On Second-Best National Saving and Game-Equilibrium Growth [1]

Nearly thirty years ago Frank Ramsey [13] pioneered in a new field of economic theory, which we now call optimal economic growth. In many respects his analysis was quite general and recently his model has been extended to cases of many capital goods, population growth, technological progress and uncertainty.[2] There has, however, been no modification of Ramsey's treatment of preferences.

Ramsey made the remarkable postulate that each generation possesses what we shall call *perfect altruism*. By this we mean that each generation's preference for their own consumption relative to the next generation's consumption is no different from their preference for any future generation's consumption relative to the succeeding generation. This is a *stationarity postulate*: the present generation's preference ordering of consumption streams is invariant to changes in their timing.[3] Thus Ramsey did not admit the possibility that the current generation would assign its own consumption a place of importance somewhat out of proportion to its proximity. In his analysis he allowed *time preference* only of an extraordinarily selfless kind: The pure time-preference or discount rate used in discounting the rate of utility from consumption t years hence is required to be constant with respect to time. A positive discount rate favours the present generation only because of and to the extent of the proximity of its consumption.

Presumably Ramsey was not so optimistic as to believe that the current population in fact experienced a pleasure from the prospect of any future generation's consumption, relative to pleasure from its own consumption, that is diminished only by its sheer futurity. He must have regarded such " preferences " as really an ethic to which all generations ought to subscribe. Indeed he termed positive utility discounting of any sort " ethically indefensible ", though he admitted a constant, positive discount rate into his analysis.[4]

But what if people do not subscribe to this ethic? Then the rate of national saving that is optimal from the standpoint of the present generation is not the Ramsey solution. If a truly democratic government attempts to cater only to the preferences of the individuals who are presently members of the body politic[5], then it is *their* optimum, rather than the Ramsey solution, in which a democratic government will interest itself. (Whether the government needs to compute this optimum instead of relying on certain fiscal rules or principles together with well-functioning markets is a separate matter.) Accordingly, this paper will investigate the optimal saving policy of an " imperfectly altruistic " present generation under various assumptions about future saving behaviour and its control.

Part I sets forth the assumptions concerning preferences and technological consumption possibilities that run throughout the paper. In Part II we analyze the " first-best " optimiz-

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[2] For a survey, some new results and many references to recent work, see Phelps [11] Chapter 5.

[3] This is Postulate 4 in Koopmans' study of ordinal utility functions which exhibit stationarity yet allow the utility discount factor to vary with the magnitude of consumption. (Note that the failure of the present generation's utility function to exhibit such stationarity does not prevent the " stationarity " of a different sort that exists if each generation's preferences are alike.) See Koopmans [5].

[4] Ramsey, [13], p. 543.

[5] This assumption is gaining ground in the theory of optimal economic growth. See, for example, Marglin [7], and Phelps [10].

ation problem that arises when the present generation can commit future generations to save the amounts which the present generation wish them to save. If, however, the present generation lacks the power to commit future generations' decisions, then the saving policies of future generations constitute additional constraints for the present generation and the optimal saving decision of the present generation becomes a problem of " second best ". In Part III we derive the second-best saving policy of the present generation when all future saving income ratios equal an arbitrary constant. Of particular interest here is the question of whether second-best saving is greater or smaller than first-best saving when given future saving is non-optimal from the standpoint of the present generation. (Clearly future generations might save non-optimally in the present generation's view if they pursued certain arbitrary fiscal rules or if they were themselves " maximizing ", imperfect altruists. In either case, the present generation would face a second-best problem—even if it were itself perfectly altruistic.)

Finally, in Part IV, we suppose that all generations are alike in their preferences: they exhibit the same imperfect altruism, the same time preference and so on. We postulate that all generations expect each succeeding generation to choose the saving ratio that is second-best in its eyes. This somewhat game-theoretic model leads to the concept of an " equilibrium " sequence of saving-income ratios having the property that no generation acting alone can do better and all generations act so as to warrant the expectations of the future saving ratios. This equilibrium is compared to the first-best optimum and its intertemporal non-optimality in the Pareto sense is shown.

I. PREFERENCES AND CONSUMPTION POSSIBILITIES

Each generation is supposed to live, save and consume over just one period. These periods are equally spaced and infinite in number. All generations are taken to be equal in size.

The preferences of the present generation are represented by the utility function

$$U = u(C_0)+\alpha\delta u(C_1)+\alpha^2\delta u(C_2)+\ldots, \quad 0<\delta<1, \quad 0<\alpha<1, \qquad \ldots(1)$$

where C_0 is the consumption of the present generation, C_1 the consumption of the next, and so on. The " period utilities ", $u(C_t)$, are identical functions of current consumption but the " utility " of consumption t periods hence is " discounted " by the factor $\delta\alpha^t$. The constant factor α reflects time preference or myopia while the constant factor δ, applied equally to all future generations regardless of timing, is a measure of the degree to which the present generation values other peoples' consumption relative to their own. " Imperfect altruism " here denotes $0<\delta<1$ while " perfect altruism " means $\delta = 1$. All the equations shown here are valid for any positive δ but for simplicity of exposition we suppose $\delta<1$. In contrast, $\alpha<1$ is often necessary for the existence of the various optima considered here.

In this paper we confine ourselves to period utilities which exhibit a constant elasticity of marginal utility:

$$u'(C_t) = C_t^{-\rho}, \quad \rho>0. \qquad \ldots(2)$$

If the present generation's preferences satisfy (1) and can be represented by an indifference map which is homothetic to the origin, then the " period utility functions " must satisfy (2).[1]

[1] Since any increasing monotonic transformation of U yields an equivalent representation of preferences there is no loss of generality in replacing each u by v, where $v = a+bu$, $b>0$. This justifies the omission of additive and multiplicative constants in (2).

It might be objected that the present generation has no interest in the time profile of future consumption; but surely it would save less if it thought that the next generation would run its capital stock to zero, leaving subsequent generations impoverished. The assumption that the intertemporal utility function is additive is equivalent to the assumption that our marginal rate of substitution of consumption in period i for consumption in period j is independent of the level of consumption in period k. An additivity assumption, together with a homogeneity assumption, implies that the intertemporal utility function is either of the form $\Sigma a_t C_t^{1-\rho}$ or $\Sigma a_t \log C_t$. Our assumptions about " discounting " and " imperfect altruism " imply $a_0 = 1$ and $a_t = \alpha^t\delta$, $t \neq 0$.

