TIME INCONSISTENT PREFERENCES AND SOCIAL SECURITY[*]

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In this paper we examine the role of social security in an economy populated by overlapping generations of individuals with time-inconsistent preferences who face mortality risk, individual income risk, and borrowing constraints. We find that unfunded social security lowers the capital stock, output, and consumption for consumers with time-consistent or time-inconsistent preferences. However, it may raise or lower welfare depending on the strength of time inconsistency.

1 Introduction

In the United States and most other developed countries, the public pension system and associated benefit payments to retirees and their families (including disability, medical, and survivor benefits) constitute the largest item in the government budget. Partly because of their scale, these programs have during the last quarter century become the subject of intense study by economists, who have become increasingly aware of the large effects such programs may have on many aspects of the economy.

The literature on unfunded public pensions has identified a variety of both costs and benefits of such systems. The costs consist largely of distortions to the labor supply and saving decisions. The major benefits arise from the fact that social security may provide avenues for risk sharing that are not otherwise available or are very costly in private markets. Depending upon the reasons for the lack of private insurance, social security might provide a lower-cost substitute for private contracts. Annuity markets provide an example. One would expect life-cycle consumers facing uncertain death dates to utilize individual annuity contracts to smooth consumption and insure against the risk of outliving their assets. Although private annuity markets exist in the United States, the volume of contracts in these markets is surprisingly small, possibly because of adverse selection (Friedman and Warshawsky, 1990).[1] By imposing a mandatory annuity system, social security might substitute for missing private annuity markets and might at least mitigate the welfare losses due to adverse selection.[2]

In addition to the benefits discussed above, some have argued that social security may provide welfare gains for agents who lack the foresight to save adequately for their retirement. For example, Diamond (1977, p. 281) states that a "justification for Social Security is that many individuals will not save enough for retirement if left to their own devices." Kotlikoff, Spivak, and Summers (1982) remark on the widely held belief that the "essential premise underlying the Social Security system . . . is that left to their own devices, large numbers of people would fail to save adequately and find themselves destitute in their old age." And according to Feldstein (1985, p. 303), the "principal rationale for such mandatory programs is that some individuals lack the foresight to save for their retirement years."

Extensive empirical evidence is cited to support the view that many households do not save adequately, although much of this evidence is subject to alternative interpretations. Studies using a wide variety of data have documented that a substantial fraction of the U.S. population accumulates very little wealth relative to its lifetime income.[3] The mere fact that many individuals fail to

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[1] Individuals might choose not to annuitize all their wealth if they have operative bequest motives or wish to self-insure against large medical or nursing home expenses.

[2] Diamond (1977) discusses various rationales for a social security system qualitatively like that in the United States. Hubbard and Judd (1987), İmrohoroğlu, İmrohoroğlu, and Joines (1995,1999), and Storesletten, Telmer, and Yaron (1999) evaluate the quantitative trade-off between the insurance benefits of social security against the saving distortion and find that the cost of social security outweighs its benefits. Also see İmrohoroğlu (1999).

[3] See Diamond (1977), Feldstein and Feenberg (1983), Avery, Elliehausen, Canner, and

accumulate large stocks of wealth does not imply that they lack foresight, however. As Bernheim, Skinner, and Weinberg (2001, p. 832) note, "if saving reflects rational, farsighted optimization, then low saving is simply an expression of preferences." They contrast this view with one in which "households are shortsighted, boundedly rational, dynamically inconsistent, impulsive, or prone to regret." Distinguishing between these two points of view requires more detailed analysis of the data. Diamond (1977), Bernheim (1995), and Kotlikoff, Spivak, and Summers (1982) provide evidence on this point by comparing observed asset holdings with asset levels required to achieve various consumption targets.[4]

Several papers that examine the behavior of the elderly report a drop in consumption at retirement (Hammermesh, 1984; Mariger, 1987). Although this pattern is sometimes taken as evidence of a lack of foresight, alternative explanations exist. For example, unforeseen circumstances, possibly related to health, may force workers into retirement, thus leading to an unanticipated reduction in lifetime resources and a sudden drop in consumption (Hausman and Paquette, 1987). In addition, leisure (or time spent in home production) may substitute for market goods in providing utility in old age. Using different and more recent data sets, Banks, Blundell, and Tanner (1998) and Bernheim, Skinner, and Weinberg (2001) confirm the drop in consumption around retirement. After considering several explanations, they conclude that part of the consumption decline must be due to the arrival of new information, with the most likely candidate being negative innovations to the income process because workers have overestimated their retirement income. Bernheim, Skinner, and Weinberg (2001) state that "a broad range of standard life-cycle considerations are collectively incapable of accounting for the observed variation in wealth" (p. 833) and that "the empirical patterns in this paper are more easily explained if one steps outside the framework of rational, farsighted optimization" (p. 855).

Despite the apparently widespread view that many individuals may lack the foresight to save adequately for their retirement, there have been few attempts to analyze the effectiveness of social security in mitigating the welfare costs of such undersaving. Feldstein (1985) examines a two-period overlapping generations economy with inelastic labor supply and no uncertainty. Individuals in his model are short-sighted in that the elderly attach greater weight to period-2 outcomes than do the young. In that framework, reductions in saving constitute the only welfare cost of social security, and providing consumption for short-sighted agents constitutes the only benefit. His findings indicate that even if every individual is substantially short-sighted it may be optimal to have either no social security system or one in which the social security replacement ratio is very low.

An alternative modeling strategy stems from the literature on time-inconsistent behavior and more specifically from the recent literature dealing with quasi-

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Gustafson (1984), Diamond and Hausman (1984), Hurd (1990), Avery and Kennickell (1991), and Hubbard, Skinner, and Zeldes (1995), among others.

[4] Gale (1997) raises questions about the estimates of the adequacy of retirement saving in some of these studies.

hyperbolic discounting.[5] Strotz (1956) argues that mechanisms that constrain the future choices of agents may be desirable if their behavior exhibits time inconsistency. Social security may be viewed as such a commitment device. According to Akerlof (1998, p. 187), the "hyperbolic model explains the uniform popularity of social security, which acts as a pre-commitment device to redistribute consumption from times when people would be tempted to overspend – during their working lives – to times when they would otherwise be spending too little – in retirement .... [S]uch a transfer is most likely to improve welfare significantly."

In this paper we examine the welfare effects of unfunded social security on individuals with time-inconsistent preferences. In addition to incorporating quasi-hyperbolic discounting, our model nests the retrospective form of time inconsistency analyzed by Feldstein (1985) and extends his framework to include a wider range of benefits and costs of social security. In order to examine the role of social security in an economy with time-inconsistent preferences, we construct a model which consists of overlapping generations of 65-period-lived individuals facing mortality risk, individual income risk, and borrowing constraints. Private annuity markets and credit markets are closed by assumption. Agents in this economy choose the number of hours worked whenever they are given the opportunity to do so. If they are not given the opportunity to work, they receive unemployment insurance. Agents in this economy accumulate assets to provide for old-age consumption and, because they face liquidity constraints, to self-insure against future income shocks. Elderly agents receive social security benefits that are financed by a payroll tax on workers. At any time after reaching the normal retirement age, they may make an irreversible decision to draw social security benefits, although collection of benefits does not preclude working. Individuals in this economy are heterogenous with respect to their age, employment status, retirement status, hours worked, and asset holdings.

In this environment social security may provide additional utility for individuals who regret their saving decisions when they find themselves with low consumption after retirement. In addition, social security may substitute for missing private annuity markets in helping agents allocate consumption in the face of uncertain life spans. On the other hand, social security distorts aggregate saving and labor supply behavior and affects the wage rate and the interest rate. Consequently, whether or not social security is welfare enhancing even for short-sighted agents is a quantitative question.

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[5]For example, see Phelps and Pollak (1968) and Laibson (1997). For time-inconsistent preferences more generally, see Strotz (1956), Pollak (1968), and Thaler and Shefrin (1981), among others.

2   A Model of Social Security

2.1   The Environment

Time is discrete and starts in the infinite past. The setup is a stationary over-lapping generations economy. At each date, a new generation is born which is n percent larger than the previous generation. Individuals face long but random lives and some live through age J, the maximum possible life span. Life-span uncertainty is described by ψ_j, the time-invariant conditional survival probability from age $j-1$ to j. Under our stationary population assumption, the cohort shares, $\{\mu_j\}_{j=1}^J$, are given by

(1)
$$\mu_j = \psi_j \mu_{j-1}/(1+n), \text{ where } \sum_{j=1}^J \mu_j = 1.$$

2.2   Preferences and Measures of Utility

Preferences are defined over sequences of lifetime consumption and labor $\{c_j, \ell_j\}_{j=1}^J$. The essence of time-inconsistent preferences is that the value agents attach to these sequences depends on the agent's vantage point. In particular, the agent may value actions differently *ex post* than at the time those actions are taken, and so may later regret those actions.

Social security can have potentially large effects on the average lifetime levels of consumption and labor and also on the allocation of consumption and labor over the life cycle. The possibility that social security can improve the welfare of time-inconsistent agents is primarily a question of whether the resulting intertemporal redistributions of consumption and labor would raise utility as viewed from at least some ages.

If preferences are time-consistent, and assuming no life-span uncertainty, the value an agent of age j^* places on the lifetime consumption and labor sequences $\{c_1, \ell_1, c_2, \ell_2, \ldots, c_{j^*}, \ell_{j^*}, \ldots, c_J, \ell_J\}$ is independent of the agent's vantage point j^*. If preferences are time-inconsistent, this valuation depends on j^*. We are concerned with a particular type of time inconsistency in preferences that can be characterized as follows. Let U_j denote the marginal utility of consumption at age j and suppose that the values of consumption and leisure in all periods of life are fixed. Also suppose that an agent's preferences are such that the ratio of marginal utilities $U_{j'}/U_{j^*}$ for some j^* and $j' > j^*$ is larger when viewed from age j' than from age j^*. If at age j^* the agent acts so as to equate this ratio of marginal utilities (as viewed at that time) to the marginal rate of transformation, then upon reaching age j' he will regret having consumed so much and saved so little at age j^*. We consider two features of preferences that can lead to this sort of regret.

