I will heavily lean on Ian Martin's paper titled "Disasters and asset pricing: evidence from option markets." I think he has all the ingredients, but has a different spin.

A preliminary remark. The research on joint dynamics of S&P 500 has arrived at three important conclusions regarding features a realistic model should have: (i) stochastic volatility; (ii) jumps in prices and volatility; (iii) prices of jump risk. The disaster story has no hope of capturing (i). We would have to introduce stochastic volatility of consumption as Bansal and Yaron (2004) and, most recently, Drechsler and Yaron (2008). I think we should abstract from this to focus on the disaster issues.

What is the drawback of abstracting from stochastic volatility? If we are to compare some data-based *conditional* likelihoods (using likelihood ratio, or KLIC, or Chernoff's entropy – I would love to use the latter!) the interpretation will be a mess because of the model misspecification. For example, is KLIC large because SV is missing or because Barro's calibration is wrong? It is hard to say.

However, we can pretend that a model without (i), but carefully calibrated otherwise, is the true DGP. Then we can check how far the Barro's version is off and what specifically does it mean. What do I mean under a carefully calibrated model? Let's take the SVJ model from BCJ2 (Broadie, Chernov, and Johanness on option returns) as a benchmark.

The model assumes that the ex-dividend index level, S_t, and its spot variance, V_t, evolve under the physical (or real-world) $\mathbb{P}$-measure according to

$$dS_t = (r + \mu - \delta) S_t dt + S_t \sqrt{V_t} dW_t^s(\mathbb{P}) + d\left(\sum_{j=1}^{N_t(\mathbb{P})} S_{\tau_{j-}} \left[e^{Z_j(\mathbb{P})} - 1\right]\right) - \omega^{\mathbb{P}} \overline{\mu}^{\mathbb{P}} S_t dt \quad (0.1)$$

$$dV_t = \kappa_v^{\mathbb{P}} \left(\theta_v^{\mathbb{P}} - V_t\right) dt + \sigma_v \sqrt{V_t} dW_t^v(\mathbb{P}), \quad (0.2)$$

where r is the risk-free rate, μ is the cum-dividend equity premium, δ is the dividend yield, W_t^s and W_t^v are two correlated Brownian motions ($E\left[W_t^s W_t^v\right] = \rho t$), $N_t(\mathbb{P}) \sim$ Poisson $\left(\omega^{\mathbb{P}} t\right)$, $Z_j(\mathbb{P}) \sim \mathcal{N}\left(\mu_z^{\mathbb{P}}, \left(\sigma_z^{\mathbb{P}}\right)^2\right)$, and $\overline{\mu}^{\mathbb{P}} = \exp\left(\mu_z^{\mathbb{P}} + (\sigma_z^{\mathbb{P}})^2/2\right) - 1$.

Options are priced using the dynamics under the risk-neutral measure $\mathbb{Q}$:

$$dS_t = (r - \delta) S_t dt + S_t \sqrt{V_t} dW_t^s(\mathbb{Q}) + d\left(\sum_{j=1}^{N_t(\mathbb{Q})} S_{\tau_{j-}} \left[e^{Z_j(\mathbb{Q})} - 1\right]\right) - \omega^{\mathbb{Q}} \overline{\mu}^{\mathbb{Q}} S_t dt \quad (0.3)$$

$$dV_t = \kappa_v^{\mathbb{P}} (\theta_v^{\mathbb{Q}} - V_t) dt + \sigma_v \sqrt{V_t} dW_t^v(\mathbb{Q}), \quad (0.4)$$

r	μ	$\omega^{\mathbb{P}}$	$\mu_z^{\mathbb{P}}$	$\sigma_z^{\mathbb{P}}$	$\sqrt{\theta_v^{\mathbb{P}}}$	$\kappa_v^{\mathbb{P}}$	σ_v	ρ
4.50%	5.41%	0.91	−3.25%	6.00%	13.51%	5.33	0.14	−0.52
		(0.34)	(1.71)	(0.99)	(1.28)	(0.84)	(0.01)	(0.04)

Table 1: $\mathbb{P}$-measure parameters. We report parameter values that we use in our computational examples. Standard errors from the SVJ estimation are reported in parentheses. Parameters are given in annual terms.

