Risk-sensitive real business cycles[☆]

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Abstract

This paper considers the business cycle, asset pricing, and welfare effects of increased risk aversion, while holding intertemporal substitution preferences constant. I show that increasing risk aversion does not significantly affect the relative variabilities and co-movements of aggregate quantity variables. At the same time, it dramatically improves the model's asset market predictions. The welfare costs of business cycles increase when preference parameters are chosen to match financial data. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Dynamic general equilibrium models have been successful in explaining some of the relative variabilities and co-movements of aggregate quantities such as output, consumption, and investment. However, despite the fact that the solutions of these models are competitive equilibria, little attention has been paid to their implications for asset returns. Typically, risk aversion and the subjective discount factor are chosen to match some interest rate, usually the average return on capital as is the case in Kydland and Prescott (1982) and King et al. (1988). Regardless of the calibration, these models are not able to generate risk premia that correspond to the to 6% annual equity premium discussed in Mehra and Prescott (1985), match the levels of returns, and replicate aggregate fluctuations all at the same time.

I use a standard stochastic growth model to study both business cycle and asset pricing issues. In particular, I show that risk aversion can be increased in a way that improves the model's performance with regard to asset pricing while not significantly diminishing its ability to account for quantity dynamics. Increased risk aversion also leads to welfare costs of business cycles much higher than those computed by Lucas (1987).

Starting from log utility, I increase risk aversion while holding the elasticity of intertemporal substitution constant, following the work of Epstein and Zin (1989, 1991). One consequence of this preference specification is that the representative agent is no longer a Von Neumann–Morgenstern expected utility maximizer. Previous work by Rouwenhorst (1995) has increased risk aversion in an expected utility context only to find that consumption becomes smoother. Unfortunately, one interpretation of the equity premium puzzle is that consumption is already too smooth.

This apparent inability to reconcile business cycle and asset market facts has resulted in macroeconomic models that account for quantity behavior while ignoring counterfactual asset market implications and asset pricing models that explain financial data while assuming that either consumption or returns are exogenous. This paper attempts to bridge the gap between these two literatures. Papers by Boldrin et al. (1995), Jermann (1998), and Lettau and Uhlig (1995) have sought to explain the equity premium in a 'business cycle' model by introducing habit formation into agents preferences. While those papers break the standard assumption of time separability in preferences, I consider preferences that are no longer separable across states of the world. Both approaches have been pursued in the asset pricing literature as a solution to the equity premium puzzle with some success.[1]

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[1] Some examples are: Constantinides (1990), Epstein and Zin (1991), Kandel and Stambaugh (1991).

Before I present the stochastic growth model with increased risk aversion, I will discuss some of the asset market and welfare effects of my preference generalization in an endowment economy. This is standard practice in much of the asset pricing literature and it seems to be a good place to start. If I cannot replicate moments from financial data when consumption is given exogenously, it is unlikely that my model will be successful when consumption becomes endogenous. In fact, given a realistic consumption process, the model is capable of generating a risk-free rate and market price of risk (the trade-off between risk and return) that are consistent with the data using reasonable parameter values. Increased risk aversion, disentangled from intertemporal substitution, works to raise the market price of risk and lower the risk-free rate simultaneously. I then use these parameter values to calculate the welfare cost of business cycles and find that once you take financial data into account, the cost of cycles is high. I also show that trend stationary and random walk models require substantially different levels of risk aversion to match the data and make sharply different predictions about the costs of cycles.

Next, I examine the business cycle and asset market behavior of a stochastic growth model based on Christiano and Eichenbaum (1992) but with these generalized preferences. Changing the level of risk aversion has virtually no impact on the second moments of aggregate quantities that are derived under expected utility. These moments are the primary focus of the real business cycle literature. I show that the risk-free rate and market price of risk move closer to the data as risk aversion is increased. The effect on the price of risk is considerably smaller in the production economy than the endowment economy for a given level of risk aversion, while the effect on the interest rate is a bit larger. Risk aversion seems to affect asset market implications and the elasticity of intertemporal substitution affects quantity dynamics. The welfare implications are even stronger in the production economy. The end result is that it is possible, and important, to study the behavior of aggregate quantities and financial variables in the same model.

Because the elasticity of intertemporal substitution has been fixed at one, the infinite horizon utility function of the representative agent can be transformed into a functional form that is very similar to the one used in Hansen et al. (1999). That paper generalizes quadratic preferences with an exponential risk functional. The result is that the objective functions in the models studied in this paper are approximated with 'risk-sensitive' linear-quadratic objectives and solved using the methods described in Hansen and Sargent (1995). One feature of this method is that certainty equivalence no longer holds so that the variance of the shocks affect the optimal decision rules. This will play an important role in the analysis.

The rest of the paper is organized as follows. Section 2 describes the exact specification for preferences that will be used here. Asset market implications and welfare costs of uncertainty are discussed in Sections 3 and 4, respectively.

I present the growth model in Section 5. Section 6 describes the approximation technique that will be used to solve the model. The implications of increased risk aversion for the quantity and asset market variables of the baseline model are presented in Section 7 as well as some additional welfare calculations. Section 8 concludes.

2. Preferences

I consider economies in which the agents have non-expected utility preferences. In particular, each agent has a utility function of the form[2]

$$U_t = \log c_t + \beta \frac{1}{(1-\beta)(1-\chi)} \log(E_t[\exp\{(1-\beta)(1-\chi)U_{t+1}\}]), \tag{1}$$

where β is the subjective discount factor, χ is the coefficient of relative risk aversion with respect to atemporal wealth gambles, and E_t is the conditional expectation operator at date t. These preferences are in the class of recursive utility functions considered by Epstein and Zin (1989, 1991). Under certainty, the elasticity of intertemporal substitution is one. When $\chi = 1$ the preferences collapse into the more familiar expected utility case. When $\chi > 1$ the agent is more risk averse relative to the expected utility case.[3] Since these preferences are homothetic and I assume that all agents have identical preferences, I can focus on a representative agent.

Define a new parameter σ as

$$\sigma \equiv 2(1-\beta)(1-\chi) \tag{2}$$

and rewrite U_t as

$$U_t = \log c_t + \beta(2/\sigma)\log(E_t[\exp\{(\sigma/2)U_{t+1}\}]). \tag{3}$$

This representation is very similar to the recursion used in Hansen et al. (1999).[4] As a result, I will use the risk-sensitive linear quadratic control methods used in that paper (from Hansen and Sargent, 1995) to compute approximate solutions for the models developed here.

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[2] For the purposes of this section and the next two, preferences will be defined over a single consumption good. Later, the utility function will be generalized to include leisure.

[3] As noted by Kreps and Porteus (1978), Epstein and Zin (1989), Weil (1990), and others, when $\chi > (<)\eta$, the agent prefers early (late) resolution of uncertainty. This aspect of preferences will not play a role in the analysis here.