Specifically, suppose that an individual of age j^* has preferences over lifetime

consumption and labor given by

$$(2) \quad U_{j^*} = \sum_{j=1}^{j^*-1} \delta_b^{j-j^*} u(c_j, \ell_j) + u(c_{j^*}, \ell_{j^*}) + \beta E_{j^*} \sum_{j=j^*+1}^{J} \delta_f^{j-j^*} u(c_j, \ell_j).$$

Here, δ_f is the agent's forward-looking discount factor and δ_b is the backward-looking discount factor. The expectations operator in the final term accounts for mortality risk, whereas δ_f incorporates discounting only for pure time preference. Note that utility depends on consumption and leisure in the past as well as in current and future periods.

Two features of this preference structure can lead to regret as defined above. If $\delta_f < \delta_b$ (which we refer to as Effect 1), then individuals place less weight on the past than they would if $\delta_f = \delta_b$. In the extreme case where $\delta_b = \infty$ they attach no weight to the past.[6] The parameter $\beta \leq 1$ allows for the possibility that, viewed from today, the discount rate between this period and next may be greater than that between any two consecutive periods further into the future, a feature of preferences that we refer to as Effect 2.[7]

Effect 2 leads not only to time-inconsistent preferences but also to time-inconsistent behavior in the sense that the optimal policy functions derived at age j^* for ages $j' > j^*$ will no longer be optimal when the agent arrives age j'. In the absence of any commitment technology, the agent's future behavior will deviate from that prescribed by the earlier policy functions. Strotz (1956) showed that, if preferences are stationary, time-consistent behavior requires that the discount factor connecting any two periods (current or future) vary exponentially as a function of the length of the interval between the two periods. For $\delta_f < 1$, a value of β less than unity results in discount factors that decline approximately hyperbolically from period j^* into the future (Laibson, 1997). Effect 1 does not lead to this sort of time inconsistency in behavior.

Effects 1 and 2 can exist either separately or in combination, and utility comparisons can be made accordingly. For example, Feldstein (1985) considers Effect 1 in isolation. While much of the recent literature on hyperbolic discounting is concerned primarily with characterizing behavior rather than making welfare comparisons, some papers also contain utility analyses. A notable example is Laibson, Repetto, and Tobacman (1998), who analyze a life-cycle model that appears to incorporate both Effect 2 and Effect 1 with $\delta_b = \infty$.[8] In

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[6]Caplin and Leahy (1999) also consider a preference structure similar to (2). They give particular attention to the case where $\delta_f < 1 < \delta_b$, implying that individuals downweight both the past and the future relative to the present. Their paper contains an extensive justification of the assumption that δ_b is finite (i.e., individuals remember and their memories matter) but greater than unity (memory may be fallible).

[7]Equation (2) contains a third effect due to mortality risk that can cause elderly individuals to regret not having saved more, even if their preferences are otherwise time-consistent ($\delta_f = \delta_b$ and $\beta = 1$). If survival probabilities are less than unity, then $U_{j'}/U_{j^*}$ viewed from age j' will in general exceed $E U_{j'}/U_{j^*}$ viewed from age j^* for $j' > j^*$. Although we will sometimes refer to this feature of the model as Effect 3, we view it as arising from the resolution of uncertainty about survival rather than from any inherent time inconsistency in preferences.

[8]We thank an anonymous referee for clarification of this point.

addition to considering Effects 1 and 2 in combination, we attempt to quantify their individual contributions to the welfare changes resulting from unfunded social security.

The preference structure in equation (2) determines individual behavior and also constitutes the basis for making welfare comparisons among alternative social security arrangements. The first summation on the right-hand-side of the equation is irrelevant for determining behavior. As Deaton (1992, p. 14) notes, however, "it is important to recognize that, at best, [the remaining expression] only represents a fragment of lifetime preferences, albeit that fragment that is 'live' or 'active' for current decision-making." An analysis of the welfare effects of policies that reallocate consumption and leisure across the life cycle requires an explicit consideration of how individuals value past outcomes. If preferences are time-consistent ($\delta_f = \delta_b$ and $\beta = 1$) and there is no uncertainty, then individuals of all ages agree on the welfare ranking of policies. Thus, one can make welfare comparisons solely on the basis of utility at birth. This procedure implicitly assumes that the elderly value the past, and in particular that they place the same value on outcomes in old age relative to those in youth as does a newborn individual. The assumption that individuals place no value on the past ($\delta_b = \infty$) constitutes time inconsistency in preferences and would seem to lead trivially to the conclusion that the elderly prefer a generous social security system.[9] This conclusion need not follow, however, if social security depresses asset accumulation enough to lower consumption even in old age.

If preferences are time-inconsistent, then a single individual can be viewed as a collection of J individuals, each of a different age and each with a different set of preferences. These J individuals need not agree on their ranking of different consumption and labor sequences. Because of the well-known difficulties in making interpersonal utility comparisons, it is unclear which of these J preference orderings should be given priority in judging the welfare consequences of various social security arrangements.[10] The welfare rankings in Feldstein's (1985) 2-period model are based on the preferences of an agent in the final period of life. While arguably reasonable in the context of a 2-period model, this retrospective welfare criterion seems quite arbitrary in the 65-period model used here. Therefore, we use equation (2) to compute welfare measures as viewed from each age, denoted by W_{j^*} for $j^* = 1, 2, \ldots, J$, and we rank policy arrangements based on these measures.[11] W_{j^*} is an average of the individual U_{j^*}, where the averaging is with respect to the stationary distribution of individuals of age j^* across employment and asset states. As might be expected, welfare measures viewed from different ages may disagree in their ranking of policy arrangements.

In addition, we compute a weighted average of the age-specific indicators W_{j^*}, with the weight on each W_{j^*} being the unconditional probability of surviving from birth to age j^*. This aggregate welfare indicator is denoted W.

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[9]See Caplin and Leahy (1999) for persuasive arguments against taking $\delta_b = \infty$.

[10]Strotz (1956) first provided such a multi-agent interpretation of time-inconsistent preferences and pointed out the difficulty of arriving at an unambiguous welfare criterion.

[11]Laibson, Repetto, and Tobacman (1998) also compute age-specific welfare measures in a multi-period life-cycle model.

With time-consistent preferences, the appropriate welfare indicator is the expected lifetime utility of a newborn individual, W_1, because all of the other indicators W_{j^*} for $j^* > 1$ are proportional to W_1. This simple proportionality relation breaks down if preferences are time-inconsistent, yet the aggregate indicator W retains a certain similarity to the expected utility of a newborn in the time-consistent case. Throughout its lifetime, each newborn individual with time-inconsistent preferences will, depending on survival, become as many as J separate individuals, each with its own preference ordering. W is simply the expected value of the age-specific indicators W_{j^*}, where the expectation is taken with respect to the unconditional survival probabilities. This criterion obeys Ramsey's (1928) stricture against pure time discounting of the wellbeing of future generations (or, in this instance, selves), which he refers to as "a practice which is ethically indefensible [that] arises merely from the weakness of the imagination." W is also an egalitarian criterion in the following sense. If a large cohort of N newborn individuals is followed through life, it will ultimately constitute N individuals of age 1, $\pi_2 N$ individuals of age 2, $\pi_3 N$ individuals of age 3, etc., where π_j denotes the unconditional probability of surviving to age j. The welfare criterion W assigns equal weights to the preferences of these $(1 + \pi_2 + \pi_3 + ... + \pi_J)N$ individuals.

Finally, we assume that the period utility function takes the form

$$(3) \qquad u(c_j, \ell_j) = \frac{\left((c_j)^\varphi (1 - \ell_j)^{1-\varphi}\right)^{1-\gamma}}{1 - \gamma},$$

where γ is the coefficient of relative risk aversion and φ is the share of consumption in utility.

2.3   Budget Constraints

Agents are subject to individual earnings uncertainty. An age-j individual faces the state vector $x_j = (a_{j-1}, s_j, \overline{e}_j, b_j)$, where a_{j-1} is the stock of assets held at the end of age $j - 1$, s_j denotes the individual's employment shock, $\overline{e}_j$ denotes the average past earnings at age j, and b_j indicates whether an individual has elected to collect social security benefits at age j. The individual employment state $s_j \in S = \{0, 1\}$ is assumed to follow a two-state, first-order Markov process. If $s_j = 1$, the agent is given the opportunity to work and if $s_j = 0$ the agent is unemployed. The transition matrix for the employment shock is given by the 2×2 matrix $\Pi(s', s) = [\pi_{kl}]$ where $\pi_{kl} = Prob\{s_{j+1} = k | s_j = l\}$. The vector of choice variables is $y_j = (a_j, c_j, \ell_j)$ where a_j indicates the stock of assets held over to the next age, c_j is consumption and ℓ_j is labor supply at age j.

The budget constraint facing an age-j individual is given by

$$(4) \qquad c_j + a_j = (1 + r)a_{j-1} + s_j w \varepsilon_j \ell_j - T_j + Q_j + M_j + \xi,$$

where r is the real interest rate, w is the wage per efficiency unit of labor, ε_j is the efficiency index of an individual of age j, T_j is taxes paid by an age-j individual,

Q_j and M_j are retirement and unemployment insurance benefits received by an age-j individual, respectively, and ξ is a lump sum, per capita government transfer received by an individual. Unemployment insurance benefits are given by

$$
(5) \qquad M_j = \left\{ \begin{array}{ll} 0 & s = 1, \\ \phi w \varepsilon_j \ell_j & s = 0, \end{array} \right.
$$

where ϕ is the unemployment insurance replacement ratio. At any age $j \geq j_R - 1$ individuals may make an irreversible decision to begin collecting social security benefits next period.

