

Harmful Addiction[†]

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February 2004

Abstract

We construct an infinite horizon consumption model and use it to define and analyze harmful addiction. Consumption is *compulsive* if it differs from what the individual would have chosen had commitment been available. A good is *addictive* if its consumption leads to more compulsive consumption of the same good. A policy is *prohibitive* if it decreases the maximal feasible drug consumption. A *price policy* is one that increases the opportunity cost of drug consumption. We show that purely prohibitive policies make the agent better-off and pure price policies make him worse-off. Our analysis of demand for commitment identifies three regimes: If commitment (rehab) is expensive the agent never utilizes it and his drug consumption increases over time. If commitment is sufficiently inexpensive he enters and stays in rehab forever. The intermediate range is characterized by a *cycle of addiction* where the agent periodically checks into rehab for a single period. Between these visits his drug consumption increases each period.

[†] This research was supported by grants from the National Science Foundation.

1. Introduction

Substantial resources are spent to reduce the availability of and the demand for drugs. These efforts are justified by the belief that drug addiction is a serious health and social problem. What is special about drugs that could justify restricting its supply and demand?

Standard economic analysis uses the individuals' choice behavior as a welfare criterion. Alternative x is deemed to be better for the agent than alternative y if and only if given the opportunity, the agent would choose x over y . Restricting the available options for an individual can never be welfare improving in a standard economic model. Such models provide a clear welfare criterion for evaluating the individuals welfare but offer no rationale for treating drugs differently than other consumption goods.

While typical in economic analysis, the identification of welfare and choice is certainly not the norm in the analysis of addiction by healthcare professionals. Instead, addiction is often viewed as a disease that impedes the agent's decision-making ability.¹ It is believed that after being struck by the disease, a person can no longer be trusted to make the right decision for his "true" self.² The role of intervention is to "cure" (i.e. induce abstinence) or at least "control" (i.e. reduce consumption) the disease.

Viewing addiction as a disease creates a wedge between choice and welfare and hence a rationale for policy interventions that modify the addict's choices. However, rejecting the equivalence between the individuals preferences and his welfare creates the need for a new welfare criterion for evaluating the costs and benefits of these interventions. Consider a costly treatment that, if successful, would remove the agent's drug dependency (i.e. cure the disease). For someone who considers addiction a disease, if the probability of success is positive, a sufficiently low cost of treatment renders is desirable regardless of whether the agent thinks so or not. Conversely, if the cost of treatment is sufficiently high then the treatment is undesirable. But how is the trade-off between the probability of success and the cost of the treatment to be made in the wide range of intermediate cases? Healthcare

¹ "Is alcoholism a disease? Yes. Alcoholism is a chronic, often progressive disease with symptoms that include a strong need to drink despite negative consequences, such as serious job or health problems." (cited from: National Institute on Alcohol Abuse and Alcoholism. <http://silk.nih.gov/silk/niaaa1/questions/q-a.htm#question2>)

² There are numerous criticisms of the disease model of drug addiction (see for example, Davies (1992)).

professionals' view of addiction assumes a need for policy but offers little guidance as to how such policies are to be evaluated.

In this paper we provide an economic model of (harmful) addiction. In our model each individual is characterized by a single preference that describes both his behavior and his welfare. Therefore, if our analysis shows that a particular policy is welfare improving then this assertion can be falsified. This contrasts with the approach of healthcare professionals and models based on quasi-hyperbolic discounting where the analyst must introduce his own criterion to investigate the welfare of the agent. At the same time, our model is consistent with the view that addicts benefit from policies that restrict drug consumption. We build on previous work (Gul and Pesendorfer (2001)) that allows the agent's welfare to depend both on what he chooses and on the set of options from which the choice is made. This set may contain tempting alternatives that reduce the agent's welfare either by distorting his choice or by necessitating costly self-control or both. In particular, drug consumption constitutes a temptation. Moreover, current drug consumption affects how the agent will respond to temptations in future periods. Specifically, the agent is more likely to give in to tempting drug consumption if he has consumed an addictive drug in the past.

We define (harmful) addiction as follows: first, we introduce the notion of *compulsive consumption*. An individual is *compulsive* if his choice differs from what he would have chosen had commitment been possible. An agent is more compulsive after consumption history A than after consumption history B if for every decision problem in which the agent is compulsive after B he is also compulsive after A. The drug is *addictive* if an increase in drug consumption makes the agent more compulsive. Hence, a harmful addiction is defined as a widening of the gap between the individual's choice and what he would have chosen before experiencing temptation. Healthcare professionals define addiction through the underlying physiological processes or by comparing the individuals consumption choices with external social standards of acceptability. In contrast, our notion of harmful addiction relies only on revealed choice and compares the individuals behavior to his own behavior under different circumstances.

To see how our model works, consider an agent who must choose between drug consumption (d) and non-drug consumption (c) from a budget set B_t in period t . For simplicity, we assume that there is no saving. The dynamic program below characterizes the agent's utility as a function of last period's drug consumption. Let $W(d_{t-1})$ denote the utility (value) function in period t , then

$$W(d_{t-1}) = \max_{\{(c,d) \in B_t\}} \{u(c, d) + \sigma(d_{t-1})v(d) + \delta W(d)\} - \max_{\{(\hat{c}, \hat{d}) \in B_t\}} \sigma(d_{t-1})v(\hat{d})$$

We interpret $\sigma(d_{t-1})v$ as the temptation utility and call $u + \delta W$ the commitment utility. To understand this terminology, note that if all options were equally tempting – that is, resulted in the same v – then the v -terms in equation 1 would drop-out. Therefore, such consumption problems would be evaluated according to $u + \delta W$. For example, when B_t consists of a single choice (c, d) , the overall utility of the current decision problem is the commitment utility $u + \delta W$ of the single option (c, d) . The commitment utility $u + \delta W$ is independent of past drug consumption while the temptation utility $\sigma(d_{t-1})v$ depends on last period's drug consumption. Addiction exerts its influence through this dependence.

The individual's choice (c, d, x) maximizes $u + \sigma(d_{t-1})v + \delta W$. This choice reflects the compromise between the commitment utility and temptation. An individual is *compulsive* if his choice (the $u + \sigma(d_{t-1})v + \delta W$ maximizer) does not maximize his commitment utility. A drug is *addictive* if an increase in drug consumption leads to more compulsive drug consumption.

In Proposition 1, we show that an increase in σ implies that the agent is more compulsive. Hence, if σ is an increasing function then the drug is addictive. An increase in σ implies that the agent's utility function places a relatively smaller weight on commitment utility and hence the gap between optimal commitment choices and actual choices widens. Proposition 2 shows that consumption of an addictive drug is reinforcing, that is, higher drug consumption in the current period leads to higher drug consumption in future periods.

Section 3 examines the effect of price changes on drug demand. We show that drug demand decreases if the current price of the drug increases. If the drug is addictive, drug demand also decreases if the future price of the drug increases.

Section 4 analyzes the welfare effects of drug policies. We assume that the agent faces a fixed budget set in every period. The government can affect this budget set by changing the price of the drug (price policy) or by reducing the maximally feasible drug consumption (prohibitive policy). Many actual policies will change the price of the drug and the maximally feasible drug consumption simultaneously. We separate these two effects in order to identify the source of welfare effects. An example of a policy with mostly prohibitive effects is a ban on drug consumption. A tax on the drug will have mostly price effects if drug consumption is a relatively small part of an agent's budget and hence the tax does not affect the maximally feasible drug consumption in a given period. A *pure price policy* refers to a policy that has only price effects whereas a *purely prohibitive policy* refers to a policy with only prohibitive effects.

We show that a pure price policy always reduces the agents welfare. A pure price policy makes it more costly to consume the drug but does not change the most tempting alternative. In response to a pure price policy the agent will consume less of the drug and exercise more costly self-control. By a simple revealed preference argument, the increased cost of self-control is always larger than the possible utility gain from reduced drug consumption. The key feature of a pure price policy is that it does not remove temptations from the agent's choice set while it increases the cost of drug consumption. Such a policy will reduce drug consumption but also decrease welfare.

To examine the effect of a prohibitive policy, we focus on the special case where the optimal drug consumption is zero when the agent can commit. We show that if the drug is addictive then a prohibitive policy increases welfare. A prohibitive policy changes the most tempting alternative without affecting the price of the drug. We also examine how drug demand changes when a prohibitive policy is introduced. If the prohibitive policy is not binding then a reduction in the maximally allowed drug consumption will increase drug demand. To see the intuition for this result note that current drug consumption makes self-control more costly in future periods. If the maximally feasible drug consumption is reduced then this cost is smaller and hence current drug consumption is more attractive.

Together, our welfare results show that welfare improvement stems from the commitment effect of a policy while the price effects reduce welfare. Moreover, if a policy reduces

drug demand then this cannot be taken as an indication that the policy “works” in the sense of welfare improvement. This is true even though the drug is unambiguously “bad”, that is, the optimal drug consumption under commitment is zero.

Section 5 analyzes a decision problem in which the agent has the option of checking into a “rehabilitation center”. In our model, rehabilitation centers provide temporary and costly commitment to zero drug consumption. Drug treatment programs offer a variety of treatments, many of them go beyond simple commitment. However, making drugs difficult to acquire seems to be a common feature of most treatment programs. Treatment programs remove patients from their familiar surroundings, closely monitor their activities, and ensure that drugs are not available on the premises. In this way, they offer temporary commitment. Clearly, voluntary treatment programs cannot offer permanent commitment since agents can leave at any moment. For our purposes the key feature is that *immediate* drug consumption is not possible.

In our analysis of the rehab problem, the agent faces a fixed budget in every period and can choose to enter a rehabilitation center. The decision to enter rehab results in a commitment to zero drug consumption for the subsequent period but is costly in terms of the agent’s non-drug consumption. Our model predicts that agents enter rehab after their drug consumption has reached its peak. Moreover, we show that after the visit to the rehabilitation center agents can be expected to follow a pattern of increasing drug consumption followed by another visit to the rehabilitation center. We also demonstrate that the expectation of attending a rehabilitation center in the next period increases current drug demand. Hence, agents will “go on a binge” just before entering rehab.

Section 6 provides a foundation for the preferences analyzed in this paper. We provide axioms that imply the representation used in the text. The key difference to the representation found in our earlier work is that preferences may depend on past (drug) consumption.

1.1 Related Literature

We organize the existing economic literature on addiction into two groups; the standard economic models and models that follow a multiselves approach. We contrast both

of these approaches to our approach. In this subsection, we compare the welfare analysis of the alternative approaches. In the next section, we focus on empirical implications.

What we have been calling the standard economic model identifies addiction with intertemporal complementarities. Becker and Murphy (1986) view the consumption of an addictive good much like an investment that affects the utility of future consumption. For Becker and Murphy, addictive consumption is beneficial if, compared to alternative consumption choices, it entails a decrease in current utility in exchange for an increase in future utility. Conversely, addictive consumption is harmful if it entails an increase in current utility in exchange for a decrease in future utility. Hence, indulging in the consumption of a beneficial addictive good is exactly like investing; the agent forgoes current reward in exchange for higher future payoffs, while consumption of a good that is defined as a harmful addiction is like disinvesting.

Regardless of whether the addiction is harmful or beneficial, the availability of drugs is never bad for the agent in the Becker-Murphy model. Be it harmful or beneficial, their agents engage in the consumption of an addictive good if and only if the perceived trade-off between current and future utility warrants the consumption. Policy interventions cannot improve such agents' welfare. This is analogous to the observation that restricting or forcing investment cannot improve the payoff of a profit maximizing firm. It is in this sense that the Becker and Murphy preferences are standard; their analysis of addiction boils down to evaluating intertemporal cross-elasticities of drug demand.

There are at least two criticisms that can be levied at the Becker-Murphy model of addiction. First, in order to use their key distinction between harmful and beneficial addictions we need to observe the timing of *utility* flows and not just of consumption. Since optimal choices rely only on the discounted present values of these flows and not on their timing, harmful and beneficial addictions cannot be distinguished through observed behavior.

Second, the Becker-Murphy formulation entails an a priori rejection of “the problem” of addiction. Their reliance on standard dynamic preferences ensures that regardless of the details of the subsequent analysis of demand, there will be no room for welfare enhancing drug policy. A harmful addiction is harmful in the same way that disinvestment is harmful;

it increases current payoff at the expense of lower future payoffs. The model offers no argument for why individuals might be more likely to struggle with harmful addictions than with any other consumption decision. Hence, the central concepts of the Becker-Murphy model rely on a distinction that is both difficult to observe and has little relevance for policy.

The inability of standard economic models to identify addiction as a problem has led researchers to seek an alternative model of the decision-maker. O'Donoghue and Rabin (1999) and Gruber and Koszegi (2002) introduce models based on Strotz's (1955) analysis of changing preferences as well as the subsequent work of Phelps and Pollak (1968) and Laibson (1997). In this literature, the decision-maker is viewed as a sequence of distinct agents – called the (multi)selves. Each self has a different preference over consumption streams. Hence, the period t self's choice of alternative x over y reflects only the fact that given the predicted behavior of the subsequent selves, x leads to a consumption stream that is better for the period t self than the one induced by y . Other selves may be and often are made worse-off by this choice. In a multiselves model, the individual selves do not value commitment per se; commitment has value only as a vehicle for one of the selves to impose his preferences on subsequent selves. Therefore, policies that restrict current consumption can be rationalized by appealing to the need for protecting the interests of the past or future selves.