[4] It also looks like the utility function considered by Weil (1993), but it is not. Weil studies preferences that exhibit constant *absolute* risk aversion while the preferences I consider exhibit constant *relative* risk aversion. The utility function in Eqs. (1) and (3) is a monotonic transformation of the utility function in Epstein and Zin (1991) for the special case of unit elasticity of intertemporal substitution. See Tallarini (1996) for details.

3. Asset market implications

Before I examine the implications of increased levels of risk aversion ($\chi > 1$) for business cycle issues, I would like to consider some of the asset market and welfare implications of the preferences described in the previous section. In this section I will study the properties of the intertemporal marginal rate of substitution in two endowment economies. Measures of the welfare cost of fluctuations will be presented in the next section.

Consumption (as well as output, investment, and other aggregate variables) is usually parameterized in one of two ways: difference stationary or trend stationary with highly serially correlated deviations from trend. From a statistical point of view the two hypotheses are hard to discriminate between since the relevant tests lose power when a series is as serially correlated as consumption. After removing a linear trend from logged consumption, the first auto-correlation is around 0.99. Christiano and Eichenbaum (1989) discuss the evidence for and against a unit root in output as well as the implications for economic theory of the presence of a unit root. Here, I will consider both specifications for consumption.

Starting with the trend stationary assumption, consider a consumption stream given by

$$c_t^{\text{TS}} = \exp(\gamma t + z_t), \tag{4}$$

where z_t follows a first-order autoregression

$$z_t = \rho z_{t-1} + \varepsilon_t, \quad \{\varepsilon_t\} \text{ i.i.d.} \sim \text{N}(0, \sigma_\varepsilon^2). \tag{5}$$

This results in a log consumption series that is composed of a linear trend with AR(1) deviations.

The random walk specification is as follows:

$$c_t^{\text{RW}} = \exp(y_t), \tag{6}$$

where y_t is a random walk with drift:

$$y_t = \gamma + y_{t-1} + \varepsilon_t, \quad \{\varepsilon_t\} \text{ i.i.d.} \sim \text{N}(0, \sigma_\varepsilon^2). \tag{7}$$

The standard way one period assets are priced is by means of a stochastic discount factor, or, more specifically in consumption based models, the intertemporal marginal rate of substitution. With a state-separable logarithmic utility specification, the marginal rate of substitution, $m_{t+1,t}$, is just β times the ratio of today's consumption to tomorrow's. In the non-state-separable case I am studying, the marginal rate of substitution is slightly more complicated. Now future utility levels enter in:

$$m_{t+1,t} = \beta \, \frac{c_t}{c_{t+1}} \frac{\exp((1 - \beta)(1 - \chi)U_{t+1})}{\text{E}_t[\exp((1 - \beta)(1 - \chi)U_{t+1})]}. \tag{8}$$

The date t conditional expectation of this random variable is the price of a risk-free claim to one unit of consumption at date $t + 1$, or the reciprocal of the one period risk-free interest rate:

$$\frac{1}{r_t^f} = E_t[m_{t+1,t}] = E_t\left[\beta\frac{c_t}{c_{t+1}}\frac{\exp((1-\beta)(1-\chi)U_{t+1})}{E_t[\exp((1-\beta)(1-\chi)U_{t+1})]}\right]. \tag{9}$$

This expectation can be interpreted as either the integral of the true IMRS with respect to the true conditional probability measure or as the integral of the standard IMRS ($\beta c_t/c_{t+1}$) with respect to a distorted probability measure. Let $V_{t+1} = \exp((1-\beta)(1-\chi)U_{t+1})$. The distorted expectation operator,

$$\hat{E}_t[\phi] \equiv E_t\left[\frac{\phi V_{t+1}}{E_t[V_{t+1}]}\right], \tag{10}$$

is similar to the risk neutral transformations used in pricing derivative securities (e.g., Harrison and Kreps, 1979). This interpretation comes from the risk-sensitive control literature, see Whittle (1990).

Following the analysis of Hansen and Jagannathan (1991), I can use the first two moments of $m_{t+1,t}$ to determine the market price of risk. This quantity is defined as the ratio of the standard deviation of the marginal rate of substitution to its mean. Hansen and Jagannathan demonstrate how this ratio is related to the mean-standard deviation frontier for asset returns. In fact, they show that the market price of risk implied by an admissible marginal rate of substitution must be greater than or equal to the absolute value of the ratio of mean-to-standard deviation of a zero-price excess return, say, ξ:

$$\frac{|E[\xi]|}{\sigma(\xi)} \leq \frac{\sigma(m)}{E[m]}, \tag{11}$$

where $\sigma(x)$ is the standard deviation of x. When no risk-free asset exists, this bound becomes a function of the value assumed for the risk-free rate.

The average risk-free rate and market price of risk can be directly computed for the two environments considered in this section. For the trend stationary model

$$E[r_t^f] = \frac{1}{\beta}\exp\left[\gamma - \frac{\sigma_\varepsilon^2}{2}\left(1 - \frac{2(1-\beta)(1-\chi)}{1-\beta\rho} - \frac{1-\rho}{1+\rho}\right)\right] \tag{12}$$

$$\frac{\sigma(m)}{E[m]} = \left\{\exp\left[\sigma_\varepsilon^2\left(\left\{\frac{(1-\beta)(1-\chi)}{1-\beta\rho} - 1\right\}^2 + \frac{1-\rho}{1+\rho}\right)\right] - 1\right\}^{1/2}. \tag{13}$$

Table 1

Asset market data. Sample moments from quarterly U.S. data. 1948:2–1993:4. r^e is the return on the value-weighted NYSE portfolio and r^f is the return on the three month Treasury bill. Returns are measured in percent per quarter

Return	Mean	Std. dev.
r^e	2.20	7.92
r^f	0.23	0.80
$r^e - r^f$	1.97	7.82
Market price of risk: 0.2525		

For the random walk model

$$r^f = \frac{1}{\beta}\exp\left[\gamma - \frac{\sigma_\varepsilon^2}{2}(2\chi - 1)\right], \qquad (14)$$

$$\frac{\sigma(m)}{\mathrm{E}[m]} = \{\exp[\sigma_\varepsilon^2\chi^2] - 1\}^{1/2}. \qquad (15)$$

Table 1 reports the mean and standard deviation of the quarterly real returns on the value-weighted New York Stock Exchange portfolio and the three month Treasury bill from 1948 to 1993. The quarterly equity premium is about 2%. The last line of the table reports the market price of risk as measured by the ratio of the standard deviation of the excess return of the value-weighted portfolio (over the treasury bill) to its mean. I will use these moments to chose my target interest rates and market prices of risk in Sections 4 and 7.

