

Preference for early resolution and commitment: A simple case

KENJI MIYAZAKI
HOSEI UNIVERSITY

MAKOTO SAITO*†
HITOTSUBASHI UNIVERSITY

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ABSTRACT. Exploring the result of Epstein (1980) under more general utility or Kreps-Porteus preference, this paper demonstrates that not only strong intertemporal substitution, but also preference for early resolution makes consumers postpone commitment to non-durable consumption and hold more liquid assets. It also discusses a potential advantage of Kreps-Porteus preference in examining the interaction between asset pricing and liquidity demand.

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* Correspondence to: Makoto Saito, Faculty of Economics, Hitotsubashi University, 2-1 Naka, Kunitachi, Tokyo, Japan, 186-8601, E-mail: makoto@econ.hit-u.ac.jp, phone: 81-42-580-8807, fax: 81-42-580-8882.

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1. Introduction Expenditures on consumption and investment require commitment of agents to the extent that such expenditures are sunk or irreversible. Non-durable consumption is completely irreversible by nature, while even durable consumption is partially sunk due to imperfect secondary markets. Investment, physical and financial, is also sunk because of either wide ranges of transaction costs or missing markets.

Several theoretical researches demonstrate that agents tend to postpone their commitment to current expenditures by carrying liquid assets when uncertainty is expected to be resolved to some extent. In other words, agents make a commitment only after uncertainty is resolved at least partially. Jones and Ostroy (1984), for example, show that liquidity demand depends on resolution of uncertainty when liquidating investment incurs transaction costs. In particular, Epstein (1980) presents a theoretical framework to investigate the interaction between resolution of uncertainty and the timing of expenditures.

As Epstein (1980) proves, a degree of intertemporal substitution is a key preference parameter in determining the timing of commitment. As the motive of intertemporal substitution is more dominant, an incentive to postpone commitment is stronger. A major reason for this consequence is that consumers with high intertemporal substitution are more interested in allocating resources to more appropriate timing.

Another natural candidate among preference parameters is preference for early resolution of uncertainty. Strong preference for early resolution may work in favor of postponement of commitment. Most existing researches, however, adopt the expected utility framework where a consumer's preference itself is completely neutral with respect to the timing of resolution; therefore, they cannot address such an issue in a proper manner.

This paper employs a class of non-expected utility where consumers are no longer neutral on the timing of resolution; this preference function was originally presented by Kreps and Porteus (1978), and had been developed by Epstein and Zin (1989), Weil (1990), and others. Then, this paper investigates the interaction among decision about commitment, the resolution of uncertainty, and preference for early resolution. In fact, we will show that strong preference for early resolution promotes a postponement of commitment together with high degrees of intertemporal substitution.

2. Model Our model basically follows the model constructed in Section 6 of Epstein (1980) except for a preference setup. Epstein (1980) examines the rational choice between non-durable consumption (requiring perfect commitment) and liquid assets (free of both risks and transaction costs) in expectation that uncertainty will resolve to some extent.

A consumer endowed with wealth w_1 in period 1 plans to allocate consumption goods among three periods. Investment in period 1 yields a fixed return r per period, while investment in period 2 yields a random return Z per period which takes positive values (z_1, \dots, z_n) with probability $p^T = (p_1, \dots, p_n)$ where $p_i = \Pr(Z = z_i)$. In period 2, he receives a message Y which takes (y_1, \dots, y_m) with probability $q^T = (q_1, \dots, q_m)$ where $q_j = \Pr(Y = y_j)$. This message Y is correlated with the period-3 realization of Z . The posterior probability distribution is denoted by $\Pi = (\pi_{ij})$ where $\pi_{ij} = \Pr(Z = z_i | Y = y_j)$. By construction, $\Pi q = p$ obtains.

The consumer maximizes the following objective function characterized by Kreps-Porteus preference:

$$\left[(w_1 - x_1)^\rho + \beta (rx_1 - x_2)^\rho + \beta^2 \sum_j q_j \left\{ \sum_i \pi_{ij} (x_2 z_i)^\alpha \right\} \frac{\rho}{\alpha} \right]^{\frac{1}{\rho}},$$

where x_1 ($0 \leq x_1 \leq w_1$) and x_2 ($0 \leq x_2 \leq rx_1$) denote savings in periods 1 and 2 respectively, β (> 0) is a discount factor, and $\rho < 1$ and $\alpha < 1$ are preference parameters. An elasticity of intertemporal substitution is defined as $\sigma = 1/(1 - \rho)$ (> 0), while a degree of relative risk aversion between period 2 and 3 is defined as $\gamma = 1 - \alpha$ (> 0). One advantage of this functional form is that a preference for early (late) resolution can be controlled. More concretely, if $\alpha < \rho$ or $\sigma\gamma > 1$, then early resolution of uncertainty is preferred, and vice versa. When $\alpha = \rho$ or $\sigma\gamma = 1$, the above setup reduces to the case of expected utility explored by Epstein (1980).

