Robustness and Optimal Contracts

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Abstract

This paper studies the design of an optimal unemployment compensation scheme in the presence of unobservable search effort and model uncertainty about re-employment opportunities. It extends the work of Hansen and Sargent on robust decision making to a dynamic principal-agent setting. The optimal contract can be characterized recursively. The results are applied to the optimal unemployment insurance mechanism of Hopenhayn and Nicolini. The features of the optimal contract under robust decision making change dramatically. A higher degree of model uncertainty on the part of the unemployed worker reduces the incentives that the unemployment agency needs to provide to elicit the optimal search effort. For a higher degree of robustness, less insurance needs to be sacrificed for incentives. The degree of model misspecification for which the optimal replacement rate is a constant function of time is plausible: It is not easily detectable with a statistical specification test and a short time-series of unemployment data. The model sheds new light on the observed dispersion in unemployment insurance schemes encountered in developed economies. If the low labor market turnover rate in many European countries is indicative of uncertain reemployment chances, then the observed constant replacement rate may be the interpreted as a constrained optimal outcome.

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1 Introduction

The aim of this paper is to study the optimal contract between a principal and an agent in a setting in which the effort of the agent is unobservable and both parties to the contract fear model misspecification. The theoretical contribution of this paper is to provide a recursive characterization of the optimal contract.

The apparatus is applied to the design of an optimal unemployment insurance mechanism between an unemployment insurance agency, the principal, and an unemployed worker, the agent in the presence of asymmetric information. The benchmark model is a simplified version of Hopenhayn and Nicolini (1997). The source of informational asymmetry is the inability of the principal to observe and control the effort level that the agent exerts in searching for a job. The informational imperfection leads to a second-best outcome imposing some risk upon the agent. The replacement rate, the ratio of unemployment benefits to the wage prior to unemployment, is a declining function of the duration of the unemployment spell. The usual interpretation is that the agent must be provided with the incentives to exert an efficient level of search effort. The first best outcome of full insurance and a constant replacement rate is not achievable. Insurance is sacrificed for incentives.

This paper shows that introducing a concern for robustness into the preferences of the contract parties can overturn these results. The optimal contract changes dramatically as agents have sufficient model uncertainty about the true likelihood of re-employment. Fear for model misspecification makes the unemployed worker exert a higher search effort. This reduces the asymmetric information friction and more insurance is optimal. For a high enough degree of model uncertainty the optimal contract prescribes a constant replacement rate, even in the presence of asymmetric information.

The model speaks to a dichotomy in unemployment insurance schemes encountered in developed economies: the benefit schemes broadly split in two groups according to the duration and level of unemployment insurance they provide (see Figure 1). The "American system" features a relatively high unemployment compensation for a short period of time, but benefits drop rapidly as the unemployment spell continues. This payment scheme seems to acknowledge the underlying incentive problem. In contrast, the "European system" features a constant replacement rate during unemployment. For higher levels of misspecification concern, a "European system" is constrained optimal; for a lower degree of model uncertainty, the optimal contract features replacement rates that look like the "American system". Robust decision analysis has the potential to rationalize both systems, in the same underlying economy.

Empirical evidence is suggestive of a higher degree of perceived uncertainty about the re-employment probability in Europe than in the US. Evidence from polls documents the discrepancies. Workers in the US perceive their job as more insecure according to various measures. American workers are more likely than European workers to be concerned about the future of their company, less likely to think that their company provides as good security as any other company, and less satisfied with their job security (1996 and 1997 OECD Employment Outlook and refer-

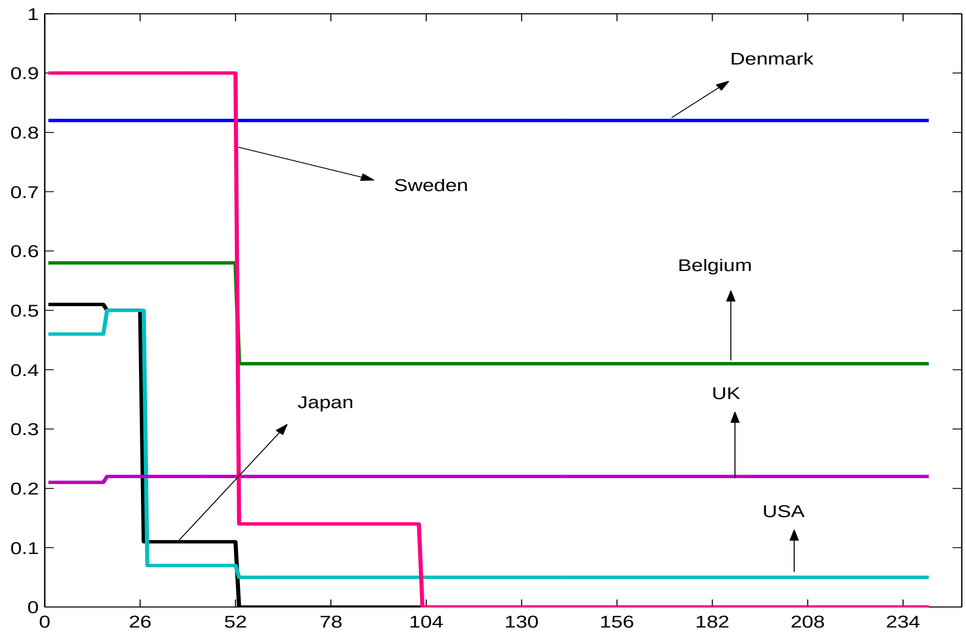

Figure 1: 1995 Replacement rate over the course of the unemployment spell for selected OECD countries. Source: OECD database on benefit entitlements and gross replacement rates. The benefits include: Unemployment Insurance (conditional on employment record and not means-tested), Unemployment Assistance (a benefit which is either conditional on employment record, means tested, or is of limited duration), Guaranteed Income (not conditional on employment record but is means tested and is of indefinite duration), Social Welfare (no defined entitlement to benefit, has only been applied to Sweden), Social Assistance (assistance in not in the form of cash payments,only used for food stamps in the United States), and Temporary Layoff benefits (awarded on the basis of the individual being classified as a temporary layoff).

ences therein). This perception is justified by the higher turnover rate of jobs in the US, lower average tenure at all tenure levels and in all industries, and lower retention rates. This evidence for workers strongly suggests that, from the perspective of the *unemployed,* there is less uncertainty about the probability of finding a job conditional on current unemployment in the US than in Europe. European unemployed have a small sample of observations on job matches and that makes misspecification of the re-employment probability harder to detect. This rationalizes the concern for model misspecification.

The paper is organized as follows. I briefly review the related literature in section 2. In section 3 I briefly review some basic concepts of robustness in a single agent environment (3.1). Then, in 3.2, I show how to adapt them to the principal-agent contract problem and provide a recursive characterization of the principal-agent problems with robustness. Section 4 applies the theory to the optimal unemployment insurance problem. I calibrate the model and numerically solve for the optimal decision rules. Detection error probabilities discipline the degree of robustness (5). Section 6 concludes. Proofs are relegated to the appendix. Tables and figures appear at the end of the text.

2 Related Literature

Rational expectations models in macroeconomics assume agents know the true model of the economy. A model is a probability distribution over a vector of sequences. Agents inside the model have no doubts about this probability distribution. Econometricians, however, experience how hard it is to detect the true law of motion of the economy. They run statistical specification tests to narrow down the class of possible models to the ones that are indistinguishable from the true data generating process. Economists accept that their models are approximations, because they have to be simple and tractable. But with tractability comes misspecification. By imputing a concern for model misspecification into agents' decision problem, the robustness literature puts the economic theorist and the agents inside the model on the same footing in terms of knowledge about the true data generating process (Hansen and Sargent (2002) and the references therein).

Agents have a reference model reflecting their best estimate of the law of motion of the economy, but they want to make decisions that are robust to small specification errors. Small means statistically indistinguishable from the true model. In other words, only candidate models that are close to the reference model are being considered. Statistical theory disciplines the extent of model misspecification that is acceptable. We use detection error probabilities (see section 5) to limit the set of acceptable models. The ability to detect model misspecification errors and hence the need for robustness will depend on the amount of available data.

This paper formulates a principal-agent model in which both the principal and the agent to have fear for model misspecification. While the principal and the agent both have the same reference model, the risk-neutral principal and the risk-averse agent will generally have different worst-case models.

Our paper also contributes to the literature related to this paper studies dynamic recursive contracts. Like many other papers in this literature, we arrive at a recursive formulation of the optimal contract using the apparatus developed in Spear and Srivastava (1987). We apply the theory to a setting with asymmetric information (Hopenhayn and Nicolini (1997)), but they could also be used in economies with a limited enforcement friction (see Kocherlakota (1996), Kehoe and Perri (2000), Alvarez and Jermann (2000)).

The Hopenhayn and Nicolini (1997) problem is based on the work of Shavell and Weiss (1979) and has recently been extended in other directions. Werning (2002) and Kocherlakota (2002) investigate the optimal unemployment compensation in the presence of hidden savings. Pavoni (2002) incorporates human capital depreciation and duration dependence in the design of the optimal unemployment contract.

3 Model misspecification in discrete time

I first review some basics about the single agent robustness setup in section 3.1, following Hansen and Sargent (2002) and Anderson, Hansen and Sargent (2000). Then, I study the principal-agent problem in section 3.2.

3.1 Robustness in single agent models

The starting point of the analysis is the reference model. It is a model that is a good approximation to the unknown true data generating process of the economy. Agents worry about a common class of model misspecification in the neighborhood of the reference model. Candidate models, perturbations around the reference model, are indexed by a strictly positive perturbation function $w(\cdot)$. Corresponding to each perturbation, there is a distorted expectations operator:

$$E^w[V(s')|s] = \frac{E[w(s')V(s')|s]}{E[w(s')|s]}$$

where $V(\cdot)$ is the value function of the agent and s denotes the state of nature. If state transitions are Markov, the distorted law of motion has state transitions that preserves the Markov structure.

The measure of distance between models is conditional relative entropy. Conditional relative entropy $I(w)$ is defined as follows:

$$I(w)(s) = E^w\left[\log\left(\frac{w(s')}{E[w(s')|s]}\right)|s\right].$$

It is the expected value of the log-likelihood ratio, where expectation is taken with respect to the probability density associated with the perturbed candidate model. This likelihood ratio is the log of the Radon-Nikodym derivative of the twisted transition law with respect to the reference law. The transition probability distribution for the perturbed candidate model must be absolutely continuous with respect to the transition probability distribution of the reference model.

Hansen and Sargent (2002) show that the optimal robust decision arises as the outcome of a zero-sum game between an economic agent and her evil counter-part. The agent maximizes her objective function by choosing the control variables. The evil agent minimizes the same objective function by choosing a perturbation function w. The evil agent minimizes:

$$J(w,s) \equiv \theta I(w(s)) + E^w[V(s')|s] \tag{1}$$

The first term of (1) reflects a penalty for choosing models that have a large entropy relative to the reference model. A decrease in $\theta > 0$ decreases the penalty, and leads the evil agent to choose a worse perturbation w. Lower parameter values of θ are associated with decision rules that are robust to a wider class of misspecifications. The parameter θ indexes the size of the set of candidate models under consideration. The evil agent's optimal perturbation satisfies $w^* = \exp\left(\frac{-V}{\theta}\right)$. Inserting the optimal perturbation in the value function gives $J(w^*(s),s) = \Re(V(s)) = -\theta \log E\left[\exp\left(\frac{-V(s)}{\theta}\right)\right]$.

The maximization problem for the agent proceeds from here. The operator $\Re$ can be interpreted as a robust evaluation of a continuation value function. It emerges naturally in a model where agents evaluate continuation values under a neighborhood of models. Alternatively, the operator $\Re$ can be interpreted as a device for injecting an additional adjustment for risk attitudes, above those captured by the curvature of the utility function, as in Epstein and Zin (1989). From this proposition, one can directly deduce a bound on the distorted expectations operator:

$$E^{w}[V(.)|s] \geq \Re(V) - \theta I(w) \tag{2}$$

When $I(w) = 0$ (this happens when w is constant), the distorted expectations operator $\Re$ yields a conservative bound for the value of future prospects under the reference model. For perturbed candidate models, θ governs the rate at which the expected value of V decreases as the discrepancy with the reference model increases. This favors an interpretation of θ as the utility price of robustness. There is a counterpart to this bound in the risk-sensitivity interpretation. It expresses risk aversion as a concern of the decision maker about tail probabilities. This interpretation allows one to bound the tail probabilities of the value function V :

$$P[V(s') \geq r|s] \leq \exp(\frac{-1}{\theta}\Re[V(s')|s]) \exp(-\frac{r}{\theta}) \tag{3}$$

The tail probability is bounded by an exponential function of r, where r is a tail event. It decreases at the rate $\frac{-1}{\theta}$. A lower value for θ, implying more preference for robustness, raises the rate at which tail probabilities go to zero. Hence a lower θ indicates a heightened concern about tail events. The operator $\Re$ influences the constant in the bound.

3.2 Robustness in Principal-Agent Models

First, I show that the contract can be written in a recursive fashion. Second, I show that the recursive contract can be simplified using results from the previous section. This multi-agent robustness setup amounts to a game with four players: the principal, the agent and an "evil deceiver" for each of them, referred to as the evil agent and evil principal. The latter two try to distort the valuation of future prospects. This setup complicates the problem because the evil principal chooses a minimizing distortion as part of a contract where the agent's and the evil agent's considerations enter through the constraints. The implications of the strategic interactions in this game are non-trivial. In section 3, I will apply the theory developed here to the optimal unemployment insurance (UI) design problem.

In an infinitely repeated setting where principal and agent discount the future, the optimal contract between a principal and an agent will, in general, depend on the entire history of the relationship. Spear and Srivastava (1987) show that the optimal contract -without robustness- has a simple recursive form which avoids the intractableness associated with history dependence. The essential ingredient is to include promised utility as a state variable in the model.[1] The contract contains a promise keeping and an incentive compatibility constraint.

