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Precautionary Savings and the Permanent Income Hypothesis

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This paper derives the explicit solution of a dynamic stochastic optimal consumption problem for infinitely-lived agents whose preferences exhibit, in the presence of non-diversifiable labour income uncertainty, a constant elasticity of intertemporal substitution and constant absolute risk aversion. The constancy of the elasticity of intertemporal substitution, which implies that marginal utility at zero consumption is infinite, guarantees that the non-negativity constraint on consumption is never binding along the optimal path. The assumption of constant absolute risk aversion allows an explicit computation of human wealth, and provides a simple representation of the precautionary savings motive.

What are the determinants of precautionary savings? What role does risk aversion play in generating prudent behaviour? Is the strength of the precautionary savings motive influenced by consumers' distaste for intertemporal substitution? How does the persistence of income shocks affect precautionary savings? What is the appropriate definition of permanent income, and the correct formulation of the permanent income hypothesis, when labour income is risky and undiversifiable?

While most of these and other elementary but fundamental questions have been answered within the context of two-period models[1] they have been only partially addressed in the multi-period case.[2] Most notably, the few available characterizations of multi-period consumption and savings under undiversifiable labour income uncertainty are all derived from frameworks which—because they specify that utility is the expected value of a discounted sum of time-additive felicities—confuse the notion of risk aversion and intertemporal substitution and thus make it impossible to address the questions posed above. Moreover, the only model (so far) of precautionary saving providing an explicit expression for the computation of human wealth in the presence of a precautionary savings motive is based on time-additive exponential utility[3] and ignores the non-negativity constraint on consumption.[4]

In this paper, building on recent advances on the representation of non-expected utility preferences and the disentanglement of risk aversion from intertemporal substitution,[5] I propose a model of precautionary savings based on hybrid non-expected utility preferences which are iso-elastic intertemporally, but exponential in their risk dimension, and I submit that this model is well suited to the analysis of the determinants of

1. See Leland (1968), Sandmo (1970), Drèze and Modigliani (1972), and Selden (1979).
2. Sibley (1975), Miller (1976), Kimball (1988), Zeldes (1989), Kimball and Mankiw (1989), Caballero (1990).
3. *Risk* preferences with a zero third derivative—quadratic as in much of the empirical literature on the permanent income hypothesis, or risk-neutral as in Farmer (1990)—do generate explicit solutions to consumption problems with random labour income, but do not give rise to precautionary savings behavior.
4. Merton (1971), Kimball and Mankiw (1989).
5. See Farmer (1990), Epstein-Zin (1989) and Weil (1989, 1990).

precautionary savings and of the permanent income hypothesis under undiversifiable labour income risk.

The advantages of the parameterization of preferences which I adopt here are threefold. First, the assumption of a constant coefficient of absolute risk aversion implies, as Kimball (1988) shows for standard time-additive, expected utility functions, constant absolute "prudence." As a result, increases in labour income uncertainty shift the consumption function but do not affect its slope at a given level of financial wealth (provided, as I show below, that the interest rate be larger than the rate of time preference and that initial wealth be large enough). This is an analytically appealing feature, as it makes possible the explicit computation of human wealth.[6] Second, the assumption of CES intertemporal tastes, which entails that the marginal utility of consumption tends to infinity as consumption goes to zero, guarantees that the non-negativity constraint on consumption is never binding in an optimum. This considerably simplifies the analysis, as dealing with such non-negativity constraints—as one would have to do if one specified that intertemporal substitution preferences are exponential—is a non-trivial task.[7] For instance, there is no way, with time-additive exponential utility, to restrict the interest rate and income process to guarantee that consumption be non-negative without imposing at the same time that the level of consumption be always rising.[8] Third, iso-elastic intertemporal preferences have the property that consumers' attitudes towards intertemporal substitution are a crucial determinant of the propensity to consume; by contrast, with exponential intertemporal preferences, the propensity to consume out of wealth is always equal to the consol rate, irrespective of the reluctance of consumers to substitute consumption intertemporally.

