Discussion of 'Policy Rules for Open Economies' by Laurence Ball

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Summary

I offer two related, apparently but not necessarily contradictory, criticisms of Laurence Ball's model. First, the model incorporates too little rational expectations; second it assumes too much rational expectations. The private sector uses too little rational expectations (at least compared to the government); and the government uses too much (in view of how little rational expectations Ball has built into the private sector's model – and in view of Ball's empirical approach to his model as an approximation). Let me explain from the beginning.

Ball ascribes a return function to the government (a weighted sum of unconditional variances of inflation and output), then solves a 'modern' single-agent control problem to deduce an optimal rule for monetary policy. He models the economy as being like *nature*, a time-invariant system of stochastic difference equations, driven by serially uncorrelated shocks, presenting state and control vectors to the policy maker.[1] The policy maker views the model economy as 'known' and 'fixed' (with respect to the policy maker's choices). Though Ball's model is estimable using modern methods (the 'duals' of the control theoretic methods he uses), Ball abstains from estimation and instead resorts to 'calibration'. In calibrating, Ball implicitly treats his model somehow as an approximation, not to be taken literally empirically. This empirical approach is compatible with Ball's motivation of his work as a sensitivity exercise designed to explore how augmenting the government's model to incorporate aspects of a foreign sector will affect the character of optimal rules that he had deduced earlier from a related closed economy model (Ball (1997)). Ball does a good job of explaining the alterations he has made to the baseline closed economy model, of interpreting how they make the optimal rule change, and of informally explaining how the optimal rule corresponds to practices observed in some small open economies. The optimal rule[2] contains the 'advice' Ball coaxes from his formal model.

Ball also evaluates six arbitrary rules. Within the confines of this single paper, comparing the operation of these rules is of less interest than just looking at the optimal rule. These comparisons acquire interest as ingredients of a robustness calculation only when they are put together with the workings of these six rules within the *different* models represented in the other papers in the conference.

To supplement Ball's acknowledgement in the text that the government's model is 'pre Lucas critique' (although cleverly constructed to embody one version of the natural rate hypothesis), I recommend Robert King and Mark Watson's 'Revisionist History' of the Phillips curve. King and Watson use theory and empirical evidence to document how the type of 'unit root' specification of the Phillips curve used by Ball reflects the serial correlation structure of inflation detected only over the last 25 years or so, which emerged hand in hand with our current fiat monetary policy regime. Our earlier monetary regime was associated with much lower serial correlation of inflation, and a different apparent intertemporal Phillips curve.[3] I recommend Larry Ball's (1995) JME paper (where the government sets a repeated economy strategy and the private economy makes predictions that reflect the government's strategy) as an example of an analysis with optimizing behavior on all sides, a setup not subject to my first criticism (though possibly to my second one).

Ball can argue somewhat convincingly that despite imperfect 'microfoundations', and despite not pretending immunity from the Lucas critique, a model of his type can somehow be a good approximation to a 'truer' (bigger? general equilibrium?) model. Accepting this defense against 'too little rational expectations in the model' strengthens my second criticism, and prompts retreat from the 'rational expectations' that Ball ascribes to the government in formulating its control problem. If the model is an approximation, the optimal policy rule ought to incorporate this attitude. This attitude recommends 'post modern' or 'robust' rather than 'modern' control theory. A quest for robustness overshadows this conference, so I want to explore how a preference for robustness would affect Ball's analysis.

Modern and robust government problems

Ball's control problem is a member of the following class. Where x_t, u_t are the state and control vectors, and z_t is a vector representing the decision maker's rewards, the government's model is

$$x_{t+1} = Ax_t + Bu_t + Cw_{t+1} \tag{1a}$$

$$u_t = -Fx_t \tag{1b}$$

$$z_t = Hx_t, \tag{1c}$$

where $\{w_{t+1}\}$ is a martingale difference sequence with unity contemporaneous covariance matrix, adapted to its own history.[4] The government chooses a feedback rule of the form (1b), to achieve an objective that Ball formulates as the unconditional variance of z_t. For the purpose of formalizing the control problem, it is useful to express the model in the form

$$z_t = G(L)\,w_t \equiv H\left[I - (A - BF)\,L\right]^{-1} Cw_t. \tag{2}$$

Equation (2) shows how the white-noise shocks w_t impact on the variables the government cares about. For Ball, the problem is to maximize minus the unconditional variance of z, by choice of F. This can be expressed in the frequency domain as choosing an F to maximize

$$H^2 = -Ez_t z_t' = -\int_\Gamma \left[G(\zeta)\,G(\zeta)'\right] d\zeta, \tag{3}$$

where ζ is a complex variable, Γ is the unit circle, and $'$ denotes matrix transposition and complex conjugation.

This problem is 'modern' in the sense that the government is assumed to know enough about the model to have rational expectations. It treats the model as known and true. This means that the government knows: (1) $G(L)$ and how it depends on F; and (2) that the shock process w_t is serially uncorrelated.