There are three types of period-utility functions satisfying (2); these correspond to $\rho<1$, $\rho = 1$ and $\rho>1$. For $\rho \neq 1$ we have

$$u(C_t) = \frac{1}{1-\rho} C_t^{1-\rho} + b. \quad ...(2a)$$

For $\rho = 1$ we have (neglecting the constant of integration)

$$u(C_t) = \log C_t \quad ...(2b)$$

When $\rho<1$, $u(0) = b$ and $u(\infty) = \infty$. When $\rho = 1$, $u(0) = -\infty$ and $u(\infty) = \infty$. When $\rho>1$, $u(0) = -\infty$ and $u(\infty) = b$.

As for the production side, we postulate a constant marginal productivity of capital, $\lambda-1>0$, and no depreciation. Capital, K, is consumable, like rabbits. The process of capital growth is described by the relation

$$K_{t+1} = \lambda(K_t - C_t), \quad \lambda>1 \quad ...(3)$$

with the initial capital stock historically given:

$$K_0 = K_o, \quad K_o>0. \quad ...(4)$$

The variable s, the present " saving ratio ", is defined by

$$s = \frac{K_0 - C_0}{K_0}, \quad 0 \leqq C_o \leqq K_o \quad ...(5)$$

and σ_t will denote the " saving ratio " t periods from the present:

$$\sigma_t = \frac{K_t - C_t}{K_t}, \quad 0 \leqq C_t \leqq K_t, \quad t = 1, 2, 3, ... \quad ...(6)$$

Hence

$$K_1 = \lambda s K_0 \quad ...(7)$$

$$K_{t+1} = \lambda\sigma_t K_t, \quad t = 1, 2, 3, ... \quad ...(8)$$

If the σ's are all equal to some constant σ we have geometric growth of capital and consumption beginning in period 1:

$$K_t = \lambda^{t-1}\sigma^{t-1}K_1, \quad t = 1, 2, 3, ... \quad ...(9)$$

$$C_t = (1-\sigma)\lambda^{t-1}\sigma^{t-1}K_1 \quad ...(10)$$

or

$$C_t = (1-\sigma)s\lambda^t\sigma^{t-1}K_0. \quad ...(11)$$

This leads to geometric growth (or decay) of the undiscounted marginal period-utilities by virtue of (2) which, as we shall see, is a property of considerable convenience. (There exists another production model in which consumption cannot exceed " current production " that also has this convenient property; the present model is merely the simplest available.)

Pigovian income, Y_t, defined as the consumption level which keeps the capital stock " intact " ($K_{t+1} = K_t$) is defined by

$$K_t = \lambda(K_t - Y_t) \quad ...(12)$$

whence

$$Y_t = \frac{\lambda-1}{\lambda} K_t. \quad ...(13)$$

If capital is not to decrease we must have $C_t \leqq Y_t$. It is readily shown that

$$\sigma_t \geqq \frac{1}{\lambda} \text{ if and only if } C_t \leqq Y_t, \quad ...(14)$$

but it would be artificial to impose such a constraint in the present model. Note finally that constancy of our σ_t is equivalent to constancy of the more familiar ratio $(Y_t - C_t)/Y_t$.

II. THE FIRST-BEST OPTIMUM

Consider now the first-best optimum. This would be realized if, for example, the present generation could control not only their own saving ratio but future saving ratios as well.

We observe that whatever the present saving ratio, an optimal programme must have the property that the σ_t's are chosen optimally with reference to the current capital stocks inherited from the past; one can determine each optimal σ_t as a function of the corresponding K_t, finding the optimal s function at the end of the problem once the policy functions governing the σ_t's have been determined. Hence, if we write

$$U = u(C_0) + \delta\alpha V \qquad \ldots(15)$$

where

$$V = \sum_{t=1}^{\infty} \alpha^{t-1} u(C_t), \qquad \ldots(16)$$

our problem is first to find the future consumption policies which maximise V (and express these in terms of σ_t). Let $V_*(K_1)$ denote the maximized value of V for given K_1. The problem can then be formulated in the usual manner of dynamic programming[1] by the recursive relation (suppressing time subscripts)

$$V_*(K) = \max_{0 \leq C \leq K} \{u(C) + \alpha V_*(\lambda(K-C))\} \qquad \ldots(17)$$

in the unknown function $V_*(K)$. Since the V-maximization problem is an infinite-horizon one and V is " stationary " in Koopmans's sense (the discount factor declines geometrically), the optimal consumption policy $C_t = C_*(K_t)$ is independent of time. (The single asterisk denotes first-best optimality.)

By solving finite N-period processes for the current C_* function and the current V_* function and by taking limits as N approaches infinity, one can find the functions $C_*(K)$ and $V_*(K)$. In the case $\rho \neq 1$, using (2a), we have

$$V_*(K) = \frac{b}{1-\alpha} + \frac{1}{1-\rho}\left[\frac{1}{1-(\alpha\lambda^{1-\rho})^{1/\rho}}\right]^{\rho} K^{1-\rho} \qquad \ldots(18)$$

$$C_*(K) = [1-(\alpha\lambda^{1-\rho})^{1/\rho}]K, \qquad \ldots(19)$$

whence the first-best future saving ratio, σ_*, is a unique constant, independent of K:

$$\sigma_* = (\alpha\lambda^{1-\rho})^{1/\rho}, \qquad \ldots(20)$$

There exists such an optimum if and only if $V_*(K)$ is finite (neglecting the b term which can always be set equal to zero). It is easy to show from (18) that this requires

$$(\alpha\lambda^{1-\rho})^{1/\rho} < 1 \text{ or equivalently, } \alpha\lambda^{1-\rho} < 1. \qquad \ldots(21)$$

If this inequality is not satisfied, every $\sigma < 1$ is inferior to a σ closer to unity, and, since there is no " closest σ ", there is no optimum σ. The existence condition then is that the calculated $\sigma_* < 1$. Note that if $\rho < 1$, condition (21) is stronger than the condition that $\alpha < 1$. If $\rho > 1$ this condition is weaker.

In the logarithmic case, $\rho = 1$, we find, using (2b),

$$V_*(K) = \frac{\log K}{1-\alpha} + \frac{1}{1-\alpha}\log(1-\alpha) + \frac{\alpha}{(1-\alpha)^2}\log(\alpha\lambda) \qquad \ldots(22)$$

$$C_*(K) = (1-\alpha)K \qquad \ldots(23)$$

[1] See Bellman [2].

whence σ_* is again independent of K:

$$\sigma_* = \alpha. \qquad \text{...(24)}$$

Note that (24) is a special case of (20) for $\rho = 1$ so that (20) is our general formula for σ_*. In this case an optimum exists if and only if $\alpha < 1$, which is postulated.[1] That these formulae are indeed solutions to the functional equation (17) the reader can easily verify by showing that the formulae satisfy

$$V_*(K) = u(C_*(K)) + \alpha V_*[\lambda(K - C_*(K))]. \qquad \text{...(25)}$$

There is no doubt that the solution is unique so that the above formulae are the only correct ones.