[1]Dynamic contracts problems are greatly simplified by making them recursive in utility promises. The main contributions are Green (1987), Spear and Srivastava (1987), Phelan and Townsend (1991), Abreu, Pearce and Stacchetti (1990), Atkeson (1991), Atkeson and Lucas (1995) . This literature is discussed in Ljungqvist and Sargent (2000), Ch. 15 and 16.

Benchmark Principal Agent Model The Spear and Srivastava (1987) model features a risk neutral principal with perfect access to credit markets. The principal's instantaneous utility is given by $U_P(y-c)$ where s is the observable stochastic output of a production process and c is the compensation the agent gets for the output. U_P is continuous, strictly increasing and defined on the entire real line. The action $a \in A$ is an input in the production process and is chosen by the agent. It is not observable by the principal. A is a compact set by assumption. The agent's instantaneous utility is given by $U_A(a,c)$, where U_A is continuous, $\frac{\partial U_A}{\partial a} < 0$, $\frac{\partial U_A}{\partial c} > 0$, $\frac{\partial^2 U_A}{\partial c^2} < 0$. The gent and principal have a common discount rate $0 < \beta < 1$. Let $s^t = \{s_t, ..., s_1\}$ denote a t-history of output realizations. The rest of this section will continue under the assumption that uncertainty follows a continuous Markov process.[2]

The timing of the standard game is as follows: the agent picks an action $a_t \in A$ unobservable to the principal, the agent receives a compensation c_t from the principal. Next, the agent and principal consume. At the end of the period output s_t is realized from a distribution $F(s_t|a_t)$. The game then moves on to the next stage. $F(.)$ is twice continuously differentiable. This informational asymmetry makes compensation a function of past output realizations only. This timing deviates from the standard one where stochastic output is realized before the principal decides on the compensation scheme. Here output is only observed at the end of the period. That will bring the timing of this section in line with the timing of the UI problem in section 3. There, the principal can make the unemployment compensation contingent only on the agent's current state of unemployment (s_{t-1}), not on whether the unemployed worker will find a job in the next period or not (s_t).

We only allow for pure strategies $c(s^t)$ and $a(s^t)$. Let S_A and S_P denote the payoffs from strategies in the subgame starting from s^t for the agent and the principal respectively.

$$S_P(s^t, c, a) = \sum_{\tau=0}^{\infty} \beta^\tau \int U_P(s_{t+\tau} - c(s^{t+\tau}))d\pi(s^{t+\tau}|s^t, a)$$

and

$$S_A(s^t, c, a) = \sum_{\tau=0}^{\infty} \beta^\tau \int U_A(a(s^{t+\tau}), c(s^{t+\tau}))d\pi(s^{t+\tau}|s^t, a)$$

where the probability distribution π is recursively defined as

$$d\pi(s^{t+\tau}|s^t, a_t) = f(s_{t+\tau}|a(s^{t+\tau-1}))d\pi(s^{t+\tau-1}|s^t, a_t) \tag{4}$$

and

$$d\pi(s^{t+1}|s^t, a) = f(s_{t+1}|a(s^t)) \tag{5}$$

An optimal contract (c, a) for this problem maximizes the principal's objective while keeping the

[2]The unemployment insurance example in section 3 is a special case in which s is a random variable that takes on the value 1 if the unemployed person finds a job and 0 if she remains in unemployment. In this example, uncertainty follows a discrete two-state process where one of the states is absorbing.

agent above a reservation value, $\bar{x}$, and ensuring that the agent has no incentive to deviate.

Definition 1. *An optimal contract under asymmetric information is a sequence* $c = \{c(s^t)\}$ *and a sequence* $a = \{a(s^t)\}$ *such that for all* s^t*,* c *maximizes* $S_P(s^t, c, a)$ *subject to* $S_A(s^t, c, a) \geq \bar{x}$ *and* $S_A(s^t, c, a) \geq S_A(s^t, c, \bar{a})$*,* $\forall s^t$ *and* $\forall \bar{a} \in A$

For this setup Spear and Srivastava (1987) proof that the optimal contracting problem can be formulated recursively by introducing the discounted expected utility promised to the agent, x, as a state variable in addition to the Markov chain s.

The problem with robust decision makers introduces concerns for model misspecification into the contract. An evil agent and an evil principal want to upset the probabilistic evaluation of the future prospects of their respective 'good' counterparts. There is nothing that prevents the principal and the agent from evaluating differently the same future income streams $s^{t+\tau}$. However, we assume they have a common reference measure π_0, which is recursively built up from the common reference marginal density $f^0 \equiv \{f^0(s_{t+\tau}|a(s^{t+\tau-1})) \in Q\}_{\tau=1}^{\infty}$. The evil agent chooses a sequence of marginal distributions $f^a \equiv \{f^a(s_{t+\tau}|a(s^{t+\tau-1})) \in Q\}_{\tau=1}^{\infty}$ and the evil principal a sequence of marginals $f^p \equiv \{f^p(s_{t+\tau}|a(s^{t+\tau-1})) \in Q\}_{\tau=1}^{\infty}$ that recursively generate the sequences $d\pi^a$ and $d\pi^p$ through (4) and (5). The goal of the "deceivers" is to minimize the objective function of their good counterpart. This minimization over marginal probability distributions within a class of measures Q will lead eventually to a worst case pdf sequence $d\tilde{\pi}^p \equiv \{d\tilde{\pi}^p(s^{t+\tau}|s^t, a)\}_{\tau=1}^{\infty}$ for the principal and $d\tilde{\pi}^a \equiv \{d\tilde{\pi}^a(s^{t+\tau}|s^t, a)\}_{\tau=1}^{\infty}$ for the agent. All probability measures in the set Q are absolutely continuous with respect to the reference measure π_0. As motivated before, agent and principal only want to consider models (measures) that are close to the reference model (measure). Other models would clearly be statistically distinguishable from it. Define two new payoff functions $\tilde{S}_A$ and $\tilde{S}_P$ which include an additive "punishment" term for probability distributions that are far away from the reference model. The measure of closeness I use in the sequential setup is the expectation under the perturbed measure of the logarithm of the Radon-Nikodym derivative of the twisted with respect to the reference pdf. This pdf is the pdf that has been recursively generated through a sequence of choices of marginal distributions. The robustness parameter $\theta_i, i = A, P$ indexes the size of the set Q.

$$\tilde{S}_A(s^t, c, a, \pi^a) = \sum_{\tau=0}^{\infty} \beta^{\tau} \int [U_A(a(s^{t+\tau}), c(s^{t+\tau})) + \theta_A \log\left(\frac{d\pi^a(s^{t+\tau}|s^t, a)}{d\pi_0(s^{t+\tau}|s^t, a)}\right)] d\pi^a(s^{t+\tau}|s^t, a) \tag{6}$$

$$\tilde{S}_P(s^t, c, a, \pi^p) = \sum_{\tau=0}^{\infty} \beta^{\tau} \int [U_P(s_{t+\tau} - c(s^{t+\tau})) + \theta_P \log\left(\frac{d\pi^p(s^{t+\tau}|s^t, a)}{d\pi_0(s^{t+\tau}|s^t, a)}\right)] d\pi^p(s^{t+\tau}|s^t, a) \tag{7}$$

At this point it is important to elicit timing protocols of the 4-player game. The evil agent moves first and chooses a function f_t^a (marginal density). Then the agent chooses an action $a_t \in A$. One can think of the evil counterparts as moving first in a Stackelberg leadership game. An outcome equivalent situation is where the agent and evil agent move simultaneously. Hansen and Sargent (2002) have shown that different timing assumptions of this subgame are outcome equivalent. I assume that there is no observability issue between the agent and the evil agent. This is behaviorally justifiable because the two players are in effect two sides of the same person. Without this assumption we would have to solve an additional robust filtering problem (with a corresponding additional avenue of deception, see Hansen and Sargent (2002)). Next, the evil principal decides on a function f_t^p. Then the principal chooses a compensation c_t. Again the last two moves can occur concurrently without affecting the results. After this, output s_t is realized from an unknown true distribution $F(s_t|a_t)$. Since this is a principal-agent contract, the 4-player game is merely a metaphor for the strategic considerations of a "robust" principal. The robust principal designs a contract in which the robust agent's decision variables are stipulated. The action a and the marginal density f^a, and by recursion π^a, are unobservable to the robust principal. They each have to satisfy an IC constraint.

Definition 2. *The optimal sequential robust contract under asymmetric information stipulates sequences* $\{\tilde{c}, \tilde{a}, \tilde{f}^p, \tilde{f}^a\}$ *such that:*
(i) f^a *recursively generates* π^a *and* f^p *recursively generates* π^p *through (4) and (5),*
(ii) For all histories s^t, $\{\tilde{c}, \tilde{a}, \tilde{\pi}^p, \tilde{\pi}^a\}$ *solves*

$$\max_{c,a} \min_{\pi^a, \pi^p} \tilde{S}_P(s^t, c, a, \pi^p)$$

subject to:

$$\tilde{S}_A(s^t, c, a, \pi^a) \geq \bar{x}, \tag{8}$$

$$\tilde{S}_A(s^t, c, a, \pi^a) \geq \tilde{S}_A(s^t, c, \bar{a}, \pi^a), \qquad \forall \bar{a} \in A \tag{9}$$

$$\tilde{S}_A(s^t, c, a, \pi^a) \leq \tilde{S}_A(s^t, c, a, \bar{\pi}^a), \qquad \forall \bar{\pi}^a \in Q \tag{10}$$

Constraint (8) says that even in the worst case probability evaluation, the agent gets at least a reservation utility $\bar{x}$. The second constraint (Equation 9) is an IC constraint on the action level of the agent and the third (Equation 10) is an IC constraint on the evil agent's choice of pdf. Note that implicit in $\tilde{S}_A$ ($\tilde{S}_P$) is a punishment on the evil agent (principal) for choosing a pdf far away from the reference model. The recursivity is stated in the following proposition and the proof is given in appendix A. This is accomplished by taking the promised utility to the agent, x, as a state variable. Let period's realization of output, s_{-1}, is also part of the state variable. Denote this state variable by $z = (x, s_{-1})$. The optimal contract is characterized by 4 policy functions: the agent's action, $a(z)$, the compensation for this action, $c(z)$, the expected payoff to the principal, $V(z)$ and next period's promised utility $v'(z)$. The principal decides upon a compensation this period $c(x)$ and gives a utility promise for next period $v'(x)$. The principal also honors his utility promise x in the current period (perfect commitment).

Proposition 3. *The sequential principal-agent contract can be formulated recursively in the state variable* $z = (x, s_{-1})$*. Furthermore the optimal contract is given by a five-tuple* (a, c, v', w^a, w^p)

satisfying:

$$V(z) = \max_{a,c,v'} \min_{w^a,w^p} \{U_P(s_{-1} - c) + \tilde{\theta}_P I(w^p) + \beta E^{w^p}[V(z')]\}$$

subject to the robust promise keeping and robust incentive compatibility constraints:

$$U_A(a,c) + \tilde{\theta}_A I(w^a) + \beta E^{w^a}[v'(z')] \geq x \tag{11}$$

$$(a(z), w^a(z)) = \arg\max_a \min_{w^a} \{U_A(a,c) + \tilde{\theta}_A I(w^a) + \beta E^{w^a}[v'(x,s)]\} \tag{12}$$

with $\tilde{\theta}_P = \frac{\theta_P}{1-\beta}$ *and* $\tilde{\theta}_A = \frac{\theta_A}{1-\beta}$

Robustness introduces the question which perturbation the principal should prescribe for the evil agent. The optimal contract has to specify a perturbation w^a which the evil agent does not want to deviate from. This is because the principal does not observe the actions of the evil agent. That implies that the IC constraint (12) is a max-min problem instead of the maximization as is the case without robustness. Hence, asymmetric information combined with robustness does not only mean that the principal cannot observe the search effort level, it also means that the principal cannot observe the perturbation w^a the agent's evil agent chooses to value tomorrow's utility promise. That is why the optimal contract stipulates an effort level a that the agent does not desire to deviate from and a perturbation function w^a that the evil agent does not desire to deviate from. Robustness hence adds an IC constraint to the formulation of the optimal contract, vis. the minimization part of the IC constraint.

It is useful to step back and think of the robust agent's decision problem in isolation. Suppose the agent is given a consumption c and promised utility $v'(z)$, then it is easy to show that the robust agent's problem is recursive in the state z.

Proposition 4. *The agent's recursive decision problem under robustness is to solve PA:*

$$V^A(z) = \max_a \min_{w^a} \left\{U_A(a,c) + \beta\tilde{\theta}_A I(w^a) + \beta E^{w^a}[v'(z')]\right\} \tag{PA}$$

The evil agent's minimization part, for a given value of z and for a given $c(z)$ and $v'(z)$, is described by:

$$\min_{w^a} J_A(z, w^a) = \min_{w^a} \{\tilde{\theta}_A I(w^a) + E^{w^a}[v'(z')]\}$$

AHS (2000) show that the minimizing sequence is $w_a^* = \exp\left(\frac{-v'(z')}{\tilde{\theta}_A}\right)$ yielding $J_A(w_a^*(z)) = \Re(v'(z)) = -\tilde{\theta}_A \log\left(E\left[\exp\left(\frac{-v'(z')}{\tilde{\theta}_A}\right)\right]\right)$. Comparing [PA] with (12) implies that the perturbation for the evil agent w^a coincides with perturbation chosen in autarky w_a^*. It is this perturbation that goes into the robust promise keeping constraint (11) when the agent values her future prospects. Any other perturbation is not incentive compatible and cannot be part of the optimal contract. Note that the expectation is now under the reference model. All this implies that, for a given state

x, the robust agent's problem simplifies to

$$v^{autrob} = \max_{a} \left\{ U_A(a,c) - \tilde{\theta}_A \beta \log \left(E \left[\exp \left(\frac{-v'(z')}{\tilde{\theta}_A} \right) \right] \right) \right\}$$

The exact analogy holds for the principal in isolation.