We model the social security system to mimic the actual U.S. system in several important respects. The piecewise-linear benefit formula incorporates a partial linkage between benefits and lifetime labor earnings, and the constant social security tax rate applies only to earnings up to a cutoff point. This cutoff point and the kink points in the benefit formula are indexed for productivity growth. Finally, elderly individuals may continue to work with no reduction of benefits.[12] The social security policy parameter varied in our experiments is the tax rate. The replacement rates along the different segments of the benefit formula are adjusted upward or downward in equal proportion so that the system's budget balances.

Taxes paid satisfy

$$
(6) \qquad T_j = \tau_c c_j + \tau_a r a_{j-1} + (\tau_\ell + \tau_s + \tau_u) w \varepsilon_j \ell_j + \kappa,
$$

where τ_c, τ_a, τ_ℓ, τ_s, and τ_u denote the tax rates for consumption, capital income, labor income, social security and unemployment insurance, respectively, and κ denotes accidental bequests.

Individuals are assumed to face borrowing constraints, so that $a_j \geq 0, \; j = 1, 2, \ldots, J$.

2.4 Individual's Dynamic Program

We will restrict attention to Markov Equilibria and therefore rely on recursive methods to characterize them. When preferences are time-consistent, i.e. $\beta = 1$, the individual's dynamic program is a standard backward recursion.[13]

When $\beta < 1$, we have to attribute a particular belief to the individual concerning how he thinks his future selves will behave. We consider two cases. In one case, we assume that the individuals are **naive** in the sense that they think that the future selves will solve the $\beta = 1$ (time-consistent) problem despite a history of violating this belief.. Let $V_j(x)$ be the (maximized) value

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[12] Although this assumption is consistent with the most recent legislation on this issue, it appears not to have a great effect on the welfare effects of social security. In some unreported experiments retirement is mandatory in the sense that agents are prohibited from working at age j_R or later. The welfare effects are qualitatively very similar to the endogenous retirement case.

[13] See Sargent (1987) and Stokey and Lucas (1989).

of the objective function of an age-j agent with state $x = (a, s, \overline{e}, b)$. $V_j(x)$ is computed as the solution to the dynamic program

$$(7) \qquad V_j(x) = \max_{y \in \Omega_j(x)} \left\{ u(c, \ell) + \beta \delta_f \psi_{j+1} E_{s'} \widetilde{V}_{j+1}(x') \right\}, \quad j = 1, 2, \ldots, J,$$

where the notation $E_{s'}$ means that the expectation is over the distribution of s', and $\Omega_j(x)$ denotes the individual's constraint set. In the program (7), the continuation payoff $\widetilde{V}_j(x)$ is computed for $j = 1, 2, \ldots, J$ from

$$\widetilde{V}_j(x) = \max_{y \in \Omega_j(x)} \left\{ u(c, \ell) + \delta_f \psi_{j+1} E_{s'} \widetilde{V}_{j+1}(x') \right\}.$$

Note that for $\beta = 1$, V_j and $\widetilde{V}_j$ coincide for all j and the decision rules are time-consistent. For $\beta < 1$, however, the behavior represented by the decision rules is time-inconsistent.[14] A stationary solution to this dynamic program will consist of a set of value functions $\{V_j(x)\}_{j=1}^{J}$, decision rules $\{A_j(x), C_j(x), L_j(x)\}_{j=1}^{J}$ and measures of agent types $\{\lambda_j(x)\}_{j=1}^{J}$. The latter are computed using the forward recursion

$$\lambda_j(x') = \sum_s \sum_{a : a' \in A_j(x)} \Pi(s', s) \lambda_{j-1}(x),$$

given an initial measure of agent types $\lambda_0(x)$.

Most of our computations rely on the alternative assumption that individuals are aware that their future selves will not compute continuation payoffs $\widetilde{V}_j(x)$ according to the recursion shown above. Instead, they assume that their future selves will also engage in quasi-hyperbolic discounting. This case requires more care in computing the value functions and the policy rules. Define the value functions from the 'sophisticated $\beta < 1$' problem by $\widehat{V}_j$ and the associated policy functions by $\widehat{c}_j, \widehat{\ell}_j$ and $\widehat{a}_j$. We can compute these functions from the recursion

$$\widehat{V}_j(x) = \max_{y \in \Omega_j(x)} \left\{ u(c_j, \ell_j) + \beta \delta_f \psi_{j+1} E_{s'} V_{j+1}^*(c_{j+1}, \ell_{j+1}) \right\},$$

where the V_j^* sequence is computed by

$$V_j^*(x) = u(\widehat{c}_j, \widehat{\ell}_j) + \delta_f \psi_{j+1} E_{s'} \widehat{V}_{j+1}(\widehat{c}_{j+1}, \widehat{\ell}_{j+1}),$$

and reflects the fact that this is not the usual continuation payoff function in the dynamic program since self j has no control over the choices of self $j + 1$ and therefore must take the future self's optimal plan as given. This explains the absence of the 'max' operator in the above computation.[15]

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[14]In recent work on time-inconsistent behavior, Gül and Pesendorfer (1999) propose an alternative preference structure by explicitly modeling disutility from commitment. They show that all the axioms of expected utility are satisfied under their specification, which allows one to utilize the Stokey and Lucas (1987) theorems on the existence and characterization of resulting dynamic programs.

[15]See İmrohoroğlu, İmrohoroğlu, and Joines (2000) for a detailed description of the computations for the 'sophisticated $\beta < 1$' case.

Given these decision rules and an initial distribution of agents, we compute the measures of agent types using the forward recursion

$$\lambda_j(x') = \sum_s \sum_{a:a' \in \hat{a}_j(x)} \Pi(s', s) \lambda_{j-1}(x).$$

2.5 Aggregate Technology

The production technology of the economy is given by a constant returns to scale Cobb-Douglas production function

$$(8) \qquad Y = BK^{1-\alpha}L^\alpha,$$

where $\alpha \in (0, 1)$ is labor's share of output, and K and L are aggregate inputs of capital and labor, respectively. The total factor productivity parameter $B > 0$ is assumed to grow at a constant, exogenously given rate, $\alpha\rho > 0$, implying that steady-state per capita output grows at rate ρ. The aggregate capital stock depreciates at the rate d. Firm maximization requires

$$(9) \qquad r = (1-\alpha)B\left(\frac{K}{L}\right)^{-\alpha} - d \qquad \text{and} \qquad w = \alpha B\left(\frac{K}{L}\right)^{1-\alpha}.$$

2.6 Government

There is an infinitely lived government that taxes consumption and income from labor and capital at constant rates of τ_c, τ_ℓ, and τ_a, respectively, and makes purchases of goods of G per period. Any excess of revenues over purchases is distributed as a lump-sum transfer to all individuals. The government also maintains pay-as-you-go social security and unemployment insurance programs with benefits financed by payroll taxes with constant rates τ_s and τ_u, respectively. The budget of each of these social insurance programs is balanced on a period-by-period basis. There is no government debt.

2.7 Stationary Equilibrium

A *government policy* is a set of parameters $\{G, \tau_c, \tau_a, \tau_\ell, \tau_s, \phi, \xi\}$. An *allocation* is given by a set of decision rules $\{A_j(x), C_j(x), L_j(x)\}_{j=1}^J$, and measures of agent types $\{\lambda_j(x)\}_{j=1}^J$. A *price system* is a pair $\{w, r\}$. A **Stationary Recursive Equilibrium** is an *allocation*, a *price system* and a *government policy* such that

- the allocation solves the dynamic program for all individuals, given the price system and government policy,

- the allocation maximizes firms' profit by satisfying (9),

- the allocation and government policy satisfy the government's budget constraints, and,

- the commodity market clears.

We specify the optimization problem of the individual as a finite-state, finite horizon, dynamic program and use numerical methods to compute stationary equilibria under alternative social security arrangements.[16] Before turning to the results of these simulations, however, we first present a stripped-down version of our general model and derive some analytical results about the ability of unfunded social security to raise the welfare of quasi-hyperbolic discounters.

3 A Simplified Example

Consider a simple economy populated by naive quasi-hyperbolic discounters who live for three periods and face no uncertainty about either their life span or their earnings, who supply labor inelastically during the first two periods of life, and who are retired during the final period.[17] Preferences are such that $\delta_b = \delta_f$ and the period utility function is logarithmic. Individuals live in an endowment rather than a production economy, and the (labor income) endowment over the life cycle is given by $\{w_1, w_2, 0\}$. They can lend, and possibly borrow, at a fixed, riskfree interest rate $r \geq 0$. For simplicity, the rates of population growth and technical progress are set to zero, so that the rate of return on unfunded social security is also zero. There is no government other than an unfunded social security system that taxes labor income at a constant rate τ_s and pays a lump-sum retirement benefit b. Because we restrict attention to steady states, we do not have time subscripts.

A newborn agent faces the following budget constraints:

$$
\begin{aligned}
c_1 + a_1 &= (1 - \tau_s)w_1 \\
c_2 + a_2 &= (1 - \tau_s)w_2 + (1 + r)a_1 \\
c_3 &= (1 + r)a_2 + b
\end{aligned}
$$

where c_1, c_2 and c_3 are consumptions at ages 1, 2, and 3, respectively, and a_1 and a_2 are asset holdings at ages 1 and 2, respectively. If there are no restrictions on borrowing, so that a_1 and a_2 can take on negative values, these three constraints can be combined into a single present-value budget constraint:

$$
c_1 + \frac{c_2}{(1+r)} + \frac{c_3}{(1+r)^2} = (1 - \tau_s)w_1 + \frac{(1 - \tau_s)w_2}{(1+r)} + \frac{b}{(1+r)^2} \equiv \Psi_1.
$$

The unfunded nature of the social security system implies that $b = \tau_s(w_1 + w_2)$.