	$\omega^{\mathbb{Q}}$	$\mu_z^{\mathbb{Q}}$	$\sigma_z^{\mathbb{Q}}$	$\sqrt{\theta_v^{\mathbb{Q}}}$
Jump risk premia	1.51	−6.85%	$\sigma_z^{\mathbb{P}}$	$\sqrt{\theta_v^{\mathbb{P}}}$
Estimation risk	1.25	−4.96%	6.99%	14.79%
BCJ1 (estimated)	$\omega^{\mathbb{P}}$	−4.82%	9.81%	$\sqrt{\theta_v^{\mathbb{P}}}$

Table 2: $\mathbb{Q}$-measure parameters for the two scenarios that we explore. In addition, in the estimation risk scenario, we value options with the spot volatility $\sqrt{V_t}$ incremented by 1%.

where $N_t(\mathbb{Q}) \sim \text{Poisson}\left(\omega^{\mathbb{Q}} t\right)$, $Z_j(\mathbb{Q}) \sim \mathcal{N}\left(\mu_z^{\mathbb{Q}}, \left(\sigma_z^{\mathbb{Q}}\right)^2\right)$, $W_t(\mathbb{Q})$ are Brownian motions, and $\overline{\mu}^{\mathbb{Q}}$ is defined analogously to $\overline{\mu}^{\mathbb{P}}$.

Table 1 provides the estimated values of the $\mathbb{P}$ parameters and Table 2 displays the calibrated $\mathbb{Q}$ parameters. Alternatively, one could use parameters estimated in BCJ1 (the JF paper). The key point is that a reasonable way to get read of stochastic volatility is, of course, to assume a constant one and to set it equal to the value in column $\sqrt{\theta_v^{\mathbb{P}}}$ of Table 1. This substitution will lead to screwed-up dynamics of the index returns, but many unconditional moments (not the serial correlation of volatility) and realistic option pricing ("average" implied volatility curves, or average option returns) will remain intact.

I started a little bit backwards, but I just wanted to motivate a Merton-style model and throw out some empirically plausible parameter values. Now, let's go back to consumption (I am trying to use notation as close to Martin's as possible).

$$dC_t = \left(\mu_c + \frac{1}{2}\sigma_c^2\right) C_t dt + \sigma_c C_t dW_t^c(\mathbb{P}) + d\left(\sum_{j=1}^{N_t(\mathbb{P})} C_{\tau_{j-}}\left[e^{Y_j(\mathbb{P})} - 1\right]\right), \qquad (0.5)$$

where μ_c is the consumption growth rate, W_t^c is a Brownian motion, $N_t(\mathbb{P}) \sim \text{Poisson}\left(\omega^{\mathbb{P}} t\right)$, $Y_j(\mathbb{P}) \sim \mathcal{N}\left(-b, \psi^2\right)$.

	μ_c	σ_c	$\omega^{\mathbb{P}}$	b	ψ
Martin's values	0.025	0.02	0.017	0.39	0.24
Chernov's values	0.018	0.02	0.017	0.20	0.10

Table 3: Two sets of $\mathbb{P}$-measure parameters of C. Martin's values are backed out of Barro's calibration. Chernov's values are selected to match the mean and variance of consumption, 0.018 and 0.035^2, respectively.

This model implies that the mean and variance of consumption growth are:

$$m = \mu_c - \omega^{\mathbb{P}}(-b) \tag{0.6}$$

$$s^2 = \sigma_c^2 + \omega^{\mathbb{P}}\left(b^2 + \psi^2\right) \tag{0.7}$$

Table 3 provides two sets of calibrations of the process.