Robust control theory would back away from the rational expectations assumption, and endow the government with the view that while 'good', its model is not 'true', only an approximation. Suspicion that the model is an approximation imparts a preference for 'robustness', i.e., acceptable performance of a rule across a range of models 'in the vicinity' of the government's; 'in the vicinity' because the model is viewed as a useful approximation. To formalize this approach, robust decision theory treats the model not as true but as a base or 'reference' model around which the government suspects approximation errors.[5] The key is to describe the form that the approximation errors can take, and how 'big' they can be. Approximation errors have fruitfully been formulated as showing up as a shock process $\{w_t\}$ that is arbitrarily serially correlated, not serially uncorrelated as specified. If the shocks have spectral density $S_w(\zeta)$ rather than I, then the variance of z_t is not given by the second equality in (3) but by

$$\tilde{H}_2 = -\int_\Gamma G(\zeta)\, S_w(\zeta)\, G(\zeta)'\, d\zeta. \tag{4}$$

The robust decision maker seeks a rule that delivers an 'acceptable' $\tilde{H}_2$ over a domain of somewhat arbitrary $S_w(\zeta)$. To construct the domain of approximation errors, some sort of 'constant variance' restraint $\int_\Gamma S_w(\zeta) d\zeta = I$ is imposed on the potentially perturbed error processes.

To design a robust rule, the decision maker performs a worst case analysis. He solves a problem of the form

$$\max_F \min_{S_w(\zeta)} \tilde{H}_2 \text{ s.t. } \int_\Gamma S_w(\zeta)\, d\zeta = I. \tag{5}$$

The F that solves this $(\max, \min)$ problem works better than the 'modern' rule across much of the domain of unknown error serial correlation specifications. The associated minimizing $S_w(\zeta)$ is the 'most pessimistic' view of the shock serial correlation process, an artifact of computing a rule designed to work well over a variety of $S_w(\zeta)$'s.[6]

Notice how this formulation makes the specification error showing up as a perturbed shock process that nevertheless feeds through the system in the way the model specifies.

This is a restrictive way of modelling 'misspecification', though as Hatanaka's (1975) work emphasized, arbitrarily serially correlated errors provide specifications so flexible that they often undermine econometric identifiability. A restrictive aspect of the present way of modelling specification errors is that the misspecified errors are supposed to feed through the system just as those in the reference model.

Remarkably, but maybe not surprisingly, because this *is* a form of Knightian uncertainty, there is a sense in which the robust decision maker turns back from the Lucas critique. Thus, for a *given* reference model, variations in the shock serial correlation process within the domain satisfying the constraint in (5) are presumed to leave the robust decision rule intact. Here I am interpreting the 'Lucas critique' as but an application of the principle, already well reflected in 'Keynesian' contributions like Kareken, Muench, and Wallace (1973),[7] stating that optimal decision rules depend on laws of motion or transition laws expressing the 'constraints', which in the modern setting always include descriptions of the serial correlation patterns of the shocks. The robust controller's behavior partly belies that principle.[8]

Problem (5) embodies a game, not 'real', but a mental one, played in the mind of a monetary authority, who as an instrument for attaining a robust rule, contemplates the reactions of a 'diabolical nature' that is not 'fixed', and that in response to *its* choice of F, responds by 'choosing' a shock process serial correlation structure to harm the monetary authority. The point is not that the policy maker expects the worst, but that by planning against it he assures acceptable performance under a range of specification errors.

Problem (5) leads to what is called the H^{∞} formulation of robust decision theory.[9] A less extreme and more convenient formulation is the 'minimum entropy' criterion, according to which the government just maximizes

$$H_{\text{entropy}} = -\log\det\int_{\Gamma}\left[\kappa I - G(\zeta)\, G(\zeta)'\right] d\,\zeta. \tag{6}$$

This criterion is defined only for $\kappa \geq \kappa_b$, where κ_b is the minimum positive scalar for which the integrand is positive semi-definite.[10] The decision rule F that maximizes criterion (6) approximates the decision rule F that solves (5) as $\kappa \searrow \kappa_b$. Further, the parameter $\kappa > 0$ is interpretable in terms of the risk-sensitive formulation of Whittle (1990) and Hansen and Sargent (1995). In particular, κ equals minus the reciprocal of the risk-sensitivity parameter, σ.[11] Formulation (6) provides a 'smooth' one-parameter family of approximators of (5). For $\sigma = 0$, we have the 'modern' case, while for σ approaching $-\kappa_b^{-1}$, we attain the case envisioned in (5).

Examples

Using criterion (6), I have computed decision rules for three values of $-\kappa^{-1} \equiv \sigma$, namely $\sigma = 0, -.1, -.2$. I prefer to parameterize in terms of σ because of its link to risk-sensitivity, but this is just a matter of taste. The value $\sigma = 0$ corresponds to Ball's calculations, and checks my calculations against his.[12] For Ball's parameter values and the case of equal weights on inflation and output variances in the government's objective, I calculated the responses of the interest rate, r to output y and the 'monetary conditions index' π^* given in table 1. Here $r = \bar{a}y + \bar{b}\pi^*$, where $\bar{a}$ and $\bar{b}$ are calculated just as in the $\sigma = 0$ case as explained by Ball. These numbers agree with Ball's in the $\sigma = 0$ case. Note how the interest rate becomes *more* sensitive to both y and π^* as the absolute value of σ rises (i.e., as the preference for robustness increases). This aspect of the rules illustrates that the 'caution' induced by activating the preference for robustness does not translate into 'doing less'. 'Caution' is relative to a worst case pattern of the shock process, which depends on the specification of the reference model. In particular, the specification of the model affects the magnitude and the serial correlation properties of the 'worst case' shocks, and thus influences how the policy maker responds to actual shocks.[13,14]