A newborn agent ranks lifetime consumption according to

$$
\ln c_1 + \beta(\delta \ln c_2 + \delta^2 \ln c_3), \quad \text{where} \quad \delta = \delta_b = \delta_f.
$$

When $\beta = 1$, preferences are time-consistent. When $\beta < 1$, preferences are time-inconsistent and an individual's preference ordering over a given set of lifetime consumption sequences will in general change as the individual ages. The three age-specific utility indicators u_{j^*} for $j^* = 1, 2, 3$ are:

$$
\begin{aligned}
u_1 &= \ln c_1 + \beta(\delta \ln c_2 + \delta^2 \ln c_3), \\
u_2 &= \delta^{-1} \ln c_1 + \ln c_2 + \beta\delta \ln c_3, \\
u_3 &= \delta^{-2} \ln c_1 + \delta^{-1} \ln c_2 + \ln c_3.
\end{aligned}
$$

We first consider the case with no restrictions on borrowing. An individual with time-consistent preferences $(\beta = 1)$ would choose the consumption sequence

$$
\begin{aligned}
c_1 &= \frac{\Psi_1}{1 + \delta(1 + \delta)}, \\
c_2 &= \delta(1 + r)c_1, \\
c_3 &= \delta(1 + r)c_2.
\end{aligned}
$$

A naive individual with $\beta < 1$ would choose the following consumption sequence at age 1:

$$
\begin{aligned}
(10) \qquad c_1^1 &= \frac{\Psi_1}{1 + \beta\delta(1 + \delta)}, \\
c_2^1 &= \beta\delta(1 + r)c_1^1, \\
c_3^1 &= \delta(1 + r)c_2^1.
\end{aligned}
$$

On reaching age 2, however, the naive individual would re-optimize and choose the following allocation for the two remaining periods of life:

$$
\begin{aligned}
(11) \qquad c_2^2 &= \frac{\Psi_2}{1 + \beta\delta}, \\
c_3^2 &= \beta\delta(1 + r)c_2^2,
\end{aligned}
$$

where

$$
\Psi_2 = (1 + r)a_1 + (1 - \tau_s)w_2 + \frac{b}{1 + r} = \frac{\beta\delta(1 + \delta)(1 + r)}{1 + \beta\delta(1 + \delta)}\Psi_1.
$$

Substituting this expression into that for period-2 consumption reveals that

$$
c_2^2 = \frac{\beta\delta(1 + \delta)(1 + r)}{1 + \beta\delta}c_1^1.
$$

It is immediately apparent that if $\beta = 1$, the re-optimized values are the same as those planned at age 1, and these are in turn equivalent to those chosen by the individual with time-consistent preferences.

With no restrictions on borrowing, consumption at each age is proportional to initial wealth, Ψ_1. The proportionality factors vary with age and depend

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on preference parameters and the interest rate, but they are independent of the parameters of the social security system. Unfunded social security affects consumption only through its effects on wealth, which can be rewritten as

$$\Psi_1 = w_1 + \frac{w_2}{1+r} - \frac{\tau_s}{(1+r)^2} \left\{ [(1+r)^2 - 1]w_1 + rw_2 \right\}.$$

If the interest rate is positive (or, more generally, greater than the growth rate of aggregate labor income subject to the social security tax), an increase in the tax rate lowers Ψ_1, resulting in equal proportionate reductions in consumption at each age, irrespective of the value of β. If the interest rate is zero (or, more generally, equal to the growth rate of aggregate labor income subject to the social security tax), then consumption at each age is independent of the social security tax rate. Thus we have demonstrated

PROPOSITION 1: *In the absence of binding constraints on borrowing during working years, an unfunded social security program does not reallocate consumption from working to retirement years.*

A naive individual who faces a binding constraint on borrowing in the first period of life chooses the consumption sequence

$$
\begin{aligned}
c_1 &= (1 - \tau_s)w_1, \\
c_2 &= \frac{\Psi_2}{1 + \beta\delta}, \\
c_3 &= \beta\delta(1 + r)c_2,
\end{aligned}
$$

where Ψ_2 is as defined above, with $a_1 = 0$. The likelihood that the period-1 borrowing constraint binds in the absence of social security depends on the preference parameters δ and β, the interest rate r, and the shape of the lifetime earnings profile.

A few simple numerical examples can be used to illustrate the scope for unfunded social security to redistribute consumption across the life cycle and to raise utility as viewed from old age. Table 1 shows the consumption profiles and age-specific utility indicators u_{j^*} associated with different earnings profiles and social security tax rates. In each case, the preference parameters are set to $\delta = 0.9$ and $\beta = 0.6$. Individuals are unable to borrow against future income, and the interest rate on private assets is assumed to be zero. Because the rate of return on social security contributions is also zero, social security does not affect initial wealth, which is equal to 1.0 in each example.

Table 1
A Numerical Example

w_1	0.65		0.5			0.35	
w_2	0.35		0.5			0.65	
τ_s	0.2	0.0	0.1	0.2	0.0	0.1	0.2
c_1	0.494	0.494	0.450	0.400	0.350	0.315	0.280
c_2	0.329	0.329	0.357	0.390	0.422	0.445	0.468
c_3	0.178	0.178	0.193	0.210	0.228	0.240	0.252
u_1	-2.15	-2.14	-2.15	-2.18	-2.23	-2.29	-2.35
u_2	-2.83	-2.83	-2.81	-2.80	-2.83	-2.86	-2.92
u_3	-3.84	-3.84	-3.78	-3.74	-3.73	-3.75	-3.79

The borrowing constraint does not bind with an earnings profile of $\{0.65, 0.35, 0\}$ and a social security tax rate of 20 percent, and it obviously would not bind with any lower social security tax rate. The resulting consumption allocation and utility indicators are thus unaffected by any social security tax rate less than 20 percent and are the same as those that would result if there were no borrowing constraint. Almost half of total consumption occurs in the first period of life.

With an earnings profile of $\{0.5, 0.5, 0\}$, the borrowing constraint just fails to bind in the absence of social security, but does bind with tax rates of 10 or 20 percent. In this case, social security redistributes consumption from period 1 to later periods. This redistribution lowers lifetime utility as viewed from the first period of life but increases utility as viewed from later periods.

In the last example, earnings rise over the working career and the borrowing constraint binds even in the absence of social security, so that period-1 consumption is about 30 percent below its unconstrained value. Social security raises consumption in periods 2 and 3, but at the cost of further reductions in the already constrained period-1 consumption. The effect of this redistribution is to lower lifetime utility as viewed from any age.[18]

These examples demonstrate that, in the absence of operative borrowing constraints, unfunded social security does not raise the old-age consumption of either exponential or quasi-hyperbolic discounters. Whether or not an unfunded social security system raises steady-state welfare viewed from old age is a quantitative question whose answer depends on the preference parameters β and δ, the shape of the age-earnings profile, and other features of the economy. In addition, unfunded social security may have general equilibrium effects on the capital stock and factor prices that are ignored in the simple examples shown above. A more detailed and carefully calibrated general equilibrium model that takes into account these and other realistic features of the U.S. economy is required to address this quantitative question. The model should incorporate a labor efficiency profile (as a function of age) that matches the one observed in the U.S. economy, and it should generate saving behavior that results in an empirically plausible wealth-output ratio. We now turn to the calibration and simulation of such a model.

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[18] This effect has been noted by Hubbard and Judd (1987) and İmrohoroğlu, İmrohoroğlu, and Joines (1995).

4   Calibration of the Model Economy

In order to obtain numerical solutions to the model, we must choose particular values for the parameters. We calibrate our model under the assumption that the model period is one year.

Individuals are assumed to be born at the real-time age of 21, and they can live a maximum of $J = 65$ years. After real-time age 85, death is certain. The sequence of conditional survival probabilities $\{\psi_j\}_{j=1}^J$ is taken from Faber (1982). The growth rate of population is taken to be 1.2 percent per year, the historical average in the United States over the last fifty years. The age at which agents become eligible for social security benefits, j_R, is taken to be equal to 45, which corresponds to a real-time age of 65. The efficiency index ε_j is intended to provide a realistic cross-sectional age distribution of wages at a point in time. This index is taken from Hansen (1993), interpolated to in-between years, extrapolated to model age 65, and then normalized to obtain an average of unity over $j = 1, 2, \ldots, 65$.

The unemployment insurance replacement ratio, ϕ, is taken to be 25 percent of the employed wage. The employment transition probabilities are chosen to make the probability of employment equal to 0.94, independent of the availability of the opportunity in the previous period.

In line with recent practice, we set the preference parameters δ_f, β, and γ so as to match the economy's observed wealth accumulation behavior as measured by an empirical wealth-output ratio of 2.52.[19] This single ratio is not sufficient to pin down the values of all three preference parameters. The wealth-output ratio in our model economy is positively related to the discount factors δ_f and β and negatively related to the risk aversion coefficient γ. Various empirical studies suggest that a coefficient of relative risk aversion in the neighborhood of 2 is a reasonable base case, and we also consider $\gamma = 1$ and $\gamma = 3$ as alternatives. We choose three values of β a priori: (i) 0.85, the value used by Laibson, Repetto, and Tobacman (1998), (ii) a value of 0.90, reflecting a milder degree of short-term impatience, and (iii) a value of 0.60, incorporating a relatively high short-term discount rate. For each combination of γ and β we search over values of δ_f to find the one which best matches the observed wealth-output ratio of 2.52, assuming a social security tax rate of 10 percent. We take the share of consumption in the utility function, φ, to be 0.33, in line with Rios-Rull (1996). This value generates an average labor input corresponding to about 30 percent of discretionary time for all values of the other preference parameters that we consider. The parameter δ_b does not affect any observable quantities, so we choose different values a priori.

The parameters describing production technology are chosen to match long-run features of the U.S. economy. The growth rate of per capita output ρ, is set to 0.0165, which is the average growth rate of output per labor hour between 1897 and 1992. The remaining technology parameters α and d are calculated

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[19]For a discussion of the empirical wealth output ratio see İmrohoroğlu, İmrohoroğlu, and Joines (1999).

from annual data since 1954. Our calculations imply a factor share of 0.690 for labor and an aggregate depreciation rate of capital of 0.044. The technology parameter B, is normalized to obtain an output of 1.0 in the model's 'base period'. Per capita quantities in this economy grow at a rate of ρ per period.

