Temptation and Taxation

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Abstract

In this paper we attempt to (i) extend the competitive equilibrium neoclassical growth model to incorporate consumer preferences that feature temptation and self-control as in the framework developed by Gul and Pesendorfer; (ii) use the model to analyze taxation and welfare; and (iii) extend and specialize the Gul-Pesendorfer temptation formulation to be dynamic and, in particular, quasi-geometric, thus providing a link to, and possibly an interpretation of, the Laibson model.

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1 Introduction

Recently, a number of researchers have begun considering economic models that capture some of the experimental evidence from the psychology literature. So-called "preference reversals"—a common occurrence in experiments—have received particular attention. These suggest, it has been argued, that consumers do not have the geometric discounting that has been used as a cornerstone for intertemporal macroeconomic analysis. In our opinion, the experimental evidence is far from conclusive. However, deviations from geometric discounting may change the positive and normative analysis of macroeconomic models significantly.[1] Therefore, we view the experimental evidence as far too potentially important to be ignored. One of the purposes of this paper is to derive positive and normative implications of a neoclassical macroeconomic equilibrium model when it is extended in a minimal way to be consistent with preference reversals.

Motivated by the psychology literature, David Laibson, in a series of recent papers (see, e.g., Laibson (1994, 1996, 1997) and Harris and Laibson (2000)), has employed models where agents have time-additive utility with quasi-geometric discounting weights. Laibson's framework has origins in Strotz (1956) and can be found in Phelps and Pollak (1968). We will refer to this model as the "Laibson model". In this model, the consumer can be viewed as having different "selves"—one for every time period. These selves disagree on the ranking of consumption paths and are assumed not to be able to commit to their future behavior. Laibson models decisions as the outcome of a dynamic game between the different selves.

Although attractive in many ways, the Laibson model is not problem-free. First, it is based on ad-hoc (not axiomatically founded) preferences. Second, since it is a nontrivial dynamic game, it allows many equilibria; even the set of Markov equilibria is infinite, as shown in Krusell and Smith (2000). Third, questions about welfare have conceptual problems: how should a hypothetical social planner evaluate welfare (which selves should this planner care about, and how)? Fourth, even with some (arbitrary) assumption about whom the planner cares about, policy analysis is not necessarily so interesting: either activism is trivially called for—if we assume that the government can provide the commitment mechanism that private consumers do not have access to—or it becomes impossible to implement. In the latter case, if we assume that the government/planner at t at least in part takes into account the preferences of the consumer's self at t, normative questions cannot be addressed, because then the government's preferences are time-inconsistent. In such a situation, we can only perform positive policy analysis: we can only study the outcome of a dynamic game between

[1]See, e.g., Krusell and Smith (2000) and Krusell, Kuruşçu, and Smith (2000a,b).

successive governments (see Krusell, Kuruşçu, and Smith (2000a,b)).

An alternative to the Laibson model has recently been developed, one that has the potential to avoid all the mentioned problems. Gul and Pesendorfer (2000a,b) provide axiomatic foundations for a utility function representation of preferences that deviates in a minimal way from standard textbook preferences in order to allow preference reversals. In their framework, a consumer for whom bundle x is feasible but who can also choose other bundles may be strictly worse off than if his choice set only contained x. The consumer might still choose x from the larger set, but be "tempted" by other elements in the choice set and therefore have to exercise "self-control" in order to refrain from choosing the tempting elements. It is this self-control that is associated with a utility loss; the preference for the smaller set over the larger one can be interpreted as a "preference for commitment". The utility function representation Gul and Pesendorfer derive consists of two functions. One of these represents the preferences a consumer would have if his choice sets were always singletons—this is the "standard" part of the utility function—and the other describes the preferences governing the temptation.

We have three main goals with the research described herein: (i) to extend the competitive equilibrium neoclassical growth model to incorporate consumer preferences that are of the Gul-Pesendorfer variety; in this sense, our paper is parallel to Barro's (1999) paper analyzing neoclassical growth using the Laibson model; (ii) to use the model in order to analyze some simple taxation and welfare issues; and (iii) investigate possible connections between the Gul-Pesendorfer setup and the Laibson setup. In terms of the last point, the model we develop attempts to draw a connection between the Gul-Pesendorfer model and the Laibson model by assuming that the temptation the consumer is faced with is to behave as a quasi-geometric discounter. The Laibson setup appears, in a certain sense to be made precise below, as a special and limiting case of our present framework.

We describe the basic temptation framework in a two-period consumption-savings model with production and competitive markets. That model, described in Section 3.2, is then extended to an infinite horizon, using a specialization of Gul and Pesendorfer (2000b). Our specialization is aimed at creating temptation behavior based on quasi-geometric discounting. Using the recursive framework, we specify two versions of these temptation preferences: one which is a true special case of Gul-Pesendorfer (2000b)'s general framework and which puts zero weight on the future, and one which puts positive weights on the future and thus is a possible extension of the Gul-Pesendorfer model. We say "possible" here because this work is not completed.

The model can be analyzed with recursive methods. However, like in Laibson's model, not only steady states but also dynamics are difficult to find. We therefore focus on special parametric cases: first, we study isoelastic utility and allow any neoclassical technology, and we then restrict attention further to logarithmic utility, Cobb-Douglas production, and 100% depreciation. In the former case, we are able to describe a set of functional equations in the aggregate state—capital—that jointly determine the global dynamics of the model.[2] Steady states of this model are nontrivial—they are, unlike in the standard model or in the Laibson model, a function for example of the elasticity of intertemporal substitution. We do not characterize global dynamics here, but we provide methods for analyzing local dynamics using linearization. For our most parameterized case, we provide full analytical solutions for the recursive competitive equilibrium.

Equipped with methods for analyzing equilibria, we then ask whether the government, somehow, can promote well-being, perhaps by diminishing the temptation and self-control problems? We pay special attention to taxation, but we also study the relevance of price taking: if the government could shut down markets and force autarky, in the sense of moving to home production, then how would allocations and welfare change? Our findings, in general terms, are that optimal government policy ought to involve a subsidy to investment and that a decentralization with price-taking behavior may or may not, depending on parameter values, be better than autarky. Our discussion of policy starts with an analysis of the two-period model. The intuition that goes along with the two-period model can then be seen to partially survive an extension of the time horizon.

Like the dynamic Gul-Pesendorfer model, our framework is recursive, so it does not express time-inconsistency of preferences; rather, it emphasizes temptation and self-control as giving rise to behavior that "looks" time-inconsistent (such as the "preference reversals" noted in experiments) . Because of the recursivity, welfare is unambiguously defined. As a result, our incorporation of quasi-geometric preferences into this framework, providing a possible interpretation of the Laibson model with different selves, also delivers a way to interpret welfare in that model. Under our interpretation, it turns out that the appropriate measure of current welfare of an agent is not utility as perceived by the current self, but that perceived by the last period's self.

Our paper is organized as follows. In Section 2, we briefly describe the Laibson model and some of the key results from Krusell and Smith (2000) and Krusell, Kuruşçu, and Smith (2000a,b). This summary sets the stage for the development of our version of the

[2]This analysis borrows from insights in Hercowitz and Krusell (2000), which analyzes steady states and dynamics in Laibson's model.

Gul-Pesendorfer model in Section 3. This section characterizes competitive equilibrium outcomes in a two-period model and begins the analysis of an infinite-horizon model with a quasi-geometric temptation. In Section 4, we use recursive tools and various parametric restrictions to study competitive equilibrium outcomes in the infinite-horizon model. In Section 5, we study the role of policy in the Gul-Pesendorfer model. Section 6 concludes.

2 Quasi-Geometric Discounting: the Laibson Model

In this section, we briefly describe the Laibson model. This description will serve as a frame of comparison for the later discussion of our version of the Gul-Pesendorfer model.

The basic setup of the Laibson model is as follows. Time is discrete and infinite and begins at time 0; there is no uncertainty. An infinitely-lived consumer derives utility from a stream of consumption at different dates. We assume that the preferences of this individual are time-additive, and that they take the form of a sequence of preference profiles:

$$\begin{aligned}
&\text{Self 0:} \quad U_0 = u_0 + \beta\left(\delta u_1 + \delta^2 u_2 + \delta^3 u_3 + \ldots\right) \\
&\text{Self 1:} \quad U_1 = u_1 + \beta\left(\delta u_2 + \delta^2 u_3 + \ldots\right) \\
&\text{Self 2:} \quad U_2 = u_2 + \beta\left(\delta u_3 + \ldots\right) \\
&\qquad \ldots
\end{aligned}$$

When $\beta = 1$, we have standard, time-consistent, geometric preferences. When $\beta \neq 1$, there is a time-inconsistency: at date 0, the trade-off between dates 1 and 2 is perceived differently than at date 1, and so on. When $\beta < 1$, we have a "bias towards the present": the individual thinks "I want to save, just not right now"; when $\beta > 1$, a bias towards the future is expressed as "I want to consume, just not right now". We refer to this class of preferences as quasi-geometric, as they are a one-period deviation from the standard geometric case.[3]

As time progresses, the individual will change his mind about the relative values of consumption at different points in time so long as $\beta \neq 1$. He would, therefore, if he could, want to commit to his future consumption levels. The standard assumption is that there is no way for the consumer to do so. Further, it is most often assumed that the consumer realizes that his preferences will change—is "sophisticated"—and makes the current decision taking this into account.[4] This means that the decision-making process becomes a dynamic game with the agent's current and future selves as players. Finally, it is typical to focus on

[3]The term quasi-hyperbolic is used in the literature as referring to the same preference setup. We prefer to use the term quasi-geometric since it is the most appropriate mathematical term.

[4]Others, such as O'Donoghue and Rabin (1999), do not make this assumption.

