

Intangible Risk?*

Lars Peter Hansen[†]

John C. Heaton[‡]

Nan Li[§]

December 9, 2003

*Conversations with Fernando Alvarez, Ravi Bansal, Susanto Basu, and Jim Heckman were valuable in completing this paper. Hansen and Heaton gratefully acknowledge support from the National Science Foundation and Li from the Olin Foundation.

[†]Department of Economics, University of Chicago and NBER

[‡]Graduate School of Business, University of Chicago and NBER

[§]Department of Economics, University of Chicago

1 Introduction

Accounting for the asset values by measured physical capital and other inputs arguably omits *intangible* sources of capital. This intangible or unmeasured component of the capital stock may result because some investments from accounting flow measures are not eventually embodied in the physical capital stock. Instead there may be scope for valuing ownership of a technology, for productivity enhancements induced by research and development, for firm specific human capital, or for organizational capital.

For an econometrician, intangible capital becomes a residual needed to account for values. In contrast to measurement error, omitted information or even model approximation error, this residual seems most fruitfully captured by an explicit economic model. It is conceived as an input into technology whose magnitude is not directly observed. Its importance is sometimes based on computing a residual contribution to production after all other measured inputs are accounted for. Alternatively it is inferred by comparing asset values from security market data to values of physical measures of firm or market capital. Asset market data is often an important ingredient in the measurement of intangible capital. Asset returns are used to convey information about the marginal product of capital and asset values are used to infer magnitude of intangible capital.

In the absence of uncertainty, appropriately constructed investment returns should be equated. With an omitted capital input, constructed investment returns across firms, sectors or enterprises will be heterogeneous because of mis-measurement. As argued by Telser (1988) and many others, differences in measured physical returns may be “explained by the omission of certain components of their ‘true’ capital.” McGrattan and Prescott (2000) and Atkeson and Kehoe (2002) are recent macroeconomic examples of this approach. Similarly, as emphasized by Hall (2001) and McGrattan and Prescott (2000), asset values should encode the values of both tangible and of intangible capital. Provided that physical capital stock can be measured there is scope for asset market data to be informative about the intangible component of the capital stock.

Following Hall (2001) we find it fruitful to consider the impact of risk in the measurement of intangible capital. Although not emphasized by Hall, there is well documented heterogeneity in the returns to equity of different types. In the presence of uncertainty, it is well known that use of a benchmark asset return must be accompanied by a risk adjustment. Historical averages of equity returns differ in systematic ways. Inferences about the intangible capital stock using security market returns necessarily must confront risk considerations or some competing interpretation for the heterogeneity in security market returns. Similarly, asset *values* reflect beliefs about the future prospects for firms, but the also reflect the riskiness of the implied cash flows.

In what follows we review the relevant investment theory (see section 2). In section 3 we review and reproduce some of the findings in the asset pricing literature by Fama and French (1992) on return heterogeneity. Risk premia can be characterized in terms of return risk or dividend or cash flow risk. We follow some recent literature in finance by exploring dividend risk. Since equity ownership of securities entitles an investor to future

claims to dividends in all subsequent time periods, quantifying dividend risk requires a time series process. We consider measurements of dividend risk using vector autoregressive (VAR) characterizations. Since asset valuation entails the study of a present-value relation, long run growth components of dividends can play an important role in determining asset values. In section 4 we reproduce the present-value approximation used in the asset pricing literature and use it to define a long-run measure of risk as a discounted impulse response. In sections 5, and 6 we use VAR methods to estimate the dividend-risk measures that have been advocated in the asset-pricing literature.

The literatures on intangible capital and asset return heterogeneity to date have been largely distinct. Our disparate discussion of these literatures will inherit some of this separation. In section 7 we conclude with some discussion of how to understand better lessons from asset pricing for the measurement of intangible capital.

2 Adjustment Cost Model

We begin with a discussion of adjustment costs and physical returns. Grunfeld (1960) shows how the market value of a firm is valuable in the explanation of corporate investment. Lucas and Prescott (1971) developed this point more fully by producing an equilibrium model of investment under uncertainty. Hayashi (1982) emphasized the simplicity that comes with assuming constant returns to scale. We exploit this simplicity in our development that follows.

Consider the following setup:

2.1 Production

Let n_t denote a variable input into production such as labor, and suppose there are two types of capital, namely $k_t = (k_t^m, k_t^u)$ where k_t^m is the measured capital and k_t^u is unmeasured or intangible capital stock. Firm production is given by

$$f(k_t, n_t, z_t)$$

where f displays constant returns to scale in the vector of capital stocks and the labor input n_t . The random variable z_t is a technology shock at date t .

Following the adjustment cost literature, there is a nonlinear evolution for how investment is converted into capital.

$$k_{t+1} = g(i_t, k_t, x_t) \tag{1}$$

where g is a two-dimensional function displaying constant returns to scale in investment and capital and x_t is a specific shock to the investment technology. We assume that there are two components of investment corresponding to the two types of capital. This technology may be separable in which case the first coordinate of g depends only on i_t^m and k_t^m while the second coordinate depends only on i_t^u and k_t^u .

Example 2.1. A typical example of the first equation in system (1) is:

$$k_{t+1}^m = (1 - \delta_m)k_t^m + i_t^m - g_m(i_t^m/k_t^m, x_t)k_t^m$$

where δ_m is the depreciation rate and g_m measures the investment lost in making new capital productive.

In the absence of adjustment costs, the function g is linear and separable.

Example 2.2. A common specification that abstracts from adjustment costs is:

$$g(k_t, i_t, x_t) = \begin{bmatrix} 1 - \delta_m & 0 \\ 0 & 1 - \delta_u \end{bmatrix} k_t + i_t$$

2.2 Firm Value

Each time period the firm purchases investment goods and produces. Let p_t denote the vector of investment good prices and w_t the wage rate. Output is the numeraire in each date. The date-zero firm value is:

$$E \left(\sum_{t=0}^{\infty} S_{t,0} [f(k_t, n_t, z_t) - p_t \cdot i_t - w_t n_t] | \mathcal{F}_0 \right) \quad (2)$$

The firm uses market determined stochastic discount factors to value cash flows. Thus $S_{t,0}$ discounts the date t cash flow back to date zero. This discount factor is stochastic and varies depending on the realized state of the world at date t . As a consequence $S_{t,0}$ not only discounts known cash flows but it adjusts for risk, see Harrison and Kreps (1979) and Hansen and Richard (1987).¹ The notation \mathcal{F}_0 denotes the information available to the firm at date zero.

Form the Lagrangian:

$$E \left(\sum_{t=0}^{\infty} S_{t,0} [f(k_t, n_t, z_t) - p_t \cdot i_t - w_t n_t - \lambda_t \cdot [k_{t+1} - g(i_t, k_t, x_t)]] | \mathcal{F}_0 \right) \quad (3)$$

where k_0 is a given initial condition for the capital stock. First-order conditions give rise to empirical relations and valuation relations that have been used previously.

Consider the first-order conditions for investment:

$$p_t = \frac{\partial g}{\partial i}(i_t, k_t, x_t)' \lambda_t \quad (4)$$

Special cases of this relation give rise to the so called q theory of investment. Consider for instance the separable specification in Example 2.1. Then

$$\frac{\partial g_m}{\partial i}(i_t^m/k_t^m, x_t) = 1 - \frac{p_t^m}{\lambda_t^m}. \quad (5)$$

¹This depiction of valuation can be thought of assigning state prices, but it also permits certain forms of market incompleteness.

This relates the investment capital ratio to what is called *Tobin's q* ($q_t = \frac{\lambda_t^m}{p_t^m}$). The Lagrange multiplier λ_t^m is the date t shadow value of the measured capital stock that is productive at date t . There is an extensive empirical literature that has used (5) to study the determinants of investment. As is well known, $\lambda_t^m = p_t^m$ and Tobin's q is equal to one in the absence of adjustment costs as in Example 2.2.

Consider next the first-order condition for capital at date $t + 1$:

$$\lambda_t = E \left(S_{t+1,t} \left[\frac{\partial f}{\partial k}(k_{t+1}, n_{t+1}, z_{t+1}) + \frac{\partial g}{\partial k}(i_{t+1}, k_{t+1}, x_{t+1})' \lambda_{t+1} \right] \middle| \mathcal{F}_t \right) \quad (6)$$

where $S_{t+1,t} \equiv S_{t+1,0}/S_{t,0}$ is the implied one-period stochastic discount factor between dates t and $t + 1$. This depiction of the first-order conditions is in the form of a one-period pricing relation. As a consequence, the implied returns to investments in the capital goods are:

$$\begin{aligned} r_{t+1}^m &\equiv \frac{\frac{\partial f}{\partial k^m}(k_{t+1}, n_{t+1}, z_{t+1}) + \frac{\partial g}{\partial k^m}(i_{t+1}, k_{t+1}, x_{t+1})' \lambda_{t+1}}{\lambda_t^m} \\ r_{t+1}^u &\equiv \frac{\frac{\partial f}{\partial k^u}(k_{t+1}, n_{t+1}, z_{t+1}) + \frac{\partial g}{\partial k^u}(i_{t+1}, k_{t+1}, x_{t+1})' \lambda_{t+1}}{\lambda_t^u}. \end{aligned}$$

The denominators of these shadow returns are the marginal costs to investing an additional unit capital at date t . The numerators are the corresponding marginal benefits reflected in the marginal product of capital and the marginal contribution to productive capital in future time periods. The shadow returns are model-based constructs and are not necessarily the same as the market returns to stock or bond holders.

In the separable case (Example 2.1), the return to the measurable component of capital is

$$r_{t+1}^m = \frac{\frac{\partial f}{\partial k^m}(k_{t+1}, n_{t+1}, z_{t+1})k_{t+1}^m + \lambda_{t+1}^m k_{t+2}^m - p_{t+1}^m i_{t+1}^m}{\lambda_t^m k_{t+1}^m}$$

An alternative depiction can be obtained by using the investment first-order conditions to substitute for λ_t^m and λ_{t+1}^m as in Cochrane (1991). In the absence of adjustment costs (Example 2.2), the return to tangible capital is:

$$r_{t+1}^m = \frac{\frac{\partial f}{\partial k^m}(k_{t+1}, n_{t+1}, z_{t+1}) + (1 - \delta_m)p_{t+1}^m}{p_t^m}. \quad (7)$$

The standard stochastic growth model is known to produce too little variability in physical returns relative to security market counterparts. In the one-sector version, the relative price p_t^m becomes unity. As can be seen in (7), the only source of variability is the marginal product of capital. Inducing variability in this term by through variability in the technology shock process z_{t+1} generates aggregate quantities such as output and consumption that are too variable.

The supply of capital is less elastic when adjustment costs exist, hence models with adjustment costs can deliver larger return variability than the standard stochastic growth

model. This motivated Cochrane (1991) and Jermann (1998) to include adjustment costs to physical capital in their attempts to generate interesting asset market implications in models of aggregate fluctuations. As an alternative, Boldrin, Christiano, and Fisher (2001) study a two sector model with limited mobility of capital across technologies. In our environment, limited mobility between physical and intangible capital could be an alternative source of aggregate return variability.

By the restricting the technology to be constant-returns-to-scale, the time zero firm value is:

$$f(k_0, n_0, z_0) - i_0 \cdot p_0 - w_0 n_0 + k_1 \cdot \lambda_0 = k_0 \cdot \left[\frac{\partial f}{\partial k}(k_0, n_0, z_0) + \frac{\partial g}{\partial k}(i_0, k_0, x_0)' \lambda_0 \right] \quad (8)$$

This relation is replicated over time. Thus the date t firm value is given by the cash flow (profit) plus the *ex-dividend* price of the firm. Equivalently it is the value of the date zero vector of capital stocks taking account of the marginal contribution of this capital to the production of output and to capital in subsequent time periods. Thus asset market values can be used to impute $k_{t+1} \cdot \lambda_t$ after adjusting for firm cash flow. When the firm has unmeasured *intangible* capital, this additional capital is reflected in the asset valuation of the firm.

