# Dynamic Inconsistency and Self-Control: A Planner-Doer Interpretation

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Abstract: *We show that Gul and Pesendorfer's (2001) representation result for preferences with temptation and self-control can be reexpressed in terms of a costly intrapersonal conflict between a Planner and Doer, as in Thaler and Shefrin (1981) and psychologists' standard view of self-control problems.*

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Faruk Gul and Wolfgang Pesendorfer (2001) develop an axiomatic theory of intertemporal choice that captures temptation and costly self-control. Building on their main result, we show that there is a natural interpretation of the class of preferences they study in terms of a costly intrapersonal conflict between two contemporaneous "subselves", or interests. Formally, a Planner and a Doer (Thaler and Shefrin (1981)) play a costly influence or lobbying game, the outcome of which stochastically determines whether the individual resists or succumbs to temptation. These two players could also be thought of as the Ego and the Id (Freud (1927)), or the brain's prefrontal cortex and limbic system. Our result shows that while Gul and Pesendorfer's approach offers a way to avoid time inconsistency by extending the space over which preferences are defined, it remains in line with psychologists' view of self-control problems as reflecting a *"divided self"*.

Gul and Pesendorfer consider an individual who takes decisions in two stages. In the first period he chooses among (or, at least, evaluates) sets of alternatives that may be available in the second, final period. The individual is sophisticated, meaning that in the first stage he understands how he will later on choose among the feasible options. This situation is modelled with a preference relation $\succcurlyeq$ over the space $\mathcal{A}$ of compact subsets of the $(n-1)$-dimensional simplex

$$\Delta = \{x = (x_1, \ldots, x_n) \in R^n_+ : ||x|| = |x_1| + \ldots + |x_n| = 1\}, \tag{1}$$

endowed with Hausdorff metric and a lottery operation:

$$\alpha A + (1-\alpha)B = \{\alpha x + (1-\alpha)y : x \in A, y \in B\}, \alpha \in [0,1]. \tag{2}$$

The preference relation over these opportunity sets is assumed to satisfy the following axioms.

**Axiom 1** *(Rationality) The preference relation $\succcurlyeq$ is a complete and transitive binary operation.*

**Axiom 2** *(Strong Continuity) The sets $\{B : B \succcurlyeq A\}$ and $\{B : B \succcurlyeq A\}$ are closed, for all $A$.*

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**Axiom 3** *(Independence)* $A \succ B$ *and* $\alpha \in (0,1)$ *implies* $\alpha A + (1-\alpha)C \succ \alpha B + (1-\alpha)C$.

**Axiom 4** *(Set Betweenness)* $A \succcurlyeq B$ *implies* $A \succcurlyeq A \cup B \succcurlyeq B$.

The last axiom allows for the traditional form of dynamic inconsistency and desire for commitment: when $A \succ A \cup B \equiv B$, the individual is better off excluding $B$ from his future options, as he knows that he would not resist the temptation to choose from $B$ instead of $A$, which he prefers ex-ante.[1] But Axiom 4 also allows for the more novel situation where the individual is able to exercise self-control, albeit at some cost. The case $A \succ A \cup B \succ B$ thus captures circumstances where, in period 2, the individual with choice set $A \cup B$ is able to resist the temptation(s) of $B$ and still choose an action in $A$, but where the availability of the options in $B$ makes this self restraint costly.

The main result in Gul and Pesendorfer's article reads as follows.

**Theorem 1** *(Gul and Pesendorfer) The binary relation* $\succcurlyeq$ *satisfies Axioms 1–4 if and only if there exist continuous linear functions* $u$ *and* $v$ *on* $\Delta$ *such that*

$$U(A) \equiv \max_{x \in A}(u(x) + v(x)) - \max_{y \in A} v(y)$$

*represents the ordering* $\succcurlyeq$ *over* $\mathcal{A}$.

To interpret this representation theorem for date-1 rankings of sets in terms of temptation and self-control at date 2, Gul and Pesendorfer rely on three additional assumptions: i) a *behavioral* assumption: in the second period, the agent always chooses so as to maximize $u + v$ over his opportunity set $A$; ii) a *cardinal* representation: when choosing any $x \in A$, the agent achieves utility level $U^*(A, x) \equiv u(x) + v(x) - \max_{y \in A} v(y)$; note that the last term in these "extended" preferences does not affect any date-2 decision, but matters for welfare when evaluating different opportunity sets; iii) a *time-consistency* assumption: in period 1, the agent's preferences over choice sets, denoted $\succcurlyeq_1$, "agree" with the way he will rank them at time 2: $A \succcurlyeq_1 B$ if and only if $\max_{x \in A} U^*(A, x) \geq \max_{x \in B} U^*(B, x)$. Theorem 1 then shows that this induced date-1 preference ordering, $\succcurlyeq_1$, coincides with the original $\succcurlyeq$ satisfying Axioms 1–4. The interpretation in terms of temptation and self-control can now be seen from the following observations. Let $A \in \mathcal{A}$, and denote by $x \in \arg\max_{x' \in A} \{u(x') + v(x')\}$ an optimal choice in the second period. If the individual in period 1 could commit to selecting $x$ in period 2, he would achieve the utility level corresponding to this singleton choice set, namely $U(\{x\}) = u(x)$. Gul and Pesendorfer thus refer to $u$ as the *commitment ordering* over outcomes. When the choice set in period 2 includes other alternatives, however, the individual's ex-ante (period 1) utility will generally be lower, even though he ends up choosing the very same $x$ : Theorem 1 implies a welfare loss of $U(\{x\}) - U(A) = \max_{y' \in A} v(y') - v(x)$ that can be interpreted as the cost of self-control, equal to the intensity of the temptation that the individual resists by choosing $x$ instead of $y \in \arg\max_{y' \in A} v(y')$. Accordingly, Gul and Pesendorfer refer to $v$ as the agent's *temptation ordering.*