Hence, in spite of limitations which I will point out below, the mixture of exponential risk preferences and CES intertemporal tastes which I adopt here is an appealing framework within which to rigorously study precautionary savings and the permanent income hypothesis in a simple intertemporal optimizing context. In this framework, uncertainty matters for savings (unlike when agents are risk-neutral or have quadratic utility risk preferences), but it does so in a tractable way. The solution for consumption which I derive does not neglect non-negativity constraints on consumption. And intertemporal substitution, parameterized independently from risk aversion, is an important determinant both of the propensity to consume and of precautionary savings.

The model I develop here is close in spirit to recent, and independent, work by van der Ploeg (1990), who studies precautionary savings in a model with hybrid exponential

6. Note, however, that Kimball (1988) presents introspective arguments in favour of *decreasing* absolute prudence. To my knowledge, no analytical solution to a multi-period consumption programme with random labour income has ever been derived for preferences exhibiting decreasing absolute prudence. To study the effect of income risk on the marginal propensity to consume in such a setting, one is therefore confined to the results of Zeldes (1989), or to Kimball's (1988) analytical results in a two-period expected utility model.

7. Lehocsky, Sethi and Shreve (1983), and Karatzas, Lehocsky, Sethi and Shreve (1986) use a dynamic programming approach to resolve these issues in a continuous-time model with complete markets, while Cox and Huang (1989) use martingale techniques. The non-negativity constraint on consumption is altogether neglected or not imposed in many cases in which it would be binding: see, for instance, Merton (1971), Caballero (1990), or Kimball and Mankiw (1989).

8. Consider the following example, which draws on Caballero (1990). Suppose that $U = -E_0 \sum_{t=0}^{\infty} \delta^t e^{-\nu c_t}/\nu$, $\nu > 0$, and let R denote the gross interest rate. Labour income follows a random walk $y_t = y_{t-1} + \zeta_t$ where ζ is an i.i.d. labour income shock. The optimal consumption process obtained by neglecting the non-negativity constraint on consumption is $c_{t+1} = c_t + (\zeta_{t+1} - \zeta^*) + \nu^{-1} \ln(\delta R)$, where ζ^* is the certainty equivalent of this income shock (i.e. $\zeta^* = -\nu^{-1} \ln E e^{-\nu\zeta}$). The only way to guarantee that this is an appropriate solution when the non-negativity constraint on consumption is imposed is to assume that the income shock distribution has a finite support with a lower bound $\underline{\zeta}$, and that $\ln(\delta R) \geqq \nu(\zeta^* - \underline{\zeta}) > 0$. This joint restriction on preferences and technology, which requires that $\delta R > 1$, implies that $c_{t+1} \geqq c_t$—i.e. it rules out the possibility of declines in consumption. This result is general to the class of expected, time-additive exponential utility.

risk and quadratic intertemporal preferences. Van der Ploeg's analysis suffers, however, from the well-known unpleasant features of quadratic intertemporal preferences (e.g. the existence of a bliss point), and shares with previous models the neglect of the non-negativity constraint on consumption (which will in general be binding if imposed because of the finiteness of marginal utility at zero consumption). My paper also owes much to pioneering work by Farmer (1990), whose discovery of RINCE utility opened a Pandora's box of preferences.[9]

In Section 1, I characterize the optimal consumption and savings behaviour of consumers endowed with the hybrid iso-elastic/exponential preferences which I introduce in this paper. I then provide the answer, in Section 2, to the questions, asked *ab initio*, which motivate this paper. In Section 3, I examine the implications of this framework for the debate on the excess volatility of consumption. I conclude by discussing some of the limitations of this paper, and by outlining some directions for future research.