The differences between the trend stationarity and random walk models as well as the differences between these preferences and the more standard expected utility specification are visible in Fig. 1. The Hansen–Jagannathan region computed with the returns on the value weighted NYSE and the three month Treasury Bill is in the upper right corner. Note that the minimum standard deviation of the admissible region is very close to the estimated market price of risk in Table 1. The figure also plots the mean-standard deviation pairs for different values of χ under three different assumptions. All three cases have $\beta = 0.995$. The circles correspond to the random walk model of consumption. The pluses correspond to the trend stationary model of consumption. The $\times$'s correspond to the expected utility case with no parametric assumptions about the consumption stream. For the non-expected utility specifications, the relevant parameters are estimated from quarterly data on the growth of consumption of non-durables and services from 1948:2 to 1993:4, then the point estimates are substituted into the formulas for the IMRSs. In all three cases, the sample mean and standard deviation are computed for the time series of IMRS's.
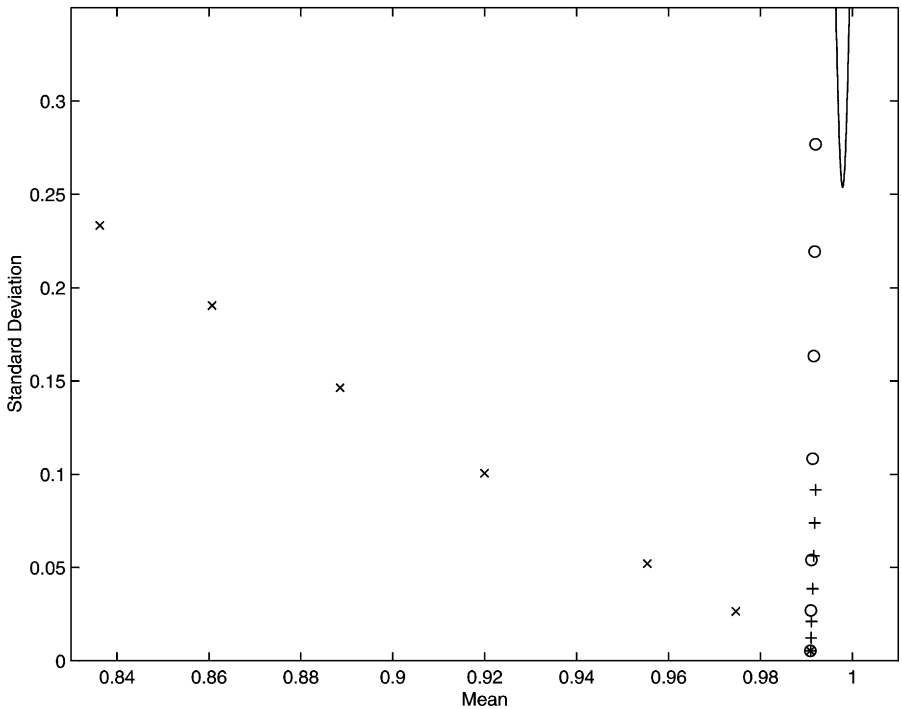
Fig. 1. *Solid line*: Hansen–Jagannathan volatility bound for quarterly returns on the value-weighted NYSE and Treasury bill, 1948–1993. *Circles*: Mean and standard deviation for intertemporal marginal rate of substitution generated by non-expected utility preferences with random walk consumption. *Pluses*: Mean and standard deviation for intertemporal marginal rate of substitution generated by non-expected utility preferences with trend stationary shocks. $\times$'s: Mean and standard deviation for intertemporal marginal rate of substitution for expected utility preferences. The coefficient of relative risk aversion, χ, takes on the values 1, 5, 10, 20, 30, 40, 50. In all cases the discount factor $\beta = 0.995$.

With non-expected utility, increasing risk aversion when consumption is trend stationary has a smaller effect on the asset market implications of the model than when consumption follows a random walk. The standard deviation for each value of χ (1, 5, 10, 20, 30, 40, 50) plotted in Fig. 1 is less than the standard deviation under a random walk. In other words, the pluses are below the circles. As β gets closer to 1, increasing χ has an even smaller effect on the standard deviation under trend stationarity. Looking at Fig. 1 again, the $\times$'s show that while increasing risk aversion increases the standard deviation of the IMRS under expected utility, it simultaneously reduces the mean since the elasticity of intertemporal substitution is decreasing. This is the 'risk-free rate' puzzle of Weil (1989). Increasing χ moves the mean-standard deviation pair even further from the admissible region. Changing β for either the random walk,

non-expected utility case or the expected utility case simply moves the mean-standard deviation pair along a ray through the origin. The market price of risk under expected utility and random walk consumption is given in Eq. (15), the formula for the non-expected utility case. The risk-free rate is given by:

$$r^{\mathrm{f}} = \frac{1}{\beta}\exp\left[\chi\left(\gamma - \frac{\sigma_{\varepsilon}^2}{2}\chi\right)\right].$$ (16)

This accounts for the differences in the figure.[5] In fact, the market price of risk, or the slope of the ray going from the origin through the mean-standard deviation pair in Fig. 1, when consumption follows a random walk is determined by the coefficient of relative risk aversion independent of the elasticity of intertemporal substitution. The risk-free rate is affected by both risk aversion and the elasticity of intertemporal substitution.

4. Welfare costs

In this section I consider the welfare costs of business cycles by performing calculations similar to those in Lucas (1987). In particular, given the preferences and endowment processes laid out in the two previous sections I compute the amount of consumption in all states and dates the representative agent would need to be indifferent between some baseline consumption process and one with a different variance. Obstfeld (1994) performs similar calculations to those which I will present later on in this section. He considers preferences which are of the same class as those here, but for $\eta \neq 1$. Both Lucas and Obstfeld consider the welfare effects of changes in consumption growth rates as well as fluctuations. With $\eta = 1$, different values of χ do not affect the growth trade-off, so I will not pursue those issues here. Dolmas (1996) computes the welfare costs of business cycles in a model similar to those in this paper and Obstfeld (1994) and also considers the effects of first-order risk aversion (see Epstein and Zin, 1990).

For the purposes of these welfare cost calculations, I need to make a small modification in each of the two specifications of consumption. Under trend

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[5] The difference between preference specifications also appears in statistical tests of whether the moments of the IMRS are in the admissible region. For β close to unity, the hypothesis that the mean-standard deviation pair is in the Hansen–Jagannathan region cannot be rejected at the 10% level for all values of $\chi \geq 1$ with the preferences I consider. On the other hand, with expected utility preferences, increasing χ makes it easier to reject the hypothesis. For details see Tallarini (1996).

stationarity, now let consumption be given by

$$c_t^{\text{TS}} = \exp\left(\lambda + \gamma t - \frac{\sigma_\varepsilon^2/2}{1-\rho^2} + z_t\right),$$ (17)

where z_t follows a first-order autoregression as in Eq. (5). The parameter λ will be explained momentarily.

The random walk specification is now:

$$c_t^{\text{RW}} = \exp(\lambda + y_t),$$ (18)

where y_t is a random walk with drift:

$$y_t = (\gamma - \tfrac{1}{2}\sigma_\varepsilon^2) + y_{t-1} + \varepsilon_t$$ (19)

and $\{\varepsilon_t\}$ has the same distribution as above.