Let us now observe that there are really *two sets of preferences* that are relevant to describe the agent at date 2. The first are those according to which he makes his actual choices from the feasible set, $u + v$. The

[1] By $A' \equiv B'$ we denote the equivalence relation: $A' \geq B'$ and $B' \geq A'$.

second are described by his temptation ranking, $v$, and correspond to the choices he is "tempted to make", in the sense that he suffers a loss from not making them. Thus, while the original, *intertemporal* form of the individual's internal conflict (time-inconsistency) has been resolved, the conflict implicitly reemerges in an *intratemporal* form, namely the divergence between the two sets of preferences needed to describe date-2 desires and actual choices (or, equivalently, date-2 choices and their welfare consequences).

Our aim here is to acknowledge this tension by explicitly modelling the situation as one of an intrapersonal conflict, drawing on the long tradition in psychology that views an individual as composed of different subselves with competing objectives (or, more recently, that emphasizes the specialization of different regions or "modules" in the brain).

This will be done in two steps. The first one (which may be of independent interest) involves formalizing the individual's date 2 behavior as characterized by a risk of *losing control* –that is, just caving in to temptation and choosing according to $v$. By contrast, in Gul and Pesendorfer's interpretation the agent always successfully exerts a (constant) measure of self-control, except in the limiting case where the cost of resisting temptation is infinite.[2]

Suppose that, in period 2, the agent with an opportunity set $A$ will either exercise some self-control and choose an $x \in \arg\max_{x' \in A} \{u(x') + v(x')\}$, or completely cave in to temptation and choose a $y \in \arg\max_{y' \in A} v(y')$, with the following probabilities:

$$\text{the agent chooses } \tilde{x} = \begin{cases} x \text{ with probability} & p_x \equiv \frac{u(x)+v(x)-u(y)-v(y)}{u(x)-u(y)} \\ y \text{ with probability} & p_y \equiv \frac{v(y)-v(x)}{u(x)-u(y)}. \end{cases} \tag{3}$$

Quite intuitively, the probability of the agent choosing at date 2 according to $u + v$ or $v$ reflects the relative intensity of these preferences, which in turn depends on how different $u$ and $v$ are. Turning now to period 1, let the individual simply evaluate date-2 lotteries by their expected utility according to his ex-ante preferences, $u$, which thus also correspond to is commitment preferences. The choice set $A$ will then result in a utility level:[3]

$$\begin{aligned} Eu(\tilde{x}) &= u(x)\left(\frac{u(x)+v(x)-u(y)-v(y)}{u(x)-u(y)}\right) + u(y)\left(\frac{v(y)-v(x)}{u(x)-u(y)}\right) \\ &= \max_{x' \in A}(u(x') + v(x')) - \max_{y' \in A} v(y') = U(A). \end{aligned} \tag{4}$$

Theorem 1 thus shows that this probabilistic choice behavior in period 2 induces *the very same* date-1 preferences $\succcurlyeq$ over choice sets as those obtained in Gul and Pesendorfer under assumptions (i)–(iii), and satisfying Axioms 1-4. On the other hand, whereas the welfare loss $\max_{y' \in A} v(y') - v(x)$ in Theorem 1 corresponds in their framework to the anticipation of a psychic cost of resisting temptation, it arises here from the individual's knowledge that, with some probability, he will actually cave in to temptation: $\max_{y' \in A} v(y') - v(x) = p_y(u(x) - u(y))$.

[2] In this limiting case, which correspond to $v/u \to +\infty$, Gul and Pesendorfer's' Theorem 1 takes a slightly different form.

[3] From period 1's point of view, it does not matter which specific $x \in \arg\max_A \{u + v\}$ and $y \in \arg\max_A \{v\}$ are chosen at date 2. Note also that $u(x) \geq u(y)$ (because $u(x) + v(x) \geq u(y) + v(y)$ and $v(y) \geq v(x)$), and that the above–defined probabilities always belong to $(0, 1)$, unless $u(x) = u(y)$, in which case they become irrelevant.

