Consistent Planning[1]

Robert H. Strotz formulated the problem of consistent planning in 1955 in a paper entitled " Myopia and Inconsistency in Dynamic Utility Maximization ".[2] In his introduction, Strotz stated the basic problem as follows:

> " An individual is imagined to choose a plan of consumption for a future period of time so as to maximize the utility of the plan as evaluated at the present moment. His choice is, of course, subject to a budget constraint. Our problem arises when we ask: If he is free to reconsider his plan at later dates, will he abide by it or disobey it——*even though his original expectations of future desires and means of consumption are verified*? Our answer is that the optimal plan of the present moment is generally one which will not be obeyed, or that the individual's future behavior will be inconsistent with his optimal plan. . . . If the inconsistency is recognized, the rational individual will do one of two things. He may " precommit " his future behavior by precluding future options so that it will conform to his present desire as to what it should be. Or, alternatively, he may modify his chosen plan to take account of future disobedience, realizing that the possibility of disobedience imposes a further constraint—beyond the budget constraint—on the set of plans which are attainable."

In his formal model, Strotz assumed that consumption takes place continuously and that the individual is free to reconsider his consumption plan at every instant rather than at a few " decision points ". He showed that inconsistency arises if and only if the individual discounts the utility of future consumption with a non-exponential discount function. If an individual's discount function is non-exponential and if he cannot precommit his future behaviour, Strotz proposed that he adopt a " strategy of consistent planning ". That is, some consumption plans which are feasible in the sense that they satisfy the budget constraint are not attainable because at some future date the remaining portion of these plans will not appear optimal. The strategy of consistent planning is to choose " the best plan among those he will actually follow ".[3] The portion of Strotz's paper thus far summarized is a major contribution to the literature on intertemporal allocation, and he argues persuasively that individuals often recognize that their preferences are inconsistent and do attempt to precommit their future behaviour.

Strotz attempted to carry the matter a step further by showing how the consumption plan corresponding to the strategy of consistent planning can be calculated. He proposed the following procedure: Substitute for the individual's true discount function the exponential discount function which has the same slope as the true discount function at $t = 0$. Then determine the consumption plan which maximizes this newly constructed utility functional subject to the budget constraint. Strotz claimed that this consumption path is the best plan among those which the individual will actually follow. This is an odd result, for, as Strotz himself pointed out, it implies that " the only relevant characteristic of his true discount function is the rate at which it changes at the *present* moment ".[4]

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This is not to be taken as an endorsement by any of the above of the views expressed in this paper; I alone am responsible for its shortcomings.

[2] *Review of Economic Studies*, **23** (3), 1955-56.

[3] *op. cit.*, p. 173.

[4] *op. cit.*, p. 175 Italics his.

In this paper I show that Strotz's result is incorrect. The plan of the paper is as follows: Section 1 is devoted largely to spelling out the assumptions of the model and defining terms. Section 2 considers the behaviour of an individual with a logarithmic instantaneous utility function under the assumption that he is allowed to reconsider his consumption only at a finite number of " decision points ". Section 3 provides a counter-example to Strotz's result. The limiting behaviour of the individual with a logarithmic instantaneous utility function is considered, as the distance between adjacent decision points approaches zero. A characterization of the best attainable consumption path is obtained, and it is shown to be inconsistent with Strotz's result. Finally, Section 4 examines Strotz's derivation of his result and discusses where and why it goes wrong.

1. THE N DECISION POINT CASE: ASSUMPTIONS AND DEFINITIONS

Consider an individual with an initial stock, K, of a homogeneous consumption good to allocate over a finite time interval $[0, T]$. At time t the individual is assumed to have a utility functional $\theta_t\{{}_t^T x(z)\}$ of the form

$$\int_t^T b(z-t)u[x(z)]dz, \qquad \ldots(1.1)$$

where $x(z)$ is the instantaneous rate of consumption at time z. u is called the instantaneous utility function and b the discount function; the discount function is normalized so that $b(0) = 1$.[1]

If the individual can precommit his future behaviour, then at $t = 0$ he will choose the consumption plan which maximizes $\theta_0\{{}_0^T x(z)\}$ subject to the budget constraint

$$\int_0^T x(z)dz = K. \qquad \ldots(1.2)$$

We call this path the " commitment optimum path " and denote it by $\{{}_0^T x(z)^c\}$.