1. CONSUMERS

1.1. *Preferences*

Consider an infinitely-lived consumer whose preferences over deterministic consumption streams exhibit a *constant elasticity of intertemporal substitution*:

$$W(c_t, c_{t+1}, \ldots) = \{(1-\delta)\textstyle\sum_{s=0}^{\infty} \delta^s c_{t+s}^{1-\alpha}\}^{1/(1-\alpha)}, \tag{1.1}$$

where $1/\alpha > 0$, $\alpha \neq 1$, denotes the constant elasticity of intertemporal substitution, and $\delta \in (0, 1)$ is, under certainty, the constant and exogenous subjective discount factor.[10] These preferences can, equivalently, be represented recursively as

$$W(c_t, c_{t+1}, \ldots) = U[c_t, W(c_{t+1}, c_{t+2}, \ldots)] \tag{1.2}$$

$$\equiv \{(1-\delta)c_t^{1-\alpha} + \delta[W(c_{t+1}, c_{t+2}, \ldots)]^{1-\alpha}\}^{1/(1-\alpha)}, \tag{1.3}$$

where $U(\cdot, \cdot)$ is, in Koopmans' (1960) terminology, an aggregator function.

Although our consumer's behaviour exhibits a constant elasticity of intertemporal substitution, I assume that his attitude towards (atemporal) risk is not characterized, as would be the case under standard time-additive expected utility, by a constant coefficient of relative risk aversion. Instead, behaviour towards risk is summarized by a *constant coefficient of absolute risk aversion*, denoted by the parameter $\beta > 0$.

For our consumer, the utility certainty equivalent, at rate β, of a lottery yielding a random utility level W' is $\hat{W}$, defined by:

$$e^{-\beta \hat{W}} = E\{e^{-\beta W'}\}. \tag{1.4}$$

Equivalently,

$$\hat{W} \equiv \frac{\ln Ee^{-\beta W'}}{-\beta}. \tag{1.5}$$

In this notation, $\hat{W}(\tilde{c}_{t+1}, \tilde{c}_{t+2}, \ldots)$ represents the certainty equivalent, conditional on time t information, of time $t+1$ utility. The limiting case $\beta = 0$ corresponds to risk-neutral agents, for whom $\hat{W} = EW'$.

9. For the sake of optimism as to the usefulness of this line of research, a not too literal interpretation of Greek mythology is required here from the reader.

10. Logarithmic intertemporal preferences obtain by letting α tend to 1, using l'Hôpital's rule.

I assume that preferences over random consumption lotteries have the recursive representation

$$W(c_t, \tilde{c}_{t+1}, \ldots) = U[c_t, \hat{W}(\tilde{c}_{t+1}, \tilde{c}_{t+2}, \ldots)], \tag{1.6}$$

with the aggregator function defined in (1.3). In other terms, current utility is an aggregate, computed by using the function $U[\cdot, \cdot]$, of current consumption and the certainty equivalent of future utility.

The utility function introduced in equation (1.6), which has the distinctive feature that it exhibits both a constant elasticity of intertemporal substitution $(1/\alpha)$ and a constant elasticity of *absolute* risk aversion (β),[11] is a member of the wider class of Kreps–Porteus (1978, 1979) preferences, as can easily be seen by performing the change of variable $Z = -\beta^{-1}e^{-\beta W}$. Kreps–Porteus theory provided the foundation which enabled Epstein-Zin (1989) and Weil (1990) to generalize iso-elastic utility; it also provides the framework within which hybrid tastes such as the ones studied here can be represented.

1.2. *Interest rate and income process*

Our consumer can competitively borrow and lend at the fixed safe gross rate of return $R > 0$,[12] and receives a random non-interest income (endowment) y in every period. Income follows an autoregressive process:

$$y_{t+1} = \rho y_t + (1-\rho)\hat{y} + \varepsilon_{t+1} \tag{1.7}$$

where y_t is income in period t (and is part of today's information set), $\hat{y}$ is a positive constant,[13] ε_{t+1} is an i.i.d. random variable with constant mean μ and variance σ^2, and $\rho > 0$ is a constant.

All income shocks are transitory if $\rho = 0$, permanent when $\rho = 1$; the parameter ρ thus measures the persistence of labor income shocks.

I impose the following joint restrictions on preferences and technology:

Assumption 1. $\rho < R$.

Assumption 2. $\delta^{1/\alpha}R^{(1-\alpha)/\alpha} < 1$.

Assumption 3. $\delta R > 1$.