To see how multiselves models generate intrapersonal conflict and hence a role for policy, consider the following simple example: for any consumptions stream $x = \{d_t\}$, $t = 1, 2, \dots$, define the utility function of the period- t self as follows:

$$U_t(x) = u(d_t) - \alpha u(d_{t-1}) + \beta \delta \sum_{\tau=t} [u(d_{\tau+1}) - \alpha u(d_{\tau})] \delta^{\tau-t} \quad (MS)$$

where $\alpha > 0$, $\delta \in (0, 1)$, $\beta \in (0, 1]$ and u is an increasing function. The functional form is interpreted as the description of a flow of utility; in period τ , this flow is $u(d_{\tau}) - \alpha u(d_{\tau-1})$. Hence, the effect of the drug consumption d_{τ} is divided into two components, $u(d_{\tau})$ at time τ and $-\alpha u(d_{\tau})$ which accrues in period $\tau + 1$ and is meant to capture the negative effect of period τ drug consumption on future well being. The self at $\tau + 1$ derives no utility from past drug consumption but suffers from the consumption in period τ .

Assume that $\beta\alpha\delta < 1 < \alpha\delta$, $u(0) = 0$ and $u(1) = 1$. Consider two consumption plans $x = \{d_t\}$, $y = \{\hat{d}_t\}$ such that $d_t = \hat{d}_t = 0$ for all $t > 1$, $\hat{d}_1 = 0$ and $d_1 = 1$. Then

$$\begin{aligned} U_1(x) &= 1 - \beta\alpha\delta > U_1(y) = 0 \\ U_2(x) &= -\alpha < U_2(y) = 0 \end{aligned}$$

The conflict between the period-1 and period-2 selves emerges from two sources; first the period-2 self suffers the negative consequences of drug consumption but does not share in the pleasures enjoyed by the period-1 self. Second, for any given increment of period-2 utility, the period-2 self is willing to give up more period 3 utility than the period-1 self. The latter effect arises from nonexponential discounting $\beta < 1$; the former is present even in the case of exponential discounting. The existence of multiple selves creates a need for a welfare criterion for evaluating policy alternatives. Finding a suitable welfare criterion for the multiselves model turns out to be a difficult task.

O'Donoghue and Rabin (1999) and Gruber and Koszegi (2002) use the following function

$$SW = (1 - \alpha\delta) \sum_{\tau=1}^{\infty} u(d_{\tau})\delta^{\tau} \quad (S)$$

as a welfare criterion for the multiselves model described by (MS). They use this function for all values of β . In effect, this corresponds to the utility function of a period-0 self that does not suffer from the presence-bias exhibited by all of the subsequent selves. Applied to example above, this social welfare function yields

$$SW(x) = (1 - \alpha\delta) < SW(y) = 0$$

Hence, the O'Donoghue-Rabin/Gruber-Koszegi planner rules in favor of the period-2 self and would prohibit the period-1 self from consuming drugs.

Consider the following alternative utility specification:

$$U_t^*(x) = (1 - \alpha\beta\delta) \left[u(d_t) + \beta\delta \sum_{\tau=t}^{\infty} u(d_{\tau+1})\delta^{\tau-t} \right] \quad (MS^*)$$

In this case, drug consumption d_t affects only the flow of utility at time t , $((1 - \alpha\beta\delta)u(d_t))$.

Note that

$$U_t^*(x) - \alpha u(d_{t-1}) = U_t(x)$$

Since the term $u(d_{t-1})$ cannot affect behavior in period t it follows that (MS) and (MS^*) lead to exactly the same behavior. The only difference is the timing of utility flows. However, applying the O'Donoghue-Rabin/Gruber-Koszegi welfare criterion for the multiselves model (MS^*) yields

$$SW^* = (1 - \alpha\beta\delta) \sum_{\tau=1}^{\infty} u(d_{\tau})\delta^{\tau} \quad (S^*)$$

Comparing x and y according the social welfare function SW^* establishes

$$SW^*(x) = (1 - \alpha\beta\delta) > 0 = SW^*(y)$$

Hence, for the agent described in (MS^*) the O'Donoghue-Rabin/Gruber-Koszegi planner would rule in favor of x over y . Yet as we noted above, the choice between the two models relies on assumptions regarding when utilities are enjoyed. This issue cannot be resolved with any choice experiment and no amount of market data can enable the researcher or planner to distinguish between the cases where (MS) or (MS^*) are appropriate. Even if we were willing to use other data besides choice behavior, it is not clear what data is relevant for answering such questions and how the planner would have access to such data.

Instead of choosing the period 0 utility function as a welfare criterion, some authors (see Laibson (1997)) have proposed the stronger requirement that *all* selves be made better-off by a welfare improving policy. This welfare criterion suffers from many of the same flaws as the O'Donoghue-Rabin/Gruber-Koszegi criterion. Moreover, it unduly biases the planners choice in favor of the outcome designated as the status quo.

A very different type of multiselves model is offered by Bernheim and Rangel (2002). In their model there are two selves identified with different states of the brain.³ In the cold state the agent makes rational, long-run optimizing choices anticipating the possibility that he may lose control to the hot state. The brain switches back and forth between these states according to some stochastic process. The welfare criterion of Bernheim and Rangel identifies the individual's true interests with the cold-state self and treats the hot state as a constraint. This approach raises questions similar to the ones discussed above within the context of the $\beta - \delta$ framework. The identification of the self with a mood of the individual

³ See also Laibson (2001), and Lowenstein (1996) for related work.

and the asymmetric manner in which the moods are treated in the welfare analysis raise a host of interesting issues that are beyond the scope of this paper.

In contrast to the multi-selves approach, our model assumes a single agent with a consistent preference who maximizes a single consistent utility function. However, our agent values commitment. Equivalently, he benefits from eliminating temptations from his choice set.

It is difficult to imagine what kind of “evidence” one could provide in favor of either the multi-selves or the single-self view. However, we note that the idea of a consistent preference corresponding to the agent’s true welfare seems to permeate our informal, everyday analysis of struggles with temptations. Consider the example of a smoker. Suppose, in period 0 he has decided to quit and thrown out his last pack of cigarettes. In period 1, he visits a friend who offers him a cigarette which he accepts. After the visit, his friend is reproached by the friend’s spouse who asks: “Why did you do that? You know he was trying to quit!” To this the friend responds: “It was his period 0 self that wanted to quit. Obviously, the period 1 self did not, since it accepted the cigarette that I offered. Why should I be concerned with the welfare of the period 0 self? After all, it was the period 1 self that was nice enough to pay us a visit.” Should we consider this an adequate defence of the friend’s actions? If we take the multi-selves view literally, we may have to. In contrast, our model takes the view that the agent is harmed by the availability of the cigarettes in *both* periods. The agent’s decision to smoke when cigarettes are available only indicates that exercising self-control is too costly. It does not invalidate his earlier desire for commitment.

1.2 Evidence

The economics literature on addiction has focused on demand analysis for drugs. The key comparative static is that the demand for the drug decreases as the future price of the drug increases. Becker, Grossman and Murphy (1994) found that sales of cigarettes in the current period decreases if future prices go up. Becker, Grossman and Murphy conclude that this reflects the complementarities between current and future consumption. Gruber and Koszegi (2001) confirm their finding after controlling for the difference between sales

and consumption. The latter research finds evidence of greater sales and lower consumption in response to an anticipated price increase, capturing both the consumers' desire to stockpile at the lower price and to avoid the increase cost of addiction. As we show in section 3, our model is consistent with the empirical analysis of Becker, Grossman and Murphy (1994) and Gruber and Koszegi (2001).

In addition to analyzing consumption, our model can be used for evaluating the demand for commitment. Our model suggests that addicts should seek commitment opportunities. We observe addicts seeking commitment by enrolling in voluntary rehabilitation programs. Treatment programs provide commitment by making drugs difficult to procure. Prohibition of certain drugs also provides a form of commitment, albeit an involuntary commitment. The fact that prohibitive drug policies have strong public support also suggests that agents benefit from commitment.

A sophisticated form of commitment is achieved through the use of the opiate antagonist naltrexone. Naltrexone blocks the opioid receptors in the brain and hence the euphoric effects of these drugs for up to 3 days after the last dose. Naltrexone is voluntarily used by some heroin and morphine addicts. Further evidence for the demand for commitment devices are the recent efforts by pharmaceutical companies to develop *vaccines* for nicotine (Pentel, et al. (2000)) and cocaine.⁴ The function of these vaccines is to prevent the drug from reaching the brain, so as to eliminate its effects and provide commitment for individuals. A novel feature of these vaccines is their long term effectiveness, and hence their ability to provide commitment over many months.

⁴ "When injected in laboratory animals, the vaccine stimulates the immune system to produce antibodies that bind tightly to nicotine. The antibody-bound nicotine is too large to enter the brain, thereby preventing nicotine from producing its effects. The antibody-bound nicotine is eventually broken down to other harmless molecules." cited from <http://pharmacology.about.com/health/pharmacology/library/99news/bl9n1217a.htm>

2. Model

We consider an environment with 2 goods and let $C = [0, 1]^2$ denote the set of possible consumption vectors. A consumption bundle is denoted (c, d) where d will be interpreted as the consumption of the addictive good, the “drug”.

An agent is confronted with a dynamic decision problem. Every period $t = 1, 2, \dots$ the agent must take an action. This action results in a consumption for period t and constrains future actions. Dynamic decision problems can be described recursively as a set of alternatives where each alternative is a lottery over current consumption and continuation decision problems.⁵ Let Z denote the set of all decision problems and let x, y or z denote generic elements of Z . Generic choices (i.e. elements of a given z) are denoted μ, ν or η . A choice μ is a lottery over $C \times Z$, where $c \in C$ represents the realization of current consumption and $x \in Z$ represents the realized continuation decision problem. A deterministic choice yields a particular consumption (c, d) and a particular deterministic continuation problem z with certainty and is denoted (c, d, z) . Most of the analysis in this paper focuses on the set of *deterministic decision problems*, $\bar{Z} \subset Z$. Each $z \in \bar{Z}$ is a (compact) set of alternatives of the form (c, d, x) where c denotes current consumption and $x \in \bar{Z}$ denotes the deterministic continuation problem.

The set of decision problems Z serves as the domain of preferences for the agent. This allows us to describe agents who struggle with temptation. For example, the agent may strictly prefer a decision problem in which some alternatives are unavailable because these alternatives present temptations that are hard to resist. Even when the agent makes the same ultimate choice from two distinct decision problems he may have a strict preference for one decision problem because making the same choice from the other requires more self-control. Choice problems are the natural domain for identifying these phenomena. Below we represent the individual’s preferences over decision problems by a utility function. This utility function is analogous to the *indirect utility function* in standard consumer theory. The traditional indirect utility function is defined for decision problems that can be represented by a budget set. In contrast, our utility function is defined for a broader class of decision problems.

⁵ See Gul and Pesendorfer (2004) for a detailed discussion of dynamic decision problems.

The preferences analyzed in this paper depend on the agent's past consumption. To capture this dependence, we index the individual's preferences by $s \in S$, the state in the initial period of the decision problem. The state s represents the relevant consumption history prior to the initial period of analysis. For simplicity, we assume that only drug consumption in the last period influences the agents preferences and set $S := [0, 1]$. We refer to the indexed family of preferences $\succeq := \{\succeq_s\}_{s \in S}$ simply as *the agent* or the preference \succeq . We say that the utility function $W : S \times Z \rightarrow \mathbb{R}$ represents the preference \succeq if, for all s , $x \succeq_s y$ iff $W(s, x) \geq W(s, y)$.

In section 6 (Theorem 2) we provide axioms for the utility function used in this paper. These axioms ensure that the decision-maker's preferences can be represented by a continuous function W of the following form:

$$W(s, z) = \max_{(c, d, x) \in z} [u(c, d) + \sigma(s)v(d)\delta + W(d, x)] - \max_{(\hat{c}, \hat{d}, \hat{x}) \in z} \sigma(s)v(\hat{d}) \quad (1)$$

where the function u is continuous and nonconstant, v is continuous and strictly increasing, σ is continuous and strictly positive, and $\delta \in (0, 1)$. Henceforth, these axioms are implicit in any reference to a preference and it is understood that W, u, v, σ, δ refer the functional form in equation (1).

Straightforward application of results from dynamic programming imply that for every (u, v, σ, δ) with u, v continuous, $\delta \in (0, 1)$, there is a unique W that satisfies equation (1). We say that (u, v, σ, δ) represents the preference \succeq if the unique W that satisfies equation (1) represents \succeq .

Equation (1) implies that if the agent is committed to a single choice (i.e., $z = \{(c, d, x)\}$) then $W(z) = u(c, d) + \delta W(x)$. Therefore, we refer to $u + \delta W$ as the *commitment utility* of a particular choice. Note that the commitment utility is independent of the state s .

