The Declining Equity Premium: What Role Does Macroeconomic Risk Play?*

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Abstract

Aggregate stock prices, relative to virtually any sensible indicator of fundamental value, soared to unprecedented levels in the 1990s. Even today, after the market declines since 2000, they remain well above historical norms. Why? We consider one particular explanation: a fall in *macroeconomic risk*, or the volatility of the aggregate economy. We estimate a two-state regime switching model for the volatility and mean of consumption growth, and find evidence of a shift to substantially lower consumption volatility at the beginning of the 1990s. We then show that there is a strong and statistically robust correlation between low macroeconomic volatility and high asset prices: the estimated posterior probability of being in a low volatility state explains 30 to 60 percent of the post-war variation in the log price-dividend ratio, depending on the measure of consumption analyzed. Next, we study a rational asset pricing model with regime switches in both the mean and standard deviation of consumption growth, where the probabilities of a regime change are calibrated to match estimates from post-war data. Plausible parameterizations of the model are found to account for almost all of the run-up in asset valuation ratios observed in the late 1990s.

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1 Introduction

It is difficult to imagine a single issue capable of eliciting near unanimous agreement among the many opposing cadres of economic thought. Yet if those who study financial markets are in accord on any one point, it is this: the close of the 20th century marked the culmination of the greatest surge in equity values ever recorded in U.S. history. Aggregate stock prices, relative to virtually any sensible indicator of fundamental value, soared to unprecedented levels. At their peak, equity valuations were so extreme that even today, after the broad market declines since 2000, aggregate price-dividend and price-earnings ratios remain well above their historical norms (Figure 1).[1]

How can such persistently high stock market valuations be justified? One possible explanation is that the equity premium has declined (e.g., Jagannathan, McGrattan, and Scherbina (2000); Fama and French (2002)). Thus, stock prices are high because future returns on stocks are expected to be lower. These authors do not address the question of why the equity premium has declined, but other researchers have pointed to reductions in the costs of stock market participation and diversification (Heaton and Lucas (1999); Siegel (1999)).

In this paper, we consider an alternative explanation for the declining equity premium and persistently high stock market valuations: a fall in *macroeconomic risk*, or the volatility of the aggregate economy. To understand intuitively why macroeconomic risk can affect asset prices, consider the following illustrative example. By the law of one price, there exists a stochastic discount factor, or pricing kernel, M_{t+1}, such that the following expression holds for any traded asset with gross return R_t at time t:

$$E_t\left[M_{t+1}R_{t+1}\right] = 1, \tag{1}$$

where E_t denotes the expectation operator conditional on information available at time t.

[1]The full run-up in valuation ratios cannot be attributed to shifts in corporate payout policies that have led many firms to substitute share repurchases for cash dividends. Although the number of dividend paying firms has decreased in recent years, large firms with high earnings actually increased real cash dividend payouts over the same period; as a consequence, aggregate payout ratios exhibit no downward trend over the last two decades (DeAngelo, DeAngelo, and Skinner (2002); Fama and French (2001)). See also Campbell and Shiller (2003). This, along with the evidence that price-earnings ratios remain unusually high, means that changes in corporate payout policies cannot fully explain the sustained high levels of financial valuation ratios.

Suppose the pricing kernel and returns are jointly lognormal. Then it follows from (1) that the Sharpe ratio, SR_t, may be written

$$SR_t \equiv \max_{\text{all assets}} \frac{E_t\left[R_{t+1} - R_{f,t+1}\right]}{\sigma_t\left(R_{t+1}\right)} \approx \sigma_t\left(\log M_{t+1}\right),$$

where $R_{f,t+1}$ is a riskless return known at time t, and $\sigma_t\left(\cdot\right)$ denotes the standard deviation of the generic argument "$\cdot$," conditional on time t information. Fixing $\sigma_t\left(R_{t+1}\right)$, the equity premium, in the numerator of the Sharpe ratio, is approximately proportional to the conditional volatility of the log pricing kernel.[2] In many asset pricing models, the pricing kernel is equal to the intertemporal marginal rate of substitution in aggregate consumption, C_t. A classic specification assumes there is a representative agent who maximizes a time-separable power utility function given by $u\left(C_t\right) = C_t^{1-\gamma}/(1-\gamma)$, $\gamma > 0$. With this specification, the Sharpe ratio may be written, to a first order approximation, as

$$SR_t \approx \gamma\sigma_t\left(\Delta \log C_{t+1}\right).$$

Thus, macroeconomic risk plays a direct role in determining the equity premium: fixing $\sigma_t\left(R_{t+1}\right)$, lower consumption volatility, $\sigma_t\left(\Delta \log C_{t+1}\right)$, implies a lower equity premium and a lower Sharpe ratio. Of course, this stylized model has important limitations, but its very simplicity serves to illustrate the crucial point: macroeconomic risk plays an important role in determining asset values. Below, we investigate these issues using a more complete asset pricing model.

Why underscore macroeconomic risk? There is now broad consensus among macroeconomists of a widespread and persistent decline in the volatility of real macroeconomic activity over the last 15 years. Kim and Nelson (1999) and McConnell and Perez-Quiros (2000) were the first to formally identify structural change in the volatility of U.S. GDP growth, occurring sometime around the first quarter of 1984. Following this work, Stock and Watson (2002) subject a large number of macroeconomic time series to an exhaustive battery of statistical tests for volatility change. They conclude that the decline in volatility has occurred broadly across sectors of the aggregate economy. It appears in employment growth, consumption growth, inflation and sectoral output growth, as well as in GDP growth. It is

[2] Conditioning (1) on time 0 information, the same expression can be stated in terms of unconditional moments.

large and it is persistent. Reductions in standard deviations are on the order of 60 to 70 percent relative to the 1970s and 1980s, and the marked change seems to be better described as a structural "break," or regime shift, than a gradual, trending decline. The macroeconomic literature is currently involved in an active debate over the cause of this sustained volatility decline.[3]

The subject of this paper is not the cause of the volatility decline, but its possible consequences for the U.S. aggregate stock market. Our investigation contains two parts. In the first part, we employ the same empirical techniques used in the macroeconomic literature to characterize the decline in volatility of various measures of aggregate consumer expenditure growth in U.S. data. In the second part, we investigate the behavior of the stock market in a theoretical asset pricing model when empirically plausible shifts in macroeconomic risk are introduced.

The empirical part of this paper follows much of the macroeconomic literature and characterize the decline in volatility by estimating a regime switching model for the standard deviation and mean of consumption growth. The estimation produces evidence of a shift to substantially lower consumption volatility at the beginning of the 1990s. We then show that there is a strong and statistically robust correlation between low macroeconomic volatility and high asset prices: the CRSP value-weighted price-dividend ratio is nearly twice as high in low volatility states as in high volatility states. Moreover, the estimated posterior probability of being in a low volatility state explains 30 to 60 percent of the post-war variation in the log price-dividend ratio, depending on the measure of consumption studied.

Of course, even the most careful empirical study cannot rule out the possibility that such a striking correlation between high stock prices and low macroeconomic volatility may be coincidental. We address this concern in two ways. First, we show that this phenomenon is not merely a feature of postwar U.S. data, but is also present in postwar international data for 10 countries, and in prewar U.S. data. Second, we investigate whether an asset pricing model that incorporates empirically plausible shifts in both the mean and volatility of consumption growth can account for the sharp run-up in aggregate stock prices during the 1990s. Using the preference specification developed by Epstein and Zin (1989, 1991) and Weil (1989), we study an asset pricing model with regime switches in both the mean and standard deviation

[3] See Stock and Watson (2002) for a survey of this debate in the literature.

of consumption growth, calibrated to match our estimates from post-war data. We assume that agents cannot observe the regime but must infer it from consumption data. Feeding in the (estimated) historical posterior probabilities of being in low and high volatility and mean states, we find parameterizations of the model that can account for almost all of the run-up in price-dividend ratios observed in the late 1990s. Our parameterizations assume fairly high risk aversion (on the order of 40 for relative risk aversion), but no more so than leading asset pricing models calibrated to match the post-war mean equity premium. The model's predicted valuation ratios move higher in the 1990s because the long-run equity premium declines, a direct consequence of the persistent decline in macroeconomic risk in the early part of the decade. A shift to a higher mean growth state also plays a role in generating the model's predicted run-up in equity values, but is far less important than the sharp decline in volatility. Finally, despite the decline in consumption volatility, the model predicts an *increase* in the volatility of the stock market in the 1990s, consistent with actual experience. Because the change in consumption volatility appears to be highly persistent, the results suggest that lower equity premia and high equity valuation ratios may be a feature of the aggregate economy for some time to come.

The literature has offered other possible explanations for the persistently high stock market valuations observed in the 1990s. One is an increase in the expected long-run growth rates of corporate earnings or dividends. The plausibility of this explanation has been questioned by academic researchers who point out that neither recent experience nor historical data provide any basis for the hypothesis (Siegel (1999); Jagannathan, McGrattan, and Scherbina (2000); Fama and French (2002); Campbell and Shiller (2003)). Other hypotheses include behavioral stories of "irrational exuberance" (Shiller (2000)), higher intangible investment in the 1990s (Hall (2000)), changes in the effective tax rate on corporate distributions (McGrattan and Prescott (2002)), the attainment of peak saving years during the 1990s by the baby boom generation (Abel (2003)), and a redistribution of rents away from factors of production towards the owners of capital (Jovanovic and Rousseau (2003)). In this paper, we do not consider any of these alternative explanations, and instead investigate the extent to which a decline in macroeconomic risk can rationalize the low-frequency behavior of stock prices characteristic of the end of the 20th century.

We emphasize that our concern in this paper is not the shorter term movements in equity valuations that may be attributable to cyclical fluctuations in the conditional (point-in-time) expected stock market return.[4] Instead, we are interested in the ultra low-frequency movements in valuation ratios corresponding to possible low-frequency movements in the equity premium, what Fama and French (2002) call the "unconditional"equity premium. Thus, the model we present below is designed to illustrate the possible impact of a regime shift in macroeconomic volatility, not to explain higher frequency fluctuations in valuation ratios.

A number of existing papers use theoretical and empirical techniques related to those employed here to investigate a range of asset pricing questions. One group of papers investigates asset pricing when there is a discrete-state Markov switching process in the conditional mean of the endowment process (Cecchetti, Lam, and Mark (1990); Kandel and Stambaugh (1991); Abel (1994); Abel (1999); Cecchetti, Lam, and Mark (2000); Wachter (2002)), or in technology shocks (Cagetti, Hansen, Sargent, and Williams (2001)). None of these studies investigate the impact of regime switches in the *volatility* of the endowment process, however, the focus of this paper. Veronesi (1999) studies an equilibrium model in which the drift in the endowment process follows a latent two-state regime switching process and finds that such a framework is better at explaining volatility clustering than a model without regime changes. Whitelaw (2000) also investigates an equilibrium economy with regime-switching in the mean of the endowment process, and he allows for time-varying transition probabilities between regimes. He finds that such a model generates a complex nonlinear relation between expected returns and volatility in the stock market. Again, however, neither of these studies investigate the impact of regime switches in the volatility of the endowment process.

The papers closest to ours in focus are Bansal and Lundblad (2002) and Bansal, Khatchatrian, and Yaron (2003). Bansal and Lundblad argue that there has been a fall in the global

[4] A large literature finds that excess stock returns on aggregate stock market indexes are forecastable, suggesting that the conditional expected excess stock return varies. Shiller (1981), Fama and French (1988), Campbell and Shiller (1988), Campbell (1991), and Hodrick (1992) find that the ratios of price to dividends or earnings have predictive power for excess returns. Harvey (1991) finds that similar financial ratios predict stock returns in many different countries. Lamont (1998) forecasts excess stock returns with the dividend-payout ratio. Campbell (1991) and Hodrick (1992) find that the relative T-bill rate (the 30-day T-bill rate minus its 12-month moving average) predicts returns, while Fama and French (1988) study the forecasting power of the term spread (the 10-year Treasury bond yield minus the one-year Treasury bond yield) and the default spread (the difference between the BAA and AAA corporate bond rates). Lettau and Ludvigson (2001) forecast returns with a proxy for the log consumption-wealth ratio.

equity risk-premium, and explore a model in which this decline is associated with a fall in the conditional volatility of the world market portfolio return. Because the conditional volatility of the world market portfolio is a magnified version of the conditional volatility of world cash flow growth in their model, they indirectly link the decline risk-premia to a decline in the volatility of underlying fundamentals. By contrast, Bansal, Khatchatrian and Yaron focus more directly on the volatility of underlying fundamentals and construct quarterly measures of volatility for aggregate consumption based on parametric models including GARCH, and from the residuals of an autoregressive specifications for consumption growth. They find that quarterly price-dividend ratios are predicted by these lagged volatility measures, with R-squared statistics as high as 25 percent. Our results are related to theirs in the sense that we both connect consumption volatility to movements in equity valuation ratios. But our analysis differs from both of these papers in that our emphasis is on the ultra low frequency movements in consumption risk that have become the subject of a large and growing body of macroeconomic inquiry, rather than on the cyclical stock market implications of quarterly fluctuations in conditional (point-in-time) consumption volatility.

The rest of this paper is organized as follows. In the next section we present empirical results documenting regime changes in the mean and volatility of measured consumption growth. We then explore their statistical relation with movements in measures of the price-dividend ratio for the aggregate stock market. Next, we turn to an investigation of whether the observed behavior of the stock market at the end of the last century can be generated from rational, forward looking behavior, as a result of the decline in macroeconomic risk. Section 3 presents an asset pricing model that incorporates shifts in regime, and evaluates how well it performs in explaining the run-up in stock prices during the 1990s. Section 4 concludes.

2 Macroeconomic Volatility and Asset Prices: Empirical Linkages

In this section we document the decline in volatility for two measures of consumer expenditure growth. For comparison with the macroeconomic literature, results for GDP growth are also reported. We consider two series of consumption: total per capita personal consumer expenditures (PCE), and per capita nondurables and services expenditures (NDS). All series are in 1996 chain-weighted dollars. The Appendix at the end of this paper gives a complete description of the data and our sources. Our data are quarterly and span the period 1952:1 to 2001:4.

We begin by looking at simple measures of the historical volatility of these series. Table 1 reports the sample standard deviation of PCE, NDS and GDP growth, for five year subsamples of the post-war period. For all series, there is a significant decline in volatility in the five-year window beginning in 1992, relative to the immediately preceding five-year window. In particular, each series is about one-half as volatile in the 1990s as it is in the whole sample.

Figure 2 provides graphical evidence of the decline in volatility. The growth rates of each series are plotted over time along with (plus or minus) two standard deviation error bands in each estimated volatility "regime," where a regime is defined by the estimated two-state markov switching process described below. (A low volatility regime is defined to be a period during which the posterior probability of being in a low volatility state is greater than 50 percent.) All figures clearly show that volatility is lower in the 1990s than previously.

Another way to see the low frequency fluctuations in macroeconomic volatility is to look at volatility estimates for non-overlapping five-year periods. Figure 3 (top panel) plots the standard deviation of NDS, PCE and GDP growth for non-overlapping five-year periods, along with the mean value of the log dividend-price ratio in each five year period.[5] The bottom panel of Figure 3 plots the same for the price-earnings ratio. Our measure of the log dividend-price ratio for the aggregate stock market is the corresponding series on the CRSP value-weighted stock market index. The data for the price-earnings ratio is taken from Robert Shiller's Yale web site.[6] The figure shows how these low frequency shifts in macroeconomic volatility are related to movements in the stock market.

Figure 3 exhibits a striking correlation between the low frequency movements in macroeconomic risk and the stock market: both volatility and the log dividend-price ratio (denoted $d_t - p_t$) are high in the early 1950s, low in the 1960s, high again in the 1970s, and then begin falling to their present low values in the 1980s. The only notable discrepancy between

[5]Replacing the mean with mid-point or end-points of $d_t - p_t$ in each five year period produces a similar picture.

[6]http://aida.econ.yale.edu/~shiller/data.htm

macroeconomic risk and the stock market is that the volatility series for NDS consumption falls more rapidly in the 1980s than does the log dividend-price ratio. But in general the correlation is quite high: the correlation between PCE volatilty and $d_t - p_t$ presented in this figure is 72 percent. A similar picture holds for the price-earnings ratio (bottom panel).

The correlations between high asset valuations and low volatility are present in countries other than the U.S. Figure 4 plots the volatility estimates for non-overlapping five-year periods, along with the mean value of the log dividend-price ratio in each five year period, for ten countries: Australia, Canada, France, Germany, Italy, Japan, the Netherlands, Sweden, Switzerland, and the United Kingdom. The international data on quarterly consumption and dividend-price ratios are from Campbell (2003), and are typically available over a shorter time period than for U.S. data.[7] Figure 4 uses the longest available sample for each country. The figure shows that international data also display a striking correlation between the low frequency movements in macroeconomic risk and the national stock market for the respective country. For every country, the figure exhibits a strong positive correlation between low frequency movements in macroeconomic risk and stock market valuation ratios, similar to that obtained for the U.S. Virtually every country also experiences a significant decline in macroeconomic volatility in the last decade of the century relative to earlier decades. The one exception is Australia, which displays no visible trend in either macroeconomic volatility or the stock market. Hence even for this observation, the correlation between macroeconomic volatility and the stock market is remarkable. More generally, Figure 3 and 4 tell the same story: for the vast majority of countries, the 1990s were a period of record-low macroeconomic volatility and record-high asset prices.

Moving back to U.S. data, Figure 5 shows that a correlation between macroeconomic volatility and the stock market is also present in prewar data. Although consistently constructed consumption data going back to the 1800s are not available, we do have access to quarterly GDP data from the first quarter of 1877 to the third quarter of 2002. The data are taken from Ray Fair's web site,[8] which provides an updated version of the GDP series

[7] The dataset uses Morgan Stanley Capital International stock market data covering the period since 1970. Data on consumption are from the International Financial Statistics of the International Monetary Fund. With the exception of a few countries, starting dates for consumption data in each country range from 1970, first quarter to 1982, second quarter.

[8] http://fairmodel.econ.yale.edu/RAYFAIR/PDF/2002DTBL.HTM

constructed in Balke and Gordon (1989). Figure 5 plots estimates of the standard deviation of GDP growth for non-overlapping ten year periods along with the mean value of the log dividend-price ratio in each ten year period, for whole decades from 1880 to 2000. The absolute value of GDP volatility in pre-war data must be viewed with caution. Volatility in this period is undoubtedly somewhat overstated relative to the postwar period due to greater measurement error, and consistent data collection methodologies were not in place until the postwar period. It is for this reason that we follow the existing literature and conduct our primary analysis using only postwar data. What Figure 5 does reveal, however, is that the strong *correlation* between macroeconomic volatilty and the stock market is not merely a feature of postwar data. Rather, it present in over a century of data spanning the period since 1880.