The presence of intangible capital alters how we interpret Tobin's q . In effect there are now multiple components to the capital stock. Tobin's q is typically measured as a ratio of values and not as a simple ratio of prices. While the market value of a firm has both contributions, a replacement value constructed by multiplying the price of new investment goods by the measured capital stock will no longer be a simple price ratio. Instead we would construct:

$$\frac{\lambda_t \cdot k_{t+1}}{p_t^m k_{t+1}^m}.$$

Heterogeneity in q across firms or groups of firms reflects in part different amounts of intangible capital not simply a price signal to conveying the profitability of investment.

The dynamics of the ex-dividend price of the firm are given by:

$$\lambda_t \cdot k_{t+1} = E(s_{t+1,t} [f(k_{t+1}, n_{t+1}, z_{t+1}) - p_{t+1} \cdot i_{t+1} - w_{t+1} n_{t+1} + k_{t+2} \cdot \lambda_{t+1}] | \mathcal{F}_t).$$

The composite return to the firm is thus

$$\begin{aligned} r_{t+1}^c &= \frac{f(k_{t+1}, n_{t+1}, z_{t+1}) - p_{t+1} \cdot i_{t+1} - w_{t+1} n_{t+1} + k_{t+2} \cdot \lambda_{t+1}}{\lambda_t \cdot k_{t+1}} \\ &= \frac{\lambda_t^m k_{t+1}^m r_{t+1}^m + \lambda_t^u k_{t+1}^u r_{t+1}^u}{\lambda_t \cdot k_{t+1}} \end{aligned} \quad (9)$$

Recall that k_{t+1} is determined at date t (but not its productivity) under our timing convention. The composite return is a weighted average of the returns to the two types of capital with weights given by the relative values of the two capital stocks.

Firm ownership includes both bond and stock holders. The market counterpart to the composite return is a weighted average of the returns to the bond holders and equity holders with portfolio weights dictated by the amount of debt and equity of the firm.

2.3 Imputing the Intangible Capital Stock

These valuation formulas have been used by others to make inferences about the intangible capital stock. First we consider a return-based approach. We then consider a second approach based on asset values.

Following Atkeson and Kehoe (2002) and others, we exploit the homogeneity of the production function and Euler's Theorem to write:

$$y_{t+1} = \frac{\partial f}{\partial k^m}(k_{t+1}, n_{t+1}, z_{t+1})k_{t+1}^m + \frac{\partial f}{\partial n}(k_{t+1}, n_{t+1}, z_{t+1})n_{t+1} + \frac{\partial f}{\partial k^u}(k_{t+1}, n_{t+1}, z_{t+1})k_{t+1}^u.$$

where $y_{t+1} = f(k_{t+1}, n_{t+1}, z_{t+1})$ is output. Thus a measure of the contribution of intangible capital to output:

$$\frac{\frac{\partial f}{\partial k^u}(k_{t+1}, n_{t+1}, z_{t+1})k_{t+1}^u}{y_{t+1}} = 1 - \frac{\frac{\partial f}{\partial k^m}(k_{t+1}, n_{t+1}, z_{t+1})k_{t+1}^m}{y_{t+1}} - \frac{\frac{\partial f}{\partial n}(k_{t+1}, n_{t+1}, z_{t+1})n_{t+1}}{y_{t+1}}.$$

To make this operational we require a measure of the labor share of output given by compensation data and a measure of the share of output attributed to measured component of capital. Using formula (7) from Example 2.2 and knowledge of the return and the depreciation rate, we can construct

$$\frac{\frac{\partial f}{\partial k^m}(k_{t+1}, n_{t+1}, z_{t+1})}{p_t^m} = r_{t+1}^m - (1 - \delta_m) \frac{p_{t+1}^m}{p_t^m}.$$

Thus

$$\frac{\frac{\partial f}{\partial k^m}(k_{t+1}, n_{t+1}, z_{t+1})k_{t+1}^m}{y_{t+1}} = \left[r_{t+1}^m - (1 - \delta_m) \frac{p_{t+1}^m}{p_t^m} \right] \frac{p_t^m k_{t+1}^m}{y_{t+1}}. \quad (10)$$

This formula avoids the need to directly measure rental income to measured capital, but it instead requires measures of the physical return, physical depreciation scaled by value appreciation, and the relative value of tangible capital to income.

The physical return to measured capital is not directly observed. Even if we observed the firm's, (or industry's or aggregate) return from security markets, this would be the composite return (9) and would include the contribution to intangible capital. As a result, a time series of return data from security markets is not directly usable. Instead Atkeson and Kehoe (2002) take a steady state approximation implying that returns should be equated to measure the importance of intangible capital in manufacturing. Income shares and price appreciation are measured using time series averages. Given the observed heterogeneity in average returns, as elsewhere in empirical studies based on the deterministic growth model, there is considerable ambiguity as to which average return to use. To their credit, Atkeson and Kehoe (2002) document the sensitivity of their intangible capital measure to the assumed magnitude of the return.² We will have more to say about return heterogeneity subsequently.

²Atkeson and Kehoe (2002) are more ambitious than what we describe. They consider some tax implications and two forms of measured capital: equipment and structures. Primarily they develop and apply an interesting and tractable model of organizational capital.

To infer the value of the intangible capital relative to output using return data, we combine equation (10) with its counterpart for intangible capital to deduce that:

$$1 - \frac{\frac{\partial f}{\partial n}(k_{t+1}, n_{t+1}, z_{t+1})n_{t+1}}{y_{t+1}} = r_{t+1}^c \frac{p_t \cdot k_{t+1}}{y_{t+1}} - (1 - \delta_m) \left(\frac{p_{t+1}^m}{p_t^m} \right) \left(\frac{p_t^m k_{t+1}^m}{y_{t+1}} \right) - (1 - \delta_u) \left(\frac{p_{t+1}^u}{p_t^u} \right) \left(\frac{p_t^u k_{t+1}^u}{y_{t+1}} \right). \quad (11)$$

To use this relation we must not only use the return r_{t+1}^c but also the growth rate in the investment prices for the two forms of capital and the depreciation rates. From this we may produce a measure of $\frac{p_t^u k_{t+1}^u}{y_{t+1}}$ using (11). McGrattan and Prescott (2000) use a similar method along with steady state calculations and a model in which $p_t^u = p_t^m = 1$ to infer the intangible capital stock.³ Instead of using security market returns or historical averages of these returns, they construct physical returns presuming that the noncorporate sector does not use intangible capital in production.⁴ Rather than making this seemingly hard to defend restriction, the return r_{t+1}^c could be linked directly to asset returns as in Atkeson and Kehoe (2002). Although the practical question of which security market return to use would still be present.⁵

In contrast to Atkeson and Kehoe (2002), McGrattan and Prescott (2000) and McGrattan and Prescott (2003), uncertainty is central in the analysis of Hall (2001). For simplicity Hall considers the case in which there is effect a single capital stock and a single investment good, but only part of capital is measured. Equivalently, the capital stocks k_t^m and k_t^u are perfect substitutes. Thus the production function is given by:

$$y_t = f^a(k_t^a, n_t, z_t)$$

where $k_t^a = k_t^m + k_t^u$. Capital evolves according to:

$$k_{t+1}^a = g^a(k_t^a, i_t^a) \quad (12)$$

with x_t excluded. The first-order conditions for investment are now given by:

$$\frac{\partial g^a}{\partial i}(k_t^a, i_t^a) = \frac{p_t^a}{\lambda_t^a}, \quad (13)$$

and

$$v_t = \frac{\lambda_t^a k_{t+1}^a}{p_t^a} \quad (14)$$

³McGrattan and Prescott (2000) also introduce tax distortions and a noncorporate sector. They also consider uncertainty, but with little gain. They use a minor variant of the standard stochastic growth model, and that model is known to produce physical returns with little variability.

⁴McGrattan and Prescott (2003) use an *a priori* restriction on preferences instead of the explicit link to returns in the noncorporate sector, but this requires independent information on the preference parameters.

⁵The measurement problem is made simpler by the fact that it is the composite return that needs to be computed and not the individual return on measured capital. The implied one-period returns to equity and bond-holders can be combined as in Hall (2001), but computing the appropriate one-period returns for bond-holders can be problematic.

is measured from the security markets using the firm value relation (8) and taking investment to be numeraire. For a given k_t^a , relations (12), (13) and (14) are three equations in the three unknowns $\frac{\lambda_t^a}{p_t^a}$, k_{t+1}^a , i_t^a . In effect they provide a recursion that can be iterated over time with the input of firm market value v_t . Instead of returns, Hall (2001) uses asset values to deduce a time series for the aggregate capital stock and the corresponding shadow valuation of that stock.⁶

While Hall (2001) applies this method to estimate a time series of aggregate capital stocks, we will consider some evidence from empirical finance on return heterogeneity that indicates important differences between returns to the tangible and intangible components of the capital stocks. This suggests the consideration of models in which intangible capital differs from tangible capital in ways that might have important consequences for measurement. This includes models that outside the adjustment cost models described here.

3 Evidence for Return Heterogeneity

We now revisit and reconstruct results from the asset pricing literature. Since the work of Fama and French (1992) and others, average returns to portfolios formed on the basis of the ratio of book value to market value are constructed. While the book to market value is reminiscent of the q measure of the ratio of the market value of a firm *vis. a. vis.* the replacement cost of its capital, here the book to market value is computed using only the equity-holders stake in the firm. Capital held by bond holdings is omitted from the analysis.

Recall from section 2 that intangible capital is reflected in only the market measure of assets but is omitted from the book measure. We are identifying firms with high intangible capital based on high book equity-to-market equity (BE/ME). It is difficult to check this identification directly because the market value of debt at the firm level is not easily observed. As a check on our interpretation of the portfolios as reflecting different levels of intangible capital we examined whether our portfolio construction would be different if we included debt. We used the book value of debt as an approximation to the market value and considered rankings of firms based on book assets-to-market assets. This resulted in essentially the same rankings of firms. In fact the rank correlation between book assets-to-market assets and BE/ME averaged 0.97 over the 53 years of our sample. This gives us confidence in identifying the high BE/ME portfolio as containing firms with low levels of intangible and the low BE/ME portfolio as containing firms with high levels of intangibles.

Fama and French form portfolios based on the ratio of book equity-to-market equity (BE/ME), and estimate the mean return of these groups. They find that low BE/ME have low average returns. Fama and French (1992) view a low BE/ME as signaling sustained high earnings and/or low risk. While we follow Fama and French (1992) in constructing portfolios ranked by BE/ME ratios, we use a coarser sort than they do. We focus on five portfolios instead of ten, but this does not change the overall nature of their findings. Each year listed

⁶Hall (2001) establishes the stability of this mapping for some adjustment cost specifications, guaranteeing that impact initializing k_0^a of the recursion the recursion at some arbitrary k_0^a decays over time.

firms are ranked by their BE/ME using information from COMPUSTAT. Firms are then allocated into five portfolios and this allocation is held fixed over the following year. The weight placed on a firm in a portfolio is proportional to its market value each month.⁷

Firms may change groups over time and the value weights are adjusted accordingly. In effect the BE/ME categories are used to form five portfolio dividends, returns and values each time period. This grouping is of course different in nature than the grouping of firms by industry SEC codes, an approach commonly used in the Industrial Organization (IO) literature. For instance firms in the low BE/ME category may come from different industries and the composition may change through time. On the other hand this portfolio formation does successfully identify interesting payout heterogeneity at the firm level as we demonstrate below.