Markov equilibria: at a moment in time, no histories are assumed to matter for outcomes beyond what is summarized in the current stock of wealth held by the agent.[5]

In some ways, the extension to quasi-geometric preferences does not represent a giant departure from the standard model. As Barro (1999) first showed, under logarithmic preferences there is observational equivalence between the standard preference class and the quasi-geometric one in the set of competitive equilibria he considers. Under isoelastic utility, there is no equivalence, but the set of steady states Barro considers is the same for the two models. For the quasi-geometric model to produce radically different output, other frictions and wrinkles seem necessary as well, such as for example borrowing constraints and idiosyncratic uncertainty (often considered in Laibson's work) or further restrictions in asset markets.

However, in some sense this similarity between the standard model and the Laibson model is superficial. The latter model has the following non-standard features. First, there is a severe indeterminacy of equilibria to the individual consumer's decision problem even when attention is restricted to Markov equilibria. As Krusell and Smith (2000) shows, for any concave utility function and any budget constraint (including budgets with decreasing returns to asset accumulation), the set of steady states is indeterminate, as is the set of paths leading in to any steady state. Thus, the uniqueness of the standard model is a knife-edge case: whenever $\beta \neq 1$, it breaks down. Relatedly, computation of equilibria is very difficult in general. As Hercowitz and Krusell (2000) show, it is possible to use linearization methods to solve for competitive equilibria, but for planning problems or utility functions outside the isoelastic class, all methods we know of fail. Both methods based on value function iteration and Euler equations produce cycling and, when convergence occurs, initial conditions matter. Our preliminary interpretation of these results is that it is the multiplicity of equilibria that makes algorithms unstable.

Second, policy analysis is difficult in the Laibson model. One reason is the lack of a natural welfare measure. How should a hypothetical social planner evaluate policy: which selves should this planner care about, and how? The framework does not suggest a natural answer to this question. Now one could decide that some particular social welfare function is desirable. Or one could simply treat the different selves as different people and use the Pareto criterion. But whenever the planner's objective is a (stationary) aggregation of the

[5]The restriction to Markov equilibria seems a natural way of restricting the set of equilibria. Laibson (1994) and Bernheim, Ray, and Yeltekin (1999) study trigger-strategy equilibria. Renegotiation arguments for the Markov assumption are particularly compelling in this "one-agent" game, although we do not know of a solid formal defense for this position.

utilities of the different selves, there is an additional unresolved issue: time inconsistency of the planner's preferences. One thus needs to decide to what extent the government can help these agent(s) commit to future consumption plans, e.g., by committing to future taxes. If it can, then policy is relatively trivially recommendable; with a bias toward the present, the government could improve outcomes (for all selves) by giving future agents incentives to save more. If it cannot, then one would need to consider time-consistent government policy by analyzing a game between a sequence of governments. But this analysis would be more of positive than normative nature. We examine such equilibria in Krusell, Kuruşçu, and Smith (2000b), where we curiously find that time-consistent policy equilibria with a benevolent government lead to lower welfare than does the laissez-faire competitive equilibrium: if the government could commit to zero taxes forever, everybody would be better off.

To summarize, even though the consideration of quasi-geometric preferences may appear as a minor extension of the standard geometric-preference model, it leads to a number of problems. Some of these problems seem to be related to a lack of axiomatic foundation: with axiomatically-based decision theory, decisions lead to unique outcomes by definition (and computation is therefore made easier) and welfare evaluation is unambiguous.

3 Temptation and Self-Control: the Gul-Pesendorfer Model

In this section, we develop our version of the Gul-Pesendorfer model. We first introduce the ideas of temptation and self-control as formalized by Gul and Pesendorfer (2000a,b). Next, before introducing a full-blown dynamic model, we study equilibrium outcomes in a two-period consumption-savings problem with a Laibson-like temptation. Finally, we study equilibrium outcomes in an infinite-horizon setting.

3.1 An Axiomatic Foundation for Temptation and Self-Control

Gul and Pesendorfer (2000a) formalize the notions of temptation and self-control by imagining that consumers have preferences over *sets* of lotteries. In addition to appropriately modified versions of the usual axioms (completeness, transitivity, continuity, and independence), Gul and Pesendorfer introduce a new axiom called "set betweenness". Letting A and B be sets, this axiom states that $A \succeq B$ implies $A \succeq (A \cup B) \succeq B$. Loosely speaking, under this axiom it is possible that a consumer strictly prefers a given set to another, *larger* set of which the first set is a *strict subset*.

These axioms allow for several possibilities. First, there is the standard case: $A \succeq B$ implies $A \sim (A \cup B)$; here, with the preferred consumption bundle being in A, the addition of the set B to the set A is irrelevant. Second, the consumer could have a preference for commitment: $A \succ B$ implies $A \succ (A \cup B)$; here, the consumer is made worse off by the addition of the set B. Third, the consumer could succumb to temptation: $A \succ B$ and $B \sim (A \cup B)$; here, the consumer prefers A to B and yet he is indifferent between B and $A \cup B$ because he knows that if his choice set is $A \cup B$ he will "succumb" to the "temptation" contained in B. Finally, the consumer could instead exert self-control: $A \succ (A \cup B)$ and $(A \cup B) \succ B$; here, the consumer has enough self-control not to succumb to the temptation contained in B, yet he nonetheless prefers the smaller set A to the larger one $A \cup B$ because exerting self-control is costly (i.e., reduces his utility).

Gul and Pesendorfer (2000a) show that these axioms imply a representation of preferences in terms of two functions $\tilde{U}$ and $\tilde{V}$:

$$\tilde{W}(A) \equiv \max_{a \in A} \{\tilde{U}(a) + \tilde{V}(a)\} - \max_{\tilde{a} \in A} \tilde{V}(\tilde{a}),$$

where $\tilde{W}(A)$ is the utility that the consumer associates with the set A. The consumer's optimal action is given by $\arg\max_{a \in A} \{\tilde{U}(a) + \tilde{V}(a)\}$, but the utility of this action depends on the amount of self-control that the consumer exerts when he makes this choice. In particular, $\tilde{V}(a) - \max_{\tilde{a} \in A} \tilde{V}(\tilde{a}) \leq 0$ can be viewed as the disutility of self-control given that the consumer chooses action a. If $\arg\max_{a \in A} \{\tilde{U}(a) + \tilde{V}(a)\} = \arg\max_{\tilde{a} \in A} \tilde{V}(\tilde{a})$ then either (i) we are in a standard case ($\tilde{U} = \tilde{V}$) where temptation and self-control play no role in the consumer's decision-making or (ii) the agent succumbs, i.e., lets $\tilde{V}$ govern his choices completely. Reiterating, when the two argmaxes are not the same, the agent exercises self-control.

3.2 The Two-Period Consumption-Savings Model with Temptation

We now specialize the general framework described in Section 3.1 to a simple two-period general equilibrium economy with production. This example allows us to illustrate the role of temptation and self-control in determining equilibrium outcomes and welfare before turning to a fully dynamic economy in the next section.

A typical consumer in the economy values consumption today (c_1) and tomorrow (c_2). Specifically, the consumer has Gul-Pesendorfer preferences represented by two functions $\tilde{u}(c_1, c_2)$ and $\tilde{v}(c_1, c_2)$; these functions are the counterparts of $\tilde{U}$ and $\tilde{V}$, respectively, in

Section 3.1. Figures 1–3 illustrate. Figure 1 describes a choice set consisting of a single point. Figure 2 depicts a number of different singleton choice sets, each giving rise to the same utility; the curve connecting these points is the standard indifference curve given by $\tilde{u}$—the one representing the consumer's preferences if temptation were not an issue. Figure 3 takes the same point as in Figure 1 but adds to the choice set the rest of a typical budget line. Now the consumer will not like this set as much as the initial singleton set, even if the singleton set represents the one point that would maximize $\tilde{u}$ over the budget set, because the larger set involves a temptation.

We specify the functions $\tilde{u}$ and $\tilde{v}$ as follows:

$$\tilde{u}(c_1, c_2) = u(c_1) + \delta u(c_2)$$

and

$$\tilde{v}(c_1, c_2) = u(c_1) + \beta\delta u(c_2),$$

where u has the usual properties and $0 < \beta \leq 1$. When $\beta = 1$, we have the standard model in which temptation and self-control do not play a role. When $\beta < 1$, however, the temptation function gives a stronger preference for present consumption. The strength of this preference increases as β increases. The decision problem of a typical consumer, then, is:

$$\max_{c_1,c_2} \{\tilde{u}(c_1, c_2) + \tilde{v}(c_1, c_2)\} - \max_{\tilde{c}_1,\tilde{c}_2} \tilde{v}(\tilde{c}_1, \tilde{c}_2) \tag{1}$$

subject to a budget constraint that we will specify below.

Each consumer is endowed with k_1 units of capital at the beginning of the first period and with one unit of labor in each period. Each consumer rents these factors of production to a profit-maximizing firm that operates a neoclassical production function. In equilibrium, given aggregate capital $\bar{k}$, the rental rate $r(\bar{k})$ and the wage rate $w(\bar{k})$ are determined by the firm's marginal product conditions. Given these prices, the consumer's budget constraint is described by the set:

$$\begin{aligned} B(k_1, \bar{k}_1, \bar{k}_2) \equiv \{(c_1, c_2) : \exists k_2 : c_1 = r(\bar{k}_1)k_1 + w(\bar{k}_1) - k_2 \text{ and} \\ c_2 = r(\bar{k}_2)k_2 + w(\bar{k}_2)\} \end{aligned}$$

where k_2 is the consumer's asset holding at the beginning of period 2 (i.e., his savings in period 1) and $\bar{k}_i$ is aggregate capital in period i. Since all consumers have the same capital holdings in period 1, $\bar{k}_1 = k_1$; in equilibrium, $\bar{k}_2 = k_2$, but when choosing k_2 the consumer takes $\bar{k}_2$ as given. Inserting the definitions of the functions $\tilde{u}$ and $\tilde{v}$ into (1) and combining terms, a typical consumer's decision problem is:

$$\max_{(c_1,c_2)\in B(k_1,\bar{k}_1,\bar{k}_2)} \{2u(c_1) + \delta(1+\beta)u(c_2)\} - \max_{(\tilde{c}_1,\tilde{c}_2)\in B(k_1,\bar{k}_1,\bar{k}_2)} \{u(\tilde{c}_1) + \beta\delta u(\tilde{c}_2)\}. \tag{2}$$

In this two-period problem, the "temptation" part of the problem (i.e., the second maximization problem in the objective function) plays no role in determining the consumer's actions in period 1. As we describe in Section 3.3, this is not true when the horizon is longer than two periods. The temptation part of the problem does, however, affect the consumer's welfare, as we discuss below.

