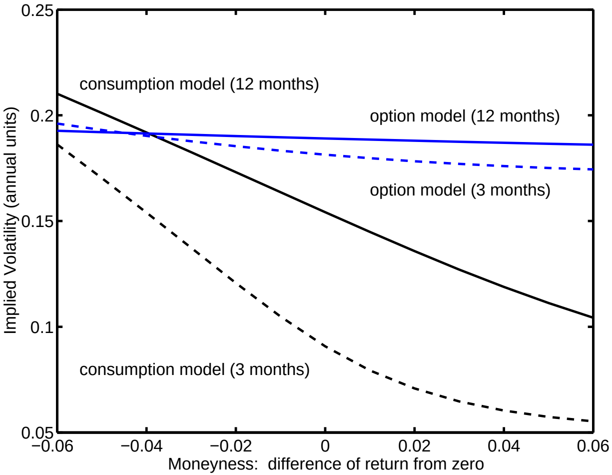

Comparing models: risk aversion

- In option model, implicit risk aversion accounts for
  - Equity premium
  - Prices of options (high entropy)
- Form differs from power utility
  - Not constant
  - Parameters imply greater aversion to adverse risks

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  - Prices of options (high entropy)
- Form differs from power utility
  - Not constant
  - Parameters imply greater aversion to adverse risks
- Computation

$$\text{RA} = -\frac{\partial \log m}{\partial \log g} = -\frac{\partial \log(p^*/p)}{\partial \log r^e} \cdot \frac{\partial \log r^e}{\partial \log g}$$

Bottom line

- Barro and Rietz
  - Disasters account for equity premium
  - Evident in macro data
- Options
  - Disasters evident in option prices
  - More modest than in macro data
  - Imply higher entropy than equity premium
  - Suggest high average risk aversion, greater aversion to bad outcomes

Open questions

- Time dependence
  - Short rate, predictable returns, stochastic volatility
  - Examples: Drechsler and Yaron (2008), Wachter (2008)
- Consumption and dividends
  - Examples: Gabaix (2009) , Longstaff and Piazzesi (2004)
- Source of apparent risk aversion
  - Exotic preferences
  - Heterogeneous agents
  - Examples: Alvarez, Atkeson, and Kehoe (2009); Bates (2008); Du (2008); Lustig and Van Nieuwerburgh (2005)