Figure 1 plots the market value relative to book value of 5 portfolios of US stocks over the period 1947 to 2001. Notice that there is substantial heterogeneity in the market value relative to book value of these portfolios. This potentially reflects substantial differences in intangible capital held by the firms that make up the portfolios. Further the value of market equity to book equity fluctuates dramatically over time.

These fluctuations can reflect changes in the relative composition of the capital stock between tangible and intangible capital. They may also reflect changes in the relative valuation of the two types of capital. Changes in valuation reflect changes in conjectured productivity of the different types of capital but may also reflect changes in how the riskiness is perceived and valued by investors.

Table 1 presents sample statistics for these portfolios of stocks. For comparison, the column labelled “Market” gives statistics for the CRSP value weighted portfolio. Consistent with Figure 1 there are substantial differences in the average value of BE/ME for these portfolios. Notice that the portfolios with lower BE/ME (high market value relative to book value of equity) are also the ones with the highest level of R&D relative to sales. This is consistent with the idea that large R&D expenditures will ultimately generate high cash flows in the future thus justifying high current market values. Also the high level of R&D by firms with high market valuation relative to book value may reflect substantial investment in intangibles.

While the five BE/ME portfolios are likely to have different compositions of capital, these portfolios also imply different risk-return tradeoffs. As in Fama and French (1992), the low BE/ME portfolios have lower mean returns but not substantially different volatility than high BE/ME portfolios. The mean returns differ and the means of implied excess returns scaled by volatility (Sharpe ratios) also differ. High BE/ME portfolios have higher Sharpe ratios. In particular, the highest BE/ME portfolio has a Sharpe ratio that is higher than that of the overall equity market. A portfolio with an even larger Sharpe ratio can be constructed by taking a long position in the high BE/ME portfolio and offsetting this with a short position in the low BE/ME portfolios. This occurs because there is substantial positive correlation across the portfolios. The spectacular Sharpe ratios that are possible have been noted by many authors. See MacKinlay (1995), for example.

⁷See Fama and French (1992) for a more complete description of portfolio construction.

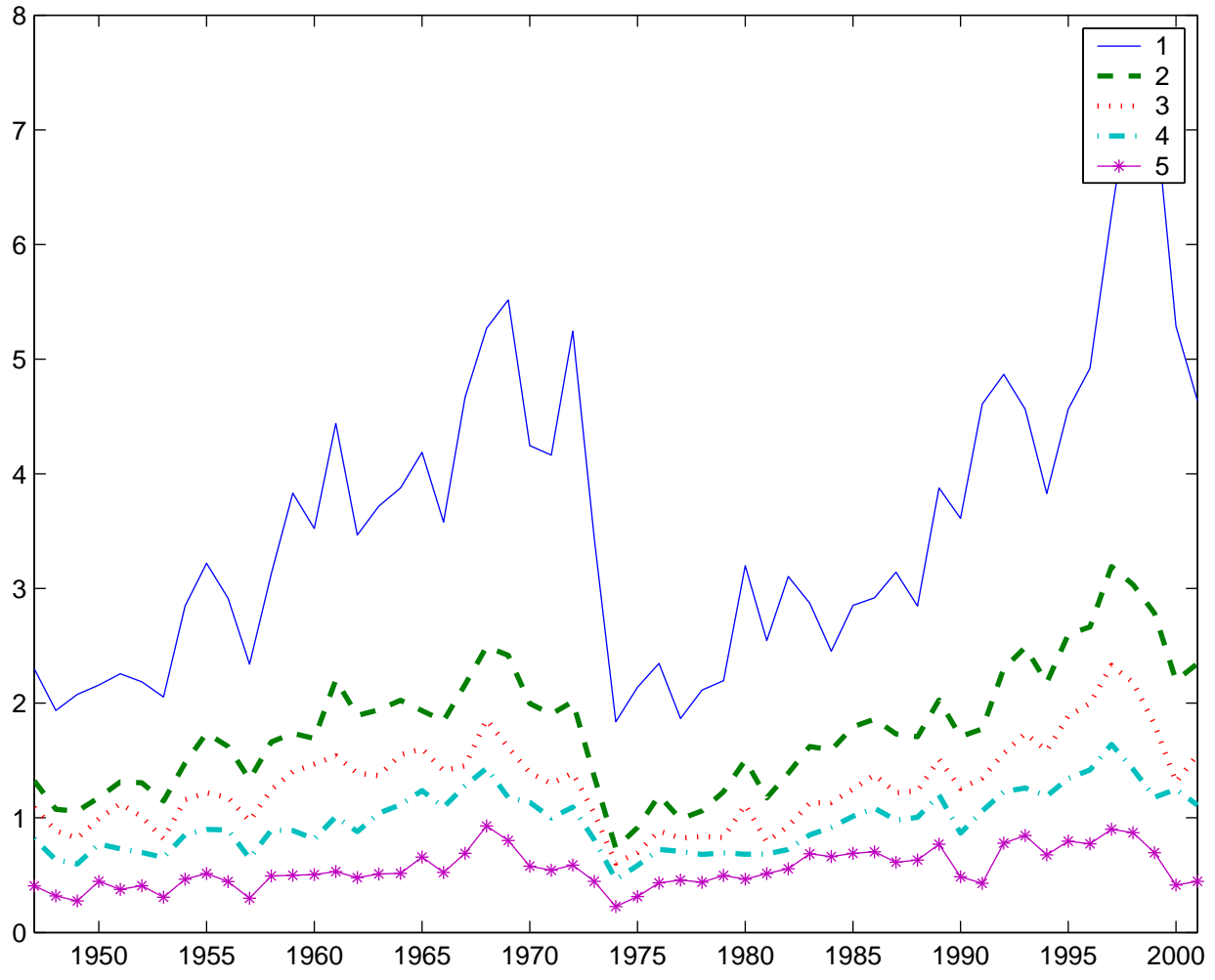


Figure 1: Market-to-Book Value of Equity for Five Portfolios of Stocks

Table 1: Properties of Portfolios Sorted by Book-to-Market

	Portfolio					Market
	1	2	3	4	5	
Avg. Return (%)	6.48	6.88	8.90	9.32	11.02	7.23
Std. Return %	37.60	32.76	29.64	31.66	35.50	32.94
Avg. B/M	0.32	0.62	0.84	1.12	2.00	0.79
Avg. R&D/Sales	0.12	0.06	0.04	0.03	0.03	0.08
Sharpe Ratio	0.18	0.20	0.28	0.28	0.30	0.21
Correlation with Consumption	0.20	0.18	0.20	0.20	0.21	0.20

Portfolios formed by sorting portfolios into 5 portfolios using NYSE breakpoints from Fama and French (1993). Portfolios are ordered from lowest to highest average book-to-market value. Data from 1947 Q1 to 2001 Q4 for returns and B/M ratios. R&D/Sales ratio is from 1950 to 2001. Returns are converted to real units using the implicit price deflator for non-durable and services consumption. Average returns and standard deviations are calculated using the natural logarithm of quarterly gross returns multiplied by 4 to put the results in annual units. Average book-to-market are averaged portfolio book-to-market or the period computed from COMPUSTAT. Average R&D/Sales also computed from COMPUSTAT. The Sharpe Ratio is based on quarterly observations. Correlation with consumption is measured as the contemporaneous correlation between log returns and log consumption growth.

The consumption-based capital asset pricing model predicts that differences in average returns across the five portfolio are due to differences in the covariances between returns and consumption. That is, portfolios may have low returns because they offer some form of consumption insurance. The last row of table 1 displays the correlation between each quarterly portfolio return and the quarterly growth rate of aggregate real expenditures on nondurables and services. Because there is little difference in the volatility across portfolios, there is little difference in the implied covariance between returns and consumption growth. This measure of risk therefore implies little differences in required returns across the portfolios. The high Sharpe ratios and small covariances with consumption is known to make the consumption insurance explanation problematic. See Hansen and Jagannathan (1991) and Cochrane and Hansen (1992) for example. We will revisit this explanation, but in the context of dividend risk instead of return risk.

Differences in BE/ME are partially reflected in differences in future cash flows. Table 2 presents some basic properties of the dividend cash flows from the portfolios. These dividends are imputed from the Center for Research in Securities Prices (CRSP) return files. Each month and for each stock, CRSP reports a return without dividends, denoted $R_{t+1}^{wo} \equiv P_{t+1}/P_t$

Table 2: Cash Flow Properties of Portfolios Sorted by Book-to-Market Value

	Portfolio					Market
	1	2	3	4	5	
Avg. (log) Div. Growth %	1.78	1.68	3.13	3.54	4.48	3.09
Std. (log) Div. Growth %	13.50	17.09	11.71	12.05	17.76	23.99
Avg. log(D/P)	-3.78	-3.41	-3.23	-3.11	-3.15	n/a
Avg. P/D	49.12	33.01	27.00	23.96	24.82	n/a

and a total return that includes dividends, denoted $R_{t+1}^w \equiv (P_{t+1} + D_{t+1})/P_t$. The dividend yield D_{t+1}/P_t is then imputed as:

$$D_{t+1}/P_t = R_{t+1}^w - R_{t+1}^{wo} .$$

Changes in this yield along with the capital gain in the portfolio are used to impute the growth in portfolio dividends. This construction has the interpretation of following an initial investment of \$1 in the portfolio and extracting the dividends while reinvesting the capital gains. From the monthly dividend series we compute quarterly averages. Real dividends are constructed by normalizing nominal dividends on a quarterly basis by the implicit price deflator for nondurable and service consumption taken from the National Income and Product accounts. Finally some adjustment must be done to quarterly dividends because of the pronounced seasonal patterns in corporate dividend payout. Our measure of quarterly dividends is constructed by taking an average of the logarithm of dividends in a particular quarter and over the previous three quarters. We average the logarithm of dividends because our empirical modelling will be linear in logs. Table 2 reports statistics for this constructed proxy of log dividends.

Notice from Table 2 that the low BE/ME portfolios also have low dividend growth. Just as there is considerable heterogeneity in the measures of average returns, there is also considerably heterogeneity in growth rates. An important measurement question that we will explore is whether these *ex post* sample differences in dividend growth is something that is fully perceived *ex ante*, or whether some of this heterogeneity is the outcome of dividend processes with low frequency components. We suspect that much of the observed heterogeneity in dividend growth was known *a priori* by investors and hence this heterogeneity will reflect potential differences in risk. Some of our calculations that follow will treat this heterogeneity as reflecting in part differences in long-run risk. In the section 4, we turn to a discussion of risk measurement for these cash flows.

A potential concern in evaluating dividends at the portfolio level is that portfolio formation could lead to artificial differences in the long-run risk properties of portfolio cash

flows that are not easily interpretable. For example it may appear that portfolios biased towards investing in stocks with low dividend growth will necessarily have low growth rates in cash flows and therefore little long run exposure to economic growth. Notice, however, that the implied dividend growth rates in the constructed portfolios depend in part on the relative prices of stocks bought and sold as the composition of the portfolios change over time. Stocks with temporarily low dividend growth rates will have relatively high price appreciation, which can offset the low growth rates. Thus the portfolio formation might actually result in a more stable dividend or cash flow.

4 Dividend Risk

In asset pricing it is common to explore risk premia by characterizing how returns co-vary with a benchmark return as in the CAPM or more generally how returns co-vary with a candidate stochastic discount factor. The focus of the resulting empirical investigations are on return risk, in contrast to dividend or cash-flow risk.