Now suppose that the individual is unable to precommit his future behaviour. Strotz focused directly on the case in which the individual reconsiders his consumption plan at every instant, but we shall not consider that case until Section 3. In this section and the next, we suppose that the individual can reconsider his consumption plan at only a finite number of " decision points " which are specified in advance.

Formally, the decision points are a set of N numbers, $\{t_i\}$, such that (a) $t_1 = 0$, (b) $t_i < t_{i+1}$ and (c) $t_N < T$. In the N decision point case, the individual begins by choosing a consumption path from the first decision point to the second, or, more technically, a consumption path for the half-open interval $[t_1, t_2)$. At t_2 the individual chooses a consumption path for the half-open interval $[t_2, t_3)$, and so on. At t_N, the individual chooses a consumption path for the closed interval $[t_N, T]$. The commitment optimum path is, by definition, the optimal path in a one decision point problem.

In the N decision points case, it simplifies the notation to append to the set of decision points a final element, t_{N+1}, which is defined by $t_{N+1} = T$. The " decision points " in the N decision point case are now a set of $N+1$ numbers, $\{t_1, \ldots, t_{N+1}\}$ which we denote by D_N.

The " naive optimum path " is defined as the path which an individual would follow if, at every decision point, he believed that he could precommit his future behaviour. Such an individual would begin by following the commitment optimum path, but, at the second decision point, he would reconsider his plan. Instead of continuing to follow the commitment optimum path, he would find the consumption path which maximizes $\theta_{t_2}\{{}_{t_2}^T x(z)\}$ subject to the budget constraint imposed by the remaining stock:

[1] Strotz used the more general instantaneous utility function $u[x(z), z]$; for present purposes nothing essential is lost by working with the less general form.

$$\int_{t_2}^{T} x(z)dz = K - \int_{t_1}^{t_2} x(z)dz. \qquad ...(1.3)$$

The naive individual believes at t_2 that he will carry out this consumption plan, but at t_3 he again reconsiders his plan and, instead of carrying out the plan he adopted at t_2, he begins to follow the path which maximizes $\theta_{t_3}\{_{t_3}^{T}x(z)\}$ subject to the budget constraint

$$\int_{t_3}^{T} x(z)dz = K - \int_{t_1}^{t_3} x(z)dz. \qquad ...(1.4)$$

The naive individual persists in assuming that he can precommit his future behaviour despite the fact that his behaviour at each decision point contradicts this assumption. We denote the naive optimum path by $\{_0^T x(z)^n\}$.

In the N decision point case, it is convenient to define consumption vectors corresponding to consumption paths. We denote the consumption vector corresponding to the consumption path $\{_0^T x(z)\}$ by $(C_1, ..., C_N)$ where

$$C_i = \int_{t_i}^{t_{i+1}} x(z)dz. \qquad ...(1.5)$$

We denote the consumption vector corresponding to the naive optimum path—the " naive optimum vector "—by $(C_1^n, ..., C_N^n)$.

A sophisticated individual, recognizing his inability to precommit his future behaviour beyond the next decision point, would adopt a strategy of consistent planning and choose the best plan among those he will actually follow. Although the mathematics is tedious, in principle the determination of the " sophisticated optimum path " is straightforward.

The definition of the sophisticated optimum path in the N decision point problem proceeds by induction on $K(t_i)$, the stock remaining at t_i. We begin by examining the allocation of a hypothetically given stock remaining at t_N over the subinterval $[t_N, T]$. That is, we determine the path, $\{_{t_N}^T x[z, K(t_N)]^s\}$, which satisfies the budget constraint

$$\int_{t_N}^{T} x(z)dz = K(t_N) \qquad ...(1.6)$$

and is preferred at t_N to all other paths satisfying (1.6). This path is, of course, a function of $K(t_N)$.

Having defined the path $\{_{t_i}^T x[z, K(t_i)]^s\}$ we define the path $\{_{t_{i-1}}^T x[z, K(t_{i-1})]^s\}$ by the requirements: (a)

$$\{_{t_i}^T x[z, K(t_{i-1})]^s\} = \{_{t_i}^T x[z, K(t_i)]^s\}, \qquad ...(1.7)$$

where

$$K(t_i) = K(t_{i-1}) - \int_{t_{i-1}}^{t_i} x[z, K(t_{i-1})]^s dz, \qquad ...(1.8)$$

and (b) $\{_{t_{i-1}}^T x(z, K(t_{i-1})]^s\}$ is preferred to all other consumption paths on $[t_{i-1}, T]$ satisfying (a).[1]

We denote the sophisticated optimum path by $\{_0^T x(z)^s\}$ and the sophisticated optimum vector by $(C_1^s, ..., C_N^s)$.