We calibrate the tax rate on consumption as 5.5 percent, the tax rate on interest earnings as 40 percent, and the tax rate on labor income as 20 percent. Government purchases of goods and services are set to 18 percent of output for the base case. These tax rates and government purchases are held constant as we vary the social security tax rate.[20]

5   Results

We start this section by examining some of the properties of an economy in which all individuals exhibit time-consistent preferences ($\delta_b = \delta_f$ and $\beta = 1$). This economy will serve as a point of comparison when we later analyze the effects of unfunded social security on economic behavior and welfare in economies populated by individuals with time-inconsistent preferences.

The time-consistent economy is calibrated to match the empirically observed capital-output ratio of 2.52 at a social security tax rate of 10 percent. Table 2 shows the properties of the steady state of this economy at various social security tax rates. With a 10 percent tax rate, the steady-state consumption-output ratio is 0.635 and the investment-output ratio is 0.183. Because this is a closed economy, the investment-output ratio is also the saving rate. As the social security tax rate is lowered toward zero, we observe a monotonic increase in the capital stock, investment, and consumption. Complete elimination of the pay-as-you-go social security system raises the saving rate to 0.216 and generates 32 percent more capital, 4 percent more work effort, 12 percent higher output, and 10 percent more consumption than an economy with a 10 percent social security tax rate. Because the change in work effort is relatively small in all of the economies we consider, we do not report it in subsequent tables.

<div align="center">

Table 2
Time-Consistent Preferences

τ_s (%)	w	r	Y	C	I	K	L	CV (%)
0	2.565	0.060	1.120	0.698	0.242	3.331	0.301	0.00
2	2.522	0.064	1.092	0.684	0.227	3.127	0.298	1.05
4	2.487	0.068	1.068	0.672	0.215	2.962	0.296	2.10
6	2.455	0.071	1.046	0.661	0.205	2.817	0.294	3.08
8	2.419	0.075	1.022	0.648	0.194	2.665	0.291	4.45
10	2.384	0.079	1.000	0.635	0.183	2.522	0.289	5.91

$\gamma = 2.0$, $\delta_f = 1.00578$, $\beta = 1.0$, $B = 1.7652$

</div>

The last column of Table 2 examines the welfare at birth of an individual born into the steady state corresponding to each social security tax rate. The

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[20]For a detailed description of the calibration strategy, see İmrohoroğlu, İmrohoroğlu, and Joines (2000).

relevant welfare criterion is expected lifetime utility as viewed from age 21, the first period of economic life in our model. This criterion, denoted W_{21}, is described in Section 2.2 and is based on equation (2). According to this criterion, welfare is maximized at a zero tax rate. We can measure the welfare cost of being born into an economy with social security as the consumption supplement (compensating variation) needed to equate the welfare of a newborn individual in that economy to the welfare of an individual born into an economy with no social security. The compensating variation is computed as a fixed percentage increase in consumption at each age. The last column of Table 2 shows these compensating variations, denoted CV. The welfare cost increases faster than linearly in the tax rate so that at a tax rate of 10 percent, individuals would require an increase in annual consumption of 5.91 percent to compensate them for living in a world with unfunded social security.

If there were no possibility of dying before the maximum possible age J, then the compensating variation for agents with time-consistent preferences would be the same when viewed from any age. The age-specific welfare criteria W_{j^*} defined in Section 2.2, however, are contingent on survival to age j^*. A pay-as-you-go social security system taxes all workers but pays benefits only to those who survive to retirement, effectively raising the rate of return to survivors. Because of this actuarial reward for survival, it is possible that individuals who reach sufficiently advanced ages might prefer social security even if a newborn individual does not, a feature of the model that we refer to as Effect 3 above. For the economy described here, however, W_{j^*} is maximized at a social security tax rate of zero for all j^*. Although individuals of all ages prefer a world without social security, the intensity of their aversion declines with age, reflecting the reward to survival. The compensating variation required to make individuals indifferent between living in an economy with a social security tax rate of 10 percent and an economy with no social security declines to 5.76 percent of lifetime consumption when viewed form age 41, to 4.87 percent when viewed from age 61, and to 2.67 percent when viewed from age 81. The fact that even the elderly do not favor unfunded social security is due primarily to the effects of such a system in lowering the aggregate capital stock and lifetime earnings and consumption.

In the remainder of this section, we first examine how social security affects economic behavior if preferences are time-inconsistent. We then report welfare effects and finally perform a couple of sensitivity analyses.

5.1 Time-Inconsistent Preferences and Behavior

Consider behavior in a world populated by quasi-hyperbolic discounters $(\beta < 1)$. Preferences of this sort are characterized by a current one-period discount rate that is higher than future one-period discount rates. This high short-term impatience leads quasi-hyperbolic discounters to postpone saving, and continual deferral may lead these individuals to enter retirement with substantially lower assets than exponential discounters. We examine how quasi-hyperbolic discounting affects aggregate capital accumulation, output, and consumption, as

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well as the allocation of consumption over the life cycle. We also examine how quasi-hyperbolic discounting affects the responses of these variables to changes in the scale of unfunded social security.

If $\beta < 1$, the optimal policy functions derived at age j^* for ages $j' > j^*$ will no longer be optimal when an individual at arrives age j'. As a consequence the age-j' individual will in general deviate from the policy rules derived at any earlier age. We assume that individuals are aware of this feature of their own behavior and that they choose current consumption, saving, and work effort optimally, taking into account the behavior of their future selves.[21]

If social security is to constitute a welfare-improving policy intervention in an economy with this sort of time inconsistency but not in a world of time-consistent preferences, one would expect to find an economically significant influence of quasi-hyperbolic discounting on observable behavior. Before reporting the results of our social security experiments we wish to establish a standard against which to compare the effects of unfunded social security, first on observable behavior and then on welfare. Our standard is a world in which individuals have a technology that allows them to commit at age 20 to a state-contingent path of lifetime consumption and work effort. From age 21 until death, these individuals follow decision rules that are the same as those implied by $\beta = 1$. Behavior is thus the same as in the exponential economy, although welfare as viewed from each age is calculated using the appropriate value of $\beta \, (< 1)$.

Table 3 summarizes the consequences of a perfect commitment technology for three configurations of preference parameters that we consider in more detail below. The table first reports the levels of capital, output, and consumption, each scaled relative to a value of 100.0 in the no-commitment case without social security. It then gives the value of the commitment technology, expressed as a fixed percentage increase in consumption at each age in the no-commitment case that makes individuals as well off as having the commitment device. These compensating variations are computed using preferences as viewed from four different ages.[22] The commitment technology results in higher steady-state levels

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[21] As with the exponential economy described in Table 2, we require that economies with quasi-hyperbolic discounters and a 10 percent social security tax rate generate a capital-output ratio that matches the historical U.S. average. We do this by appropriately choosing the standard discount factor δ_f for each value of β so that each (δ_f, β) pair results in a capital-output ratio of 2.52.

[22] The fact that these compensating variations are not monotonic in age is not an anomaly but is rather a natural feature of the preference structure we employ. With $\delta_b = \delta_f$, individuals of ages 21 and 85 have preference orderings over lifetime consumption sequences that are quite similar except for the effects of mortality risk. For example, assume that $\delta_b = \delta_f = 1.0$, $\beta = 0.8$, and the conditional survival probabilities are $\psi_j = 1.0$ for all ages up to the maximum possible age of 85. With these preferences, an 85-year-old attaches a weight of 1.0 to the utility of consumption at each age. A 21-year-old has the same preferences except that he attaches a weight of $1/\beta = 1.25$ to current utility. These two preference orderings are more similar to each other than either is to the preferences of, e.g., a 41-year-old, who attaches a weight of 1.0 to the utility of consumption at ages 42-85 and a weight of 1.25 to the utility of consumption at ages 21-41. These considerations suggest that the compensating variation numbers (i) should vary non-monotonically as a function of age and (ii) should be similar in the first and last periods of life. Introducing mortality risk overturns the second of these
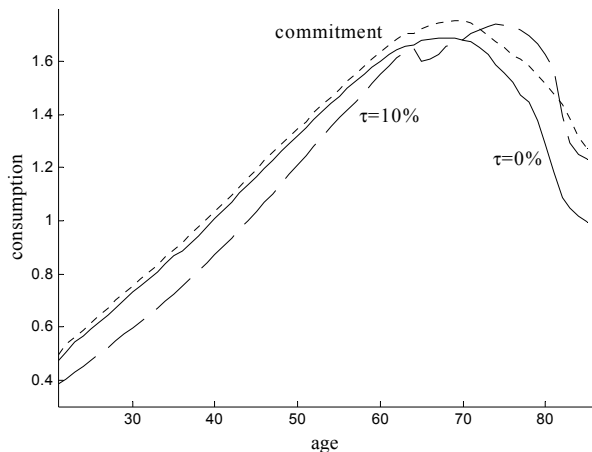
Figure 1: Age-Consumption Profiles

of aggregate capital, output, and consumption. The curve labeled "$\tau = 0\%$" in Figure 1 is the age-consumption profile of individuals with $\gamma = 2.0$, $\beta = 0.90$, no social security, and no commitment technology. The figure also shows the age-consumption profile for the same individuals with a perfect commitment technology. Such a technology increases consumption at all ages, and the increase is most pronounced during retirement years. The increase in consumption throughout the entire life cycle is due to the higher wage rate resulting from a larger capital stock. Both behavior and welfare seem more sensitive to the quasi-hyperbolic discounting parameter than to the inverse elasticity of intertemporal substitution. These results indicate that the steady-state welfare costs to quasi-hyperbolic discounters of their time-inconsistent behavior are substantial.[23]

_____

predictions since 21-year-olds discount future utilities because of such risk, whereas elderly individuals evaluate lifetime utility conditional on having survived to old age. Thus, the compensating variation is substantially higher for the extremely elderly than for the extremely young.