Recently there has been an interest in understanding cash-flow risk using linear time series methods. Examples include the work of Bansal, Dittmar, and Lundblad (2002a), Bansal, Dittmar, and Lundblad (2002b), and Cohen, Polk, and Vuoteenaho (2002). We follow this literature by using linear time series methods to motivate and construct a measure of dividend risk.

To use linear time series methods requires a log-approximation for present-discounted value formulas as developed by Campbell and Shiller (1988a, 1988b).⁸ Write the one-period return in an equity as:

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{(1 + P_{t+1}/D_{t+1})D_{t+1}/D_t}{P_t/D_t}$$

where P_t is the price and D_t is the dividend. Take logarithms and write

$$r_{t+1} = \log(1 + P_{t+1}/D_{t+1}) + (d_{t+1} - d_t) - (p_t - d_t)$$

where lower case letters denote the corresponding logarithms. Next approximate:

$$\log(1 + P_{t+1}/D_{t+1}) \approx \log[1 + \exp(\overline{p - d})] + \frac{1}{1 + \exp(\overline{d - p})}(p_{t+1} - d_{t+1} - \overline{p - d})$$

where $\overline{p - d}$ is the average logarithm of the price dividend ratio. Use this approximation to write:

$$r_{t+1} - (d_{t+1} - d_t) = \chi + \rho(p_{t+1} - d_{t+1}) - (p_t - d_t) \quad (15)$$

where

$$\rho \equiv \frac{1}{1 + \exp(\overline{d - p})}.$$

⁸Santos and Veronesi (2001) suggest a models for studying cash flow risk that avoids linear approximation by instead adopting a nonlinear model of income shares.

As shown by Campbell and Shiller (1988a), this approximation is reasonably accurate in practice.

Treat (15) as a difference equation in the log price dividend ratio and solve this equation forward:

$$p_t - d_t = \sum_{j=0}^{\infty} \rho^j (r_{t+1+j} - d_{t+1+j} + d_{t+j}) - \frac{\chi}{1 - \rho}.$$

This relation says that a time $t + 1$ shock to current and future dividends must be offset by the same shock to returns in the sense of a present-discounted value. The discount factor ρ will differ depending on the average logarithm of the dividend/price ratio for the security or portfolio. The present discounted value restriction is mathematically the same as that developed by Hansen, Roberds, and Sargent (1991) in their examination of the implications of present-value budget balance.

To understand this restriction, posit a moving-average representation for the dividend growth process and the return process:

$$\begin{aligned} d_t - d_{t-1} &= \eta(L)w_t + \mu_d \\ r_t &= \kappa(L)w_t + \mu_r. \end{aligned}$$

Here $\{w_t\}$ is a vector, *iid* standard normal process and

$$\eta(z) = \sum_{j=0}^{\infty} \eta_j z^j, \quad \kappa(z) = \sum_{j=0}^{\infty} \kappa_j z^j$$

where η_j and κ_j are row vectors.

Since $p_t - d_t$ depends only on date t information, future shocks must be present-value neutral:

$$\kappa(\rho) - \eta(\rho) = 0. \tag{16}$$

For instance if returns are close to being iid, but not dividends then

$$\kappa(0) \approx \eta(\rho)$$

The discounted dividend response should equal the return response to a shock. Whereas Hansen and Singleton (1983) looked at return risk, Bansal, Dittmar, and Lundblad (2002a) and Bansal, Dittmar, and Lundblad (2002b) look at the discounted dividend risk. We follow Bansal, Dittmar, and Lundblad (2002a) and treat $\eta(\rho)$ as a measure of risk in dividend growth. We refer to this measure as discounted dividend risk. As ρ tends to 1, we refer to the limit $\eta(1)$ as long run risk.

To evaluate the riskiness of each portfolio's exposure to the shocks w_t , we also measure the impact of the shocks on consumption growth:

$$c_t - c_{t-1} = \gamma(L)w_t + \mu_c \tag{17}$$

where c_t is the logarithm of aggregate consumption. The economic magnitude of the discounted dividend responses can then be assessed based on the Euler equation from the familiar representative agent model with CRRA utility:

$$E \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\theta} R_{t+1}^j | \mathcal{F}_t \right] = 1, \quad (18)$$

where θ is the coefficient of relative risk aversion.

Under the (counterfactual) approximation that the return to each portfolio is iid, (16) implies that the return to portfolio j satisfies:

$$r_{t+1}^j = \eta(\rho^j)w_{t+1} + \mu_r^j.$$

Euler equation (18) then implies that μ_r^j satisfies:

$$E[r_{t+1}^j | \mathcal{F}_t] - r_{t+1}^f = \mu_r^j - r_{t+1}^f = -\frac{\sigma_{t,j}^2}{2} + \theta\gamma(0) \cdot \eta^j(\rho^j) \quad (19)$$

where $\sigma_{t,j}^2$ is the variance of return of portfolio j .

We will not include returns in our vector autoregressions for the reasons explained by Hansen, Roberds, and Sargent (1991).⁹ We will sometimes include dividend/price ratios in the vector autoregressive systems. These ratios are known to be informative about future dividends. Write implied moving-average representation as:

$$p_t - d_t = \xi(L)w_t + \mu_p.$$

We may then back out a return process (approximately) as:

$$r_t = \kappa(L)w_t + \mu_r$$

where

$$\kappa(z) = (\rho - z)\xi(z) + \eta(z).$$

It follows from this formula for κ that the present-value-budget balance restriction (16) is satisfied by construction and is not testable.

To summarize, we use $\eta(\rho)$ as our measure of discounted dividend risk. When dividend-prices ratios are also included in the VAR system, the present-value-budget-balance restriction (16) is automatically satisfied. By construction, discounted return risk and discounted dividend risk coincide.

⁹Hansen, Roberds, and Sargent (1991) show that when returns are included in a VAR, restrictions (16) cannot be satisfied for the shocks identified by the VAR unless the VAR system is stochastically singular.

5 Measuring Dividend Risk Empirically

In this section we evaluate the riskiness of the five BE/ME portfolios using the framework of section 4. Riskiness is measured by the sensitivity of portfolio cash flows and prices to different assumptions made to identify aggregate shocks. Since we are interested in the long-run impact of aggregate shocks we consider several VAR specifications that make different assumptions above the long-run relationships between consumption, portfolio cash flows and prices. In particular we examine the effects of moving from the assumption of little long-run relationship between aggregates and portfolio cash flows to the assumption that there is a cointegrated relationship between aggregate consumption and cash flows.

5.1 Empirical Model of Consumption and Dividends

To measure dividend risk we require estimates of γ and η . We describe how to obtain these using vector autoregressive (VAR) methods for consumption and dividends. The least restrictive specification we consider is:

$$A_0x_t + A_1x_{t-1} + A_2x_{t-2} + \dots + A_\ell x_{t-\ell} + B_0 = w_t$$

where consumption is the first entry of x_t and the dividend level is the second entry. The vectors B_0 and B_1 are two dimensional, and similarly the square matrices $A_j, j = 1, 2, \dots, \ell$ are two by two. The shock vector w_t has mean zero and covariance matrix I . We normalize A_0 to be lower triangular with positive entries on the diagonals. Form:

$$A(z) \doteq A_0 + A_1z + A_2z^2 + \dots + A_\ell z^\ell.$$

We are interested in specification in which $A(z)$ is nonsingular for $|z| < 1$.

We identify the first shock as the consumption innovation, and our aim is to measure the discounted average responses:

$$\begin{aligned} \gamma(\beta) &= (1 - \beta) \begin{bmatrix} 1 & 0 \end{bmatrix} A(\beta)^{-1} \\ \eta(\rho) &= (1 - \rho) \begin{bmatrix} 0 & 1 \end{bmatrix} A(\rho)^{-1}. \end{aligned}$$

We use these formulas for to produce long-run risk measures for each B/M portfolio.

We also compute the limiting responses as ρ tends to unity. While we want to allow for $A(z)$ to be singular at unity, we presume that $(1 - z)A(z)^{-1}$ has a convergent power series for a region containing $|z| \leq 1$. This is equivalent to assuming that both consumption and dividends are (at least asymptotically) stationary in differences. The limiting responses are thus contained in the matrix

$$(1 - z)A(z)^{-1}|_{z=1}.$$

When $A(1)$ is nonsingular, the limiting response matrix is identically zero, but it will be nonzero when $A(1)$ is singular. The matrix $A(1)$ is nonsingular when the VAR does not have stochastic growth components. When it is singular, the vector time series will be *cointegrated* in the sense of Engle and Granger (1987). We will explore specifications singular

specifications of $A(1)$ in which difference between log consumption and log dividends is presumed to be stationary.

5.2 Data Construction

For our measure of aggregate consumption we use aggregate consumption of nondurables and services taken from the National Income and Product Accounts. This measure is quarterly from 1947 Q1 to 2002 Q4, is in real terms and is seasonally adjusted. Portfolio dividends were constructed as discussed in section 3. For portfolio prices in each quarter we use end of quarter prices.

Motivated by the work of Lettau and Ludvigson (2001) and Santos and Veronesi (2001), in several of our specifications we allow for a second source of aggregate risk that captures aggregate exposure to stock market cash flows. This is measured as the share of corporate cash flows in aggregate consumption and is measured as the ratio of corporate earnings to aggregate consumption. Corporate earnings are taken from NIPA.

In all of the specifications reported below the VAR models were fit using five quarters of lags.¹⁰ See appendix A for more details of the data construction.

5.3 Bivariate Model of Consumption and Dividends

First we follow Bansal, Dittmar, and Lundblad (2002b) and consider bivariate regressions that include aggregate consumption and the dividends for each portfolio separately. Table 3 reports for the case where the state variable x_t is given by:¹¹

$$x_t = \begin{bmatrix} c_t \\ d_t^j \end{bmatrix}. \quad (20)$$

For notational convenience we do not display the dependence of x_t on j .

To simplify the interpretation of the shock vector w_t , we initially restrict the matrix $A(z)$ to be lower triangular. Under this restriction, consumption depends only on the first shock while dividends depend on both shocks. This recursive structure presumes that consumption is not “caused” by dividends in the sense of Granger (1969) and Sims (1972).¹²

The first row of panel A shows that according to the discounted measure of dividend risk, the high book-to-market returns have a larger measure of dividend *vis-a-vis* the low book-to-market returns. The differences are quite striking in that the response to a consumption shock increases almost ten times in comparing portfolio 1 and portfolio 5. This ordering was noted by Bansal, Dittmar, and Lundblad (2002a), using a different set of restrictions on

¹⁰We also conducted some runs with eight lags. With the exception of the results for portfolio 1 when using aggregate earnings, the results were not greatly effected.

¹¹Notice that we consider separate specifications of the state variable for each portfolio. Ideally estimation with all of the portfolio cash flows would be interesting but because of data limitations this is not possible.

¹²When this restriction on $A(z)$ is relaxed the measured discounted responses that we report below to a consumption shock are essentially the same.