Proposition 5. *The principal's recursive decision problem under robustness is to solve PP:*

$$\max_{c,v'} \min_{w^p} \left\{ U_P(s_{-1} - c) + \tilde{\theta}_P I(w^p) + E^{w^p} \left[V(z') \right] \right\} \tag{PP}$$

The same considerations lead to a simplified principal's problem for a given z.

$$\max_{c,v'} \left\{ U_P(s_{-1} - c) - \tilde{\theta}_P \beta \log \left(E \left[\exp \left(\frac{-V(z')}{\tilde{\theta}_P} \right) \right] \right) \right\}$$

Note that the principal's problem does not depend on a nor f^a because both are unobservable to the robust principal.

In the optimal contract, the evil principal's minimization problem is unaffected by the robust promise keeping constraint, as the latter does not depend on w^p. Hence w^p coincides with $w_p^* = \exp(\frac{-V(z')}{\tilde{\theta}_P})$. Of course, the evil principal indirectly affects the choice of the robust agent because its choice of distortion influences the principal's choice of compensation and next period's promised utility. There is a complex strategic interaction between the players.

Corollary 6. *The optimal principal-agent contract under robustness simplifies to a triple* (a, c, v') *satisfying:*

$$V(z) = \max_{c,v'} \left\{ U_P(s_{-1} - c) - \tilde{\theta}_P \beta \log \left(E \left[\exp \left(\frac{-V(z')}{\tilde{\theta}_P} \right) \right] \right) \right\}$$

subject to the robust promise keeping and robust incentive compatibility constraints:

$$U_A(a,c) - \tilde{\theta}_A \beta \log \left(E \left[\exp \left(\frac{-v'(z')}{\tilde{\theta}_A} \right) \right] \right) \geq x \tag{13}$$

$$a(z) = \arg\max_{a} \left\{ U_A(a,c) - \tilde{\theta}_A \beta \log \left(E \left[\exp \left(\frac{-v'(z')}{\tilde{\theta}_A} \right) \right] \right) \right\} \tag{14}$$

Under the timing assumptions made, the formulation of the principal-agent problem with robustness does not add numerical complexities. The problem simplifies in a tractable way. The corollary shows how the worst case perturbations are embedded in the risk-sensitivity formulation above. For parameter values $\theta_A = \theta_P = +\infty$ it is infinitely costly to choose a model away from the reference model. We recover the reference model but the decision rules are still designed with fear of model misspecification in mind. Note that this parameter assumption does not recover the model without robustness.

Other assumptions on the timing of the sequential game between the four players involved may lead to an alternative formulation of the recursive contract. In general, however, the problem does not simplify as above, making analytical solutions less elegant or even not existent. Or the sequential contract may have no recursive counterpart. Section 3 applies these results to an unemployment insurance (UI) design problem.

4 Robustness and optimal contracts with asymmetric information

The problem studied here is to find the optimal unemployment insurance scheme (UI) in the presence of asymmetric information. The setup of the UI problem is taken from Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997). As in Ljungqvist and Sargent (2000), we make one simplifying assumption. Employment is an absorbing state: once the agent has found a job she is beyond the grasp of the unemployment agency and remains employed forever.

4.1 The Benchmark Case: No Robustness

An unemployed worker (agent) has a probability distribution of finding a job, p, given a level of search effort, a, exerted with properties $p(a) \in [0,1]$ for $a \geq 0$, $p(0) = 0$. The function $p(\cdot)$ is increasing, twice continuously differentiable and strictly concave in a. In the numerical results I assume $p(a;r) = 1 - e^{-ra}$, with fixed hazard rate r. Once employed, all jobs are alike and pay wages $n > 0$ units of the consumption good each period forever. In each period, the principal observes whether the agent found a job or not, but it cannot observe the search effort exerted. The informational imperfection leads to a second-best contract imposing some risk upon the agent in the form of a declining replacement rate (henceforth RR). The interpretation is that the agent needs to be provided with the right incentives to search. The first best outcome, full insurance and associated constant RR, is generally not achievable.

Workers are risk averse and have separable preferences over consumption and search effort

$$E_0 \sum_{t=0}^{\infty} \beta^t (u(c_t) - a_t) \tag{15}$$

with $0 < \beta < 1$ and $u(c)$ strictly increasing, twice differentiable and strictly concave. In the numerical results I assume $u(c_t) = \frac{c^{1-\sigma}}{1-\sigma}$. The consumption good is non-storable. Consumption and search effort are restricted to be nonnegative. The unemployed person cannot borrow or lend and holds no assets. Consumption smoothing exclusively takes place through a contract with an unemployment agency responsible for UI policies. As a consequence the consumption of the unemployed agent is entirely given by the unemployment benefits. Wang and Williamson (1999) investigate the situation where workers have access to an incomplete asset market in addition to an unemployment insurance scheme.

The autarky problem studies the situation without an unemployment agency. We assume that there exists a minimal welfare system that provides the agent in autarky with a consumption level c^w in every period. When the coefficient of relative risk aversion σ is greater than one, giving the agent no consumption makes the problem ill-defined as $u(0) = -\infty$ and giving the agent zero utility would make him very well off. To make the problem well-defined one needs to take a stance on a minimum consumption level. For the case $\sigma < 1$ (this is the only case discussed in Hopenhayn and Nicolini) this is not true because $u(c^w) = 0$ for $c^w = 0$, and the autarky problem is well-defined. Let $V^e \equiv \frac{u(n)}{1-\beta}$ denote the value of employment. The fate of an unemployed worker who does not get an unemployment benefit but receives a welfare consumption level c^w and has to choose a search effort level can be recursively formulated as:

$$v^{aut} = \max_{a \geq 0} \left\{ u(c^w) - a + \beta \left[p(a)V^e + (1 - p(a))v^{aut} \right] \right\}$$

The first order condition (necessary and sufficient) is

$$\beta p'(a) \left[V^e - v^{aut} \right] \leq 1 \qquad = 1 \;\; if \;\; a > 0$$

I now introduce a government agency (principal) who designs an insurance contract with the unemployed worker (agent). The principal enters the period with a utility promise v. He decides on the agent's current consumption c and a utility promise from tomorrow onwards v'. The principal is assumed to always honor his utility promises. This is reflected in the promise keeping constraint (17) below. The government agency tries to minimize the costs V of the insurance mechanism, given in (16). In the asymmetric information (AI) case the principal cannot observe the search effort exerted by the agent. He only observes and controls the agent's consumption. Hence, the optimal contract has to prescribe the search effort level that the worker would not deviate from. That a obeys the FOC of the worker's problem under autarky (necessary and sufficient). This incentive compatibility constraint is reflected in (18) and (19). If search is unobserved, the agent chooses a search effort level that is lower than optimal because she fails to internalize the costs of the UI scheme. In the full insurance case (FI), the principal can fully control and enforce the search effort of the agent. This implies that the FI contract prescribes the agent's search effort. Without the IC constraint we are in the FI case. I do not consider imperfect or costly observability of work (job retention) effort because I assume the worker is beyond the grasp of the unemployment agency once employed. Wang and Williamson (1999) treat both sources of asymmetric information.

Hopenhayn and Nicolini (1997) show that the principal's cost function V is strictly convex so that necessary FOC are also sufficient. Because the state variable s is a binary random variable and because employment is an absorbing state, we do not need to keep track of s as a state variable. The following equations summarize the recursive contract design. The principal minimizes the cost of paying unemployment benefits while living up to current utility promises (v) and by recognizing the incentive compatibility that the contract needs to satisfy because of unobservable search effort.

$$V(v) = \min_{c,a,v'} \{c + \beta E[V(v')|v]\} = \min_{c,a,v'} \{c + \beta(1 - p(a))V(v')\} \tag{16}$$

subject to

$$u(c) - a + \beta \left[(1 - p(a))v' + p(a)V^e\right] \geq v \tag{17}$$

$$\beta p'(a)[V^e - v'] \leq 1 \qquad or \tag{18}$$

$$a \in \arg\max\{u(c) - a + \beta \left[(1 - p(a))v' + p(a)V^e\right]\} \tag{19}$$

The welfare maximizing UI program has the interesting feature that the replacement rate (RR), $\frac{c}{n}$, is decreasing monotonically during the unemployment spell. The informational asymmetry makes it optimal for the principal to provide the agent with an incentive to exert a search effort that is increasing with the duration of the unemployment spell. It is not a punishment for wrong behavior but a mechanism to induce proper search incentives. The decreasing unemployment compensation -and hence consumption- for the unemployed worker reflects the trade-off between incentives and insurance. This outcome contrasts with complete insurance in the full insurance case: a constant RR and a constant search effort.

Figure 1 shows the RR under the optimal contract with asymmetric information. The horizontal axis shows the duration of the unemployment spell, the vertical axis the RR. The top line is the RR for an initially high level of promised utility v_0. The middle line for an initially intermediate level and the lower line for an initially low level. Figure 3 does the same for the optimal search intensity. Now, the top line is for a low initial level of promised utility, the lowest line for an initially high level.

4.2 The Autarky case with Robustness Considerations

In order to consider the effects of robustness on the optimal consumption level, search effort and replacement rate I will impute model uncertainty. This uncertainty reflects fear for model misspecification. I will assume that the probability of finding a job given a certain search effort level is uncertain. This uncertainty proxies for doubts about the phase of the business cycle, about the true quality of a job match, about job interviews, etc. The true likelihood $p^*(a;r)$ is not know to the government insurance agency and the worker. The principal and the agent put forward a common reference pdf $p(a;r) = 1 - e^{-ra}$ instead which, they think, is a simple but good approximation to the unknown true model. The hazard rate r is fixed to be the same as under no robustness. Principal and agent worry about a common class of potential misspecifications in the form of perturbations around the reference model. As outlined before, I index the class of candidate models by the parameters $\tilde{\theta}_P$ and $\tilde{\theta}_A$ respectively. The principal and agent make decisions that are robust to these misspecifications. A third model under consideration is the worst case model $\widetilde{p}(a;\tilde{r})$. It is the likelihood of finding a job when the evil agent and the evil principal choose the worst perturbation possible. I show below how to obtain this distorted probability measure. In this section I consider the autarky situation and use the robustness results of section 2.2.

The robust autarky problem becomes:

$$v^{autrob} = \max_{a \geq 0} \left\{ u(c^w) - a - \tilde{\theta}_A \beta \log \left[p(a) \exp\left(\frac{-V^e}{\tilde{\theta}_A} \right) + (1 - p(a)) \exp\left(-\frac{v^{autrob}}{\tilde{\theta}_A} \right) \right] \right\}$$

with FONC

$$\beta p'(a) \left\{ \frac{-\tilde{\theta}_A \left[\exp\left(\frac{-V^e}{\tilde{\theta}_A} \right) - \exp\left(-\frac{v^{autrob}}{\tilde{\theta}_A} \right) \right]}{\left[p(a) \exp\left(\frac{-V^e}{\tilde{\theta}_A} \right) + (1 - p(a)) \exp\left(-\frac{v^{autrob}}{\tilde{\theta}_A} \right) \right]} \right\} \leq 1$$

$$= 1 \quad if \quad a > 0$$

Iterating on the two previous equations until convergence will give us the value of unemployment along with the optimal search effort under autarky. The agent uses her reference model to assess the likelihood of finding a job next period and the evil agent chooses the worst perturbation function. The value of autarky decreases with the agent's robustness parameter $\tilde{\theta}_A$. Doubts about the true chances of finding a job make the agent search harder for a job in the absence of a unemployment insurance. Figure 4 shows how the value of autarky increases (right scale, dashed line) and the agent's search intensity decreases (left scale, full line) with decreasing robustness.

This result can be understood by backing out the worst case probability distribution $\widetilde{p}(a; \tilde{r}) = 1 - e^{-a\tilde{r}}$. The hazard rate $\tilde{r}$ solves the autarky Bellman equation without robustness evaluated at the search effort that is optimal for the autarky case with robustness, a^{rob}, and the corresponding value of being unemployed v^{autrob}.

$$v^{autrob} = u(c^w) - a^{rob} + \beta \left[\widetilde{p}(a^{rob}; \tilde{r}) V^e + (1 - \widetilde{p}(a^{rob}; \tilde{r})) v^{autrob} \right]$$

The hazard rate $\tilde{r}$ associated with $\widetilde{p}(a; \tilde{r})$ decreases with increasing robustness (see Figure 5). Ceteris paribus, when the agent faces a lot of model uncertainty, she perceives her chances of finding a job to be lower and she will exert a high search effort. Hence, fear for model misspecification makes the agent search more intensively because her value of autarky is lower than when she has no doubts about the model. This behavior will turn out to be a very important ingredient of the optimal contract.

4.3 The Optimal Contract under Robustness

The asymmetric information contract design problem under robustness is the subject of this subsection. The principal minimizes the costs of the UI mechanism V subject to a robust PK and a robust IC constraint. As outlined in section 2.2, the evil principal tries to distort the principal's evaluation of the costs of the insurance by distorting the continuation value function. Simultaneously the evil agent affects the contract by distorting the promise keeping and the incentive compatibility constraint. There is no production s taking place in this problem. Rather s is a binary random variable denoting whether the unemployed found a job or not. In a sense this problem is a simpler case than described in section 2.2. The optimal contract under asymmetric information and with robustness solves:

$$V(v) = \min_{c,a,v'}\{c + \beta\Re[V(v')|v]\} = \min_{c,a,v'}\left\{c + \tilde{\theta}_P \beta \log\left[(1-p(a))\exp\left(\frac{V(v')}{\tilde{\theta}_P}\right) + p(a)\exp\left(\frac{0}{\tilde{\theta}_P}\right)\right]\right\} \tag{20}$$

subject to

$$u(c) - a - \tilde{\theta}_A \beta \log\left[(1-p(a))\exp\left(\frac{-v'}{\tilde{\theta}_A}\right) + p(a)\exp\left(\frac{-V^e}{\tilde{\theta}_A}\right)\right] \geq v \tag{21}$$

$$\beta p'(a)\left\{\frac{-\tilde{\theta}_A\left[\exp\left(\frac{-V^e}{\tilde{\theta}_A}\right) - \exp\left(\frac{-v'}{\tilde{\theta}_A}\right)\right]}{\left[(1-p(a))\exp\left(\frac{-v'}{\tilde{\theta}_A}\right) + p(a)\exp\left(\frac{-V^e}{\tilde{\theta}_A}\right)\right]}\right\} \leq 1 \tag{22}$$

$$= 1 \quad if \quad a > 0$$

The FONC with respect to c, a, v' are derived in appendix B.