Consider a deterministic decision problem that does not offer commitment and assume that (c, d, x) is the unique maximizer of the commitment utility $u + \delta W$ and (\hat{c}, \hat{d}, y) is the unique maximizer of v in z . In this case, it follows from equation (1) that removing (\hat{c}, \hat{d}, y) from the choice set would increase the agent's welfare. We refer to alternatives (\hat{c}, \hat{d}, y) that have this property as *temptations*. Temptations create a *preference for commitment*; that

is, situations where the agent strictly prefers the decision problem x over z even though $x \subset z$.

The agent's choice from z in state s maximizes the function

$$u + \sigma(s)v + \delta W$$

If (c, d, x) is the choice from z and (\hat{c}, \hat{d}, y) maximizes v in z then the agent incurs a self-control cost of

$$-\sigma(s)[v(d) - v(\hat{d})]$$

This cost is zero if the choice maximizes v . Otherwise it is positive. In our model, past consumption affects current behavior by changing the cost of self-control.

The optimal choices from z are denoted $\mathcal{D}(s, z)$ while $\mathcal{C}(z)$ denotes the maximizers of commitment utility. For any function $f : C \times Z \rightarrow \mathbb{R}$, let $E_\mu[f]$ be the expectation of f with respect to μ . Then,

$$\mathcal{D}(s, z) := \{\mu \in z | E_\mu[u + \sigma(s)v + \delta W] \geq E_\nu[u + \sigma(s)v + \delta W], \forall \nu \in z\}$$

$$\mathcal{C}(z) := \{\mu \in z | E_\mu[u + \delta W] \geq E_\nu[u + \delta W], \forall \nu \in z\}$$

When the agent chooses alternatives that do not maximize commitment utility it means that behavior is affected by temptations. We call such choices *compulsive*. This motivates the following definition.

Definition: \succeq_s is *compulsive* at z iff $\mathcal{D}(s, z) \setminus \mathcal{C}(z) \neq \emptyset$.

The notion of compulsive consumption plays a central role in the clinical definition of addiction and in the definition we present below. What distinguishes addiction from other types of compulsive behavior is the fact that the compulsiveness associated with an addictive substance is “caused” (or made worse) by past consumption (or higher levels of past consumption) of the *same* substance. Below, we offer criteria for ranking states with respect to the compulsiveness of the agent. This criterion provides a formal, choice-theoretic definition of what it means for compulsiveness to get worse.

Definition: A preference \succeq is *more compulsive* at \bar{s} than at s (denoted $\bar{s}\mathbf{C}s$) if \succeq_s is *compulsive* at z implies $\succeq_{\bar{s}}$ is *compulsive* at z .

Note that our notion of more compulsive is analogous to the familiar notion of more risk averse: first we provide a criterion for a consumption choice to be compulsive. Then we define the agent to be more compulsive in situation \bar{s} than in s if the set of choice problems in which he makes a compulsive choice at s is contained in the set of choice problems in which he makes a compulsive choice at \bar{s} .

Psychologists and healthcare professionals commonly refer to an individual as addicted if, after repeated self-administration of a drug, the individual develops a pattern of compulsive drug seeking and drug-taking behavior.⁶ The clinical definition emphasizes a lack of control on the part of addicted subjects and suggest a conflict between what the addict *ought to consume* and what he *actually consumes*.

In our model, the agent is compulsive when the choice is different from the $u + \delta W$ optimal alternative. Thus, an agent is compulsive if behavior would change were commitment possible. Similar to the clinical definition above, we say that the drug is addictive if higher current drug consumption makes the individual more compulsive; that is, following the increase in drug consumption there are more situations in which the agent makes a choice that does not maximize U . The definition below expresses this idea.

Definition: *The drug is addictive if $\bar{s}\mathbf{C}s$ for all $\bar{s} > s$ and $\succeq_1 \neq \succeq_0$.*

Proposition 1: *(i) $\bar{s}\mathbf{C}s$ if and only if $\sigma(\bar{s}) > \sigma(s)$; (ii) the drug is addictive if and only if σ is non-decreasing with $\sigma(1) > \sigma(0)$.*

Proposition 1 relates our definition of addiction to our representation of preferences. It shows that the function σ measures how compulsive the agent is and therefore the drug is addictive when σ is increasing.⁷ The proof of Proposition 1 is in the appendix. Note that the "if" part of part (i) is straightforward since a higher σ implies a larger weight on the temptation utility. For the "only if" part we must show that when $\sigma(\bar{s}) > \sigma(s)$ there

⁶ See Robinson and Berridge (1993), pg. 248.

⁷ Straightforward extensions of known uniqueness arguments (for example, the uniqueness result in Gul and Pesendorfer (2004)) ensure that if (u, v, σ, δ) represents some \succeq satisfying the conditions of Theorem 2 then $(\hat{u}, \hat{v}, \hat{\sigma}, \hat{\delta})$ represents the same \succeq if and only if there exist constants $a, b > 0$, c and e such that $\hat{u} = au + c$, $\hat{v} = bv + e$, $\hat{\sigma} = \frac{a}{b}\sigma$, and $\hat{\delta} = \delta$ with $e = 0$ whenever σ is nonconstant (i.e., if the drug is addictive). Hence all of our assumptions (such as monotonicity, increasingness and differentiability etc.) and conclusions regarding (u, v, σ, δ) are properties of the underlying preference and not the particular representation.

is a decision problem with the property that the agent is compulsive at \bar{s} but not at s . Such a decision problem can be constructed as long as $u + \delta W$ and $u + \sigma v + \delta W$ are not positive affine transformations of each other. Since u and v are not constant, $\delta > 0, \sigma > 0$ this holds in our case.

For $z \in \bar{Z}$, let $D(s, z)$ denote the individual's current period drug demand in state s ; that is, $d \in D(s, z)$ if and only if there exists c, x such that $(c, d, x) \in \mathcal{D}(s, z)$. We write $D(\bar{s}, x) \geq D(s, y)$ if $\bar{d} \in D(\bar{s}, x), d \in D(s, y)$ implies $\bar{d} \geq d$. Proposition 2 shows that an increase in σ leads to higher drug demand in every decision problem.

Proposition 2: *If $\sigma(\bar{s}) \geq \sigma(s)$ then $D(\bar{s}, z) \geq D(s, z)$ for all $z \in \bar{Z}$.*

Proof: Let $(c, d, x) \in \mathcal{D}(s, z)$ and $(\bar{c}, \bar{d}, \bar{x}) \in \mathcal{D}(\bar{s}, z)$. Then,

$$\begin{aligned} u(c, d) + \sigma(s)v(d) + \delta W(d, x) &\geq u(\bar{c}, \bar{d}) + \sigma(s)v(\bar{d}) + \delta W(\bar{d}, \bar{x}) \\ u(\bar{c}, \bar{d}) + \sigma(\bar{s})v(\bar{d}) + \delta W(\bar{d}, \bar{x}) &\geq u(c, d) + \sigma(\bar{s})v(d) + \delta W(d, x) \end{aligned}$$

Hence,

$$(\sigma(\bar{s}) - \sigma(s))(v(\bar{d}) - v(d)) \geq 0$$

and therefore $\sigma(\bar{s}) \geq \sigma(s)$ implies $D(\bar{s}, z) \geq D(s, z)$. \square

Psychologists use the term *reinforcement* to describe the fact that an increase in current drug consumption leads to an increase in future drug consumption. If $\epsilon > 0$ and $\sigma(d + \epsilon) > \sigma(d)$ then the ϵ increase is reinforcing. In particular, an addictive increase in drug consumption is always reinforcing.

3. Addiction and Drug Demand

Our next objective is to analyze the implications of addiction on drug demand. In order to facilitate the comparative statics results in this and the subsequent sections, the following assumptions will sometimes be used. Assumption 1 requires u to not depend on drug consumption. It implies that the agent would commit to zero drug consumption if commitment were possible.

Assumption 1: *$u(\cdot, d)$ is strictly increasing with $u(\cdot, d) = u(\cdot, \hat{d})$ for all d, \hat{d} .*

When Assumption 1 is satisfied we write $u(c)$ instead of $u(c, d)$. Assumption 2 imposes curvature restrictions on u, v and σ . These restrictions are analogous to the standard curvature and differentiability assumptions in demand theory. The function σ plays a role similar to a cost function in a standard optimization problem. Therefore concavity of the objective function in the decision problems below is guaranteed when σ is convex.

Assumption 2: $u, v, -\sigma$ are twice differentiable and strictly concave.

We consider a simple stationary consumption problem. The individual cannot borrow or lend and can consume at most 1 unit of the drug in every period. Let $\mathbf{p} = p_1, \dots, p_t, \dots$ denote the sequence of prices. Given the price sequence \mathbf{p} , after date $\tau \geq 0$ the agent faces the price sequence $p_\tau, \dots, p_{\tau+k}, \dots$. We let \mathbf{p}_τ denote this sequence. Hence, $\mathbf{p}_1 = \mathbf{p}$. The individual is endowed with one unit of wealth and must choose consumption (c, d) from the budget set

$$B_t = \{(c, d) \in C \mid c + p_t d \leq 1\}$$

We assume that $p_t < 1$ for all t . Since $d \leq 1$ the maximally feasible drug consumption is 1 in every period independent of the price of the drug. Let $x(\mathbf{p}_\tau)$ denote the dynamic decision problem confronting an agent who faces the price sequence \mathbf{p} . It is easy to verify that there is a unique optimal consumption plan for every $x(\mathbf{p}_\tau)$. In particular, the current period drug demand $D(s, x(\mathbf{p}))$ is a singleton. We use $d(s, x(\mathbf{p}))$ to denote this demand. We define the period τ demand of the agent facing \mathbf{p} recursively, as follows:

$$\begin{aligned} d_1(s, \mathbf{p}) &= d(s, \mathbf{p}) \\ d_{\tau+1}(s, \mathbf{p}) &= d(d_\tau(s, \mathbf{p}), \mathbf{p}_{\tau+1}) \end{aligned}$$

To see how the two assumptions above facilitate comparative static analysis of addiction recall that by Proposition 1, addictiveness implies that σ is non-decreasing. If, in addition, Assumption 2 holds then σ is a strictly increasing function. Assumption 1 ensures that u depends only on non-drug consumption. Hence, the objective function for the agent's choice from $x(\mathbf{p}_t)$ simplifies to

$$u(c_t) + \sigma(d_{t-1})v(d_t) + \delta W(d_t, x(\mathbf{p}_{t+1}))$$

Then, Assumption 2 enables us to identify the following first order condition for an interior solution for the optimal choice of d_t :

$$p_t u'(c_t) = \sigma(d_{t-1})v'(d_t) + \delta\sigma'(d_t)(v(d_{t+1}) - v(1))$$

To understand the above equation consider a marginal increase in drug consumption. This implies a reduction in current (non-drug) consumption and the left-hand side captures the utility consequence of this reduction. The first term on the right hand side captures the current period utility change from the increase in drug consumption. Since v is increasing this is positive. The second term captures the effect of the increase in drug consumption on future utility. This effect works through a change in the cost of self-control in the next period. If σ is increasing (as in the case of an addictive drug) then the increase in the current drug consumption implies a higher self-control cost next period and the second term on the left hand side is negative. If σ is decreasing, then the increase in drug consumption implies a smaller self-control cost in the next period and the term is positive.

The next proposition analyzes the change in demand as a function of current and future prices.

Proposition 3: *Suppose that the drug is addictive and Assumptions 1 and 2 are satisfied. Then, $d(s, \mathbf{p})$ is nondecreasing in p_t for $t \geq 1$. If $0 < d_\tau(s, \mathbf{p}) < 1$ for $\tau \in \{1, \dots, t\}$ then $d(s, p_t, \dots, p_{t-1}, \cdot, p_{t+1}, \dots)$ is differentiable at p_t and $\partial d(s, \mathbf{p})/\partial p_t < 0$.*

Proof: First we prove the result for $t = 1$. Assume that $0 < d(s, \mathbf{p}) < 1$. The first order condition is

$$-p_1 u'(1 - p_1 d_1) + \sigma(s)v'(d_1) + \delta\sigma'(v(d_2) - v(1)) = 0$$

Taking the total derivative we find

$$\partial d/\partial p_1 = \frac{p_1^2 u''(c_1) + \sigma(s)v''(d_1) + \delta\sigma''(d_1)(v(d_2) - v(1))}{u'(c_1) - p_1 u''(c_1)} < 0$$

by Assumption 2. The weak monotonicity for boundary solutions is equally straightforward.

Next, assume that $d_\tau := d_\tau(s, \mathbf{p}), d_{\tau+1} = d_{\tau+1}(s, \mathbf{p})$ are interior for some τ such that $1 \leq \tau < t$. Note that

$$\frac{dd_\tau}{dd_{\tau+1}} = \frac{p_t^2 u''(c_\tau) + \sigma(d_{\tau-1})v''(c_\tau) + \delta\sigma''(v(d_{\tau+1}) - v(1))}{-\sigma'(d_\tau)v'(d_{\tau+1})} > 0$$

by Assumption 2. Then, the fact that $\partial d(s, \mathbf{p})/\partial p_1 < 0$ and an inductive argument implies the result for the interior case. Weak monotonicity for the case of boundary solutions is equally straightforward. \square

Proposition 3 applied to the case of $t = 1$ shows that the drug is a normal good under our assumptions. For $t > 1$, Proposition 3 shows that drug demand decreases if the future price of the drug increases. This connection between current demand and future prices has been documented in the literature on drug demand. Hence, Proposition 3 shows that our model is consistent with this empirical finding.