In the second stage of our interpretation, we shall derive these choice probabilities as the outcome of an *intrapersonal game* played among subselves of the individual in period 2. As pointed out for instance by Thaler and Shefrin (1981), a division of the self into conflicting subselves coexisting at the same point in time is how psychologists usually think about self-control. Indeed, the very etymology of the world "self-control" suggests such a modelling strategy. Thaler and Shefrin thus divide the self into one *Planner* (common to all moments in time) and many *Doers* (one per moment in time), and allow the Planner some influence over the Doer's decisions. This is achieved through the costly control of a "preference modification parameter," which Thaler and Shefrin offer as a reduced-form representation of more concrete incentives (rewards, punishments) or rules put into place by the Planner. We shall draw on and extend their model, which focuses on consumption-savings decisions and does not explicitly formalize the process through which the intrapersonal conflict is resolved.

Let us thus consider the agent's self at date 2 as consisting of two contemporaneous subselves, actors, or interests:

- A short-sighted *Doer,* who only takes into account the second-period temptation preferences $v$.
- A long-lived *Planner,* who takes into account both the commitment and the temptation preferences (first and second-period utilities) $u$ and $v$, weighting them equally. His utility function is thus $u + v$.

In a multi-period context $u$ could for instance reflect long-run preferences over date 2 actions, while $v$ reflected short-run preferences, marked by a "salience of the present".[4]

As in standard models of social or political conflict (e.g., Esteban and Ray (1999)), let the Doer and Planner spend resources –nervous impulses, energy, Freudian "libido," etc.– to obtain their preferred outcome; we denote these resources as $r_D$ and $r_P$ One can also think of the two subselves as lobbying the brain's motor control areas, in the same way as interest groups lobby the government in Becker (1985). The technology of conflict or influence is assumed to be such that the outcome reflects relative resource expenditures: the Doer wins with probability $p_y \equiv \frac{r_D}{r_D + r_P}$, while the Planner prevails with probability $p_x \equiv \frac{r_P}{r_D + r_P}$.

The short-sighted Doer thus chooses $r_D$ to solve

$$\max_{r_D} \left\{ \left( \frac{r_D}{r_D + r_P} \right) v(y) + \left( \frac{r_P}{r_D + r_P} \right) v(x) - r_D \right\}, \tag{5}$$

while the long-sighted Planner's problem is

$$\max_{r_P} \left\{ \left( \frac{r_P}{r_D + r_P} \right) (u + v)(x) + \left( \frac{r_D}{r_D + r_P} \right) (u + v)(y) - r_P \right\}, \tag{6}$$

[4] Thus in the familiar consumption–savings problem with quasi–hyperbolic discounting, $u = \Sigma_{t=1}^{T-1} \delta^t c_{t+1}$ and $v = \delta \left( c_2 + \beta \Sigma_{t=2}^{T-1} \delta^{t-1} c_{t+1} \right)$, where $0 < \beta, \delta < 1$ and $\{c_t\}_{t=1}^{T}$ denotes the sequence of consumption levels.

where $x$ and $y$ were defined above. The Nash equilibrium of this game is given by the first-order conditions:

$$r_P = \frac{(r_D + r_P)^2}{v(y) - v(x)} \tag{7}$$

$$r_D = \frac{(r_D + r_P)^2}{(u+v)(x) - (u+v)(y)} \tag{8}$$

The unique solution $(r_D, r_P)$ is easily seen to yield the same probabilities $(p_x, p_y)$ that were postulated in (3), and result in ex-ante utility $u(\tilde{x}) \equiv p_x u(x) + p_y u(y) = U(A)$.[5] Hence the result.

Our model is easily extended to capture the role of cues and other salient stimuli that affect the intensity of temptation, even though the choice set remains constant. These can be thought of as influencing the Doer's relative power in the struggle over decision-making, so that the resource costs $(r_D, r_P)$ now translate into outcome probabilities $p_y = 1 - p_x \equiv \frac{\theta r_D}{\theta r_D + r_P}$, with $\theta \geq 0$ measuring salience. The equilibrium of the influence game is now easily seen to imply odds of caving in to temptation, $p_y/p_x$, that rise proportionally to $\theta$. This, in turn, increases the individual's ex-ante welfare loss.

We close this note with a conjecture, namely that the close correspondence established here in a two-period context between "temptation preferences with self-control" and multi-selves models is quite general, and likely to extend to dynamic settings, such as those considered in Gul and Pesendorfer (2000).

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[5] Recall that in our interpretation where the cost of a wider choice set is purely the risk of succumbing to temptation, the individual at date 1 cares only about the actual choices made in period 2 (or their long-run consequences). Thus the influence costs $r_P$ and $r_D$ spend by Planner and Doer in period 2 are not subtracted from his period 1 welfare $u(\tilde{x})$.