Assumption 4. $(1-\rho)\hat{y} + \underline{\varepsilon} \geqq \{[R-(\delta R)^{1/\alpha}]/[(\delta R)^{1/\alpha} - 1]\}(u - \underline{\varepsilon})$, where $\underline{\varepsilon}$ denotes the (possibly negative) lower bound of the income shock distribution.

The first assumption guarantees below the finiteness of human wealth, while the second (common) assumption is necessary for the existence of an optimal programme. Assumption 3 constrains the interest rate to be larger than the subjective discount rate; under certainty, our consumer's optimal consumption would thus *grow* at the gross rate $(\delta R)^{1/\alpha} > 1$. Finally, Assumption 4 limits the extent of downward income risk; in

11. In Appendix A, I show that there is a tight connection between the concept of risk aversion used here (which is specified in utility terms) and the more standard definition in terms of consumption.

12. R can be viewed as rate of return on a safe storage technology, or as the world interest rate in a small open economy.

13. As long as one does not deal with equilibrium issues, there is no loss of generality in assuming that $\hat{y}$ is constant, for all deterministic components of labour income can be viewed as accruing to the consumer at the beginning of his life.

particular it ensures, in conjunction with Assumptions 2 and 3, that consumption never becomes negative.[14] I show below that Assumptions 3 and 4, while undoubtedly restrictive as they limit the generality of the consumer's problem, are jointly sufficient to guarantee that it has a solution, and that this solution can be written in closed form.

1.3. *Budget set*

I assume that market structure is such that labour income risk is undiversifiable, for reasons I do not make explicit here.[15]

Letting w_t denote financial wealth at the beginning of the current period, w_{t+1} financial wealth at the beginning of next period, and c_t current consumption, the period-by-period budget constraint faced by our consumer can be written as

$$w_{t+1} = Rw_t + y_t - c_t, \tag{1.8}$$

with $w_0 = 0$. Equivalently, letting

$$a_t = Rw_t + y_t \tag{1.9}$$

denote total resources or "cash on hand" (the sum of financial wealth, interest and labour income) available at the beginning of the current period, the period-by-period budget constraint is

$$a_{t+1} = R(a_t - c_t) + y_{t+1}, \tag{1.10}$$

with initial resources at time 0, $a_0 = y_0$, given. I assume that current income, y_t, is known when current consumption, c_t, is decided upon.

To avoid trivial solutions, the consumer is constrained to be solvent for all possible (and in particular the worst) realizations of the income process, in the sense that it is required that

$$\lim_{i\to\infty} R^{-i} a_{t+i} \geqq 0. \tag{1.11}$$

1.4. *Maximization problem*

Our consumer's objective is to find a contingent consumption plan which maximizes his utility (1.6), subject to the budget constraint (1.10), the constraint that consumption be non-negative at all dates and in all states, and the transversality constraint (1.11).

The optimal solution to this problem is characterized most simply in terms of a value function $V(a, y)$ which must satisfy, using equations (1.3), (1.4), (1.6), and (1.10), the following functional equation:

$$V(a_t, y_t) = \max_{c_t \geqq 0} \left\{ (1-\delta) c_t^{1-\alpha} + \delta \left[\frac{\ln E_t e^{-\beta V[R(a_t - c_t) + y_{t+1}, y_{t+1}]}}{-\beta} \right]^{1-\alpha} \right\}^{1/(1-\alpha)}, \tag{1.12}$$

where E_t denotes mathematical expectation conditional on information available at time t.

14. I do not consider it appropriate to specify an income process which violates this constraint.

15. The study of the incentive problems which, under asymmetric information, could give rise to this market incompleteness is outside the scope of this paper.