To characterize the decline in macroeconomic volatility more formally, we follow Hamilton (1989), and much of the macroeconomic literature (e.g., McConnell and Perez-Quiros (2000)), in using our postwar data set to estimate a regime-switching model based on the following discrete-state Markov process.[9] The estimates from this model will also serve as a basis for calibrating the asset pricing model we explore in the next section. Consider a time-series of observations on some variable X_t and let x_t denote $\log X_t$. A common empirical specification takes the form

$$\begin{aligned} \Delta x_t &= \mu(S_t) + \phi\left(\Delta x_{t-1} - \mu(S_{t-1})\right) + \epsilon_t \\ \epsilon_t &\sim N\left(0, \sigma^2\left(V_t\right)\right), \end{aligned} \tag{2}$$

where S_t and V_t are latent state variables for the states of mean and variance, respectively, each of which can assume a value of 1 or 2. We assume that the probability of changing mean states is independent of the probability of changing volatility states, and vice versa. In our empirical application, Δx_t will be the log difference of either PCE, NDS or GDP, and we allow the mean and variance of each series to follow independent, two-state Markov switching processes. It follows that there are two mean states, $\mu_t \equiv \mu(S_t) \in \{\mu_l, \mu_h\}$ and two volatility states $\sigma_t \equiv \sigma\left(V_t\right) \in \{\sigma_l, \sigma_h\}$. We denote the transition probabilities of the Markov

[9] We focus on U.S. on data for this procedure, as it is known to require a large number of data points to produce stable results.

chains

$$
\begin{aligned}
P\left(\mu_t = \mu_h | \mu_{t-1} = \mu_h\right) &= p^{\mu}_{hh} \\
P\left(\mu_t = \mu_l | \mu_{t-1} = \mu_l\right) &= p^{\mu}_{ll} \\
P\left(\mu_t = \mu_h | \mu_{t-1} = \mu_l\right) &= p^{\mu}_{hl} = 1 - p^{\mu}_{ll} \\
P\left(\mu_t = \mu_l | \mu_{t-1} = \mu_h\right) &= p^{\mu}_{lh} = 1 - p^{\mu}_{hh}
\end{aligned}
$$

and

$$
\begin{aligned}
P\left(\sigma_t = \sigma_h | \sigma_{t-1} = \sigma_h\right) &= p^{\sigma}_{hh} \\
P\left(\sigma_t = \sigma_l | \sigma_{t-1} = \sigma_l\right) &= p^{\sigma}_{ll} \\
P\left(\sigma_t = \sigma_h | \sigma_{t-1} = \sigma_l\right) &= p^{\sigma}_{hl} = 1 - p^{\sigma}_{ll} \\
P\left(\sigma_t = \sigma_l | \sigma_{t-1} = \sigma_h\right) &= p^{\sigma}_{lh} = 1 - p^{\sigma}_{hh}.
\end{aligned}
$$

Denote the transition probability matrices

$$
\begin{aligned}
\mathbf{P}^{\boldsymbol{\mu}} &= \begin{bmatrix} p^{\mu}_{hh} & p^{\mu}_{hl} \\ p^{\mu}_{lh} & p^{\mu}_{ll} \end{bmatrix}, \\
\mathbf{P}^{\boldsymbol{\sigma}} &= \begin{bmatrix} p^{\sigma}_{hh} & p^{\sigma}_{hl} \\ p^{\sigma}_{lh} & p^{\sigma}_{ll} \end{bmatrix}.
\end{aligned}
$$

The parameters $\Theta = \{\mu_h, \mu_l, \sigma_h, \sigma_l, \phi, \mathbf{P}^{\boldsymbol{\mu}}, \mathbf{P}^{\boldsymbol{\sigma}}\}$ are estimated using maximum likelihood, subject to the constraints $p^k_{ij} \geq 0$ for $i = l, h,\ j = l, h$ and $k = \{\mu, \sigma\}$.

Let lower case s_t represent a state variable that takes on one of $2^3 = 8$ different values representing the eight possible combinations for S_t, S_{t-1} and V_t. The model above may then be written as a function of the single state variable s_t:

$$
\Delta x_t = \mu(s_t) + \sigma(s_t)\epsilon_{t+1}.
$$

Since the state variables are latent, information about the unobserved regime must be inferred from observations on x_t. Such inference is provided by estimating the posterior probability of being in state s_t, conditional on estimates of the model parameters Θ and observations on Δx_t. Let $\mathbf{Y}_T = \{\Delta x_0, \Delta x_1, ..., \Delta x_T\}$ denote all observations in a sample of size T, and $\mathbf{Y}_t = \{\Delta x_0, \Delta x_1, ...\Delta x_t\}$ denote observations based on data available through time t. We call the posterior probability $P\left\{s_t = j | \mathbf{Y}_T; \widehat{\Theta}\right\}$, where $\widehat{\Theta}$ is the maximum likelihood estimate of Θ, the *smoothed probability* of being in state $s_t = j$. Similarly, we call the posterior probability $P\left\{s_t = j|\mathbf{Y}_t; \widehat{\Theta}\right\}$, the *unsmoothed probability* of being in state $s_t = j$. The smoothed probabilities are computed recursively from the unsmoothed probabilities using an algorithm developed by Kim (1994).

The estimation results are reported in Table 2. For PCE expenditure, the regime represented by $\mu(S_t) = \mu_h$ has average consumption growth equal to 0.722% per quarter, whereas the regime represented by $\mu(S_t) = \mu_l$, has an average growth rate of -0.121% per quarter. Thus, the high growth regime is an expansion state and the low growth regime a contraction state. The corresponding numbers for NDS growth are 0.536, -0.550. The results for GDP growth are qualitatively similar. These fluctuations in the conditional mean growth rate of consumption mirror cyclical variation in the macroeconomy.

The volatility estimates give a sense of the degree to which macroeconomic risk varies across regimes. For example, for PCE consumption, the high volatility regime represented by $\sigma(V_t) = \sigma_h$, has residual variance of 0.513 per quarter, whereas the low volatility regime represented by $\sigma(V_t) = \sigma_l$ has the much smaller residual variance of 0.113 per quarter. The corresponding numbers for NDS growth are 0.233 and 0.059; this corresponds to a 53 percent and 50 percent reduction in the standard deviation of PCE and NDS expenditure growth, respectively. The results for GDP growth are again qualitatively similar.

How persistent are these regimes? The probability that high mean growth will be followed by another high mean growth state is 0.955 for PCE consumption, implying that the high mean state is expected to last on average about 20 quarters. The volatility states are more persistent than the mean states. The probability that a low volatility state will be followed by another low volatility state is 0.984 for PCE consumption growth and 0.962 for NDS consumption growth, while the probability that a high volatility state will be followed by another high volatility state is 0.981 for NDS consumption and .992 for PCE consumption. A 95% confidence interval includes unity for these values, so we cannot rule out the possibility that the low macroeconomic volatility regime is an absorbing state. This characterization is consistent with that in the macroeconomic literature, which has generally viewed the shift toward lower volatility as a very persistent, if not permanent, break.

Figure 6 shows time-series plots of the smoothed and unsmoothed posterior probabilities of being in a low volatility state, $P(\sigma_t = \sigma_l)$, along with the smoothed and unsmoothed probabilities of being in a high mean state, $P(\mu_t = \mu_h)$, for our two measures of consumption growth.[10] PCE consumption exhibits a sharp increase in the probability of being in a low volatility state at the beginning of the 1990s. The probability of being in a low volatility state switches from essentially zero, where it resided for most of the post-war period prior to 1991, to unity, where it remains for the rest of the decade. For NDS growth, the posterior probability of being in a low volatility state is hump-shaped: it is close to zero until about 1982, increases to one by 1985, falls back to zero by 1990, and then increases again to one in the early 1990s where it stays for the rest of the decade. Both series show a marked decrease in volatility in the 1990s relative to previous periods.

Figure 7 displays the posterior probabilities along with the price-dividend ratio on the CRSP value-weighted index. The posterior probability of being in a high mean state also increases in the early 1990s, and becomes very close to one by 1992. Thus, in the last half of the 1990s, the posterior probabilities of being in *both* a high mean growth state and a low macroeconomic volatility state shift to very near unity. The 1990s were the only period in post-war data in which a shift to a high mean states occurred along side a shift to a low volatility state. Figure 6 shows that this dual transition coincides with the beginning of the sustained run-up in asset prices in the 1990s.

How much of the variation in price-dividend ratios can be explained by these posterior probabilities? Table 3 presents the results of regressing the log price-dividend ratio on the posterior probability of being in a low volatility state, $P\left(\sigma_t = \sigma_l\right)$, and the posterior probability of being in a high mean growth state, $P\left(\mu_t = \mu_l\right)$. For comparison, the posterior probabilities are computed for NDS consumption, PCE consumption, and GDP growth, and the regressions run for all three cases. In each regression, the probability of being in a low volatility state is a highly statistically significant explanatory variable for the aggregate stock market, with serial correlation and heteroskedasticity robust $t-$statistics in excess of 8 for all cases. The probability of being in a high mean growth state also has a positive affect on asset prices, but is not statistically different from zero. The regression adjusted R-square statistics imply that these two variables explain any where from 35 to 70 percent of the post-war variation in the price-dividend ratio of the aggregate stock market, depending on the measure of aggregate consumption used. Results (not reported) show that virtually all

[10] $P\left(\sigma_t = \sigma_l\right)$ is calculated by summing the joint probabilities of all states s_t associated with being in a low volatility state. $P\left(\mu_t = \mu_h\right)$ is calculated by summing the joint probabilities of all states s_t associated with being in a high mean growth state.

of this explanatory power comes from the posterior probability of being in a low volatility state, rather than the probability of being in a high mean state.

With estimates from the regression output presented in Table 3 in hand, we can also compute the fitted-value of the price-dividend ratio, when the posterior probability of being in a low volatility state and high mean state are both equal to unity, as they were in the late 1990s. These are displayed in Table 4. Using the unsmoothed PCE probabilities, the predicted price-dividend ratio is about 67, which is relatively close to the actual value of 80 reached during the peak of the bull market. Note also that, regardless of which measure of consumption is used, the fitted values of the price-dividend ratio vary more with changes in the volatility probability than with changes in the mean probabilities. For example, using the estimates from PCE consumption, smoothed probabilities, fixing the probability of being in a low volatility state at zero, a change in the probability of being in a high mean state from zero to one increases the predicted price-dividend ratio from 23 to 29, an increase of 26%. But fixing the probability of being in a high mean state at one, a change in the probability of being in a low volatility state from zero to one more than doubles the predicted price-dividend ratio,from 23 to 48, or an increase of 108%. Taken together, these results suggest that low macroeconomic volatility is associated with higher stock prices and explains a vast majority of its post-war variation.

The regression results just reported suggest that the perceived shift towards lower macroeconomic volatility may be associated with higher equity valuation ratios. A possible statistical concern with the regression results involves the degree of persistence of the variables studied. Although there are theoretical reasons to think that these variables are stationary (clearly they are bounded), the regressor and regressand are nevertheless persistent; for example, the first-order autocorrelation of the smoothed probability of being in a low volatility state is 0.97, while that of being in a high mean state is 0.76. This is not surprising since the regimes are estimated to be very persistent. Nevertheless, it is possible that such persistence could lead to a finite sample bias is the coefficient estimates and other regression statistics. To address this potential concern, we performed a Monte-Carlo excercise, the results of which are presented in Table 5.[11] We performed 50,000 simulations equal to the

[11]Note that it would not be appropriate to include the lagged price-dividend ratio in the regressions of $p_t - d_t$ on the posterior probabilities. The lagged value of $p_t - d_t$ should be a sufficient statistic for precisely the type of low-frequency variation in the level of the stock market we are trying to explain. Put another

size of our post-war data set, under the null hypothesis that there is no relation between $p_t - d_t$ and the posterior probability of being in either a low volatility or a high mean growth state. The data generating process for $d_t - p_t$ is assumed to be a first-order autoregressive process, with autoregressive coefficient set to its full sample estimate of 0.98. All innovations are drawn from a normal distribution, with innovation variance set to their sample estimates. We simulate a Markov chain for the state variables S_t and V_t using the transition probabilities presented in Table 2 for PCE, NDS, and GDP growth. Thus, S_t and V_t are the underlying state variables that can take on the value of either 0 or 1 depending on whether the state is high or low. We then regress the simulated values of $p_t - d_t$ on the state variables S_t and V_t. This is not exactly what is done in the actual empirical estimation, since there the state is latent, and the regression therefore run on posterior probabilities rather than the underlying state. Nevertheless, the approach of using the underlying states as regressors is far more tractable to implement numerically as a repeated simulation. The differences are unlikely to be of substantive importance, since the filtering problem inherent in the latent state case merely serves to smooth the posterior probabilities relative to the full information case in which S_t and V_t are observable.

To summarize, this procedure poses the following question: under the null hypothesis of no relation between the log price-dividend ratio and variation in mean and volatility of consumption growth across discrete states, how large can we expect the coefficient estimates and R-square statistics to be in finite samples of the size we currently encounter? The 90, 95 and 99 percentiles of the distribution for the coefficient estimates and R-squared statistics are presented in Table 4. For each case, the 95th percentile of the coefficient estimates on the state variables are different from zero, but are not as large as those reported in Table 3 from the data. Moreover, the R-squared statistics are positive, but again not large enough to explain the values obtained using actual data. For example, the 95th percentile of the R-squared statistic for a regression of $p_t - d_t$ on the state variables is 0.35 for PCE consumption, whereas it is 0.68 for the regression on posterior probabilities computed from data. These results suggest that the explanatory power of the posterior probabilities presented in Table

way, including lagged $p_t - d_t$ as a regressor is more akin to regressing, not the level of the stock market, but its growth rate onto the posterior probabilities. We cannot address the fundamental question of what macroeconomic factors might explain the low-frequency variation in equity valuation ratios if we remove that persistent variation by looking at first differences in $p_t - d_t$.

2 cannot be attributed to any finite-sample biases associated with the use of persistent variables.

3 An Asset Pricing Model With Shifts in Macroeconomic Risk

The results in the previous section suggest an important empirical relation between macroeconomic volatility and the stock market. In particular, the shift toward lower macroeconomic risk coincides with a sharp increase in the stock market in the 1990s. To better assess whether there may be a causal link between the low macroeconomic volatility and high equity values, we now investigate whether such a relation can be generated in a model of rational, forward-looking agents. To do so, we consider an asset pricing model augmented to account for regime switches in both the mean and standard deviation of consumption growth, with the shifts in regime calibrated to match our estimates from post-war data.

Modeling such shifts as changes in regime is an appealing device for addressing the potential impact of declining macroeconomic risk on asset prices, for two reasons. First, the macroeconomic literature has characterized the moderation in volatility as a sharp break rather than a gradual downward trend, a phenomenon that is straightforward to capture in a regime-switching framework (e.g., McConnell and Perez-Quiros (2000); Stock and Watson (2002)). Second, changes in regime can be readily incorporated into a rational, forward-looking model of behavior without regarding them as purely forecastable, deterministic events, by explicitly modeling the underlying probability law governing the transition from one regime to another. The probability law can be readily calibrated from our previous estimates from post-war consumption data.

Consider a representative agent who maximizes utility defined over aggregate consumption. To model utility, we use the more flexible version of the power utility model developed by Epstein and Zin (1989, 1991) and Weil (1989). Let C_t denote consumption and $R_{w,t}$ denote the simple gross return on the portfolio of all invested wealth. The Epstein-Zin-Weil objective function is defined recursively as

$$U_t = \left\{ (1-\delta) C_t^{\frac{1-\gamma}{\alpha}} + \delta \left(E_t U_{t+1}^{1-\gamma} \right)^{\frac{1}{\alpha}} \right\}^{\frac{\alpha}{1-\gamma}}, \tag{3}$$

where $\alpha \equiv (1-\gamma)/(1-1/\psi)$, ψ is the intertemporal elasticity of substitution in consumption (IES), and γ is the coefficient of relative risk aversion.

We consider a model of complete markets in which all wealth (including human capital) is tradeable. In this case, the aggregate wealth return $R_{w,t}$ can be interpreted as the gross return to an asset that represents a claim to aggregate consumption, C_t, and aggregate consumption is the dividend on the portfolio of all invested wealth. Following Campbell (1986) and Abel (1999), we assume that the dividend on equity, D_t, equals aggregate consumption raised to a power λ:

$$D_t = C_t^{\lambda}.$$

When $\lambda > 1$, dividends and the return to equity are more variable than consumption and the return to aggregate wealth, respectively. Abel (1999) shows that $\lambda > 1$ can be interpreted as a measure of leverage. We refer to the dividend claim interchangeably as the levered consumption claim. In what follows, we use lower case letters to denote log variables, e.g., $\log(C_t) \equiv c_t$.

The decline in volatilty present in consumption data is also present in cash-flow data. For example, according to quarterly data available on Robert Shiller's Yale web site, the standard deviation of dividend growth is 57 percent lower in the 1990s than in the period 1946 to 1989, about the same percentage decline present in PCE consumption growth. Nevertheless, we follow the common practice of calibrating the model based on estimates of the consumption process, and model dividends as a scale transformation of consumption. This practice has an important advantage: we do not need to model the short-run dynamics of cash-flows, which have been especially affected in the 1990s by changes in accounting practices and corporate payout policies.

To incorporate regime shifts in the mean and volatility of consumption growth, consider the following model for the first difference of log consumption:

$$\Delta c_t = \mu(s_t) + \sigma(s_t)\epsilon_t, \tag{4}$$

where $\epsilon_t \sim N(0,1)$ and s_t again represents a state variable that takes on one of N different values representing the possible combinations for the mean state S_t and the volatility sate V_t. This model is the same as the empirical model (2), except that we do not allow for autocorrelation in the conditional mean process, $\mu(s_t)$. As the results in Table 2 suggest, the estimated autocorrelation coefficient for $\mu(s_t)$ is not large for either measure of consumption and is likely to be inflated by time-averaging of aggregate consumption data. The more parsimonious framework (4) is far more manageable, as it reduces the number of states over which the model must be solved numerically.

Define the $N \times 1$ vector $\hat{\xi}_{t+1|t}$ of unsmoothed posterior probabilities in the following manner, where its jth element is given by

$$\hat{\xi}_{t+1|t}(j) = P\left\{s_{t+1} = j \mid \mathbf{Y}_t; \Theta\right\}.$$

As before, $\mathbf{Y}_t$ denotes a vector of all the data up to time t and Θ contains all the parameters of the model. Throughout it will be assumed that a representative agent knows Θ, which consequently will be dropped from conditioning statements unless essential for clarity.

The derivations below are presented using the unsmoothed probabilities, $\hat{\xi}_{t+1|t}(j)$, but we also feed the model historical data on the smoothed probabilities, defined analogously as

$$\hat{\xi}_{t+1|T}(j) = P\left\{s_{t+1} = j \mid \mathbf{Y}_T; \Theta\right\},$$

where $\mathbf{Y}_T$ denotes the vector of all observations on consumption in the sample of size T. The model implications using smoothed probabilities are relevant if we assume that agents have more information about their consumption than is captured in the available data on consumption itself, $\mathbf{Y}_t$; the smoothed probabilities give an upper bound on the results corresponding to the case of perfect information. Below, we present theoretical results based on both smoothed and unsmoothed probabilities.