The remaining problem is to optimize with respect to the present saving ratio, given that future saving ratios are to be set at their optimal value, σ_*. Thus we want to maximize U, as given below, with respect to s:

$$U = u[(1-s)K_0] + \delta\alpha V_*[\lambda s K_0]. \qquad \text{...(26)}$$

By virtue of the strict concavity of u and V_*, the stationary value at which $\partial U/\partial s = 0$ is a unique maximum. We shall let s_* denote the maximizing value of s; the single asterisk denotes the fact that it is the *first-best* present saving ratio, the appropriate present saving ratio when the present generation can control the future saving ratios in its own interest.

Using (2), (18), (22) and

$$0 = -K_0 u'[(1-s_*)K_0] + \delta\alpha\lambda K_0 V_*'(\lambda s_* K_0) = \frac{\partial U}{\partial s}, \qquad \text{...(27)}$$

we calculate that

$$\left(\frac{1-s_*}{s_*}\right)^{-\rho} = \delta\left(\frac{(\alpha\lambda^{1-\rho})^{1/\rho}}{1-(\alpha\lambda^{1-\rho})^{1/\rho}}\right)^{\rho} \qquad \text{...(28a)}$$

or equivalently

$$s_* = \frac{1}{1+\delta^{-1/\rho}\left(\dfrac{(\alpha\lambda^{1-\rho})^{1/\rho}}{1-(\alpha\lambda^{1-\rho})^{1/\rho}}\right)^{-1}}, \qquad \text{...(28b)}$$

which are valid for all $\rho > 0$. From (28b) we see that $0 < s_* < 1$ so the maximum is an interior one—on the condition, of course, that an optimum exists, hence that (21) is satisfied.

It is immediately apparent from (28a) and (20) that s_* would equal σ_* if δ were equal to one; in that case we would have the standard Ramsey problem, with stationarity, so that present saving would be the same function of current capital as future saving and, since our special utility function makes the saving ratios constants, all present and future saving ratios would be equal.

In our model, with $\delta < 1$, one sees from (28a) and (20) that

$$s_* < \sigma_*. \qquad \text{...(29)}$$

Imperfect altruism causes the present generation to choose a present saving ratio that is smaller than the future saving ratio it would like future generations to select. It can easily be verified that s_* increases monotonically with δ and that $\partial\sigma_*/\partial\lambda$ and $\partial s_*/\partial\lambda$ have the same sign as $1-\rho$.

III. THE SECOND-BEST OPTIMUM FOR ARBITRARY σ

We suppose now that future saving behaviour is beyond the present generation's control. In particular, we postulate that the future σ's are known constants and are equal:

$$\sigma_t = \sigma = \text{constant}, \quad 0 < \sigma < 1, \quad \text{for all } t \geqq 1. \qquad \text{...(30)}$$

[1] The above formulae are presented for the case in which λ is uncertain in Phelps [9].

The inequalities imply that consumption will be positive in all future periods if $K_1 > 0$. It is conceivable that the economy might exhibit constancy of the future saving ratio through the interaction of certain government fiscal policies with certain private saving propensities. As we shall show in the subsequent section, such behaviour could also come from certain maximizations by future generations. We note that the formulae of this section are valid for $\delta = 1$ as well as $\delta < 1$ on the part of the present generation.

Using (1) and (11) one sees that the second-best present saving ratio, to be denoted s_{**}, maximizes

$$U = u[(1-s)K_0] + \alpha\delta u[(1-\sigma)\lambda s K_0] + \alpha^2\delta u[(1-\sigma)\lambda^2\sigma s K_0] + \ldots + \alpha^t\delta u[(1-\sigma)\lambda^t\sigma^{t-1}sK_0] + \ldots \qquad \ldots(31)$$

with respect to s subject to (30) and the constraint $0 \leqq s \leqq 1$ in (5). Upon calculating the partial derivative $\partial U/\partial s$ and substituting the marginal utility formula in (2) we obtain

$$\frac{\partial U}{\partial s} = [-(1-s)^{-\rho} + \delta(1-\sigma)^{1-\rho}s^{-\rho}\sigma^{\rho-1}M]K_0^{-\rho} \qquad \ldots(32)$$

where

$$M = \alpha\lambda^{1-\rho}\sigma^{1-\rho} + (\alpha\lambda^{1-\rho}\sigma^{1-\rho})^2 + \ldots + (\alpha\lambda^{1-\rho}\sigma^{1-\rho})^t + \ldots.$$

This infinite series converges if and only if

$$\alpha\lambda^{1-\rho}\sigma^{1-\rho} < 1. \qquad \ldots(33)$$

On that condition we have

$$M = \frac{\alpha\lambda^{1-\rho}\sigma^{1-\rho}}{1-\alpha\lambda^{1-\rho}\sigma^{1-\rho}}. \qquad \ldots(34)$$

Equating the derivative in (32) to zero and using (34) yields our basic equation

$$\left(\frac{1-s_{**}}{s_{**}}\right)^{-\rho} = \delta\left(\frac{1-\sigma}{\sigma}\right)^{1-\rho}\left(\frac{\alpha\lambda^{1-\rho}\sigma^{1-\rho}}{1-\alpha\lambda^{1-\rho}\sigma^{1-\rho}}\right) \qquad \ldots(35a)$$

or equivalently

$$s_{**} = \frac{1}{1+\delta^{-1/\rho}\left(\frac{1-\sigma}{\sigma}\right)^{(1-\rho)/-\rho}\left[\frac{\alpha\lambda^{1-\rho}\sigma^{1-\rho}}{1-\alpha\lambda^{1-\rho}\sigma^{1-\rho}}\right]^{-1/\rho}}. \qquad \ldots(35b)$$

Our use of the double asterisk in (35) indicates that the value of s which satisfies this equation is the second-best value of s. That s_{**} is utility-maximizing rather than minimizing follows from the fact that $\partial^2 U/\partial s^2 < 0$. The maximum is clearly unique and an interior one. The common-sense explanation of the latter is that when $s = 1$ the marginal utility of present consumption is infinite while future marginal utilities are finite (and their sum converges) so $s = 1$ cannot be optimal; similarly, when $s = 0$ future marginal utilities are infinite.