The unconditional variance is included in Eq. (17) to preserve the mean of the consumption process. Without this correction, an increase in the variance would imply in an increase in the mean for consumption, reducing any welfare losses from the increased variance. Similarly, the innovation variance is included in the drift term in Eq. (19). Now, it is the conditional mean of consumption that I am holding constant in the face of changing variance.

The parameter λ will be used to measure the welfare costs of fluctuations. Changing λ alters the level of consumption in all states and dates. For instance, for a consumption process with $\lambda = \lambda_1$, consumption is $e^{\lambda_1 - \lambda_0}$ times the consumption associated with a consumption process with $\lambda = \lambda_0$.

The next step involves the choice of a baseline consumption stream. One option is to take the unconditional mean of consumption as the benchmark. This is the choice made by Lucas (1987). This would be fine if I were only considering the trend stationary assumption. Since the random walk process for consumption does not have an unconditional mean, I will use the *conditional* mean of consumption as my benchmark process. So λ will be computed to make the representative agent indifferent between a variable consumption stream and the deterministic path given by $\{\text{E}_t[c_{t+j}]\}$.

Now I can compute welfare cost functions. For the trend stationary model:

$$\hat{\lambda}_{\text{DT}}(\rho, \sigma_\varepsilon^2; \beta, \chi) = \frac{\sigma_\varepsilon^2}{2}\left[\frac{\beta}{1-\beta\rho^2} + \frac{\beta(1-\beta)(\chi-1)}{(1-\beta\rho)^2}\right].$$ (20)

For the random walk model:

$$\hat{\lambda}_{\text{RW}}(\sigma_\varepsilon^2; \beta, \chi) = \frac{\sigma_\varepsilon^2}{2}\frac{\beta}{1-\beta}\chi.$$ (21)

The limit of $\hat{\lambda}_{DT}$ as ρ goes to 1 is $\hat{\lambda}_{RW}$. This follows from my choice of the conditional mean as the baseline specification. Holding the innovation variance constant, $\hat{\lambda}_{DT}$ is strictly increasing in ρ when ρ is positive. Both cost functions are linear in the risk aversion parameter, χ, and independent of the current level of consumption.

At this point I can compute the welfare costs associated with different values for χ and β, given estimates of the parameters for the two consumption processes. However, I think it is worthwhile to use some of the asset market implications presented in the previous section as a guide for choosing relevant values for β and χ. Given an average growth rate of 0.42% per quarter, a standard deviation of 0.55% for the innovations under either assumption of stationarity, and a value of 0.99 for the serial correlation parameter in the trend stationary case, I can compute risk free rates, market prices of risk, and compensation functions for various values of β and χ under the two specifications for consumption.

Conversely, I can compute values for β and χ that produce a particular combination of risk-free rate and market price of risk, given parameter values for the endowment processes, and then compute welfare costs. Empirical values are reported in Table 1 above. Table 2 reports (β,χ) pairs that match the interest rate and market price of risk. I consider interest rates from 0.25% per quarter (roughly the sample average) up to 1% and market prices of risk from 0.125 to 0.5. It is not possible, given the parameter estimates for the consumption processes, to match a low market price of risk and a low interest rate with values of $\beta < 1$.

Note that the values of β are the same under the two different specifications for consumption. The degree of risk aversion, χ, on the other hand differs greatly between the specifications. From Eq. (15) with random walk consumption the market price of risk determines χ, independent of β. With trend stationary consumption, this is no longer the case. The levels of risk aversion, for a given value of the market price of risk, are decreasing in the interest rate and much higher than those needed in the random walk case. While these values for χ may appear to be 'too high' I should point out that some of the counterfactual implications associated with high risk aversion are not present here. For example, high risk aversion is often associated with high interest rates (see Weil, 1989 and below), which is not the case here since I chose parameters to match low interest rates. Also, high risk aversion is associated with excessively smooth consumption in production economies (see Rouwenhorst, 1995). Section 7 shows how that argument does not apply here either. See Cochrane (1997) and Kandel and Stambaugh (1991) for additional support of high risk aversion.

Given these (β,χ) pairs, I calculate the welfare cost of fluctuations. Table 3 reports the amount consumption would have to be increased in all dates and states, in percentage terms, to make consumers indifferent between a deterministic consumption process and what we actually observe. In computing

Table 2

(β,χ) *Pairs*. Values of the discount factor, β, and risk aversion parameter, χ, that produce the indicated market price of risk, M.P.R., and average risk-free rate, $E[r^f]$, in the endowment economies. '—' indicates combination not feasible for $\beta < 1$. *Panel* (a): random walk economy; *Panel* (b): trend stationary economy

$E[r^f]$	M.P.R.			
	0.125	0.25	0.375	0.5
(a) Random walk				
0.25	—	—	(0.9997, 66.0)	(0.9991, 85.9)
0.50	(0.9985, 22.6)	(0.9979, 44.8)	(0.9972, 66.0)	(0.9966, 85.9)
0.75	(0.9961, 22.6)	(0.9954, 44.8)	(0.9948, 66.0)	(0.9942, 85.9)
1.00	(0.9936, 22.6)	(0.9929, 44.8)	(0.9923, 66.0)	(0.9917, 85.9)
(b) Trend stationary				
0.25	—	—	(0.9997, 2236)	(0.9991, 1036)
0.50	(0.9985, 171)	(0.9979, 250)	(0.9972, 300)	(0.9966, 337)
0.75	(0.9961, 77.4)	(0.9954, 140)	(0.9948, 189)	(0.9942, 230)
1.00	(0.9936, 56.2)	(0.9929, 106)	(0.9923, 150)	(0.9917, 187)

these costs, I assume that preferences are described by the parameter values in Table 2. These costs are much larger than those reported in Lucas (1987). However, Lucas did not consider asset market implications in his experiment. The parameter values I use have both a higher discount factor and higher level of risk aversion.

Welfare costs under a random walk are much higher than with a deterministic trend despite lower risk aversion. This is especially clear for an interest rate of 0.25 and price of risk of 0.375. In that case $\beta = 0.9997$. The cost here is much larger than for the same case under trend stationarity. The difference is that with a random walk, variance grows without bound and the future is very highly weighted: for example, the discount factor on consumption 75 years (300 periods) in the future is $0.9997^{300} = 0.914$. Under trend stationarity, the variance is finite and therefore not as much of a concern. The costs, and the differences, would be much smaller if I had considered the finite horizon case.[6]

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[6] Campbell and Cochrane (1995) find even higher costs of fluctuations that vary over the business cycle.