Bayes' Law implies that the posterior probability $\hat{\xi}_{t+1|t}$ evolves according to

$$\hat{\xi}_{t+1|t} = \mathbf{P}\frac{(\hat{\xi}_{t|t-1} \odot \eta_t)}{\mathbf{1}'(\hat{\xi}_{t|t-1} \odot \eta_t)} \tag{5}$$

where $\odot$ denotes element-by-element multiplication, $\mathbf{P}$ is the $N \times N$ matrix of transition probabilities and

$$\eta_t = \begin{bmatrix} f(\Delta c_t \mid s_t = 1, \mathbf{Y}_{t-1}) \\ \vdots \\ f(\Delta c_t \mid s_t = N, \mathbf{Y}_{t-1}) \end{bmatrix}$$

is the vector of likelihood functions conditional on the state.[12] Given the distributional assumptions stated in (4), the likelihood functions are based on the normal distribution and

[12] See Hamilton (1994), Chapter 22.

take the form

$$f(\Delta c_t \mid s_t, \mathbf{Y}_{t-1}) = \frac{1}{\sqrt{2\pi}\sigma(s_t)} \exp\left\{\frac{-(\Delta c_t - \mu(s_t))^2}{2\sigma(s_t)^2}\right\}.$$

We again assume that there are two possible values for the mean, μ, and two possible values for the variance, σ, of consumption growth, implying four possible combinations of the two:

$$\begin{aligned}
\mu(1) &= \mu_h, \quad \sigma(1) = \sigma_h \\
\mu(2) &= \mu_h, \quad \sigma(2) = \sigma_l \\
\mu(3) &= \mu_l, \quad \sigma(3) = \sigma_h \\
\mu(4) &= \mu_l, \quad \sigma(4) = \sigma_l.
\end{aligned}$$

Thus, s_t takes on one of 4 different values representing the $2^2 = 4$ possible combinations for the mean state S_t,and the variance state V_t.

Finally, we assume that probability of a switch from a high to a low mean state is independent of the variance state, and that the probability of a switch from a high to a low variance state is independent of the mean state. As above, let $\mathbf{P}^{\boldsymbol{\sigma}}$ be the transition matrix for the variance and $\mathbf{P}^{\boldsymbol{\mu}}$ be the transition matrix for the means. Then the full 4×4 transition matrix is given by

$$\mathbf{P} = \begin{bmatrix} p^{\mu}_{hh}\mathbf{P}^{\sigma} & p^{\mu}_{hl}\mathbf{P}^{\sigma} \\ p^{\mu}_{lh}\mathbf{P}^{\sigma} & p^{\mu}_{ll}\mathbf{P}^{\sigma} \end{bmatrix}.$$

The elements of the four-state transition matrix (and the eight-state transition matrix in Section 2) can be calculated from the two-state transition matrices $\mathbf{P}^{\mu}$ and $\mathbf{P}^{\sigma}$. The theoretical model can therefore be calibrated to match our estimates of $\mathbf{P}$, $\hat{\xi}_{t+1|t}$ and Θ from the regime switching model for aggregate consumption data, and closed as an general equilibrium exchange economy in which a representative agent receives the endowment stream given by the consumption process (4).

3.1 Pricing the Consumption and Dividend Claims

This section discusses how we solve for the price of a consumption and dividend claim. The Appendix gives a detailed description of the solution procedure; here we give only a broad outline.

Let P_t^D denote the ex-dividend price of a claim to the dividend stream measured at the end of time t, and P_t^C denote the ex-dividend price of a share of a claim to the consumption stream. From the first-order condition for optimal consumption choice and the definition of returns

$$E_t\left[M_{t+1}R_{t+1}\right] = 1, \qquad R_{t+1} = \frac{P_{t+1}^D + D_{t+1}}{P_t^D} \tag{6}$$

where M_{t+1} is the stochastic discount factor, given under Epstein-Zin-Weil utility as

$$M_{t+1} = \left(\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\right)^{\alpha} R_{w,t+1}^{\alpha-1}. \tag{7}$$

Again, $R_{w,t+1}$ is the simple gross return on the aggregate wealth portfolio, which pays a dividend equal to aggregate consumption, C_t. The return on a risk-free asset whose value is known with certainty at time t is given by

$$R_{t+1}^f \equiv \left(E_t\left[M_{t+1}\right]\right)^{-1}.$$

We assume that investors cannot observe the state s_t directly, but must instead infer it from observable consumption data. Because innovations to consumption growth are i.i.d. conditional on regime, the posterior probabilities $\hat{\xi}_{t+1|t}$ summarize the information upon which conditional expectations are based. The price-dividend ratio for either claim may be computed by summing the discounted value of future expected dividends across states, weighted by the posterior probabilities of being in each state. It follows from the first order conditions that the price-dividend ratio of a claim to the dividend stream satisfies

$$E_t\left[M_{t+1}\left(\frac{P_{t+1}^D}{D_{t+1}}(\hat{\xi}_{t+2|t+1}) + 1\right)\frac{D_{t+1}}{D_t}\right] = \frac{P_t^D}{D_t}(\hat{\xi}_{t+1|t}), \tag{8}$$

and the price-consumption ratio for the consumption claim satisfies

$$E_t\left[M_{t+1}\left(\frac{P_{t+1}^C}{C_{t+1}}(\hat{\xi}_{t+2|t+1}) + 1\right)\frac{C_{t+1}}{C_t}\right] = \frac{P_t^C}{C_t}(\hat{\xi}_{t+1|t}). \tag{9}$$

Notice that $\frac{P_t^C}{C_t}$ is the wealth-consumption ratio, where wealth here is measured on an ex-dividend basis. The posterior probabilities $\hat{\xi}_{t+1|t}$ are the only state variables in this framework, so the price-dividend ratio is a function only of $\hat{\xi}_{t+1|t}$. We substitute for M_{t+1} from (7) and solve these functional equations numerically on a grid of values for the state variables $\hat{\xi}_{t+1|t}$.

Given the price-dividend ratio as a function of the state, we calculate the model's predicted price-dividend ratio over time by feeding in our time-series estimates of $\hat{\xi}_{t+1|t}$ and

$\hat{\xi}_{t+1|T}$ presented above.[13] We also compute an estimate of the M year-ahead equity premium (the difference between the equity return and the risk-free rate over an M-year period) as a function of time t information. For M large, this "long-run" equity premium is analogous to what Fama and French (2002) call the unconditional equity premium, as of time t. The Appendix provides details about how these quantities are computed.

3.2 Choosing Model Parameters

We calibrate the model above at a quarterly frequency. The rate of time-preference is set to $\delta = 0.995$. The parameters of the consumption process, (4), are set to match the empirical estimates reported in Table 2 for PCE consumption. The goal is to determine whether parameter values can be found for which the model above can account for a significant fraction of the run-up in stock prices in the 1990s. The key parameters in this regard are the leverage parameter, λ, the coefficient of relative risk aversion, γ, the IES, ψ, and the transition probability of staying in a high or low volatility state. We discuss these in turn.

To calibrate the transition probabilities, we use the estimates presented in Table 2. The probability of remaining in the same volatility state next period is quite close to one regardless of whether the volatility state is high or low, and indeed a 95% confidence interval for these estimates includes unity. These values coincide with evidence from the macroeconomic literature that the shift to lower macroeconomic volatility is well described as an extremely persistent, if not permanent, break (Kim and Nelson (1999), McConnell and Perez-Quiros (2000), Stock and Watson (2002)). To capture the notion that reduced macroeconomic volatility represents a near-permanent shift, we set $p^{\sigma}_{hh} = p^{\sigma}_{ll} = 0.9999$. The transition probabilities for the mean state, p^{μ}_{hh} and p^{μ}_{ll} are set to their samples estimates for PCE consumption.

To calibrate λ, we compare the standard deviation of consumption growth with that of dividend growth. In post-war data, the percent standard deviation of real, per capita dividend growth is 12.2 at an annual rate, about 8 times as high as that of real, per capita

[13] The posterior probabilities from the empirical model (2) are eight rather than four-dimensional because the mean-state of consumption growth in the previous model is also an element of the state vector. The four dimensional vector $\hat{\xi}_{t+1|t}$ fed into the model is created by summing the appropriate elements of the eight dimensional vector estimated from data. For example, $P\left\{\mu_{t+1}, \sigma_{t+1} = h \mid \mathbf{Y}_t; \Theta\right\}$ is created by summing $P\{S_{t+1} = 1, V_{t+1} = 1, S_t = 1 \mid \mathbf{Y}_t; \Theta\}$ and $P\{S_{t+1} = 1, V_{t+1} = 1, S_t = 2 \mid \mathbf{Y}_t; \Theta\}$.

PCE consumption growth, equal to 1.52 percent. For our benchmark results we set $\lambda = 5$, but we also considered values for λ as low as 3 and as high as 7.

We calibrate the coefficient of relative risk aversion, γ, in order to roughly match the average level of the price-dividend ratio simultaneously with a low average risk-free rate. To do so, the model presented above requires fairly high risk aversion, around $\gamma = 40$. Risk aversion of this magnitude is a feature of leading asset pricing models. A common objection to high risk aversion is that it delivers counterfactual implications for the mean risk-free rate in asset pricing models with power utility and time-separable preferences. But leading asset pricing models that depart from power utility resolve these difficulties by delivering reasonable implications for the risk-free rate even with high risk aversion. For example, the broadly successful habit-based asset-pricing model explored by Campbell and Cochrane (1999) has steady state relative risk aversion of 80, and rises to values in the hundreds away from steady state. The asset pricing model with idiosyncratic risk explored by Constantinides and Duffie (1996) requires risk aversion of about 50 to match key asset pricing facts if cross-sectional uncertainty about individual income is restricted to empirically plausible levels (Cochrane (2001)). Both of these models do a good job of matching the mean and volatility of the risk-free rate. The framework explored here also delivers reasonable implications for the risk-free rate when risk aversion is high. Thus, we set $\gamma = 40$ for the baseline results reported in this paper.

Finally, to choose parameter values for the IES, ψ, we consider how macroeconomic volatility influences the behavior of the equilibrium price-dividend ratio in the model presented above. A change in macroeconomic volatility has two affects on the equilibrium price-dividend ratio. First, regardless of the IES, lower macroeconomic volatility reduces the long-run equity premium because it lowers consumption risk; this effect drives up the price-dividend ratio. Second, lower macroeconomic volatility reduces the precautionary motive for saving, increasing the desire to borrow and therefore the equilibrium risk-free rate; this effect drives down the price-dividend ratio. The magnitude of this second effect relative to the first depends on the value of ψ and γ. If $\gamma > 1$, the first effect will dominate the second effect (so that lower macroeconomic volatility leads to higher asset prices) only if $\psi > 1$.

Empirical estimates of ψ using aggregate data often suggest that the IES is relatively small, and in many cases statistically indistinguishable from zero (e.g. Campbell and Mankiw (1989), Ludvigson (1999), Campbell (2003)). But there are several reasons to think that the relevant IES for our purposes may be larger than estimates from aggregate data suggest. First, other researchers have found higher values for ψ using cohort level data (Attanasio and Weber (1993), Beaudry and van Wincoop (1996)), or when the analysis is restricted to asset market participants using household-level data (Vissing-Jørgensen (2002)). More recently, Vissing-Jørgensen and Attanasio (2003) estimate the IES using the same Epstein-Zin framework employed in this study and find that this parameter for stockholders is typically above 1 (depending on the specification), with the most common values ranging from 1.17 to 1.75.[14] Second, Bansal and Yaron (2000) suggest that estimates of ψ based on aggregate data will be biased down if the usual assumption that consumption growth and asset returns are homoskedastic is relaxed. Third, Guvenen (2003) points out that macroeconomic models with limited stock market participation imply that properties of aggregate variables directly linked to asset wealth are almost entirely determined by stockholders who have empirically higher values for ψ. For the results reported below, we set $\psi = 1.5$, in the mid-range of the estimates reported by Vissing-Jørgensen and Attanasio (2003).[15]

3.3 Model Results

In this section we present results from solving the model numerically. We focus on how stock prices are influenced by the break in macroeconomic volatility documented in the empirical macroeconomic literature. To this end, we characterize the behavior of the equilibrium price-dividend ratio of a claim to the dividend stream by plotting the model's solution for this quantity as a function of the posterior probabilities, and by feeding the model the historical values of $\hat{\xi}_{t+1|t}$ and $\hat{\xi}_{t+1|T}$, estimated and presented above.

Figure 8 (left plot) presents the model's predicted log price-dividend ratio of the dividend claim as a function of the posterior probability of being in a low volatility state, $\sigma = \sigma_l$, when the mean state is high either with probability one or with probability zero. The price-dividend ratio increases with the posterior probability of being in a low volatility state,

[14] Unlike the estimates reported in Vissing-Jørgensen and Attanasio (2003), which are typically greater than one, the estimates of the IES reported in Vissing-Jørgensen (2002) are close to but slightly less than one. Vissing-Jørgensen (2002) explains the reason for this discrepancy: Vissing-Jørgensen (2002) ignored taxes, which biases estimates of the IES down. Vissing-Jørgensen and Attanasio (2003) address the downward bias by assuming a marginal tax rate of 30 percent.

[15] This value is also used in Bansal and Yaron (2000).

whether the mean state is high or low. The increasing function is not linear, but is instead a convex function of investor's posterior probability of being in the low volatility state.

The intuition for this convexity is similar to that given in Veronesi (1999) for an asset pricing model with regime shifts in the mean of the endowment process. Suppose the probability of being in a low consumption volatility state is initially zero. News that causes an increase in the posterior probability of being in a low volatility state has two effects on the price-dividend ratio. First, because investors believe that the probability of being in a low volatility state has risen, consumption risk is perceived to be lower, which works to decrease the equilibrium risk-premium and raise the price-dividend ratio. Second, because the probability of being in a low volatility state is farther from zero, investors are more uncertain about which volatility regime the economy is in, which works to lower the equilibrium price-dividend ratio. The two effects are offsetting. Consequently, as the posterior probability of being in a low volatility state increases from zero, the price-dividend ratio rises only modestly.

Conversely, suppose the probability of being in a low consumption volatility state is initially at unity. News that causes a decrease in this posterior probability again has two effects on the price-dividend ratio. First, consumption risk is perceived to be higher, which works to increase the equilibrium risk-premium and lower the price-dividend ratio. Second, because the probability of being in a low volatility state is now farther from unity, investors are more uncertain about which volatility regime the economy is in, which works to further lower the equilibrium price-dividend ratio. In this case, the two effects are reinforcing rather than offsetting. Consequently, as the posterior probability of being in a low volatility state declines from unity, the price-dividend ratio falls dramatically. This explains why the equilibrium price-dividend ratio is a convex function of the posterior probabilities. In addition, because investors are quite risk averse, the posterior probability of being in a low volatility state must be sufficiently close to one before it has a noticeable affect on the equilibrium price-dividend ratio.

Figure 8 (right plot) displays the log price-dividend ratio of the dividend claim as a function of the posterior probability of being in a high mean growth state, $\mu = \mu_h$, when the volatility state is high either with probability one or with probability zero. The price-dividend ratio increases with the posterior probability of being in a high mean state. For reasons similar to those just given, the function is again convex in the investor's posterior probability of being in the high mean state, but is substantially less convex than the function plotted against the low volatility probability. The effect of a change in mean probability on the price-dividend ratio is also much smaller than the effect of a change in the volatility probability on the price-dividend ratio. These differences appear to be attributable to the lower persistence of the mean regimes compared to the volatility regimes. For example, the probability that a low mean (contraction) state will be followed by another period of contraction is 0.8 for PCE growth, so that this regime will persist on average for only 5 quarters. The estimated high mean, or expansion, regime is more persistent, but is still only expected to last 20 quarters on average. By contrast, the volatility regimes we estimate are far more persistent, and we have calibrated them so that the shift to lower macroeconomic volatility in the early 1990s is expected to persist almost indefinitely, consistent with this characterization in the macro literature. Because of this persistence in regime, asset prices can rise dramatically as investors become increasingly certain that a low macroeconomic volatility state has been reached.

Figure 9 shows that the convexity in the relation between the model's implied price-dividend ratio and the posterior probability of being in a low volatility state, displayed in Figure 8, is also present in the data. The scatter plot displays the corresponding observations from U.S. data, with the theoretical relation implied by the model overlaid on the figure. The figure shows clearly that the convex relation present in the model is mimicked by the data. The darkened circles in the scatter plot signify "recession"periods, in which the posterior probability of being in a high growth state is less than one-half. With the exception of the most recent (2001) recession, recession periods correspond to high-volatility states (the probability of being in a low volatility state is close to zero). The most recent recession does not deliver a high-volatility state because the economic contraction in GDP and consumption was brief and mild by historical standards.

How well does this model capture the run-up in asset prices observed in the late 1990s? To address this question, we feed the model historical values of $\hat{\xi}_{t+1|t}$ and $\hat{\xi}_{t+1|T}$ for our post-war sample, 1951:Q4 to 2001:Q4. Figure 10 presents the actual log price-dividend ratio on the CRSP value-weighted index, along with the post-war history of the price-dividend ratio on the dividend claim implied by the model. The graph plots the model's prediction for $p_t - d_t$ using the estimated smoothed posterior probabilities (top panel) and unsmoothed posterior probabilities (bottom panel). Note that the "model" line on the graph is produced using only the posterior probabilities estimated from consumption data; no asset market data is used.

Using the historical values of the smoothed probabilities, the top panel of Figure 10 shows that the benchmark model provides a remarkable account of the longer-term tendencies in stock prices. In particular, it captures virtually all of the boom in equity values that began in the early 1990s and continued through the end of the millenium. In addition, the modest decline in the price-dividend ratio since 2000 is mimicked by the model's predicted price-dividend ratio as a result of the small decline in the posterior probability of being in low volatility state during the last few quarters of our sample.

Figure 10 shows that there are some differences in the behavior of the equilibrium price-dividend ratio depending on whether smoothed or unsmoothed probabilities are used. When we feed in the smoothed probabilities, the run-up in equity values predicted by the model is of the right magnitude but is a bit less gradual than observed in the data. When we feed in the unsmoothed probabilities, the increase in equity values is more gradual, but the boom is choppier than that displayed in the data.

We noted above that the model considered here delivers plausible implications for the risk-free rate of interest. If we feed the model historical values of $\hat{\xi}_{t+1|t}$ and $\hat{\xi}_{t+1|T}$ we may compute the post-war history of the risk-free rate predicted by the model. This rate has a has a mean of 1.1 percent and a standard deviation of 0.7 percent per annum, in line with actual values for an estimate of the real rate of return on a short-term Treasury bill.