To see why drug demand decreases in response to an increase in future drug prices, note that since the drug is a normal good, consumption in period t decreases as p_t increases. As a result the period t cost of self-control, given by $\sigma(d_{t-1})(v(1) - v(d_t))$, increases. Since the drug is addictive, σ is increasing. But this implies that drug consumption in period $t - 1$ becomes less attractive, since it is associated with a greater marginal increase in the cost of self-control in period t . Hence, drug demand in period $t - 1$ decreases. Proceeding inductively, we conclude that drug demand in period 1 must decrease.

Empirical work on drug demand has found support for the result described in Proposition 3. Becker, Grossman and Murphy (1994) and Gruber and Koszegi (2001) find support for the prediction that drug demand decreases as future prices increase. Note that results analogous to Proposition 3 have been shown for other models of addiction. Becker, Grossman and Murphy (1994) show this result for quadratic utility for the Becker-Murphy model. Gruber and Koszegi (2001) analyze a decision problem very similar to the one analyzed in this section. They give conditions under which Proposition 3(ii) holds in a $\beta - \delta$ model of addiction with quadratic utility.

4. Drug Policy and Welfare

Drug policies affect consumers along two dimensions. On the one hand, they affect the availability of the drug – and hence the feasible level drug consumption. On the other hand, they affect the price of the drug and hence the opportunity cost of drug consumption.

Since a typical consumer can afford the maximally feasible cigarette consumption in a period before and after a moderate tax increase, a moderate tax on a drug such as cigarettes will affect the opportunity cost of drug consumption without changing the feasible drug consumption in the current period. In contrast, the prohibition of drug consumption will affect the maximally feasible drug consumption for a typical consumer. Often prohibitive policies will be accompanied by a higher opportunity cost of drug consumption. For analytical clarity we separate the prohibitive effects of a policy from the price effects in the analysis below.

A drug policy is a pair (τ, q) where $\tau \geq 0$ is a per unit tax on the drug and $q \in [0, 1]$ is the maximum feasible drug consumption. Let

$$B(\tau, q) = \{(c, d) \in [0, 1]^2 \mid c + (p + \tau)d \leq 1, d \leq q\}$$

denote the individual's opportunity set under the policy (τ, q) . We assume that the agent faces a stationary decision problem in which he must choose (c, d) from $B(\tau, q)$ in every period. Note that in this section we assume that prices (and the parameter q) are constant across time. This is done for simplicity. We denote with $y(\tau, q)$ the corresponding decision problem.

Any policy $(0, q)$ with $q < 1$ is a purely prohibitive policy since it reduces the maximum feasible drug consumption but does not affect the opportunity cost of drugs. A pure price policy is a policy $(\tau, 1)$ with $p + \tau \leq 1$. In this case, the maximum feasible drug consumption remains 1 in every period but the opportunity cost of the drug is increased to $p + \tau$. If the tax is high enough, in particular, if $p + \tau > 1$ then the policy $(\tau, 1)$ also has a prohibitive effect since it decreases the maximal drug consumption to $\frac{1}{p + \tau}$.

Propositions 4, 5 and 6 examine the welfare effects of prohibitive and price policies. Proposition 4 considers the case where the drug is addictive and u does not depend on drug

consumption (Assumption 1). It shows that under those circumstances, a more restrictive prohibitive policy leads to higher welfare than a less restrictive prohibitive policy.

Proposition 4: *If the drug is addictive and Assumption 1 is satisfied then $\bar{q} > q$ implies $W(s, y(\tau, q)) > W(s, y(\tau, \bar{q}))$.*

Proof: Let $s = d_0 = \bar{d}_0$ be the initial state let $\{(c_t, d_t)_{t \geq 1}\}$ denote the optimal consumption plan for the decision problem $y(0, q)$ at state s . Similarly, let $\{(\bar{c}_t, \bar{d}_t)_{t \geq 1}\}$ denote the optimal consumption plan for the decision problem $x(0, \bar{q})$ at state s . Since Assumption 1 is satisfied we write $u(c)$ instead of $u(c, d)$. Define $\hat{d}_t = \min\{\bar{d}_t, q\}$ and set $\hat{c}_t = 1 - p\hat{d}_t$ for all $t \geq 1$. Clearly, $\hat{d}_t \leq \bar{d}_t$ for all $t \geq 1$ and therefore we have $u(\hat{c}_t) \geq u(\bar{c}_t)$, $\sigma(\hat{d}_t) \leq \sigma(\bar{d}_t)$. If $\bar{d}_t = \bar{q}_t$, then we have $\hat{d}_t = q_t$; if $\bar{d}_t < \bar{q}_t$ then $v(\hat{d}_t) - v(q) < v(\bar{d}_t) - v(\bar{q})$. In the former case, $u(\hat{c}_t) > u(\bar{c}_t)$; in the latter case $\sigma(\hat{d}_{t-1})[v(\hat{d}_t) - v(q)] > \sigma(\bar{d}_{t-1})[v(\bar{d}_t) - v(\bar{q})]$. Hence,

$$\begin{aligned} W(s, y(0, q)) &\geq \sum_{t=0}^{\infty} \delta^t [u(\hat{c}_t, \hat{d}_t) + \sigma(\hat{d}_{t-1})v(\hat{d}_t) - \sigma(\hat{d}_{t-1})v_0(q)] \\ &> \sum_{t=0}^{\infty} \delta^t [(u(\bar{c}_t, \bar{d}_t) + \sigma(\bar{d}_{t-1})v(\bar{d}_t) - \sigma(\bar{d}_{t-1})v(\bar{q}))] \\ &= W(s, y(0, \bar{q})) \end{aligned}$$

□

A prohibitive policy has two effects; it reduces self-control costs and it may render the previous level of drug consumption infeasible. The reduction in self-control costs always increases welfare. Assumption 1 ensures that the level of drug consumption that maximizes the commitment utility is zero. Hence, the reduction in consumption increases utility in the current period. Moreover, if the good is addictive, this reduction in consumption lowers future self-control costs. Thus, a purely prohibitive policy on an addictive drug always increases welfare.

To see why it is important for the drug to be addictive, consider an agent who is in state $s = .5$ in period 1. Suppose that abstaining ($d = 0$) or binging ($d = 1$) for one period will cause all temptation to go away in the next period but consuming intermediate levels will cause temptation to persist. Moreover, assume that the cost of self-control in the current state is very high. Then, it may be optimal for the agent to binge in the current

period and abstain thereafter. In such a situation, a policy that reduces the maximal feasible level of drug consumption from 1 to $q = .5$ may reduce the agents welfare by forcing him to either incur the (reduced but still) high cost of self-control in the current period or remain addicted.

Proposition 5 below shows that a pure price policy can never increase welfare. This result is derived with a simple revealed preference argument and therefore does not require any additional assumptions.

Proposition 5: *If $p + \tau < 1$ and $\bar{\tau} > \tau$ then $W(s, y(\tau, q)) \geq W(s, y(\bar{\tau}, q))$ for all s .*

Proof: Let $s = d_0$ denote the initial state and $\{(\bar{c}_t, \bar{d}_t)_{t \geq 1}\}$ be the optimal consumption plan for the problem $y(\bar{\tau}, 1)$. Since $\{(\bar{c}_t, \bar{d}_t)_{t \geq 1}\}$ is a feasible choice from $y(\tau, 1)$ we have

$$\begin{aligned} W(s, y(\tau, 1)) &\geq \sum_{t=0}^{\infty} \delta^t (u(\bar{c}_t, \bar{d}_t) + \sigma(d_{t-1})v(\bar{d}^t) - \sigma(\bar{d}^{t-1})v(1)) \\ &= W(s, y(\bar{\tau}, 1)) \end{aligned}$$

□

A pure price policies does not affect the maximal feasible drug consumption and therefore does not reduce self-control costs. This implies that it cannot improve the agent's welfare.

Proposition 5 stands in contrast to the findings of Gruber and Koszegi (2001) for the $\beta - \delta$ model of addiction. As discussed in section 1.1, the $\beta - \delta$ model and our approach lead to very different welfare conclusions. The reason why pure price policies cannot improve welfare in our setting is that pure price policies cannot eliminate temptations. By definition a pure price policy does not change the maximally feasible drug consumption. Since we assume that the temptation utility depends only on current drug consumption this implies that a price policy cannot eliminate temptations. More generally, a policy can improve welfare *only if* it can eliminate temptations.

In section 6, Theorem 1, we provide a representation theorem that allows for a more general specification of the temptation utility. In that model, the temptation utility depends not only on current drug consumption but also on non-drug consumption and the

continuation problem. For that general model, it is possible to find specifications under which a price policy can increase welfare. We analyze the simpler model to capture the idea that the temptation associated with drugs is focused on current consumption of the drug.

Ultimately, determining the best specification of temptation utility is an empirical issue. To distinguish various specifications for temptation utility one needs to examine the (policy) choices of consumers. Our specification would predict that smokers voluntarily seek commitment but vote against an increase in cigarette taxes. In contrast, a formulation for temptation utility that renders a price policy welfare improving leads to the prediction that smokers seek voluntary commitment but also vote for an increase in cigarette taxes. Such a relationship between behavior and welfare analysis cannot be established within the $\beta - \delta$ framework.

Next, we analyze the impact of prohibitive policies on the demand for drugs. Current period drug demand in state s under the policy (τ, q) is denoted $D(s, y(\tau, q))$. Consider a purely prohibitive policy $(0, q)$. If the prohibitive policy is binding, that is, if $D(s, y(0, q)) = q$ then a reduction in the maximum allowed drug consumption q will obviously lead to a reduction in drug demand. Proposition 6 shows that if the policy is not binding then a reduction in q will lead to an *increase* in drug demand.

Proposition 6: *Suppose that the drug is addictive, Assumptions 1, 2 are satisfied, and $0 < d(s, y(0, q)) < q$ then $d(s, y(0, \cdot))$ is differentiable at q and $\partial d(s, y(0, q))/\partial q < 0$.*

Proof: Since the optimal consumption is interior, the first order necessary condition is

$$0 = -pu'(1 - pd_1) + \sigma(s)v'(d_1) + \sigma'(d_1)(v(d_2) - v(q)) \equiv A(d_1)$$

Taking the total derivative we get

$$dd_1A'(d_1) - dq\sigma'(d_1)v'(q) = 0$$

Assumptions 1, 2 and addictiveness (see Proposition 1) implies that $A'(d_1) < 0$. Since $\sigma' > 0$ and $v' > 0$, the desired result follows. \square

A prohibitive policy reduces the utility cost of drug consumption by reducing the future self-control costs associated with current drug consumption. For this reason, drug demand increases as the prohibitive policy becomes more stringent. In contrast, as Proposition 3 shows, a price policy reduces demand, that is, drug demand is decreasing in τ .

Assumption 1 implies that the agent (in period 0) would choose not to consume the drug in any period if perfect commitment were available. Hence, the fact that the drug is available is unambiguously “bad” for the consumer. Nevertheless, as our results show, policies that reduce drug consumption may reduce welfare while policies that increases drug consumption may increase welfare.

5. Rehabilitation

In this section we analyze a situation where the agent can choose to check into a rehabilitation center. In our interpretation, rehabilitation centers offer short term commitment to zero drug consumption.

As in the previous sections, we consider a simple decision problem that rules out intertemporal transfers of resources. The agent is either *in* or *out* of the rehabilitation center. If the agent is *out* he faces the budget set

$$B^o := \{(c, d) | c + pd = 1\}$$

If the agent is *in* then he is committed to zero drug consumption and hence the choice set is

$$B^i(a) := \{(c, d) | c = 1 - a, d = 0\}$$

The parameter $a \in [0, 1]$ represents the cost of commitment.

The agent’s decision problem is as follows. In each period $t \geq 1$ he finds himself either *in* and hence choosing from $B^i(a)$ or *out* and choosing from B^o . In addition, the agent must choose *in* or *out* for the next period. The decision problems $x^i(a), x^o(a)$ represent the two situations.

$$x^o(a) := \{(c, d, x) | (c, d) \in B^o, x \in \{x^o(a), x^i(a)\}\}$$

$$x^i(a) := \{(c, d, x) | (c, d) \in B^i(a), x \in \{x^o(a), x^i(a)\}\}$$

In period 1 the agent faces the decision problem $x^o(a)$.

The following propositions characterize optimal rehabilitation strategies for the addict. To simplify the notation below, we write (c, d, j) with $j \in \{i, o\}$ for a choice from $x^k(a)$, $k \in \{i, o\}$. An optimal policy for $x^k(a)$ is a sequence (c_t, d_t, j_t) , $t = 1, 2, \dots$. We assume that the drug is addictive and that there is a unique optimal policy. Proposition 7 establishes that under these conditions only three patterns of behavior can emerge. If the cost of rehab is too high the addict never utilizes the program. If rehab is very inexpensive, then agent eventually enters rehab and once he is in he stays in. Between these two extremes, we observe a cycle of addiction and rehabilitation where the agent increases his drug consumption as long as he is not in rehab, then he enters rehab for one period and afterwards restarts the cycle of increasing drug consumption.