[1] The essence of this procedure for determining the sophisticated optimum path can be illustrated most simply by a discrete, three period example. Consider an individual whose present utility function is given by $U_1(C_1, C_2, C_3)$ and whose next period utility function is given by $U_2(C_2, C_3)$. Any amount of his initial stock, K, which the individual does not consume during the first period will be allocated between C_2 and C_3 so as to maximize U_2. Hence, $C_2 = h^2(K_2)$ and $C_3 = h^3(K_2)$, where K_2 is the portion of the initial stock remaining at the beginning of the second period. The sophisticated individual will recognize that his future preferences impose a constraint on his ability to select a consumption path, and that only consumption paths of the form $[C_1, h^2(K-C_1), h^3(K-C_1)]$ are attainable. Hence, he will choose C_1 so as to maximize $U_1[C_1, h^2(K-C_1), h^3(K-C_1)]$. Notice that the budget constraint has been absorbed into the utility function.

2. THE N DECISION POINT CASE: THE LOGARITHMIC INSTANTANEOUS UTILITY FUNCTION

This section is devoted to establishing a theorem which underlies the construction of the counter-example in Section 3. The theorem can be stated in one sentence: In the N decision point case, if the instantaneous utility function is logarithmic, then the naive optimum path and the sophisticated optimum path coincide. The proof, unfortunately, is not so simple.

2.1. We begin by observing that, if the naive optimum vector coincides with the sophisticated optimum vector, then the naive optimum path coincides with the sophisticated optimum path. In order to maximize $\theta_{t_i}\{_{t_i}^T x(z)\}$ the naive individual will allocate C_i^n over the subinterval $[t_i, t_{i+1})$ so as to maximize

$$\int_{t_i}^{t_{i+1}} b(z-t_i)u[x(z)]dz \qquad \ldots(2.1)$$

subject to

$$\int_{t_i}^{t_{i+1}} x(z)dz = C_i. \qquad \ldots(2.2)$$

The sophisticated individual knows that how he allocates C_i^s over $[t_i, t_{i+1})$ will not alter the allocation of $\sum_{k=i+1}^{N} C_k$ over $[t_{i+1}, T]$, so maximizing $\theta_{t_i}\{_{t_i}^T x(z)\}$ implies allocating C_i^s over $[t_i, t_{i+1})$ so as to maximize (2.1) subject to (2.2).

2.2. We next assert a result about intertemporal decision problems in which consumption takes place in N discrete periods instead of continuously. Suppose that the individual's utility function at the beginning of period j is given by

$$U_j(C_j, \ldots, C_N) = \sum_{i=j}^{N} a_{ji} \log C_i. \qquad \ldots(2.3)$$

Then the discrete sophisticated optimum vector is given by

$$\begin{aligned} C_1^s &= A_1 K, \\ C_j^s &= A_j K \prod_{i=1}^{j-1} (1-A_i), \quad j = 2, \ldots, N-1 \qquad \ldots(2.4) \\ C_N^s &= K \prod_{i=1}^{N-1} (1-A_i), \end{aligned}$$

where

$$A_i = \frac{a_{ii}}{\sum_{k=i}^{N} a_{ik}}.^{1} \qquad \ldots(2.5)$$

The reader can easily verify that (2.4) holds for $N = 3$. The general result is established by induction, and we omit the proof.[2]

2.3. We now establish a relation between an N-decision point problem in which the instantaneous utility function is logarithmic and an N period decision problem in which the period utility function is logarithmic: the sophisticated optimum vector corresponding to the utility functional

[1] The meaning of the discrete sophisticated optimum vector should be obvious from the discussion of Section 1 and the example of the previous footnote. We call the solution to a discrete N period decision problem a " discrete sophisticated optimum vector " to distinguish between it and the sophisticated optimum vector constructed from the sophisticated optimum path of the continuous N decision point problem.