[23]Laibson (1997, p. 467) reports compensating variations that are much smaller than those in Table 5. There appear to be two reasons for the difference. First, Laibson's welfare analysis is for a partial commitment technology that takes the form of an illiquid asset. Second, his analysis is for an infinitely-lived representative agent and includes the change in consumption during the transition from one steady state to another, whereas our comparison is only of the two steady states. Barro (1999, p. 1139) examines the value of perfect commitment. Because he takes into account the transition between steady states, he reports a smaller welfare effect for a given change in steady-state capital than we do. Assuming log utility, he finds that the value of commitment is small unless the degree of short-term impatience is high.

Table 3
Perfect Commitment Technology

	$\gamma = 2.0$		$\gamma = 1.0$
	$\beta = 0.90$	$\beta = 0.85$	$\beta = 0.90$
A. Behavior			
K	114.0	121.3	116.1
Y	105.2	107.6	105.8
C	103.4	104.9	103.7
B. Compensating Variation			
W_{21}	3.28	4.49	3.74
W_{41}	3.20	4.62	3.75
W_{61}	3.25	4.50	3.73
W_{81}	4.68	7.07	5.09

We now examine the effectiveness of unfunded social security as a substitute for a perfect commitment technology in maintaining old-age consumption. Table 4 summarizes the aggregate economic effects of a 10-percent social security tax rate in economies in which the preference parameters δ_f, γ, and β take on different values. The last two parameters are specified a priori, and δ_f is then chosen to yield a capital-output ratio of 2.52. Our central value for γ, the inverse elasticity of intertemporal substitution, is 2.0. Given $\gamma = 2.0$, the quasi-hyperbolic discounting parameter β takes on values of 1.00 (the exponential case from Table 2), 0.90, and 0.85. In addition, we consider values of γ of 1.0 and 3.0, each paired with a β of 0.90. The table is normalized so that capital, output, and consumption are all 100.0 in the absence of social security.

Social security reduces the steady-state values of capital, output, and consumption in each of the economies considered. The results for $\gamma = 2.0$ indicate that social security reduces the capital stock by about 25 percent. The magnitude of this effect is similar across the three values of β, although it is somewhat more pronounced with quasi-hyperbolic discounting. A lower elasticity of intertemporal substitution ($\gamma = 3.0$) implies a larger effect of social security on steady-state capital, while $\gamma = 1.0$ implies a smaller effect. The smaller the elasticity of substitution, the greater the reduction in the saving of workers when the government attempts to reallocate consumption toward retirement years through the payroll tax.

Table 4
Effects of Social Security with Quasi-Hyperbolic Discounting

γ		2.0		3.0	1.0
β	1.00	0.90	0.85	0.90	0.90
δ_f	1.0058	1.0117	1.0146	1.0261	0.9980
B	1.7652	1.7740	1.7880	1.7766	1.7625
K	0.757	0.746	0.733	0.687	0.812
Y	0.891	0.886	0.877	0.859	0.914
C	0.910	0.905	0.899	0.888	0.923

Whereas a perfect commitment technology results in higher steady-state

values of capital, output, and consumption, unfunded social security lowers each of these three variables, and the reductions are if anything larger with quasi-hyperbolic discounting than in a pure exponential economy. Thus, any steady-state welfare gains from unfunded social security must come from a reallocation of consumption over the life cycle.

Figure 1 shows age-consumption profiles for individuals with social security tax rates of zero and 10 percent, each without a commitment technology. With social security, simulated consumption exhibits a discrete drop at retirement similar to that documented by Banks, Blundell, and Tanner (1998) and Bernheim, Skinner, and Weinberg (2001). In our model, the institutional features of social security cause an increase in the effective labor income tax rate at age 65 which is similar to that occurring when people reach their early 60s in the U.S. system. This increase in the effective tax rate causes a discrete reduction in hours worked which is not observed in the absence of social security. Because individuals smooth a composite of leisure and market goods, a sudden increase in leisure is accompanied by a drop in consumption expenditures. Bernheim, Skinner, and Weinberg (2001) have noted that any drop in consumption at retirement could be associated with a reduction in work-related expenditures that might be more properly deducted from earnings rather counted as consumption. Other explanations suggested in the literature include adverse shocks to health and over-estimation of post-retirement income. While all of these effects may exist empirically, our findings show that none of them is necessary to generate a discontinuous drop in consumption at retirement.

As with the commitment technology, social security raises old-age consumption, but unlike the commitment technology, which results in higher consumption at each age, it does so at the cost of noticeably reduced consumption during working years. Table 5 reports average consumption levels for the last decade before retirement, the first decade after retirement, and two periods of extreme old age for each of the three economies with $\gamma = 2.0$. Consumption is normalized to 100.0 in the decade before retirement in the regime without social security, and consumption in the other cells is scaled relative to this.

Table 5
Old-Age Consumption

| Age | $\beta = 1.00$ | | $\beta = 0.90$ | | $\beta = 0.85$ | |
	$\tau_s = 0.0$	$\tau_s = 0.1$	$\tau_s = 0.0$	$\tau_s = 0.1$	$\tau_s = 0.0$	$\tau_s = 0.1$
55-64	100.0	97.7	100.0	96.7	100.0	95.3
65-74	106.6	108.3	105.3	105.8	105.3	104.3
75-81	95.8	112.6	88.9	106.1	85.6	100.2
82-85	81.4	101.3	65.5	81.7	62.9	76.0

Without social security, consumption peaks in the first decade after retirement and then declines below pre-retirement levels. Consumption of quasi-hyperbolic discounters aged 82-85 is less than two thirds the pre-retirement level. Social security reduces pre-retirement consumption in each economy,

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reflecting the effects of the payroll tax and lower lifetime earnings, and the reduction is greater for quasi-hyperbolic discounters. Consumption in the first decade after retirement rises or falls slightly as a result of the 10 percent tax rate, depending on the value of β. Social security has its greatest effect on consumption in extreme old age. In all three economies, a tax rate of 10 percent results in consumption at ages 75-81 that is noticeably above the pre-retirement level.[24] The story is different among the very oldest individuals, however. Although social security raises the consumption of quasi-hyperbolic discounters aged 82-85, the effect is not large enough to prevent a substantial shortfall relative to pre-retirement levels. Individuals with $\beta = 0.90$ drive their asset holdings to less than 20 percent of annual consumption by age 81, and those with $\beta = 0.85$ reach even lower asset levels even sooner. In contrast, the consumption of exponential discounters at ages 82-85 remains above the pre-retirement level.[25] Quasi-hyperbolic discounters have higher consumption in extreme old age with a perfect commitment technology than with a 10 percent social security tax rate.

5.2   Welfare Analysis

We now analyze the effects of unfunded social security on welfare in economies with time-inconsistent preferences. The preference structure in equation (2) incorporates features that potentially lead individuals of different ages to attach different rank orderings to various lifetime sequences of consumption and leisure. We referred to these features as Effect 1 ($\delta_b > \delta_f$) and Effect 2 ($\beta < 1$). In addition, if the survival probability at each age is less than unity, the rank orderings of young individuals may differ from those of the elderly, conditional on survival to old age. The presence of any of these effects implies the existence of age-specific welfare indicators W_{j*} for each age j^*. The welfare consequences of unfunded social security depend on the strength of Effects 1 and 2, which can exist either separately or in combination. In general, either of these effects could be expected to cause older individuals to look more favorably on unfunded social security than younger individuals.

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[24]Consumption in our model, either with or without social security, seems to peak later than in the data. One potential explanation is age dependence in the period utility function $u(c_j, \ell_j)$, possibly due to changes in household composition. We have not attempted to incorporate age-specific taste shifters because measuring them would be difficult and because it seems unlikely that they would qualitatively affect the discrepancy between the optimal and actual consumption paths.

[25]It is curious that the consumption of quasi-hyperbolic discounters drops so sharply a few years before the certain death date of 85. One explanation for this phenomenon is that these individuals simply require about 15 years to exhaust their retirement assets. An alternative is that impending mortality exacerbates their high short-term impatience, causing them to run down their assets a few years before certain death. To help us distinguish between these two explanations, we simulated a version of the model with $\beta = 0.90$ and 75 periods (a maximum real-time age of 95). Individuals in this model drove their asset levels to 16 prcent of annual consumption by age 82, tending to support the former explanation. Thus, the precipitous drop in the consumption of quasi-hyperbolic discounters in their early 80s does not seem to be merely an artifact of the certain death date.

We begin by considering Effect 1 in isolation. In this specification $\beta = 1$, so that behavior is time-consistent, but $\delta_b > \delta_f$ so that, even apart from differences due to mortality risk, an individual places more weight on $u(c_{j'}, \ell_{j'})$ relative to $u(c_{j*}, \ell_{j*})$ when looking back from age j' than when looking forward from age j^*, where $j' > j^*$. Thus, an old individual may regret having consumed so much when young. We define the degree of this type of retrospective time inconsistency, $\widehat{\sigma} \geq 0$, implicitly by $\delta_b = 1/(1 + \sigma - \widehat{\sigma})$, where σ is itself implicitly defined by $\delta_f = 1/(1 + \sigma)$.

Behavior depends on the preference parameters δ_f, γ, and β but not on δ_b. We take $\gamma = 2$, $\beta = 1$, and $\delta_f = 1.00578$ to ensure a capital-output ratio of 2.52 with a 10 percent social security tax rate. Thus, the behavior of this economy is as displayed in Table 2. Within this environment we examine the welfare effects, as viewed from different ages, of varying the social security tax rate. We restrict our experiment to tax rates of zero, $2, 4, \ldots, 10$ percent and ask the following questions: For different degrees of retrospective time inconsistency, $\widehat{\sigma}$, what is the earliest vantage point j^* from which lifetime welfare is higher with a positive social security tax rate than with a rate of zero and what is the earliest vantage point j' from which a 10 percent tax rate is preferred to any of the other tax rates under consideration?