To shed light on how a preference for robustness surfaces, it is useful to display the optimized $G(\zeta)$ functions in the frequency domain. Figure 1 displays the maximum singular value of $G(\zeta)G(\zeta)'$ by frequency for $\sigma = 0, -.1, -.2$. As σ rises in absolute value, the maximum singular value across frequencies falls, but the area under the curve rises.[15] This pattern captures how 'worst case' performance improves as σ rises in absolute value, at the cost of worse performance at the reference model. In particular, think about how the 'great deceiver' feared in problem (5) would respond to a $G(\zeta)$ having the pattern of maximum singular values associated with the $\sigma = 0$ case in figure 1. The deceiver could lower the utility of the monetary authority (raise the weighted average of variances) by concentrating the spectral power of the shock process near the frequencies at which the maximum singular value is highest. To protect itself against such a 'malevolent nature', the monetary authority can design a rule to lower the maximum singular value, although this causes the average level of the maximum singular values across frequencies to drift upward (which lowers the value of the monetary authority's objective under the assumption that the reference model is *true*). In Ball's model, flattening the maximum singular values across frequencies can be achieved by making the interest rate respond more to both y and π^*.

Figures 2, 3, and 4 depict the impulse response functions with respect to the w_t's (i.e., the $G(L)$'s) for the three values of $\sigma = 0, -.1, -.2$, respectively. Close inspection of these figures confirms how making σ more negative leads to 'whiter' impulse responses. The 'less colorful' impulse responses reflect the 'flattening' across frequencies of the maximum singular values as σ (which is nonpositive) is raised in absolute value. Thus, the 'more active' responses in the rules for more negative σ's, depicted in table 1, deliver whiter spectral outcomes (assuming the reference model to be true).

Thus, this particular way of framing the class of misspecifications to be protected against causes 'more caution' to translates into 'more activity'. How can this happen? One way to approach this question is to study how the 'worst case' conditional means of the shocks, call them $\hat{w}_{t+j}$, respond to the true shocks w_t. Hansen, Sargent, and Tallarini (1997) report a formula for such worse case conditional means. The formula shows how the worst case conditional means are linear functions of the state of the system x_t. This makes it possible to compute impulse responses of the worst case shocks with respect to the w_t's. Figures 5 and 6 depict the impulse responses of these worst case conditional means in the shocks with respect to the w_t's for $\sigma = -.1$ and $-.2$; for $\sigma = 0$, these 'worst case' means are identically zero. The impulse responses in figures 5 and 6 shows how for Ball's model, the worst case conditional means in the shocks alter both in size and in their serial correlation properties as σ becomes more negative. Figures 5 and 6 show how the worst case shocks become larger as σ grows in absolute value, and that they are serially correlated.[16] This makes the robust decision maker respond to what the reference model asserts are white noise shocks as though they were positively serially correlated. To guard against the worst case, the robust decision maker exercises 'caution' by 'pressing the brakes' or 'pushing on the accelerator' more than would the modern decision maker. This is reasonable because a shock is interpreted as being larger and more persistent than it is supposed to be by the reference model. This helps to explain the pattern of the table 1 responses.

Thus, in the context of Ball's model, the 'caution' exercised by the robust decision maker does not manifest itself in the form of 'moving less' in response to shocks. Robust decision theory teaches that what 'caution' means depends on the model with respect to which caution is being exercised.

Unanswered Questions

The preceding response to my 'two criticisms' of Ball's general approach opens more questions than it answers. If we imagine the monetary authority to be using robust decision theory as I have outlined, what should be done to 'close' the model, in the spirit of seeking some counterpart to a 'rational expectations equilibrium', only now where both the government and the 'market' are robust decision makers? More generally, if policy makers are entertaining the same doubts across models as we are at this conference, how are we to formulate the forecasting problems facing private agents? Along with the papers presented in this conference, my 'robust decision' analysis of Ball's model proceeds without any sort of 'robust equilibrium' concept.[17] Maybe such an equilibrium concept could help us.[18]

Table 1

Robust Policy Rules

σ	$\bar{a}$	$\bar{b}$
0	1.04	.82
-.1	1.17	1.13
-.2	1.55	2.08

List of Legends

Figure 1 Logarithm of maximum singular values of $G(\zeta)$ for three $\sigma = 0, -.1, -.2$ (straight, dashed, and dashed-dotted lines, respectively). Angular frequency (from 0 to π) is the ordinate, the logarithm of the maximum singular value the coordinate.