Assuming power utility with SDF equal to $\exp(-\rho t)(C_t)^{-\gamma}$, returns on the levered asset (λ) evolve according to:

$$\begin{aligned} R_\lambda dt &= \left[\rho + \gamma\mu_c - \frac{1}{2}\gamma(\gamma - 2\lambda)\sigma^2 - \omega^{\mathbb{P}}\left(e^{b(\gamma-\lambda)+\frac{1}{2}(\gamma-\lambda)^2\psi^2} - 1\right)\right] dt \\ &+ \lambda\sigma_c dW_t + d\left(\sum_{j=1}^{N_t(\mathbb{P})} C_{\tau_{j-}}\left[e^{\lambda Y_j(\mathbb{P})} - 1\right]\right). \end{aligned} \tag{0.8}$$

In particular,

$$R_f = R_0 = \rho + \gamma\mu_c - \frac{1}{2}\gamma^2\sigma^2 - \omega^{\mathbb{P}}\left(e^{b\gamma+\frac{1}{2}\gamma^2\psi^2} - 1\right) \tag{0.9}$$

$$EP_\lambda = \lambda\gamma\sigma^2 + \omega^{\mathbb{P}}\left(e^{-b\lambda+\frac{1}{2}\lambda^2\psi^2} - 1\right) + \omega^{\mathbb{P}}\left(e^{b\gamma+\frac{1}{2}\gamma^2\psi^2} - 1\right) - \omega^{\mathbb{P}}\left(e^{b(\gamma-\lambda)+\frac{1}{2}(\gamma-\lambda)^2\psi^2} - 1\right)$$

$$s_\lambda = \lambda s \tag{0.10}$$

$$Z_j(\mathbb{P}) = \lambda Y_j(\mathbb{P}) \sim \mathcal{N}\left(-\lambda b, (\lambda\psi)^2\right) \equiv \mathcal{N}\left(\mu_z^{\mathbb{P}}, \left(\sigma_z^{\mathbb{P}}\right)^2\right). \tag{0.11}$$

The calibrated values of the three extra parameters are provided in Table 4.

Finally, option pricing requires the knowledge of $\mathbb{Q}$ parameters. The Naik and Lee

	ρ	γ	λ
Martin's values	0.03	4.00	1
Chernov's values	0.03	6.75	4

Table 4: Two sets of preference parameters.

	s_λ	EP	R_f	$\omega^{\mathbb{P}}$	$\mu_z^{\mathbb{P}}$	$\sigma_z^{\mathbb{P}}$
Data	0.150	0.054	0.045	0.910	-0.03	0.06
Martin	0.063	0.054	0.016	0.017	-0.39	0.24
Chernov	0.141	0.054	0.077	0.017	-0.80	0.40

Table 5: The implications for $\mathbb{P}$ parameters.

(1990) results imply:

$$\omega^{\mathbb{Q}} = \omega^{\mathbb{P}} \exp\left(-\mu_z^{\mathbb{P}}\gamma + \frac{1}{2}\gamma^2(\sigma_z^{\mathbb{P}})^2\right) \quad (0.12)$$

$$\mu_z^{\mathbb{Q}} = \mu_z^{\mathbb{P}} - \gamma(\sigma_z^{\mathbb{P}})^2, \quad (0.13)$$

What are the implications of these calibrations? Table 5 provides the answer for $\mathbb{P}$ parameters. As you can see, I have calibrated the preference parameters to better match the volatility and risk-free rate of the index returns. However, I could not match the jump parameters estimated in the data (see Table 1). Table 6 displays implications for the $\mathbb{Q}$ parameters. I do it in two ways. "Moving from C" means that I start with a calibration of the consumption process. "Moving from S" means starting from the empirically-based process for S, assuming that $S = W$, that is the index coincides with the wealth portfolio, and, finally, that preferences are over wealth. Comapring these with Table 2, we see that everything is way off.

		$\omega^{\mathbb{Q}}$	$\mu_z^{\mathbb{Q}}$
Moving from C	Martin	0.128	-0.62
	Chernov	144.092	-1.88
Moving from S	Martin	1.067	-0.05
	Chernov	1.23	-0.06

Table 6: The implications for $\mathbb{Q}$ parameters.