Table 3

Compensation function. Percentage increase of consumption in all states and dates to make the representative agent indifferent between a non-stochastic economy and a stochastic economy. Preference parameters were chosen to match the indicated market price of risk, M.P.R., and risk-free rate, $E[r^f]$. '—' indicates combination not feasible for $\beta < 1$. (See Table 2) *Panel* (a): random walk economy; *Panel* (b): trend stationary economy

$E[r^f]$	M.P.R.			
	0.125	0.25	0.375	0.5
(a) Random walk				
0.25	—	—	3572	338
0.50	26.5	37.4	43.3	46.9
0.75	9.1	15.8	20.9	24.8
1.00	5.5	10.0	13.7	16.8
(b) Trend stationary				
0.25	—	—	10.1	12.6
0.50	3.0	5.7	8.1	10.1
0.75	2.4	4.7	6.7	8.5
1.00	2.1	4.0	5.7	7.3

5. The production economy

I now present the model I will use to study the effects of increased levels of risk aversion on business cycles. The model is a version of the one-sector stochastic growth model studied by Christiano and Eichenbaum (1992). Now preferences will be defined over both consumption and leisure. Rewrite Eq. (1) as

$$U_t = \log C_t + \theta \log L_t$$
$$+ \beta \frac{1 + \theta}{(1 - \beta)(1 - \chi)} \log\left(E_t\left[\exp\left(\frac{(1 - \beta)(1 - \chi)}{1 + \theta} U_{t+1} \right) \right] \right). \tag{22}$$

where L_t is the amount of leisure enjoyed at date t and $\theta > 0$. Redefine σ as

$$\sigma \equiv 2\frac{(1 - \beta)(1 - \chi)}{1 + \theta} \tag{23}$$

and rewrite U_t as

$$U_t = \log C_t + \theta \log L_t + \beta(2/\sigma)\log(E_t[\exp((\sigma/2)U_{t+1})]). \tag{24}$$

When preferences are independent of leisure as in Sections 2–4, χ is interpreted as the coefficient of relative risk aversion for atemporal consumption

wealth gambles. Now with leisure in the utility function, that interpretation does not always hold. χ is still the coefficient of relative risk aversion for atemporal wealth gambles, but now wealth is measured in terms of the composite commodity $C^{1/(1+\theta)}L^{\theta/(1+\theta)}$. Tallarini (1996) shows that the coefficient of relative risk aversion for atemporal wealth gambles, with deterministic leisure, is $(\chi + \theta)/(1 + \theta)$. Thus, under expected utility ($\chi = 1$), incorporating leisure into the utility function does not affect preferences over consumption gambles. However, when $\chi > 1$ leisure in the utility function results in a coefficient of relative risk aversion that is less than χ.

There is a single good produced in this economy. It is produced according to a constant returns to scale neoclassical production function. In particular, it has the Cobb–Douglas form

$$Y_t = K_{t-1}^{1-\alpha}(N_t X_t)^\alpha, \tag{25}$$

where K_{t-1} is the fixed stock of capital carried into date t, N_t is the labor input at t and X_t is an aggregate productivity shock. The logarithm of the productivity shock follows a random walk with drift

$$\log X_t = \gamma + \log X_{t-1} + \varepsilon_t, \tag{26}$$

where $\{\varepsilon_t\}$ is a sequence of i.i.d. random variables distributed $N(0,\sigma_\varepsilon^2)$.

The capital stock evolves according to

$$K_t = (1 - \delta)K_{t-1} + I_t, \tag{27}$$

where I_t is gross investment and δ is the depreciation rate of capital.

The representative agent is endowed with one unit of time which can be taken as leisure or used as labor, i.e.

$$L_t + N_t \leq 1. \tag{28}$$

Resources used for consumption and investment cannot exceed those produced in any given period, i.e.

$$C_t + I_t \leq Y_t. \tag{29}$$

The economy grows at the rate γ. Output, consumption, and investment all follow a stochastic trend with an average growth rate of γ. In order to solve the social planning problem associated with the equilibrium of this model for $\gamma \geq 0$ it must be transformed into a stationary problem. This is accomplished by dividing all variables by the contemporaneous level of aggregate productivity: $y_t = Y_t/X_t$, $c_t = C_t/X_t$, etc. Labor, and therefore leisure, is stationary and does not need to be transformed. Upper-case (non-stationary) variables are replaced by their lower-case (stationary) counterparts with two exceptions. First, the capital evolution equation, Eq. (27). This is now given by

$$k_t = \exp[-(\gamma + \varepsilon_t)](1 - \delta)k_{t-1} + i_t. \tag{30}$$

Second, the single-period utility function is now written as

$$u(c_t, L_t, \varepsilon_t) = \log c_t + \theta \log L_t + \frac{\varepsilon_t}{1 - \beta}, \tag{31}$$

to account for the effect of the permanent shocks. When $\chi = 1$, the presence of ε_t in the utility function is irrelevant. For values of $\chi \neq 1$ this is no longer the case. Under expected utility, the shocks appear in the objective function as a discounted sum, independent of consumption and leisure (see Christiano and Eichenbaum, 1992). When $\chi \neq 1$, preferences are no longer state-separable and therefore the effects of the innovations on the date t objective function are non-trivial.

Given an initial value for the capital stock, K_{-1}, and the level of productivity, X_{-1}, the social planner maximizes (22) subject to (25)–(29). This Pareto optimum can be decentralized in the usual manner into a competitive equilibrium.

I will calculate the risk-free rate as in Section 3. I will use the return to owning the capital stock as my equity return. Think of a continuum of identical firms who own the capital stock, produce output, invest, and pay wages and dividends to the workers. Since units do not matter, let there be one infinitely divisible share in each firm. Shares are traded at the end of each period, after investment decisions have been made. The share price can be computed easily; it is just the price of a unit of capital times the amount of capital owned by the firm. The price of a unit of capital is constant at unity. This is obvious from looking at the resource constraint after substituting for investment with the capital evolution equation. The dividend paid by the firm is whatever is left over after wages have been paid and investment goods purchased:

$$D_t = Y_t - w_t(N_t X_t) - I_t, \tag{32}$$

where w_t is the wage rate at date t and equal to the marginal product of effective labor, $w_t = \alpha K_{t-1}^{1-\alpha}(N_t X_t)^{\alpha - 1}$.[7] The equity return is then computed from date t to date $t + 1$ as

$$r_{t+1}^e = \frac{K_{t+1} + D_{t+1}}{K_t}. \tag{33}$$

Substituting several times yields a much simpler expression

$$r_{t+1}^e = (1 - \delta) + v_{t+1}, \tag{34}$$

where $v_t = (1 - \alpha)K_{t-1}^{-\alpha}(N_t X_t)^{\alpha}$ is the rental rate on capital in period t. Both the wage rate w_t and the rental rate v_t are stationary variables.

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[7] Alternatively, the wage could be considered to be the marginal product of actual labor, $W_t = \alpha K_{t-1}^{1-\alpha} N_t^{\alpha-1} X_t^{\alpha} = w_t X_t$. Eq. (32) would then read $D_t = Y_t - W_t N_t - I_t$.