Proposition 7: *Suppose the drug is addictive and that (c_t, d_t, j_t) is the unique optimal policy for the decision problem $x^o(a)$ in state $s = 0$. Then, (c_t, d_t, j_t) satisfies one of the following:*

- (i) $j_t = i$ for all t and $d_t = 0$ for all $t > 1$;
- (ii) $j_t = o$ for all t and $d_t \leq d_{t+1}$ for all t ;
- (iii) there is $N \in \{2, 3, \dots\}$ and $(\hat{c}_n, \hat{d}_n, \hat{j}_n)$, $n = 1, \dots, N$ such that for all $t = kN + n$, $k = 0, 1, \dots$, $(c_t, d_t, j_t) = (\hat{c}_n, \hat{d}_n, \hat{j}_n)$, where, $j_{N-1} = i$, $\hat{d}_N = 0$ and $0 < \hat{d}_1 < \dots < \hat{d}_{N-1}$.

Proof: Let (c_t, d_t, i_t) denote the unique optimal policy. Note that

$$W(s, x^i(a)) = W(0, x^i(a))$$

since the agent is committed to zero drug consumption in $x^i(a)$. Note also that $u(c, d) + \sigma(s)(v(d) - v(1)) + \delta \max[W(d, x^i(a)), W(d, x^o(a))]$ is non-increasing in s since σ is increasing and $v(d) \leq v(1)$ for all d . Therefore, $W(s, x^o(a))$ is non-increasing in s .

First, consider the case where $W(0, x^i(a)) > W(0, x^o(a))$. Hence, the agent prefers to be “in” when the state is 0. By the above argument it follows that $W(s, x^i(a)) >$

$W(s, x^o(a))$ for all s . This implies that the agent chooses $j_t = i$ for all $t = 1, 2, \dots$. Hence, case (i) applies.

Next, consider the case where $W(d_t, x^i(a)) < W(d_t, x^o(a))$ for all d_t . In that case the agent chooses $j_t = o$ for all $t = 1, 2, \dots$. Note that the agent's consumption plan is optimal for the stationary decision problem in which he faces the budget set B^o in every period. Let $x = \{(c, d, x) | (c, d) \in B^o\}$ denote the corresponding decision problem. By Proposition 2, the drug demand from x is monotonically increasing in s . This in turn implies that d_t is non-decreasing and case (ii) applies.

Since we assumed a unique optimal solution, it remains to show that if $W(0, x^o(a)) > W(0, x^i(a))$ and $W(d_t, x^i(a)) > W(d_t, x^o(a))$ for some d_t , then case (iii) applies. It follows from $W(0, x^o(a)) > W(0, x^i(a))$ that $j_1 = o$. Let $N = t + 1$ where t is the smallest integer such that $W(d_t, x^i(a)) > W(d_t, x^o(a))$. Hence, $j_{N-1} = i$. Note that $W(0, x^o(a)) > W(0, x^i(a))$ implies $j_N = o$ and therefore, in period $N + 1$ the agent makes an optimal choice from the decision problem $x^o(a)$ in state 0, the same state and decision problem that the agent faced in period 1. From the uniqueness of the optimal policy it follows that optimal choices in periods $N + 1, \dots, 2N$ are identical to the choices in periods $1, \dots, N$ and that $j_n = o$ for $n < N - 1$. It remains to show that $0 < d_1 < \dots < d_{N-1}$. In spite of the difference in the choice problems, the argument of Proposition 2 applies to ensure that the drug demand in $x^o(a)$ is non-decreasing in the state. Hence, it follows that $0 \leq d_1 \leq \dots \leq d_{N-1}$. Because the optimal policy is unique, the drug demand must be strictly increasing. To see this first suppose $d_t = d_{t+1}$, $t < N - 1$. Then (c_t, d_t, o) is an optimal choice from $x^o(a)$ at state d_t . But this contradicts the uniqueness of the optimal choice and the fact that $j_t = i$ for some t . If $d_1 = 0$ then it must be that $W(d, x^i(a)) \geq W(d, x^o(a))$ again contradiction. \square

In Proposition 7, we have assumed that the optimal solution is unique. Alternatively, we could have imposed Assumptions 1 and 2, which would have rendered drug demand strictly increasing in the state. Then, the optimal solution would be unique, for generic values of a .

The following proposition demonstrates that cheaper rehabilitation centers may increase drug consumption in some periods. More precisely, suppose the initial cost of rehab

is so high that in the current state it is not optimal to choose i . Now assume that this cost is lowered so that i becomes the optimal choice. Then, drug consumption in the current period increases.

Proposition 8: *Suppose the drug is addictive and Assumptions 1 and 2 are satisfied. If $\bar{a} > a$, (c, d, i) is an optimal choice from $x^o(a)$ in state s and (\bar{c}, \bar{d}, o) is an optimal choice from $x^o(\bar{a})$ in state s then $d \geq \bar{d}$. A strict inequality holds if $d > 0$.*

Proof: Assumptions 1 and 2 imply that both $W(s, x^o(a))$ and $W(s, x^o(\bar{a}))$ have unique maximizers (c, d) , (\bar{c}, \bar{d}) respectively. Hence,

$$u(c) + \sigma(s)v(d) > u(\bar{c}) + \sigma(s)v(\bar{d})$$

since next period the agent is committed to a zero drug consumption. Similarly,

$$u(\bar{c}) + \sigma(s)v(\bar{d}) + \delta\sigma(\bar{d})(v(\hat{d}) - v(1)) > u(c) + \sigma(s)v(d) + \delta\sigma(d)(v(\hat{d}) - v(1))$$

where \hat{d} is the optimal drug consumption in the next period. If $\hat{d} = 1$, then the two inequalities above yield

$$u(1 - \bar{a}d) - u(1 - \bar{a}\bar{d}) < u(1 - ad) - u(1 - a\bar{d})$$

Then, the concavity strict increasingness of u ensures that $\bar{d} < d$. Hence, assume $\hat{d} < 1$.

Then,

$$u' + \sigma(s)v' + \delta\sigma'(v(\hat{d}) - v(1)) < u' + \sigma(s)v'$$

If $d = 1$ then we are done. If $0 < d < 1$ then the first order condition at d holds with equality and hence $\bar{d} > d$. If $d = 0$ then $\bar{d} = 0$. \square

Although drug demand in period 0 may increase as a result of less expensive rehab the agent's welfare increases as rehab becomes cheaper.

Proposition 9: *If $u(\cdot, d)$ is nondecreasing and $\bar{a} > a$ then $W(s, x^o(a)) \geq W(s, x^o(\bar{a}))$.*

Proof: Let $(\bar{c}_t, \bar{d}_t, \bar{j}_t)$ denote an optimal policy for the decision problem $x^o(\bar{a})$. We have

$$W(s, x^o(\bar{a})) = \sum_{t=1}^{\infty} \delta^{t-1} (u(\bar{c}_1, \bar{d}_1) + v(\bar{d}_1) - \bar{v}_t^{\max})$$

where \bar{v}_t^{\max} denotes the maximally feasible drug consumption in period t . Note that $\bar{v}_t^{\max} = 1$ in period 1. In all other periods it is 1 if $\bar{j}_{t-1} = o$ and 0 if $\bar{j}_{t-1} = i$. Since $\bar{a} > a$ there is a feasible policy $(\hat{c}_t, \hat{d}_t, \hat{j}_t)$ for $x^o(a)$ with $\hat{d}_t = \bar{d}_t, \hat{j}_t = \bar{j}_t$ and $\hat{c}_t \geq \bar{c}_t$. The utility of this policy is

$$\sum_{t=1}^{\infty} \delta^{t-1} (u(\hat{c}_t, \hat{d}_t) + v(\hat{d}_t) - \bar{v}_t^{\max}) \geq W(s, x^o(a))$$

Hence $W(s, x^o(a)) \geq W(s, x^o(\bar{a}))$. □

As in the previous section we find that the success of policy or treatment options cannot be determined by examining their effect on drug consumption. For a small enough, checking into a rehabilitation center is unambiguously welfare improving for the agent even though it may actually increase overall drug consumption.

6. Representation Theorems

In this section we provide two representation theorems. Theorem 1 axiomatizes a representation that is more general than the one used in the applications above. In particular, the representation allows for a more general state space and a more general specification of the temptation utility. Theorem 2 provides additional axioms that yield the representation throughout the previous sections.

The set of consumptions in each period is $C = [0, 1]^2$ and $b \in C$ denotes a generic consumption vector. For any subset X of a metric space, we let $\Delta(X)$ denote the set of all probability measures on the Borel σ -algebra of X and $\mathcal{K}(X)$ denote the set of all nonempty compact subsets of X . An infinite horizon decision problem (denoted $z \in Z$) can be identified with an element in $\mathcal{K}(\Delta(C \times Z))$ and conversely each element in $\mathcal{K}(\Delta(C \times Z))$ identifies a decision problem $z \in Z$. For formal definitions of Z and the map that associates each element of Z with its equivalent recursive description as an element of $\mathcal{K}(\Delta(C \times Z))$, we refer the reader to Gul and Pesendorfer (2004). In what follows only the recursive definition is used and hence without risk of confusion we identify the sets Z and $\mathcal{K}(\Delta(C \times Z))$. In

Gul and Pesendorfer (2004) we note that since C is a compact metric space it follows that $Z, \Delta(C \times Z)$ and $\mathcal{K}(\Delta(C \times Z))$ are compact metric spaces as well. To abbreviate notation, we write Δ instead of $\Delta(C \times Z)$ when there is no risk of confusion.

The individual's preferences are defined on Z and are indexed by $s \in S$, the state in the initial period of the decision problem. The state s represents the relevant consumption history prior to the initial period. We assume that there is a finite number K such that consumption in only the last K periods influences the agents preferences. Therefore, without loss of generality we set $S := C^K$ where K is the minimal length of the individual's consumption history that allows us to describe \succeq .⁸ We refer to the indexed family of preferences $\succeq := \{\succeq_s\}_{s \in S}$ simply as *the agent* or the preference \succeq .

For any state $s = (b_1, \dots, b_K)$ let sb denote the state (b_2, \dots, b_K, b) . We impose the following axioms on \succeq_s for every $s \in S$.

Axiom 1: (*Preference Relation*) \succeq_s is a complete and transitive binary relation.

Axiom 2: (*Strong Continuity*) The sets $\{x \mid x \succeq_s z\}$ and $\{x \mid z \succeq_s x\}$ are closed in Z .

Axiom 3: (*Independence*) $\{\mu\} \succ_s \{\nu\}$ implies $\{\alpha\mu + (1-\alpha)\eta\} \succ_s \{\alpha\nu + (1-\alpha)\eta\} \forall \alpha \in (0, 1)$.

Axioms 1 – 3 are standard. In Axiom 4 we deviate from standard choice theory and allow for the possibility that adding options to a decision problem makes the consumer strictly worse-off. For a detailed discussion of Axiom 4, we refer the reader to our earlier paper (Gul and Pesendorfer 2001).

Axiom 4: (*Set Betweenness*) $x \succeq_s y$ implies $x \succeq_s x \cup y \succeq_s y$.

Next, we make a separability assumption. For $z \in Z$ let $bz \in Z$ denote the decision problem $\{(b, z)\}$, that is, the degenerate decision problem that yields c in the current period and the continuation problem z . Thus $b_1 b_2 \dots b_K z$ is a degenerate decision problem that yields the consumption (b_1, \dots, b_K) in the first K periods and the continuation problem z in period $K + 1$. For $s = (b_1, \dots, b_K)$ we write sz instead of $b_1 b_2 \dots b_K z$. Axiom 5 considers decision problems of the form $\{(b, sz)\}$ and requires that preferences are not affected by

⁸ That is, there is a pair of states, $(s = (b_1, \dots, b_K), \hat{s} = (\hat{b}_1, \dots, \hat{b}_K))$ that differ only in their first component $(b_1 \neq \hat{b}_1, b_t = \hat{b}_t, t \geq 2)$ and lead to different preferences $(\succeq_s \neq \succeq_{\hat{s}})$.

the correlation between current consumption c and the $K + 1$ period continuation problem z .

Axiom 5: (*Separability*) $\{\frac{1}{2}(b, sz) + \frac{1}{2}(\hat{c}, s\hat{z})\} \sim_s \{\frac{1}{2}(b, s\hat{z}) + \frac{1}{2}(\hat{c}, sz)\}$.

Axiom 6 requires preferences to be stationary. Consider the degenerate lotteries, (b, x) and (b, y) , each leading to the same period 1 consumption c . Stationarity requires that $\{(b, x)\}$ is preferred to $\{(b, y)\}$ in state s if and only if the continuation problem x is preferred to the continuation problem y in state sb .

Axiom 6: (*Stationarity*) $\{(b, x)\} \succeq_s \{(b, y)\}$ iff $x \succeq_{sb} y$.

Note that Axiom 6 implies that the conditional preferences at time $K + 1$ after consuming s in the first K periods is the same as the initial preference \succeq_s . Together, Axioms 5 and 6 restrict the manner in which past consumption influences future preferences. Axiom 5 ensures that correlation between consumption prior to period $t - K$ and the decision problem in period t does not affect preferences whereas Axiom 6 ensures that the realization of consumption prior to period $t - K$ does not affect the individuals ranking of consumption flows after period t .