A degree of resolution of uncertainty is defined as below. The following definition is originally proposed by Marschak and Miyasawa (1968), and subsequently discussed by Epstein (1980), Jones and Ostroy (1984), and others. Consider another message Y' which takes (y'_1, \dots, y'_m) with probability $q'^T = (q'_1, \dots, q'_m)$ where $q'_j = \Pr(Y' = y'_j)$. Π' is defined such that the prior distribution is fixed or $\Pi'q' = p$. The message Y' is called ‘more

informative' than Y if

$$\sum q'_j \Phi(\pi'_j) \geq \sum q_j \Phi(\pi_j) \quad (1)$$

for any convex function Φ , where π_j and π'_j are the j th columns of Π and Π' respectively. Obviously, the reverse inequality of (1) holds if Φ is concave.

Solving the above maximization problem backward, the first order condition with respect to x_2 leads to

$$x_2 = rx_1(1 + \beta^{\frac{1}{\rho-1}}(\sum_i \pi_{ij}z_i^\alpha)^{\frac{\rho}{\alpha(\rho-1)}})^{-1} = rx_1(1 + \beta^{-\sigma}(\sum_i \pi_{ij}z_i^{1-\gamma})^{\frac{1-\sigma}{1-\gamma}})^{-1}.$$

Then, we obtain from the first order condition with respect to x_1 together with the above equation,

$$w_1 - x_1 = \beta^{\frac{1}{\rho-1}} r^{\frac{\rho}{\rho-1}} x_1 \left[\sum_j q_j h(\pi_j) \right]^{\frac{1}{\rho-1}} = \beta^{-\sigma} r^{1-\sigma} x_1 \left[\sum_j q_j h(\pi_j) \right]^{-\sigma}, \quad (2)$$

where

$$h(\pi_j) = (1 + \beta^{\frac{1}{1-\rho}}(\sum_i \pi_{ij}z_i^\alpha)^{\frac{\rho}{\alpha(1-\rho)}})^{1-\rho} = (1 + \beta^\sigma(\sum_i \pi_{ij}z_i^{1-\gamma})^{\frac{\sigma-1}{1-\gamma}})^{\frac{1}{\sigma}}.$$

Equation (2) allows us to investigate the effect of resolution of uncertainty on period-1 savings, or the choice between consumption commitment and liquid assets. Suppose that Y' is more informative than Y . Let x_1^* (x_1^{**}) be the optimum savings based on Y (Y'). From the definition of informativeness of Y based on inequality (1), it immediately follows that if $h(\pi_j)$ for each j is convex (concave), then $x_1^{**} \geq x_1^*$ ($x_1^{**} \leq x_1^*$).

As proved in the appendix, the following proposition holds depending on whether $h(\pi_j)$ is convex or concave.

Proposition: More resolution of uncertainty raises savings in period 1 (x_1) if

$$(\sigma > 1, \sigma\gamma \geq 1) \quad \text{or} \quad (0 < \sigma < 1, \sigma + \gamma < 2),$$

and more resolution reduces savings x_1 if

$$(\sigma > 1, \sigma + \gamma < 2) \quad \text{or} \quad (0 < \sigma < 1, \sigma\gamma \leq 1).$$

It is under the former condition that a consumer postpones a commitment to current expenditures on consumption goods with earlier resolution of uncertainty.

3. Discussion We have a couple of remarks on the above result. First, the result of Epstein (1980) corresponds to the case where improving resolution of uncertainty leads to an increase in savings when $\sigma > 1$. In his case, the magnitude of an elasticity of intertemporal substitution plays an essential role in determining the choice between consumption commitment and liquid assets.

Second, our case adds another preference factor. When $\sigma > 1$, preference for early resolution or $\sigma\gamma > 1$ jointly promotes a postponement of consumption commitment with earlier resolution of uncertainty. In other words, a consumer with strong preference for both early resolution and intertemporal substitution tends to increase savings in order to wait for uncertainty to be resolved to some extent.

Third, one interesting result is that even if intertemporal substitution is rather low ($0 < \sigma < 1$), a postponement of consumption commitment takes place with $\sigma + \gamma < 2$. Since $\sigma + \gamma < 2$ immediately leads to preference for late resolution or $\sigma\gamma < 1$, one possible interpretation of this consequence is that preference for late resolution may reverse the impact of low intertemporal substitution on optimum savings. In any case, not a magnitude of risk aversion, but a combination of intertemporal substitution and preference for early (late) resolution does matter in determining the choice between consumption commitment and liquid assets. One caveat about our result is that the above proposition concerns sufficient conditions; therefore, a postponement of consumption commitment may take place at least locally even if $\sigma\gamma < 1$ and $\sigma + \gamma \geq 2$.