1.5. *Value function*

The Bellman equation (1.12) can be solved explicitly. In Appendix B, I show that it takes the simple linear form

$$V(a, y) = \phi a + \psi y + \lambda, \tag{1.13}$$

where

$$\phi = (1-\delta)^{1/(1-\alpha)}\left[1 - \frac{(\delta R)^{1/\alpha}}{R}\right]^{\alpha/(\alpha-1)} > 0, \tag{1.14}$$

$$\psi = \frac{\rho}{R-\rho}\phi > 0, \tag{1.15}$$

$$\lambda = \frac{R}{(R-\rho)(R-1)}[(1-\rho)\hat{y} + \varepsilon^*]\phi > 0, \tag{1.16}$$

and

$$\varepsilon^* = -\frac{R-\rho}{\beta\phi R}\ln Ee^{-[\beta\phi R/(R-\rho)]\varepsilon'} \leq \mu \tag{1.17}$$

One should interpret ε^* as the *certainty equivalent* of the labour income disturbance, which (1.17) instructs us to compute at a rate of absolute risk aversion $\beta\phi R/(R-\rho)$ which depends—because labour income risk is *not* atemporal[16]—not only on attitude towards atemporal risk (through β) but also on attitudes towards intertemporal substitution (through ϕ), persistence, and the interest rate.[17]

1.6. *Consumption function*

In Appendix B, I establish that the optimum consumption function can be written in feedback form as

$$c_t = \omega[a_t + h_t], \tag{1.18}$$

where

$$\omega = 1 - \delta^{1/\alpha}R^{(1-\alpha)/\alpha} \tag{1.19}$$

denotes the marginal and average propensity to consume,[18] and

$$h_t = \frac{1}{R-\rho}\left\{\rho y_t + \frac{R}{R-1}[(1-\rho)\hat{y} + \varepsilon^*]\right\} \tag{1.20}$$

represents human wealth, defined as the certainty equivalent, computed at rate $\beta\phi R/(R-\rho)$, of the present discounted value of future labour income.

16. Wealth can be reallocated to smooth out labor income fluctuations, and thus affects the amount of risk borne by the consumer. See Drèze-Modigliani (1972) for related points in a non-additive two-period expected utility setting.

17. The inequalities in (1.14) and (1.15) follow from Assumption 1, while that in (1.16) follows from both the property that ε^* cannot fall below the smallest possible realization $\underline{\varepsilon}$ of the income shock and Assumption 4. Note also that, using l'Hôpital's rule, $\lim_{\alpha\to 1}\phi = (1-\delta)(\delta R)^{\delta/(1-\delta)}$.

18. Assumption 2 ensures that $\omega \in (0, 1)$.

Written in the perhaps more familiar terms of financial wealth, the consumption function is, using (1.9),

$$c_t = \omega[Rw_t + H_t], \tag{1.21}$$

where

$$H_t = h_t + y_t = \frac{R}{R-\rho}\left\{y_t + \frac{1}{R-1}[(1-\rho)\hat{y} + \varepsilon^*]\right\} \tag{1.22}$$

is an alternative measure of human wealth which includes current labour income.

Optimal consumption is proportional to total (non-human plus human) wealth. The constancy of the propensity to consume results jointly from the assumptions of infinite horizon, constant interest rate, and i.i.d. labour income shocks. It is noteworthy that the propensity to consume out of total wealth does not depend, under our assumptions, on attitudes towards risk, and is, therefore, equal to that which would prevail under certainty (in particular $\omega = 1 - \delta$ for logarithmic intertemporal preferences—i.e. when $\alpha = 1$).[19] Thus changes in the variance of labour income shocks do not affect the propensity to consume.

Labour income risk does affect, however, the level of consumption. If agents are risk neutral, $\varepsilon^* = \mu$ and H_t is the *expected* present discount value of current and future labour income. When consumers are risk averse, however, $\varepsilon^* < \mu$ and perceived human wealth is smaller than the expected present discount value of labour income. For any given level of financial wealth, consumption is therefore smaller than it would be if agents were risk neutral: consumers engage in precautionary savings—because the third derivative of their risk-preference function is positive.

Finally, the marginal propensity to consume out of current income, $\omega R/(R-\rho)$, is larger the more persistent income shocks are. Therefore, consumption reacts more to permanent than to transitory innovations in labour income.

1.7. *A simple representation*

A two-step decomposition of the consumer's optimal plan helps understand the economics of the solution derived above.