Table 6 contains the answers to these questions. Individuals as young as 40 prefer some social security when the degree of retrospective time inconsistency is as great as 8 percent per year. It turns out in our experiments that if an individual of age j^* prefers a given social security tax rate to zero, individuals of any greater age also prefer that tax rate. In all of the cases reported in Table 6, the optimal tax rate as viewed from any age is either zero or at least 6 percent, with more modest rates never being preferred. In order for a majority of the population to view a tax rate of 10 percent or more as optimal, the degree of retrospective time inconsistency must be at least 8 percent per year. Given our assumed value of δ_f, retrospective time inconsistency of 8 percent implies $\delta_b = 1.0938$, which in turn implies that the weight on past outcomes declines to $2/3$ the weight on current outcomes after about 4.5 years and to $1/3$ after 12 years. While we know of no empirical evidence on the magnitude of retrospective discounting, an annual rate of 8 percent is substantially larger than is generally assumed for ex ante discounting.

Table 6
Retrospective Time Inconsistency

	Positive tax rate preferred		10% tax rate preferred	
$\widehat{\sigma}$ (%)	Age	Share (%)	Age	Share (%)
0	–	0.0	–	0.0
2	57	26.7	62	19.5
4	46	45.0	50	37.9
6	42	52.6	45	46.9
8	40	56.6	42	52.6
10	38	60.6	40	56.6

The most extreme form of Effect 1 occurs when $\delta_b = \infty$, implying that individuals place no weight on the past. In this case individuals of age 31 and above (71.3 percent of the population) prefer a positive social security tax rate, and individuals of age 34 and above (69.1 percent) prefer a social security tax rate of 10 percent to any lower rate.

Most if not all of the previously published welfare analyses in the quasi-hyperbolic discounting literature seem to combine Effect 2 ($\beta < 1$) with this extreme form of Effect 1. Adding Effect 2 causes a slight increase in the preference for social security. For example, if $\gamma = 2.0$, $\beta = 0.90$, $\delta_f = 1.0117$, and $\delta_b = \infty$, individuals of age 27 and higher (43.2 percent of the population) prefer a positive social security tax rate, while 69.1 percent of the population (ages 34 and above) continue to prefer a tax rate of 10 percent to any lower rate.

At first glance, these results seem to indicate that the ability of social security to increase the welfare of time-inconsistent agents arises primarily from Effect 1, with only a small incremental contribution from Effect 2. These results do not necessarily imply that Effect 2 is trivial, however. Both effects operate by causing young agents to place relatively larger weight on consumption early in life than do the elderly, and Effect 1 can indeed generate much greater disagreement between young and old selves than can Effect 2.[26] In addition to influencing the valuation of given sequences of lifetime consumption and leisure, Effect 2 alters those sequences in ways documented in section 5.1 above. Even without generating intrapersonal disagreements, the resulting reductions in saving and old-age consumption may lower welfare as viewed from any age and may create scope for unfunded social security to increase welfare. The extreme intrapersonal disagreements generated by Effect 1 may mask these welfare changes brought about by Effect 2. In addition, Deaton (1992) and Caplin and Leahy (1999) offer persuasive arguments against completely discounting the past when making lifetime welfare comparisons. For these reasons, we devote the remainder of the paper to an examination of how unfunded social security influences lifetime welfare in the presence of Effect 2 alone.

Table 7 summarizes the welfare effects of varying the social security tax rate

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[26]For example, normalize the weight placed on consumption at age 85 to 1.0 and consider the weight placed on age-21 consumption by a 21-year-old and an 85-year-old. Take the parameter values used to generate Table 11, which result in intrapersonal disagreement due only to Effect 1. Assume that the degree of myopia due to Effect 1 is $\hat{\sigma} = 0.08$, the amount required for a majority of the population to prefer a social security tax rate as high as 10 percent. These parameter values imply that the 21-year-old places a weight of 0.692 on age-21 consumption, whereas the 85-year-old assigns a weight of only 0.003. Effect 1 causes the young agent to attach roughly 23,000 percent more weight to current consumption than his elderly self would do in retrospect.

As was shown in footnote 21 above, when $\delta_b = \delta_f$, so that Effect 1 is absent, individuals of ages 21 and 85 have preference orderings over lifetime consumption sequences that are much more similar, even in the presence of Effect 2. In the example presented there, a 21-year-old attaches 25 percent more weight to current consumption than his 85-year-old self would do in retrospect.

In both of these examples we have ignored mortality risk by assuming that the conditional survival probabilities are $\psi_j = 1.0$ for all ages up to the maximum possible age of 85.

in three of the economies examined in Table 4 above. For social security tax rates between 2 and 10 percent, we determine the first age $\widehat{j}$ at which $W_{\widehat{j}}$ is greater with social security than without. (It turns out in our experiments that if social security raises welfare as viewed from age $\widehat{j}$, it also raises welfare as viewed from any age $j > \widehat{j}$.) We also calculate the fraction of the population falling into ages $j \geq \widehat{j}$. It should be emphasized that these welfare comparisons are between zero and positive social security tax rates in a world with no other commitment device. They do not involve a comparison between these economies and an economy with a commitment technology. We omit the exponential economy, where even a 2 percent social security tax rate lowers welfare as viewed from all ages, and the economy with $\gamma = 3.0$, where a social security tax rate as high as 10 percent lowers welfare as viewed from any age.[27]

In the remaining economies, we find that social security raises welfare as viewed from sufficiently advanced ages. With $\gamma = 2.0$, the fraction of the population falling into these cohorts never exceeds about 6 percent, however. With $\gamma = 1.0$, a 2 percent social security tax rate increases welfare as viewed from ages 71 and greater, corresponding to more than 9 percent of the population. The aggregate welfare measure W, which weights each of the age-specific indicators W_{j*} by the unconditional probability of surviving to age j^*, is always higher without social security than with any of the tax rates considered here. Overall, these results indicate that unfunded social security is not particularly effective in correcting for the undersaving resulting from quasi-hyperbolic preferences for short-term discount rates in the neighborhood of 10 to 15 percent.

<div align="center">

Table 7
Who Prefers Social Security?

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| | $\gamma = 2.0$ | | | | $\gamma = 1.0$ | |
| | $\beta = 0.90$ | | $\beta = 0.85$ | | $\beta = 0.90$ | |
$\tau_s(\%)$	Age	Share (%)	Age	Share (%)	Age	Share (%)
2	80	2.50	80	2.50	71	9.22
4	75	5.77	77	4.32	72	8.29
6	75	5.77	77	4.32	74	6.56
8	80	2.50	78	3.67	77	4.32
10	–	0.00	82	1.51	83	1.08

In light of the apparently widely held view that social security may raise the welfare of short-sighted individuals who fail to save adequately for their retirement, the question arises as to why such a system is not more effective in offsetting the behavioral consequences and lifetime utility losses due to Effect 2 ($\beta < 1$). The answer has already been suggested by the simulation results reported in Tables 4 and 5 and Figure 1. Unfunded social security depresses the aggregate capital stock in economies with $\beta < 1$, just as in economies with $\beta = 1$, thus exacerbating any undersaving due to a low β. Although social

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[27] With $\gamma = 3.0$, welfare as viewed from age 85 is maximized with a tax rate of 6 percent, but welfare as viewed from any other age is higher with no social security than with any of the tax rates we have examined.

security raises consumption during retirement, it does not prevent a significant decline during extreme old age. As viewed from most points in the life cycle (including substantial portions of retirement), the utility gains from increased old-age consumption are too small to offset the losses from reduced consumption earlier in life. The contrast with the perfect commitment technology is particularly instructive. That technology raises aggregate saving as well as consumption at all ages, particularly during retirement. Our findings suggest that policies that closely mimic a commitment device more closely than unfunded social security apparently does might also be more effective in offsetting any welfare losses arising from high short-term discount rates.

5.3   Sensitivity Analysis

The ineffectiveness of unfunded social security in offsetting the welfare losses due to Effect 2 ($\beta < 1$) arises because social security exacerbates the undersaving brought about by high short-term discount rates. The economies we have considered thus far are closed, and it is possible that the reduction in the capital stock is thus overstated. To examine the sensitivity of our findings to the closed-economy assumption, we analyze open-economy variants of the three economies of Table 7, in each of which social security led to welfare gains as viewed from old age. We assume that the world capital-output ratio is equal to 2.52, the value to which each of these economies is calibrated when the social security tax rate is 10 percent. The domestic capital-output ratio remains fixed at this level, and the wage rate and the interest rate remain fixed as we vary the tax rate. Changes in the financial wealth of domestic residents cause one-to-one changes in net foreign assets.

Our results indicate that the depressing effect of social security on total asset holdings is roughly three times as large in an open economy as in a closed one. As the social security tax rate is raised in a closed economy, the reduction in the capital stock raises the interest rate, which in turn mitigates the reduction in saving. The return to saving is fixed in the small, open economy, however, and does not tend to damp the change in asset accumulation. The effect of social security on GNP and aggregate consumption is about the same in these small, open economies as in their closed-economy counterparts.

The effects of unfunded social security in enhancing welfare and reallocating consumption from working to retirement years are weaker in an open economy than in a closed one. In each of the open economies, lifetime utility as viewed from all ages is higher without social security than with any tax rate we have examined.