[2] It is quite easy to show by induction that for (2.3) the discrete naive optimum vector coincides with the discrete sophisticated optimum vector. This result is highly suggestive, but it does not actually help us prove the continuous time result which we need.

$$\theta_t\{_t^T x(z)\} = \int_t^T b(z-t)\log x(z)dz \qquad \ldots(2.6)$$

is equal to the discrete sophisticated optimum vector corresponding to (2.3), where

$$a_{ij} = \int_{t_i}^{t_{i+1}} b(z-t_j)dz. \qquad \ldots(2.7)$$

To show this, it is necessary to derive an explicit expression for the utility of an arbitrary consumption vector, $(C_j, \ldots, C_N)$. Since C_i will be allocated over $[t_i, t_{i+1})$ so as to maximize (2.1) subject to (2.2), it is easily verified that

$$x(z) = \frac{b(z-t_i)C_i}{\int_{t_i}^{t_{i+1}} b(w-t_i)dw} \quad t_i \leqq z < t_{i+1}. \qquad \ldots(2.8)$$

Hence

$$\theta_{t_j}\{_{t_j}^T x(z)\} = \int_{t_j}^T b(z-t_j)\log x(z)dz = \sum_{i=j}^N \int_{t_i}^{t_{i+1}} b(z-t_j)\log x(z)dz$$

$$= \sum_{i=1}^N \left[\int_{t_i}^{t_{i+1}} b(z-t_j)dz\right]\log C_i + \sum_{i=j}^N \int_{t_i}^{t_{i+1}} b(z-t_j)\log\frac{b(z-t_i)C_i}{\int_{t_i}^{t_{i+1}} b(w-t_i)dw}\,dz. \qquad \ldots(2.9a)$$

Since the second term is a constant, the utility functional is equivalent to

$$\sum_{i=j}^N \left[\int_{t_i}^{t_{i+1}} b(z-t_j)dz\right]\log C_i = \sum_{i=j}^N a_{ji}\log C_i \qquad \ldots(2.9b)$$

as asserted above.

Hence, the sophisticated optimum vector corresponding to (2.6) is equal to the discrete sophisticated optimum vector corresponding to (2.3). Using this result, we can now adopt a two-stage procedure for determining the sophisticated optimum path corresponding to (2.6). First, we set up the corresponding N period problem and use (2.4) to obtain the sophisticated optimum vector; then we use (2.8) to allocate the C_i's over the appropriate subintervals. We remark that the conditions which determine the allocation of K among the C_i's are quite different from those which determine the allocation of the C_i's within subintervals.

2.4. It remains to be shown that if the instantaneous utility function is logarithmic, then the naive optimum vector coincides with the sophisticated optimum vector.

The naive individual begins by following the commitment optimum path, and it is easily verified that

$$x(z)^c = \frac{b(z)K}{\int_0^T b(w)dw} \quad 0 \leqq z \leqq T. \qquad \ldots(2.10)$$

Hence

$$x(z)^n = \frac{b(z)K}{\int_0^T b(w)dw} \quad 0 \leqq z < t_2, \qquad \ldots(2.11)$$

so

$$C_1^n = \int_{t_1}^{t_2} x(z)^n dz = \frac{K\int_0^{t_2} b(z)dz}{\int_0^T b(w)dw} = \frac{a_{11}K}{\sum_{i=1}^N a_{1i}} = A_1 K \qquad \ldots(2.12)$$

as required by (2.4). By means of a simple induction proof it can easily be shown that the naive optimum vector is (2.4) and hence coincides with the sophisticated optimum vector. By 2.1, the corresponding paths also coincide.

3. THE CONTINUOUS DECISION CASE: A COUNTER-EXAMPLE

Corresponding to each set of decision points, D_N, is a naive optimum path, $\{{}_0^T x(z)^{nD_N}\}$. We define the "continuous decision naive optimum path" as the limit of these paths as the norm of D_N approaches zero, and we denote this limit path by $\{{}_0^T x(z)^{n^*}\}$.[1] Similarly, if there is a sophisticated optimum path $\{{}_0^T x(z)^{sD_N}\}$ corresponding to each set of decision points, we define the "continuous decision sophisticated optimum path" as the limit of these paths as the norm of D_N approaches zero; we denote this limit path by $\{{}_0^T x(z)^{s^*}\}$.[2]

We want to characterize the continuous decision sophisticated optimum path of an individual whose utility functional is of the form (2.6). From Section 2 we know that in this case the N-decision point sophisticated optimum path coincides with the N-decision point naive optimum path. Since these two paths coincide for every N, their limits must also coincide. We begin, then, by considering the continuous decision naive optimum path.

