NOTE ON VON NEUMANN-MORGENSTERN'S STRONG INDEPENDENCE AXIOM

By E. Malinvaud

Professor Samuelson noticed that no explicit axiom in von Neumann-Morgenstern's treatment of behaviour under uncertainty seemed to correspond to his "strong independence axiom." He then made the conjecture that the authors had implicitly introduced in their mathematical setup some assumption that was carrying the true content of this axiom. The present note is intended as supporting such a view.

The axioms in von Neumann-Morgenstern are written in terms of equivalence classes $u, v, w, \cdots \epsilon U$. An equivalence class $u \epsilon U$ is defined as a set of events. Two events x_1 and x_2 belong to the same equivalence class if and only if the individual in question is indifferent between them; i. e., if $x_1 I x_2$.

Before they write any axiom, the authors define an operation on equivalence classes by:

$$\alpha u + (1 - \alpha)v = w$$

where α is any positive real number smaller than 1.

From the heuristic comments it follows that w must be understood as being the equivalence class of all events which may be obtained as a probability combination of any event in u, with frequence α, and any event in v, with frequence $1 - \alpha$.

Now, why should such an operation make sense? Why should w be an equivalence class? Assuming this, as von Neumann-Morgenstern implicitly did, amounts to postulating the Samuelson "strong independence axiom." Indeed, if the operation is valid, then for any x_1, $x_2 \epsilon u$ and $y \epsilon v$, we must have:

$$\alpha x_1 + (1 - \alpha)y \epsilon w,$$
$$\alpha x_2 + (1 - \alpha)y \epsilon w.$$

This must be true for any u and v; hence, if $x_1 I x_2$, then for any y:

$$[\alpha x_1 + (1 - \alpha)y] \; I \; [\alpha x_2 + (1 - \alpha)y],$$

which is Samuelson's "strong independence axiom."

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[Ed. Note: The above is one of several related contributions published in this issue. Reference should be made to the editorial note which appears on page 661.]