Let H^*_{t+1} denote the certainty equivalent, at rate $\beta\phi R/(R-\rho)$, of time $t+1$ human wealth. It then follows that

$$H^*_{t+1} = R(H_t - y_t). \tag{1.23}$$

This expression—formally analogous to the one which would obtain under certainty—emphasizes that the appropriately computed certainty equivalent rate of return on human capital is equal to the safe rate.

Using the budget constraint (1.0), a simple calculation then establishes that

$$c_{t+1} = (\delta R)^{1/\alpha} c_t + \omega \frac{R}{R-\rho}(\varepsilon_{t+1} - \varepsilon^*). \tag{1.24}$$

In the absence of uncertainty ($\varepsilon_{t+1} = \varepsilon^* = \mu$), consumption evolves according to $c_{t+1} = (\delta R)^{1/\alpha} c_t$—i.e. according to the usual Euler equation necessary for optimality under CES intertemporal preferences. Since $\varepsilon^* < \mu$ under uncertainty, labour income risk thus has the effect of tilting, relative to the certainty case or the risk neutral case, the consumption profile towards the future—which is the essence of the precautionary savings motive.

19. This property is of course specific to the preferences studied here.

Equation (1.24) implies that no contemporaneous variable beyond current consumption has predictive power for future consumption. This is the appropriate version, in this model, of Hall's (1978) results. In contrast with Hall's formulation—which is based on quadratic time-additive utility and thus neglects precautionary savings—note, however, that the persistence of income shocks affects the sensitivity of consumption growth to labor income surprises.[20]

Letting c^*_{t+1} denote the certainty equivalent of tomorrow's consumption, computed (like that of ε) at rate $\beta\phi R/(R-\rho)$, one can rewrite (1.24) to yield the following expression:

$$c^*_{t+1} = (\delta R)^{1/\alpha} c_t. \tag{1.25}$$

Hence, it is a feature of the optimal consumption plan that the certainty equivalent of tomorrow's planned consumption is $(\delta R)^{1/\alpha}$ times larger than today's consumption.

Therefore,

$$c_{t+1} - c^*_{t+1} = \omega \frac{R}{R-\rho}(\varepsilon_{t+1} - \varepsilon^*). \tag{1.26}$$

Consumption thus is lower (higher) than its certainty equivalent whenever income falls short of (exceeds) its certainty equivalent.

Equations (1.25) and (1.26) provide an extremely simple characterization of the optimal consumption programme. Given today's optimal consumption c_t, the computation of tomorrow's contingent consumption plan can be performed in two steps:

- use (1.25) to compute the optimal certainty equivalent c^*_{t+1} of tomorrow's consumption;
- from (1.26), calculate the optimal deviation of actual consumption at $t+1$ away from c^*_{t+1} as a function of the deviation of the labour income shock away from its certainty equivalent.

1.8. *Non-negativity of consumption*

The crucial role played by Assumptions 3 and 4 follows from considering the implications for the non-negativity of consumption of equations (1.25) and (1.26).[21]

First, if the interest rate were lower than δ^{-1}, and if the solution exhibited above were still valid, the certainty equivalent of consumption would tend to zero asymptotically after a long sequence of low income realizations, and planned consumption would become negative in finite time with positive probability—which is inconsistent with the fact that utility is not defined for negative consumption. For our solution to be valid, it is thus necessary, when the consumers' planning horizon is infinite, to impose Assumption 3.

Second, even if $\delta R > 1$ and consumption has an upward trend, initial consumption might be too low, relative to the size of the *constant* precautionary savings effect, to prevent c_t from becoming negative after a series of "bad" income shocks realizations. Assumption 4 is therefore required to restrict the downward riskiness of the income process.[22] Instead of requiring Assumption 4, one could alternatively assume that initial financial wealth, instead of being zero as assumed here, is positive and large. By reducing

20. A similar result is derived by van der Ploeg (1990).

21. Appendix C presents a formal argument.

22. By inspection of (1.24), it is clear that the closer $(\delta R)^{1/\alpha}$ is to 1 and the less tilted towards the future the consumption path is, the more stringent the restriction imposed by Assumption 4.

the share of risky human wealth in total wealth, a sufficiently large initial financial wealth would suffice to guarantee the validity of the solution presented above.[23]

Thus, the main simplifying feature of the solution—a precautionary savings motive whose strength, measured by ε^*, is independent of the stock of financial wealth—becomes infeasible when consumption is too close to zero, either asymptotically or at the origin of time.[24] In other terms, the precautionary savings behaviour of consumers with extremely low wealth levels cannot be independent of their wealth level. But the behaviour of consumers with high wealth levels is, as the solution presented here establishes.