The consumer's intertemporal first-order condition is:

$$\frac{2}{(1+\beta)\delta}\frac{u'(c_1)}{u'(c_2)} = r(\bar{k}_2).$$

It is straightforward to see that the intertemporal consumption allocation (which, in effect, maximizes $\tilde{u} + \tilde{v}$) represents a compromise between maximizing $\tilde{u} = u(c_1) + \delta u(c_2)$ and maximizing $\tilde{v} = u(c_1) + \beta\delta u(c_2)$. In the former case, the first-order condition is:

$$\frac{1}{\delta}\frac{u'(c_1)}{u'(c_2)} = r(\bar{k}_2),$$

whereas in the latter case, the first-order condition is:

$$\frac{1}{\beta\delta}\frac{u'(c_1)}{u'(c_2)} = r(\bar{k}_2).$$

Since

$$\frac{1}{\beta\delta} \geq \frac{2}{(1+\beta)\delta} \geq \frac{1}{\delta},$$

the consumer's consumption allocation is tilted towards the present relative to maximizing $u(c_1)+\delta u(c_2)$ and is tilted towards the future relative to maximizing the temptation function $u(c_1) + \beta\delta u(c_2)$. Figure 4 illustrates these points.

To determine the competitive equilibrium allocation, set $\bar{k}_2 = k_2$ in the consumer's first-order condition and recognize that $r(\bar{k}) = f'(\bar{k})$, where f is the firm's production function (which has the standard properties). To wit,

$$\frac{2}{(1+\beta)\delta}\frac{u'(\bar{c}_1)}{u'(\bar{c}_2)} = f'(\bar{k}_2),$$

where $\bar{c}_i$ is aggregate (per capita) consumption in period i. It is easy to see that this is the same first-order condition that obtains in autarky, i.e., in an environment without markets in which each consumer operates his own "backyard" technology. In this case, the consumer's problem is to maximize (2) subject to the technological constraints $c_1 = f(k_1) - k_2$ and $c_2 = f(k_2)$. Thus the competitive equilibrium and autarky allocations coincide in the two-period model.

Nonetheless, the consumer is better off in autarky than in competitive equilibrium. This happens because the temptation is weaker in autarky, implying that the disutility of self-control is smaller. The temptation is weaker in autarky because the production possibilities frontier (which plays the role of the consumer's "budget constraint" in autarky) lies strictly inside the (triangular) budget set faced by a consumer in competitive equilibrium. In other words, the utility that the consumer could gain by succumbing to the temptation is larger under competitive equilibrium than in autarky. Since the allocations are identical, the consumer must exert more self-control (i.e., incur a larger disutility of self-control) under competitive equilibrium than in autarky. Consequently, the consumer is worse off in competitive equilibrium than in autarky, even though the outcomes are the same! Figure 5 illustrates these points.

3.3 The Infinite-Horizon Consumption-Savings Model with Temptation

In this section, we study temptation and self-control in an infinite-horizon setting. First, we describe how Gul and Pesendorfer (2000b) extend the static (or two-period) framework developed in Gul and Pesendorfer (2000a) to an infinite-horizon setting. We then modify the Gul-Pesendorfer model to incorporate a Laibson-like quasi-geometric temptation. Finally, we study competitive equilibrium outcomes in this framework under various functional form and parameter restrictions.

3.3.1 The Gul-Pesendorfer Infinite-Horizon Model

Gul and Pesendorfer (2000b) provide additional suitable axioms to build a foundation for preferences over a domain which they define to be sets of pairs of current consumption and a continuation problem.[6] That is, an agent has a set of possible choices in the current period, and each of these choices specifies a current consumption level and a set of choices (of the same nature) for next period. In a simple price-taking framework with a constant net interest rate r for transforming consumption today into consumption tomorrow, a set could summarized by the agent's present-value wealth, which we denote w. The characterization

[6] A continuation problem is itself an infinite-horizon consumption problem, that is, a set of alternatives within which the consumer can choose.

theorem in Gul and Pesendorfer (2000b) applied to this price-taking example implies that the agent's consumption preferences and associated consumption problem can be represented using two functions u and v and a discount rate δ together with a recursive functional equation:

$$W(w) = \max_{w' \geq 0}\{u(w - w'/(1+r)) + \delta W(w') + v(w - w'/(1+r))\} - \max_{w' \geq 0}\{v(w - w'/(1+r))\}, \quad (3)$$

where $W(w)$ denotes the lifetime utility that the consumer assigns to the set of intertemporal consumption bundles defined by present-value wealth w.

In this formulation, the temptation function v can be viewed as a "free parameter". Gul and Pesendorfer (2000b) study a function v that is convex and increasing. They show that this framework can produce (apparent) "preference reversals" of the kind documented in experimental studies.

3.3.2 The Gul-Pesendorfer Model with a "Laibson" Temptation

We now introduce a Laibson-like temptation function v into Gul and Pesendorfer's infinite-horizon setup. This can be viewed as an attempt to understand the Laibson model described in Section 2.

Before studying competitive equilibrium outcomes, we first develop our ideas in the context of an "autarky" (or Robinson Crusoe) model. As in the two-period model in Section 3.2, we use capital rather than present-value wealth as a state variable (we do this merely for analytical convenience). In our framework, the counterpart of the functional equation (3) is:

$$W(k) = \max_{k' \leq f(k)}\{u(f(k) - k') + \delta W(k') + V(k, k')\} - \max_{\tilde{k}' \leq f(k)}\{V(k, \tilde{k}')\}, \quad (4)$$

where the temptation function V is *quasi-geometric*:

$$V(k, k') \equiv \gamma\{u(f(k) - \tilde{k}') + \beta\delta W(k')\}.$$

Notice that when $\gamma = 0$ or $\beta = 1$, we have as a special case the standard model without temptation or self-control. When $\beta < 1$, we can approach the Laibson model by making γ large: as γ goes to infinity, the consumer puts so much weight on the temptation that he succumbs. This limiting case is appealing because it provides one way to evaluate policy in the Laibson model. That is, this case provides a potential resolution to the problem of which of the consumer's "selves" should be used when assessing welfare.

A potential pitfall of this formulation is that the operator $TW(w)$ defined by the right-hand side of the functional equation (4) does not satisfy Blackwell's sufficient conditions for

T to be a contraction mapping. Although T does satisfy discounting, it is not monotone in general. A failure of monotonicity can occur because a greater utility from future wealth may imply lower total utility since the disutility from self-control may increase with the higher temptation resulting from a higher utility from future wealth. Figure 6 illustrates this point.

In fact, the functional equation (4) can admit multiple solutions for W, in a manner reminiscent of the findings of Krusell and Smith (2000) for the Laibson model. It is straightforward, for example, to find examples of multiplicity (provided $\beta < 1$) when the state space is discrete and γ is sufficiently large. Moreover, it is possible to find examples where there is a unique solution for W but where iteration backwards does not converge to this W. Unlike in Krusell and Smith (2000), "mixing" solutions typically do not appear to be regular equilibria. It should also be possible to generalize these examples of multiplicity to indeterminacy in the case of a continuous state space (as Krusell and Smith (2000) do for the Laibson model).

This multiplicity suggests that, in moving outside of the framework of Gul and Pesendorfer (2000b) (by making the temptation depend on future consumption as well as today's consumption), the theoretical framework developed in Gul and Pesendorfer (2000b) no longer applies. In particular, it is possible that not all of the solutions to the functional equation represent well-defined recursive preferences over sets of consumption bundles. We suspect, but have not yet been able to prove, that continuous solutions do represent a set of Axioms very close to those in Gul and Pesendorfer's analysis.

When $\beta = 0$, however, T is a contraction mapping, in which case the functional equation does have a unique solution. When $\beta = 0$, the consumer is tempted to consume all of his wealth immediately, so that this special case is similar in some respects to the case developed in Gul and Pesendorfer (2000b) (in which the temptation depends only on current consumption). Here, however, the temptation function is a multiple γ of u, where u is the "usual" concave period utility function rather than an arbitrary convex function of current consumption. The free parameter γ can then be varied to calibrate the strength of the temptation (just as β can be varied in the Laibson model).

These remarks notwithstanding, when $\beta > 0$, there are special parametric assumptions that "work" in the sense that we can find analytical solutions to the functional equation (4). Before studying these parametric examples, however, it might be instructive to derive the "Euler equations" associated with the problem defined by (4). Since we must determine two "decision rules" (one for each of the maximization problems on the right-hand side of (4)), there are two Euler equations:

$$u'(c_t) = \delta \frac{1+\beta\gamma}{1+\gamma} f'(k_{t+1})\{u'(c_{t+1}) + \gamma[u'(c_{t+1}) - u'(\tilde{c}_{t+1})]\},$$

where $\tilde{c}_{t+1}$ refers to temptation consumption in the next period, and

$$u'(\tilde{c}_t) = \beta\delta f'(\tilde{k}_{t+1})\{u'(\tilde{c}_{t+1}) + \gamma[u'(\tilde{c}_{t+1}) - u'(\tilde{\tilde{c}}_{t+1})]\},$$

where $\tilde{c}_t$ is current temptation consumption, $\tilde{c}_{t+1}$ is next period's actual consumption given that you succumb today, and $\tilde{\tilde{c}}_{t+1}$ is next period's temptation consumption given that you succumb today.