The convergence condition in (33) is thus sufficient for the existence of this (second-best) optimum. But it is not always a necessary condition. If (33) does not hold—it must hold when $\rho = 1$, given $\alpha < 1$—then total utility diverges either to plus infinity (when $\rho < 1$) or to minus infinity (when $\rho > 1$) for all s. Let us however adopt the over-taking criterion according to which one policy is preferred to another if it produces greater cumulative utility over T periods for every T greater than some $T^\circ \geqq 1$, and according to which a feasible decision is optimal if it is preferred or indifferent to all others.[1] On that criterion, when (33) does not hold, every increase of s, $s < 1$, is an improvement. If $\rho < 1$, $s = 1$ is best of all since such a policy will " overtake " any policy of $s < 1$; thus the second-best optimum exists in this case and gives $s_{**} = 1$. If $\rho > 1$, the policy $s = 1$ gives a present-period utility of minus infinity by (2) and hence cannot overtake policies making $s < 1$; in

[1] See Weizsäcker [14] and Atsumi [1]. An exposition is contained in Phelps [11].

this case there exists no optimum since there is no value of s which is nearest to one yet not equal to it.

As a matter of notation, let $F(\sigma, \delta, \alpha, \rho, \lambda)$ denote the right-hand side of (35b). Then our results can be stated as follows:

$$s_{**} = \begin{cases} F(\sigma, \delta, \alpha, \rho, \lambda) \text{ if } \alpha\lambda^{1-\rho}\sigma^{1-\rho} < 1, \\ 1 \text{ if } \alpha\lambda^{1-\rho}\sigma^{1-\rho} \geqq 1 \text{ and } \rho < 1 \\ \text{Does not exist otherwise.} \end{cases} \quad \ldots(36)$$

In what range must σ lie in order to satisfy the convergence condition in (33)? If $\rho = 1$, the condition reduces to $\alpha < 1$ so that all values of σ satisfy the condition. If $\rho \neq 1$ we solve for the value of σ, denoted by $\bar{\sigma}$, which gives equality in (33), i.e., the σ value which just fails to satisfy the convergence condition:

$$\bar{\sigma} = (\alpha\lambda^{1-\rho})^{-1/(1-\rho)}. \quad \ldots(37)$$

Then, if $\rho < 1$, convergence will occur if and only if $\sigma < \bar{\sigma}$. We note in this case that

$$\bar{\sigma} > 1 \text{ if and only if } \alpha\lambda^{1-\rho} > 1 \text{ when } \rho < 1. \quad \ldots(38)$$

Hence, if this latter inequality holds we have convergence for all σ, $0 < \sigma < 1$. Note that this condition is identical to the condition for the existence of the first-best optimum.

Figure 1 illustrates the case with $\bar{\sigma} > 1$ so that convergence occurs and the F function exists for all admissible σ. In Figure 2 we illustrate the $\bar{\sigma} < 1$ case in which F exists only for $0 < \sigma < \bar{\sigma}$ and $s_{**} = 1$ for σ such that $\bar{\sigma} \leqq \sigma < 1$. (The case $\bar{\sigma} = 1$ requires separate treatment which we omit.)

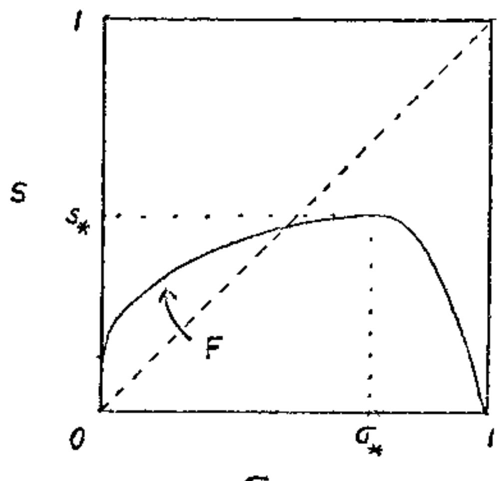

FIGURE 1

$\rho < 1, \bar{\sigma} > 1$

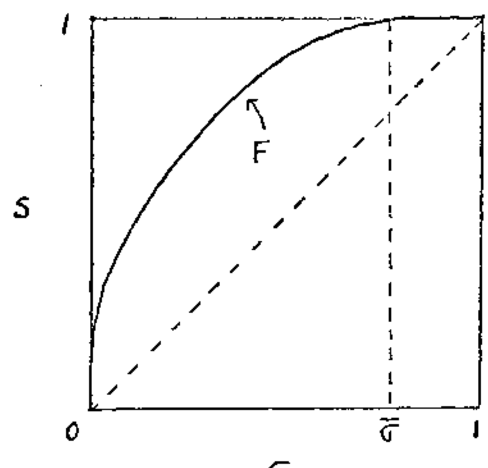

FIGURE 2

$\rho < 1, \bar{\sigma} < 1$

If $\rho > 1$ convergence will occur if and only if $\sigma > \bar{\sigma}$ where

$$0 < \bar{\sigma} < 1 \text{ when } \rho > 1. \quad \ldots(39)$$

This is illustrated in Figure 3 where s_{**} exists only in the interval $\bar{\sigma} < \sigma < 1$.

Note that, when $\rho > 1$, $\bar{\sigma} < \lambda^{-1}$ so that, by (14), capital will be shrinking toward zero for all $\sigma \leqq \bar{\sigma}$; this makes the region of Figure 3 in which no optimum exists one of little interest. In contrast, when $\rho < 1$, $\bar{\sigma} > \lambda^{-1}$ so the region in Figure 2 where $s_{**} = 1$ is one of future capital growth (as is the adjacent interval between λ^{-1} and $\bar{\sigma}$).

Finally, Figure 4 illustrates the logarithmic case with $\rho = 1$ where convergence occurs, so that the F function is defined, for all σ.