6. Approximation method

The model described in the previous section has a closed-form solution only for the case of full depreciation, $\delta = 1$. In that case, the optimal decision rules are the same regardless of the value of χ; only the constant term in the value function is affected. Since I am interested in values of δ close to zero, I must use an approximation to solve the model. The approach I take is to follow Christiano (1990a,b) by using a log-linear quadratic approximation of the single period utility function. This requires taking a second-order Taylor expansion of Eq. (31) in terms of the state variables, $\log k_{t-1}$ and ε_t, and the control variables, $\log k_t$ and $\log N_t$. The approximate utility function is then substituted into Eq. (22) for $u(c, L)$. This creates a discounted linear exponential quadratic Gaussian optimal control problem which can be solved using the methods in Hansen and Sargent (1995). The variance of the technology shock now affects the solution of the approximation as well as the original model.[8]

Since certainty equivalence no longer holds, the deterministic steady state is not necessarily the appropriate point around which to center the approximation. Precautionary savings motives as discussed in Hansen et al. (1999) result in stochastic steady-state means that exceed the corresponding deterministic values. Therefore when I approximate risk-sensitive versions of the model ($\chi > 1$) the point around which the approximation is centered must be computed as part of the approximation algorithm. This results in a fixed-point problem in the steady-state mean of capital.

More explicitly, for a proposed steady-state mean of capital, say, $\bar{k}_j$, corresponding values are computed for the other variables in the system.[9] The resource constraints are substituted into $u(c_t, L_t, \varepsilon_t)$ and the utility function is rewritten as a function of $(\log k_t, \log N_t, \log k_{t-1}, \varepsilon_t)$. Then the single-period utility function is approximated by a second-order Taylor expansion around $(\log \bar{k}_j, \log \bar{N}_j, \log \bar{k}_j, 0)$. This allows the approximate single period utility function to be written as

$$\tilde{u}[\log k_t, \log N_t, \log k_{t-1}, \varepsilon_t] = v_t' Q v_t + x_t' R x_t + 2 x_t' S v_t, \tag{35}$$

where $v_t = [\log k_t \ \log N_t]'$, $x_t = [\log k_{t-1} \ \varepsilon_t \ 1]'$, and Q, R, and S are matrices of derivatives evaluated at the centering point. The state vector follows the

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[8] Jacobson (1973) and Whittle (1981,1990) showed how to break certainty equivalence in the context of a linear quadratic model. Hansen and Sargent (1995) showed how to incorporate discounting in an infinite horizon setting in an economically sensible manner.

[9] For each value of β, θ is chosen so that the steady state value of labor is $\bar{N} = 0.2305$ (from Christiano and Eichenbaum (1992)) in the expected utility, $\chi = 1$, case. As χ is varied, the steady-state mean for labor is allowed to change with θ held constant.

law of motion

$$x_{t+1} = Ax_t + Bv_t + Cw_{t+1}, \quad t = 0,1, \tag{36}$$

for appropriate matrices A, B, and C. Substituting Eq. (35) into Eq. (24) allows me to write an approximate objective function:

$$\tilde{U}_t = \tilde{u}(k_t, N_t, k_{t-1}, \varepsilon_t) + \beta(2/\sigma)\log E_t[\exp((\sigma/2)\tilde{U}_{t+1})]. \tag{37}$$

This is now in the form of an LEQG control problem which I can solve for an optimal control rule $v_t = -Fx_t$. This yields a closed-loop law of motion for the state vector given by

$$x_{t+1} = (A - BF)x_t = A^{\circ}x_t. \tag{38}$$

With this law of motion I can compute the stochastic steady-state mean of capital, call it k_j^*. If $k_j^* = \bar{k}_j$, then my approximation is complete. If not, then I choose a new $\bar{k}_{j+1}$ which lies between k_j^* and $\bar{k}_j$. This algorithm implicitly defines an operator T that maps steady-state means into themselves. The approximation is complete when the fixed point of T is found. This fixed point is computed by iterating on the operator T, ensuring the internal consistency of the approximation.

Once the optimal decision rules have been computed, the logarithms of output, consumption, investment, etc. can be expressed as linear functions of the state vector. The risk-free rate, the average return on equity, and the market price of risk are calculated using this linear representation and the transformed expectations operators discussed in Hansen et al. (1999).

7. Results for the production economy

I will now discuss the implications of increased risk aversion on the business cycle and asset market properties of the model I described in Section 5. Table 4 contains the values I chose for the parameters of the model. These parameters are taken, with the exception of the innovation standard deviation σ_ε, from Christiano and Eichenbaum (1992). I chose σ_ε to match the variance of consumption growth to the data under expected utility.[10] Following King et al. (1988), the mean level of hours, $\bar{N}$, will be treated as a parameter for the purpose of determining θ when $\chi = 1$ for different values of β. For $\chi \neq 1$, $\bar{N}$ will be determined endogenously given the value of θ determined above.

Again, I need to specify values for the preference parameters β and χ. First I present results for several values of χ (1, 10, 25, 100) for two different values of β (0.9926 and 0.9995). I chose 0.9926 since that is the value Christiano and

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[10] Later, I will allow σ_ε to vary to keep the variance of consumption growth constant.

Table 4
Parameter values for the production economy. α is the income share of labor, δ is the depreciation rate of the capital stock, $\bar{N}$ is the mean of labor supply under expected utility ($\chi = 1$), γ is the mean growth rate of the productivity shock, and σ_ε is the innovation standard deviation for the productivity shock

Parameter	Value
α	0.661
δ	0.021
$\bar{N}$ ($\chi = 1$)	0.2305
γ	0.004
σ_ε	0.0115

Eichenbaum used to fix the annualized subjective discount rate to 3 per cent ($0.9926 = 1.03^{-0.25}$) before they estimated the other parameters of the model. Since I have used their values for almost all other parameters, this seems to be a reasonable value to use. I chose 0.9995 to make the risk-free rate closer to the data. Later, I will perform the same experiment as in Section 4: find a (β, χ) pair to match an interest rate and market price of risk and then compute the welfare costs of fluctuations.

Steady-state mean values for capital, output, consumption, investment and labor supply are reported in Table 5. These also happen to be the centering points for the approximation. Increasing χ increases the mean of all quantity variables. The increase is roughly 10% when χ is increased from 1 (expected utility) to 100. This is the result of precautionary savings that is now captured in the approximation. This is consistent with Kimball and Weil (1992) who show that increasing risk aversion without changing intertemporal substitution preferences results in an increase in precautionary saving in a two period model.

Table 6 contains sample moments for quarterly data on output, consumption, investment, and labor hours for the United States from 1948:2 to 1993:4 from CITIBASE. The moments reported are a subset of those reported in King, Plosser, and Rebelo (1988b), Tables 1 and 2. The output series is gross domestic product. The consumption series is non-durables and services, as in Section 4. The investment series is gross fixed investment. The hours series is constructed as in King et al. (1988a): total employment multiplied by average weekly hours worked divided by the civilian non-institutional population 16 years and older. Since the last two series are monthly, the three observations each quarter are averaged to create a quarterly series.