Both these expressions look like standard Euler equations except in two places: (i) the discount factors; and (ii) the added term $\gamma(u'_{t+1} - \tilde{u}'_{t+1})$. The discount factors, which are ranked (given that $1 > \frac{1+\beta\gamma}{1+\gamma} > \beta$), show that the temptation discount factor is lower than that of the actual one, provided $\beta < 1$. The term $\gamma(u'_{t+1} - \tilde{u}'_{t+1})$ is the derivative of the disutility/cost of self-control, $\gamma(u_{t+1} - \tilde{u}_{t+1})$, with respect to wealth. This term is positive, since temptation consumption is higher than actual consumption and the utility function is strictly concave. The interpretation of these equations is that the marginal benefit from wealth tomorrow exceeds u'_{t+1}, because the self-control cost gets smaller as wealth increases in this model. This is an effect which is parallel to an effect in the Laibson model. The Laibson model yields an Euler equation which reads

$$u'(c_t) = \beta\delta u'(c_{t+1})\{f'(k_{t+1}) + (1/\beta - 1)g'(k_{t+1})\},$$

where $g(k)$ is the savings given a capital stock of k and, thus, $g'(k_{t+1}) > 0$ is the marginal savings propensity in the next period. Here, there is an added benefit to savings, too: so long as $\beta < 1$, the benefit arises due to the disagreement between the current and the next selves. Any unit of wealth next period will decrease consumption that period (by an amount g'_{t+1}, so $g'_{t+1}u'_{t+1}$ measured in $t+1$ utils). In return, it will increase consumption thereafter. The future consumption increase is normally worth the same amount in present value ($g'_{t+1}u'_{t+1}$) from the first-order condition of next period's savings choice: the envelope theorem. In the Laibson model, however, it is valued higher by the current self by a factor $1/\beta$, since the next self, who makes the savings decision in question, has an additional weight β on every utility flow from $t+2$ and on. So like in the Gul-Pesendorfer model, the Laibson agents perceive a cost involving future savings—they are too low—and higher current savings decrease this cost.

4 Solving for Recursive Competitive Equilibria with a Quasi-Geometric Temptation

We now study competitive equilibrium with a quasi-geometric temptation. The economic environment is the same as for the two-period model studied in Section 3.2, appropriately extended to an infinite horizon. We use recursive tools to characterize the equilibrium. A typical consumer takes factor prices and an aggregate law of motion $\bar{k}' = G(\bar{k})$ as given. His decision problem can be characterized recursively as follows:

$$\begin{aligned} W(k,\bar{k}) &= \max_{k'}\{u(r(\bar{k})k + w(\bar{k}) - k') + \delta W(k',\bar{k}') + V(k,k',\bar{k},\bar{k}')\} - \\ &\quad \max_{\tilde{k}'}\{V(k,\tilde{k}',\bar{k},\bar{k}')\}, \end{aligned} \tag{5}$$

where

$$V(k,k',\bar{k},\bar{k}') = \gamma\left(u(r(\bar{k})k + w(\bar{k}) - k') + \beta\delta W(k',\bar{k}')\right).$$

Substituting the temptation function into (5) and combining terms, the consumer's problem becomes:

$$\begin{aligned} W(k,\bar{k}) &= \max_{k'}\{(1+\gamma)u(r(\bar{k})k + w(\bar{k}) - k') + \delta(1+\beta\gamma)W(k',\bar{k}')\} - \\ &\quad \gamma \max_{\tilde{k}'}\{u(r(\bar{k})k + w(\bar{k}) - \tilde{k}') + \beta\delta W(\tilde{k}',\bar{k}')\}. \end{aligned}$$

Ignoring possible multiplicity for the moment, this problem determines a "realized" decision rule $g(k,\bar{k})$ which solves the first maximization problem and a "temptation" decision rule $\tilde{g}(k,\bar{k})$ which solves the second maximization problem. In equilibrium, we require $g(\bar{k},\bar{k}) = G(\bar{k})$.

We will now consider two special parametric examples. First, we consider the case of isoelastic utility: $u(c) = (1-\sigma)^{-1}c^{1-\sigma}$, where $\sigma > 0$. Later, we consider what we call the "log-Cobb" model: u is logarithmic, capital depreciates fully in one period, and the production function is Cobb-Douglas. For the case of isoelastic utility, we can obtain a complete characterization of the steady state in the competitive equilibrium and a partial characterization of the dynamic behavior of the competitive equilibrium. For the "log-Cobb" model, we can obtain a complete characterization of both steady states and dynamics.

4.1 Isoelastic Utility and Any Convex Technology

In this section, we study competitive equilibrium when utility is isoelastic. Proposition 1 characterizes equilibrium dynamics for this case.

Proposition 1 Suppose that $u(c) = (1-\sigma)^{-1}c^{1-\sigma}$, where $\sigma > 0$, and that f is a standard neoclassical production function. In competitive equilibrium, the realized decision rule $g(k,\bar{k}) = \lambda(\bar{k})k + \mu(\bar{k})$ and the temptation decision rule $\widetilde{g}(k,\bar{k}) = \widetilde{\lambda}(\bar{k})k + \widetilde{\mu}(\bar{k})$, where the functions λ, μ, $\widetilde{\lambda}$, and $\widetilde{\mu}$ solve the following functional equations:

$$\mu(\bar{k}) + \frac{w(\bar{k}') - \mu(\bar{k}')}{r(\bar{k}') - \lambda(\bar{k}')} = \frac{w(\bar{k}) - \mu(\bar{k})}{r(\bar{k}) - \lambda(\bar{k})}\lambda(\bar{k}) \tag{6}$$

$$\widetilde{\mu}(\bar{k}) + \frac{w(\bar{k}') - \mu(\bar{k}')}{r(\bar{k}') - \lambda(\bar{k}')} = \frac{w(\bar{k}) - \widetilde{\mu}(\bar{k})}{r(\bar{k}) - \widetilde{\lambda}(\bar{k})}\widetilde{\lambda}(\bar{k}) \tag{7}$$

$$\frac{1+\gamma}{\delta\left(1+\beta\gamma\right) r\left(\bar{k}'\right)} = (1+\gamma)\left(\frac{(r(\bar{k}') - \lambda(\bar{k}'))\lambda(\bar{k})}{r(\bar{k}) - \lambda(\bar{k})}\right)^{-\sigma} - \gamma\left(\frac{(r(\bar{k}') - \widetilde{\lambda}(\bar{k}'))\lambda(\bar{k})}{r(\bar{k}) - \lambda(\bar{k})}\right)^{-\sigma} \tag{8}$$

$$\frac{1}{\delta\beta r\left(\bar{k}'\right)} = (1+\gamma)\left(\frac{(r(\bar{k}') - \lambda(\bar{k}'))\widetilde{\lambda}(\bar{k})}{r(\bar{k}) - \widetilde{\lambda}(\bar{k})}\right)^{-\sigma} - \gamma\left(\frac{(r(\bar{k}') - \widetilde{\lambda}(\bar{k}'))\widetilde{\lambda}(\bar{k})}{r(\bar{k}) - \widetilde{\lambda}(\bar{k})}\right)^{-\sigma} \tag{9}$$

where $\bar{k}' = G(\bar{k}) = g(\bar{k},\bar{k}) = \lambda(\bar{k})\bar{k} + \mu(\bar{k})$.
Proof: See the Appendix.

The fact that a consumer's decision rules are linear in his consumer capital implies that the steady-state wealth distribution is indetermined. This result stands in contrast to Gul and Pesendorfer (2000b) who find that the steady-state wealth distribution is uniquely determined when the temptation function is convex.

4.1.1 Steady States

Let $\bar{k}_{ss}$ denote the steady-state aggregate capital stock in competitive equilibrium; by definition, $\bar{k}_{ss} = G(\bar{k}_{ss}) = \lambda(\bar{k}_{ss})\bar{k}_{ss} + \mu(\bar{k}_{ss})$. This equation, together with equations (6)–(9) evaluated at $\bar{k}' = \bar{k} = \bar{k}_{ss}$, jointly determine the steady-state capital stock and the values of the functions λ, μ, $\widetilde{\lambda}$, and $\widetilde{\mu}$ at the steady-state capital stock. Using these equations, it is straightforward to verify that $\lambda(\bar{k}_{ss}) = 1$, $\mu(\bar{k}_{ss}) = 0$, and $\bar{k}_{ss}$ solves:

$$\frac{1+\gamma}{r(\bar{k}_{ss})\delta(1+\beta\gamma)} = 1+\gamma-\gamma\left(1 - \frac{1-\left(\frac{\beta(1+\gamma)}{1+\beta\gamma}\right)^{1/\sigma}}{r(\bar{k}_{ss})}\right)^{\sigma}. \tag{10}$$

Given these values, equation (8) determines $\widetilde{\lambda}(\bar{k}_{ss})$ and equation (9) determines $\widetilde{\mu}(\bar{k}_{ss})$.

An interesting feature of equation (10) is that the steady-state capital stock, and hence the steady-state rate of interest, depends on the preference parameter σ. This dependence disappears if $\beta = 1$ or if $\gamma = 0$, in which case equation (10) simplifies to the familiar formula

$r(\bar{k}_{ss}) = \delta^{-1}$. In addition, this dependence disappers in the limit as γ goes to infinity. In particular, the limiting steady-state interest rate is

$$\frac{1-\delta(1-\beta)}{\beta\delta}.$$

This is the same steady-state interest that obtains in the Laibson model with isoelastic utility.