The first order conditions with respect to v' in the problem without robustness can be shown to be (see Hopenhayn and Nicolini (1997)):

$$V'(v') = V'(v) - \eta\frac{p'(a)}{1-p(a)}$$

where η is the Lagrange multipliers on the IC constraint. In the no-robustness case $V'(v') < V'(v)$ because the costliness of the insurance implies that $\eta = \frac{1}{r}\frac{V(v')}{V^e - v'} > 0$ and $\frac{p'(a)}{1-p(a)} = r$. If V is convex this implies that $v' < v$, i.e. promised utilities are declining as the unemployment spell continues. This is ultimately the incentive to induce the agent to exert the optimal search effort. The equivalent condition for the robustness case is

$$V'(v') = T_1\left\{V'(v) - \tilde{\eta}\frac{p'(a)}{1-p(a)}\right\} \tag{23}$$

where

$$\tilde{\eta} = \frac{1}{r}\frac{\theta_P}{\theta_A}\left(\frac{\frac{1}{1-D} - e^{-ra}}{\frac{1}{1-E} - e^{-ra}}\right)$$

$$T_1 = \left(\frac{D}{1-(1-D)e^{-ra}}\right)\left(\frac{1-(1-E)e^{-ra}}{E}\right)$$

$$D = \exp\left(\frac{V^e - v'}{\tilde{\theta}_A}\right) \qquad E = \exp\left(\frac{V(v')}{\tilde{\theta}_P}\right)$$

Using the definition of T_1 and $\tilde{\eta}$, equation (23) becomes

$$V'(v') = T_1 V'(v) - T_6$$

with

$$T_6 = \frac{\theta_P}{\theta_A}\left(\frac{D}{1-D}\right)\left(\frac{1-E}{E}\right)$$

For $\tilde{\theta}_A = \tilde{\theta}_P = +\infty$, we recover the no-robustness result where $T_1 = 1$ and $T_6 = \frac{V(v')}{V^e - v'} > 0$. Once robustness is turned on, T_1 and T_6 are still positive but they are changing in the degree of robustness. This implies that $V'(v')$ can be either greater or smaller than $V'(v)$. The claims stated below reflect important characteristics of the optimal contract. The claims rely on numerical results, fond to be qualitatively unaffected for different parameter values.

As in Hopenhayn and Nicolini (1997) the constraints (21) and (22) may well lead to a non-convex constraint set. Convexity of V then becomes difficult to show. If V is not convex one needs to use lotteries over controls to convexify the problem as in Phelan and Townsend (1991) and use linear programming to solve the DP problem. In our numerical example lotteries turn out not be necessary.

Claim 1. *The slope of the replacement rate curve and the search intensity curve under the optimal contract change sign when the agent's preference for robustness is increased sufficiently.*

This implies there is a degree of robustness that delivers perfect insurance in spite of the presence of asymmetric information.

Claim 2. *There is a degree of robustness that corresponds to a flat consumption pattern.*

The previous claim and -claimed- continuity of V in $\tilde{\theta}$ and compactness of the set of robustness parameters Θ imply by the intermediate value theorem that there exists a unique level of robustness for which the replacement rate is constant over time.

Claim 3. *The replacement rate converges over time and the level it converges to rises with the preference for robustness*

The long-run level of unemployment, or the level of unemployment given to unemployed of long duration, is increasing as the fear for model misspecification increases.

Claim 4. *The agent's preference for robustness has a larger impact on the features of the optimal contract than the principal's for a given degree of robustness.*

4.4 The Worst-Case Probability Distributions

It is interesting to determine explicitly the worst case probability law for the principal $\tilde{p}^P(a, \tilde{r}^P)$ and the agent $\tilde{p}^A(a, \tilde{r}^A)$ implied by the contract with robustness. For a given pair $(\tilde{\theta}_A, \tilde{\theta}_P)$, the previous section showed how to compute the optimal contract under robustness (a, c, v') for each v. The worst case probability law for the agent $(\tilde{r}^A)$ is such that it makes the agent equally well off in a situation without robustness considerations but with the twisted probability law $\tilde{p}^A$. That is, for the optimal robust contract (a, c, v') and a given v, $\tilde{r}^A$ solves

$$v = u(c) - a + \beta\left[(1 - \tilde{p}^A(a, \tilde{r}^A))v' + \tilde{p}^A(a, \tilde{r}^A)V^e\right] \tag{24}$$

The fact that the principal may also be robust is fully incorporated in the contract (a, c, v'). The worst case probability law for the principal $(\tilde{r}^P)$ is such that it makes the principal equally well off in a situation without robustness considerations but with the twisted probability law $\tilde{p}^P$. That is, for the optimal robust contract (a, c, v') and a given v, $\tilde{r}^A$ solves

$$C(v) = c + \beta(1 - \tilde{p}^P(a, \tilde{r}^P))C(v') \tag{25}$$

The fact that the agent may also be robust is fully incorporated in the contract (a, c, v'). I solve for $(\tilde{r}^A, \tilde{r}^P)$ by using a bracketing algorithm on equation (25) and (24) separately. I find that the implied worst case hazard rates $\tilde{r}^A$ and $\tilde{r}^P$ are decreasing for increasing robustness levels. The worst-case hazard rate for the agent is better with an employment insurance agency than without. Figures 6 and 7 display the model implied worst-case models for the principal and the agent.

4.5 Calibration and Results

4.5.1 Parameters

The model is calibrated to weekly data. I assume $p(a) = 1 - e^{-ra}$ where the hazard rate $r = 0.0001$. This corresponds to a 4.25% chance of finding a job in the first week of unemployment (and a 89.5% probability in the first year) under the optimal search effort in autarky in the case without robustness ($a^* = 433.8$). The other parameter values in the calibration are the period consumption level of being employed ($n = 100$), the discount factor ($\beta = 0.999$), the coefficient of relative risk aversion σ in $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. We consider two cases: $\sigma = 0.5 < 1$ and $\sigma = 1.5 > 1$. The low value of σ can be motivated by a higher intertemporal elasticity of substitution for weekly than for the standard quarterly consumption data. These two cases are two computationally different problems. Hopenhayn and Nicolini (1997) only consider the simpler case where $\sigma < 1$. That will be the baseline model. When $\sigma < 1$, I set the "welfare" level of consumption in autarky equal to zero. This gives the agent zero current period utility when unemployed. When $\sigma > 1$, giving the agent zero consumption in autarky leads to a utility level of -∞; giving zero utility makes the agent extremely well off under autarky. Therefore, I endow the agent in autarky with a welfare compensation c^w in every period of unemployment. This level is chosen to yield the same 4.25% chance of finding a job in autarky in the case without robustness as in the $\sigma < 1$ case. For $\sigma = 1.5$, $c^w = 0.01$. I also solve the model for $\sigma = 3$ ($c^w = 0.1582$). I consider parameter values $\theta_A \geq 10$ to restrict the solutions to be real numbers (as mentioned below, we need $\tilde{\theta}_A \beta r > 1$). No such restrictions apply for the robustness parameter of the principal. The lower bound for the parameter θ_P is 0.

4.5.2 Value Function Approximation

The numerical dynamic programming problem is to solve equation (20) s.t. equations (21) and (22). The state and control spaces are continuous, and hence discretization is not a good option.

Also, the value (cost) function V is not known. I use Chebyshev orthogonal polynomials to approximate V. Least squares Chebyshev approximation is almost as good as the best polynomial approximation under the L_∞ norm (see Judd (1998)). Since this method does not necessarily preserves the convex shape of the cost function, I check the shape of numerical approximation to V. The degree of approximation I use is 15 and I evaluate this polynomial at 60 Chebyshev nodes on the interval $[v_{\min}, v_{\max}]$. Chebyshev regression (using more zeros than polynomial parameters) makes the approximation method better behaved. This number of Chebyshev zeros and this degree of approximation guarantee the convexity of V.

The Chebyshev method works only on a compact interval The minimum continuation value is the value under autarky, $v_{min} = v^{autrob}$. This value decreases in the degree of robustness as outlined in section 4.2. The upper bound to the set of continuation values, $v_{\max} = V^e - \tilde{\theta}_A \log\left(\frac{\tilde{\theta}_A\beta r}{\tilde{\theta}_A\beta r-1}\right)$, can be found from the FOC under autarky by imposing a positive search effort (see appendix C). The upper bound also decreases in the degree of robustness but not as fast as the lower bound, so that the length of the optimization interval increases.

We reduce the contract problem to a problem with one state variable (v) and one control variable (v'). This is possible by first solving for the optimal search intensity a from equation (22):

$$a = \max\left\{0, \frac{1}{r}\log\left[e^{\frac{V^e - v'}{\tilde{\theta}_A}}\left(\tilde{\theta}_A\beta r - 1\right)\right]\right\}$$

and then solving for the optimal unemployment compensation c from the promise keeping constraint (21). In the case where $\sigma < 1$, whenever the right hand side of equation (26) is negative, the PK constraint can be satisfied by setting $c = 0$

$$u(c) \geq v + a + \tilde{\theta}_A\beta\log\left[(1-p(a))\exp(\frac{-v'}{\tilde{\theta}_A}) + p(a)\exp(\frac{-V^e}{\tilde{\theta}_A})\right] \tag{26}$$

Using this observation, the optimal consumption level can be recovered from

$$c = \left[(1-\sigma)\max\left\{0, v + a + \tilde{\theta}_A\beta\log\left[(1-p(a))\exp(\frac{-v'}{\tilde{\theta}_A}) + p(a)\exp(\frac{-V^e}{\tilde{\theta}_A})\right]\right\}\right]^{1-\sigma}$$

In the case $\sigma > 1$, when the right hand side of equation (26) is positive, the PK constraint can never be satisfied because $u(c) < 0, \forall c$. In this case we need to restrict the choice of next period's promised utility to ensure that the right hand side is negative. After some algebra this leads to the restriction $v' \in [v'_{\min}, v_{\max}]$ where

$$v'_{\min} = V^e - \tilde{\theta}_A\log\left(\frac{\exp\left[\frac{1}{\tilde{\theta}_A\beta}\left(\beta V^e - v - a\right)\right] - p(a)}{1-p(a)}\right)$$

is evaluated at the optimal search intensity a.

4.5.3 Results

I vary the robustness parameter for the agent and for the principal and study the effect on the replacement rate (RR). The RR is an indication of the amount of risk-sharing society can achieve in the presence of an unemployment agency. I compare the robust contract results with the benchmark case of no robustness. I focus on the case in which the agent starts the contract with an intermediate initial promised utility. Since there is only one government agency there is no unique way to pin down the initial utility promise. Monge Naranjo (2000) and Krueger and Uhlig (2000) determine initial promised utilities from competition between 'principals'. Alternatively one could close the a model with a continuum of agents, a stationary fraction of which is unemployed. The unemployment agency would tax the unemployed as soon as they find a job. The initial promised utility could then be determined to ensure that the government breaks even in expectation under the worst case model. Since this addition does not provide extra intuition about the optimal contract (it would lower V^e proportionately), I do not further discuss it.

Recall that the agent is induced to search harder in autarky because his perceived hazard rate of finding a job becomes lower as he is more robust. Since search effort is unobservable, the principal has to stipulate an optimal search effort in the contract. Otherwise the agent wants to deviate from it. This is captured by the incentive compatibility constraint. The same force that is active in autarky will be present through the IC constraint. As the agent becomes more robust, her search intensity goes up because she believes she has a lower probability of finding a job. Importantly, this reduces the incentive problem. The trade-off between insurances and incentives becomes "less binding". It enables the principal to give more insurance to the agent in terms of a flatter consumption pattern and a higher final level of consumption. As the agent becomes increasingly robust, the replacement rate becomes less steeply downward sloped, then flat and eventually upward sloping. This is optimal for the agent and the principal. The agent is willing to shift unemployment benefits into the future because she thinks she will be in unemployment for a long time. More robustness leads to a lower worst case hazard $\tilde{r}^A$ and therefore acts like a lower discount rate on future unemployment compensation. The more risk averse the agent is, the stronger the tendency to avoid future consumption disasters associated with low future unemployment benefits. The principal is also willing to shift unemployment benefits more into the future. This is because the "stick-and-carrot strategy" of lowering unemployment compensation to set search incentives right has become less necessary because of the agent's fear for model uncertainty. The principal is risk-neutral and does not care about the timing of the unemployment compensation, only about the expected value of the compensation.

As the principal becomes more robust, he fears that the agent's hazard rate of finding a job is low, resulting in a more costly contract. This is reflected in a decreasing worst-case hazard rate $\tilde{r}^P$ as the principal's robustness concern is increasing. The effective rate at which the principal discounts future cost is higher and that induces him to shift unemployment compensation from the future to the present. The principal's concern for model misspecification worsens the informational asymmetry problem. To understand why, consider the following experiment. An increase in unemployment compensation today and an offsetting reduction in the utility promise tomorrow induces a higher search effort and hence a higher probability of finding a job, p. When the principal is not robust, the marginal benefit (cost reduction) from the change in p is constant. More robustness makes the principal's future cost concave in the probability of finding a job (see Figure 8). Hence, when the principal is robust the marginal benefit is the lowest for a low p. That's why the principal fears a low hazard rate and why he shifts unemployment compensation even more into the present.