What drives up the price-dividend ratio in the 1990s in this model? Although the shift to a higher mean growth state during this period generates a small part of the increase, the vast majority of the boom is caused by a decline in the equity premium as a consequence of the shift to reduced macroeconomic volatility. Figure 9 shows that–fixing the volatility state–variation in the equilibrium price-dividend ratio across mean states is quite modest. For example, fixing the probability of being in a low volatility state at one, the log price-dividend ratio ranges between 3.32 (when the probability of being in a high mean state is zero), to 3.49 (when the probability of being in a high mean state is one). Thus, the maximum possible variation in $p_t - d_t$ across mean states is about 5 percent. Fixing the probability of being in a low volatility state at zero, the maximum possible variation in in $p_t - d_t$ across mean states is even smaller, about 4 percent. This variation should be contrasted with the results for variation across volatility states. Fixing the probability of being in a high mean state at one, the log price-dividend ratio ranges between 3.49 (when the probability of being in a low volatility state is zero), to 4.47 (when the probability of being in a low volatility state is one), a range of variation of over 28 percent. Fixing the probability of being in a high mean state at zero, the maximum possible variation in in $p_t - d_t$ across volatility states is about 30 percent. In short, large swings in the price-dividend ratio in this model are generated not by shifts in the mean of the endowment process, but by changes in the posterior probability of being exposed to a less volatile endowment process.

The same message is conveyed by Figure 11, which plots the predicted behavior of the price-dividend ratio when the posterior probability of being in a low volatility state is counterfactually set at its pre-1990 level throughout the sample.[16] As the figure clearly demonstrates, the shift toward a higher mean growth state in the 1990s fails to capture the stock market boom during this period.

To understand what happens to the equity premium in the model, Figure 12 plots the $M = 50$, 75, and 100-year equity premia implied by the model, computed recursively from the one period equity premium. The M-period equity premium is the expected value of the sum of M future one-period log excess return on the dividend claim from t to $t + M$, reported at an annual rate. (The Appendix provides details about how this quantity is computed.) We use this quantity for M large to capture the model's predicted value for the "unconditional" equity premium as of time t. The figure presents the predicted equity premium of the dividend claim as a function of the posterior probability of being in a low volatility state, $\sigma = \sigma_l$, when the mean state is high either with probability one or with probability zero. In all cases, the predicted equity premium declines as the probability of being in a low volatility state increases, and drops off sharply once that probability exceeds 90 percent.

Figure 13 plots the post-war history of the log annual (100-year) equity premium on the dividend claim implied by the model, feeding in this history of the posterior probabilities.

[16] Some normalization is necessary in order to insure that the relevant probabilities continue to sum to one in this counterfactual. To do so, we fix the probability of being in a low volatility, high mean state and low volatility, low mean state at its value in the first quarter of 1990 (a value close to zero), and then re-weight the probabilities of being in a high volatility, high mean state, and high volatility, low mean state, so that the probability of being in a low mean state (regardless of the volatility state) is the same as it is in actual data, and the probability of being in a high mean state (regardless of the volatility state) is the same as it is in actual data.

The model equity premium is relatively flat for most of the post-war period, but begins to decline in the early 1990s. For the benchmark model, premium declines by a little over two percentage points from peak to trough. We should not be surprised that the percentage decline is not greater; even small changes in the equity premium can have a large impact on asset values if they are sufficiently persistent. The modest increase in the model's predicted equity premium during the last few quarters of our sample corresponds to the small decrease in the predicted price-dividend ratio discussed above.

The level of the long-run equity premium predicted by this model is a bit higher than estimates based on the historical average return on stocks in excess of returns on money market instruments. For example, the average annual excess return on the Standard and Poor 500 Stock market index in excess of a short-term Treasury bill rate is about 8 percent in data from 1951 to 2000, whereas the model's predicted premium declines from around 12.7 percent at an annual rate, to 10.7 percent at its trough. Nevertheless, several researchers have pointed out that the historical average excess return may not result in a good estimate of the long-run premium that investors actually expect to earn in the future. This is because such a computation misses any change in stock prices that result in an unexpected decline in the equity premium (Jagannathan, McGrattan, and Scherbina (2000); Fama and French (2002)). Jagannathan, McGrattan and Scherbina provide a computation of the expected stock return (by decade) that takes such changes into account, and find that the expected return for the aggregate stock market in data ending in 1990, ranges from about 10 percent to 11.4 percent, depending on which measure of the aggregate stock market is studied. These values for the expected (long-run) equity return are close to those predicted by the model explored here, using benchmark parameter values.

We emphasize an additional aspect of this model: although the volatility of consumption declines in the 1990s, the volatility of stock returns does not–consistent with actual experience. In fact, the model predicts a temporary *increase* in conditional volatility in the last half of the 1990s, a result of heightened uncertainty about which volatility regime the economy is in during the transition period from a high to low volatility. This prediction of the model is consistent with evidence that stock market volatility was indeed higher in the last half of the 1990s than during most of the post-war period prior.[17] We illustrate this in

[17]Updated plots of volatility of aggregate stock market indexes are provided by G. William Schwert at his University of Rochester web site: http://schwert.ssb.rochester.edu/volatility.htm.

Figure 14 which plots the post-war history of the conditional quarterly standard deviation of the log stock market return implied by the model, reported at an annual rate. Using the smoothed probabilities (top panel), there is an upward spike in the conditional volatility of the stock market in the 1990s, before it returns to levels more common prior to the 1990s. The up-tick in volatility at the far right of the figure is again attributable to the small decline in the posterior probability of being in a low volatility state in the last few quarters of our sample. Using the unsmoothed probabilities (bottom panel), volatility increases starting in the mid 1990s and for the rest of the sample remains, on average, higher than in previously in the post-war sample.

Some of the model's predictions are changed by departing from the benchmark parameter values for λ, δ, and p^{σ}_{jj}, the probability that next period's volatility state is j, given that this period's volatility state is j. To get a sense of how the shift toward lower volatility influences the model's asset pricing predictions as these parameter values change, Table 6 exhibits model predictions for the price-dividend ratio, the long-run (100-year) equity premium, and the long-run (100-year) risk-free rate in 1990:Q1 (before the volatility shift) and in 2001:Q4 (after the volatility shift). Also shown is the one-period risk-free rate in 1990:Q1 and 2001:Q4. The values in each time period are computed by feeding the model the historical values of the posterior probabilities for the relevant quarters in our sample. The first row of Table 6 presents the results from our benchmark parameter values, those results reported in Figure 10. Subsequent rows show results when we depart from the benchmark values for the parameters indicated in the first three rows.

Several notable aspects of the model are exhibited in Table 6. First, the mean price-dividend ratio in the data from 1952:Q4 to 1990:Q1 is 28; the first row of Table 6 shows that the benchmark case predicts that the price-dividend ratio in 1990:Q1 was 28 using the smoothed probabilities, and 29.7 using the unsmoothed probabilities. These values are roughly equal to the predicted mean values for the entire period 1952:Q4 to 1990:Q1. This shows that the benchmark model produces about the right mean price-dividend ratio for the post-war sample. Second, row 2 shows that when $p^{\sigma}_{hh} = p^{\sigma}_{ll} = 0.99999$, rather than the benchmark $p^{\sigma}_{hh} = p^{\sigma}_{ll} = 0.9999$, and all other parameters are set at their benchmark values, the model predicts an even greater increase in the price-dividend ratio. For example, using the unsmoothed probabilities, the model's predicted price-dividend ratio increases from 27.97 to 71.36 over the period, compared to an increase from 28 to 59.30 using benchmark parameter values. This demonstrates the importance of the perceived permanence of the volatility decline for generating the predicted surge in equilibrium price-dividend ratio. Even a modest decrease in macroeconomic volatility can cause a dramatic boom in stock prices when the decrease is perceived to be sufficiently permanent. Third, rows three and four show that when λ is lowered to from 5 to 3, the model predicts a smaller fraction of the run-up in price-dividend ratios than in the benchmark case, but still captures about 60% of the boom. This case also delivers a lower long-run equity premium than the benchmark case. For example, row 3 shows that when $\lambda = 3$ and all other parameter values are held at their benchmark values, the predicted equity premium is 7.51% in 1990:Q1, and falls to 6.15% in 2001:Q4, as a result of the shift toward lower volatility. At these parameter values the mean price-dividend ratio is too high, but this can be remedied by lowering δ from 0.9950 to 0.9925 (row 4).

Fourth, Table 6 shows that, for all parameter-value combinations, price-dividend ratios rise not because the long-run risk-free rate falls, but because the long-run equity premium falls. In fact, the long-run risk-free rate actually rises modestly in each case, but not by enough to offset the decline in the equity premium and cause an increase in the total rate of return. Finally, the last two columns show the model's prediction for the one-period risk-free rate in 1990:Q1 and 2001:Q4. In every case, the model predicts a small decline in the one-period real risk-free return. Unlike the long-run risk-free rate, the one-year risk-free rate is strongly affected by a rise in short-term precautionary demand for saving, a result of the uncertainty created by transitioning from a high- to low-variance Hamilton state probability; hence the predicted decline in one-period interest rates. By contrast, the long-run risk-free rate is dominated by a *decline* in precautionary saving motives, owing to the long-term lower consumption volatility; hence the slight increase in the predicted long-term risk-free rate.

As the analysis above demonstrates, the model explored here predicts a surge in the price-dividend ratio on the dividend claim in the 1990s. What about the price-dividend ratio of the consumption claim (the wealth-consumption ratio)? It turns out that this quantity is far less affected by the shift to lower consumption volatility. This occurs because the price of an unlevered consumption claim is much less sensitive to swings in consumption risk than is the price of a levered claim. This result is depicted in Figure 15, which feeds the model historical values of $\hat{\xi}_{t+1|t}$ and $\hat{\xi}_{t+1|T}$ and plots the post-war history of the wealth-consumption ratio implied by the model, for both the estimated smoothed and unsmoothed posterior probabilities. As the figure demonstrates, the wealth-consumption ratio is hardly affected by the shift toward lower volatility. This prediction is consistent with empirical evidence in Lettau and Ludvigson (2003), which shows that–unlike the log price-dividend ratio–the log wealth-consumption ratio has been largely restored to its sample mean with the broad market declines since 2000, and with evidence in Lettau and Ludvigson (2003), which suggests the presence of a low frequency component in the sampling variation of the log dividend-price ratio that is not evident in the wealth-consumption ratio.

Of course, these considerations also imply that some short-term fluctuations in asset market valuation ratios are not as well captured by the model studied here. The focus in this paper is on the low-frequency movements in the stock market and the unconditional equity premium, and in particular in understanding the boom in the 1990s, an episode that dominates the post-war sampling variation of the stock market. As such, the model we investigate is not designed to capture the higher frequency fluctuations observed in the log price-dividend ratio prior to 1990 and displayed in Figure 11. Although both the mean and the general direction of these higher frequency fluctuations is well captured, the magnitude of these cyclical fluctuations is not. For example, the model correctly predicts a decline in the price-dividend ratio during the early 1950s and throughout most of the 1970s, but it misses the magnitude of both declines. One model that is successful at capturing the shorter-term, cyclical fluctuations in equity values and forecastable variation in the equity risk-premium is that explored by Campbell and Cochrane (1999). Nevertheless, the cyclical fluctuations in stock market valuation ratios prior to 1990 are shown to be mere wiggles relative to the extraordinary stock market boom in the 1990s. Thus, we emphasize that the behavior of stock prices at the close of the last century is dominated by extremely low frequency movements in the price-dividend ratio, and such variation is far better captured by the model presented here with shifting macroeconomic volatility than by the model explored by Campbell and Cochrane, which delivers its worst performance in the 1990s and fails to account for the magnitude of the surge in equity values during this period (Figure 16).

4 Conclusions

This paper considers the low-frequency behavior of post-war equity values relative to measures of fundamental value. Such longer-term movements are dominated by the stock market boom of the 1990s, an extraordinary episode in which price-dividend ratios on aggregate stock market indices increased three-fold over a period of five years. Indeed, Figure 1 shows this period to be the defining episode of postwar financial markets. A growing body of literature is now working to understand this phenomenon, and explanations run the gamut from declining costs of equity market participation and diversification, to irrational exuberance, to changes in technology and demography.

In this paper, we consider a different explanation. We ask whether the phenomenal surge in asset values that dominated the close of the 20th century can be plausibly described as a rational response to macroeconomic factors, namely the sharp decline in macroeconomic risk. We find that it can. There is a strong correlation between low macroeconomic volatility and high asset prices in post-war data, both in the US and internationally. We show that, when such a shift toward decreased consumption risk is perceived to be highly persistent, as empirical estimates suggest it is, an otherwise standard asset pricing model can deliver a surge in equity valuation ratios of about the same magnitude observed in U.S. data. In the model economy, a boom in stock prices occurs because the decline in macroeconomic risk leads to a fall in expected future stock returns, or the equity risk-premium. An implication of these findings is that multiples of price to earnings or dividends could remain above previous historical norms into the indefinite future.

Of course, in the final analysis, the complexities of modern financial markets leave little doubt that several factors outside of our model are likely to have contributed to the surge in asset values relative to measures of fundamental value during the final part of the last century. Nevertheless, the analysis here suggests that the well documented break in macroeconomic volatility could be a quantitatively important factor.

5 Appendix

I. Data Description

The sources and description of each data series we use are listed below.

GDP

GDP is gross domestic product, measured in 1996 chain-weighted dollars. Our source is the U.S. Department of Commerce, Bureau of Economic Analysis.

CONSUMPTION

Consumption is measured as either total personal consumption expenditure, or expenditures on nondurables and services, excluding shoes and clothing. The quarterly data are seasonally adjusted at annual rates, in billions of chain- weighted 1996 dollars. For the latter measure, the components are chain-weighted together, and this series is scaled up so that the sample mean matches the sample mean of total personal consumption expenditures. Our source is the U.S. Department of Commerce, Bureau of Economic Analysis.

POPULATION

A measure of population is created by dividing real total disposable income by real per capita disposable income. Consumption, wealth, labor income, and dividends are in per capita terms. Our source is the Bureau of Economic Analysis.

PRICE DEFLATOR

Real asset returns are deflated by the implicit chain-type price deflator (1996=100) given for the consumption measure described above. Our source is the U.S. Department of Commerce, Bureau of Economic Analysis.

PRICE-DIVIDEND RATIO

The price-dividend ratio is that of the CRSP value-weighted index, constructed as in Campbell (2003). Our source is the Center for Research in Security Prices, University of Chicago.

II. Pricing the Consumption and Dividend Claims, Computation of Unconditional Equity Premium

This appendix describes the algorithm used to solve for prices.

Equation (6) can be rewritten as:

$$E_t\left[M_{t+1}\left(\frac{P^D_{t+1}}{C^\lambda_{t+1}}+1\right)\left(\frac{C_{t+1}}{C_t}\right)^\lambda\right]=\frac{P^D_t}{C^\lambda_t}, \tag{10}$$

where M_{t+1} is given in (7). Applying the definition of returns with $\lambda = 1$, M_{t+1} can be rewritten as

$$M_{t+1}=\delta^\alpha\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\alpha}{\psi}+\alpha-1}\left(\frac{P^C_{t+1}}{C_{t+1}}+1\right)^{\alpha-1}\left(\frac{P^C_t}{C_t}\right)^{1-\alpha}. \tag{11}$$

Plugging (11) into (10) we obtain

$$E_t\left[\delta^\alpha\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\alpha}{\psi}+\alpha-1}\left(\frac{P^C_{t+1}}{C_{t+1}}+1\right)^{\alpha-1}\left(\frac{P^C_t}{C_t}\right)^{1-\alpha}\left(\frac{P^D_{t+1}}{C^\lambda_{t+1}}+1\right)\left(\frac{C_{t+1}}{C_t}\right)^\lambda\right]=\frac{P^D_t}{C^\lambda_t} \tag{12}$$

The price-dividend ratio on equity, $\frac{P^D_t}{C^\lambda_t}$, is defined recursively by (12).

We write the price-dividend ratio for a levered consumption claim as a function of the state vector $\hat{\xi}_{t+1|t}$:

$$\frac{P^D_t}{C^\lambda_t}=F_D(\hat{\xi}_{t+1|t}). \tag{13}$$

Similarly, the price-dividend ratio for the unlevered consumption claim can be written

$$\frac{P^C_t}{C_t}=F_C(\hat{\xi}_{t+1|t}), \tag{14}$$

for some function F_C. Notice that the price-dividend ratio in (14) is simply the wealth-consumption ratio, where wealth is defined to be ex-dividend wealth.

The wealth-consumption ratio is defined as the fixed point of (12) for $\lambda = 1$ and $P^D = P^C$ everywhere. Applying this case to (12) and substituting $\frac{P^C_t}{C_t} = F_C(\hat{\xi}_{t+1|t})$, the Euler equation for the consumption claim may be written

$$E_t\left[\delta^\alpha\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\alpha}{\psi}+\alpha}\left(F_C(\hat{\xi}_{t+2|t+1})+1\right)^\alpha\right]=\left(F_C(\hat{\xi}_{t+1|t})\right)^\alpha.$$

From the definition of the conditional expectation, the left-hand-side of the expression above is given by

$$E_t \left[\delta^\alpha \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\alpha}{\psi}+\alpha} \left(F_C(\hat{\xi}_{t+2|t+1}) + 1 \right)^\alpha \right] =$$

$$\sum_{j=1}^{N} P\left\{ s_{t+1} = j \mid \mathbf{Y}_t \right\} E\left[\delta^\alpha \left(\frac{C_{t+1}}{C_t} \right)_j^{-\frac{\alpha}{\psi}+\alpha} \left(F_C(\hat{\xi}_{t+2|t+1}) + 1 \right)^\alpha \mid s_{t+1} = j, \mathbf{Y}_t \right], \quad (15)$$

where $\exp\left[\left(\frac{C_{t+1}}{C_t} \right)_j \right] \equiv \mu(s_j) + \sigma(s_j)\epsilon_{t+1}$ denotes consumption growth in state j. From the evolution equation (5) and the stochastic model (4), the distribution of $\hat{\xi}_{t+2|t+1}$ conditional on time-t data Y_t and on the state s_{t+1} depends only on $\hat{\xi}_{t+1|t}$ and the state. Using the definition of $\hat{\xi}_{t+1|t}$, (15) can be written

$$E_t \left[\delta^\alpha \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\alpha}{\psi}+\alpha} \left(F_C(\hat{\xi}_{t+2|t+1}) + 1 \right)^\alpha \right] =$$

$$\sum_{j=1}^{N} \hat{\xi}_{t+1|t}(j) E\left[\delta^\alpha \left(\frac{C_{t+1}}{C_t} \right)_j^{-\frac{\alpha}{\psi}+\alpha} \left(F_C(\hat{\xi}_{t+2|t+1}) + 1 \right)^\alpha \mid s_{t+1} = j, \hat{\xi}_{t+1|t} \right].$$

Thus, the wealth-consumption ratio, $F_C(\hat{\xi}_{t+1|t})$, is defined by the recursion:

$$\left(F_C(\hat{\xi}_{t+1|t}) \right)^\alpha = \sum_{j=1}^{N} \hat{\xi}_{t+1|t}(j) E\left[\delta^\alpha \left(\frac{C_{t+1}}{C_t} \right)_j^{-\frac{\alpha}{\psi}+\alpha} \left(F_C(\hat{\xi}_{t+2|t+1}) + 1 \right)^\alpha \mid s_{t+1} = j, \hat{\xi}_{t+1|t} \right]. \quad (16)$$

It is straightforward to show that a similar recursion defines the price-dividend ratio of a levered equity claim, $F_D(\hat{\xi}_{t+1|t})$, by allowing λ to take on arbitrary values greater than unity. From (12), we have

$$\frac{P_t^D}{D_t} = F_C(\hat{\xi}_{t+1|t})^{1-\alpha} \delta^\alpha E_t \left[\left(\frac{C_{t+1}}{C_t} \right)_j^{-\frac{\alpha}{\psi}+\alpha-1+\lambda} \left(F_C(\hat{\xi}_{t+2|t+1}) + 1 \right)^{\alpha-1} \left(\frac{P_{t+1}^D}{D_{t+1}} + 1 \right) \right].$$

Substituting (13) into the above, we obtain

$$F_D(\hat{\xi}_{t+1|t}) = F_C(\hat{\xi}_{t+1|t})^{1-\alpha} \delta^\alpha \times$$

$$\sum_{j=1}^{N} \hat{\xi}_{t+1|t}(j) E\left[\left(\frac{C_{t+1}}{C_t} \right)_j^{-\frac{\alpha}{\psi}+\alpha-1+\lambda} \left(F_C(\hat{\xi}_{t+2|t+1}) + 1 \right)^{\alpha-1} \left(F_D(\hat{\xi}_{t+2|t+1}) + 1 \right) \mid s_{t+1} = j, \hat{\xi}_{t+1|t} \right].$$

The expectation above is computed by numerical integration under the assumption that innovations to consumption growth are i.i.d., conditional on the state j.