For a particular numerical example, Table 1 illustrates how the steady-state interest rate $r(\bar{k}) - 1$ varies as β and σ vary. For these calculations, we assume that the technology is Cobb-Douglas, so that $r(\bar{k}) = \alpha\bar{k}^{\alpha-1} + 1 - \eta$, where η is the rate of depreciation of capital, and $w(\bar{k}) = (1-\alpha)\bar{k}^{\alpha}$. We set $\alpha = 0.36$, $\eta = 0.1$, $\delta = 0.95$, and $\gamma = 1$.

Table 1:
Steady-State Interest Rate

	$\sigma = 0.5$	$\sigma = 1$	$\sigma = 2$	$\sigma = 3$	$\sigma = 5$	$\sigma = 10$
$\beta = 0.4$	8.724%	7.519%	7.123%	7.012%	6.930%	6.872%
$\beta = 0.7$	6.303%	6.192%	6.142%	6.127%	6.114%	6.105%

Table 1 shows that the steady-state capital stock rises (and consequently the steady-state interest falls) as β and σ increase.

Needless to say, the determination of the steady state is more complex in this model than in the standard model. Moreover, without isoelastic utility, we do not know how to find steady states without simultaneously solving for the decision rules globally.

4.1.2 Dynamics

Global dynamics can be characterized numerically by solving the four functional equations (6)–(9) for the four unknown functions. A variety of standard numerical methods can be used to accomplish this task.

Local dynamics around a steady state can be determined by differentiating the four equations with respect to $\bar{k}$, imposing the steady-state condition $\bar{k}' = \bar{k} = \bar{k}_{ss}$, and then solving for $\lambda'(\bar{k}_{ss})$, $\mu'(\bar{k}_{ss})$, $\tilde{\lambda}'(\bar{k}_{ss})$, and $\tilde{\mu}'(\bar{k}_{ss})$. The steady state is thus locally stable if $\lambda'(\bar{k}_{ss})\bar{k}_{ss} + \lambda(\bar{k}_{ss}) + \mu'(\bar{k}_{ss})$ is less than one in absolute value.

In particular, we have implemented the following numerical algorithm for examining local dynamics numerically. First, postulate that

$$\lambda(\bar{k}) \quad = \quad 1 + \lambda'(\bar{k}_{ss})(\bar{k} - \bar{k}_{ss})$$

$$\begin{aligned} \mu(\bar{k}) &= \mu'(\bar{k}_{ss})(\bar{k} - \bar{k}_{ss}) \\ \widetilde{\lambda}(\bar{k}) &= \widetilde{\lambda}(\bar{k}_{ss}) + \widetilde{\lambda}'(\bar{k}_{ss})(\bar{k} - \bar{k}_{ss}) \\ \widetilde{\mu}(\bar{k}) &= \widetilde{\mu}(\bar{k}_{ss}) + \widetilde{\mu}'(\bar{k}_{ss})(\bar{k} - \bar{k}_{ss}), \end{aligned}$$

where $\widetilde{\lambda}(\bar{k}_{ss})$, $\widetilde{\mu}(\bar{k}_{ss})$, and $\bar{k}_{ss}$ are determined as described in Section 4.1.1 and the four first derivatives $\lambda'(\bar{k}_{ss})$, $\mu'(\bar{k}_{ss})$, $\widetilde{\lambda}'(\bar{k}_{ss})$, and $\widetilde{\mu}'(\bar{k}_{ss})$ are unknown parameters to be determined by the numerical algorithm. Second, rewrite each of the equations (6)–(9) so that the right-hand side is zero. Third, given values of the four first derivatives, use a finite-difference method to approximate the derivatives with respect to $\bar{k}$ of the left-hand sides of each of the equations (6)–(9). Fourth, vary the four first derivatives in order to set the derivatives of the left-hand sides each of the four equations equal to zero.

For a particular numerical example, Table 2 illustrates how the local speed of convergence to the steady state (i.e., the quantity $\lambda'(\bar{k}_{ss})\bar{k}_{ss} + \lambda(\bar{k}_{ss}) + \mu'(\bar{k}_{ss})$) varies as σ and β vary, holding the steady state constant. For these calculations, we assume, as we did in Section 4.1.1, that the technology is Cobb-Douglas. We set $\alpha = 0.36$, $\eta = 0.1$, and $\gamma = 1$; for each pair (σ, β), we choose δ so that the steady-state interest rate is the one that prevails when $\beta = 1$ and $\delta = 0.95$ (recall that when $\beta = 1$, the steady-state interest rate does not depend on σ).

Table 2:
Speed of Adjustment to the Steady State

	$\beta = 0.25$	$\beta = 0.5$	$\beta = 0.75$	$\beta = 1$
$\sigma = 0.5$	0.79093	0.79757	0.80155	0.80477
$\sigma = 1$	0.86039	0.86039	0.86039	0.86039
$\sigma = 3$	0.93254	0.93075	0.92854	0.92643

Table 2 shows that, holding the interest rate fixed, the (local) speed of convergence to the steady state increases as β increases and as σ increases. Because the interest rate is fixed in Table 2, these results indicate that the model in which temptation and self-control play a role is not observationally equivalent to the standard model (i.e., the one that obtains when $\beta = 1$), except in the special case that $\sigma = 1$, i.e., the case of logarithmic utility (see Section 4.1.4 below on the "log-Cobb" model).

4.1.3 The Special Case $\beta = 0$

When $\beta = 0$, one can show that $\widetilde{\lambda}(\bar{k}) = 0$ and $\widetilde{\mu}(\bar{k}) = -w(\bar{k})/(r(\bar{k} - 1)$: the temptation is to consume all present-value wealth. In this case, equation (10), which determines the steady-state interest rate, simplifies to:

$$\frac{1}{\delta r(\bar{k}_{ss})} = 1 - \frac{\gamma}{1+\gamma}\left(\frac{r(\bar{k}_{ss})}{r(\bar{k}_{ss})-1}\right)^{-\sigma}.$$

Hence, even when $\beta = 0$, the steady-state interest rate depends on the preference parameter σ.

4.1.4 The "Log-Cobb" Model

We turn now to the "log-Cobb" model: logarithmic u, full depreciation, and Cobb-Douglas production (i.e., $f(k) = Ak^{\alpha}$, $0 < \alpha < 1$). Under these parametric restrictions, we can completely characterize the competitive equilibrium by means of analytical expressions for the decision rules. Specifically,

$$W(k,\bar{k}) = A_0 + A_1 \log(\bar{k}) + A_2 \log(k + \varphi\bar{k}),$$

where A_0, A_1, A_2, and φ are undetermined coefficients that can be computed using a standard guess-and-verify method.[7] Since u is isoelastic, the "realized" decision rule has the same form as above, with

$$\lambda(\bar{k}) = \frac{\delta}{\delta + (1-\delta)\frac{1+\gamma}{1+\beta\gamma}}$$

and $\mu(\bar{k}) = 0$, implying that

$$G(\bar{k}) = g(\bar{k},\bar{k}) = \frac{\alpha\delta}{\delta + (1-\delta)\,\frac{1+\gamma}{1+\beta\gamma}}A\bar{k}^{\alpha}.$$

The "temptation" decision rule also has the same form as above, with

$$\widetilde{\lambda}(\bar{k}) = \frac{\delta\beta}{1-\delta+\delta\beta}r(\bar{k})$$

and

$$\widetilde{\mu}(\bar{k}) = \frac{\delta\beta}{1-\delta+\delta\beta}w(\bar{k}) - \frac{\varphi(1-\delta)}{1-\delta+\delta\beta}G(\bar{k}).$$

If $\gamma = 0$ or $\beta = 1$, then we have the standard model: the savings rate out of current output is $\alpha\delta$. If $\gamma > 0$ and $\beta < 1$ (i.e., if there are self-control problems), then the savings rate falls relative to the standard model. As γ goes to infinity, the realized decision rule converges to the decision rule in the Laibson model under log-Cobb functional form restrictions (see Krusell, Kuruşçu, and Smith (2000b)).

[7] In particular, $A_1 = \frac{\alpha-1}{(1-\delta)(1-\alpha\delta)}$ $A_2 = \frac{1}{1-\delta}$, and $\varphi = \frac{1-\alpha}{\alpha}\frac{[(1-\delta)(1+\gamma)+\delta(1+\beta\gamma)]}{(1-\delta)(1+\gamma)}$.

For the log-Cobb model, it is straightforward to compare competitive equilibrium outcomes to those that prevail in autarky (i.e., when consumers simply operate their own technologies). In this case, $W(k) = a + b\log(k)$, where a and b can be determined using a guess-and-verify method.[8] The "realized" decision rule is

$$g(k) = \frac{\alpha\delta}{\alpha\delta + (1-\alpha\delta)\frac{1+\gamma}{1+\beta\gamma}} Ak^{\alpha}$$

and the "temptation" decision rule is

$$\tilde{g}(k) = \frac{\alpha\beta\delta}{1-\alpha\delta+\alpha\beta\delta} Ak^{\alpha}.$$

Unlike in the two-period model, the competitive equilibrium and autarky allocations no longer coincide in the "log-Cobb" infinite-horizon model: in particular, if there are problems of self-control, the savings rate is higher in competitive equilibrium than in autarky. To understand the origin of this result, consider a 3-period model first. In period 2, the temptation consumption is higher in a competitive equilibrium than in autarky (this was shown in Section 3.2). Therefore, the marginal disutility of saving in period 1 that arises because of the increase in temptation in period 2 is lower in the competitive equilibrium, since u is strictly concave. As a result, the savings in a competitive equilibrium will be higher. Welfare, however, is not necessarily higher under competitive equilibrium. For low values of γ, autarky delivers higher welfare than competitive equilibrium, but this ranking is reversed for high values of γ.