Quantitatively, the effects of robustness of the principal are summarized in Table 1 and Figures 9 and 10. I gradually increase the principal's fear for misspecification by decreasing $\tilde{\theta}_P$, while keeping $\tilde{\theta}_A = +\infty$. Only high degrees of robustness for the principal change the contract noticeably. Only in the range $\tilde{\theta}_P < 10$ does the principal's objective function become more concave in the probability of finding a job. Recall, $\theta_P = \frac{\tilde{\theta}_P}{1-\beta}$. A principal who is robust beyond this point will negotiate a contract with an downward sloping (increasingly steep) replacement rate. The initial RR (R_i) goes up and the replacement rate after 100 periods of unemployment (R_f) goes down when $\tilde{\theta}_P$ decreases. Correspondingly, the initial search effort (A_i) increases slightly and the final search intensity (A_f) increases by a larger amount.

The effects of robustness of the agent are summarized in Table 2 and Figures 11 and 12. I look at the agent's decision rule when the principal has no preference for robustness: ($\tilde{\theta}_P = +\infty, \tilde{\theta}_A < +\infty$). The optimal contract (starting again from an intermediate value for the initial utility promise) changes as follows with a decreasing value for $\tilde{\theta}_A$. The optimal initial RR (R_i) becomes gradually lower and the RR after 100 periods (R_f) is increasing. These tendencies are sharpened for $\tilde{\theta}_A$ ≤ 40. The RR curve between these two endpoints slowly shifts from a convex decreasing into a concave increasing function of unemployment duration. For a value of $\tilde{\theta}_A$ around 25, a constant replacement rate is constrained optimal. The concave increasing shape of the search effort curve gradually becomes a convex decreasing curve.

I finally consider the case where agent and principal are both robust. Table 3 presents how robustness of principal and agent interact.when they share the same doubt about the model ($\tilde{\theta} = \tilde{\theta}_A = \tilde{\theta}_P$). The robustness of the principal counteracts the changes in the RR and the search intensity induced by the agent's robustness. Comparing the three tables it is clear that introducing robustness for the principal has comparatively small effects. The preference for robustness of the agent is the dominant force in the optimal contract for robustness values $\tilde{\theta} > 16$. Only for very low values of $\tilde{\theta}_P$ does the principal's robustness concern off set the agent's. For $(\tilde{\theta}_A, \tilde{\theta}_P) = (20, 1)$ the RR is flat. Decreasing $\tilde{\theta}_P$ below this level makes the replacement rate decreasing again, because the principal's concern for robustness kicks in.

4.5.4 Discussion

The simulations imply that the patterns of optimal employment compensation can change dramatically. Over time the optimal contract in the no-robustness case prescribes the agent to increase her search effort as the unemployment spell continues. The optimal contract should give the unemployed agent an incentive to search harder by reducing the RR as the unemployment spell continues.

This is necessary because the worker has an incentive to misrepresent her search effort. At the risk of oversimplification, the unemployment insurance system in the USA reflects this incentive-based mechanism. On average, US states provide a RR of about 50% for a period of about half a year, after which the RR virtually drops to zero. A similar system is in place in Japan and to a lesser extent in Sweden, Canada, Greece, Italy and Spain. Figure (1) plots replacements rates for some of these countries. The unemployment compensation reported is for a single person and consists of a variety of benefits. It is from the OECD database on Benefit Entitlements for 1995.

With enough model uncertainty imputed into the agent's preferences, however, the unemployment agency should offer a much flatter or even constant RR as the unemployment spell continues. Robustness not only calls for a flatter pattern of unemployment compensation, but the RR also converges to a higher long-run level. Some OECD countries have such features present in the design of their UI mechanism. Examples are Denmark, Austria, the UK, New Zealand, Germany, Ireland and to a lesser extent Belgium and France. Some of these countries' unemployment programs have been widely criticized for their generosity and their perverse effects on reemployment incentives. This paper showed that a constant replacement rate may well be a feature of a constrained optimal contract. Thus, introducing model uncertainty in a contract with asymmetric information can revert "conventional wisdom" about the optimal compensation scheme. In such situation, the policy advice is to keep replacement levels sufficiently flat and relatively high. More risk-sharing is realized in an economy where unemployed workers are robust.

4.5.5 Sensitivity Analysis

The baseline model investigated the case $\sigma = 0.5$. By modifying the problem as described above, I also compute the optimal contract for a more risk averse agent.($\sigma = 1.5, \sigma = 3$). The results remain qualitatively unaltered. The higher degree of risk aversion makes the agent want to smooth consumption. The replacement rate of the optimal contract without robustness is less steeply sloped. When the agent's degree of robustness is high, she thinks that she will be in unemployment for a long time and she will work harder to avoid consumption disasters. The optimal replacement rate becomes constant for a lower degree of robustness than in the baseline case. The principal's robustness is only relevant for even lower values than before.

5 Detection Error Probabilities

An important advantage of treating model uncertainty within the robustness framework is that it provides a mechanism to discipline the degree of model misspecification. Limiting the set of models under consideration amounts to a restriction on the robustness parameters. The aim is not to make decisions that are robust to any form of model misspecification. Rather, agent and principal are only interested in being robust to candidate models that cannot be distinguished from the reference model using statistical discrimination theory. There is no need for robust decision making when misspecifications can be detected with great confidence and with a limited amount of data. Such misspecification concerns would be hard to justify. An important device to rule out unreasonable levels of robustness are detection error probabilities. They are a measure of distance between the reference model and the worst-case model model. When principal and agent are not robust, the reference and the worst-case models coincide. Their distance is zero, and the probability of making a classification (detection) error between the two models is 0.5. For higher robustness levels, the distance between the reference and worst-case model models gradually increases. The probability of not being able to distinguish between them decreases: the detection error probability falls below 0.5. As long as the detection error probability is lower than the traditional cutoff value $\alpha = 0.05$ or $\alpha = 0.10$, we fail to reject the null hypothesis that the reference and the worst case-model are different. We should be considering all candidate models with detection error probability greater than α. Following Hansen, Sargent and Wang (2000), the detectability criterion (test statistic) we use is the log likelihood ratio.

Applied to the optimal UI example of the previous section, I proceed in four steps. First, I calculate the optimal contract ($\tilde{a}$) where the principal and the agent are robust ($\tilde{\theta}^P < +\infty$, $\tilde{\theta}^A < +\infty$). Second, I compute the worst-case hazard rate for the principal ($\tilde{r}^P$) and the worst-case hazard rate for the agent ($\tilde{r}^A$) associated with the robust contract as described above. Third, I calculate the likelihood that a draw of length T from a Bernoulli distributed variable with parameter p_1 comes from the reference model

$$L_{11} = p_1^{N_1}(1-p_1)^{T-N_1}$$

and compare it to the likelihood that the same draw came from the worst case model of principal, p_2, and agent, p_3, respectively.

$$L_{12} = p_2^{N_1}(1-p_2)^{T-N_1} \qquad L_{13} = p_3^{N_1}(1-p_3)^{T-N_1}$$

where the worst-case models are evaluated at the search intensity that arises in the robust model: $p_2 = 1-\exp(-\tilde{r}^P\ \tilde{a})$ and $p_3 = 1-\exp(-\tilde{r}^A\ \tilde{a})$ and $p_1 = 1-\exp(-r\ \tilde{a})$. The number of times that one comes up in the random variable generated by parameter p_i is N_i. Vice versa, I calculate the likelihood that a draw of length T from a Bernoulli distributed variable with parameter p_2 and a draw from the model p_3 come from the respective worst-case model

$$L_{22} = p_2^{N_2}(1-p_2)^{T-N_2} \qquad L_{33} = p_3^{N_3}(1-p_3)^{T-N_3}$$

and compare them to the likelihood that the same draw came from the reference model.

$$L_{21} = p_1^{N_2}(1-p_1)^{T-N_2} \qquad L_{31} = p_1^{N_3}(1-p_1)^{T-N_3}$$

I calculate the log-likelihood ratio's $R_1^A = \log(\frac{L_{11}}{L_{12}}), R_2^A = \log(\frac{L_{22}}{L_{21}}), R_1^P = \log(\frac{L_{11}}{L_{13}}), R_2^P = \log(\frac{L_{33}}{L_{31}})$. Fourth, I repeat the third step 50,000 times and calculate the frequency of the event $\left\{R_i^j \leq 0\right\}$

for $i = 1, 2$ and $j = A, P$. Call this frequency r_i^j. The decision maker j commits a classification or detection error when $R_i^j \leq 0$ for either i. The detection error probability weighs each of these errors equally and is defined by

$$D^P(\tilde{\theta}^P) = \frac{1}{2}(r_1^P + r_2^P)$$

for the principal and similarly for the agent

$$D^A(\tilde{\theta}^A) = 0.5(r_1^A + r_2^A)$$

Figure 13 plots the principal's detection error probabilities for a large range of robustness values $5 \leq \theta^P \leq 10^{10}$ while keeping the agent not robust ($\tilde{\theta}_A = +\infty$). Applying the cut-off criterion $D^P(\tilde{\theta}^P) \geq \alpha = 0.10$ we can justify robustness values $\theta^P \geq 230$. Figure 14 plots the agent's detection error probabilities for all admissible robustness values $14.75 \leq \tilde{\theta}^A \leq 10^7$ while keeping the principal not robust ($\tilde{\theta}_p = +\infty$). Applying the cut-off criterion $D^A(\tilde{\theta}^A) \geq \alpha = 0.10$ one can justify all robustness values $\tilde{\theta}^A \geq 15$. Finally, we look at the optimal contract in which principal and agent are equally robust ($\tilde{\theta} = \tilde{\theta}_P = \tilde{\theta}_A$). The above log-likelihood ratio analysis goes through. The detection error probability for $\tilde{\theta} = 20$ is 0.31 for the agent and 0.49 for the principal. For this level of robustness, the replacement rate curve is upward sloping. Hence, the contracts prescribing a flat or increasing RR are in the set of admissible outcomes (using detectability restrictions on the parameter space Θ). However, the values of $\tilde{\theta}_P$ for which the principal's robustness starts to matter have a detection error probability below $\alpha = 0.10$. This reinforces the conclusion that the RR in the optimal contract is flat or even increasing for reasonably robust principal and agent.

6 Conclusion

This paper investigated the optimal contract between a principal and an agent who have concerns about model misspecification in the presence of asymmetric information. The main theoretical contribution of this paper is to incorporate robustness concerns into the standard dynamic contracting problem and to proof that there is a relatively simple recursive formulation.

The application under study is the design of an optimal dynamic unemployment compensation scheme between an unemployment agency and an unemployed worker in the presence of unobservable search effort. As is well understood, the optimal contract sacrifices insurance for incentives. When the parties to the contract have model uncertainty, the constrained optimal contract changes dramatically. The replacement rate gradually becomes flatter and eventually upward sloping for an unemployed worker with a high degree of model uncertainty. There is a level of robustness for which the optimal contract shows full insurance in spite of the presence of asymmetric information. The main force behind this result is that uncertainty about the model induces the unemployed worker to perceive her chances of finding a job as more pessimistic and hence to exert a higher search effort. It allows the government unemployment agency to provide more insurance because it partially overcomes the asymmetric information problem. I use a likelihood-ratio test to rule out unacceptable levels of model uncertainty. A flat replacement rate can be obtained for acceptable robustness parameter values.

We investigate the differential effect of model misspecification for principal and agent. When the government has a very high degree of model uncertainty, the asymmetric information problem is worsened and a sharply declining replacement rate is optimal. For mutually acceptable pairs of robustness parameters, this force is dominated by the risk-averse agent's fear for model misspecification.

The results of the paper may be used to argue that both the steeply downward sloping "American" replacement rate schedule and the much flatter "European" schedule are both constrained efficient, albeit for different levels of robustness. Empirical evidence on job turnover rates and workers' job insecurity polls supports the robustness explanation for the cross-country difference in observed unemployment insurance schedules.

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A Appendix

A.1 Recursive Characterization of the Robust Principal-Agent Problem

Let $\{\tilde{c}, \tilde{a}, \tilde{\pi}^p, \tilde{\pi}^a\}$ be an optimal contract. I suppress the dependence of $\tilde{S}_P$ on (c, a, π^p) and of $\tilde{S}_A$ on (c, a, π^a) for notational simplicity. First, recall equation (7) and write out the sequence of payoffs as a first period payoff and the sequel.

$$\tilde{S}_P(s^t) = \sum_{\tau=0}^{\infty} \beta^\tau \int [U_P(s_{t+\tau} - c(s^{t+\tau})) + \theta_P \log\left(\frac{d\pi^p(s^{t+\tau}|s^t,a)}{d\pi_0(s^{t+\tau}|s^t,a)}\right)]d\pi^p(s^{t+\tau}|s^t,a)$$

$$\tilde{S}_P(s^t) = U_P(s_t - c(s^t)) + \beta \sum_{\tau=1}^{\infty} \beta^{\tau-1} \int \left[U_P(s_{t+\tau} - c(s^{t+\tau})) + \theta_P \log\left(\frac{d\pi^p(s^{t+\tau}|s^t,a)}{d\pi_0(s^{t+\tau}|s^t,a)}\right)\right] d\pi^p(s^{t+\tau}|s^t,a)$$

$$\tilde{S}_P(s^t) = U_P(s_t - c(s^t)) +$$
$$\beta \sum_{\tau=1}^{\infty} \beta^{\tau-1} \int \left[U_P(s_{t+\tau} - c(s^{t+\tau})) + \theta_P \log\left(\frac{d\pi^p(s^{t+\tau}|s^{t+1},a)}{d\pi_0(s^{t+\tau}|s^{t+1},a)}\right) + \theta_P \log\left(\frac{f^p(s_{t+1}|a(s^t))}{f_0(s_{t+1}|a(s^t))}\right)\right] d\pi^p(s^{t+\tau}|s^t,a)$$

Now use

$$d\pi(s^{t+\tau}|s^t,a_t) = f(s_{t+1}|a(s^t))d\pi(s^{t+\tau}|s^{t+1},a_t)$$

and

$$\tilde{\theta}_P = \frac{\theta_P}{1-\beta}$$

to rewrite the payoff as:

$$\tilde{S}_P(s^t) = U_P(s_t - c(s^t)) + \beta\tilde{\theta}_P \int \log\left(\frac{f^p(s_{t+1}|a(s^t))}{f_0(s_{t+1}|a(s^t))}\right) f^p(s_{t+1}|a(s^t))dy_{t+1} + \beta\tilde{S}_P(s^{t+1}) \tag{27}$$

Analogously,

$$\tilde{S}_A(s^t) = U_A(a_t(s^t), c(s^t)) + \beta\tilde{\theta}_A \int \log\left(\frac{f^a(s_{t+1}|a(s^t))}{f_0(s_{t+1}|a(s^t))}\right) f^a(s_{t+1}|a(s^t))dy_{t+1} + \beta\tilde{S}_A(s^{t+1}) \tag{28}$$

where

$$\tilde{\theta}_A = \frac{\theta_A}{1-\beta}$$

I will refer to the equations (27) and (28) in the proof below. Recall the timing assumption that s is realized at the end of the period and is then public knowledge. At the beginning of the period s_{-1} is known to principal and agent. that is, the principal can only observe the output of the agent's effort at the end of the period and only act on it in the next period.