Table 3: Discounted Responses of Portfolio Dividends in a Log-Level VAR

	Portfolio				
	1	2	3	4	5
Discount Factor	0.9943	0.9918	0.9902	0.9889	0.9894
<i>Panel A: Consumption Shock</i>					
OLS Estimator	0.14	0.34	0.22	0.53	1.32
10 percentile	-0.05	0.06	0.01	0.38	0.98
30 percentile	0.07	0.20	0.15	0.47	1.16
median	0.14	0.30	0.22	0.53	1.33
70 percentile	0.21	0.43	0.30	0.61	1.53
90 percentile	0.34	0.72	0.41	0.75	1.92
<i>Panel B: Discounted Responses, Dividend Shock</i>					
OLS Estimator	0.68	1.23	0.75	0.58	1.19
10 percentile	0.45	0.66	0.56	0.42	0.78
30 percentile	0.56	0.87	0.65	0.50	0.98
median	0.67	1.09	0.74	0.58	1.18
70 percentile	0.85	1.45	0.86	0.69	1.48
90 percentile	1.32	2.23	1.12	0.96	2.29

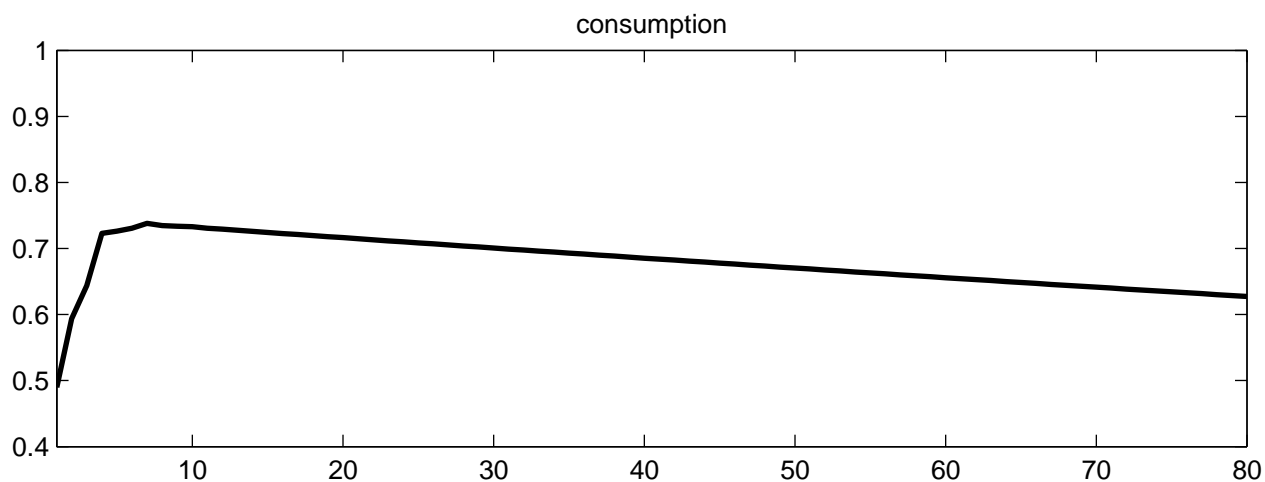
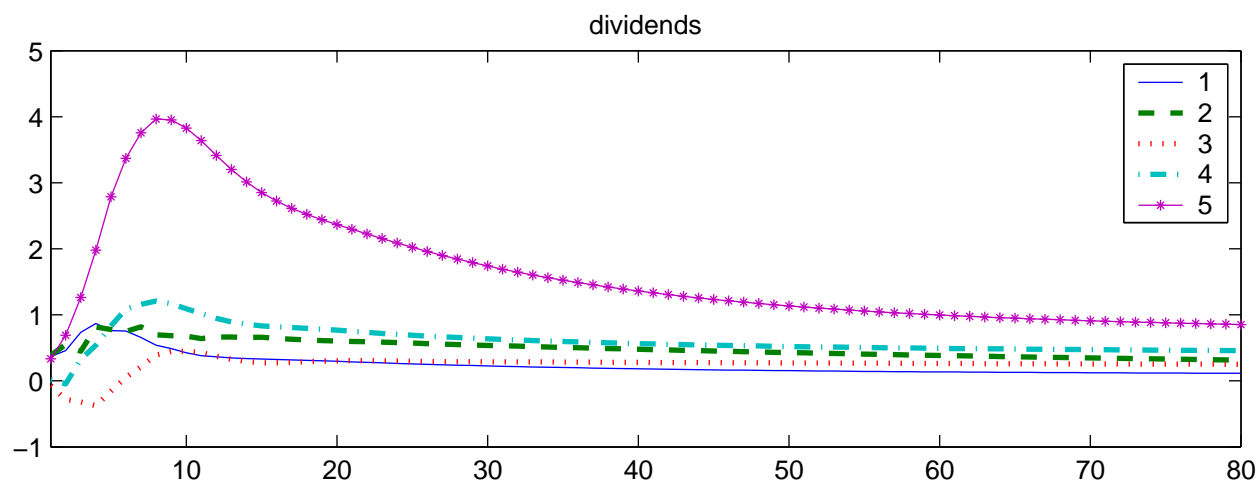


Figure 2: Impulse Response to a Consumption Shock. This figure reports the impulse response functions for each of the portfolios and for consumption obtained by estimating the log-level version of the VAR in which x_t has entries c_t and d_t . The matrix $A(z)$ is restricted to lower triangular.

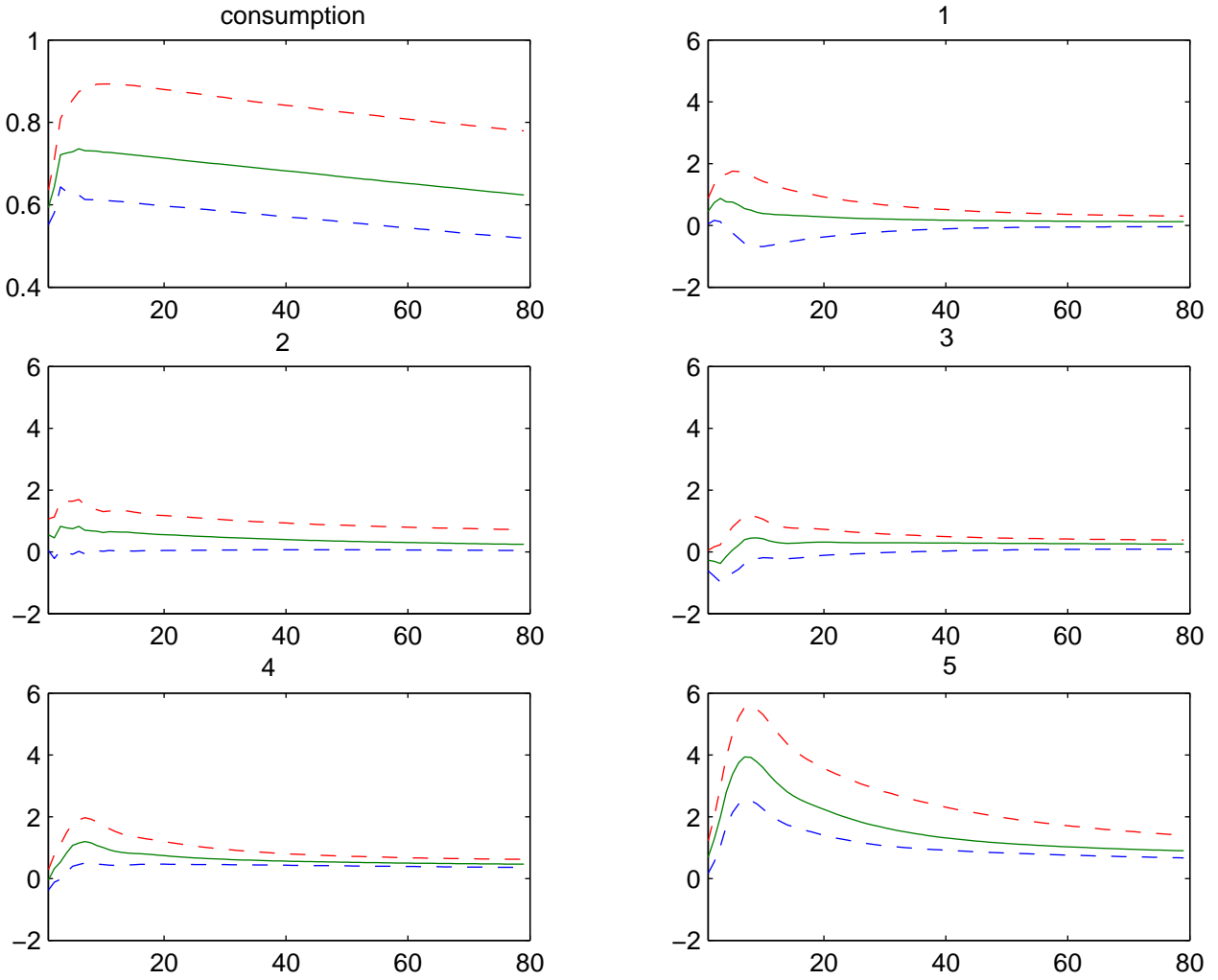


Figure 3: Bayesian Percentile for Impulse Responses to a Shock to Consumption. This figure gives the 10 percent, 50 percent and 90 percent percentile for the impulse response function depicted in Figure 2. The upper left panel depicts the consumption response and the other five panels depict the responses for each of the five portfolio cash flows.

the VAR.¹³ To illustrate the portfolio differences more fully consider Figure 2. This figure displays the implied responses of log dividends to a consumption shock. The discounted measure of risk reported in Table 3 is a weighted average of the responses depicted in Figure 2. Notice in particular that portfolio 5 has a substantially different response to a consumption shock with a pronounced peak response at about the ten quarter horizon. The half-lives of the discount factors range between 15.6 years for portfolio 4 to 30.4 years for portfolio 1. As result, the discounted average responses weight heavily tail responses.

Table 3 also reports Bayesian posterior percentile for the discounted consumption risk computed using the method described in B. These percentile provide a measure of accuracy. Figure 3 gives plots the 10%, 50% and 90% percentile for the individual impulse responses. Notice that these measures of accuracy imply substantial sampling error in the estimated discounted responses. For example consider the results for portfolios 1 and 5 as displayed in figure 3. Although the estimated short-run response to a consumption shock is quite different across these two portfolios, the confidence intervals narrow this difference substantially.

Next we explore specifications singular specifications of $A(1)$ in which difference between log consumption and log dividends is presumed to be stationary. Again we use VAR methods but now the first variable is the first-difference in logarithms of consumption and the second is difference between log consumption and log dividends. This specification is in effect a restriction on $A(z)$. We continue to assume that $A(z)$ is lower triangular. Thus the long-run response of dividends to a consumption shock is the same for all portfolios by construction. This discounted response can still differ, however. It is only when ρ is one that the response heterogeneity vanishes.

As Table 4 demonstrates, when dividends and consumption are restricted to respond the same way to permanent shocks, the discounted risk measures increase relative to those computed without restricting the rank of $A(z)$. The limiting response is about .82 for all portfolios. The discounted responses of portfolios 1, 2 and 3 to a consumption shock are all pulled towards this value. The discounted risk measures for portfolios 4 and 5 are also increased by imposing this limiting value on the impulse response. In Figure 4 we depict the impulse responses when cointegration is imposed and consumption is restricted. Comparing the impulse responses to a consumption shock in this figure to those in Figure 2, we see that while tail properties of the impulse responses have been altered, portfolio 5 continues to have a peak response at about ten quarters.¹⁴

For the cointegrated systems, we consider an alternative identification scheme. We do not restrict $A(z)$ to be lower triangular, but we instead identify a permanent and transitory shock following an approach suggested by Blanchard and Quah (1989). The long-run response to consumption is given by the first row of $A(1)^{-1}$. We now transform the shocks so that $A(1)^{-1}$

¹³Bansal, Dittmar, and Lundblad (2002a) consider two types of regressions. In the first, dividend growth is regressed on an eight quarter moving average of past consumption growth. In the second, detrended dividends are regressed on contemporaneous detrended consumption and four leads and lags of consumption growth.

¹⁴Given the data transformation, the Bayesian posterior percentile for the VAR are based on a different specification of the prior coefficient distribution over comparable coefficients.