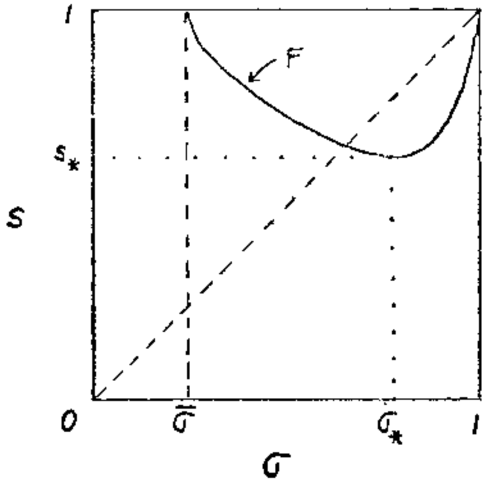

FIGURE 3

$\rho > 1$

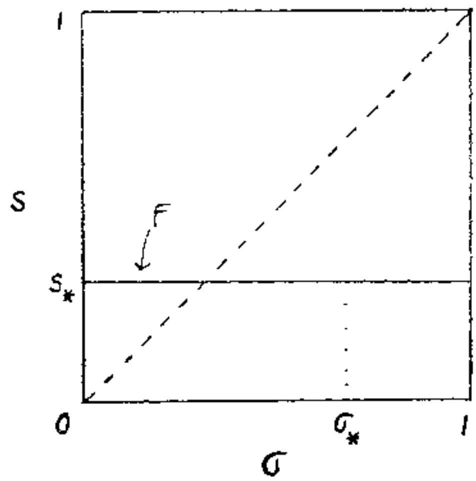

FIGURE 4

$\rho = 1$

We shall now discuss the interesting, essential properties of these diagrams and compare second-best to first-best saving behaviour.

The logarithmic case is simple. Letting $\rho = 1$ we obtain from (35b) the result

$$s_{**} = \frac{\delta\alpha}{1-\alpha(1-\delta)} \text{ for all } \sigma. \qquad \ldots(40)$$

Thus s_{**} is independent of σ in this special case. It is therefore clear, as a comparison of (28b) and (35b) confirms, (using $\rho = 1$), that $s_{**} = s_*$. Hence a departure of σ from its optimal value, σ_*,—optimal from the viewpoint of the present generation—while reducing the present generation's utility, should not cause any adjustment of present saving. This logarithmic case must be added to the curious list of examples in which first-best and second-best decisions do not differ.[1]

Consider now the other cases, where $\rho \neq 1$. It is straightforward to prove that the F function approaches the boundary values shown in Figures 1-3 as σ approaches the values zero, $\bar{\sigma}$, and one.

As for the slope, we calculate

$$\frac{\partial F}{\partial \sigma} = \left(\frac{1-\rho}{\rho}\right)\frac{s_{**}(1-s_{**})}{\sigma}\left[\frac{1}{1-\alpha\lambda^{1-\rho}\sigma^{1-\rho}} - \frac{1}{1-\sigma}\right]. \qquad \ldots(41)$$

Of course, this derivative is meaningful only for σ such that the convergence condition, $\alpha\lambda^{1-\rho}\sigma^{1-\rho} < 1$, is satisfied. (41) shows that $\partial F/\partial\sigma = 0$ if and only if

$$\sigma = \alpha\lambda^{1-\rho}\sigma^{1-\rho} \text{ or equivalently, } \sigma = (\alpha\lambda^{1-\rho})^{1/\rho}. \qquad \ldots(42)$$

Recalling (20), we see therefore that a stationary value occurs at $\sigma = \sigma_*$, the " optimal " value of σ, and only there. If σ_* exists, the convergence condition must be satisfied at

[1] These cases have been characterized in terms of " separability ". See Davis and Whinston [4], and Pollak [12].

least for σ in the neighbourhood of σ_*, so that there must be a stationary value of F at $\sigma = \sigma_*$. Recalling (21), σ_* must exist if $\rho > 1$ (Figure 3) and, when $\rho < 1$, if $\bar{\sigma} > 1$ (Figure 1). Since σ_* is unique, the stationary value is unique if it exists, which is to say, if σ_* exists.

The insensitivity of s_{**} to σ in the neighbourhood of σ_* may come as a surprise to those who would have expected F always to be monotone in σ. Analogous results arise, however, in short-run, least-cost production theory.

We can now deduce that the stationary value, if it exists, must occur below the 45-degree line. For it is intuitively obvious and easy to show that

$$F(\sigma_*, \delta, \alpha, \lambda, \rho) = s_*; \qquad \ldots(43)$$

i.e., when $\sigma = \sigma_*$, the second-best saving ratio is equal to the first-best ratio that would be chosen were the present generation able to choose σ. Since $s_{**} = s_*$ at $\sigma = \sigma_*$ and $s_* < \sigma_*$ [(29)] we have

$$F(\sigma_*, \delta, \alpha, \rho, \lambda) < \sigma_*. \qquad \ldots(44)$$

The stationary value is a maximum or a minimum according as $1-\rho$ is positive or negative. If $\rho < 1$ then (41) shows that $\partial F/\partial\sigma$ is positive if $\sigma < \alpha\lambda^{1-\rho}\sigma^{1-\rho}$—which is to say, if $\sigma < \sigma_*$—and negative if $\sigma > \alpha\lambda^{1-\rho}\sigma^{1-\rho}$, i.e., $\sigma > \sigma_*$; thus the stationary value is a maximum in this case. Similarly, if $\rho > 1$, the stationary value must be a minimum. (Actually these results follow simply from the uniqueness of the stationary value and the values of the end-points of the F function.) These results explain the shape of the F functions in Figures 1 and 3.

We now consider the case of $\rho < 1$ when σ_* does not exist. This means that $\alpha\lambda^{1-\rho} > 1$. (This inequality can occur only if $\rho < 1$.) In this case $\partial F/\partial\sigma$ is clearly positive for all σ such that $\sigma < \alpha\lambda^{1-\rho}\sigma^{1-\rho}$ or $\sigma < (\alpha\lambda^{1-\rho})^{1/\rho}$, hence for all admissible σ for which F is defined. But when $\alpha\lambda^{1-\rho} > 1$ then $\bar{\sigma} < 1$ by (37) so that F is defined only for $\sigma < \bar{\sigma}$. This case is illustrated in Figure 2.

The significance of the inverted-U-shaped curve in Figure 1, with its maximum at $\sigma = \sigma_*$, is obvious. It means that $s_{**} < s_*$ for all $\sigma \neq \sigma_*$. When $\rho < 1$, the non-optimality of future σ in our eyes should cause us to save less than we would if we could impose our desires on future generations. When $\rho > 1$, as in Figure 3, $s_{**} > s_*$ for all $\sigma \neq 1$ so the non-optimality of σ is a reason for saving more. (Of course, in Figure 2 no first-best optimum exists so no comparison of first- and second-best present saving can be made.)