The proof will proceed in three steps.

Step 1: Time 1 situation. Claim: $\tilde{S}_P(s_1)$ extremizes (maxmin) the robust principal's expected utility given that the robust agent is promised $\tilde{S}_A(s_1)$ and the first period IC constraints hold. I proof this claim by contradiction.

Let $a_1 = a(s_0)$ and idem for the other three controls. The initial $s_0 = s_0$ is given. From (27) the principal's payoff at time 1 is given by:

$$\tilde{S}_P(s_0) = U_P(s_0 - c(s_0)) + \beta\tilde{\theta}_P \int \log\left(\frac{f^p(s_1|a_1)}{f_0(s_1|a_1)}\right) f^p(s_1|a_1)dy_1 + \beta\tilde{S}_P(s_1)$$

Similarly from (28);

$$\tilde{S}_A(s_0) = U_A(a_1, c(s_1)) + \beta\tilde{\theta}_A \int \log\left(\frac{f^a(s_1|a_1)}{f_0(s_1|a_1)}\right) f^a(s_1|a_1)dy_1 + \beta\tilde{S}_A(s_1)$$

The penalties to the evil agent for choosing a model that is not equal to the reference model accumulate over time as suggested by the parameters $\tilde{\theta}$. Suppose there was an alternative scheme $(c', a', \pi^{a\prime}, \pi^{p\prime})$ that was IC and gave the robust agent an expected utility of at least $\bar{x}$, and that stipulated the same compensation in the first period, $c'(s_1) = c(s_1)$, but an expected payoff for the principal that is not a saddle point, $\tilde{S}'_P(s_1) \neq \tilde{S}_P(s_1)$. Then a_1 and f_1^a would still be incentive compatible at t=2 for the agent and the evil agent respectively given the robust agent is offered $\tilde{S}_A(s_1)$. That implies that there the original contract was not optimal, which is a contradiction. This proofs the claim.

Step 2: Construction of the optimal contract starting from time 1.

Define $\int_P = \{\tilde{S}_P(s^t)\}_{t=1}^{\infty}$ and $\int_A = \{\tilde{S}_A(s^t)\}_{t=1}^{\infty}$ to represent the sets of values under the optimal contract for the robust principal and the robust agent.

For each $x \in \int_A$, optimize the robust principal's time 1 payoff subject to the robust agent getting payoff x (today's utility promise), given an output realization s_0 and subject to incentive compatibility. Denote the solution to this problem by: $V(x, s_0) = V(x)$ -time 1 payoff to the robust principal-, $a(x, s_0) = a(x)$ -time 1 action of the agent-, $c(x, s_0) = c(x)$ -compensation schedule at time 1 by principal-, $w^a(x, s_0) = w^a(x)$ -the evil agent's perturbation function (marginal distribution) at time 1- and $w^p(x, s_0) = w^p(x)$ -the evil principal's marginal at time 1-. Let $v'(x, s_0) = v'(x)$ be the payoff promised to the robust agent at t=2. Then by step 1, if $x = \tilde{S}_A(s^t)$, it follows that $V(x) = \tilde{S}_P(s^{t+1})$, $a(x) = a(s^t)$, $c(x) = c(s^{t+1})$, $w^a(x) = f^a(s^{t+1})$, $w^p(x) = f^p(s^{t+1})$, and $v'(x) = \tilde{S}_A(s^t)$. The same argument applies to the reference model -which is a special choice for w^p for $\theta_P = \infty$- and hence $w_0(x, s_0) = w_0(x) = f_0(s^{t+1})$.

Start at t=1 and consider the following strategy for the principal: offer $c(\bar{x}, s_0)$ to the agent and specify $w^a(\bar{x}, s_0)$ to the evil agent today and promise $v'(\bar{x}, s_0)$ tomorrow to the robust agent (i.e. the "joint" person). We know the agent will work $a(\bar{x}, s_0)$, and the robust principal will get the optimum payoff $V(\bar{x}, s_0)$. If there is a unique saddle point to $\tilde{S}_P$, there will be a unique optimal perturbation $w^p(\bar{x}, s_0)$. At t=2, the output realization s_1 is known. The principal has to honor his utility promise of $v'(\bar{x}, s_0)$ to the robust agent. Consider offering $c(v'(\bar{x}, s_0), s_1)$ and specifying $w^a(v'(\bar{x}, s_0), s_1)$ today and $v'(v'(\bar{x}, s_0), s_1)$ tomorrow. Then the agent works $a(v'(\bar{x}, s_0), s_1)$, and the robust principal gets $V(v'(\bar{x}, s_0), s_1)$, which is the optimal payoff by construction given the robust agent has to be promised $v'(\bar{x}, s_0)$. From $c(v'(\bar{x}, s_0), s_1)$ and $V(v'(\bar{x}, s_0), s_1)$, we can identify $w^p(v'(\bar{x}, s_0), s_1)$. Given realization s_2 the compensation scheme at t=3 will be $c(v'(v'(\bar{x}, s_0), s_1), s_2)$ etc...

Step 3: The above argument implies the existence of the following policy functions:

$$\begin{array}{rcccll}
v' : \int_A & X & \Re & \rightarrow & \int_A & \\
V : \int_A & X & \Re & \rightarrow & \int_P & \\
c : \int_A & X & \Re & \rightarrow & \Re & \\
a : \int_A & X & \Re & \rightarrow & A & \\
w^i : \int_i & X & \Re & \rightarrow & \Re^+ & \quad i = a, p, 0
\end{array}$$

that satisfy $\forall z = (x, s_{-1})$, $x \in \int_A$, $s_{-1} \in \Re$:

$$x = U_A(a(z), c(z)) + \beta \int \left\{ \tilde{\theta}_A \log \left(\frac{w^a(z'|a(z'))}{w_0(z'|a(z'))} \right) + v'(z') \right\} w^a(z'|a(z))dy$$

$$v'(z) = \int \left\{ U_A(a(z'), c(z')) + \beta\tilde{\theta}_A \log \left(\frac{w^a(z'|a(z'))}{w_0(z'|a(z'))} \right) + v'(v'(z), s) \right\} w^a(z'|a(z'))dy$$

$$V(z) = U_P(s_{-1} - c(z)) + \beta \int \left\{ \tilde{\theta}_P \log \left(\frac{w^p(z'|a(z'))}{w_0(z'|a(z'))} \right) + \beta V(z') \right\} w^p(z'|a(z'))dy$$

$$(a(z), w^a(z)) \in \arg\max_{a \in A} \min_{w^a \in \Re^+} U_A(a, c(z)) + \beta \int \left\{ \tilde{\theta}_A \log \left(\frac{w^a}{w_0(z'|a(z'))} \right) + \beta v'(z') \right\} w^a dy$$

Where I used $z' = (x', s) = (v'(z), s)$.

The above implies that we should solve the following contract, recursive in the state variables $(x, s_{-1}) = z$:

$$V(z) = \max_{a,c,v'} \min_{w^a, w^p} U_P(s_{-1} - c(z)) + \beta\tilde{\theta}_P E \left[\log \left(\frac{w^p(z'|a(z'))}{w_0(z'|a(z'))} \right) w^p(z'|a(z')) \right]$$
$$+ \beta E[V(z')w^p(z'|a(z'))]$$

subject to a robust promise keeping constraint ($RPKC$):

$$U_A(a(z), c(z)) + \beta\tilde{\theta}_A E\left[\log\left(\frac{w^a(z'|a(z'))}{w_0(z'|a(z'))}\right) w^a(z'|a(z'))\right] + \beta E\left[v'(z')w^a(z'|a(z'))\right] \geq x \tag{29}$$

and subject to a robust incentive compatibility constraint ($RICC$):

$$(a(z), w^a(z)) \in \arg\max_a \min_{w^a} U_A(a, c) + \beta\tilde{\theta}_A E[\log\left(\frac{w^a}{f_0(z'|a(z'))}\right) w^a] + \beta E[v'(z')w^a] \tag{30}$$

Use the definition of conditional relative entropy to write:

$$V(z) = \max_{a,c,v'} \min_{w^a, w^p} \{U_P(s-c) + \beta\tilde{\theta}_P I(w^p) + \beta E^{w^p}[V(v'(z), s)]\}$$

subject to the RPKC and RICC:

$$U_A(a, c) + \beta\tilde{\theta}_A I(w^a) + \beta E^{w^a}[v'(z')] \geq x$$

$$(a(z), w^a(z)) = \arg\max_a \min_{w^a} \{U_A(a, c) + \beta\tilde{\theta}_A I(w^a) + \beta E^{w^a}[v'(z')]\} \tag{31}$$

The fact that the evil persons choose marginal distributions that recursively build up the measure π makes the problem recursive in the same state variables as the model without robustness. The discrete time analogue is transition probability matrices of a Markov chain. For an analogous proof of the discrete time version, see **?**.

A.2 First Order Conditions of the Optimal Contract

The First order conditions of the robust optimal contract are:

w.r.t. c

$$\lambda = \frac{1}{u'(c)} \tag{32}$$

w.r.t. a

$$\frac{1}{\eta} = T_3 T_5 \tag{33}$$

w.r.t. v'

$$V'(v') = T_1\{\lambda + \eta\frac{p'(a)}{1-p(a)}T_2\} \tag{34}$$

where λ and η are the Lagrange multipliers on the PK and IC constraint respectively. The envelope condition gives

$$V'(v) = \lambda$$

The details of the calculation are presented below.

The second FOC (w.r.t. a) of the optimal contract with asymmetric info and robustness can be manipulated as follows:

$$0 = \frac{\theta_P\beta p'(a)\left[1 - \exp\left(\frac{V(v')}{\theta_P}\right)\right]}{(1-p(a))\exp\left(\frac{V(v')}{\theta_P}\right) + p(a)} + \lambda\left\{1 + \frac{\theta_A\beta p'(a)\left[\exp\left(\frac{-V^e}{\theta_A}\right) - \exp\left(-\frac{v'}{\theta_A}\right)\right]}{\left[p(a)\exp\left(\frac{-V^e}{\theta_A}\right) + (1-p(a))\exp\left(-\frac{v'}{\theta_A}\right)\right]}\right\} +$$
$$\eta\left\{p''(a)\frac{-\theta_A\beta\left[\exp\left(\frac{-V^e}{\theta_A}\right) - \exp\left(-\frac{v'}{\theta_A}\right)\right]}{\left[p(a)\exp\left(\frac{-V^e}{\theta_A}\right) + (1-p(a))\exp\left(-\frac{v'}{\theta_A}\right)\right]} + [p'(a)]^2\frac{\theta_A\beta\left[\exp\left(\frac{-V^e}{\theta_A}\right) - \exp\left(-\frac{v'}{\theta_A}\right)\right]^2}{\left[p(a)\exp\left(\frac{-V^e}{\theta_A}\right) + (1-p(a))\exp\left(-\frac{v'}{\theta_A}\right)\right]^2}\right\}$$

The second term drops out because of the incentive compatibility constraint.

$$0 = \frac{\theta_P \beta p'(a)\left[1 - \exp\left(\frac{V(v')}{\theta_P}\right)\right]}{(1-p(a))\exp\left(\frac{V(v')}{\theta_P}\right) + p(a)} + \eta.$$

$$\left\{ p''(a) \frac{-\theta_A \beta \left[\exp(\frac{-V^e}{\theta_A}) - \exp(-\frac{v'}{\theta_A})\right]}{\left[p(a)\exp\left(\frac{-V^e}{\theta_A}\right) + (1-p(a))\exp\left(-\frac{v'}{\theta_A}\right)\right]} + [p'(a)]^2 \frac{\theta_A \beta \left[\exp\left(\frac{-V^e}{\theta_A}\right) - \exp\left(-\frac{v'}{\theta_A}\right)\right]^2}{\left[p(a)\exp\left(\frac{-V^e}{\theta_A}\right) + (1-p(a))\exp\left(-\frac{v'}{\theta_A}\right)\right]^2} \right\}$$

Collecting terms:

$$\frac{\theta_P \beta p'(a)\left[1 - \exp\left(\frac{V(v')}{\theta_P}\right)\right]}{(1-p(a))\exp\left(\frac{V(v')}{\theta_P}\right) + p(a)} + \eta \left[\frac{\theta_A \beta \left[\exp\left(\frac{-V^e}{\theta_A}\right) - \exp\left(-\frac{v'}{\theta_A}\right)\right]}{\left[p(a)\exp\left(\frac{-V^e}{\theta_A}\right) + (1-p(a))\exp\left(-\frac{v'}{\theta_A}\right)\right]} \right]$$
$$\left\{ -p''(a) + [p'(a)]^2 \frac{\left[\exp\left(\frac{-V^e}{\theta_A}\right) - \exp\left(-\frac{v'}{\theta_A}\right)\right]}{\left[p(a)\exp\left(\frac{-V^e}{\theta_A}\right) + (1-p(a))\exp\left(-\frac{v'}{\theta_A}\right)\right]} \right\} = 0$$

