Econometrica, Vol. 48, No. 6 (September, 1980)

TEMPORAL RESOLUTION OF UNCERTAINTY IN STAPLETON AND SUBRAHMANYAM'S "MULTIPERIOD EQUILIBRIUM ASSET PRICING MODEL"[1]

BY DAVID M. KREPS AND ROBERT WILSON

IN THEIR DERIVATION of a multiperiod asset pricing model, Stapleton and Subrahmanyam [**1**] make an implicit assumption concerning the temporal resolution of uncertainty about dividends. This is that all uncertainty concerning dividends paid at time t resolves at time t (except insofar as the values of previous dividends yield information about the dividends paid at time t).

The following example illustrates this. Consider a model with two future time periods $t = 1, 2$ and two firms $j = 1, 2$. The streams of dividends paid by the two firms (X_1^j, X_2^j) are independent of one another and have identical joint normal distributions: $X_1^j \equiv 0$ and X_2^j is normal with mean μ and variance σ^2. But the two streams differ in one respect. At time one, when the time one consumption decision must be made, no information concerning X_2^1 has been received: Its conditional distribution is still normal with mean μ and variance σ^2. But at time one all uncertainty concerning X_2^2 has resolved: Its value is known with certainty. Note that this difference is not encoded in the joint distribution of the matrix (X_t^j) so that the Stapleton–Subrahmanyam model values the two streams identically. But it is intuitively clear that the two streams will not be priced identically; one expects that the early resolution in the second stream permits better choice of consumption plans, thus making that stream worth more.

The reason that the Stapleton–Subrahmanyam model values these two streams identically is that they implicitly rule out the second one. They assume that the information available at any time is exactly the information generated by previous dividend payments. (See the first sentence of Stapleton and Subrahmanyam [**1**, p. 1094, paragraph 4].) This assumption is not implied by any of their explicit assumptions and is somewhat restrictive (cf. Wilson [**2**, Part IIA]). And with virtually no effort, this implicit assumption can be dropped. The mechanics follow.

Suppose that each firm j generates cash flows $(X_1^j, \ldots, X_n^j)$ where the matrix (X_t^j) has a joint normal distribution. Further suppose that each X_t^j can be written

$$X_t^j = \sum_{s=1}^{t} Z_{t,s}^j$$

where the matrix $(Z_{t,s}^j)$ has a joint normal distribution and *where the information available at any time τ is exactly the information generated* by $\{Z_{t,s}^j; s \leqslant \tau, t\}$. In other words, $Z_{t,s}^j$ is that bit of X_t^j which resolves at time s. (Stapleton and Subrahmanyam are considering the special case where $Z_{t,s}^j \equiv 0$ if $t > s$.) Holding to the assumption of nonstochastic interest rates $r_t - 1$, receiving $Z_{t,s}^j$ at time t is identical economically with receiving $(\pi_{\tau=s+1}^t r_\tau)^{-1} Z_{t,s}^j$ at time s. This is because one can use the market mechanisms available to agents to turn one into the other. Thus if we define

$$Y_s^j = \sum_{t=s}^{n} (\pi_{\tau=s+1}^t r_\tau)^{-1} Z_{t,s}^j,$$

the stream $X^j = (X_1^j, \ldots, X_n^j)$ and the stream $Y^j = (Y_1^j, \ldots, Y_n^j)$ are economically identical and have the same total value. Of course, the information structure associated with the stream Y^j is that implicitly assumed by Stapleton and Subrahmanyam, so that their formulae can be applied to this stream. (Since the $(Z_{t,s}^j)$ are assumed to have a joint normal distribution, so do the (Y_s^j).)

[1] This work was supported by National Science Foundation Grant SOC77-07741 A01 at the Institute for Mathematical Studies in the Social Sciences, Stanford University.

In the example given above, we would write

$$X_2^1 = Z_{2,1}^1 + Z_{2,2}^1 \quad \text{where} \quad X_2^1 \equiv Z_{2,2}^1 \quad \text{and} \quad Z_{2,1}^1 \equiv 0, \quad \text{and}$$
$$X_2^2 = Z_{2,1}^2 + Z_{2,2}^2 \quad \text{where} \quad X_2^2 \equiv Z_{2,1}^2 \quad \text{and} \quad Z_{2,2}^2 \equiv 0.$$

Of course, $X_1^1 \equiv Z_{1,1}^1 \equiv X_1^2 \equiv Z_{1,1}^2 \equiv 0$. Thus

$$Y_1^1 \equiv Z_{1,1}^1 + \frac{Z_{2,1}^1}{r_2} \equiv 0, \quad Y_2^1 \equiv Z_{2,2}^1 \equiv X_2^1,$$

$$Y_1^2 \equiv Z_{1,1}^2 + \frac{Z_{2,1}^2}{r_2} \equiv \frac{X_2^2}{r_2}, \quad Y_2^2 \equiv Z_{2,2}^2 \equiv 0.$$

The reader may now apply the Stapleton–Subrahmanyam formulae to the streams Y^1 and Y^2. In general, they will be valued differently; thus the total value of the streams X^1 and X^2 ought to be different.

Stanford University

Manuscript received March, 1979; revision received November, 1979.

REFERENCES

[1] STAPLETON, R. C., AND M. G. SUBRAHMANYAM: "A Multiperiod Equilibrium Asset Pricing Model," *Econometrica*, 46 (1978), 1077–1096.

[2] WILSON, R.: "Risk Measurement of Public Projects," Economic Series, Technical Report No. 240, Institute for Mathematical Studies in the Social Sciences (IMSSS), Stanford University, June, 1977.