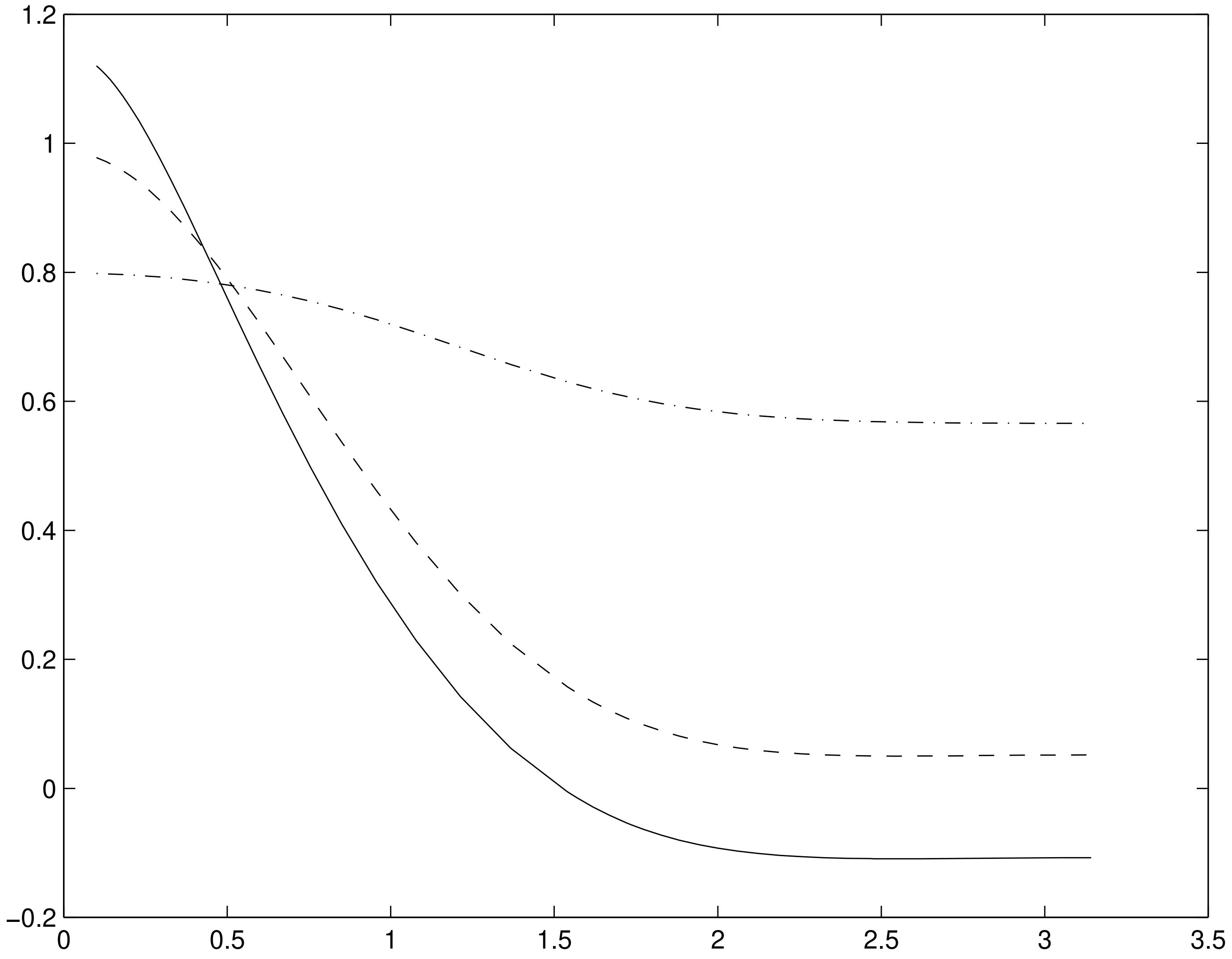
Figure 2: Impulse response of $[\, y \quad \pi \quad e_{-1} \quad \nu \,]'$ to $w_t \equiv [\, \epsilon_t \quad \eta_t \quad (1/\sqrt{2})\nu_t \,]'$ for Ball's model with $\sigma = 0$. The shocks correspond to columns, the variables to rows.

Figure 3: Impulse response of $[\, y \quad \pi \quad e_{-1} \quad \nu \,]'$ to $w_t \equiv [\, \epsilon_t \quad \eta_t \quad (1/\sqrt{2})\nu_t \,]'$ for Ball's model with $\sigma = -.1$. The shocks correspond to columns, the variables to rows.

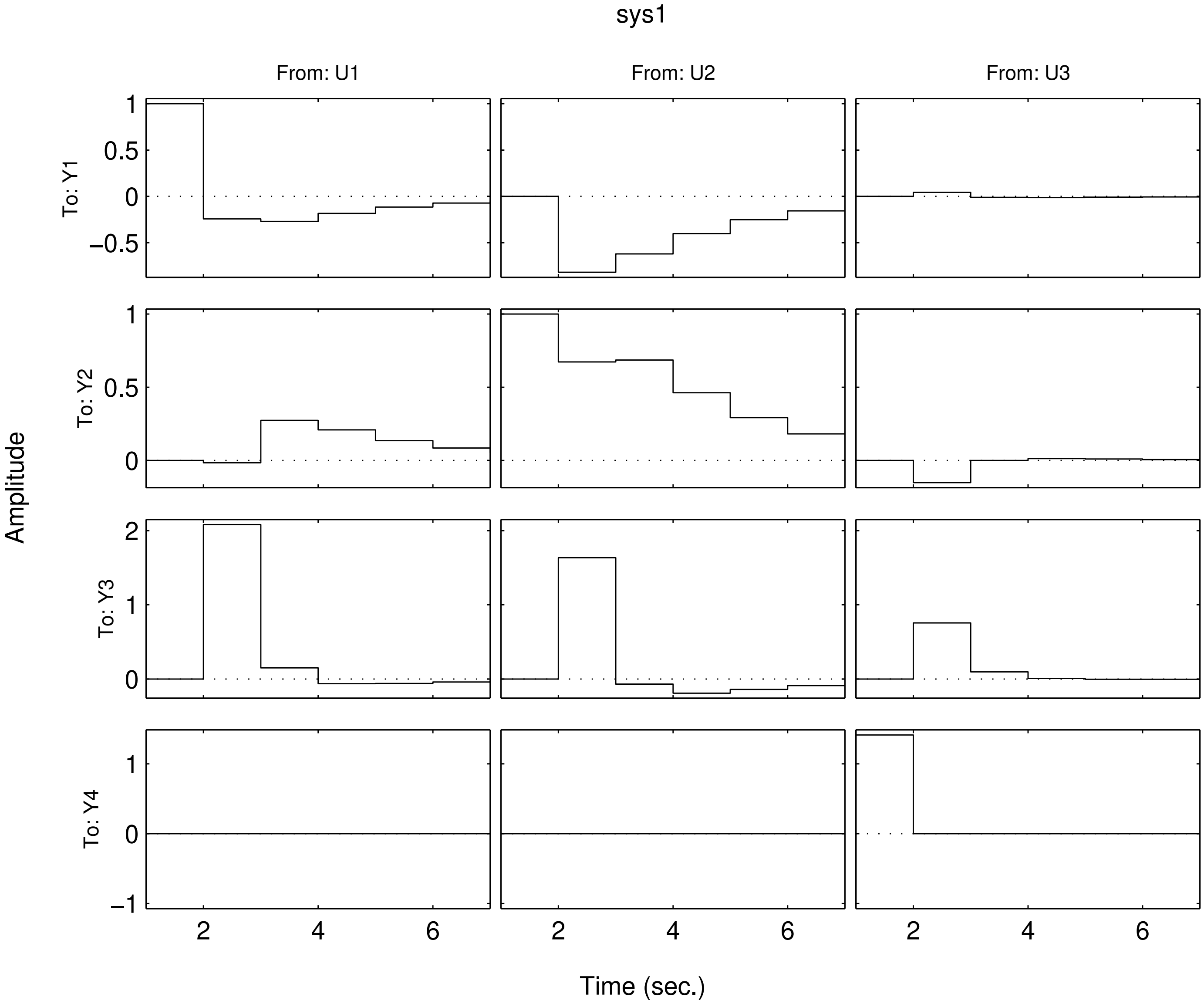
Figure 4: Impulse response of $[\, y \quad \pi \quad e_{-1} \quad \nu \,]'$ to $w_t = [\, \epsilon_t \quad \eta_t \quad 1/\sqrt{2}\nu_t \,]'$ for Ball's model with $\sigma = -.2$. The shocks correspond to columns, the variables to rows.