Dividing through both sides by $\theta_A \beta$, bringing the first term to the other side of the equality sign and using the definition of D and E:

$$\frac{-\theta_P}{\theta_A}\left[\frac{1-E}{p(a)(1-E)+E}\right] = \eta \left[\frac{1-D}{p(a)(1-D)+D}\right] \left\{ -\frac{p''(a)}{p'(a)} + p'(a)\frac{1-D}{p(a)(1-D)+D} \right\}$$

where D and E are defined by:

$$\begin{aligned} D &= \exp\left(\frac{V^e - v'}{\theta_A}\right) > 1 \\ E &= \exp\left(\frac{V(v')}{\theta_P}\right) > 1 \end{aligned}$$

We can now solve for η

$$\eta = \frac{-\theta_P}{\theta_A}\left(\frac{1-E}{1-D}\right)\left(\frac{p(a)(1-D)+D}{p(a)(1-E)+E}\right)\left\{ p'(a)\frac{1-D}{p(a)(1-D)+D} - \frac{p''(a)}{p'(a)} \right\}^{-1}$$

$$\begin{aligned} \frac{1}{\eta} &= -T_3 \left\{ p'(a) T_4 - \frac{p''(a)}{p'(a)} \right\} \\ &= -T_3 \; T_5 \end{aligned}$$

where:

$$\begin{aligned} T_3 &= \frac{\theta_A}{\theta_P}\left(\frac{1-D}{1-E}\right)\left(\frac{p(a)(1-E)+E}{p(a)(1-D)+D}\right) \\ T_4 &= \frac{1-D}{p(a)(1-D)+D} \\ T_5 &= p'(a)\left(\frac{1-D}{p(a)(1-D)+D}\right) - \frac{p''(a)}{p'(a)} \end{aligned}$$

The third FONC that appears above is derived as follows:

$$(1-p(a))V'(v')\frac{\exp\left(\frac{V(v')}{\theta_P}\right)}{\left[(1-p(a))\exp\left(\frac{V(v')}{\theta_P}\right)+p(a)\right]}-(1-p(a))\lambda\frac{\exp\left(\frac{-v'}{\theta_A}\right)}{\left[p(a)\exp\left(\frac{-V^e}{\theta_A}\right)+(1-p(a))\exp\left(\frac{-v'}{\theta_A}\right)\right]}+$$
$$\eta p'(a)\left\{\frac{-(1-p(a))\exp\left(\frac{-v'}{\theta_A}\right)\left[\exp\left(\frac{-V^e}{\theta_A}\right)-\exp\left(\frac{-v'}{\theta_A}\right)\right]}{\left[p(a)\exp\left(\frac{-V^e}{\theta_A}\right)+(1-p(a))\exp\left(\frac{-v'}{\theta_A}\right)\right]^2}+\frac{-\exp\left(\frac{-v'}{\theta_A}\right)}{\left[p(a)\exp\left(\frac{-V^e}{\theta_A}\right)+(1-p(a))\exp\left(\frac{-v'}{\theta_A}\right)\right]}\right\}=0$$

$$V'(v')\left[\frac{E}{p(a)+(1-p(a))E}\right]=\lambda\left[\frac{D}{p(a)+(1-p(a))D}\right]+$$
$$\eta\frac{p'(a)}{1-p(a)}\left[\frac{D}{p(a)+(1-p(a))D}\right]\left\{\frac{(1-p(a))\left[\exp\left(\frac{-V^e}{\theta_A}\right)-\exp\left(\frac{-v'}{\theta_A}\right)\right]}{\left[p(a)\exp\left(\frac{-V^e}{\theta_A}\right)+(1-p(a))\exp\left(\frac{-v'}{\theta_A}\right)\right]}+1\right\}$$

$$V'(v')\left[\frac{E}{p(a)+(1-p(a))E}\right]=\lambda\left[\frac{D}{p(a)+(1-p(a))D}\right]+$$
$$\eta\frac{p'(a)}{1-p(a)}\frac{D}{p(a)+(1-p(a))D}\left\{\frac{\exp\left(\frac{-V^e}{\theta_A}\right)}{\left[p(a)\exp\left(\frac{-V^e}{\theta_A}\right)+(1-p(a))\exp\left(\frac{-v'}{\theta_A}\right)\right]}\right\}$$

$$V'(v')=\frac{D}{E}\left(\frac{p(a)+(1-p(a))E}{p(a)+(1-p(a))D}\right)\left\{\lambda+\eta\frac{p'(a)}{1-p(a)}\frac{1}{p(a)+(1-p(a))D}\right\}$$

$$V'(v')=T_1\{\lambda+\eta\frac{p'(a)}{1-p(a)}T_2\}$$

where

$$\begin{aligned}T_1 &= \frac{D}{E}\left(\frac{p(a)+(1-p(a))E}{p(a)+(1-p(a))D}\right)\\ T_2 &= \frac{1}{p(a)+(1-p(a))D}\end{aligned}$$

When $p(a)=1-e^{-ra}$, T_1 simplifies to:

$$\begin{aligned}T_1 &= \left(\frac{D}{E}\right)\left(\frac{1-(1-E)e^{-ra}}{1-(1-D)e^{-ra}}\right)\\ &= \left(\frac{\frac{1}{E}-1}{\frac{1}{D}-1}\right)\left(\frac{\frac{1}{1-E}-e^{-ra}}{\frac{1}{1-D}-e^{-ra}}\right)\end{aligned}$$

Using the envelope condition and substituting in the second FOC we can write:

$$V'(v')=T_1\left\{V'(v)-\tilde{\eta}\frac{p'(a)}{1-p(a)}\right\} \tag{35}$$

This is the condition that appears in the text, where

$$\begin{aligned}\tilde{\eta} &= -\eta\ T_2=\frac{T_2}{T_3\ T_5}\\ \tilde{\eta}^{-1} &= \left[\frac{\theta_A}{\theta_P}\left(\frac{1-D}{1-E}\right)\left(\frac{p(a)(1-E)+E}{p(a)(1-D)+D}\right)\right]\left[p'(a)\left(\frac{1-D}{p(a)(1-D)+D}\right)-\frac{p''(a)}{p'(a)}\right][p(a)+(1-p(a))D]\\ \tilde{\eta}^{-1} &= \frac{\theta_A}{\theta_P}\left(\frac{1-D}{1-E}\right)(p(a)(1-E)+E)\left[p'(a)\left(\frac{1-D}{p(a)(1-D)+D}\right)-\frac{p''(a)}{p'(a)}\right]\end{aligned}$$

For $p(a) = 1 - e^{-ra}$ we have that

$$
\begin{aligned}
\tilde{\eta}^{-1} &= r\frac{\theta_A}{\theta_P}(1-D)\left(\frac{1-e^{-ra}(1-E)}{1-E}\right)\left\{\left(\frac{e^{-ra}\,(1-D)}{1-e^{-ra}(1-D)}\right)+1\right\} \\
\tilde{\eta} &= \frac{1}{r}\frac{\theta_P}{\theta_A}\left(\frac{1-e^{-ra}(1-D)}{1-D}\right)\left(\frac{1-E}{1-e^{-ra}(1-E)}\right) \\
\tilde{\eta} &= \frac{1}{r}\frac{\theta_P}{\theta_A}\left(\frac{\frac{1}{1-D}-e^{-ra}}{\frac{1}{1-E}-e^{-ra}}\right)
\end{aligned}
$$

Finally we can rewrite equation (35) using $p(a) = 1 - e^{-ra}$ and the expression for $\tilde{\eta}$

$$V'(v') = T_1 V'(v) - T_6$$

with

$$
\begin{aligned}
T_6 &= \frac{\theta_P}{\theta_A}\left(\frac{D}{1-D}\right)\left(\frac{1-E}{E}\right) \\
&= \frac{\theta_P}{\theta_A}\left(\frac{\frac{1}{E}-1}{\frac{1}{D}-1}\right) \\
&= \frac{\theta_P}{\theta_A}\left(\frac{\exp\left(\frac{-V(v')}{\theta_P}\right)-1}{\exp\left(\frac{v'-V^e}{\theta_A}\right)-1}\right)
\end{aligned}
$$

A.3 Derivation of upper bound on value of being unemployed

The equation (22) can be rewritten and subsequently manipulated as follows:

$$-\exp\left(\frac{-V^e}{\tilde{\theta}_A}\right)+\exp\left(-\frac{v'}{\tilde{\theta}_A}\right) \le \frac{p(a)\exp\left(\frac{-V^e}{\tilde{\theta}_A}\right)+(1-p(a))\exp\left(-\frac{v'}{\tilde{\theta}_A}\right)}{\tilde{\theta}_A\beta p'(a)}$$

$$\tilde{\theta}_A\beta p'(a)\exp\left(-\frac{v'}{\tilde{\theta}_A}\right)-(1-p(a))\exp\left(-\frac{v'}{\tilde{\theta}_A}\right) \le p(a)\exp\left(\frac{-V^e}{\tilde{\theta}_A}\right)+\tilde{\theta}_A\beta p'(a)\exp\left(\frac{-V^e}{\tilde{\theta}_A}\right)$$

$$\exp\left(-\frac{v'}{\tilde{\theta}_A}\right) \le \frac{[p(a)+\tilde{\theta}_A\beta p'(a)]}{[\tilde{\theta}_A\beta p'(a)-1+p(a)]}\exp\left(\frac{-V^e}{\tilde{\theta}_A}\right)$$

where the division is by a strictly positive term because $\tilde{\theta}_A\beta r > 1$ for all values of $\tilde{\theta}_A$ we consider. This gives:

$$v' \ge V^e - \tilde{\theta}_A \log\left(\frac{p(a)+\tilde{\theta}_A\beta p'(a)}{p(a)+\tilde{\theta}_A\beta p'(a)-1}\right)$$

with strict inequality for $a = 0$. In order to rule out zero search effort we require:

$$
\begin{aligned}
v' &\le V^e - \tilde{\theta}_A \log\left(\frac{p(0)+\tilde{\theta}_A\beta p'(0)}{p(0)+\tilde{\theta}_A\beta p'(0)-1}\right) \\
&= V^e - \tilde{\theta}_A \log\left(\frac{\tilde{\theta}_A\beta r}{\tilde{\theta}_A\beta r-1}\right) \equiv v_{\max}
\end{aligned}
$$

B Tables and Figures

.

θ_P	$+\infty$	$10,000$	$1,000$	100	10
R_i	0.23	0.24	0.30	0.47	0.51
R_f	0.05	0.05	0.04	0.01	0.00
A_i	222	222	223	225	226
A_f	331	334	354	394	417

Table 1: Only The principal is Robust.

.

θ_A	$+\infty$	$100,000$	$40,000$	$20,000$	$16,000$
R_i	0.23	0.19	0.13	0.04	0.00
R_f	0.05	0.07	0.08	0.06	0.04
A_i	222	226	233	251	389
A_f	331	313	280	212	176

Table 2: Only Agent is Robust

.

θ	$+\infty$	$100,000$	$40,000$	$20,000$	$16,000$
R_i	0.23	0.19	0.13	0.04	0.00
R_f	0.05	0.07	0.07	0.06	0.04
A_i	222	226	233	251	390
A_f	331	313	282	214	178

Table 3: Symmetric Robustness

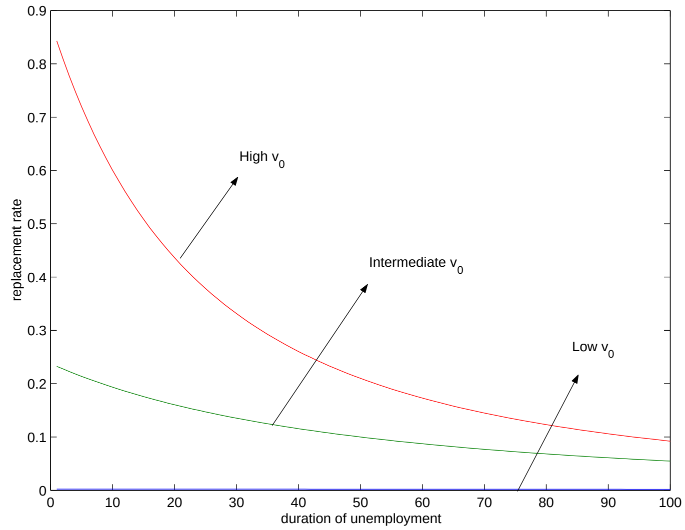

Figure 2: Replacement rate under optimal contract without robustness

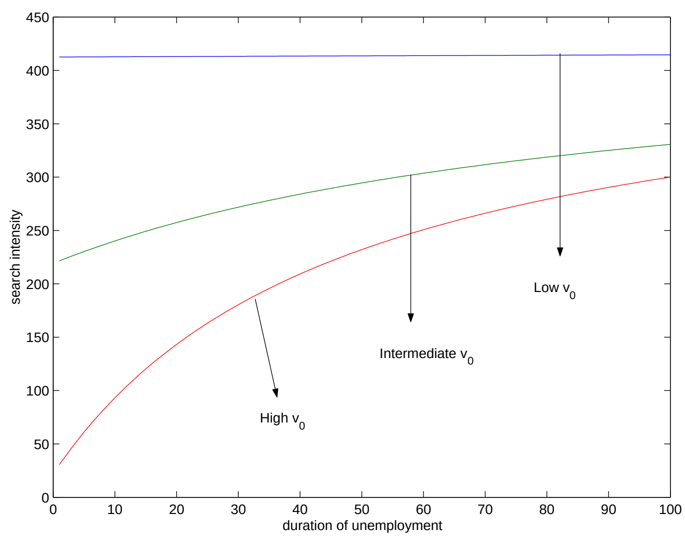

Figure 3: Search intensity under optimal contract without robustness

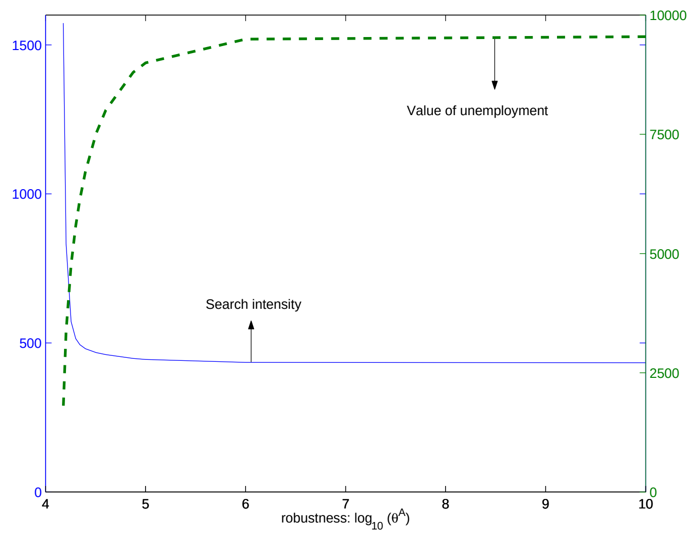

Figure 4: Optimal search intensity and value of unemployment for various levels of robustness