It can be shown that $\partial F/\partial\lambda$ has the same sign as $1-\rho$. Hence, where both first- and second-best optima exist, so that comparisons are possible, a divergence of σ from its optimal value will decrease (increase) optimal present saving if and only if an increase of λ would increase (decrease) optimal present saving. The ubiquitous conflict between income and substitution-effects will produce one pair of results or the other according as the marginal utility in (2) is elastic or inelastic.

It can also be shown that $\partial F/\partial\delta$ is everywhere positive, meaning that an increase of altruism will increase s_{**} (where an interior maximum exists) for any given σ.

IV. " EQUILIBRIUM " SAVING IN THE COURNOT-NASH SENSE

The concept and calculation of the second-best optimum is of interest even if that analysis does not explain actual national saving because society as a whole has no notion of such an optimum. Let us suppose now that the present society (or eventually some generation) acquires the notion of the optimum and that it becomes a conscious, calculating maximizer. Then the present generation will want to know what the future saving ratios are going to be, for its optimal s is not generally independent of σ.

A theory of future saving ratios is suggested by the observation that if the present generation has eaten from the fruit of knowledge, it is reasonable for this generation to expect that subsequent generations will likewise seek to optimize (in *their* eyes), and similarly for each future generation. Hence our problem is to look for a sequence (or sequences) of saving ratios each one of which is second-best from the point of view of the generation that chooses it. Such a sequence will be called an *equilibrium*.

We shall suppose that the consumption-possibility relation implicit in (3) is known and common to all generations. In addition, we postulate that the preferences of each generation for its own consumption and consumptions one, two, three . . . periods subsequent to it are identical to such preferences for every other generation; i.e., each generation has the same imperfect altruism, the same time preference and the same u function. Thus the next generation maximizes

$$u(C_1)+\alpha\delta u(C_2)+\alpha^2\delta u(C_3)+\dots$$

(subject to the subsequent saving ratios that it takes as given), the following generation similarly maximizes

$$u(C_2)+\alpha\delta u(C_3)+\alpha^2\delta u(C_4)+\dots$$

and so on, where the subscripts denote the dates of the consumptions. There is still an infinite number of periods.

Let us note that this problem possesses " stationarity " in a relevant sense (even though every generation's preferences are non-stationary in Koopmans's sense): If the present generation thinks it faces future saving ratios ($\sigma_1 = x_1$, $\sigma_2 = x_2$, ...) and, say, the next generation thinks (not always compatibly) that it faces the identical sequence of saving ratios ($\sigma_2 = x_1$, $\sigma_3 = x_2$, ...) then they will adopt the same second-best policy, independently of the fact that their dates in history differ. Since a second-best policy makes the saving ratio independent of current capital stocks they will adopt the same saving ratio—even if, unlike our earlier assumption, the future σ's are unequal.

It will now be clear that there may very well exist at least one equilibrium having the simple form that all the saving ratios are equal. Such an equilibrium exists if there is a number, say $\hat{\sigma}$, such that, if every generation expects all subsequent generations to choose a saving ratio equal to $\hat{\sigma}$, every generation will find that its own second-best saving ratio is equal to $\hat{\sigma}$. Each generation's assumption that subsequent generations will save the fraction $\hat{\sigma}$ of their respective capital stocks is self-warranting in that if the generations make this assumption they will act so as to validate it. The resulting sequence of saving ratios is an " equilibrium " in the sense (customary in other contexts) that expectations are fulfilled—albeit posthumously. It is also an equilibrium in the game-theoretic sense, used previously, that *ex post facto* no generation acting alone could have increased its total utility, given the saving policies of the other generations.

Thus we say that a sequence of equal saving ratios, $s = \sigma_1 = \sigma_2 = \dots = \sigma_t = \dots = \hat{\sigma}$, is an equilibrium one if and only if

$$\hat{\sigma} = F(\hat{\sigma}, \delta, \alpha, \rho, \lambda) \qquad \dots(45)$$

Such a " fixed point " occurs at the intersection of the F function with the 45-degree line in Figs. 1-4. In one case, as we shall show, there may be two such fixed points or none.

This concept of equilibrium was, of course, discussed (with reference to duopoly) by Cournot [3] in terms of the intersection of " reaction curves " such as our F function. Nash [8] in the past decade proved the existence of at least one " equilibrium point " in n-person, non-cooperative games in which each player has available to him a finite set of pure strategies—where an equilibrium point is a collection of strategies (possibly mixed strategies), one for each player, such that no player is able to increase his payoff when the others hold their strategies fixed.[1] The type of game here clearly differs somewhat from that studied by Nash. Nevertheless the equilibrium concept here does appear to be essentially that used by Cournot, Nash and other game theorists; hence the term " Cournot-Nash equilibrium " in this part title. Such an equilibrium is not necessarily of the sort customarily meant by many growth theorists.

[1] Nash [8]. For a survey of non-cooperative games, see especially Chapters 5 and 7 in Luce and Raiffa [6].

Using (45) and thus replacing s_{**} and σ by $\hat{\sigma}$ in (35) yields the following equation determining $\hat{\sigma}$

$$\frac{\hat{\sigma}}{1-\hat{\sigma}} = \delta\left(\frac{\alpha\hat{\lambda}^{1-\rho}\hat{\sigma}^{1-\rho}}{1-\alpha\lambda^{1-\rho}\hat{\sigma}^{1-\rho}}\right) \qquad \ldots(46a)$$

or equivalently

$$\hat{\sigma}^{\rho} = \alpha\lambda^{1-\rho}[\delta+(1-\delta)\hat{\sigma}]. \qquad \ldots(46b)$$

We shall now briefly discuss the existence and uniqueness of such fixed points. We then compare the fixed point(s) with the first-best optimum and test for Pareto-optimality.

If $\rho>1$ and $\sigma>1$ it is clear that the F function (Fig. 1) must intersect the 45-degree line at least once. In fact, we shall show that in this case the F function intersects the 45-degree line only once, so that the fixed point is unique. If $\rho<1$ and $\bar{\sigma}>1$, the F function need not intersect the 45-degree line at all (Fig. 2); but we shall show that it is also possible for the F function to intersect the 45-degree line twice or to be tangent to the 45-degree line for some σ. If $\rho>1$, the F function (Fig. 3) clearly intersects the 45-degree line at least once. We shall show that in this case there is always exactly one fixed point. In the logarithmic case, $\rho = 1$, there clearly exists a unique fixed point (Fig. 4).