Axiom 7 requires individuals to be indifferent as to the timing of resolution of uncertainty. In a standard, expected utility environment this indifference is implicit in the assumption that the domain of preference is the set of lotteries over consumption paths. Our domain of preferences are decision problems and in this richer structure a separate assumption is required to rule out agents that are not indifferent to the timing of resolution of uncertainty, such as the ones described by Kreps and Porteus (1978).

Consider the lotteries $\mu = \alpha(b, x) + (1 - \alpha)(b, y)$ and $\nu = (b, \alpha x + (1 - \alpha)y)$. The lottery μ returns the consumption c together with the continuation problem x with probability α and the consumption c with the continuation problem y with probability $1 - \alpha$. In contrast, ν returns c together with the continuation problem $\alpha x + (1 - \alpha)y$ with probability 1. Hence, μ resolves the uncertainty about x and y in the current period whereas ν resolves this uncertainty in the future. If $\{\mu\} \sim_s \{\nu\}$ then the agent is indifferent as to the timing of the resolution of uncertainty.

Axiom 7: (*Indifference to Timing*) $\{\alpha(b, x) + (1 - \alpha)(b, y)\} \sim_s \{(b, \alpha x + (1 - \alpha)y)\}$.

Definition: \succeq_s is regular if there exists $x, \hat{x}, y, \hat{y} \in Z$ such that $\hat{x} \subset x, \hat{y} \subset y, x \succ_s \hat{x}$ and $\hat{y} \succ_s y$. \succeq is regular if each \succeq_s is regular.

Hence \succeq is not regular if it either displays no preference for commitment (i.e., is standard) or if it always prefers fewer options. Theorem 1 below establishes that all regular preferences that satisfy Axioms 1 – 7 can be represented as a discounted sum of state-dependent utilities minus state-dependent self-control costs. We say that the function $W : S \times Z \rightarrow \mathbb{R}$ represents \succeq when $x \succeq_s y$ iff $W(s, x) \geq W(s, y)$ for all s . For any $\mu \in \Delta$, let μ^1 denote the marginal of μ on C .

Theorem 1: *If \succeq is regular and satisfies Axioms 1 – 7, then there exists $\delta \in (0, 1)$, continuous functions $u : S \times C \rightarrow \mathbb{R}, V : S \times C \times Z \rightarrow \mathbb{R}, W : S \times Z \rightarrow \mathbb{R}$ such that*

$$W(s, z) = \max_{\mu \in \mathcal{Z}} \int [u(s, b) + \delta W(sb, z) + V(s, b, z)] d\mu(b, z) - \max_{\nu \in \mathcal{Z}} \int V(s, b, z) d\nu(b, z)$$

for all $s \in S, \nu \in \Delta$ and W represents \succeq . For any $\delta \in (0, 1)$, continuous u, V there exists a unique function W that satisfies the equation above and the \succeq represented by this W satisfies Axioms 1 – 7.

The two main steps of the proof of Theorem 1 entail showing that a preference relation (over decision problems) that satisfies continuity, independence, set betweenness, stationarity and indifference to timing of resolution of uncertainty has a representation of the form

$$W(s, z) = \max_{\mu \in \mathcal{Z}} \{U(s, \mu) + V(s, \mu)\} - \max_{\nu} V(s, \nu) \quad (2)$$

and then using stationarity and separability to show that U is of the form $U = u + \delta W$. In Gul and Pesendorfer (2004) we offer a related proof under stronger stationarity and separability axioms, yielding a representation of state-independent preferences

Next, we provide additional assumptions that are needed to characterize the preferences used in our analysis of addiction; that is, those represented by a utility function W satisfying equation (1).

Assumption I below is taken from Gul and Pesendorfer (2004). It requires that two alternatives, ν, η , offer the same temptation if they have the same marginal distribution over current consumption. For any $\mu \in \Delta(C \times Z)$ we denote by μ^1 the marginal on the

first coordinate (current consumption) and by μ^2 the marginal on the second coordinate (the continuation problem).

Assumption I: (*Temptation by Immediate Consumption*) For $\mu, \nu \in \Delta$ suppose $\nu^1 = \eta^1$. If $\{\mu\} \succ_s \{\mu, \nu\} \succ_s \{\nu\}$ and $\{\mu\} \succ_s \{\mu, \eta\} \succ_s \{\eta\}$ then $\{\mu, \nu\} \sim_s \{\mu, \eta\}$.

To understand Assumption I, note that $\{\mu\} \succ_s \{\mu, \nu\} \succ_s \{\nu\}$ represents a situation where the agent is tempted by ν but chooses μ from $\{\mu, \nu\}$.⁹ Similarly, $\{\mu\} \succ_s \{\mu, \eta\} \succ_s \{\eta\}$ means that the agent is tempted by η but chooses μ . Hence, the agent makes the same choice in both situations. If $\nu^1 = \eta^1$ then immediate temptation means that the agent experiences the same temptation in the two situations and therefore is indifferent between them; $\{\mu, \nu\} \sim_s \{\mu, \eta\}$.

Assumption N below ensures that goods other than d are *neutral*, i.e., cause no temptation and have no dynamic effects. That is, only good d is tempting and only past consumption of d affects future rankings of decision problems.

Assumption N: Let $b = (c, d)$ and $\hat{b} = (\hat{c}, \hat{d})$. If $d = \hat{d}$ then $\{(b, z), (\hat{b}, \hat{z})\} \succeq_s \{(b, z)\}$ and $\succeq_{sb} = \succeq_{s\hat{b}}$. If $\hat{d} > d$ and $\{(b, z)\} \succ_s \{(\hat{b}, \hat{z})\}$ then $\{(b, z)\} \succ_s \{(b, z), (\hat{b}, \hat{z})\}$.

The first statement in Assumption N ensures that there is no temptation so long as the options differ only with respect to current consumption of non-drugs. The second statement means that future preferences are the same so long as the current state and current consumption of drugs are the same. Finally, the third statement implies that higher current drug consumption is always tempting.

To state the final assumption, we first define what it means for the agent to have the same preference for commitment at two states.

Definition: \succeq_s has a preference for commitment at z if there is $x \subset z$ such that $x \succ_s z$; \succeq has the same preference for commitment at \hat{s} and at s if \succeq_s has a preference for commitment at z iff $\succeq_{\hat{s}}$ has a preference for commitment at z .

Assumption P says that the agent's preference for commitment does not change as the state changes. In other words, which alternatives constitute a temptation is independent of the state.

⁹ This follows from a straightforward application of the representation in (2).

Assumption P: *The agent has the same preference for commitment at all s .*

Theorem 2: \succeq is regular and satisfies Axioms 1–7, *I*, *N* and *P* if and only if (i) $S = [0, 1]$, and (ii) there are continuous functions $v, \sigma : [0, 1] \rightarrow \mathbb{R}$, $u : C \rightarrow \mathbb{R}$, and $\delta \in (0, 1)$, such that for $z \in \bar{Z}$

$$W(s, z) = \max_{(c, d, x) \in z} \{u(c, d) + \sigma(s)v(d) + \delta W(c, d, x)\} - \max_{(\hat{c}, \hat{d}, y) \in z} \sigma(s)v(\hat{d})$$

and W represents \succeq . (iii) u is nonconstant, v is strictly increasing, $\sigma > 0$, and s is the previous period’s drug consumption.

Proof: See Appendix.

To illustrate the role of the assumptions in Theorem 2, consider the representation provided in Theorem 1. Adding Assumption *I* ensures that $V(s, \cdot)$ depends only on current consumption. Then, Assumption *N* guarantees that $V(s, \cdot)$ depends only on current *drug* consumption and is strictly increasing in d . Finally, Assumption *P* implies that $U = u + \delta W$ is independent of the state and that the state is equal to last period’s drug consumption.

7. Conclusion

Most studies on drug abuse assert that addiction should be considered a disease.¹⁰ In our approach drug abuse is identified with the discrepancy between what the agent would want to commit to, as reflected by maximizing U , and what he ends-up consuming by maximizing $U + V$. We provide straightforward choice experiments for measuring this discrepancy. Our approach is silent on the question of whether addiction is a disease or a part of the “normal” variation of preferences across individuals.

While our approach is compatible with the disease conception of addiction, there are important differences between our formulation and the typical disease model. Consider the following example: the opiate antagonist naltrexone blocks the opioid receptors in the brain and hence the euphoric effects of these drugs for up to 3 days after the last dose. Naltrexone is used in the treatment of heroin and morphine. However, with the exception of highly motivated addicts such as parolees, probationers and healthcare professionals,

¹⁰ To emphasize the organic basis of the condition the term “disease of the brain” is often used.

most addicts receiving naltrexone tend to stop taking their medicine and relapse. Addicts often report that they stop taking naltrexone because it prevents “getting high”. Doctors call this as a “compliance problem” with naltrexone. For them, this is simply a limitation on the usefulness naltrexone, the same way that toxicity might be a limitation on the usefulness of some other medication.

In our model, there can be two reasons for an addict to discontinue naltrexone and resume heroin consumption: either 3 days is not the right time horizon for commitment or the addict does not wish to commit. The former would suggest a need for longer acting versions of Naltrexone while the latter would mean that there is neither a need nor any room for treatment of this addict. In fact, by our definition, an individual who is unwilling to commit to reducing his drug consumption, for any length of time, at any future date is not an addict. Hence, where the disease model of addiction finds a compliance problem our model suggests that there may be no problem at all.

The fact that naltrexone continues to be used by the most motivated addicts, those who are more likely to abstain even without commitment, suggests a reduction of the cost of self-control as a possible motive taking naltrexone.

Economists interpret behavior as a reflection of the agents’ stable interests and desires. In standard economic analysis there is no room for the notion of a behavioral problem, except to the extent that the behavior is a problem for someone else. Consequently, there is no role for therapy aimed at controlling problem behavior. In contrast, psychologists often view behavior to be independent of and even an impediment to the agent’s welfare. Our model of temptation and self-control provides a potential bridge between these two approaches. Like standard models in economics, we take as given agents’ interests and desires (i.e., utility functions) and accept the hypothesis that behavior is motivated by these interests and desires (i.e., utility maximization). But, we extend the domain of utility functions to include temptation. Without the aid of some outside agency, it is difficult and often very costly for the individual to commit; that is, reduce temptation. Hence, our model leaves room for welfare enhancing treatments and policy. In our interpretation, the role of treatment and policy is to develop commitment devices and opportunities for the agent.

Our model provides a framework for the analysis of both the purposeful actions (e.g. decisions made in the stock market) studied by most economists as well as the compulsive and detrimental behavior (e.g. addiction) studied by many psychologists and healthcare professionals. We have analyzed the interaction of these two types of behavior and evaluated policy alternatives. Our focus was on psychoactive drugs but the model presented in this paper can also be applied to other types of compulsive behavior such as over-eating and other forms of dependency.

8. Appendix

Proof of Proposition 1: To prove the “if” part, let $\sigma(\bar{s}) \geq \sigma(s)$ and let $\mu \in \mathcal{D}(\bar{s}, z) \cap \mathcal{C}(z)$.

Then

$$\int (u(\hat{c}, \hat{d}) + \sigma(\bar{s})v(\hat{d}) + \delta W(\hat{d}, \hat{z})) d\mu(\hat{c}, \hat{d}, \hat{z}) \geq \int (u(\hat{c}, \hat{d}) + \sigma(s)v(\hat{d}) + \delta W(\hat{d}, \hat{z})) d\nu(\hat{c}, \hat{d}, \hat{z})$$

and

$$\int (u(\hat{c}, \hat{d}) + \delta W(\hat{d}, \hat{z})) d\mu(\hat{c}, \hat{d}, \hat{z}) \geq \int (u(\hat{c}, \hat{d}) + \delta W(\hat{d}, \hat{z})) d\nu(\hat{c}, \hat{d}, \hat{z})$$

for all $\nu \in z$. Since $\sigma > 0$ there is $\alpha \in (0, 1]$ such that $\sigma(s) = \alpha\sigma(\bar{s})$. Taking a convex combination of the above two inequalities we conclude that $\mu \in \mathcal{D}(s, z) \cap \mathcal{C}(z)$. Hence, if $\succeq_{\bar{s}}$ is not compulsive then \succeq_s is not compulsive. Obviously $\succeq_s \neq \succeq_{\bar{s}}$ if $\sigma(\bar{s}) > \sigma(s)$.

To prove the “only if” part we can repeat the argument of Lemma 12 from Gul and Pesendorfer (2001) in the current setting to obtain the following fact.

Fact: \succeq is more compulsive at \bar{s} than at s only if for some $\beta_1, \beta_2 \in \mathbb{R}_+$ and β_3 ,

$$U + \sigma(s)v = \beta_1 U + \beta_2 (U + \sigma(\bar{s})v) + \beta_3$$

for all μ .

Note that

$$U + \sigma(s)v = U + \frac{\sigma(s)}{\sigma(\bar{s})} \sigma(\bar{s})v$$

Hence, $\beta_1 + \beta_2 = 1$ and $\beta_2 = \frac{\sigma(s)}{\sigma(\bar{s})}$. Since, $\beta_1, \beta_2 > 0$, we conclude $\sigma(\bar{s}) > \sigma(s)$.