5 Policy in the Gul-Pesendorfer Model

In this section, we ask whether government policy can help consumers overcome their self-control problems. We address these questions first in the context of the two-period model. We then turn to the role of policy in the infinite-horizon model.

5.1 Policy in the Two-Period Model

We compare the welfare implications of two kinds of policies: command policy and optimal taxation policy. By command policy, we mean that the government chooses *for* the consumer. In this case, since the consumer's choice set is, in effect, reduced to a single point, issues of temptation and self-control are eliminated. Command policy therefore delivers a first-best outcome: an intertemporal allocation $c^{\star}$ tailored to u (i.e., such that $u(c_1) + \delta u(c_2)$ is maximized).

[8] In particular, $b = \frac{\alpha}{1-\alpha\delta}$.

Command policy is almost certainly infeasible. Instead, the government could attempt to design taxation policy in order to help the consumer with his self-control problems. We consider two kinds of taxation in a competitive equilibrium setting with arbitrary utility function u and production function f (as in Section 3.2, these functions satisfy the usual properties). Specifically, in the first period, there is a proportional tax τ_y on income and a proportional tax τ_i on investment. The consumer's budget set, then is:

$$\begin{aligned} B(k_1, \bar{k}_1, \bar{k}_2) \equiv \{(c_1, c_2) : \exists k_2 : c_1 = (r(\bar{k}_1)k_1 + w(\bar{k}_1))(1 - \tau_y) - (1 + \tau_i)k_2 \text{ and} \\ c_2 = r(\bar{k}_2)k_2 + w(\bar{k}_2)\} \end{aligned}$$

We assume that the government balances its budget in each period. Since the government has no exogenous expenditures to finance, its budget constraint reads: $\tau_y f(\bar{k}_1) = -\tau_i \bar{k}_2$. The consumer's problem, then, is:

$$\max_{(c_1,c_2)\in B_\tau(k_1,\bar{k}_1,\bar{k}_2)} \{2u(c_1) + \delta(1+\beta)u(c_2)\} - \max_{(\tilde{c}_1,\tilde{c}_2)\in B_\tau(k_1,\bar{k}_1,\bar{k}_2)} \{u(\tilde{c}_1) + \beta\delta u(\tilde{c}_2).\} \qquad (11)$$

In equilibrium, $k_2 = \bar{k}_2$ and the tax rates are such that the government's budget balances (in other words, the government has only one free tax instrument at its disposal).

The government's objective is to choose the two tax rates so that an individual's welfare is maximized in equilibrium (subject to the government's budget constraint). Proposition 2 states that, when $\beta < 1$, the government can improve an individual's welfare by imposing a positive tax on income and a negative tax (i.e., a subsidy) on investment.

Proposition 2 In the two-period model, the optimal investment tax τ_i is negative.
Proof: See the Appendix.

With optimal taxation, then, the consumer is induced to save more, so that his intertemporal consumption allocation is tilted more towards the future than in the absence of taxation. At the same, the change in the slope of the consumer's budget constraint reduces (other things equal) the temptation faced by the consumer. The net result is to increase the consumer's welfare.

These results can be made especially clear by imposing log-Cobb functional form restrictions. In this case, we can explicitly compute the competitive equilibrium outcome with optimal taxation. It turns out that the optimal consumption allocation is precisely the one that obtains under the command policy. Figure 7 illustrates this result. Although the command outcome is identical (given log-Cobb assumptions) to the competitive equilibrium outcome with optimal taxation, welfare is still higher under the command outcome because the consumer does not incur a self-control cost.

Taxation in the autarky case will improve welfare as well. Like in the competitive equilibrium, the command policy allocation—that is, the policy that maximizes welfare using the commitment utility function (with $\beta = 1$)—is the best allocation when taxation is allowed. This allocation delivers welfare that lies between that of the command policy (i.e., when the government chooses *for* the consumer) and that of the best tax policy in competitive equilibrium.

5.2 Policy in the Infinite-Horizon Model

When the horizon is infinite, the government's command policy would give the consumer a consumption path that coincides with the one that the consumer would choose if he had no self-control problems and his discount rate were equal to δ. Given "log-Cobb" functional form assumptions, optimal taxation policy in competitive equilibrium has the same qualitative effects as in the two-period model studied in Section 5.1 under log-Cobb functional form assumptions. In particular, the government chooses a subsidy on investment and a tax on income so that the consumer has the same savings rate that he does under the command policy. Thus optimal taxation policy increases welfare, but the consumer is still worse off than under the command policy since he incurs the cost of self-control.

Although perhaps not a clean "policy experiment", suppose that the government could choose between competitive equilibrium and autarky (that is, home production): these decentralizations embody very different amounts of price-taking. Then we saw above that price-taking (assuming no taxes) will be preferable when γ is not too small. With taxation in addition, price-taking is worse, since it delivers higher temptations but the same consumption allocation as under autarky.

6 Conclusion

We try to extend the neoclassical growth model to incorporate self-control problems in the consumption-savings choice à la Gul and Pesendorfer (2000a,b). With a larger cake, the consumer is tempted to eat more, exercises self-control in order not to eat the whole cake, but still ends up eating more than in the absence of the temptation. The model with temptation leads to a significantly more complicated analysis than the standard model. Steady states are no longer straightforward to pin down, and only in the case of isoelastic utility can we arrive at relatively simple algebraic expressions. Outside this class, the steady states fundamentally are supported by the dynamics of temptation, and they need to be solved for jointly with these dynamics. When steady states can be solved for, they depend on the curvature of the utility function, which they do not in the standard model (nor in the Laibson model). We show that global dynamics can be described with a set of functional equations, and that local dynamics can be derived using differentiation of these equations.

The analysis of taxation, for which we provide intuition using a two-period model and then solve using a parametric example allowing closed-form solutions, suggests that the government ought to subsidize investment. This result was not a foregone conclusion; it turns out that the temptation utility is decreased with a subsidy to investment.

Finally, we just wish to point out that one fundamental question remains unanswered in the policy analysis we provide. We find that government policy helps; but given that it helps, why don't agents themselves create "governments"? If living under commands is good for me, why can't I buy it in the market? Or can I? Religion, for example, may be viewed as a market-provided vehicle for alleviating self-control problems. When the market can provide such help, government policy may not be desirable. We need a deeper way of drawing distinctions between what the market can do and the government can do.

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Appendix

Proof of Proposition 1: The two Euler equations are:

$$(1+\gamma)u'(r(\bar{k})k + w(\bar{k}) - g(k,\bar{k})) =$$

$$\delta(1+\beta\gamma)r(g(\bar{k},\bar{k}))((1+\gamma)u'(r(g(\bar{k},\bar{k}))g(k,\bar{k}) + w(g(\bar{k},\bar{k})) - g(g(k,\bar{k}), g(\bar{k},\bar{k})))-$$
$$\gamma u'(r(g(\bar{k},\bar{k}))g(k,\bar{k}) + w(g(\bar{k},\bar{k})) - \tilde{g}(g(k,\bar{k}), g(\bar{k},\bar{k}))))$$

and

$$u'(r(\bar{k})k + w(\bar{k}) - \tilde{g}(k,\bar{k})) =$$
$$\delta\beta r(g(\bar{k},\bar{k}))((1+\gamma)u'(r(g(\bar{k},\bar{k}))\tilde{g}(k,\bar{k}) + w(g(\bar{k},\bar{k})) - g(\tilde{g}(k,\bar{k}), g(\bar{k},\bar{k})))$$
$$-\gamma u'(r(g(\bar{k},\bar{k}))\tilde{g}(k,\bar{k}) + w(g(\bar{k},\bar{k})) - \tilde{g}(\tilde{g}(k,\bar{k}), g(\bar{k},\bar{k})))).$$

Inserting the guesses for $\tilde{g}(k,\bar{k})$ and $g(k,\bar{k})$ into these equations and using the fact that u is isoelastic, we obtain:

$$\frac{(1+\gamma)}{\delta(1+\beta\gamma)r(g(\bar{k},\bar{k}))} =$$

$$(1+\gamma)\left[\frac{(r(\bar{k}')-\lambda(\bar{k}'))\lambda(k)}{r(\bar{k})-\lambda(\bar{k})}\frac{k+\frac{\mu(\bar{k})}{\lambda(\bar{k})}+\frac{w(\bar{k}')-\mu(\bar{k}')}{(r(\bar{k}')-\lambda(\bar{k}'))\lambda(\bar{k})}}{k+\frac{w(\bar{k})-\mu(\bar{k})}{r(\bar{k})-\lambda(\bar{k})}}\right]^{-\sigma}$$
$$-\gamma\left[\frac{(r(\bar{k}')-\tilde{\lambda}(\bar{k}'))\lambda(k)}{r(\bar{k})-\lambda(\bar{k})}\frac{k+\frac{\mu(\bar{k})}{\lambda(\bar{k})}+\frac{w(\bar{k}')-\tilde{\mu}(\bar{k}')}{(r(\bar{k}')-\tilde{\lambda}(\bar{k}'))\lambda(\bar{k})}}{k+\frac{w(\bar{k})-\mu(\bar{k})}{r(\bar{k})-\lambda(\bar{k})}}\right]^{-\sigma}$$

$$\frac{1}{\delta\beta r(g(\bar{k},\bar{k}))} =$$

$$(1+\gamma)\left[\frac{(r(\bar{k}')-\lambda(\bar{k}'))\tilde{\lambda}(k)}{r(\bar{k})-\tilde{\lambda}(\bar{k})}\frac{k+\frac{\tilde{\mu}(\bar{k})}{\tilde{\lambda}(\bar{k})}+\frac{w(\bar{k}')-\mu(\bar{k}')}{(r(\bar{k}')-\lambda(\bar{k}'))\tilde{\lambda}(\bar{k})}}{k+\frac{w(\bar{k})-\tilde{\mu}(\bar{k})}{r(\bar{k})-\tilde{\lambda}(\bar{k})}}\right]^{-\sigma}$$
$$-\gamma\left[\frac{(r(\bar{k}')-\tilde{\lambda}(\bar{k}'))\tilde{\lambda}(k)}{r(\bar{k})-\tilde{\lambda}(\bar{k})}\frac{k+\frac{\tilde{\mu}(\bar{k})}{\tilde{\lambda}(\bar{k})}+\frac{w(\bar{k}')-\tilde{\mu}(\bar{k}')}{(r(\bar{k}')-\tilde{\lambda}(\bar{k}'))\tilde{\lambda}(\bar{k})}}{k+\frac{w(\bar{k})-\tilde{\mu}(\bar{k})}{r(\bar{k})-\tilde{\lambda}(\bar{k})}}\right]^{-\sigma}$$