To examine the existence and uniqueness of the fixed point of the F function, we define two new functions. We let $L(\sigma)$ denote the left-hand side of (46b) and $R(\sigma)$ the right hand side:

$$L(\sigma) = \sigma^{\rho}, \qquad \ldots(47)$$

$$R(\sigma) = \delta\alpha\lambda^{1-\rho}+\alpha\lambda^{1-\rho}(1-\delta)\sigma. \qquad \ldots(48)$$

A value of σ is a fixed point if and only if it is admissible (between the zero and one) and $L(\sigma) = R(\sigma)$. We begin by calculating the first and second derivatives of L and R:

$$L'(\sigma) = \rho\sigma^{\rho-1}, \quad L''(\sigma) = \rho(\rho-1)\sigma^{\rho-2}, \qquad \ldots(49)$$

$$R'(\sigma) = \alpha\lambda^{1-\rho}(1-\delta), \quad R''(\sigma) = 0. \qquad \ldots(50)$$

If $\rho<1$ and $\bar{\sigma}>1$, $R(\sigma)$ is an increasing linear function of σ, and lies above $L(\sigma)$ at $\sigma = 0$ and below $L(\sigma)$ at $\sigma = 1$. Because its second derivative does not change sign, the monotonically increasing L function can intersect the R function only once. Hence, for $\rho<1$ and $\bar{\sigma}>1$, the F function has a unique fixed point, $\hat{\sigma}$.

As the geometry of Fig. 1 leads us to expect,

$$\hat{\sigma}<s_*<\sigma_*, \qquad \ldots(51)$$

since s_* is the maximum value assumed by the F function and lies below the 45-degree line.

If $\rho>1$, the fixed point must occur for a value of σ greater than $\bar{\sigma}$, so we examine the behaviour of L and R for $\bar{\sigma} \leqq \sigma \leqq 1$. Again, $R(\sigma)$ is an increasing linear function of σ whose initial value, $R(\bar{\sigma})$, lies above the initial value of L, $(\bar{\sigma})$, and whose terminal value, $R(1)$, lies below the terminal value of L, $L(1)$. The L function is monotonically increasing, and since its second derivative does not change sign, there is clearly one and only one value of σ for which $L(\sigma) = R(\sigma)$. Thus, for $\rho>1$ the function has exactly one fixed point.

As the geometry of Fig. 3 suggests,

$$s_*<\hat{\sigma}<\sigma_*. \qquad \ldots(52)$$

The first inequality follows from the fact that s_* is the minimum value assumed by the F function and is not on the 45-degree line. The second is a consequence of the fact that for $\rho>1$ the F function must have a negative slope at the fixed point and that the F function has a negative slope for and only for values of σ between $\bar{\sigma}$ and σ_*.

If $\rho<1$ and $\bar{\sigma}<1$, a fixed point is a value of σ, $0<\sigma<\bar{\sigma}$, such that $L(\sigma) = R(\sigma)$. It is easily shown that $R(\sigma)$ is an increasing linear function of σ and that it both begins and ends above the L function; that is, $R(0)>L(0)$ and $R(\bar{\sigma})>L(\bar{\sigma})$. L is a monotonically increasing function with a negative second derivative. From this we may conclude that there are three possible cases: (i) there may be no fixed point, (ii) there may be one fixed point, if the L function is tangent to the R function for some σ, and (iii) there may be two fixed points.

By returning to the F function itself, it is possible to say considerably more about the existence or non-existence of fixed points in this case. From (41), it can be shown that the slope of the F function at a fixed point is less than one if (but not only if) $\rho+\delta>1$. But there can be two fixed points only if the slope of the F function at the second fixed point is greater than one, and one fixed point only if the slope of the F function at the fixed point is equal to one. Hence, if $\rho+\delta>1$ there can be no fixed point. If $\rho+\delta<1$ it is possible to have two fixed points, as the reader can verify by taking $\rho = \frac{1}{2}$, $\alpha = \frac{4}{5}$, $\lambda = \frac{16}{9}$, $\delta = \frac{11}{60}$, and computing the values of the F functions at $\sigma = \frac{1}{16}$ and $\sigma = (\frac{44}{49})^2$.

Note that in this case ($\rho<1$, $\bar{\sigma}<1$) there is no first-best optimum to be compared to the fixed point(s).

In the logarithmic case ($\rho = 1$), as already remarked, it is immediately clear that a unique fixed point exists and that

$$s_* = \hat{\sigma} < \sigma_*, \qquad \ldots(53)$$

In this analysis we have confined ourselves to fixed-points described by a constant saving ratio over time. We are unsure whether or not there may exist fixed-point sequences with non-constant saving ratios. It is possible therefore that the discussion which follows is somewhat incomplete.

We shall next show that the Cournot-Nash-fixed point equilibrium is not Pareto-optimal and further that there is a sense in which the equilibrium point displays " under-saving ". Non-Pareto-optimality is not surprising for the basic situation has much in common with the " prisoners' dilemma " of game theory in which the equilibrium strategy of every partner-in-crime is to " confess ". The question of under-saving is much subtler but if we consider only alternative *constant* saving ratios then, within this class of paths, there is under-saving at the game-equilibrium point. For we show now that there exists at least one point on the 45-degree line above the fixed point (or above both fixed points if there are two) which dominates the fixed point and dominates every point on the 45-degree line below the fixed point. This, of course, implies that the equilibrium point is non-Pareto-optimal.[1]

Total utility of the present generation in (31) depends upon s and σ so that we may write $U = U(s, \sigma)$, given initial capital and the four parameters. We wish to calculate $dU(s, \sigma)/ds$, subject to the side relation $\sigma = s$; this is given by

$$\frac{dU(s, \sigma)}{ds} = \frac{\partial U(s, \sigma)}{\partial s} + \frac{\partial U(s, \sigma)}{\partial \sigma} \qquad \ldots(54)$$

when evaluated at $\sigma = s$.