Denote the log return of the dividend claim from t to $t+1$ as $\log(R_{D,t+1}) = r_{D,t+1}$. It is also possible to compute moments of log returns:

$$E_t[r_{D,t+1}] = E_t\left[\log\left(\frac{F_D(\hat{\xi}_{t+2|t+1})+1}{F_D(\hat{\xi}_{t+1|t})}\left(\frac{D_{t+1}}{D_t}\right)\right)\right] \tag{17}$$

$$\sigma_t^2[r_{D,t+1}] = E_t\left[\left(\log\left(\frac{F_D(\hat{\xi}_{t+2|t+1})+1}{F_D(\hat{\xi}_{t+1|t})}\left(\frac{D_{t+1}}{D_t}\right)\right)\right)^2\right] - \left(E_t\left[r_{D,t+1}\right]\right)^2$$

Equation (17) points the way to calculating the expected M period return. For example, suppose we were interested in the annualized compound rate of return from investing in the levered consumption claim for thirty years. Because the model is calibrated to t equals a quarter, we would compute

$$\frac{4}{120}E_t[r_{D,t+1} + r_{D,t+2} + \cdots + r_{D,t+120}] \tag{18}$$

To compute the thirty-year equity premium, we could subtract out the return from rolling over investments in the risk-free rate. Let $r^f_{t+1} = \log R^f_{t+1}$, then

$$\frac{4}{120}E_t[r^f_{t+1} + r^f_{t+2} + \cdots + r^f_{t+120}] \tag{19}$$

Note that r^f_{t+1} is known at time t and so could be brought outside the expectation. Subtracting (19) from (18) gives the thirty-year ahead risk premium.

The question is how to compute the elements in the sums of (18) and (19)? We show that these quantities can be computed recursively. Because the one-period ahead expected return and risk-free rate are functions of $\hat{\xi}_{t+1|t}$, we can write

$$G_1(\hat{\xi}_{t+1|t}) = E_t(r_{D,t+1})$$

Define

$$G_2(\hat{\xi}_{t+1|t}) = E_t(G_1(\hat{\xi}_{t+2|t+1}))$$

By the law of iterated expectations

$$G_2(\hat{\xi}_{t+1|t}) = E_t(G_1(\hat{\xi}_{t+2|t+1})) = E_t(E_{t+1}(r_{D,t+2})) = E_t(r_{D,t+2})$$

More generally, define G_m recursively as

$$G_m(\hat{\xi}_{t+1|t}) = E_t(G_{m-1}(\hat{\xi}_{t+2|t+1}))$$

Note that assuming $G_{m-1}(\hat{\xi}_{t+1|t}) = E_t[r_{t+m-1}]$ implies,

$$G_m(\hat{\xi}_{t+1|t}) = E_t(G_{m-1}(\hat{\xi}_{t+2|t+1})) = E_t(E_{t+1}(r_{D,t+m})) = E_t(r_{D,t+m}). \tag{20}$$

So by induction, (20) holds for all m.

Similarly define

$$H_1(\hat{\xi}_{t+1|t}) = r^f_{t+1}$$

and

$$H_m(\hat{\xi}_{t+1|t}) = E_t(H_{m-1}(\hat{\xi}_{t+2|t+1}))$$

If $H_{m-1}(\hat{\xi}_{t+1|t}) = E_t[r^f_{t+m-1}]$,

$$H_m(\hat{\xi}_{t+1|t}) = E_t(H_{m-1}(\hat{\xi}_{t+2|t+1})) = E_t(E_{t+1}(r^f_{t+m})) = E_t(r^f_{t+m}). \tag{21}$$

So (21) holds for all m. Thus the M-period ahead expected return can be found by recursively calculating G_m for $m = 1, \ldots M$, summing up, and multiplying by $4/M$. The M-period ahead risk premium can be found by calculating H_m for $m = 1, \ldots M$, summing up the differences $G_m - H_m$, and multiplying by $4/M$.

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Table 1: Standard Deviation of Growth Rates

5-year Window	NDS	PCE	GDP
1952Q1-1956Q4	0.557	0.969	1.198
1957Q1-1961Q4	0.541	0.834	1.421
1962Q1-1966Q4	0.447	0.658	0.716
1967Q1-1971Q4	0.364	0.648	0.867
1972Q1-1976Q4	0.634	1.019	1.145
1977Q1-1981Q4	0.530	0.914	1.271
1982Q1-1986Q4	0.304	0.517	0.932
1987Q1-1991Q4	0.485	0.640	0.621
1992Q1-1996Q4	0.263	0.326	0.439
1997Q1-2001Q4	0.273	0.375	0.578
1952Q1-2001Q4	0.474	0.752	0.968

Notes: This table reports standard deviation of consumption of nondurables and services (NDS), total consumption expenditures (PCE) and GDP for different subsamples. The data are quarterly and span the period from 1952 to 2001.

Table 2: A Markov-Switching Model

x_t	μ_h	μ_l	σ_h^2	σ_l^2	ϕ	p_{hh}^{μ}	p_{ll}^{μ}	p_{hh}^{σ}	p_{ll}^{σ}
NDS	0.536	-0.550	0.233	0.059	0.345	0.986	0.229	0.981	0.962
	(0.042)	(0.348)	(0.061)	(0.013)	(0.074)	(0.028)	(0.335)	(0.016)	(0.034)
PCE	0.722	-0.121	0.513	0.113	0.102	0.955	0.807	0.992	0.984
	(0.062)	(0.204)	(0.076)	(0.037)	(0.090)	(0.026)	(0.101)	(0.009)	(0.025)
GDP	0.890	0.100	1.102	0.211	0.247	0.961	0.738	0.995	0.993
	(0.098)	(0.314)	(0.154)	(0.048)	(0.099)	(0.039)	(0.182)	(0.006)	(0.009)

Notes: This table reports the maximum likelihood estimates of the model

$$\begin{aligned}\Delta x_t &= \mu(S_t) + \phi(\Delta x_{t-1} - \mu(S_{t-1})) + \epsilon_t \\ \epsilon_t &\sim N(0, \sigma^2(V_t)).\end{aligned}$$

We allow for two mean states and two volatility states. μ_h denotes the growth rate in the high mean state, while μ_l denotes the growth rate in the low mean state. σ_h^2 denotes the variance of the shock in the high volatility state and σ_l^2 denotes the variance of the shock in the low volatility state. S_t and V_t are latent variables that are assumed to follow independent Markov chains. The probabilities of transiting to next period's state j given today's state i are p_{ij}^{μ} and p_{ij}^{σ}, respectively. The model is estimated for the growth rates of consumption of nondurables and services (NDS), total consumption expenditures (PCE) and GDP, respectively. Standard errors are in parentheses. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2001.

Table 3: Regressions of the log Price-Dividend Ratio on State Probabilities

Probabilities	α	β_μ	β_σ	R^2
NDS, smoothed	3.18	0.13	0.50	0.32
	(15.82)	(0.61)	(9.47)	
NDS, unsmoothed	2.96	0.31	0.63	0.35
	(14.04)	(1.44)	(10.23)	
PCE, smoothed	3.13	0.23	0.72	0.68
	(69.79)	(4.25)	(17.96)	
PCE, unsmoothed	3.13	0.19	0.87	0.67
	(56.81)	(2.92)	(18.01)	
GDP, smoothed	3.06	0.28	0.44	0.35
	(27.05)	(2.18)	(10.14)	
GDP, unsmoothed	2.93	0.40	0.52	0.42
	(25.44)	(3.05)	(11.56)	

Notes: This table reports results from the regression

$$p_t - d_t = \alpha + \beta_\mu \, P(\mu_t = \mu_h) + \beta_\sigma \, P(\sigma_t^2 = \sigma_l^2) + e_t$$

The state probabilities P(.) for the growth rates of consumption of nondurables and services (NDS), total consumption expenditures (PCE) and GDP are estimated using the Hamilton Markov-switching model reported in Table 2. t-statistics are in parentheses. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2001.

Table 4: Regressions of the Price-Dividend Ratio on Probabilities: A Monte Carlo Simulation

Percentiles	β_μ	β_σ	$\bar{R}^2$
	NDS		
90%	0.26	0.34	0.15
95%	0.35	0.44	0.21
99%	0.54	0.63	0.34
	PCE		
90%	0.18	0.49	0.26
95%	0.24	0.62	0.35
99%	0.35	0.88	0.51
	GDP		
90%	0.17	0.61	0.34
95%	0.28	0.77	0.45
99%	0.34	1.04	0.62

Notes: This table reports results of a Monte Carlo simulation of the small sample properties of the estimators reported in Table 4. We simulate an AR(1) model for the log P/D ratio and Markov chains for the state probabilities. The parameters of the AR(1) are set to the sample estimates. The innovations are drawn from a normal distribution. The state probabilities are approximated by two independent Markov chains. In each period the Markov chains have a value of 0 or 1. The transition probabilities are set to the estimated values from the Hamilton model. We run 50,000 regressions of simulated log P/D ratios on the simulated state probabilities and save the parameter estimates as well as the $\bar{R}^2$'s. The table reports the 90, 95 and 99 percentiles of these three statistics.

Table 5: Effects of State Probabilities on the P/D Ratio

	fitted $exp(p-d)$ when			
	$P(\mu_t = \mu_h) = 0$ $P(\sigma_t^2 = \sigma_l^2) = 0$	$P(\mu_t = \mu_h) = 1$ $P(\sigma_t^2 = \sigma_l^2) = 0$	$P(\mu_t = \mu_h) = 0$ $P(\sigma_t^2 = \sigma_l^2) = 1$	$P(\mu_t = \mu_h) = 1$ $P(\sigma_t^2 = \sigma_l^2) = 1$
NDS, smoothed	24.17	27.41	39.92	45.28
NDS, unsmoothed	19.32	26.37	36.25	49.47
PCE, smoothed	23.03	29.05	47.76	60.23
PCE, unsmoothed	23.08	28.03	55.34	67.21
GDP, smoothed	21.48	28.32	33.49	44.15
GDP, unsmoothed	18.90	28.05	31.84	47.27

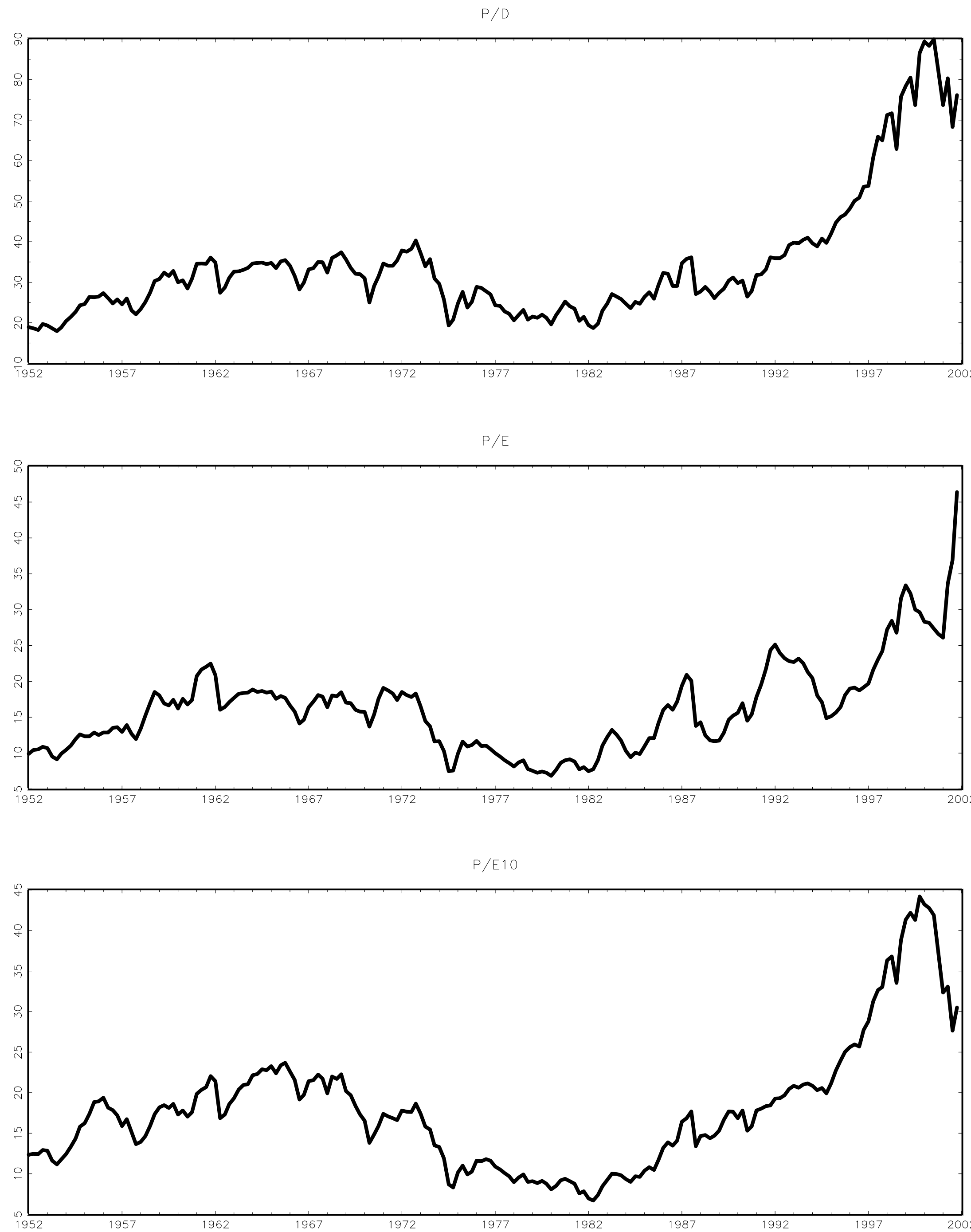
Notes: This table reports fitted values from the regressions reported in Table 3. The state probabilities P(.) are estimated using the Hamilton Markov-switching model reported in Table 2. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2001.

Table 6: Model Implications in 1990Q1 and 2001Q4

λ	δ	$P^{\sigma}_{j,j}$	Prob.	P/D_{90}	P/D_{01}	$r^p_{90}(100)$	$r^p_{01}(100)$	$r^f_{90}(100)$	$r^f_{01}(100)$	$r^f_{90}(1)$	$r^f_{01}(1)$
5	0.9950	0.99990	u	30.76	64.48	12.73	11.15	1.65	2.04	1.13	-0.67
			s	28.95	64.11	12.94	11.16	1.56	2.04	0.77	-0.59
5	0.9950	0.99999	u	30.80	81.25	12.71	10.77	1.65	2.08	1.12	-1.31
			s	29.00	80.48	12.94	10.79	1.55	2.08	0.77	-1.25
3	0.9950	0.99990	u	39.20	58.30	7.57	6.50	1.65	2.04	1.13	-0.67
			s	37.86	58.11	7.73	6.50	1.56	2.04	0.77	-0.59
3	0.9925	0.99990	u	28.72	39.72	7.40	6.16	2.66	3.06	2.15	1.70
			s	27.71	39.69	7.65	6.16	2.57	3.06	1.77	1.81

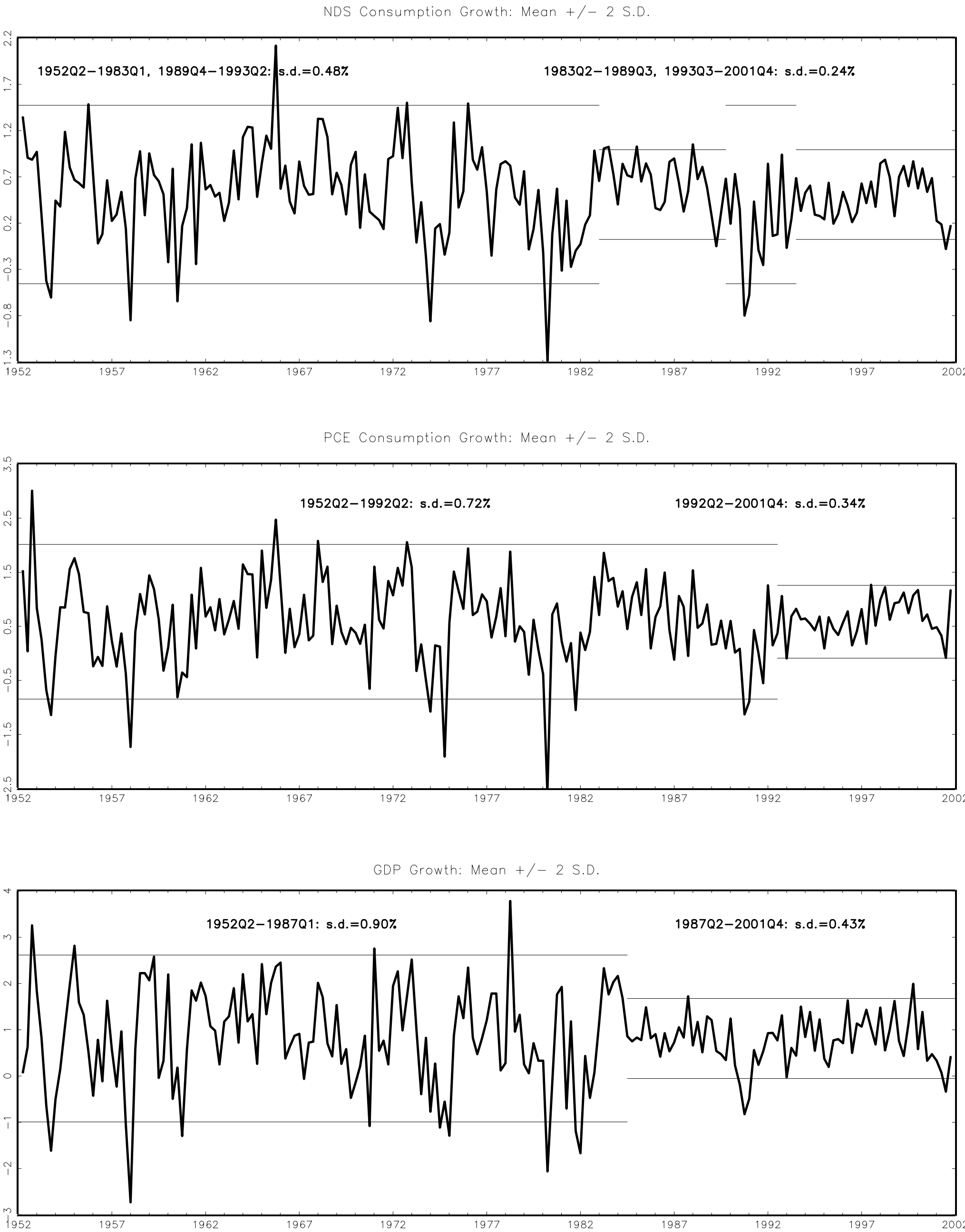
Notes: This table reports the model implications for asset prices using the estimated state probabilities in 1990:Q1 and 2001:Q4. λ is the leverage factor, δ is the discount rate and $P^{\sigma}_{j,j}$ is the probability that next period is a volatility state j given that today's state is volatility state j, $j \in \{l, h\}$. Risk aversion is set to 40 and the elasticity of intertemporal substitution is set to 2 for all cases. Asset prices are reported for unsmoothed ("u") and smoothed ("s") probabilities. $P/D, r^p(100), r^f(100)$ and $r^f(1)$ are the price-dividend ratio, the 100-year risk premium, the 100-year risk-free rate, and the 1-period risk-free rate, respectively. All returns are annualized in percent. The variables with subscript "90" ("01") report the model's predictions using historical state probabilities in 1990:Q1(2001:Q4).