Note that these two Euler equations must hold for all k. This implies that:

$$\frac{\mu(\bar{k})}{\lambda(\bar{k})} + \frac{w(\bar{k}')-\mu(\bar{k}')}{(r(\bar{k}')-\lambda(\bar{k}'))\lambda(\bar{k})} = \frac{w(\bar{k})-\mu(\bar{k})}{r(\bar{k})-\lambda(\bar{k})} \tag{12}$$

$$\frac{\mu(\bar{k})}{\lambda(\bar{k})} + \frac{w(\bar{k}')-\tilde{\mu}(\bar{k}')}{(r(\bar{k}')-\tilde{\lambda}(\bar{k}'))\lambda(\bar{k})} = \frac{w(\bar{k})-\mu(\bar{k})}{r(\bar{k})-\lambda(\bar{k})} \tag{13}$$

$$\frac{\tilde{\mu}(\bar{k})}{\tilde{\lambda}(\bar{k})} + \frac{w(\bar{k}') - \mu(\bar{k}')}{(r(\bar{k}') - \lambda(\bar{k}'))\tilde{\lambda}(\bar{k})} = \frac{w(\bar{k}) - \tilde{\mu}(\bar{k})}{r(\bar{k}) - \tilde{\lambda}(\bar{k})} \tag{14}$$

$$\frac{\tilde{\mu}(\bar{k})}{\tilde{\lambda}(\bar{k})} + \frac{w(\bar{k}') - \tilde{\mu}(\bar{k}')}{(r(\bar{k}') - \tilde{\lambda}(\bar{k}'))\tilde{\lambda}(\bar{k})} = \frac{w(\bar{k}) - \tilde{\mu}(\bar{k})}{r(\bar{k}) - \tilde{\lambda}(\bar{k})} \tag{15}$$

Imposing these conditions in the Euler equations, we obtain:

$$\begin{aligned}
\frac{1+\gamma}{\delta\left(1+\beta\gamma\right) r\left(\bar{k}'\right)} &= (1+\gamma)\left[\frac{(r(\bar{k}') - \lambda(\bar{k}'))\lambda(\bar{k})}{r(\bar{k}) - \lambda(\bar{k})}\right]^{-\sigma} - \gamma\left[\frac{(r(\bar{k}') - \tilde{\lambda}(\bar{k}'))\lambda(\bar{k})}{r(\bar{k}) - \lambda(\bar{k})}\right]^{-\sigma} \\
\frac{1+\gamma}{\delta\beta r\left(\bar{k}'\right)} &= (1+\gamma)\left[\frac{(r(\bar{k}') - \lambda(\bar{k}'))\tilde{\lambda}(\bar{k})}{r(\bar{k}) - \tilde{\lambda}(\bar{k})}\right]^{-\sigma} - \gamma\left[\frac{(r(\bar{k}') - \tilde{\lambda}(\bar{k}'))\tilde{\lambda}(\bar{k})}{r(\bar{k}) - \tilde{\lambda}(\bar{k})}\right]^{-\sigma}
\end{aligned}$$

Note that we now have 6 equations but only 4 unknowns (i.e., the functions $\tilde{\lambda}$, $\tilde{\mu}$, λ, μ). We can show, however, that equations (12) and (14) imply equations (13) and (15), thereby reducing the number of equations to 4. Q.E.D.

Proof of Proposition 2: The first-order conditions for the competitive consumer's maximization problem are:

$$\begin{aligned}
(1+\tau_i)(1+\gamma)u'(c_1) &= \delta(1+\beta\gamma)r(\bar{k}')u'(c_2) \\
(1+\tau_i)u'(\tilde{c}_1) &= \delta\beta r(\bar{k}')u'(\tilde{c}_2) \\
c_1 &= (r(\bar{k})k + w(\bar{k}))(1-\tau_y) - k'(1+\tau_i) \\
c_2 &= r(\bar{k}')k' + w(\bar{k}') \\
\tilde{c}_1 &= (r(\bar{k})k + w(\bar{k}))(1-\tau_y) - \tilde{k}'(1+\tau_i) \\
\tilde{c}_2 &= r(\bar{k}')\tilde{k}' + w(\bar{k}')
\end{aligned}$$

Let the solution to the first-order conditions be denoted by $k' = g(k, \bar{k}, \tau)$ and $\tilde{k}' = \tilde{g}(k, \bar{k}, \tau)$. We can use the government budget constraint, $-g(\bar{k}, \bar{k}, \tau)\tau_i = f(\bar{k})\tau_y$, to write decision rules as a function only of τ_i. In addition,

$$\begin{aligned}
\tilde{c}_1 &= (r(\bar{k})\bar{k} + w(\bar{k}))(1-\tau_y) - \tilde{k}'(1+\tau_i) \\
&= f(\bar{k}) - \tilde{k}' + (\bar{k}' - \tilde{k}')\tau_i.
\end{aligned}$$

The value function for a typical consumer in competitive equilibrium is:

$$\begin{aligned}
V(\bar{k}, \bar{k}, \tau_i) &= (1+\gamma)u(f(\bar{k}) - \bar{k}') + \delta(1+\beta\gamma)u(f(\bar{k})) - \\
&\quad \gamma\left\{u(f(\bar{k}) - \tilde{k}' + (\bar{k}' - \tilde{k}')\tau_i) + \delta\beta u(r(\bar{k}')\tilde{k}' + w(\bar{k}'))\right\}.
\end{aligned}$$

Taking the derivative of the value function with respect to τ_i and imposing the first-order conditions above, we get

$$\left.\frac{dV}{d\tau_i}\right|_{\tau_i=0} = -\gamma\left(u'(\tilde{c}_1)[\bar{k}' - \tilde{k}'] + \delta\beta u'(\tilde{c}_2)[r'(\bar{k}')\tilde{k}' + w'(\bar{k}')]\frac{d\bar{k}'}{d\tau_i}\right).$$

Again using the first-order conditions, we obtain

$$\frac{d\bar{k}'}{d\tau_i} = \frac{(1+\gamma)u'(c_1)}{(1+\tau_i)(1+\gamma)u''(c_1) + \delta(1+\beta\gamma)f''(\bar{k}')u'(c_2) + \delta(1+\beta\gamma)f'(\bar{k}')^2 u''(c_2)} < 0.$$

This result, together with the facts that $r'(\bar{k}')\tilde{k}' + w'(\bar{k}') = r'(\bar{k}')(\tilde{k}' - \bar{k}')$ and $\bar{k}' - \tilde{k}' > 0$ implies that

$$\left.\frac{dV}{d\tau_i}\right|_{\tau_i=0} < 0.$$

Q.E.D.