Figure 5: Impulse response of the 'worst case' $[\, \epsilon_t \quad \eta_t \quad (1/\sqrt{2})\nu_t \,]'$ with respect to $[\, \epsilon_t \quad \eta_t \quad (1/\sqrt{2})\nu_t \,]'$ for Ball's model with $\sigma = -.1$. The shocks correspond to columns, the worst case shocks to rows.

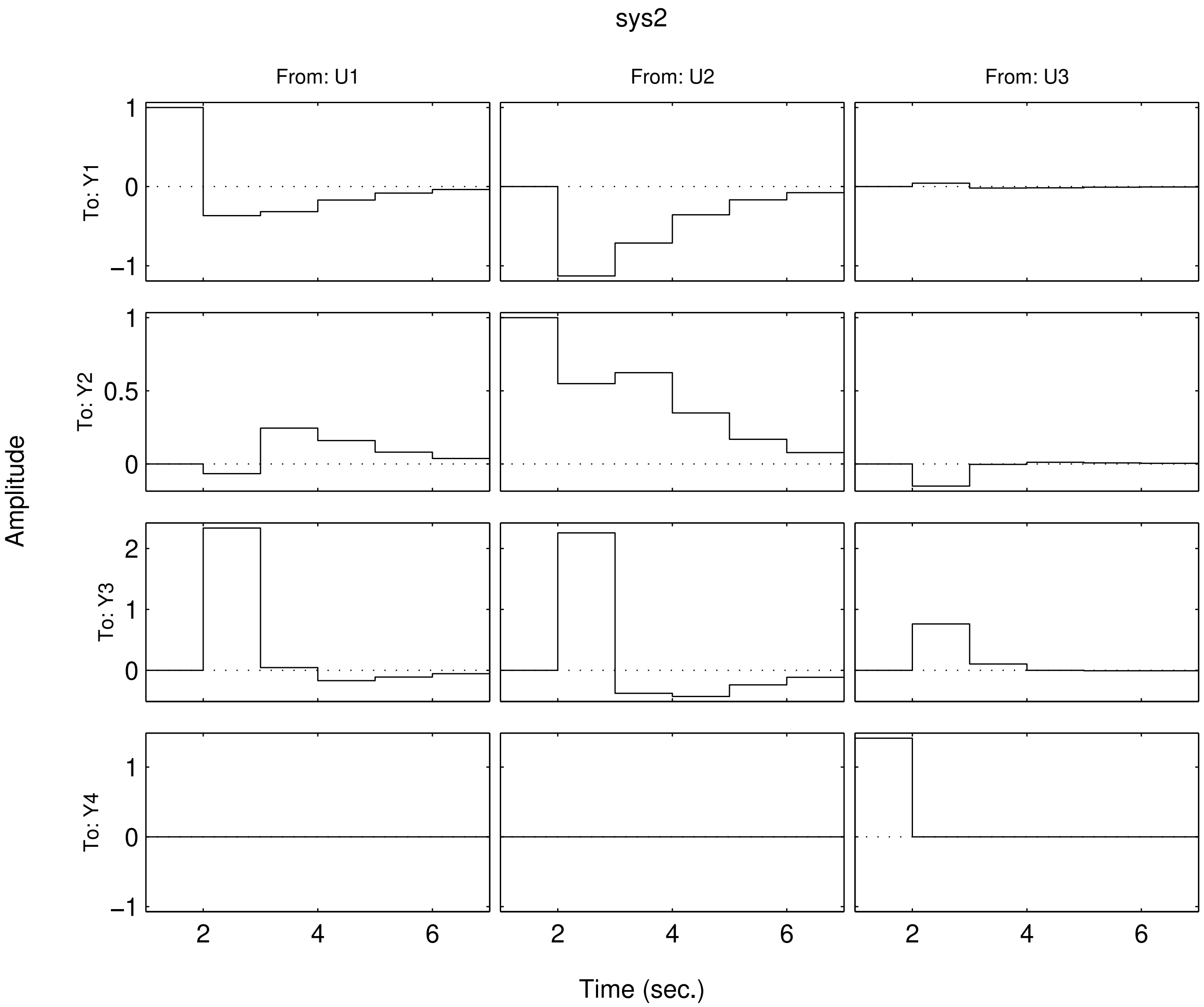
Figure 6: Impulse response of the 'worst case' $[\, \epsilon_t \quad \eta_t \quad (1/\sqrt{2})\nu_t \,]'$ with respect to $[\, \epsilon_t \quad \eta_t \quad (1/\sqrt{2})\nu_t \,]'$ for Ball's model with $\sigma = -.2$. The shocks correspond to columns, the worst case shocks to rows.