At time t the naive individual plans to allocate the stock remaining at time t, $K(t)$, over $[t, T]$ so as to maximize

$$\int_t^T b(z-t)\log x(z)dz, \qquad \ldots(3.1)$$

subject to the budget constraint

$$\int_t^T x(z)dz = K(t). \qquad \ldots(3.2)$$

It is easy to show that the path which the naive individual plans to follow is given by

$$x(z) = \frac{b(z-t)K(t)}{\int_t^T b(w-t)dw} \quad t \leqq z \leqq T. \qquad \ldots(3.3)$$

This implies that at time t the individual's rate of consumption is given by

$$x(t) = \frac{K(t)}{\int_t^T b(w-t)dw}. \qquad \ldots(3.4)$$

The consumption path of a naive individual must satisfy (3.4) for all t.

We can eliminate $K(t)$ from (3.4) by observing that

$$K(t) = K(0) - \int_0^t x(z)dz. \qquad \ldots(3.5)$$

Substituting (3.5) into (3.4) we readily obtain

$$x(t)\int_t^T b(z-t)dz = K(0) - \int_0^t x(z)dz. \qquad \ldots(3.6)$$

Differentiating (3.6) with respect to t and solving for $\dfrac{\dot{x}(t)}{x(t)}$ yields

[1] We have not shown that this limit exists for any particular sequence of sets of decision points, nor that, if it exists, it is the same for all admissible sequences.

[2] The reservations of the previous footnote apply here as well.

$$\frac{\dot{x}(t)}{x(t)} = \frac{\int_t^T \dot{b}(z-t)dz}{\int_t^T b(z-t)dz} = \frac{b(T-t)-1}{\int_t^T b(z-t)dz}, \qquad \ldots(3.7)$$

where $\dot{x}(t)$ denotes the derivative of $x(t)$ with respect to t. (3.7), together with the budget constraint, uniquely determines the continuous decision naive optimum path. And since the continuous decision naive optimum path coincides with the continuous decision sophisticated optimum path, (3.7) also characterizes the continuous decision sophisticated optimum path.

This contradicts Strotz's assertion that the continuous decision sophisticated optimum path coincides with the commitment optimum path of the utility functional constructed by replacing the true discount function by a properly chosen exponential discount function. For if we replace the true discount function in (2.6) by the exponential discount function e^{-rt} and compute the commitment optimum path we obtain

$$x(t) = \frac{e^{-rt}K(0)}{\int_0^T e^{-rz}dz} \quad 0 \leqq t \leqq T. \qquad \ldots(3.8)$$

This implies

$$\frac{\dot{x}(t)}{x(t)} = -r, \qquad \ldots(3.9)$$

where r is a constant. It is easily verified that (3.9) is not satisfied by (3.7) unless b is itself exponential.

4. STROTZ'S "PROOF"

This section examines Strotz's derivation of his result and shows where it went wrong. Unfortunately, the criticisms are not constructive and do not lead to an alternative general formula for the continuous decision sophisticated optimum path. Indeed, the whole thrust of this section is that such a formula will be very hard to come by, except in certain exceptional cases.

Strotz derived his result from the following argument: Suppose that the individual can precommit himself over small intervals, and suppose that we know the total amount he will consume on the subinterval $[\bar{t}, t^*)$; call this amount C. If the individual's utility functional is (1.1) he will allocate C over $[\bar{t}, t^*)$ so as to maximize

$$\int_{\bar{t}}^{t^*} b(z-\bar{t})u[x(z)]dz, \qquad \ldots(4.1)$$

subject to the budget constraint

$$\int_{\bar{t}}^{t^*} x(z)dz = C. \qquad \ldots(4.2)$$

By a simple calculus of variations argument, it can easily be shown that the optimal path must satisfy

$$\frac{\dot{u}'[x(z)]}{u'[x(z)]} = -\frac{\dot{b}(z-\bar{t})}{b(z-\bar{t})}, \quad \bar{t} \leqq z \leqq t^*. \qquad \ldots(4.3)$$