Consider first Figs. 1 and 4 where there is a unique fixed point. At $s = \hat{\sigma}$, i.e., at the fixed point $(\hat{\sigma}, \hat{\sigma})$, $\partial U/\partial s = 0$ since the fixed point lies on the F function. For all $s<\hat{\sigma}$, i.e., for all points (s, s) below $(\hat{\sigma}, \hat{\sigma})$, $\partial U/\partial s>0$ since such points must be below the F function and $\partial^2 U/\partial s^2>0$ for all σ. Hence

$$\frac{\partial U(s, \sigma)}{\partial s} \geqq 0 \text{ for all } \sigma = s \leqq \hat{\sigma}. \qquad \ldots(55)$$

Thus $\partial U/\partial\sigma>0$ for all $s \leqq \hat{\sigma}$ suffices to show that $dU(s, s)/ds$ in (54) is positive for all $s \leqq \hat{\sigma}$. Differentiation of (31) with respect to σ yields

$$\frac{\partial U(s, \sigma)}{\partial \sigma} = -\frac{\delta s^{1-\rho}\alpha\lambda^{1-\rho}(1-\sigma)^{-\rho}(1-\alpha\lambda^{1-\rho}\sigma^{-\rho})K_0^{-\rho}}{(1-\alpha\lambda^{1-\rho}\sigma^{1-\rho})^2}. \qquad \ldots(56)$$

[1] In the preliminary version it was shown that no point on the F function other than the first-best point (s_*, σ_*) is Pareto-optimal.

It is easily verified that this expression is positive for all $\sigma < \sigma_*$. Since $\hat{\sigma} < \sigma_*$ in all cases, $\partial U/\partial\sigma > 0$ at the fixed point and below it on the 45-degree line. Hence (54) is positive at the fixed point and below it. Thus the present generation and any future generation would be willing to increase its own saving ratio beyond $\hat{\sigma}$ by some amount if every succeeding generation were bound to imitate it. Further, such an increase of the saving ratio makes succeeding generations better off for an additional reason for each succeeding generation will inherit more capital if past generations have saved more. Similarly, a reduction of the common saving ratio below $\hat{\sigma}$ will make all generations worse off. So there may be said to be under-saving at the fixed point.

The Fig. 3 case requires only slight modification of the above argument. There $\partial U/\partial s$ and $\partial U/\partial\sigma$ are not defined for $\sigma \leqq \bar{\sigma}(<\hat{\sigma})$. But all points in this region yield infinite, negative utility so none of them can be preferred to the fixed points or to any point (s, s) above the fixed point; hence the undersaving argument carries over to this case.

In the Fig. 2 case, the above argument goes through if $\hat{\sigma}$ is unique, upon replacing σ_* in the argument by the number one (so that $\partial U/\partial\sigma$ is everywhere positive). If there exist two fixed points the above argument is invalid but the conclusion remains. For as we choose 45-degree points closer to $(\bar{\sigma}, \bar{\sigma})$, total utility goes to infinity so there is always a point (s, s) sufficiently close to $(\bar{\sigma}, \bar{\sigma})$ that dominates the fixed point and all points on the 45-degree line below it.

Hence, if we confine ourselves to constant saving ratio sequences, thus sticking to our 45-degree line, there can be said to be under-saving at any fixed point equilibrium. But we have not and shall not attempt to rule out the existence of some non-constant saving-ratio sequence which is both Pareto-optimal and which causes $\sigma_t < \hat{\sigma}$ for some t. So the under-saving hypotheses has not been completely sustained and possibly cannot be.

Our final topic is the consequence for the equilibrium saving ratio of an increase of altruism. If we write (46) in the form

$$\hat{\sigma} = G(\delta, \alpha, \rho, \lambda), \qquad \ldots(57)$$

where we consider only unique fixed points then, by definition of $\hat{\sigma}$,

$$G(\delta, \alpha, \rho, \lambda) = F[G(\delta, \alpha, \rho, \lambda), \delta, \alpha, \rho, \lambda] \qquad \ldots(58)$$

whence

$$\frac{\partial G}{\partial \delta} = \frac{\partial F/\partial\delta}{1-(\partial F/\partial\sigma)}, \qquad \ldots(59)$$

the right-hand side of which is to be evaluated at $\sigma = \hat{\sigma}$.

Since $\partial F/\partial\delta > 0$ everywhere, $\partial G/\partial\delta > 0$ if and only if $\partial F/\partial\sigma < 1$ at $\sigma = \hat{\sigma}$. The latter inequality clearly holds in Figs. 1, 3 and 4 where $\hat{\sigma}$ is unique. In the case $\rho < 1$, $\sigma < 1$ (Fig. 2), if two fixed points exist, an increase of δ, since it shifts up the F function, will increase the lower of the two fixed points while decreasing the upper fixed point. As δ increases, the fixed points come together at a tangency point corresponding to some $\delta < 1$; for larger δ no fixed point exists.

We remark that as $\delta \to 1$, s_* and $\hat{\sigma}$ approach σ_* so that the first-best sequence of saving ratios and the equilibrium sequence merge, both approaching σ_* which is, of course, the Ramsey solution for all σ_t (in our model) on his assumption that $\delta = 1$. In Fig. 2, the fixed points disappear which is as it should be since σ_* does not exist in that case.

V. CONCLUDING REMARKS

In studying the first-best optimization problem under imperfect altruism, we found that the present generation would save less as a proportion of income or capital than it would have future generations save. If the present generation cannot control future generations' saving and it expects future generations to choose a common saving ratio that is non-optimal in its view, the second-best present saving ratio will be smaller or greater than the first-best amount according as marginal utility is consumption-inelastic or consumption elastic. Then, upon imputing to each generation the expectation that succeeding generations would likewise seek a second-best optimum saving policy, we investigated an " equilibrium " sequence of saving ratios in the game-theoretic sense. We showed that such an equilibrium, where it existed, is not Pareto-optimal and that there is at least a natural and limited sense in which any such equilibrium entails " under-saving ".[1] We showed that the game-equilibrium saving ratio, if unique, was greater the greater is each generation's altruism and we remarked that the equilibrium sequence of saving ratios and the first-best sequence merge and become equivalent to the Ramsey-optimal sequence as altruism becomes perfect.

If we are right that the approach here represents a gain over previous approaches, then much more work needs to be done. One would like to suppose diminishing returns to saving, that capital consumption is possible only within limits, that population grows and the technology improves. Uncertainty about future decisions and even future existence need to be introduced. The over-lapping of generations should be treated. And ultimately one wants to know the implications of more general utility functions.

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[1] We saw that the present generation would increase its saving ratio if all future generations were bound to do likewise. Can the present generation so bind future generations? In the context of the present model the answer would appear to be no. Presumably a constitution establishing a saving ratio exceeding the game-equilibrium saving ratio would have no defenders and would be amended; but perhaps sentimental attachment to the constitution would save it. In a different model with over-lapping generations, the anticipated survival of people for two or more generations (" periods ") may lend stability to a constitution requiring national saving in excess of the game-equilibrium amount implied by that model. Further, in a model with people of varying altruism, the more altruistic may be able to block constitutional amendment.

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