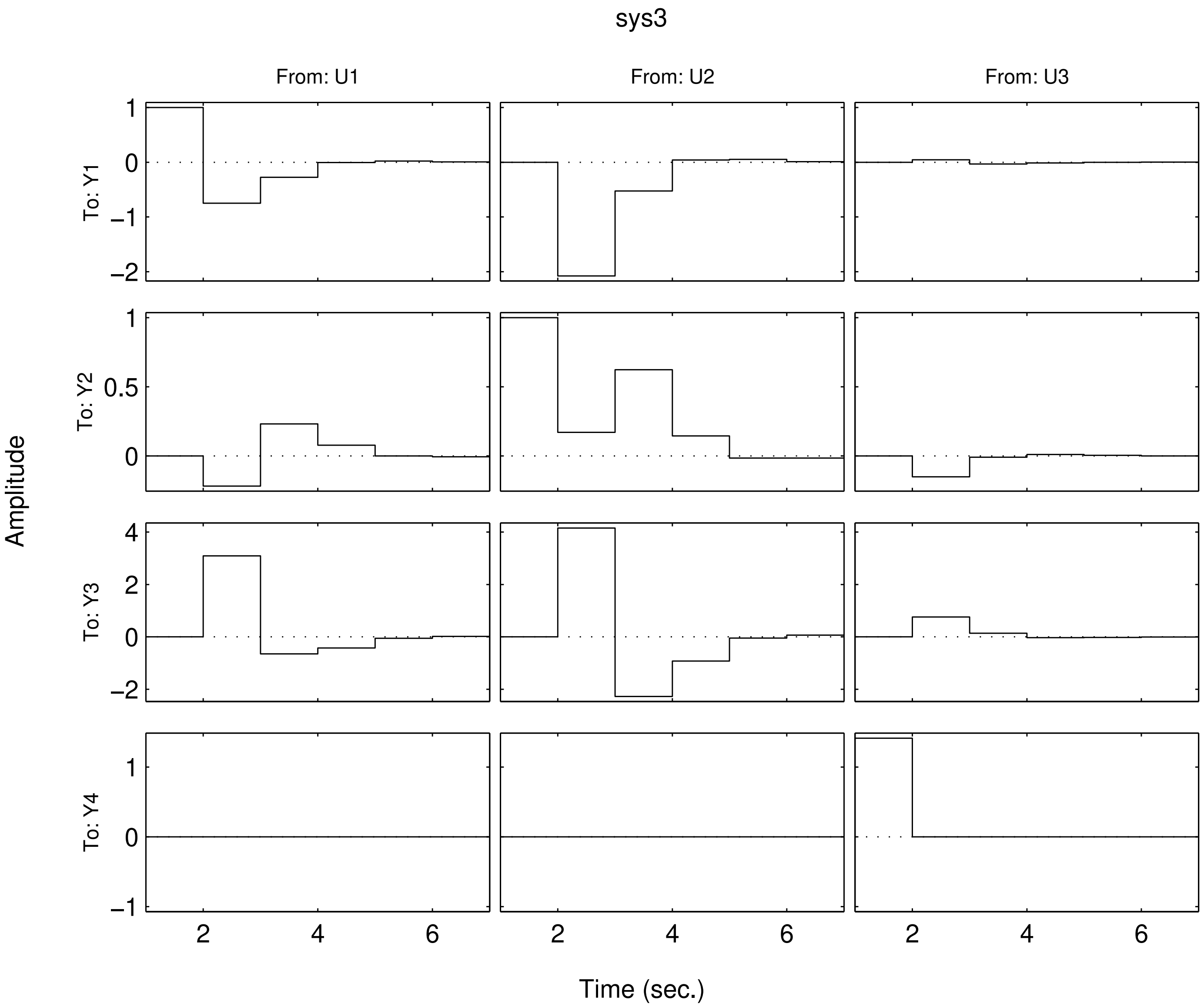
Sargent, comment on Ball, Figure 1



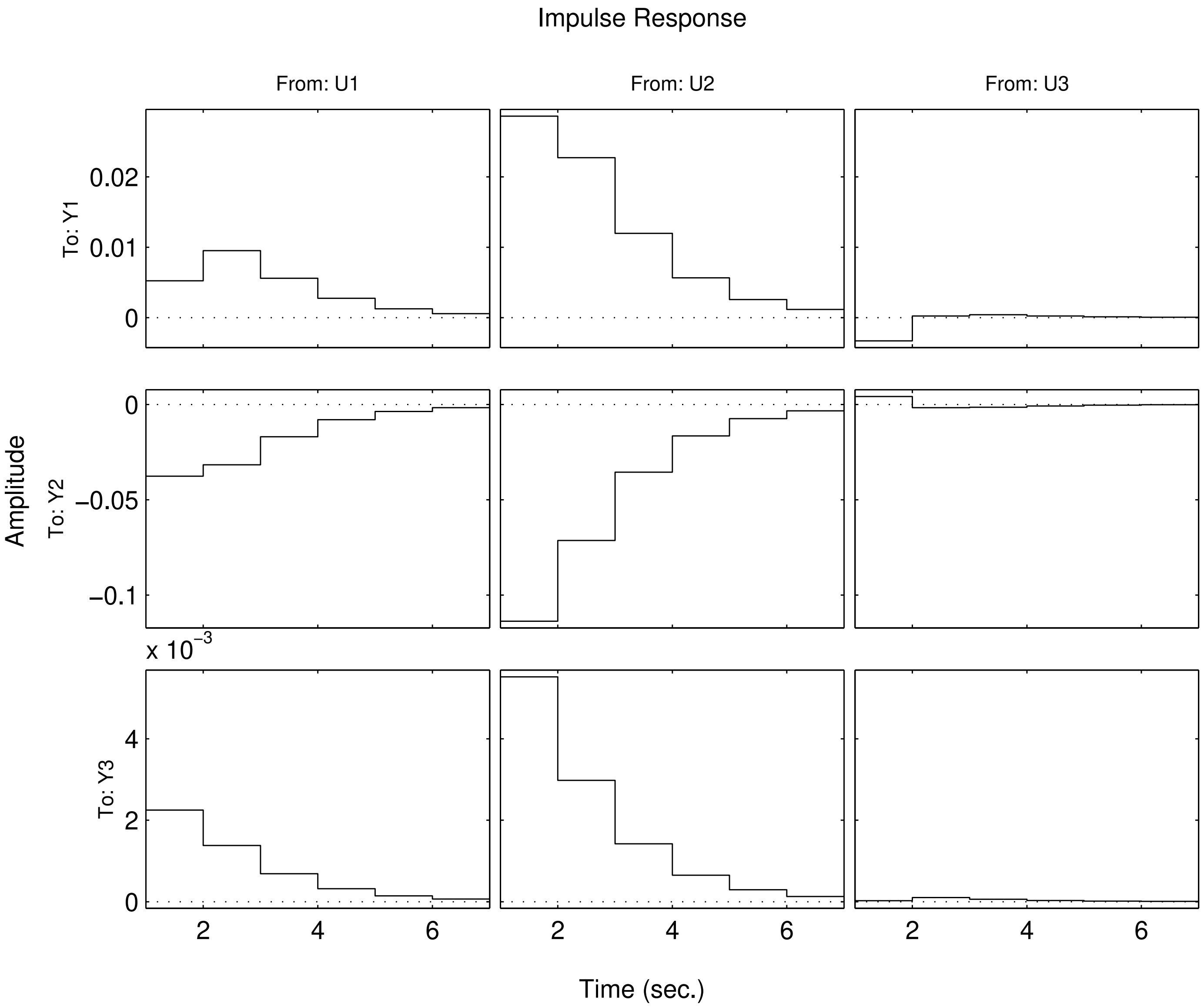
Sargent, comment on Ball, Figure 2



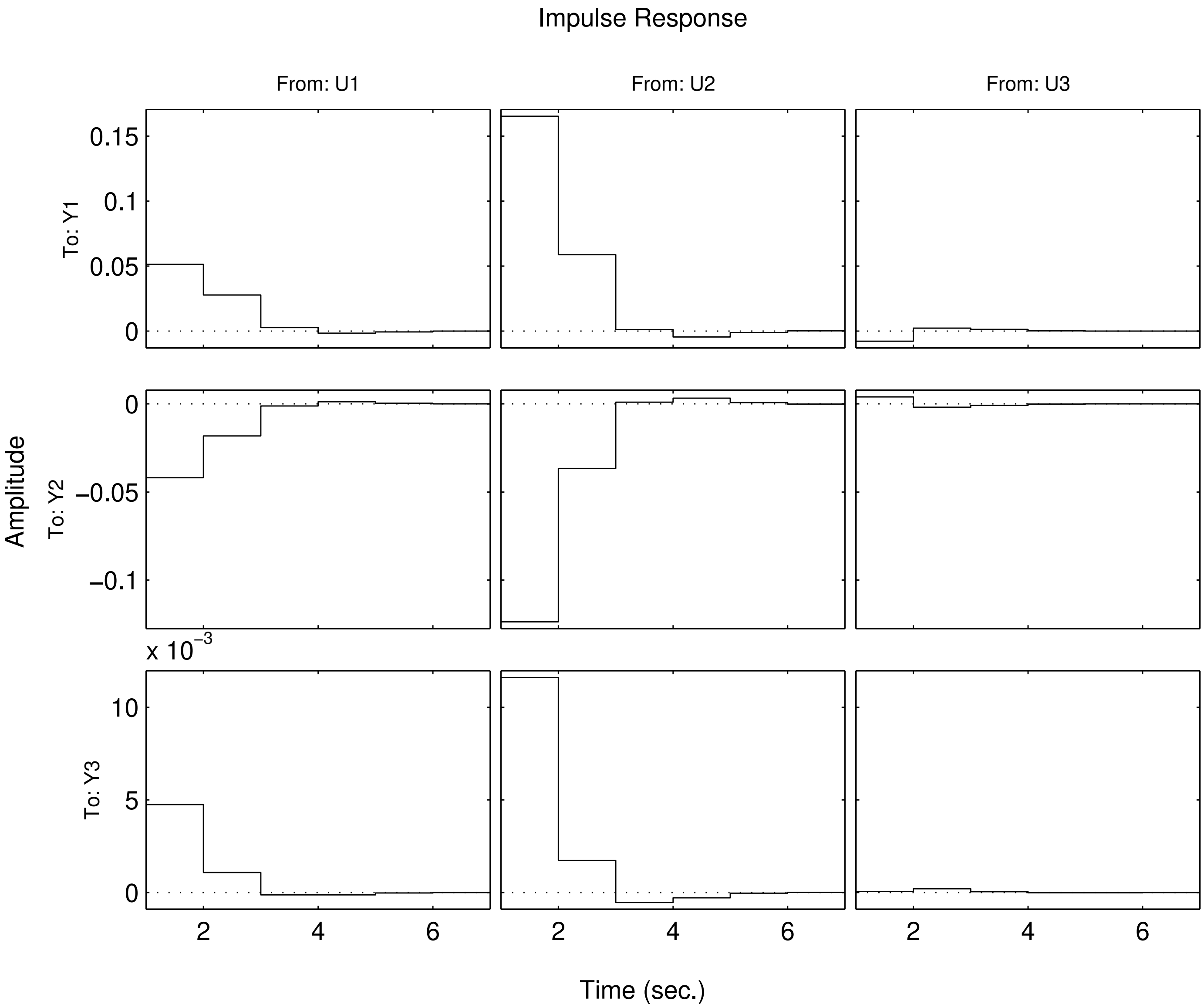
Sargent, comment on Ball, Figure 3



Sargent, comment on Ball, Figure 4



Sargent, comment on Ball, Figure 5



Sargent, comment on Ball, Figure 6

Notes

1. Ball thus adheres to Einstein's dictum that while nature might be complicated, it is not cruel. Contrast this modern view with the malevolent view of 'nature' embraced by the 'robust' monetary authority to be described shortly.

2. He computes one optimal rule for each of his relative weight parameterizing the government's objective function.

3. Robert Hall makes related and more extensive observations in his contribution to this conference volume.

4. For Ball's model, we can take $x_t = [\, y_t \quad \pi_t \quad e_{t-1} \quad \nu_t \,]'$, $u_t = e_t$, $A = \begin{bmatrix} \lambda & 0 & 0 & \beta/\theta \\ \alpha & 1 & \gamma & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $B' = [\, -(\beta/\theta + \delta) \quad -\gamma \quad 1 \quad 0 \,]$, $C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sqrt(2) \end{bmatrix}$.

5. For general background on robust control, see Zhou, Glover, and Doyle (1995), Başar and Bernhard (1995), and Mustafa and Glover (1990). See Whittle (1990) for an account of risk-sensitive control, and Hansen and Sargent (1995) for a recursive reformulation of risk sensitivity designed to accommodate discounting.