□

9. Proof of Theorems 1 and 2

9.1 Proof of Theorem 1

It is easy to show that if \succeq satisfies Axioms 3, 6 and 7 then it also satisfies the following stronger version of the independence axiom:

Axiom 3*: $x \succ_s y, \alpha \in (0, 1)$ implies $\alpha x + (1 - \alpha)z \succ_s \alpha y + (1 - \alpha)z$.

Theorem 1 of Gul and Pesendorfer (2001) establishes that \succeq_s satisfies Axioms 1,2, 4 and 3* if and only if there exist $W(s, \cdot), U(s, \cdot), V(s, \cdot)$ such that

$$W(s, z) := \max_{\mu \in z} \{U(s, \mu) + V(s, \mu)\} - \max_{\nu \in z} V(s, \nu) \quad (*)$$

for all $z \in Z$ and \hat{W} represents \succeq . Moreover, the functions $W(s, \cdot), U(s, \cdot), V(s, \cdot)$ are continuous and linear in their second arguments. We refer to the triple $(U(s, \cdot), V(s, \cdot), W(s, \cdot))$ as a representation of \succeq_s . The additional content of Theorem 1 is that we may choose functions (U, V, W) that are continuous in s such that $(U(s, \cdot), V(s, \cdot), W(s, \cdot))$ is a representation of \succeq_s for each s and $U(s, \cdot)$ satisfies

$$U(s, \mu) = \int [u(s, b) + \delta W(sb, z)] d\mu(c, z)$$

for some continuous function u and $\delta \in (0, 1)$.

Fix \bar{s} and let $(\hat{W}(\bar{s}, \cdot), \hat{U}(\bar{s}, \cdot), \hat{V}(\bar{s}, \cdot))$ be a representation of $\succeq_{\bar{s}}$. Define W to be the following function:

$$W(s, y) := \hat{W}(\bar{s}, sy) \quad (**)$$

Observe that W is well defined and continuous in both arguments since \hat{W} is continuous in its second argument. In the following Lemmas, the function W is the function defined in (**).

Lemma 1: W represents \succeq . Moreover, there exist continuous functions U, V such that

$$W(s, z) := \max_{\mu \in z} \{U(s, \mu) + V(s, \mu)\} - \max_{\nu \in z} V(s, \nu)$$

and W, U, V are linear in their second arguments.

Proof: Axiom 6 implies $W(s, x) \geq W(s, y)$ iff $\hat{W}(s, x) \geq \hat{W}(s, y)$. Therefore, W represents \succeq . Note that \hat{W} is linear in its second argument. Let $z = \alpha x + (1 - \alpha)y$. Axiom 7 and linearity of \hat{W} in its second argument imply that

$$\begin{aligned} W(s, z) &= \hat{W}(\bar{s}, sz) \\ &= \hat{W}(\bar{s}, \{\alpha sx + (1 - \alpha)sy\}) \\ &= \alpha \hat{W}(\bar{s}, sx) + (1 - \alpha) \hat{W}(\bar{s}, sy) \\ &= \alpha W(s, x) + (1 - \alpha)W(s, y) \end{aligned}$$

Thus, W is linear in its second argument. It follows that $W(s, z) = \alpha(s)\hat{W}(s, z) + \beta(s)$ for some $\alpha, \beta : S \rightarrow \mathbb{R}$ such that $\alpha(s) \geq 0$. Since \succeq is regular, $\alpha(s) > 0$ for all s . Hence, $U = \alpha\hat{U} + \beta, V = \alpha\hat{V}$ and the W have the desired properties. \square

Lemma 2: Let $\hat{W}(s, \cdot)$ represent \succeq_s . Then,

$$\hat{W}(s, b_1 \dots b_l \hat{s} \bar{z}) - \hat{W}(s, b_1 \dots b_l \hat{s} z) = \hat{W}(s, \bar{b}_1 \dots \bar{b}_l \hat{s} \bar{z}) - \hat{W}(s, \bar{b}_1 \dots \bar{b}_l \hat{s} z)$$

for all $l, (\bar{b}_1, \dots, \bar{b}_l), (b_1, \dots, b_l) \in C^{l+1}, \hat{s} \in C^K, z, \bar{z} \in Z$.

Proof: Note that by Axiom 5,

$$\frac{1}{2}(\bar{b}_1, b_2 \dots b_l \hat{s} \bar{z}) + \frac{1}{2}(b_1, b_2 \dots b_l \hat{s} z) \sim_s \frac{1}{2}(b_1, b_2 \dots b_l \hat{s} \bar{z}) + \frac{1}{2}(\bar{b}_1, b_2 \dots b_l \hat{s} z)$$

Assume that the assertion holds for $l' \leq l - 1$. Then, Axiom 6 implies that

$$\frac{1}{2}(\bar{b}_1, \bar{b}_2 \dots \bar{b}_l \hat{s} \bar{z}) + \frac{1}{2}(\bar{b}_1, b_2 \dots b_l \hat{s} z) \sim_s \frac{1}{2}(\bar{b}_1, b_2 \dots b_l \hat{s} \bar{z}) + \frac{1}{2}(\bar{b}_1, \bar{b}_2 \dots \bar{b}_l \hat{s} z)$$

Since \hat{W} represents \succeq_s , we conclude

$$\begin{aligned} \hat{W}(s, b_1 \dots b_l \hat{s} \bar{z}) - \hat{W}(s, b_1 \dots b_l \hat{s} z) &= \hat{W}(s, \bar{b}_1 b_2 \dots b_l \hat{s} \bar{z}) - \hat{W}(s, \bar{b}_1 b_2 \dots b_l \hat{s} z) \\ &= \hat{W}(s, \bar{b}_1 \dots \bar{b}_l \hat{s} \bar{z}) - \hat{W}(s, \bar{b}_1 \dots \bar{b}_l \hat{s} z) \end{aligned}$$

and hence the assertion holds for $l' \leq \bar{l}$. Observe that Axiom 5 implies that the Lemma holds for $l = 1$. \square

Lemma 3: $W(s', sx) - W(s', sy) = W(s'', sx) - W(s'', sy)$ for all s', s'', x, y .

Proof: Recall that

$$W(s', sx) = \hat{W}(\bar{s}, s' sx)$$

for some \hat{W} such that $\hat{W}(\bar{s}, \cdot)$ represents $\succeq_{\bar{s}}$. Lemma 2 implies that

$$\hat{W}(\bar{s}, s' sx) - \hat{W}(\bar{s}, s' sy) = \hat{W}(\bar{s}, s'' sx) - \hat{W}(\bar{s}, s'' sy) \quad (\ddagger)$$

Substituting W for \hat{W} in equation (\ddagger) then proves the Lemma. \square

Lemma 3: *There exist $\delta : S \times C \rightarrow (0, \infty)$ and $u : S \times C \rightarrow \mathbb{R}$ such that $U(s, \nu) = \int [u(s, b) + \delta(s, b)W(sb, z)]d\nu(b, z)$ for all $s \in S, \nu \in \Delta$.*

Proof: Since $U(s, \cdot)$ is linear and continuous, it has an integral representation. That is;

$$U(s, \nu) = \int U(s, b, z)d\nu(b, z)$$

By Axiom 6, $U(s, b, \cdot)$ and $W(sb, \cdot)$ yield the same linear preferences over Z . By regularity, neither function is constant. It follows that $U(s, b, \cdot)$ is a strictly positive affine transformation of $W(sb, \cdot)$. Hence, for some u, δ ,

$$U(s, b, \cdot) = u(s, b) + \delta(s, b)W(sb, y)$$

where $\delta(s, b) > 0$ for all $s \in S, b \in C$. Therefore,

$$U(s, \nu) = \int [u(s, b) + \delta(s, b)W(sb, y)]d\nu(b, z)$$

as desired \square

Lemma 4: *The function $\delta(\cdot)$ in Lemma 3 is constant.*

Proof: Suppose δ is not constant. Let $k \in 1, \dots, K + 1$ denote the smallest integer such that $\delta(b_1, \dots, b_{K+1}) = \delta(\tilde{b}_1, \dots, \tilde{b}_{K+1})$ for all $(b_1, \dots, b_{K+1}), (\tilde{b}_1, \dots, \tilde{b}_{K+1})$ with $b_n = \tilde{b}_n$ for $n \leq k$. Then, it is straightforward to show that there exist $(s, b_{K+1}) = (b_1, \dots, b_{K+1})$ and $(s^*, b_{K+1}^*) = (b_1^*, \dots, b_{K+1}^*)$ such that $b_n = b_n^*, n \neq k$ and $\delta(b_1, \dots, b_{K+1}) > \delta(b_1^*, \dots, b_{K+1}^*)$.

Pick any $b \in C$. Let $s' = (b, \dots, b, b_1, b_2, \dots, b_{k-1})$. Fix any \hat{s} . By regularity there are $y_h, y_l \in Z$ such that $W(\hat{s}, y_h) > W(\hat{s}, y_l)$. Let $y_{hh} = b_k \dots b_{K+1} \hat{s} y_h, y_{hl} = b_k \dots b_{K+1} \hat{s} y_l$

and $y_{lh} = b_k^* \dots b_{K+1}^* \hat{s}y_h$, $y_{ll} = b_k^* \dots b_{K+1}^* \hat{s}y_l$. Let $x = .5y_{hh} + .5y_{ll}$ and $z = .5y_{hl} + .5y_{lh}$. By Lemma 2, $W(s', x) = W(s', z)$.

Applying Lemma 3 repeatedly and using the fact that $\delta(s, b) = \delta(\tilde{s}, \tilde{b})$ for $(s, b), (\tilde{s}, \tilde{b})$ with $b_n = \tilde{b}_n, n \leq k$ establishes $W(s', x) - W(s', z) = 0$ iff

$$\begin{aligned} & \delta(s, b_{K+1})W(sb_{K+1}, \hat{s}y_h) + \delta(s^*, b_{K+1}^*)W(s^*b_{K+1}^*, \hat{s}y_l) = \\ & \delta(s, b_{K+1})W(sb_{K+1}, \hat{s}y_l) + \delta(s^*, b_{K+1}^*)W(s^*b_{K+1}^*, \hat{s}y_h) \end{aligned}$$

Rearranging, this implies

$$\begin{aligned} & \delta(s, b_{K+1})(W(sb_{K+1}, \hat{s}y_h) - W(sb_{K+1}, \hat{s}y_l)) = \\ & \delta(s^*, b_{K+1}^*)(W(s^*b_{K+1}^*, \hat{s}y_h) - W(s^*b_{K+1}^*, \hat{s}y_l)) \end{aligned}$$

Observe that $W(s, \hat{s}y_h) - W(s, \hat{s}y_l) > 0$ by construction and hence Lemma 3 implies the desired contradiction. \square

Lemma 5: *Let $\delta \in \mathbb{R}$ denote the constant function in Lemma 4. Then, $0 < \delta < 1$.*

Proof: That $\delta > 0$ has already been established. Pick any $b \in C$ and let $s = (b, b, \dots, b)$. Let z_b denote the unique $z \in Z$ such that $z = \{b, z\}$. Pick $y_1 \in Z$ such that $W(s, y_1) \neq W(s, z)$. By regularity, such a y_1 exists. Define $y_n \in Z$ inductively as $y_n = \{(b, y_{n-1})\}$ and note that y_n converges to z . Hence, by continuity, $W(s, z) - W(s, y_n)$ must converge to 0. But, by Lemma 3, $W(s, z) - W(s, y_n) = \delta^{n-1}(W(s, y_1) - W(s, z)) \neq 0$. Hence, $\delta < 1$. \square

Lemmas 1 – 5 establish that there is a continuous representation (U, V, W) that satisfies $U(s, \mu) = u(\mu^1) + \int \delta W(sb, z) d\mu(b, z)$.

To conclude the proof, let $\delta \in (0, 1)$ and $u : S \times C \rightarrow \mathbb{R}$ and $V : S \times C \times Z \rightarrow \mathbb{R}$ be continuous functions.

Lemma 6 (A Fixed-Point Theorem): *If B is a closed subset of a Banach space with norm $\|\cdot\|$ and $T : B \rightarrow B$ is a contraction mapping (i.e., for some integer m and scalar $\alpha \in (0, 1)$, $\|T^m(W) - T^m(W')\| \leq \alpha \|W - W'\|$ for all $W, W' \in B$), then there is a unique $W^* \in B$ such that $T(W^*) = W^*$.*

Proof: See [Bertsekas and Shreve (1978), p. 55] who note that the theorem in Ortega and Rheinbold (1970) can be generalized to Banach spaces. \square

Let \mathcal{W} be the Banach space of all continuous, real-valued functions on $S \times Z$ (endowed with the sup norm). The operator $T : \mathcal{W} \rightarrow \mathcal{W}$, where

$$TW(s, z) = \max_{\mu \in z} \int [u(s, b) + V(s, b, z) + \delta W(sb, x)] d\mu(b, x) - \max_{\nu \in z} \int V(s, b, z) d\nu(b, z)$$

is well-defined and is a contraction mapping. Hence, by Lemma 6, there exists a unique $W \in \mathcal{W}$ such that $T(W) = W$.