Table 4: Discounted Responses of Portfolio Dividends in a Cointegrated VAR

	Portfolio				
	1	2	3	4	5
Discount Factor	0.9943	0.9918	0.9902	0.9889	0.9894
<i>Panel A: Discounted Responses, Consumption and Permanent Shock</i>					
OLS Estimator	0.75	0.89	0.75	1.11	2.03
10 percentile	0.13	0.40	0.27	0.80	1.24
30 percentile	0.59	0.71	0.60	0.97	1.62
median	0.82	0.91	0.80	1.12	1.97
70 percentile	1.04	1.11	0.99	1.28	2.43
90 percentile	1.39	1.44	1.31	1.61	3.41
<i>Panel B: Discounted Responses, Dividend and Transitory Shock</i>					
OLS Estimator	2.60	2.06	2.18	1.14	2.43
10 percentile	1.31	1.40	1.24	0.72	1.49
30 percentile	1.72	1.68	1.59	0.91	1.90
median	2.19	1.96	1.92	1.10	2.30
70 percentile	2.86	2.30	2.36	1.40	2.89
90 percentile	4.15	2.91	3.13	2.09	4.21

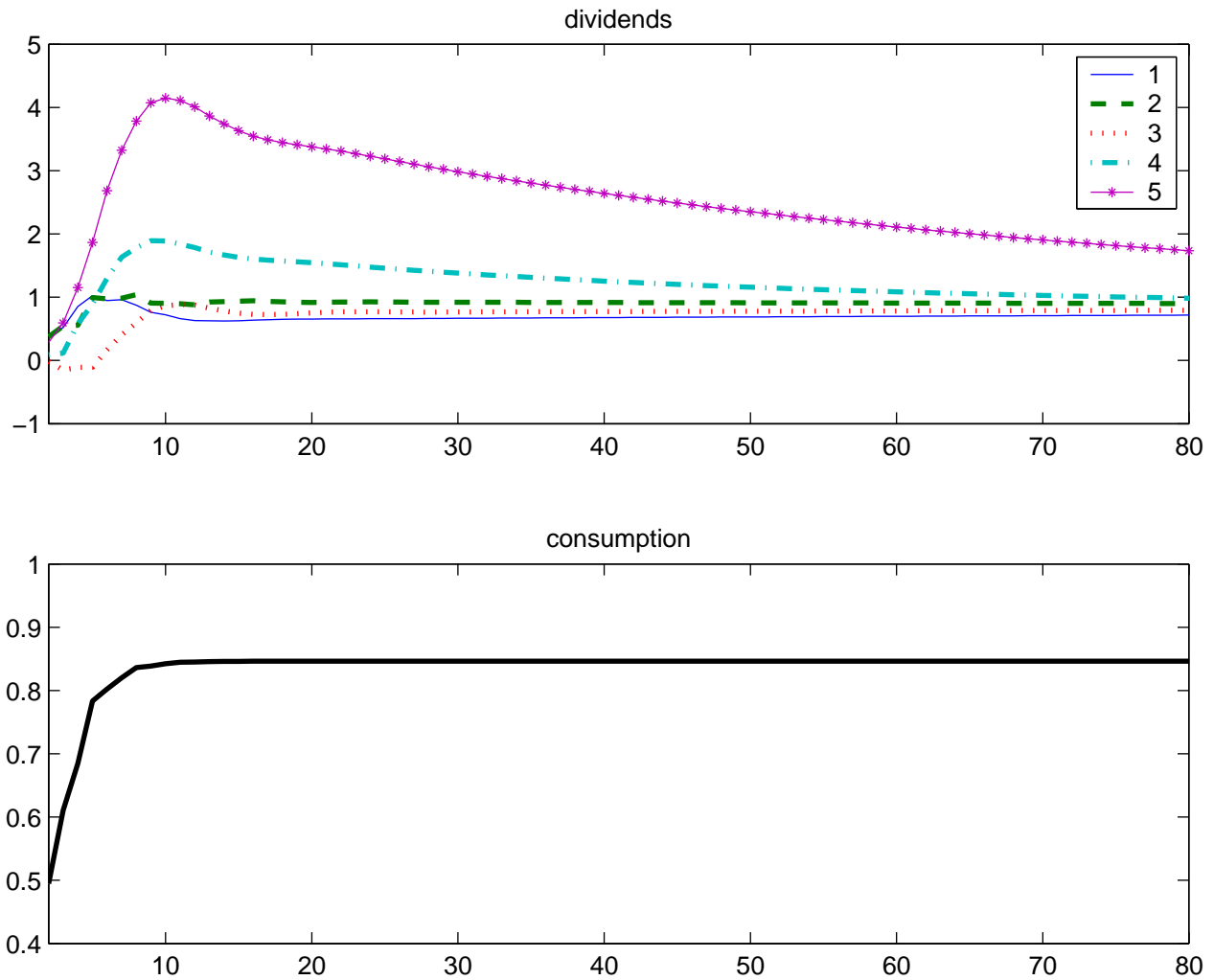


Figure 4: Impulse Response to a Consumption Shock for the Cointegrated Specification. The impulse response are identified by a VAR estimated with $c_t - c_{t-1}$ and $c_t - d_t$ as the components of x_t . The matrix $A(z)$ is restricted to be lower triangular.

is lower triangular while preserving the restriction that the shocks continue to uncorrelated with each other and have unit variances. Thus we find an orthogonal matrix Q such that $A(1)^{-1}Q$ is lower triangular. The shocks of interest are now constructed as $Q'w_t$ and the impulse responses are responses to these transformed shocks. We also normalize the shocks so that positive movements in both shocks induce positive movements in consumption. By construction, only the first shock has a permanent impact on consumption and dividends. The impact of the second shock is transitory. Since both shocks influence consumption, both shocks are pertinent in assessing the riskiness of the implied cash flows.

The results are reported in Table 5 and the impulse responses are depicted in Figure 5. The discounted responses to the permanent consumption shock differ from the response to the previously identified consumption shocks. For instance, portfolio five now has an initial negative response to permanent consumption shock and this persists for many periods. The discounted response remains negative for this portfolio even though the limiting response is by construction positive. Thus holding portfolio five appears to provide some insurance against consumption risk, which makes the large mean return appear puzzling. The other portfolios dividends respond positively to this consumption shock. The portfolio five response to transitory shock is always positive, however. This is in contrast to the other four portfolios, which have negative responses to this shock. The transitory shock contributes to the discounted riskiness of the dividends.

A defect of this identification scheme is that the identified shocks differ depending upon which portfolio we use in the empirical investigation. To address this concern the final panel of table 5 reports values of the term $\gamma(0)'\eta(\rho)$ for each portfolio. This gives the conditional covariance between consumption growth and the discounted dividends. This accumulation of the effects of the two shocks mirrors our previous results. Risk increases from portfolio 1 to portfolio 5, although the largest increase is from portfolio 3 to 4 and then from portfolio 4 to 5.

While the dividend risk measures suggest that the high book-to-market returns have mean returns that are larger and more longer run covariation with consumption. As emphasized by Bansal, Dittmar, and Lundblad (2002a) this provides an qualitative explanation for the heterogeneity in mean returns. The discounted dividend riskiness of the high book-to-market returns must be compensated for by a higher mean return. We now examine the more ambitious quantitative question of how much risk aversion is required for this explanation to work.

Using (19) we can compute an implied value of θ from the difference in the risk premia between any two portfolios. In what follows we use the risk premium of each portfolio relative to that of portfolio 5 to calculate θ . This calculation clearly ignores the restriction that the same value of θ should be used in explaining the entire cross-section of average returns. Our goal is merely to provide a convenient metric to evaluate the quantitative significance of observed differences in discounted dividend responses. The results are reported in table 6 based on the risk measures of table 3. The discounted dividend responses do imply higher returns for the higher book-to-market portfolios but the magnitude of the risks are small. As a result, we calculate high implied coefficients of relative risk aversion. Similar observations

Table 5: Discounted Responses of Portfolio Dividends to Permanent and Transitory Shocks

	Portfolio				
	1	2	3	4	5
Discount Factor	0.9943	0.9918	0.9902	0.9889	0.9894
<i>Panel A: Discounted Responses, Permanent Shock</i>					
OLS Estimator	3.62	2.86	2.76	1.50	-0.14
10 percentile	1.61	1.62	1.16	0.45	-1.41
30 percentile	2.36	2.20	0.60	0.98	-0.47
median	3.08	2.67	0.80	1.35	-0.05
70 percentile	4.07	3.22	0.99	1.89	0.30
90 percentile	5.88	4.07	1.31	3.09	0.86
<i>Panel B: Discounted Responses, Transitory Shock</i>					
OLS Estimator	-0.63	-0.56	-0.66	-1.01	1.68
10 percentile	-1.36	-1.26	-1.49	-1.45	1.13
30 percentile	-0.92	-0.87	-0.97	-1.03	1.34
median	-0.68	-0.62	-0.70	-0.83	1.52
70 percentile	-0.45	-0.33	-0.42	-0.64	1.76
90 percentile	0.02	0.15	0.08	-0.38	2.25
<i>Panel C: Covariance Between Consumption and Discounted Response</i>					
OLS Estimator	0.29	0.36	0.32	0.58	0.63