Since this holds for all z, $\bar{t} \leqq z < t^*$, at $z = \bar{t}$ we have

$$\frac{\dot{u}'[x(\bar{t})]_{\bar{t}}}{u'[x(\bar{t})]} = -\frac{\dot{b}(0)}{b(0)}. \qquad \ldots(4.4)$$

In the continuous decision case, where every point is a decision point, this must hold for all t, $0 \leqq t \leqq T$, so the continuous decision sophisticated optimum path must satisfy

$$\frac{\dot{u}'[x(t)]}{u'[x(t)]} = -\frac{\dot{b}(0)}{b(0)} \qquad \ldots(4.5)$$

for all t.[1]

Strotz's argument fails at several points. First, the N decision point sophisticated optimum path need not be continuous, much less differentiable, at the decision point $\bar{t}$. Second, it is not in general true that

$$\lim_{\| D_N \| \to 0} \left(\frac{\dot{x}(t)^{sD_N}}{x(t)^{sD_N}} \right) = \frac{\dot{x}(t)^{s^*}}{x(t)^{s^*}}. \qquad \ldots(4.6)$$

Since the derivative of a function is, by definition, a limit, there are two limits involved in (4.6); in general, the order in which these two limits are evaluated cannot be interchanged. To find $\frac{\dot{x}(t)^{s^*}}{x(t)^{s^*}}$, the proper procedure, in general, is to calculate $\{_0^T x(z)^{s^*}\}$ as the limit of $\{_0^T x(z)^{sD_N}\}$ and then to calculate $\frac{\dot{x}(t)^{s^*}}{x(t)^{s^*}}$ from $\{_0^T x(z)s^*\}$.

Strotz's result does tell us something about the N decision point sophisticated optimum path and naive optimum path when N is " large " and the maximum distance between adjacent decision points is " small ". It is true that $\dot{u}'[x(t)^{sD_N}]/u'[x(t)^{sD_N}]$ and $\dot{u}'[x(t)^{nD_N}]/u'[x(t)^{nD_N}]$ are close to $\frac{\dot{b}(0)}{b(0)}$ at almost every point, and that the approximation becomes closer as N becomes larger. Strotz's condition tells us a good deal about how $C_i^{sD_N}$ and $C_i^{nD_N}$ are allocated over $[t_i, t_{i+1})$; but it does not tell us anything about how K would be allocated among the C_i's by either a naive or a sophisticated individual.[2]

As we saw in Section 2, the conditions which determine the allocation of the C_i's within a subinterval are quite different from those which determine the allocation of K among the C_i's. Furthermore, the conditions which determine the allocation of the C_i's within a subinterval are the same for a naive and a sophisticated individual; but, in general, the conditions which determine the allocation of K among the C_i's are different for naive and sophisticated individuals.[3] Strotz's result suggests that as the size of the subintervals approaches zero, the conditions which determine the allocation of the C_i's within subintervals become all-important and the conditions which determine the allocation of K among subintervals drop out entirely. Intuition leads us to expect just the opposite, and our result confirms this.[4]

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[1] If the instantaneous utility function is logarithmic, (4.5) becomes (3.9) since $\dot{b}(0)/b(0)$ is a constant. We showed in Section 3 that (3.9) is not a correct characterization of the continuous decision sophisticated optimum path in the logarithmic case.

[2] Think of the sequence of N decision point sophisticated optimum paths as a sequence of functions approximating $\{_0^T x(z)^{s^*}\}$. Strotz's condition (4.5) is essentially a restriction on the slopes of these approximating functions. But these slopes imply nothing about the slope of $\{_0^T x(z)^{s^*}\}$.

Suppose, for example, that we required the individual to allocate C_i over the ith subinterval so that the rate of consumption was constant, instead of allowing him to allocate C_i optimally over the subinterval. This certainly does not imply that the corresponding continuous decision sophisticated optimum path would exhibit a constant rate of consumption (i.e. zero slope) on $[0, T]$.

[3] The logarithmic case is, of course, an exception.

[4] This also suggests a further difficulty. It would seem that Strotz's derivation applies as well to the continuous decision naive optimum path as to the continuous decision sophisticated optimum path. Thus, if one accepts Strotz's derivation of the sophisticated optimum path, one must argue that (a) the derivation is not valid for the continuous decision naive optimum path or (b) the continuous decision naive optimum path coincides with the continuous decision sophisticated optimum path regardless of the form of the instantaneous utility function.