For any W, u, v, δ such that

$$W(s, z) = \max_{\mu \in z} \left\{ \int [u(s, b) + V(s, b, z) + \delta W(sb, x)] d\mu(b, x) \right\} - \max_{\nu \in z} \int V(s, b, z) d\nu(b, z)$$

define \succeq_s by $x \succeq_s y$ iff $W(s, x) \geq W(s, z)$. Verifying that \succeq_s satisfies Axioms 1 – 7 is straightforward. \square

9.2 Proof of Theorem 2

By Theorem 1, \succeq can be represented by a continuous \hat{W} where

$$\hat{W}(s, z) = \max_{\mu \in z} \left\{ \int [\hat{u}(s, \mu^1) + \delta \hat{W}(sb, x)] d\mu(b, x) + \hat{V}(s, \mu) \right\} - \max_{\nu \in z} \hat{V}(s, \nu) \quad (3)$$

for some continuous u, v and $\delta \in (0, 1)$. Moreover, $\hat{W}, \hat{u}, \hat{V}$ are linear in their second argument. Let $\hat{U}(s, \mu) = \int (\hat{u}(s, b) + \delta \hat{W}(sb, x)) d\mu(b, x)$.

Lemma 6: $\hat{V}(s, \mu) = \hat{V}(s, \nu)$ if $\mu^1 = \nu^1$.

Proof: If $\hat{V}(s, \cdot) = \alpha \hat{U}(s, \cdot) + \beta$ for some $\alpha \leq -1$, then $x \succeq_s y$ for all $x \subset y$ contradicting regularity. If $\hat{V}(s, \cdot) = \alpha \hat{U}(s, \cdot) + \beta$ for some $\alpha \geq 0$ then $x \succeq_s y$ for all $y \subset x \in Z$ and \succeq is not regular. Hence, for each $s \in S$ there are two possibilities: either $\hat{V}(s, \cdot)$ is not an affine transformation of $\hat{U}(s, \cdot)$ or there exists $\alpha \in (-1, 0)$ such that $\hat{V}(s, \cdot) = \alpha \hat{U}(s, \cdot) + \beta$. In either case, regularity implies that there exist $\mu^s, \nu^s \in \Delta$ such that $\hat{U}(s, \mu^s) + \hat{V}(s, \mu^s) > \hat{U}(s, \nu^s) + \hat{V}(s, \mu^s)$ and $\hat{V}(s, \mu^s) < \hat{V}(s, \nu^s)$.

Take any $\nu, \hat{\nu} \in \Delta$ such that $\nu^1 = \hat{\nu}^1$. By continuity, there exists $\alpha > 0$ small enough so that

$$\begin{aligned}\hat{U}(s, \mu^s) + \hat{V}(s, \mu^s) &> \hat{U}(s, \alpha\nu + (1 - \alpha)\nu^s) + \hat{V}(s, \alpha\nu + (1 - \alpha)\nu^s) \\ \hat{U}(s, \mu^s) + \hat{V}(s, \mu^s) &> \hat{U}(s, \alpha\hat{\nu} + (1 - \alpha)\nu^s) + \hat{V}(s, \alpha\hat{\nu} + (1 - \alpha)\nu^s) \\ \hat{V}(s, \mu^s) &< \hat{V}(s, \alpha\nu + (1 - \alpha)\nu^s) \\ \hat{V}(s, \mu^s) &< \hat{V}(s, \alpha\hat{\nu} + (1 - \alpha)\nu^s)\end{aligned}$$

Then, Assumption I implies $\{\alpha\nu + (1 - \alpha)\nu^s, \mu^s\} \sim_s \{\alpha\hat{\nu} + (1 - \alpha)\nu^s, \mu^s\}$. Since \hat{W} represents \succeq we have $\hat{V}(s, \alpha\nu + (1 - \alpha)\nu^s) = \hat{V}(s, \alpha\hat{\nu} + (1 - \alpha)\nu^s)$ and since \hat{V} is linear, we conclude $\hat{V}(s, \nu) = \hat{V}(s, \hat{\nu})$ as desired. \square

By Lemma 6, there is a function $\hat{v} : S \times \Delta(C) \rightarrow \mathbb{R}$ such that $\hat{V}(s, \mu) = \hat{v}(s, \mu^1)$. Regularity implies that neither $\hat{U}(s, \cdot)$ nor $\hat{v}(s, \cdot)$ is constant. Moreover, (since $\delta > 0$) this implies that $\hat{v}(s, \cdot)$ is not an affine transformation of $\hat{U}(s, \cdot)$. Hence, we may apply Theorem 7 of Gul and Pesendorfer (2001) to yield the following implications:

Fact: (Theorem 7 (Gul and Pesendorfer (2001))) $\hat{s}Ps$ iff for some $\alpha_u, \alpha_v \in [0, 1], \gamma > 0, \gamma_u, \gamma_v \in \mathbb{R}$

$$\begin{aligned}\gamma\hat{U}(s, \mu) &= \alpha_u\hat{U}(\hat{s}, \mu) + (1 - \alpha_u)\hat{v}(\hat{s}, \mu^1) + \gamma_u \\ \gamma\hat{v}(s, \mu^1) &= \alpha_v\hat{U}(\hat{s}, \mu) + (1 - \alpha_v)\hat{v}(\hat{s}, \mu^1) + \gamma_v\end{aligned}$$

for all μ .

Pick any $s_0 \in S$. It follows from Assumption P and the Fact above that for all $s \in S$

$$\begin{aligned}\hat{U}(s, \mu) &= \alpha(s)\hat{U}(s_0, \mu) + \hat{\gamma}_u(s) \\ \hat{v}(s, \mu^1) &= \beta(s)\hat{v}(s_0, \mu^1) + \hat{\gamma}_v(s)\end{aligned}\tag{4}$$

for some functions $\alpha, \beta, \hat{\gamma}_u, \hat{\gamma}_v$ such that $\alpha(s) > 0, \beta(s) > 0$ for all s . Note that \hat{U} and \hat{v} are continuous and hence $\alpha, \beta, \hat{\gamma}_u, \hat{\gamma}_v$ are continuous.

Combining (3) and (4) yields,

$$\begin{aligned}\int [\hat{u}(s, b) + \delta\hat{W}(sb, z)]d\nu(b, z) &= \\ \int [\alpha(s)\hat{u}(s_0, b) + \hat{\gamma}_u(s) + \alpha(s)\delta\hat{W}(s_0b, z)]d\nu(b, z)\end{aligned}\tag{5}$$

The only terms on either side of (5) that depend on ν^2 are $\delta\hat{W}(sb, z)$ and $\alpha(s)\delta\hat{W}(s_0b, z)$. Since regularity implies that neither of these terms is constant it follows that

$$\hat{W}(sb, \cdot) = \alpha(s)\hat{W}(s_0b, \cdot) + A(s, b)$$

Lemma 2 (in the proof of Theorem 1) then implies that $\alpha(s) = 1$ for all s . It follows that $\hat{W}(s_0b, \cdot)$ represents \succeq_{sb} . Hence, $K = 1$. That is, $sb = b$ for all s, b . Henceforth, we write b instead of sb .

Let $W(b, z) = \hat{W}(b, z) - \gamma_u(b)$, $u(b) = \hat{u}(s_0, b) + \delta\gamma_u(b)$ for all b . Let $v(\cdot) = \hat{v}(s_0, \cdot)$. Then,

$$\begin{aligned} W(b, z) &= \hat{W}(b, z) - \gamma_u(b) = \max_{\mu \in z} \{\hat{U}(b, \mu) + \hat{v}(b, \mu)\} - \max_{\nu \in z} \hat{v}(b, \nu) - \gamma_u(b) \\ &= \max_{\mu \in z} \{\hat{U}(s_0, \mu) + \beta_v(b)\hat{v}(s_0, \mu)\} - \max_{\nu \in z} \beta_v(b)\hat{v}(s_0, \nu) \\ &= \max_{\mu \in z} \int [\hat{u}(s_0, b') + \beta_v(b)\hat{v}(s_0, b') + \delta\hat{W}(b', x)] d\mu(b', x) \\ &\quad - \max_{\nu \in z} \beta_v(b)\hat{v}(s_0, \nu) \\ &= \max_{\mu \in z} \int [u(b') + \beta_v(b)v(b') + \delta W(b', x)] d\mu(b', x) \\ &\quad - \max_{\nu \in z} \beta_v(b)\hat{v}(\nu^1) \end{aligned}$$

Define $\sigma(b) := \beta_v(b)$. By Assumption N, $v(c, d) = v(\hat{c}, \hat{d})$ if $d = \hat{d}$. Assumption N also implies that $v(c, d)$ is strictly increasing in d . Finally, Assumption N implies that $\sigma(c, d) = \sigma(\hat{c}, \hat{d})$ if $d = \hat{d}$. Hence, u, v, σ, δ satisfy all the desired properties.

Establishing that the preference represented by W with (u, v, σ, δ) satisfying the conditions (i – iii) of the theorem satisfies Axioms 1 – 7, I, N and P is straightforward. Hence, to conclude the proof of the converse, we need to show only that the \succeq represented is regular. Since u is nonconstant, there exists (\bar{c}, \bar{d}) and (c, d) such that $u(c, d) > u(\bar{c}, \bar{d})$. Pick any $x \in Z$ and let $\bar{z} = \{(\bar{c}, \bar{d}, x)\}$ and $z = \{(c, d, x)\}$. Then, it follows from the representation of Theorem 2 that $W(s, z \cup \bar{z}) = W(s, \bar{z}) > W(s, z)$. Next, let $\bar{y} = (\hat{c}, \hat{d}, \bar{z})$ and $y = (\hat{c}, \hat{d} + \epsilon, z)$ for \bar{z}, z as defined above and some \hat{c}, \hat{d} and some $\epsilon > 0$. It follows from the continuity and increasingness of v that for ϵ sufficiently small, $W(s, \bar{y}) > W(s, y \cup \bar{y})$ proving that \succeq is regular. \square

References

1. Bertsekas D. P. and S. E. Shreve, *Stochastic Optimal Control: The Discrete Time Case*, 1978, Academic Press, New York.
2. Becker, G. S., and K. M. Murphy, "A Theory of Rational Addiction," *Journal of Political Economy*, 1988, 675–700.
3. Becker, G. S., Grossman M. and K. M. Murphy, "An Empirical Analysis of Cigarette Addiction," *American Economic Review*, 1994, 84, 396–418.
4. Bernheim, B. D., and A. Rangel, "Addiction and Cue-Conditioned Cognitive Processes," 2002, working paper. Stanford University.
5. Chick, J., "Emergent Treatment Concepts", *Annual Review of Addictions Research and Treatment*, 1992, 297–312.
6. Davies, J. B. *The Myth of Addiction; the Application of the Psychological Theory of Attribution to Illicit Drug Use*, 1992, Harwood Academic Publishers, Chur.
7. Gruber, J. and B. Koszegi, "Is Addiction Rational? Theory and Evidence," *Quarterly Journal of Economics*, 116, 1261–1303.
8. Gul, F. and W. Pesendorfer, "Temptation and Self-Control," *Econometrica*, 2001, 69, 1403–1436..
9. Gul, F. and W. Pesendorfer, "Self-Control and the Theory of Consumption," *Econometrica*, 2004, 72, 119–158.
10. Laibson, D., "Golden Eggs and Hyperbolic Discounting," *Quarterly Journal of Economics*, 1997, 112, 443–477.
11. Laibson, D., "A Cue Theory of Consumption," *Quarterly Journal of Economics*, 2001, 66, 81–120.
12. Loewenstein G., "Out of Control: Visceral Influences on Behavior," *Organizational Behavior and Human Decision Processes*, 1996, 65, 272–292.
13. O'Donoghue, T. and M. Rabin, "Addiction and Self-Control," in *Addiction: Entries and Exits*, 1999, J. Elster, ed., Russel Sage Foundation.
14. Ortega J.M., and W.L. Rheinbold, *Iterative Solutions of Nonlinear Equations in Several Variables*, 1970, Academic Press, New York.
15. Pentel, P.R.; Malin, D.H.; Ennifar, S.; Hieda, Y.; Keyler, D.E.; Lake, J.R.; Milstein, J.R.; Basham, L.E.; Coy, R.T.; Moon, J.W.D.; Naso, R.; Fattom, A. , "A Nicotine Conjugate Vaccine Reduces Nicotine Distribution to Brain and Attenuates Its Behavioral and Cardiovascular Effects in Rats - Role of Dose and Dose Interval," *Pharmacology Biochemistry & Behavior*, 2000, 65, 191–198.

16. Phelps, E. S. and R. A. Pollak “On Second-Best National Saving and Game Equilibrium Growth,” *Review of Economic Studies*, 1968, 35, 185–199.
17. Robinson T. E. and K. C. Berridge, “The Neural Basis of Drug Craving: an Incentive-Sensitization Theory of Addiction”, *Brain Research Reviews*, 1993, 18, 247–291.
18. Strotz, R. H., “Myopia and Inconsistency in Dynamic Utility Maximization,” *Review of Economic Studies*, 1955, 23, 165–180.