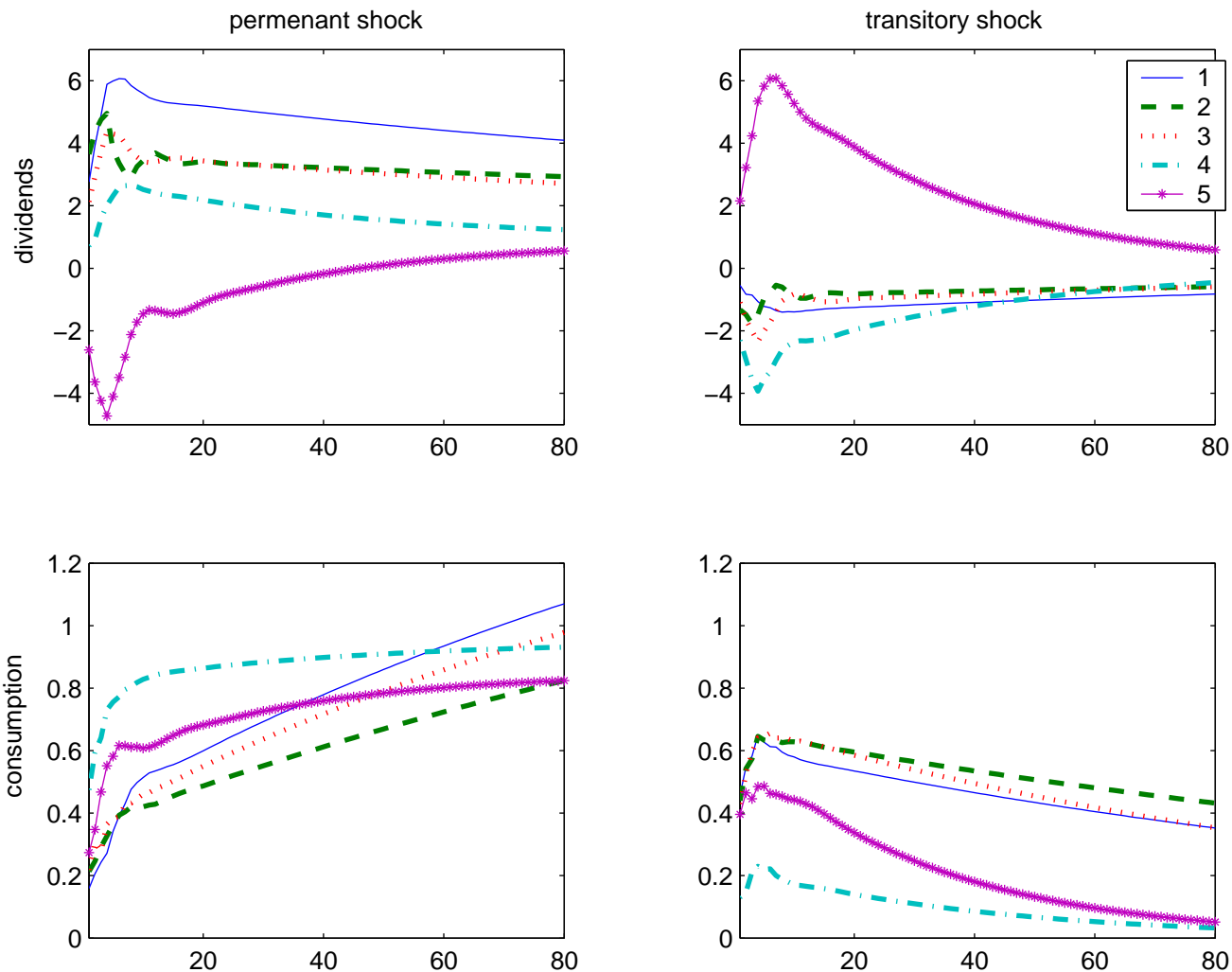


Figure 5: Impulse Response to a Permanent and Transitory Shock. The impulse response are identified by a VAR estimated with $c_t - c_{t-1}$ and $c_t - d_t$ as the two components of x_t . The permanent shock is identified as the shock that alters consumption permanently but is exactly offset in the long run by a movement dividends. The transitory shock is uncorrelated with the permanent shock and has a transitory impact on both consumption and dividends. A solid line — is used to depict the portfolio one response, a dashed line - - - for portfolio two, a dotted line \cdots for portfolio three, a dashed/dotted line - \cdot - for portfolio four and a line of asterisks * * * for portfolio five.

Table 6: Implied Value of Risk Aversion Coefficient θ

	Portfolio			
	1	2	3	4
θ	199	217	101	113

These values of θ are computed using the discounted risk measures reported in table 3 to a consumption shock.

have been made using the equity premium.¹⁵

6 Results with Additional Aggregate Shock

In our final VAR specification we consider what happens when an additional aggregate shock is added. In this specification x_t is given by:

$$x_t = \begin{bmatrix} c_t \\ e_t \\ p_t \\ d_t \end{bmatrix}, \quad (21)$$

where e_t is corporate profits at time t . To avoid parameter proliferation, we restrict the dynamics of the aggregate variables c_t and e_t to not be Granger-caused by the individual portfolio dividends and ratios of price to dividends. That is we restriction $A(z)$ to block lower triangular.

We consider the discounted response to a shock to consumption and to a shock to the share of corporate profits in aggregate consumption. We are led to consider this latter variable by the empirical investigations of Lettau and Ludvigson (2001) and Santos and Veronesi (2001). These authors argue for the addition of an aggregate share variable to help account for asset values. For example Santos and Veronesi (2001) argue that exposure to stock market risk is affected by the contribution of corporate payouts to aggregate consumption. We restrict $A(0)$ to be lower-triangular matrix with positive entries on the diagonal. We refer the first shock as the consumption shock and the second one as the aggregate profits shock.

The implied responses from this four-variable system are reported in table 7. The impulse response of prices and portfolio dividends to a consumption shock and to an aggregate earnings shock are displayed in figure 6. We no longer find that the high book-to-market portfolios have larger discounted responses to a consumption shock.

¹⁵See, for example, Mehra and Prescott (1985).

The discounted dividend responses to a consumption shock are very similar to those we reported in 3. Consistent with this we see from 6 that the the new earnings shock has very little impact on consumption. While portfolio three has the largest discounted earnings response, the earnings innovation may be of little consequence for measuring aggregate risk since this innovation accounts for very little of the consumption variation. A robust finding across all of the runs is the distinctive response of dividends from portfolio five to a consumption shock in the first ten time periods.

7 Conclusions

In this paper we reviewed two findings pertinent for using asset market data to make inferences about the intangible capital stock. We presented evidence familiar from the empirical finance literature that returns are heterogeneous when firms are grouped according to their ratio of market equity to book equity. This evidence suggests that there are important differences in the riskiness of investment in measured capital *vis a vis* intangible capital. This has potentially important ramifications for how to build explicit economic models to use in constructing measurements of the intangible capital stock.

A risk-based interpretation of return heterogeneity requires more than just a model with heterogeneous capital. It also requires a justification for the implied risk premia. There has been much interest recently in the finance literature on using vector autoregressive (VAR) methods to understand riskiness of serially correlated cash flows or dividends. The discounted dividend risk-measures using VAR methods find that high-book-to-market portfolio returns have more economically relevant risk. The discounted responses are larger for these returns. Moreover, the dividend response to a consumption shock for portfolio five, a portfolio of the highest book-to-market returns, stands out relative to other dividend responses. The impulse response for this portfolio has a pronounced peak at around ten quarters. This peak is present in three of our four VAR specifications of shocks, including a bivariate, log-level specification, a bivariate cointegrated specification and a four variate log-level specification. The shock responses differ very different when we identify a permanent-transitory decomposition of shocks to consumption and dividends, but portfolio five still stands out relative to the other portfolios.

The empirical evidence we report follows the finance literature by focusing on the claims of equity-holders. As emphasized by Hall (2001), what is pertinent for measurement purposes is the combined claims of bond-holders and equity-holders. It is the overall value of the firm or enterprize that is pertinent. Similarly, this analysis focuses on dividends as the underlying claims of equity-holders and not on overall cash flows of the firms. The risk associated with broader-based cash flow measures are of considerable interest for future research.

Table 7: Discounted Responses in Four Variate VAR with Aggregate Profit Share

	Portfolio				
	1	2	3	4	5
Discount Factor	0.9943	0.9918	0.9902	0.9889	0.9894
<i>Panel A: Dividends to Consumption Shock</i>					
OLS Estimator	0.05	0.17	0.42	0.57	1.11
10 percentile	-0.34	-0.08	0.25	0.38	0.75
30 percentile	-0.04	0.09	0.34	0.48	0.95
median	0.07	0.18	0.41	0.57	1.12
70 percentile	0.16	0.26	0.50	0.68	1.33
90 percentile	0.29	0.42	0.64	0.90	1.81
<i>Panel B: Dividends to Earning Shock</i>					
OLS Estimator	0.02	0.08	0.32	0.00	0.04
10 percentile	-0.43	-0.26	0.10	-0.26	-0.47
30 percentile	-0.09	-0.03	0.23	-0.11	-0.17
median	0.05	0.09	0.33	0.00	0.05
70 percentile	0.15	0.19	0.44	0.12	0.31
90 percentile	0.30	0.37	0.64	0.38	0.85
<i>Panel C: Prices to Consumption Shock</i>					
OLS Estimator	0.37	0.59	1.07	0.83	1.48
10 percentile	-0.30	0.04	0.66	0.35	1.12
30 percentile	0.20	0.40	0.90	0.67	1.32
median	0.39	0.60	1.07	0.83	1.48
70 percentile	0.56	0.81	1.27	1.01	1.68
90 percentile	0.83	1.17	1.64	1.30	2.07
<i>Panel D: Prices to Earning Shock</i>					
OLS Estimator	0.05	0.18	0.29	0.38	0.57
10 percentile	-0.71	-0.58	-0.38	-0.21	0.10
30 percentile	-0.15	-0.07	0.04	0.17	0.38
median	0.08	0.19	0.28	0.38	0.59
70 percentile	0.29	0.43	0.52	0.59	0.83
90 percentile	0.61	0.85	0.93	0.99	1.31

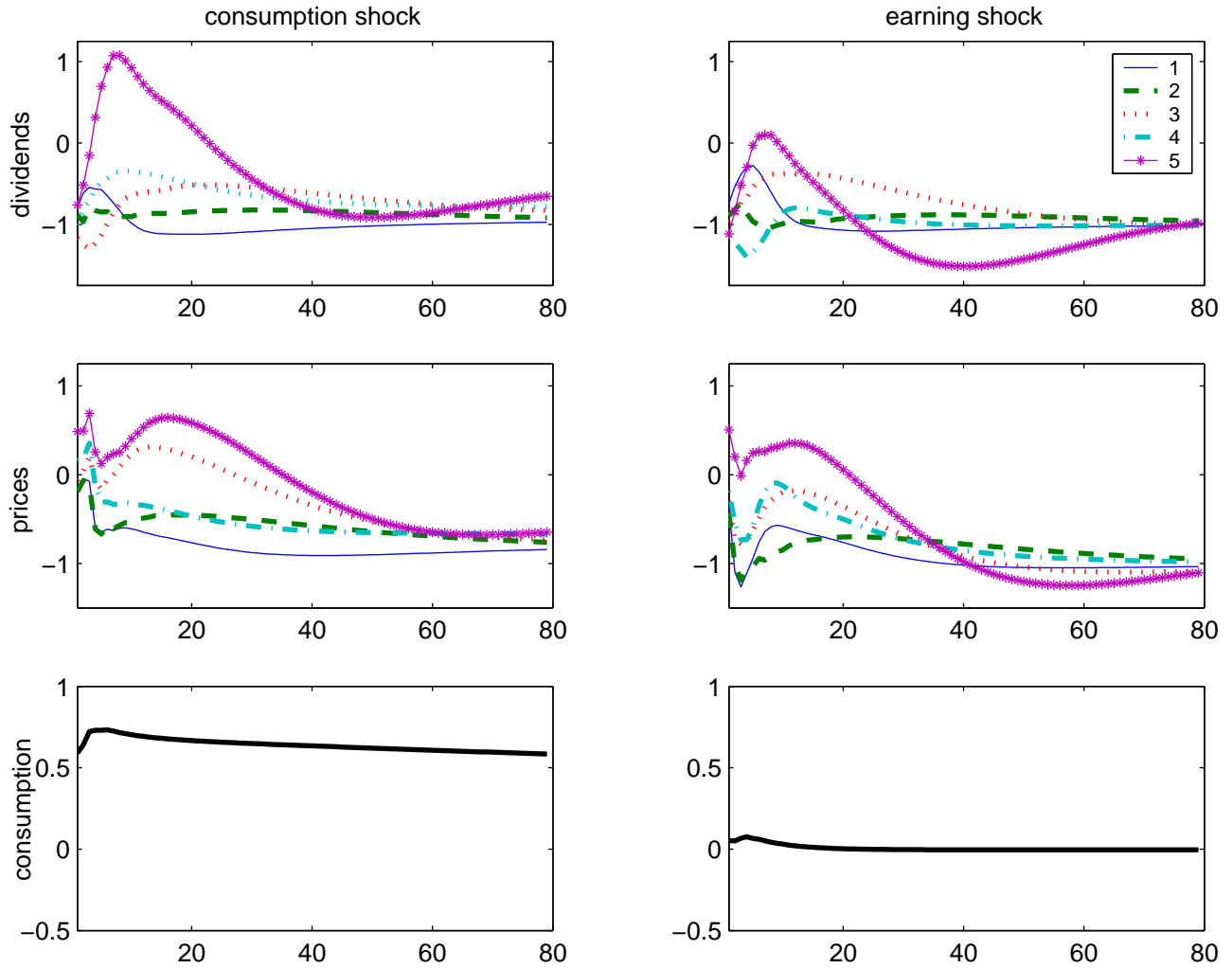


Figure 6: Impulse Response to a Consumption and Earnings Shock. These estimates were constructed from a VAR with entries c_t , e_t , p_t and d_t . The matrix $A(z)$ is restricted to be lower triangular in blocks of two and $A(0)$ is lower triangular. A solid line — is used to depict the portfolio one response, a dashed line — — — for portfolio two, a dotted line \cdots for portfolio three, a dashed/dotted line — \cdot — for for portfolio five.