Figure 1: Price Ratios



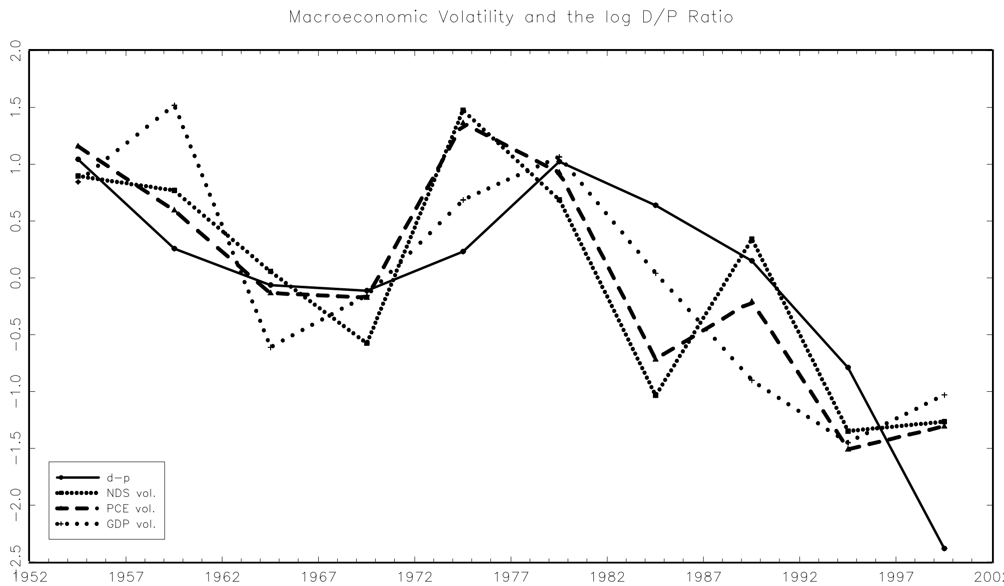
Notes: This figure plots the CRSP-VW price-dividend ratio in the top panel, the S&P500 price-earnings ratio where earnings are a one-year lagged moving average in the middle panel and the S&P500 price-earnings ratio where earnings are a 10-year lagged moving average in the bottom panel. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2001.

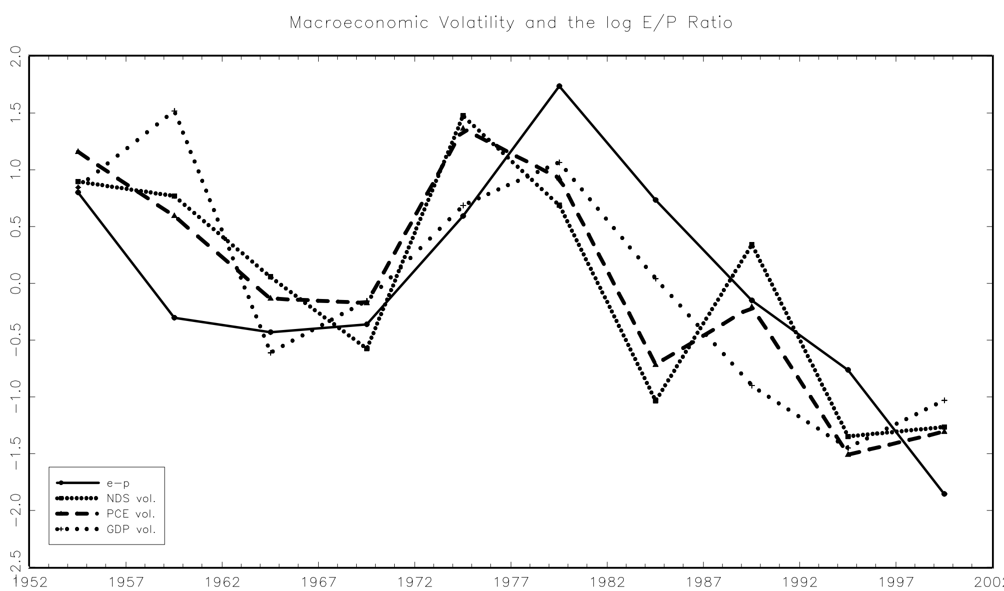
Figure 2: Growth Rates



Notes: This figure shows the growth rates for consumption of nondurables and services (NDS) in the top panel, the growth rates for personal consumption expenditures (PCE) in the middle panel and the growth rate of GDP in the bottom panel. The lines in the plot correspond to the volatility regimes estimated from the Hamilton regime switching model. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2001.

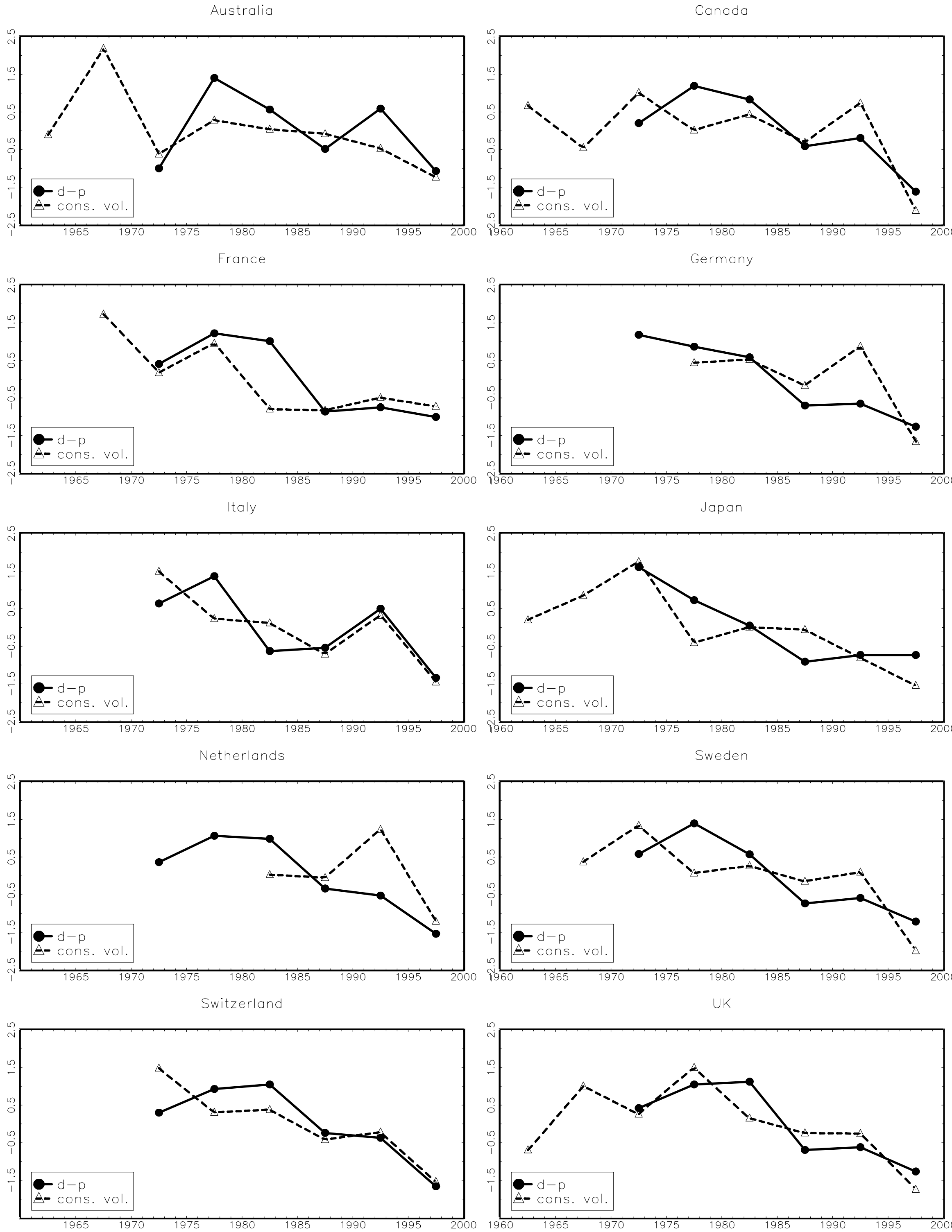
Figure 3: 5-Year Volatility Estimates and the log D/P Ratio





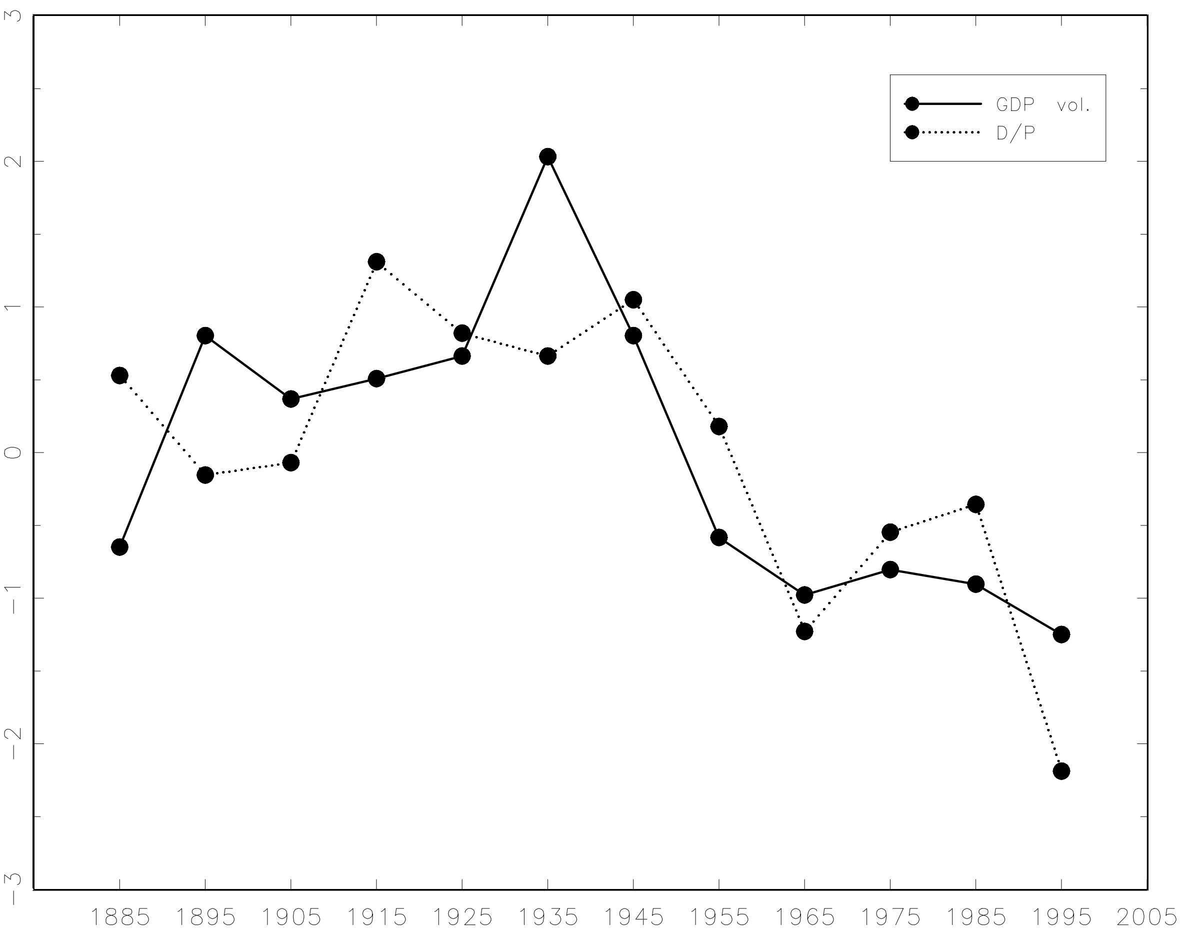
Notes: This figure plots the standard deviation of NDS growth, PCE growth, GDP growth as well as the average CRSP-VW log dividend-price ratio in 5-year windows. All series are demeaned and divided by their standard deviation. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2001.

Figure 4: 5-Year Volatility Estimates and the log D/P Ratio – International Evidence



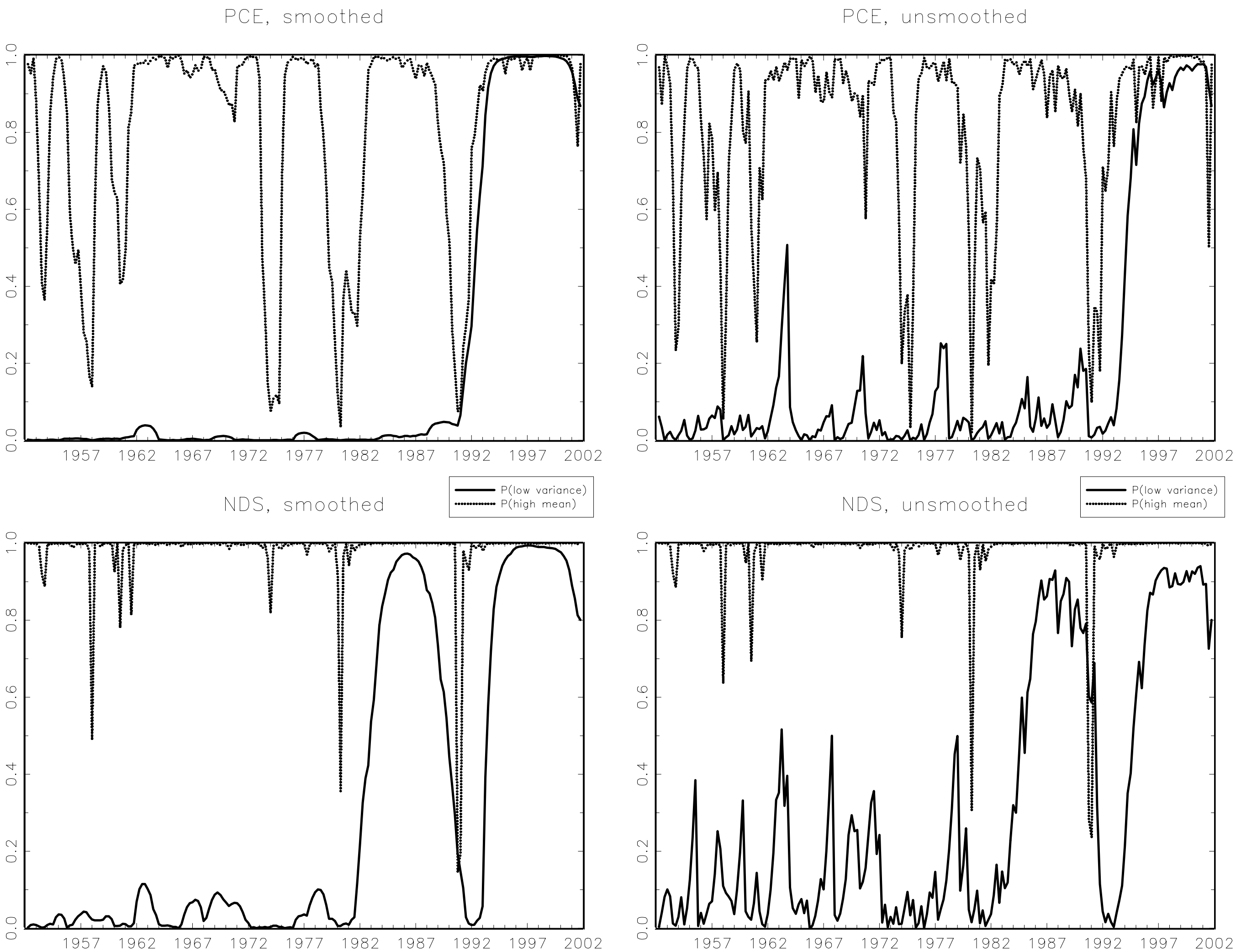
Notes: This figure plots the standard deviation of consumption growth and the average log dividend-price ratio in 5-year windows for ten countries. The plots in each each panel use the longest available data in each country. The data are from Campbell (2003).