Figure 2 shows the age-consumption profiles for tax rates of zero and 10 percent with $\gamma = 2.0$ and $\beta = 0.90$. Because the interest rate is unchanged, these two consumption profiles have approximately the same shape, and social security results in a roughly proportional decline in consumption at all ages. Compare this with the closed-economy profiles in Figure 1. It appears that the effect of social security in reallocating consumption to retirement years in the closed economy is largely an endogenous response to the increase in the interest
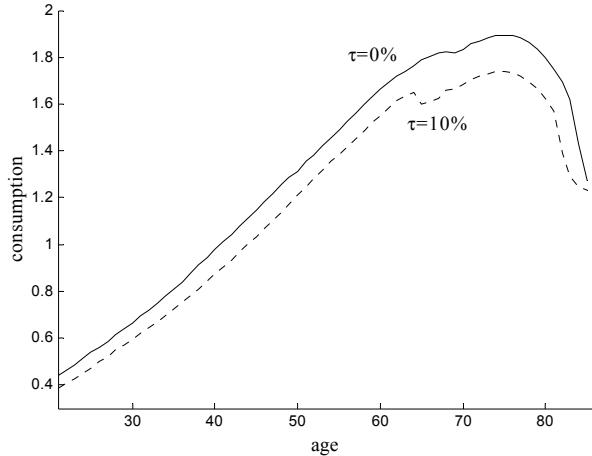
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Figure 2: Small, Open Economy Case

rate, which steepens the age-consumption profile, rather than because social security is a particularly effective commitment device. In the simple three-period model of section 3, unfunded social security could reallocate consumption toward working years only in the presence of binding constraints on borrowing. Our results suggest that, given our calibration, borrowing constraints are not severe enough for social security to effect much of a reallocation of lifetime consumption.

It might be argued that the individuals we have considered thus far are not very short-sighted. First, they are rather sophisticated in recognizing the time inconsistency resulting from their preference structure, and they optimize given those preferences. In this sense, they are not short-sighted at all. Second, the degree of time inconsistency as represented by β might not be large enough to generate serious welfare consequences. Concerning this second point, the results in Table 3 above suggest that both the behavioral and the welfare consequences of a β in the neighborhood of 0.85 to 0.90 can be significant. Nevertheless, Laibson, Repetto, and Tobacman (1998) have argued that experimental evidence supports values of β closer to 0.60, and the scope for social security to improve welfare might be substantially greater with a lower β. As Laibson, Repetto, and Tobacman (p. 119) have also pointed out, however, "a value of 0.6 generates pathologies in discrete time simulations: strongly non-monotonic and noncontinuous consumption functions. Such effects are commonplace in dynamic games such as the intrapersonal game that we consider."

Laibson, Repetto, and Tobacman suggest simulating the model using two alternative values of β for which solutions can be obtained and using these solutions to extrapolate to lower values of β. We now consider an alternative approach. Specifically, we consider individuals with preferences as given in

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equation (2) who naively think that their future selves will adhere to optimal plans derived today. These naive agents do not play the same intrapersonal game as sophisticated agents, and the behavior of naive agents can be obtained as the solution to a straightforward programming problem. Because naive agents have the same preferences as the relatively sophisticated agents considered above, social security will have different welfare effects on the two types of individuals only to the extent that they behave differently. If naive and sophisticated agents behave sufficiently similarly, then social security will have similar effects on their welfare. Therefore, we allow for more severe time inconsistency by also considering economies populated by naive individuals with $\beta = 0.60$.[28]

It is not clear a priori whether naive agents save less than sophisticated ones with the same preferences. A sophisticated individual will save more than a naive one in order to achieve any target level of wealth farther than one period into the future, because the sophisticated individual realizes that his profligate intermediate self will tend to consume any assets set aside for the more distant future. On the other hand, a dollar of saving is more valuable relative to a dollar of current consumption if the assets will be consumed optimally (as viewed by the current individual) over time rather than dissipated on consumption in the near future. Therefore, saving will appear more attractive to a quasi-hyperbolic discounter who naively believes that his future selves will optimally allocate additional resources over time than to a more sophisticated agent who realizes that this is not the case.

To isolate the consequences of the computational mistakes made by quasi-hyperbolic discounters who fail to take into account their future behavior, we first examine three economies populated by naive agents with the same preferences as those in Table 7. The difference in behavior between naive and sophisticated individuals is greater without than with social security, and among the three sets of preference parameters from Table 7, the difference is greatest with $\gamma = 2$ and $\beta = 0.85$. With those preferences and no social security, however, the capital stock is only 3.4 percent lower and aggregate consumption is 0.9 percent lower in an economy with naive agents than with sophisticated ones. Furthermore, the two types of agents allocate consumption across the life cycle in an almost identical manner. As a result, the welfare consequences of social security are qualitatively the same in the two economies. A 10 percent social security tax rate raises the welfare of naive individuals as viewed from ages 80 and above (2.5 percent of the population), compared with ages 82 and above (1.5 percent of the population) for sophisticated individuals.

We now consider a world populated by naive quasi-hyperbolic discounters with preference parameters $\gamma = 2$ and $\beta = 0.60$. In addition, we must recalibrate the standard time discount factor δ_f to 1.03905 so that this economy

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[28]Some of the literature has made a distinction between optimizing but time-inconsistent behavior of the sort exhibited by sophisticated agents and a more fundamental failure to plan for the future, and this distinction constitutes another reason for examining naive agents. For example, Laibson (1997, p. 449) and Barro (1999, p. 1127) distinguish between the model with sophisticated quasi-hyperbolic discounters and that of Akerlof (1991), in which "the standard assumption of rational, forward-looking, utility maximizing is violated."

generates a capital-output ratio of 2.52 when the social security tax rate is 10 percent.

In this economy a 10-percent social security tax rate increases the steady-state values of capital, output, and consumption by 16.6, 7.9, and 7.3 percent, respectively. The decline in saving is less with $\beta = 0.60$ than with the higher values considered above. As was the case for sophisticated agents with β of 0.85 or 0.90, consumption drops substantially in old age even in the presence of social security. Without social security, however, consumption would drop much more dramatically than was the case with the agents considered previously. Thus, social security is more successful in raising old-age consumption than was the case with higher values of β. This fact, combined with the smaller effect on the capital stock, means that a tax rate of 10 percent raises welfare as viewed from all ages. Individuals in an economy with a 10 percent tax rate would sacrifice a substantial fraction of annual consumption rather than give up social security entirely. The compensating variation is 10.11% of lifetime consumption as viewed through age-21 preferences, 6.08% as viewed from age 41, 7.2% as viewed from age 61, and 14.51% as viewed from age 81.[29] Furthermore, the optimal tax rate as viewed from all ages is substantially higher than the 10 percent value that approximates the current U.S. system.

In summary, our model indicates that there is little scope for unfunded social security to offset the welfare losses due to Effect 2 for values of β in the neighborhood of 0.85 to 0.90. Finding a welfare-enhancing role requires more extreme time inconsistency. Simply replacing sophisticated agents with non-optimizing counterparts who fail to recognize the implications of their own future preferences scarcely increases the beneficial effects of social security, at least for β in the range of 0.85 to 0.90. Social security does significantly raise welfare with $\beta = 0.60$, however. We do not know whether this result would carry over to a world of sophisticated agents with the same β, as we have thus far been unable to solve such a model. The fact that naive agents with higher values of β behave much like sophisticated agents suggests that welfare effects for the two types of agents might continue to be similar even at lower values of β.

6   Concluding Remarks

In this paper we examine the welfare effects of unfunded social security on individuals with time-inconsistent preferences. Our model incorporates retrospective time inconsistency whereby older individuals place less weight on outcomes early in life than do young individuals ex ante (Effect 1). Feldstein (1985), among others, has analyzed this effect, and our model extends his framework

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[29]We do not compare these welfare gains to those from a commitment technology. Because naive agents are unaware of their own future shortsightedness, they would have no reason to avail themselves of such a technology.

to include a wider range of benefits and costs of social security. We also consider preferences where the short-term discount rate is higher than longer-term discount rates (Effect 2), as modeled by Phelps and Pollock (1968), Laibson (1997), and others.

In this environment social security may provide additional utility for short-sighted agents who regret their saving decisions when they find themselves with low consumption after retirement. In addition, social security may substitute for missing private annuity markets in helping agents allocate consumption in the face of uncertain life spans. On the other hand, social security distorts aggregate saving and labor supply behavior and affects the wage rate and the interest rate. Consequently, whether or not social security is welfare enhancing even for short-sighted agents is a quantitative question. Because individuals have time-inconsistent preferences, it is necessary to evaluate lifetime welfare from different vantage points in the life cycle.

Our findings can be summarized as follows:

- Quasi-hyperbolic discounting at the rate of 15 percent lowers the capital stock by about 20 percent at any social security tax rate, and there are substantial steady-state welfare costs to quasi-hyperbolic discounters of their time-inconsistent behavior.

- For both exponential and quasi-hyperbolic discounters, a social security system like that in the United States results in a discontinuous drop in consumption at retirement similar to that observed in the data. This drop does not occur in the absence of social security, and it does not depend on several previously suggested explanations, including work-related consumption expenditures, adverse shocks to health, and misestimation of post-retirement income.

- Social security is a poor substitute for a perfect commitment technology in maintaining the old-age consumption of sophisticated quasi-hyperbolic discounters; the capital stock would be about one-third larger in the absence of social security than with a tax rate of 10 percent.

- If preferences exhibit Effect 2 only, unfunded social security generally does not raise welfare for short-term discount rates of up to 15 percent for either naive or sophisticated agents. Social security does raise the welfare of naive agents with a short-term discount rate of 40 percent.

- With retrospective time inconsistency (Effect 1 alone), the ex ante annual discount rate must be at least 8 percent greater than seems warranted ex post in order for a majority of the population to prefer a social security tax rate as high as 10 percent. In the extreme case where individuals place no weight on past outcomes, 69 percent of the population prefer a tax rate of at least 10 percent. Adding a high short-term discount rate (Effect 2) slightly increases the preference for social security.

We know of no empirical evidence on the degree of retrospective time inconsistency in preferences. Citing experimental evidence, much of the literature on quasi-hyperbolic discounting favors a short-term discount rate in the neighborhood of 40 percent. With a discount rate this high, we find that social security substantially raises old-age consumption. This finding suggests that future simulations using such high short-term discount rates should explicitly account for the existence of social security in evaluating the effects of other policies and institutions on life-cycle consumption and saving behavior.

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