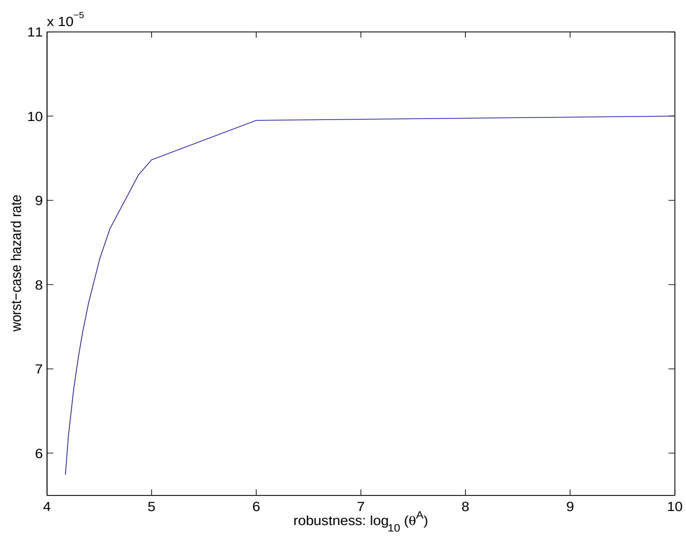

Figure 5: Worst case hazard rate of finding a job in autarky for various levels of robustness ($log_{10}(\theta_A)$).

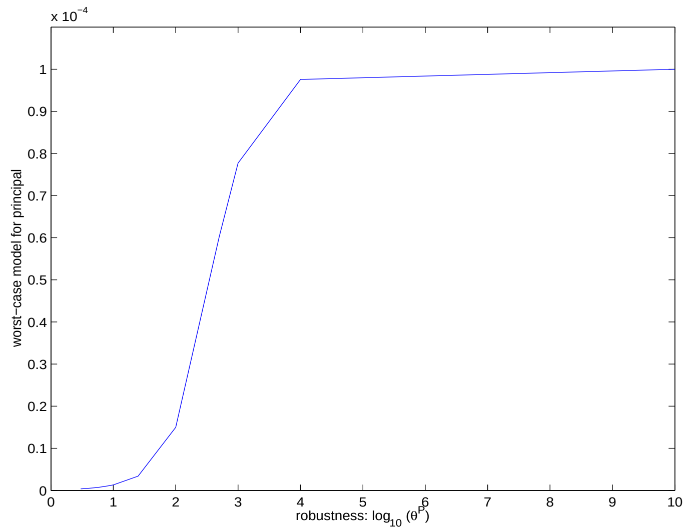

Figure 6: Model implied worst-case hazard rate for the principal

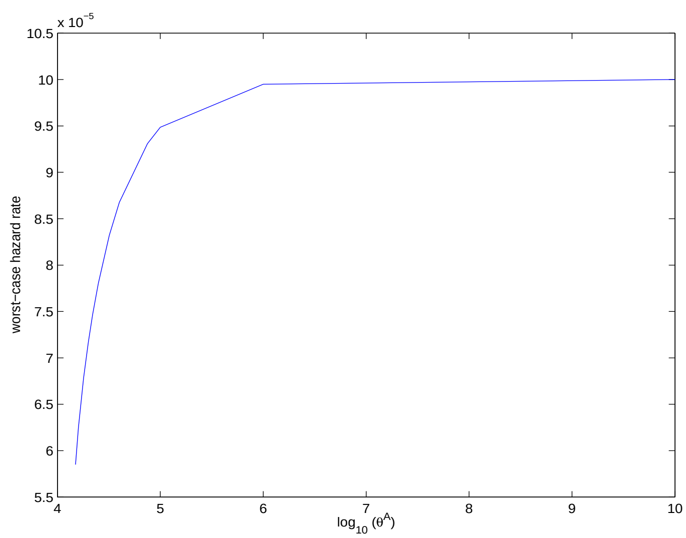

Figure 7: Model implied worst-case hazard rate for the agent

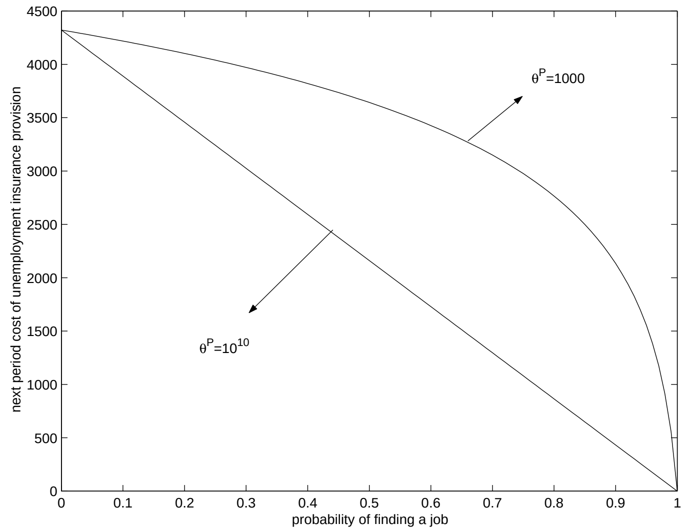

Figure 8: Next period's part of the principal's objective function as a function of the agent's probability of finding a job

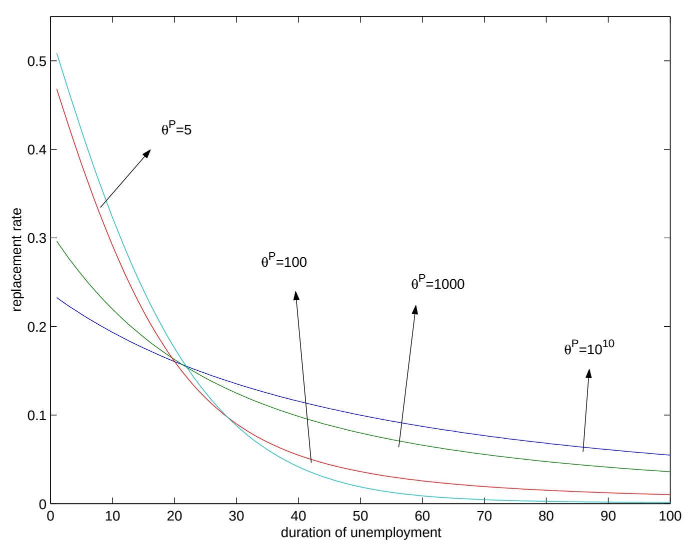

Figure 9: Optimal replacement rate in contract when only principal is robust

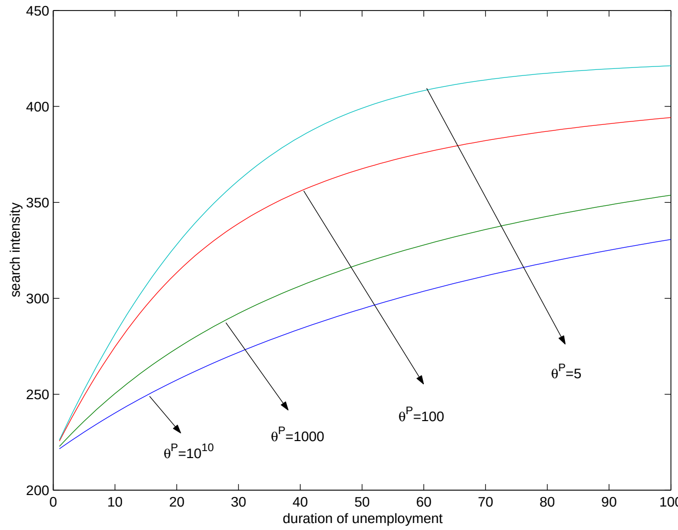

Figure 10: Optimal search intensity in contract when only principal is robust

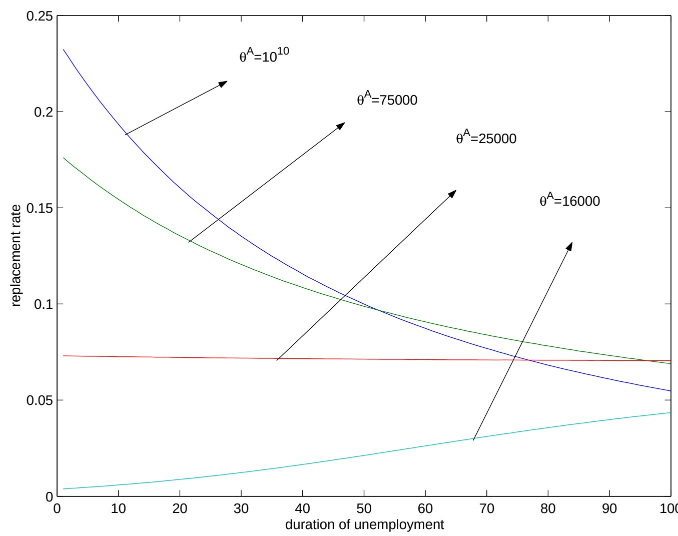

Figure 11: Optimal replacement rate in contract when only agent is robust

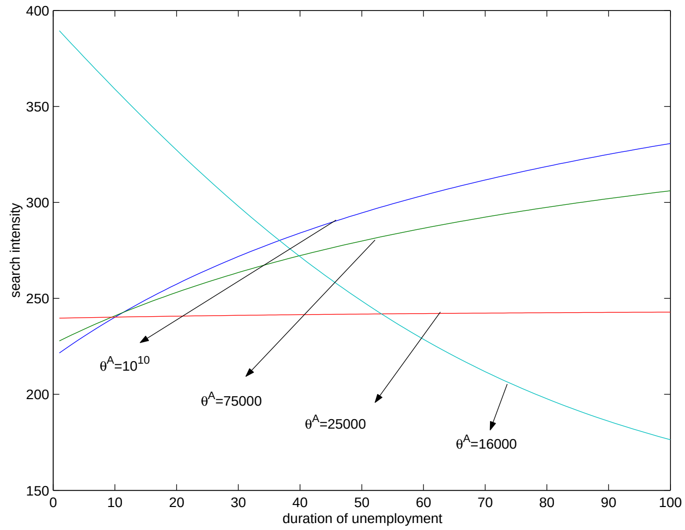

Figure 12: Optimal search intensity in contract when only agent is robust

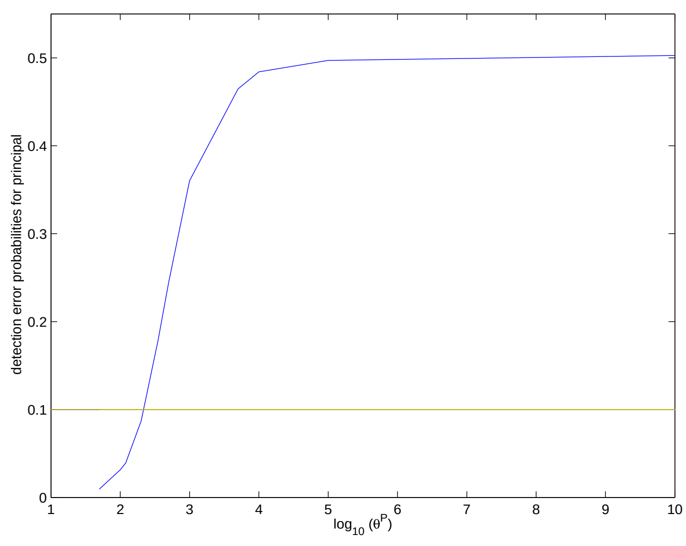

Figure 13: Detection error probabilities for the principal

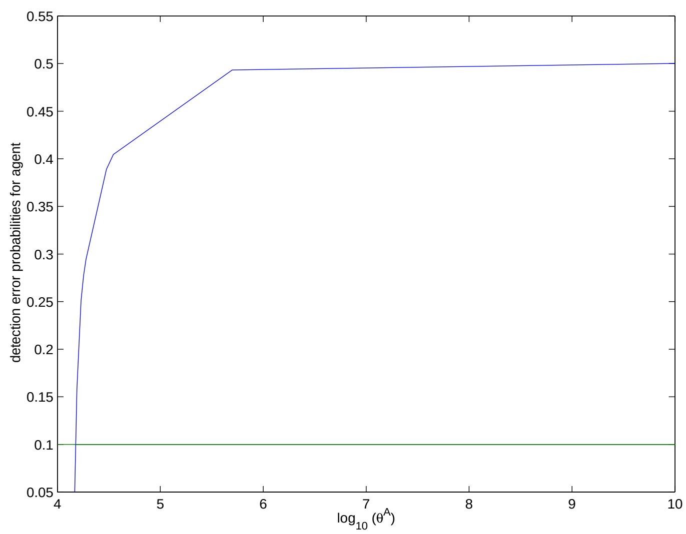

Figure 14: Detection error probabilities for the agent