A Data Appendix

CONSUMPTION

We use aggregate consumption of nondurables and services taken from the National Income and Product Account¹⁶ (NIPA Table 2.2). The quarterly data are seasonally adjusted at annual rates, deflated by the implicit price deflator for nondurable and services consumption.

CORPORATE EARNINGS

Corporate earnings is measured as corporate profits with inventory valuation and capital consumption adjustments from NIPA (Table 1.14), the quarterly data are deflated by the implicit price deflator for nondurable and services consumption.

BE/ME PORTFOLIOS¹⁷

We follow Fama and French (1992) in constructing portfolios ranked by book-to-market ratios. Five BE/ME portfolios are formed at the end of each June using NYSE breakpoints. The BE used in June of year t is the book equity for the last fiscal year end in $t - 1$. ME is price times shares outstanding at the end of December of $t - 1$. We use all NYSE, AMEX, and NASDAQ stocks for which we have ME for December of $t - 1$ and June of t , and BE for $t - 1$. For each stock, monthly returns with and without dividends and monthly market values are taken from CRSP monthly stock dataset. We take annual BE data from 1950 to 2001 from CRSP/COMPUSTAT merged industrial dataset¹⁸. We thank Kenneth French for providing us with the annual BE data from 1926 to 1950. The two datasets are merged together with the CRSP dataset using CRSP Permanent Company Number (PERMNO). For each portfolio, the monthly returns with and without dividends from July of year t to June of year $t - 1$ are weighted average of the stock returns with and without dividends in the same period, using use the ME in June of year t as the weights. Portfolio book-to-market ratios in year t is the weighted average of the stock book-to-market ratios in year t . R&D and sales data are taken from COMPUSTAT, available from 1950 to 2001. In year t , portfolio R&D/Sales ratios is the weighted average of the R&D/Sales of each firms in year t .

DIVIDENDS¹⁹

The dividends yield D_{t+1}/P_t is imputed from portfolio returns with dividend $R_{t+1}^w \equiv (P_{t+1} + D_{t+1})/P_t$ and returns without dividend $R_{t+1}^{wo} \equiv (P_{t+1} - D_{t+1})/P_t$ as following

$$\frac{D_{t+1}}{P_t} = R_{t+1}^w - R_{t+1}^{wo}$$

Change in this yield along with the capital gain in the portfolio are used to impute the

¹⁶Our source is the U.S. Department of Commerce, Bureau of Economic Analysis

¹⁷The SAS codes used to construct the portfolio monthly returns, BE/ME and R&D/Sale are available upon request.

¹⁸CRSP monthly stock dataset and CRSP/COMPUSTAT merged dataset are from Wharton Research Data Services, University of Pennsylvania

¹⁹Portfolio dividends, prices and returns series used in this paper are available upon request.

growth in portfolio dividends

$$\frac{D_{t+1}}{D_t} = \frac{D_{t+1}/P_t}{D_t/P_{t-1}} \cdot \frac{P_t}{P_{t-1}} = \frac{R_{t+1}^w - R_{t+1}^{wo}}{R_t^w - R_t^{wo}} R_t^{wo}$$

From the dividend growth we impute the dividend level except for the initial value,

$$\frac{D_{t+1}}{P_0} = \prod_{s=1}^t \frac{D_{s+1}}{D_s} \cdot \frac{D_1}{P_0}$$

From monthly dividend series we compute the quarterly average. We initialize the dividends in 1947Q1 such that the dividend for market portfolio in 1947Q1 is same as the corporate earning in 1947Q1, and the BE/ME portfolio dividends are proportional to the market portfolio with respect to the market value. We then take 12 months trailing average because of the pronounced seasonal pattern in the corporate dividend payout. Our measure of quarterly dividends in quarter t is constructed by taking an average of the logarithm of dividends in quarter t and over the previous three quarters $t-3$, $t-2$, $t-1$. We average the logarithm of dividends instead of levels because our empirical modelling will be linear in logs. This construction has the interpretation of following an initial investment of \$1 in the portfolio and extracting the dividends while reinvesting the capital gains.

Returns and dividends are converted to real units using the implicit price deflator for nondurable and services consumption.

PRICE DEFLATOR

The nominal consumption, corporate earning, portfolio returns and portfolio dividends are deflated by the implicit price deflator for nondurable and services consumption, which is the weighted average of the personal nondurable consumption implicit price deflator P_t^{CN} (1996=100) and personal services consumption implicit price deflator P_t^S (1996=100), taken from NIPA Table 7.1. The weights are determined by the relative importance of nominal nondurable consumption (CN_t) and service consumption (CS_t), that is

$$P_t^C = \frac{P_t^{CN} CN_t + P_t^{CS} CS_t}{CN_t + CS_t}$$

B Bayesian Confidence Intervals

Consider the VAR:

$$A(L)y_t + C_0 = w_t$$

where y_t is d -dimensional. The matrix $A(0)$ is lower triangular. We base inferences on systems that can be estimated equation-by-equation. The w_t is normal random vector with mean zero and covariance matrix I . We follow Sims and Zha (1999) and Zha (1999) by considering a uniform prior on the coefficients. Given the recursive nature of our model, we may follow Zha (1999) by building the joint posterior for all parameters across all equations as a corresponding product. This requires that we include the appropriate contemporary

Table 8: Unstable Fraction of Simulation

	Portfolio				
	1	2	3	4	5
Log-Level VAR	2%	14%	1%	1%	3%
Cointegrated VAR	22%	11%	21%	6%	10%
Cointegrated Unrestricted VAR	20%	14%	19%	8%	2%
Four Variate VAR	11%	9%	2%	7%	5%

variables on the right-hand side of the equation to ensure that w_{t+1} has the identity as the covariance matrix. In effect we have divided the coefficients of the VAR into blocks that have independent posteriors given the data. We construct posterior confidence intervals for the objects that interest us as nonlinear functions of the VAR coefficients.²⁰

We computed the posterior confidence intervals using Monte Carlo methods using characterizations in Zha (1999) and Box and Tiao (1973). Confidence intervals are centered around the posterior median computed in our simulation, the error bands are computed using the 10th and 90th percentile. Our numbers are based on 100,000 simulations, taking out the unstable systems. The unstable fractions of the simulated systems for different models we used are reported in Table 8.

²⁰In making the prior uniform over all coefficients, we follow a suggestion by but not the actual practice in Sims and Zha (1999). There is a minor difference evident in the discussion in Sims and Zha (1999). See page ?.

References

- Atkeson, A. and P. J. Kehoe (2002). Measuring organizational capital. Federal Reserve Bank of Minneapolis Research Department Staff Report 291.
- Bansal, R., R. Dittmar, and C. Lundblad (2002a). Consumption, dividends, and the cross-section of equity returns. Duke University.
- Bansal, R., R. Dittmar, and C. Lundblad (2002b). Interpreting risk premia across size, value and industry portfolios. Duke University.
- Blanchard, O. J. and D. Quah (1989). The dynamic effects of aggregate demand and supply disturbances. *American Economic Review* 79, 655–673.
- Boldrin, M., L. Christiano, and J. Fisher (2001). Habit persistence, asset returns and the business cycle. *American Economic Review* 91, 149–166.
- Box, G. E. P. and G. C. Tiao (1973). *Bayesian Inference in Statistical Analysis*. Reading, Massachusetts: Addison-Wesley.
- Campbell, J. and R. Shiller (1988a). The dividend-price ratio and expectations of future dividends and discount factors. *Review of Financial Studies* 1, 195–227.
- Campbell, J. and R. Shiller (1988b). Stock prices, earnings, and expected dividends. *Journal of Finance* 43, 661–676.
- Cochrane, J. (1991). Production-based asset pricing and the link between stock returns and economic fluctuations. *Journal of Finance* 46, 209–237.
- Cochrane, J. H. and L. P. Hansen (1992). Asset pricing explorations for macroeconomics. *NBER macroeconomics annual*, 115–65.
- Cohen, R., C. Polk, and T. Vuoteenaho (2002). Does risk or mispricing explain the cross-section of stock prices? . Harvard University.
- Engle, R. F. and C. W. J. Granger (1987). Co-integration and error correction: Representation, estimation, and testing. *Econometrica* 55(2), 251–276.
- Fama, E. and K. French (1992). The cross-section of expected stock returns. *Journal of Finance* 47, 427–465.
- Granger, C. W. J. (1969). Investigating causal relations by econometric models and cross-spectral methods. *Econometrica* 37(2), 424–438.
- Grunfeld, Y. (1960). The determinants of corporate investment. In A. C. Harberger (Ed.), *The Demand for Durable Goods*. Chicago: University of Chicago Press.
- Hall, R. E. (2001). The stock market and capital accumulation. *American Economic Review* 91, 1185–1202.
- Hansen, L. P. and R. Jagannathan (1991). Implications of security market data for models of dynamic economies. *Journal of Political Economics* 99(2), 225–262.

- Hansen, L. P. and S. J. Richard (1987). The role of conditioning information in deducing testable implications of asset pricing models. *Econometrica* 55(3), 587–613.
- Hansen, L. P., W. Roberds, and T. J. Sargent (1991). Time series implications of present value budget balance and of martingale models of consumption and taxes. In L. P. Hansen and T. J. Sargent (Eds.), *Rational Expectations Econometrics*, pp. 121–161. Westview Press.
- Hansen, L. P. and K. Singleton (1983). Stochastic consumption, risk aversion, and the temporal behavior of asset returns. *Journal of Political Economy* 91, 249–265.
- Harrison, J. and D. Kreps (1979). Martingales and arbitrage in multiperiod security markets. *Journal of Mathematical Economics* 20, 381–408.
- Hayashi, F. (1982). Tobin’s marginal q and average q: A neoclassical interpretation. *Econometrica* 50, 213–224.
- Jermann, U. J. (1998). Asset pricing in production economies. *Journal of Monetary Economics* 41, 257–275.
- Lettau, M. and S. Ludvigson (2001). Resurrecting the (c)capm: A cross-sectional test when risk premia are time-varying. *Journal of Political Economy* 109, 1238–1287.
- Lucas, R. E. and E. C. Prescott (1971). Investment under uncertainty. *Econometrica* 39, 659–681.
- MacKinlay, A. (1995). Multifactor models do not explain deviations from the capital asset pricing model. *Journal of Financial Economics* 38, 3–28.
- McGrattan, E. R. and E. C. Prescott (2000). Is the stock market overvalued? *Federal Reserve Bank of Minneapolis Quarterly Review* 24, 20–40.
- McGrattan, E. R. and E. C. Prescott (2003). Taxes, regulations, and the value of u.s. and u.k. corporations. *Staff Report, Federal Reserve Bank of Minneapolis*.
- Mehra, R. and E. Prescott (1985). The equity premium: A puzzle. *Journal of Monetary Economics* 15, 145–161.
- Santos, T. and P. Veronesi (2001). Labor income and predictable stock returns. Graduate School of Business, University of Chicago.
- Sims, C. A. (1972). Money, income, and causality. *American Economic Review* 62(4), 540–552.
- Sims, C. A. and T. Zha (1999). Error bands for impulse responses. *Econometrica* 67, 1113–1156.
- Telser, L. G. (1988). *Theories of Competition*, Chapter Eight: Some Determinants of the Returns to Corporate Investment, pp. 335–392. North Holland.
- Zha, T. (1999). Block recursion and structural vector autoregression. *Journal of Econometrics* 90, 291–316.