When either Assumption 3 or 4 is violated (or, alternatively, if initial wealth is too small), the solution to the consumers' problem cannot be written in closed form,[25] and one would have to resort to numerical simulations (very much as Zeldes (1989) did) to elucidate the savings behaviour of consumers.

Despite the limitations which they entail, Assumptions 3 and 4 do not rule out all "interesting" economies. First, equilibrium considerations will often impose $R > 1/\delta$; for instance, in an exchange economy with undiversifiable idiosyncratic income risk but with no aggregate risk, it suffices that the aggregate endowment grows at a positive rate to guarantee that this assumption be satisfied. Second, Assumption 4 is satisfied whenever downward income risk is small or, *a fortiori*, when income shocks are always positive.

Finally, one may note that the joint restriction on tastes and technology which is imposed here does not imply that consumption be always increasing—in contrast with the assumptions required, under time-additive exponential expected utility—to guarantee the non-negativity of consumption.

1.9. *Summary*

The solution of our consumer's problem takes a particularly simple form. Consumption is a fixed fraction of total wealth. The marginal propensity to consume depends only on the interest rate, the rate of time preference under certainty, and attitudes towards intertemporal substitution. Human wealth is measured as the appropriately computed certainty equivalent of the present discounted value of future labour income, whose value depends on attitudes towards risk, intertemporal substitution, persistence and the interest rate. The optimal consumption plan admits a simple recursive representation.

2. COMPARATIVE STATICS

In this section, I consider the effects on consumption and savings of labour income risk, persistence, risk aversion, intertemporal substitution, and interest rates.

2.1. *Labour income risk*

Our consumers, because their risk-preference function is exponential and thus has a positive third derivative, engage in precautionary savings. Labour income uncertainty, as

23. It is easy to show—by working out the appropriate solution—that if consumers were finitely-lived, imposing that initial wealth is large enough would make it possible to dispense with *both* Assumptions 3 and 4: it would suffice to choose w_0 so that both initial and final consumption be positive if the worst possible income state is realized in every period.

24. This fact is of course not contradictory with the statement that, with CES intertemporal preferences, the non-negativity constraint on consumption never binds at an optimum. It is only evidence that the solution presented in the text is simply not valid for $\delta R \leqq 1$ or when Assumption 4 is violated.

25. At least by me.

I showed above, does not affect, in this economy, the marginal propensity to consume out of total wealth. Instead, as equation (1.18) shows, labour income risk decreases the perceived (certainty equivalent) present discounted value of future non-traded income. Formally, the amount by which a mean-preserving increase in income uncertainty decreases the level of consumption is approximately proportional to the variance σ^2 of labour income shocks, since, using a second-order Taylor expansion of (1.17),[26]

$$\varepsilon^* \approx \mu - \frac{\beta\phi R}{R-\rho}\frac{\sigma^2}{2}. \tag{2.1}$$

As a consequence,

$$\frac{\partial \varepsilon^*}{\partial \sigma^2} \approx -\frac{\beta\phi R}{2(R-\rho)} < 0. \tag{2.2}$$

Alternatively, the effect of labour income risk on the consumption *profile* is exhibited by (1.24): more uncertainty results in a larger consumption growth rate, i.e. it leads consumers to postpone consumption.

2.2. *Persistence*

Although it does not affect the propensity to consume out of wealth, the persistence of labour income shocks is a crucial determinant of the strength of precautionary savings motive. It follows from equation (2.1) that

$$\frac{\partial \varepsilon^*}{\partial \rho} \approx -\frac{\beta\phi R}{(R-\rho)^2} < 0. \tag{2.3}$$

Hence, the more persistent income shocks are (the larger ρ), the smaller ε^*, and thus the smaller perceived human wealth: more persistence in income shocks leads to a stronger precautionary savings motive.