The effects on the relative variabilities and correlations are even smaller. Table 7 reports some of the second moments of the model for $\beta = 0.9926$ and $\chi = 1$ and 100. Recall that the coefficient of relative risk aversion for

Table 5

Stochastic steady-states. Steady-state mean values for capital, output, consumption, investment, and labor supply for different values of the risk aversion parameter χ, the discount factor β, and the labor supply parameter θ. Note that θ is chosen so that the mean of labor supply is 0.2305 in the expected utility, $\chi = 1$ case

χ	$\bar{k}$	$\bar{y}$	$\bar{c}$	$\bar{i}$	$\bar{N}$
$\beta = 0.9926, \theta = 2.9869$					
1	8.052	0.768	0.567	0.201	0.2305
10	8.105	0.770	0.568	0.202	0.2307
25	8.193	0.774	0.570	0.204	0.2311
100	8.657	0.793	0.577	0.216	0.2331
$\beta = 0.9995, \theta = 3.3050$					
1	11.590	0.869	0.580	0.289	0.2305
10	11.699	0.873	0.581	0.291	0.2309
25	11.884	0.879	0.583	0.296	0.2317
100	12.878	0.913	0.593	0.321	0.2356

Table 6

Business cycle data. Sample moments from quarterly U.S. data, 1948:2–1993:4

Variable	Std. dev.	Relative std. dev.	Autocorrelations			Cross corr. $w/\Delta \log Y_t$
			1	2	3	
$\Delta \log Y_t$	1.036	1.000	0.402	0.266	0.085	1.000
$\Delta \log C_t$	0.546	0.527	0.200	0.193	0.157	0.518
$\Delta \log I_t$	2.789	2.691	0.497	0.227	0.001	0.648
$\Delta \log N_t$	0.912	0.880	0.215	0.094	0.010	0.541
$\log N_t$	3.545	3.420	0.967	0.920	0.864	0.074
						Cross corr. $w/\log N_t$
$\log C_t - \log Y_t$	3.770	1.064	0.972	0.940	0.900	− 0.138
$\log I_t - \log Y_t$	6.490	1.831	0.940	0.828	0.689	0.134

consumption gambles is $(\chi + \theta)/(1 + \theta)$ which in this case is 25.8. The model obviously falls short with regard to labor input and the propagation of shocks, but this is a well-known short-coming of this simple specification of the growth model.[11] Clearly, the effects on the second moments are almost negligible, despite a coefficient of relative (consumption) risk aversion of 26. Kimball (1993)

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[11] See Hansen (1985), Burnside et al. (1993), among others, for models that address these issues more directly.

Table 7
Population moments for quantity variables. $\beta = 0.9926$, $\chi = 1$ (expected utility) and 100 (increased risk aversion)

Variable	Standard deviation		Autocorrelation		Cross corr. $w/\Delta \log Y_t$	
	$\chi = 1$	$\chi = 100$	$\chi = 1$	$\chi = 100$	$\chi = 1$	$\chi = 100$
$\Delta \log Y_t$	0.992	0.995	0.009	0.008	1.000	1.000
$\Delta \log C_t$	0.546	0.541	0.085	0.081	0.988	0.988
$\Delta \log I_t$	2.284	2.240	− 0.018	− 0.017	0.995	0.995
$\Delta \log N_t$	1.501	1.506	− 0.010	− 0.010	0.998	0.998
$\log N_t$	1.128	1.155	0.951	0.906	0.334	0.331
					Cross corr. $w/\log N_t$	
$\log C_t - \log Y_t$	1.466	1.506	0.951	0.906	− 1.000	− 1.000
$\log I_t - \log Y_t$	4.145	4.031	0.951	0.906	1.000	1.000

finds similarly small effects of increased risk aversion on fluctuations. This result is in contrast to Rouwenhorst (1995) who finds that higher risk aversion results in smoother consumption. However, since Rouwenhorst only considers expected utility preferences, which force the elasticity of intertemporal substitution to decline when risk aversion increases, it seems that risk aversion is not what is smoothing out consumption.

Table 8 reports population moments for the risk-free rate and the equity return. It also reports the market price of risk, the effective coefficient of relative risk aversion, and a measure of the welfare cost of fluctuations. Note that the largest equity premium in the table is 0.01% per quarter, as opposed to 2% per quarter in the data. The primary reason for the lack of an equity premium is the low variability of the excess return. In the data, the excess return has a standard deviation of almost 8% per quarter whereas in the model the standard deviation is much less than one tenth of one per cent. This extremely small standard deviation leads to high Sharpe ratios (the ratio of the mean to standard deviation) for the excess return. The excess return has a low variance since the only source of variability is variation in the marginal product of capital. The price of a unit of capital is constant since there are no frictions in the capital accumulation process. Adding adjustment costs or other frictions would generate price variation which would increase the equity premium. While such frictions are an interesting subject of future research, I will go beyond the low equity premium and focus on the market price of risk.

The results reported in Table 8 indicate that increasing risk aversion reduces the interest rate and increases the market price of risk as in the endowment economies of Section 4. Increasing risk aversion has a stronger effect on the risk-free rate in the production economy. This is because of the precautionary savings motive that increases the mean capital stock, reducing the average

Table 8

Population moments for asset market variables. Mean and standard deviation for the risk-free rate, r^f, the return on equity, r^e, and the excess return of equity over the risk-free rate ($z = r^e - r^f$), the Sharpe ratio for the excess return, the average market price of risk, and the implied coefficient of relative risk aversion for consumption gambles. Returns are measured in per cent per quarter. χ: risk aversion parameter, β: discount factor, θ: labor supply parameter

χ	$E[r^f]$	$\sigma(r^f)$	$E[r^e]$	$\sigma(r^e)$	$E[z]$	$\sigma(z)$	S.R.	M.P.R.	R.R.A.	Welfare
$\beta = 0.9926, \theta = 2.9869$										
1	1.148	0.099	1.151	0.104	0.003	0.032	0.095	0.005	1.0	0.02
10	1.135	0.099	1.139	0.104	0.004	0.032	0.118	0.029	3.3	2.22
25	1.114	0.099	1.119	0.103	0.005	0.032	0.157	0.067	7.0	6.01
100	1.011	0.098	1.022	0.101	0.011	0.031	0.352	0.263	25.8	27.02
$\beta = 0.9995, \theta = 3.3050$										
1	0.450	0.086	0.453	0.089	0.003	0.025	0.122	0.006	1.0	0.63
10	0.436	0.086	0.440	0.089	0.004	0.025	0.146	0.029	3.1	33.15
25	0.415	0.086	0.419	0.089	0.005	0.025	0.186	0.069	6.6	112.33
100	0.306	0.085	0.316	0.086	0.009	0.024	0.388	0.272	24.0	2087.65

return on equity and consequently the risk-free rate. However, the effect of increasing χ on the market price of risk is less than in the endowment economy because now leisure enters into the preferences of the representative agent. The first difference of single-period utility has a standard deviation of about one-half that of consumption which results in a less variable value function. Variation in the value function, scaled by $(1 - \beta)(1 - \chi)/(1 + \theta)$ (see Eq. (8)), is what causes the market price of risk to increase. Notice that the market price of risk is roughly the same for given values of χ.