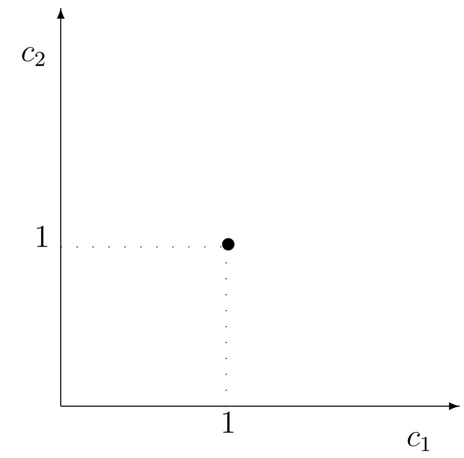

Figure 1

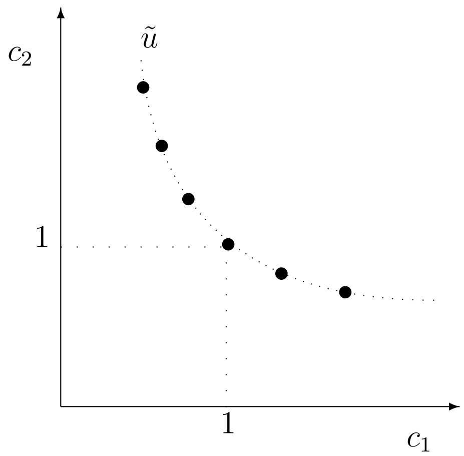

Figure 2

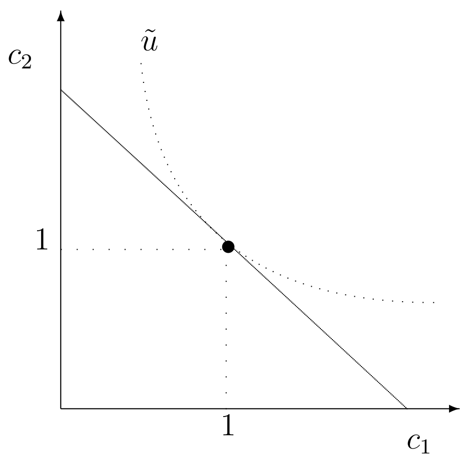

Figure 3

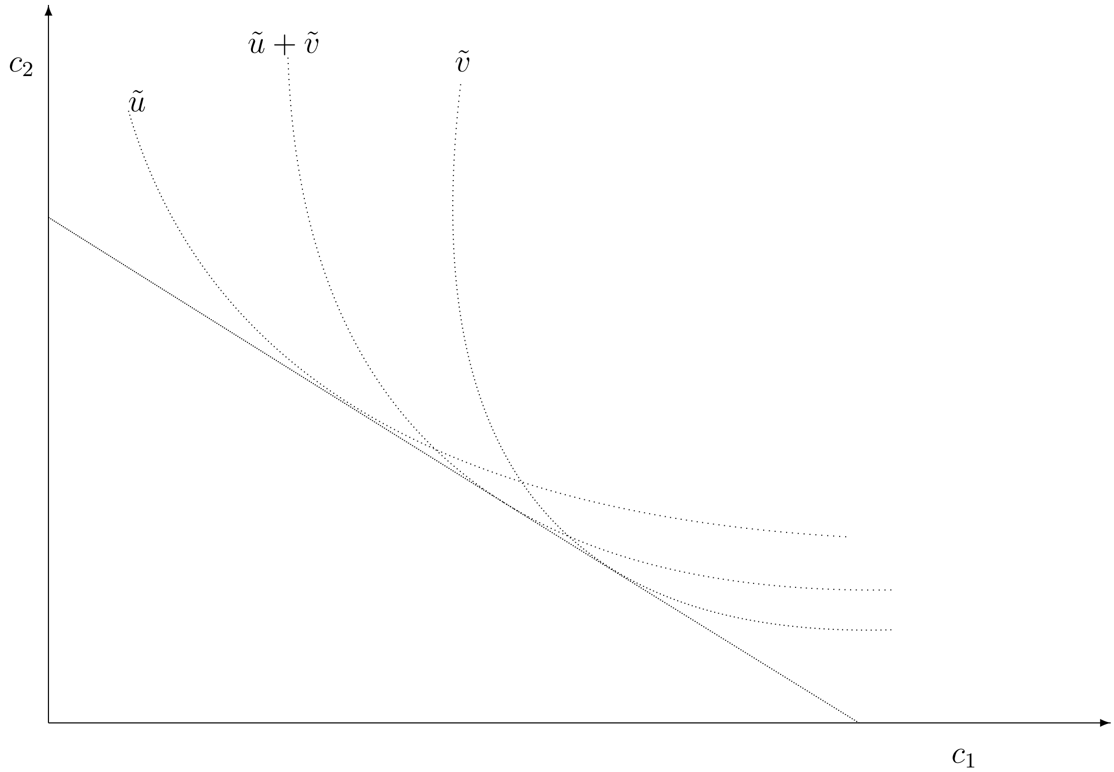

Figure 4

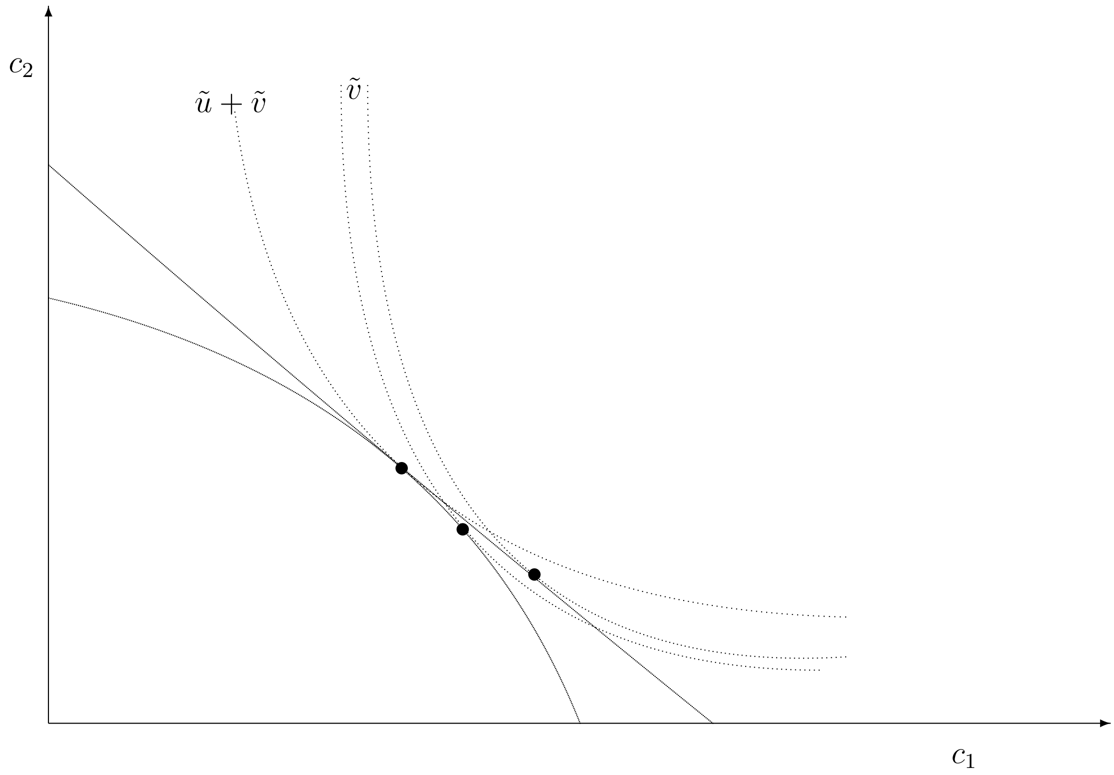

Figure 5

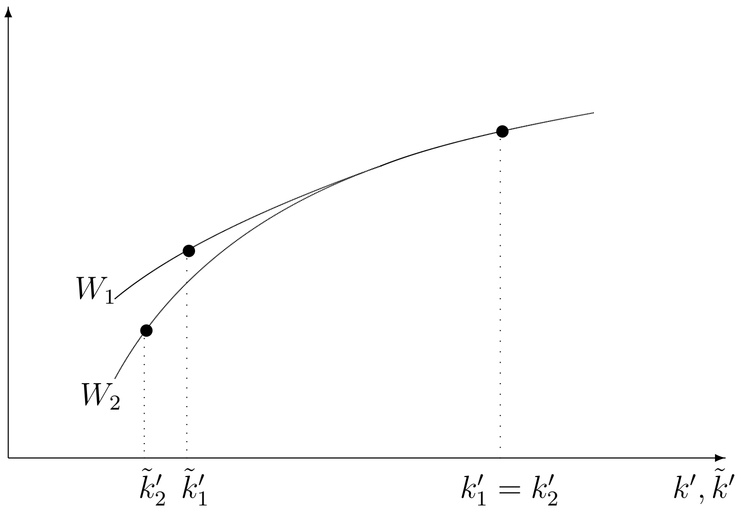

Figure 6

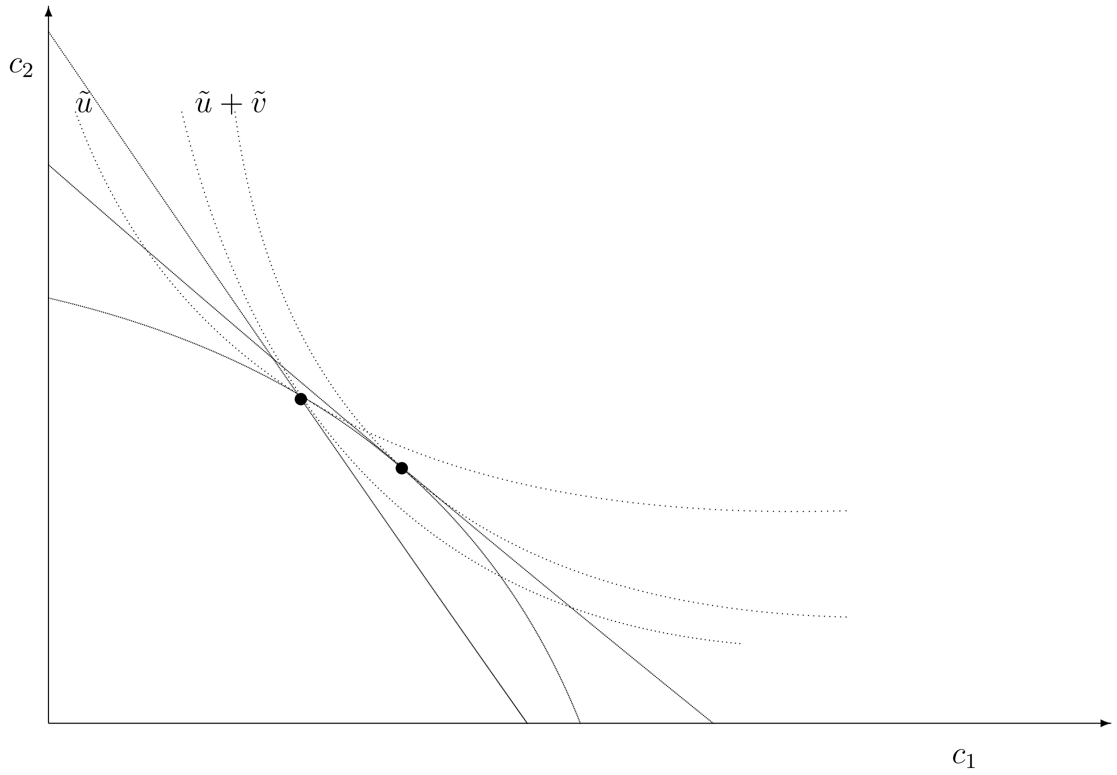

Figure 7