Finally, one potential application of our result may be that the non-expected utility framework allows us to investigate the interaction between liquidity demand and asset pricing in a reasonable manner. Using the expected utility setup, we often face the following dilemma: strong intertemporal substitution ($\sigma \gg 1$) enhances liquidity demand, but

weakens aversion to risk ($\gamma \ll 1$), thereby making every asset return close to risk-free rates or narrowing risk premiums. As demonstrated by Miyazaki and Saito (2003), consequently, strong liquidity demand is likely to yield fairly weak effects on asset pricing under expected utility. The above proposition suggests that strong liquidity demand may generate significant impacts on asset pricing when both intertemporal substitution ($\sigma > 1$) and risk aversion ($\gamma > 1$) are strong, or early resolution is preferred ($\sigma\gamma > 1$). In other words, premiums commanded by liquidity demand may be non-negligible under Kreps-Porteus preference.

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Appendix This appendix demonstrates a sufficient condition for convexity (concavity) of $h(\pi_j)$ so as to prove the proposition in the text. Consider the positive-valued function of $\pi_j^T = (\pi_{1j}, \dots, \pi_{nj})$

$$h(\pi_j) = [1 + \beta^\sigma (\sum_i \pi_{ij} z_i^{1-\gamma})^{\frac{\sigma-1}{1-\gamma}}]^{\frac{1}{\sigma}}.$$

Notice that $z_i^{1-\gamma}$, π_{ij} , and β^σ are all positive for all i . The first derivative with respect to π_{ij} is

$$\frac{\partial f(\pi_j)}{\partial \pi_{ij}} = \frac{\sigma-1}{\sigma(1-\gamma)} z_i^{1-\gamma} \beta^\sigma [1 + \beta^\sigma (\sum_i \pi_{ij} z_i^{1-\gamma})^{\frac{\sigma-1}{1-\gamma}}]^{\frac{1}{\sigma}-1} (\sum_i \pi_{ij} z_i^{1-\gamma})^{\frac{\sigma-1}{1-\gamma}-1},$$

while the second derivative with respect to π_{ij} and π_{kj} is

$$\begin{aligned} \frac{\partial^2 f(\pi_j)}{\partial \pi_{ij} \partial \pi_{kj}} &= z_i^{1-\gamma} z_k^{1-\gamma} \beta^\sigma [1 + \beta^\sigma (\sum_i \pi_{ij} z_i^{1-\gamma})^{\frac{\sigma-1}{1-\gamma}}]^{\frac{1}{\sigma}-2} (\sum_i \pi_{ij} z_i^{1-\gamma})^{\frac{\sigma-1}{1-\gamma}-2} \\ &\quad \cdot \frac{\sigma-1}{\sigma(1-\gamma)} \left\{ \frac{\sigma\gamma-1}{\sigma(1-\gamma)} \beta^\sigma (\sum_i \pi_{ij} z_i^{1-\gamma})^{\frac{\sigma-1}{1-\gamma}} + \frac{\sigma+\gamma-2}{1-\gamma} \right\} \\ &\equiv z_i^{1-\gamma} z_k^{1-\gamma} \omega. \end{aligned}$$

The Hessian matrix is thus defined as

$$\omega \begin{bmatrix} z_1^{1-\gamma} \\ \vdots \\ z_n^{1-\gamma} \end{bmatrix} [z_1^{1-\gamma} \dots z_n^{1-\gamma}].$$

The Hessian is positive definite if and only if $\omega > 0$. Hence, a sufficient condition for convexity of $h(\pi_j)$ is as follows:

$$\begin{aligned} \frac{\sigma-1}{\sigma(1-\gamma)} > 0, \frac{\sigma\gamma-1}{\sigma(1-\gamma)} \geq 0, \text{ and } \frac{\sigma+\gamma-2}{1-\gamma} > 0; \text{ or} \\ \frac{\sigma-1}{\sigma(1-\gamma)} < 0, \frac{\sigma\gamma-1}{\sigma(1-\gamma)} \leq 0, \text{ and } \frac{\sigma+\gamma-2}{1-\gamma} < 0. \end{aligned}$$

The above condition can be rewritten as

$$(\sigma > 1, \sigma\gamma \geq 1) \quad \text{or} \quad (0 < \sigma < 1, \sigma + \gamma < 2).$$

A sufficient condition for concavity of $h(\pi_j)$ is obtained in a similar manner.