There are several factors that help explain the small effect of high risk aversion on the second moments of the quantity variables. First, consider the case of full depreciation. In that case, the optimal choices for capital and labor are the same regardless of the level of risk aversion. This can be shown analytically. Similar intuition applies in the low depreciation case. There are some changes in the average level of work effort due to precautionary saving (increased capital leads to a higher wage), but the labor-leisure choice is still pinned down primarily by the shape of the single-period utility function since in a decentralized world $w = u_L/u_C$.

Second, the elasticity of intertemporal substitution is unchanged. So for a given change in the interest rate, under certainty, the resulting change in savings is not affected. In this model, the only saving instrument is the (not very) risky capital stock. By saving more, the agent can reduce the conditional variance of the return on capital (see Eq. (34)). In fact, this is what happens. Consider instead a constant consumption plan. This results in a much more variable investment process, and therefore a more variable capital stock and

equity return. The agent prefers the variable consumption plan. Another implication of increased risk aversion is a rise in the average time spent working. This results in a less variable wage. As the innovation variance of the productivity shock increases, these precautionary responses are magnified and the variability of consumption relative to output shrinks. However, for this parameterization, the innovation variance is small.

Preferences are defined over consumption and leisure. Increasing risk aversion reduces the variability of the first difference of $u(C, L)$, which is to be expected, but not by very much. As mentioned above, the inclusion of leisure in the utility function results in smaller gains from risk aversion with regard to the market price of risk.

Finally, the value function of the optimal resource allocation problem is almost linear in the state variables, $\log k_{t-1}$ and ε_t. So it is not surprising that increased risk aversion does not affect the behavior of the aggregate variables very much. Increasing β does affect the relative variabilities since the steady-state level of capital changes considerably while steady-state labor is constrained.

I conduct a welfare experiment similar to the one in Section 4 by choosing (β,χ) pairs to match risk-free rate-market price of risk combinations. Since high levels of risk aversion result in less variable consumption streams, I adjust σ_ε to keep the variance of consumption growth constant. Table 9 reports the (β,χ) pairs that match the asset market moments. Comparing these results to those in Panel (a) of Table 2 shows that for a given risk-free rate-market price of risk combination the discount factor β in the production economy is close to that in the endowment economy. As before, not all risk-free rate-market price of risk combinations are feasible for $\beta < 1$. In addition, the risk aversion parameter χ in the production economy is roughly twice the value in the endowment economy. Table 10 shows that the welfare costs of uncertainty are even higher in this context. This is due to the higher levels of χ required to match the asset moments as well as the increases in the fraction of time devoted to labor.

The experiment here is slightly different than before. Starting with a deterministic environment, I calculate the value function at the steady state. I then introduce uncertainty into the technology process and, as before, adjust the drift to correct for uncertainty. In the endowment economy the adjustment was one half of the variance of the consumption innovation which preserved the conditional mean of the consumption process under varying degrees of uncertainty. Now the adjustment is based on the variance of the first difference of the flow of utility, $\log C_t + \theta \log(1 - N_t)$. The consumption and leisure processes are not perfect random walks in the production economy so this adjustment does not preserve the conditional mean of the utility flow exactly, but since the degree of serial correlation in the first difference is so small, it is a reasonable approximation. I then calculate the value function at the new steady state. Before comparing this value to the value under perfect certainty, I make an additional

Table 9

(β,χ) *Pairs.* Values of the discount factor, β, and risk aversion parameter, χ, that produce the indicated market price of risk, M.P.R., and average risk-free rate, $E[r^f]$, in the production economy. The innovation variance of the productivity shock, σ_ε, was adjusted to keep the variance of the first difference of consumption constant. '—' indicates combination not feasible for $\beta < 1$

$E[r^f]$	M.P.R.			
	0.125	0.25	0.375	0.5
0.25	—	—	(0.9995, 136.2)	(0.9990, 177.3)
0.50	(0.9984, 46.5)	(0.9977, 92.7)	(0.9971, 136.9)	(0.9965, 178.5)
0.75	(0.9959, 46.6)	(0.9952, 93.1)	(0.9946, 137.5)	(0.9940, 179.4)
1.00	(0.9934, 46.7)	(0.9928, 93.4)	(0.9921, 138.1)	(0.9915, 180.2)

Table 10

Compensation function. Percentage increase of consumption in all states and dates to make the representative agent indifferent between a non-stochastic economy and a stochastic economy. Preference parameters and the productivity shock innovation variance were chosen to match the indicated market price of risk, M.P.R., and risk-free rate, $E[r^f]$. '—' indicates combination not feasible for $\beta < 1$. (See Table 9)

$E[r^f]$	M.P.R.			
	0.125	0.25	0.375	0.5
0.25	—	—	10,283.3	1402.9
0.50	56.0	93.5	120.0	140.3
0.75	21.1	40.6	57.6	72.4
1.00	13.4	26.5	38.5	49.3

adjustment. The value functions are written in terms of capital that has been transformed to be a stationary random variable. Therefore, then I must adjust the value functions to account for the difference between the steady states. The values in the table are the percentage by which consumption would have to be increased in each state and date in the uncertain world to make the representative agent indifferent between the optimally chosen variable consumption stream and one with no variance.

   The last column of Table 8 shows the welfare costs when σ_ε is held constant. These costs are comparable to those in the endowment economy for the same coefficient of relative risk aversion with respect to consumption gambles (χ in the endowment economy, $(\chi + \theta)/(1 + \theta)$ in the production economy). Clearly the higher costs reported in Table 10 are the result of choosing parameters, including σ_ε, to match the asset market moments, holding the variance of consumption growth constant.

8. Summary

I have demonstrated the effects of a particular generalization of logarithmic preferences in models of aggregate fluctuations. Increasing risk aversion independent of the elasticity of intertemporal substitution results in higher values for both the mean and standard deviation of the IMRS unlike the expected utility case which results in a higher standard deviation, but a lower mean. In fact, the market price of risk is controlled exclusively by risk aversion while the risk-free rate depends on both risk aversion and the elasticity of intertemporal substitution. In an endowment economy I found the welfare costs of fluctuations to be much larger than Lucas (1987). I did this while choosing the preference parameters so as to be consistent with observed asset market data.

In the context of a stochastic growth model, increased risk aversion does not significantly affect the second moment properties of aggregate variables but it does improve the asset market implications of the model. This is in contrast to previous papers that have shown that increasing risk aversion under expected utility, and therefore decreasing the elasticity of intertemporal substitution, results in dramatically smoother consumption. Increased risk aversion, by itself, leads to precautionary savings, but only modest consumption smoothing. It also improves model performance with regard to the risk-free rate and the market price of risk. The welfare costs of fluctuations are even higher than in the endowment economy.

While the effects of capital adjustment costs and elasticities of intertemporal substitution different than one are left for future research, I have shown that a single model can and should be used to account for both business cycle and asset pricing facts.

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