Figure 5: GDP volatility and the D/P Ratio - Pre-war Evidence



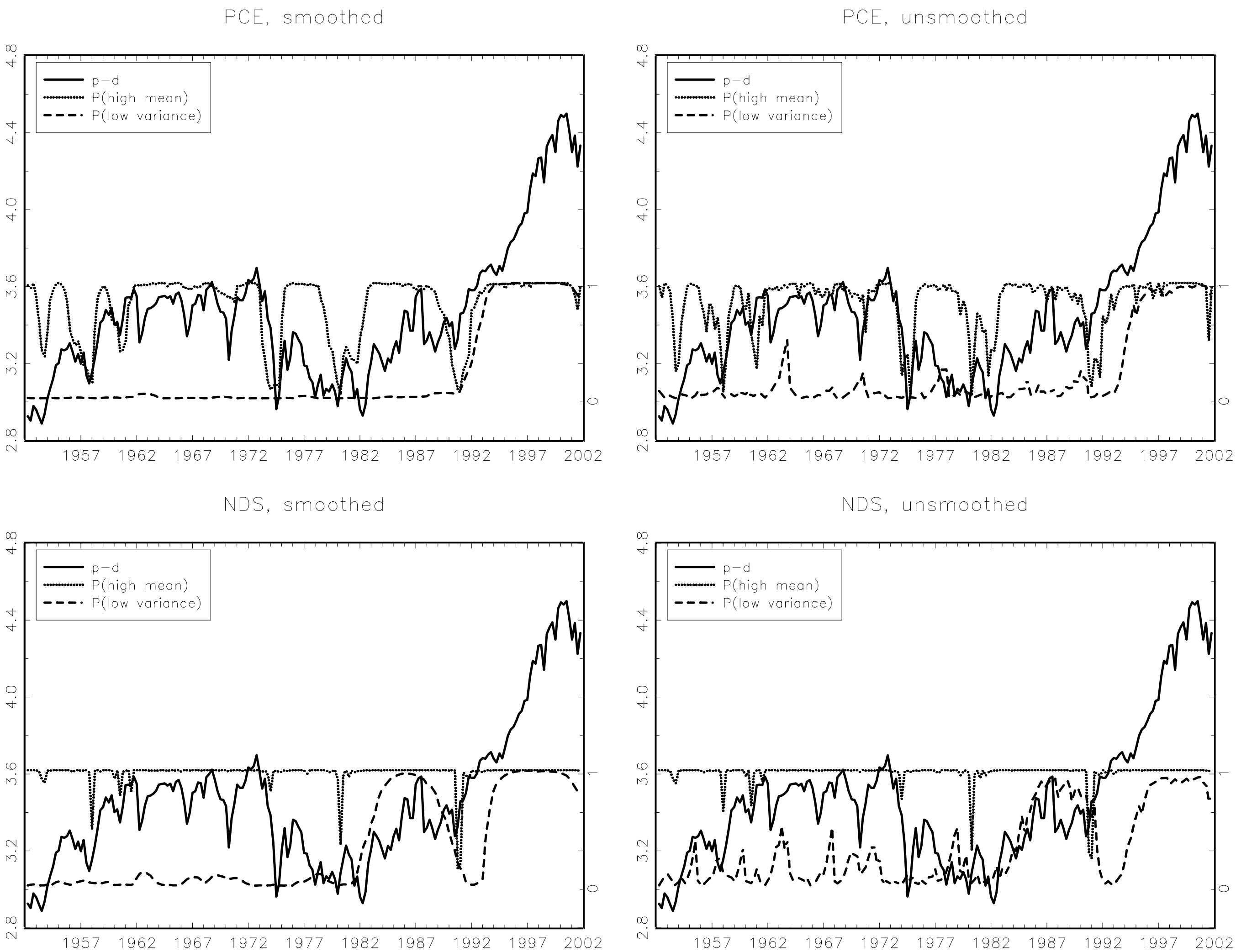
Notes: This figure plots the standard deviations of GDP growth and the mean D/P ratio by decade starting in 1880 until 2000. Both series are demeaned and divided by their standard deviation. The GDP data is from Ray Fair's website (http://fairmodel.econ.yale.edu/RAYFAIR/PDF/2002DTBL.HTM) based on Balke and Gordon (1989). The dividend yield data is from Robert Shiller's website (http://aida.econ.yale.edu/∼shiller/data/ie_data.htm).

Figure 6: State Probabilities



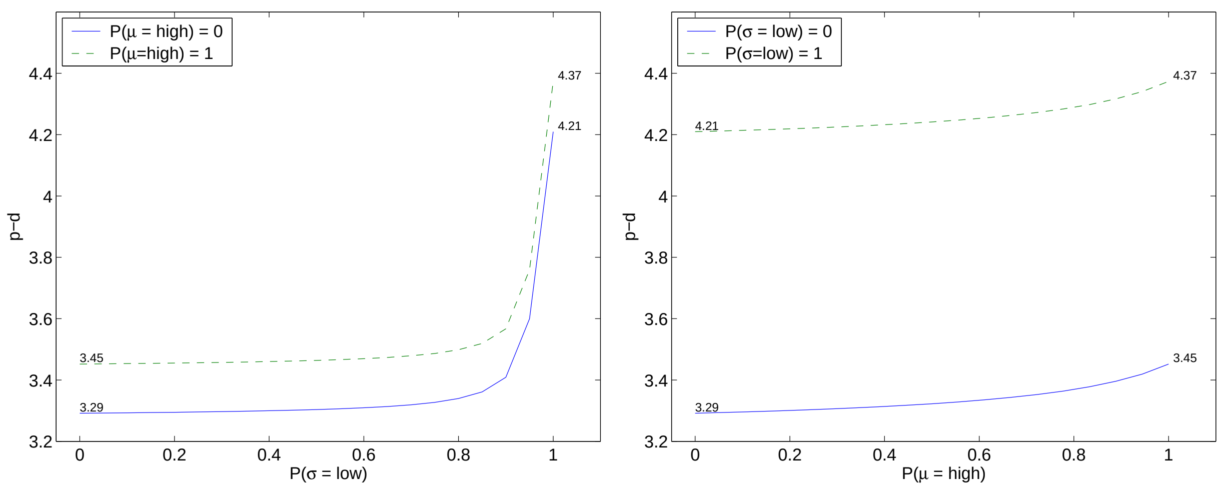
Notes: This figure plots the time series of estimated state probabilities. P(low variance) is the unconditional probability of being in a low consumption volatility state next period, calculated be summing the probability of being in a low volatility state and high mean state, and the probability of being in a low volatility state and low mean state. P(high mean) is calculated analogously. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2001.

Figure 7: State Probabilities and the log P/D Ratio



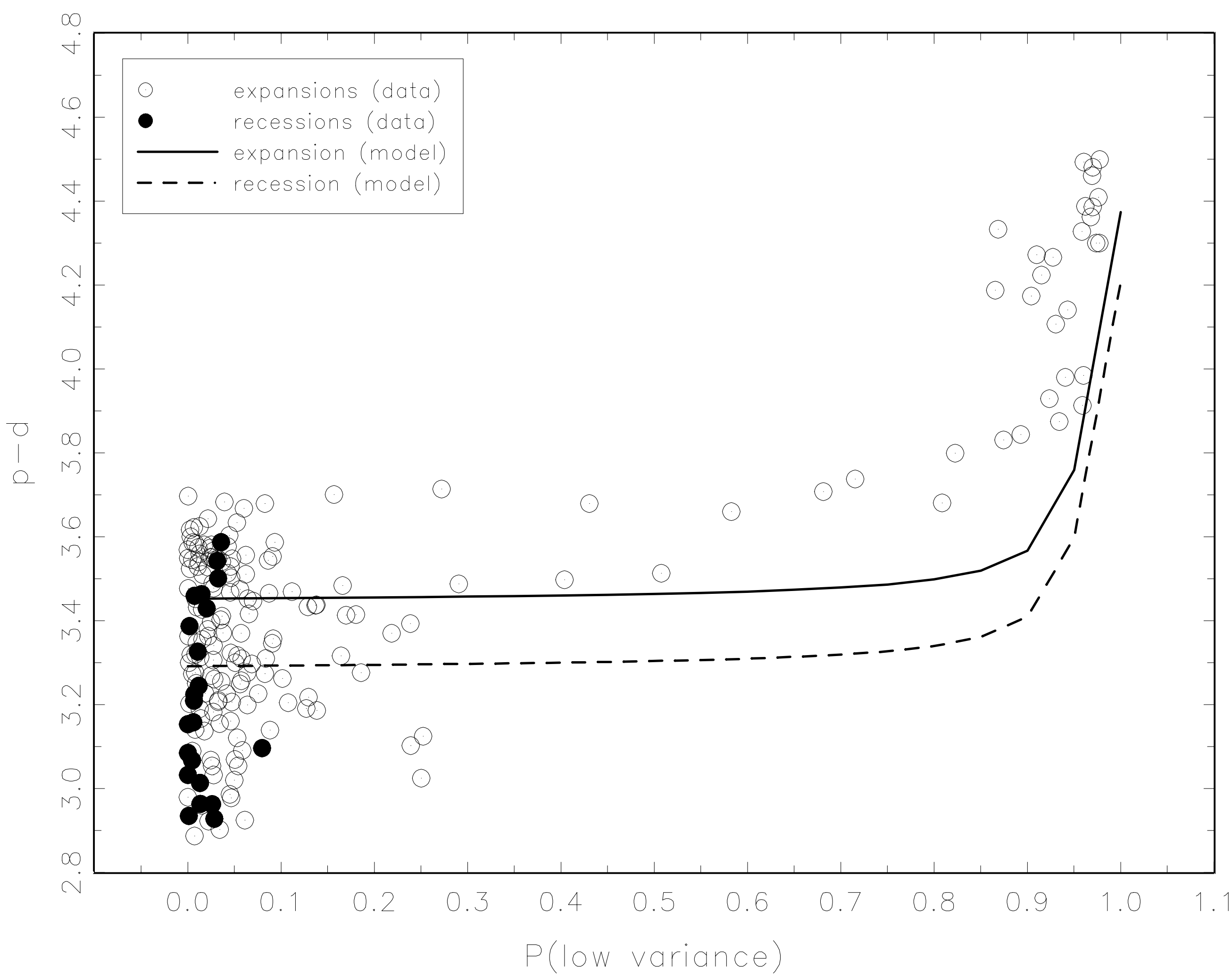
Notes: This figure shows the CRSP-VW log price-dividend ratio and the smoothed regime probabilities for PCE growth in the top left panel and the unsmoothed regime probabilities in the top left panel. The bottom panels show plots of the regime probabilities for NDS growth along with the log price-dividend ratio. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2001.

Figure 8: The Price-Dividend Ratio



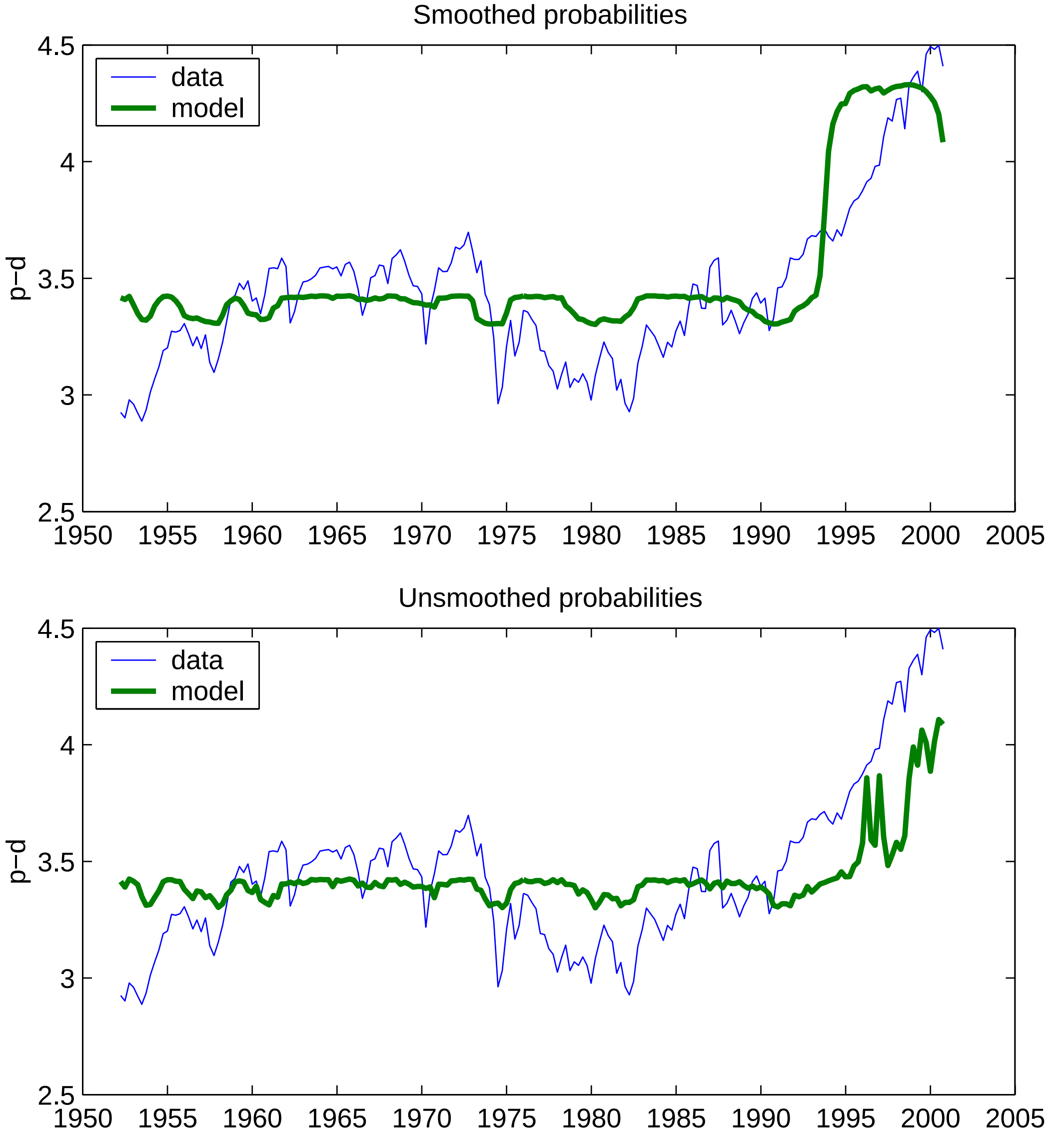
Notes: The price-dividend ratio as a function of the probability that consumption volatility is low (left panel) and the probability that consumption mean is high (right panel). In the left panel, the probability that the consumption mean is high is set to be zero (solid line) or one (dashed line). In the right panel, the probability that consumption volatility is low is set to be zero (solid line) and one (dashed line). The probability of a change in the consumption volatility state is assumed to be .0001; otherwise the parameters of the endowment process are set equal to their maximum likelihood estimates. The rate of time preference $\delta = .995$, the elasticity of intertemporal substitution, $\psi = 1.5$, risk aversion $\gamma = 40$ and leverage $\lambda = 5$.

Figure 9: The Price-Dividend Ratio: Data and Model



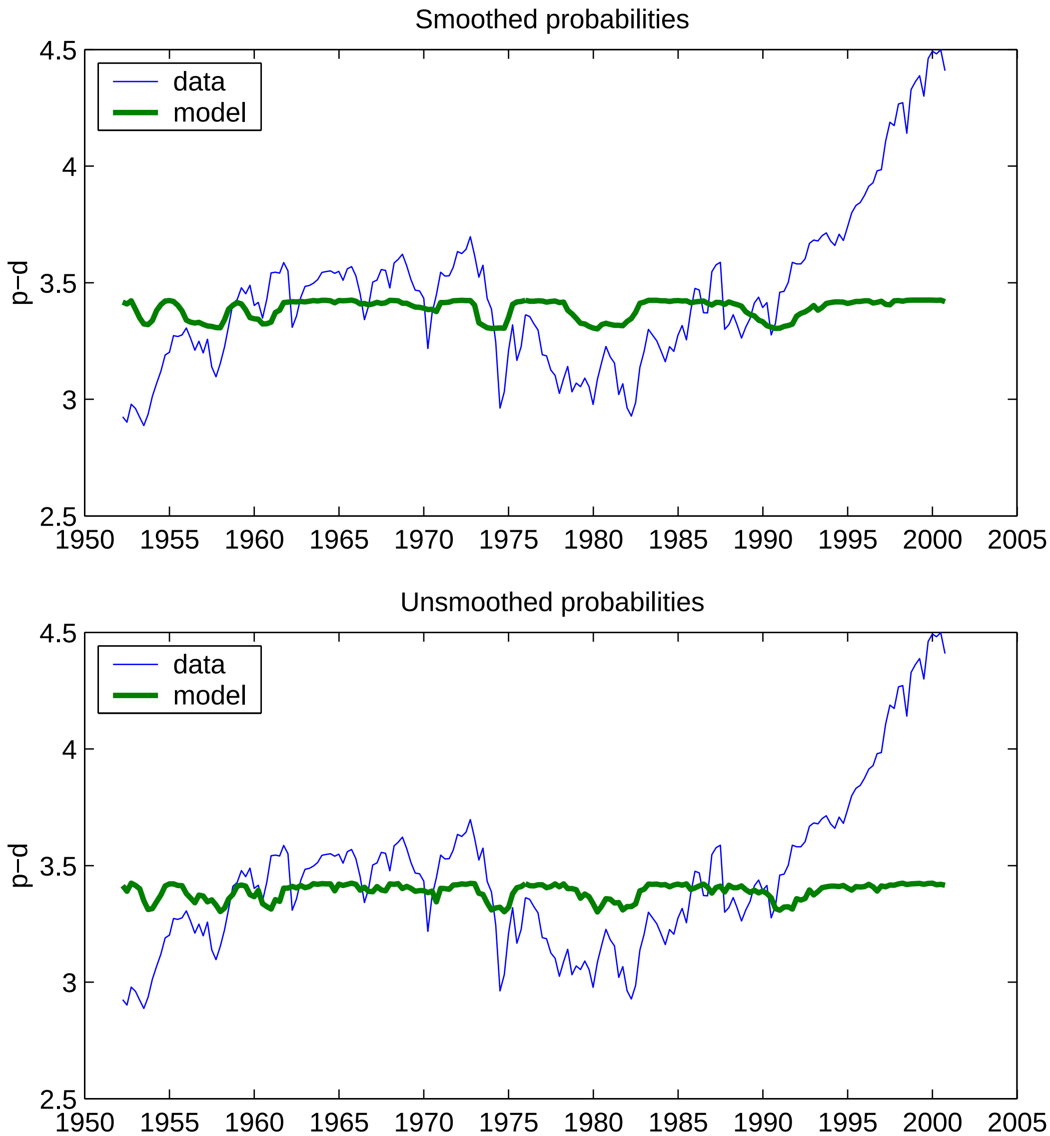
Notes: This plot shows the relationship of the probability that consumption volatility is low and the log price-dividend ratio in the model and in the data. The two lines are those plotted in Figure and show the price-dividend ratio as a function of the probability that consumption volatility is low as implied by the model. The solid line corresponds to $P(\mu = \text{high}) = 1$ and the dashed line corresponds to $P(\mu = \text{high}) = 0$. The dots represent the data estimates from the Hamilton procedure. Expansion periods, defined as periods in which the probability of high growth exceeds one half, are indicated as circles. Recession periods, defined as periods in which the probability of high growth is less than one half, are indicated as solid dots. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2001.