6. Max-min problems of this type occur widely in analyses of robustness. For a related problem in quite a different context, see Fudenberg and Levine (1995).

7. Their monetary authority has rational expectations and recommends 'look at everything' rules, just like Ball's.

8. The robust decision makers behavior *does* vary systematically with respect to variations in his 'reference model', the version of the Lucas critique that survives under robust decision theory.

9. See Mustafa and Glover (1990); see Hansen and Sargent (1998) for the formulations of H^∞ control modified to incorporate discounting. Hansen and Sargent also describe the 'minimum entropy' formulation below, and how it relates to the H^∞ criterion. In an interesting example, Kasa (1998) studies how a robust controller would behave in ways that an outsider might interpret as responding to adjustment costs.

10. Hansen and Sargent (1998) derive this criterion for discounted problems relate it to the H^∞ and risk-sensitivity formulations.

11. See Anderson, Hansen, and Sargent (1997) and Hansen and Sargent (1998). These papers also relate the risk-sensitive formulation to measures of model misspecification like those used by White (1982) and Sims (1972).

12. They agree where they should.

13. That robust control can be 'more active' comes through in results reported by Zhou, Glover, and Doyle (1996, pp. 438-439).

14. While the optimal rules remain linear, their dependence on innovation variances reflects a break down of 'certainty equivalence'.

15. The L^∞ norm for $\sigma = 0, -.1, -.2$ are $(3.13, 2.68, 2.22)$; the L^2 norms are $1.57, 1.61, 1.97$.

16. The figures also show that the worst case shocks become are less serially correlated for the σ that is larger in absolute value. In interpreting this diminished serially correlation, it has to be remembered that these objects are the outcome of the

'game' associated with a min, max problem, and that the 'great deceiver' picking these worst case errors faces a different feedback rule F for each σ.

17. For example, the agents inside the four models compared in of Levin, Wieland, and Williams' paper all live in secure rational expectations environments, and have no need to trouble themselves with the doubts over models that proccupy Levin, Wieland, and Williams'. I believe that the three authors' comforting finding of robustness across models might mainly reflect the proximity of the four models. Note that the four models compared are all rational expectations models with 'forward looking' private sectors, and all have been thoughtfully designed to fit recent U.S. data and to incorporate similar monetary control channels. It would be more challenging for rules also to be robust against the non-forward looking models illustrated by those of Ball and Rudebusch and Svensson.

18. Evan Anderson (1998) studies a multiple agent environment with robust decision makers whose preferences for robustness diverge.

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