Figure 10: Time Series of the P/D Ratio



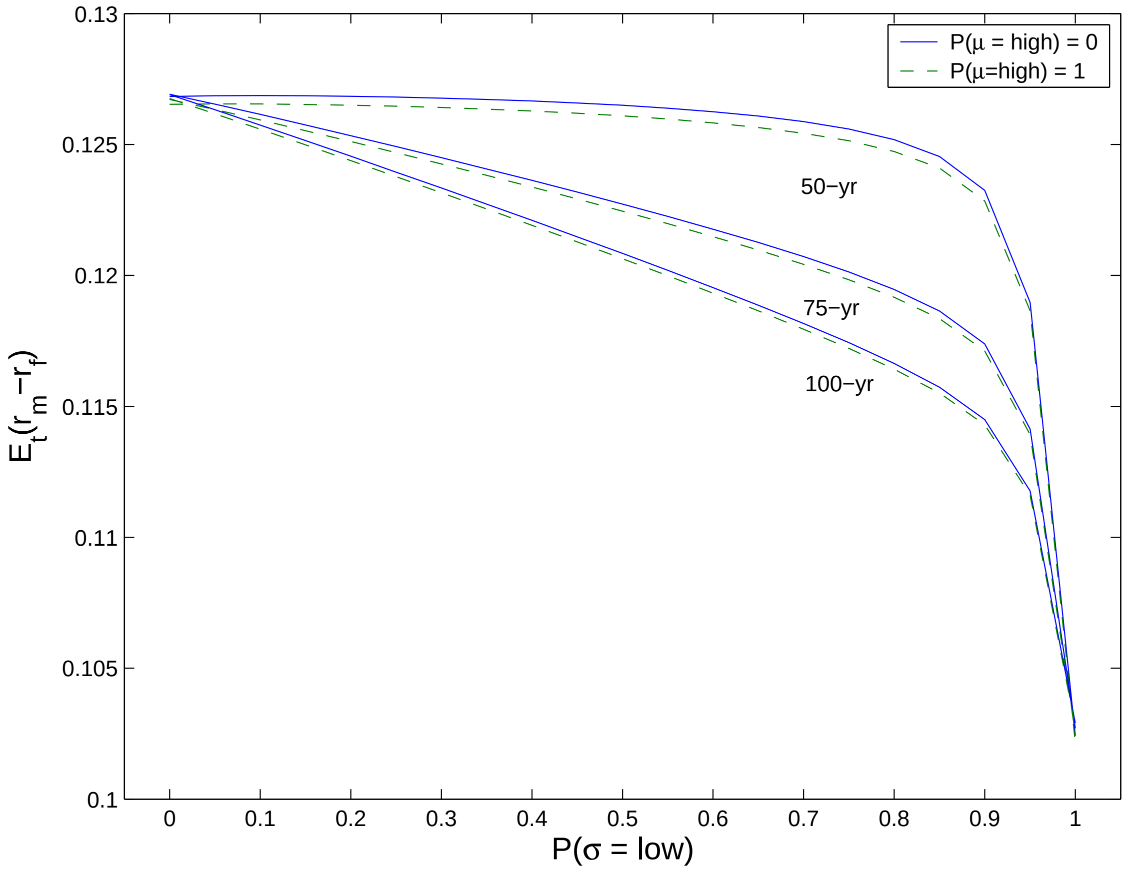
Notes: Time series of the log price-dividend ratio from the data and implied by the model. The top figure assumes the representative agent calculates the smoothed probabilities, the bottom figure assumes the representative agent calculates unsmoothed probabilities. The probability of a change in the consumption volatility state is assumed to be .0001; otherwise the parameters of the endowment process are set equal to their maximum likelihood estimates. The rate of time preference $\delta = .995$, the elasticity of intertemporal substitution, $\psi = 1.5$, risk aversion $\gamma = 40$ and leverage $\lambda = 5$. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2001.

Figure 11: Counterfactual: P/D without Volatility Shift



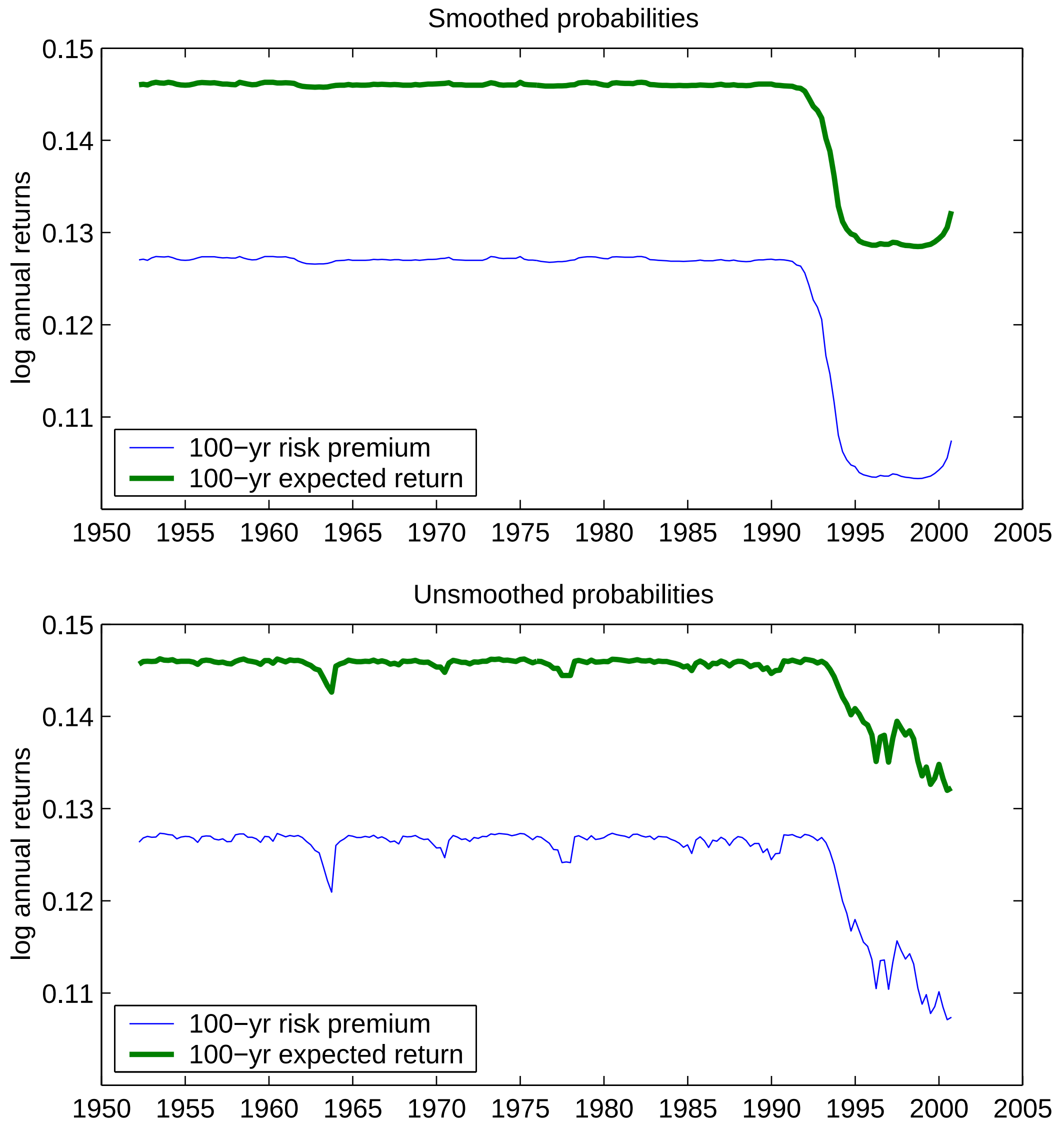
Notes: This plot shows the time series of the log P/D ratio under the counterfactual assumption that the probabilities of the high volatility states for the quarters following 1990Q1 are fixed at their estimates in 1990Q1 (the unsmoothed probability of the high volatility state is 76%, the smoothed probability is 93%.) The model parameters are the same as in Figure 10. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2001.

Figure 12: The Long-run Equity Premium



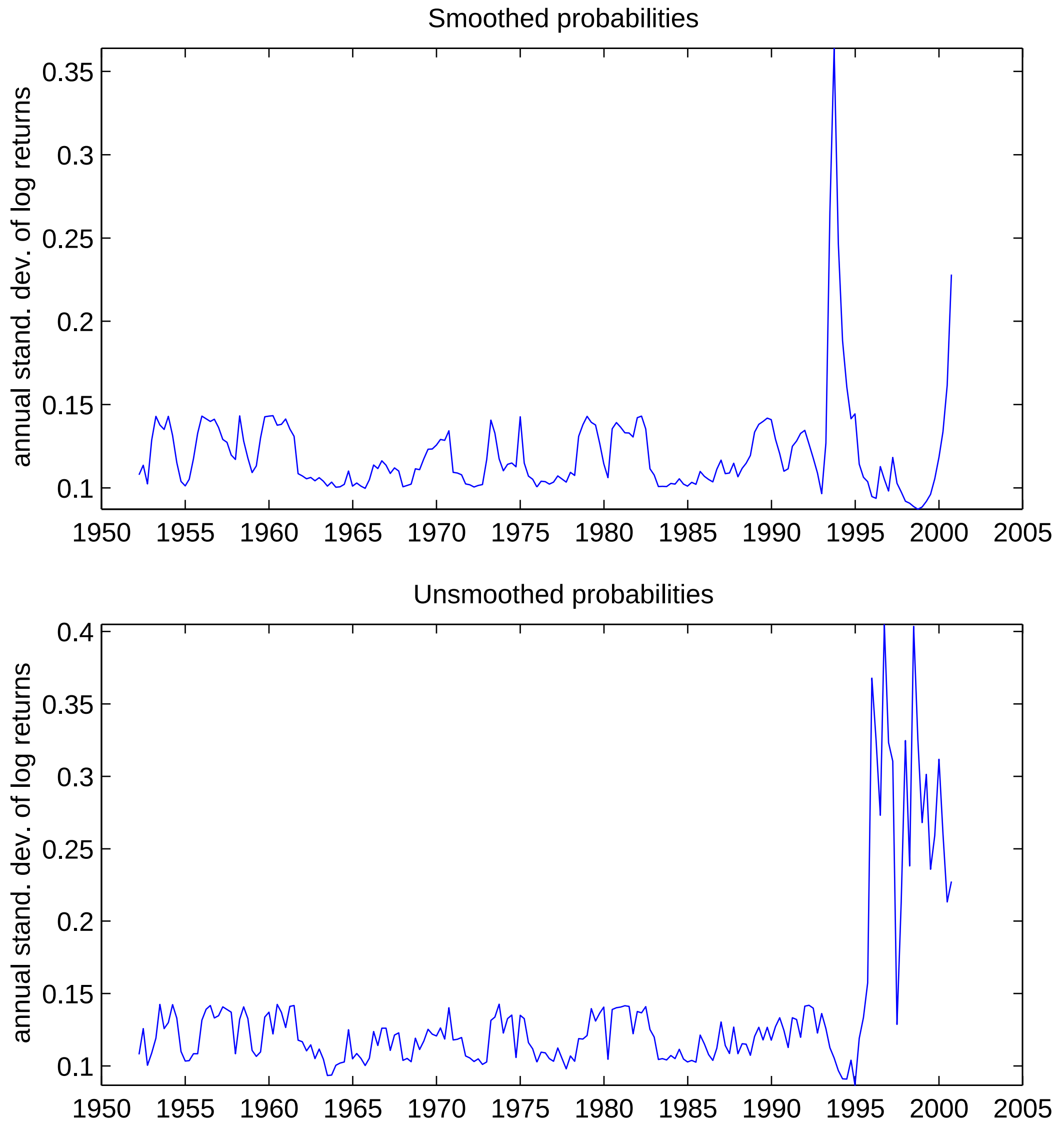
Notes: The conditional long-run equity premium as a function of the probability that consumption volatility is low. The probability that the consumption mean is high is set to be zero (solid line) or one (dashed line). The means is reported in annual terms. The probability of a change in the consumption volatility state is assumed to be .0001; otherwise the parameters of the endowment process are set equal to their maximum likelihood estimates. The rate of time preference $\delta = .995$, the elasticity of intertemporal substitution, $\psi = 1.5$, risk aversion $\gamma = 40$ and leverage $\lambda = 5$.

Figure 13: Time Series of the Long-run Expected Return and Equity Premium



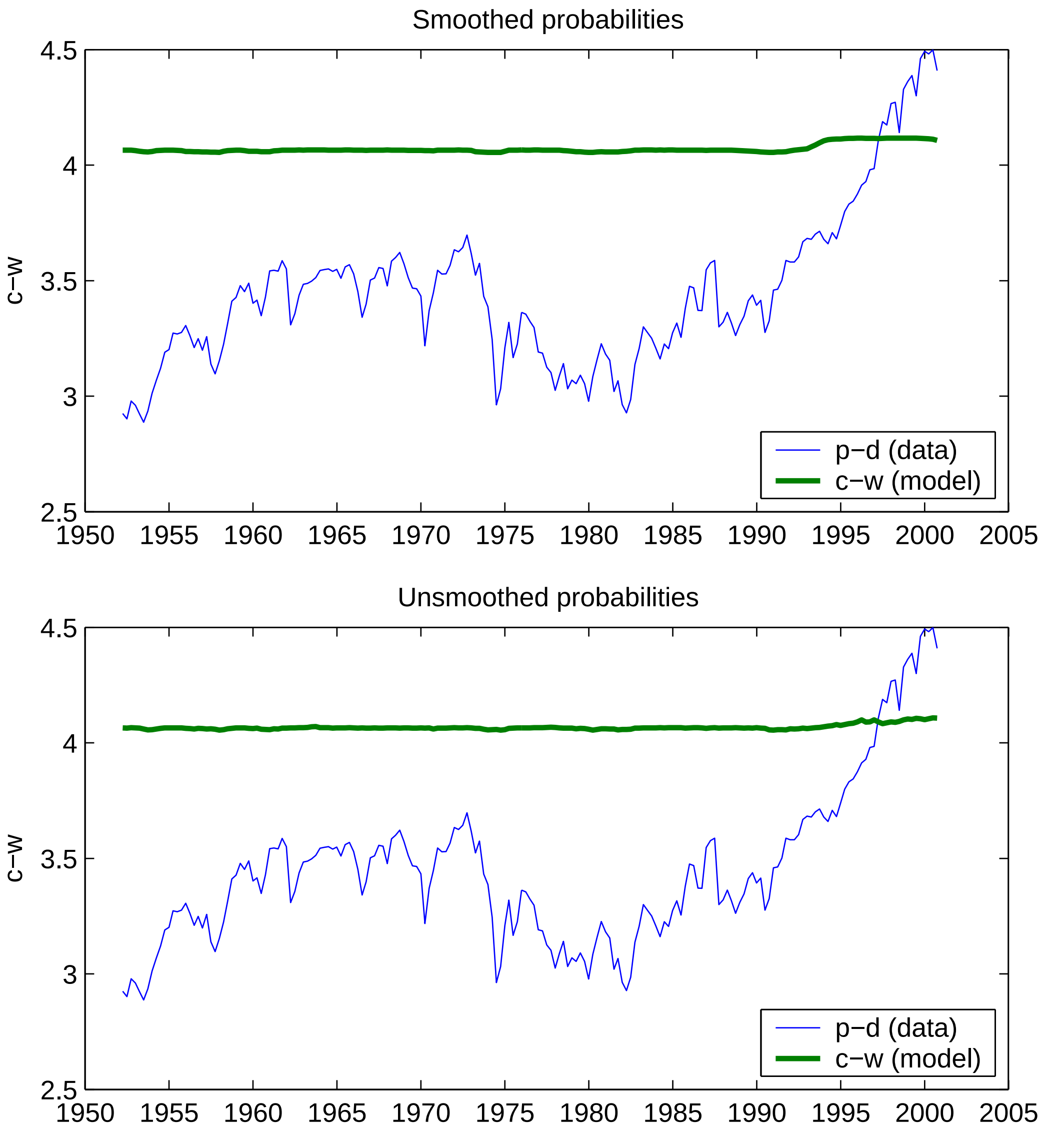
Notes: Time series of the 100-year expected equity return and equity premium implied by the model. The top panel assumes the representative agent calculates the smoothed probabilities, the bottom figure assumes the representative agent calculates unsmoothed probabilities. The probability of a change in the consumption volatility state is assumed to be .0001; otherwise the parameters of the endowment process are set equal to their maximum likelihood estimates. The rate of time preference $\delta = .995$, the elasticity of intertemporal substitution, $\psi = 1.5$, risk aversion $\gamma = 40$ and leverage $\lambda = 5$. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2001.

Figure 14: Time series of volatility



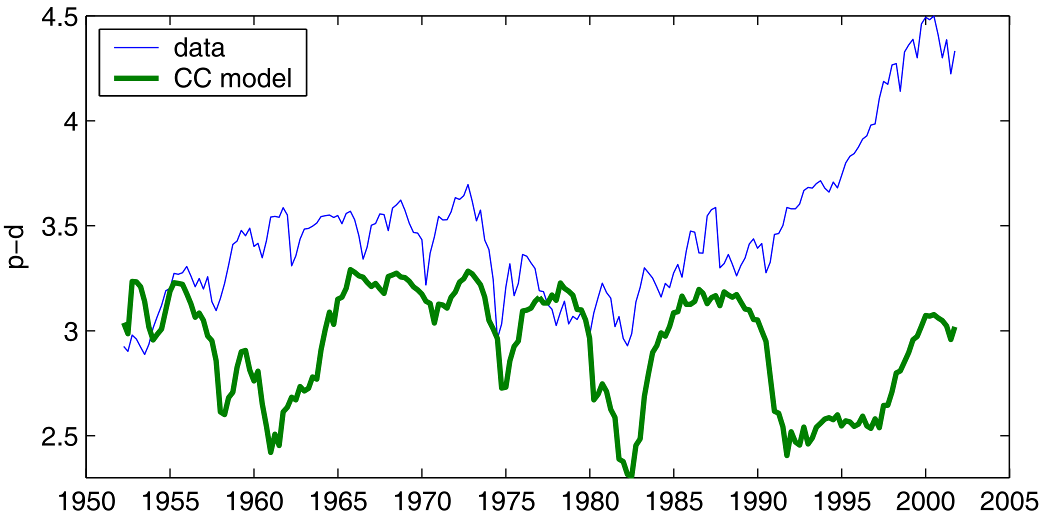
Notes: Time series of conditional volatility of one-quarter ahead equity returns. Volatility is annualized, i.e. $2\sigma_t$. The top panel assumes the representative agent calculates the smoothed probabilities, the bottom figure assumes the representative agent calculates unsmoothed probabilities. The probability of a change in the consumption volatility state is assumed to be .0001; otherwise the parameters of the endowment process are set equal to their maximum likelihood estimates. The rate of time preference $\delta = .995$, the elasticity of intertemporal substitution, $\psi = 1.5$, risk aversion $\gamma = 40$ and leverage $\lambda = 5$. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2001.

Figure 15: The Wealth-Consumption Ratio



Notes: Time series of the log price-dividend ratio from the data and the consumption-wealth ratio implied by the model. The top figure assumes the representative agent calculates the smoothed probabilities, the bottom figure assumes the representative agent calculates unsmoothed probabilities. The probability of a change in the consumption volatility state is assumed to be .0001; otherwise the parameters of the endowment process are set equal to their maximum likelihood estimates. The rate of time preference $\delta = .995$, the elasticity of intertemporal substitution, $\psi = 1.5$, risk aversion $\gamma = 40$. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2001.

Figure 16: The P/D-Ratio in the Campbell-Cochrane Model



Notes: Time series of the log price-dividend ratio from the data and implied by the model of Campbell and Cochrane (1999). For the implied price-dividend ratio, quarterly PCE data are used to calculate an implied series of surplus consumption. The parameters of the model are the same as those used by Campbell and Cochrane, except that the mean and standard deviation of consumption growth are changed to match the mean and standard deviation of quarterly log PCE growth. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2001.