Moreover, from equations (1.21) and (1.22), the marginal propensity to consume out of current income is

$$\frac{\partial c_t}{\partial y_t} = \omega R/(R-\rho). \tag{2.4}$$

Not unexpectedly, the more persistent the income process (i.e. the larger ρ), the more responsive current consumption to fluctuations in current income.

2.3. *Risk aversion*

Stronger aversion to atemporal risk (a larger β) decreases, from (2.1), the certainty equivalent income shock ε^*, and hence leads to a lower level of consumption. Hence, the more risk averse our consumer is, the stronger his precautionary savings motive.

Two special polar cases are of interest. For risk-neutral consumers ($\beta = 0$),[27] $\varepsilon^* = \mu$. As a consequence, human wealth is, as in "standard" formulations of the permanent income hypothesis, the *expected* present discounted value of future income, and labour

26. This approximation would be exact if we allowed ε^* to be normally distributed.

27. The following statements can be extended to risk-preference functions which exhibit a zero third derivative.

income risk does not affect the level of savings.[28] Note that, because of the assumption of iso-elastic intertemporal tastes, intertemporal choice is not degenerate when consumers are risk-neutral. For an infinitely risk-averse consumer ($\beta=+\infty$), $\varepsilon^*=\underline{\varepsilon}$. Our consumer then chooses consumption in every period according to a worst-case scenario: he computes human wealth as the worst possible realization of the present discounted value of future labour income, and only deviates upward from his certainty-equivalent consumption plan.

2.4. *Intertemporal substitution*

Aversion to intertemporal substitution affects all aspects of our consumer's behaviour.

A decrease in the elasticity of intertemporal substitution increases, under Assumption 3, the propensity to consume out of wealth and out of current income since, letting $\chi=1/\alpha$ denote the elasticity of intertemporal substitution,

$$\frac{\partial\omega}{\partial\chi}=-(1-\omega)\ln(\delta R)<0. \tag{2.5}$$

Equivalently, from equation (1.24), the consumption profile becomes flatter, *ceteris paribus*, when consumers become more averse to intertemporal substitition. These results are identical to those which obtain under certainty for time-additive iso-elastic preferences.

More interestingly, a stronger distaste for intertemporal substitution affects the precautionary savings motive through the parameter ϕ. From equations (1.19), (1.14), (2.1) and (2.5),

$$\frac{\partial\ln\phi}{\partial\chi}=\frac{1}{(1-\chi)^2}\ln\left[\frac{\omega}{1-\delta}\right]+\frac{1}{1-\chi}\frac{1}{\omega}\frac{\partial\omega}{\partial\chi}. \tag{2.6}$$

While the sign of this derivative cannot be determined analytically,[29] it is easy to establish that ϕ is globally increasing in χ, since:

$$\lim_{\chi\to 0}\phi=1-\frac{1}{R}>1 \tag{2.7}$$

and

$$\lim_{\chi\to\bar{\chi}}\phi=+\infty, \tag{2.8}$$

where $\bar{\chi}\equiv\ln(R)/(\ln(\delta R)>1$ denotes the maximum elasticity of substitution consistent, from Assumptions 2 and 3, with the existence of a solution for given δ and R. Numerical analysis shows that this pattern is local as well: for all parameter configurations tried, ϕ is strictly increasing in χ over the interval $[0,\bar{\chi})$. Figures 1 and 2 illustrate the typical response of ϕ to changes in the elasticity of intertemporal substitution for the cases $(\delta,R)=(0{\cdot}98, 1{\cdot}05)$ and $(0{\cdot}99, 1{\cdot}04)$. Hence, the weaker the distaste for intertemporal substitution, the larger ϕ and the stronger the precautionary savings motive. This result is best explained by considering heuristically the effect on the time-pattern of consumption of more prudent behaviour. From equation (1.24), it follows that

$$E_tc_{t+1}=(\delta R)^{1/\alpha}c_t+\omega\frac{R}{R-\rho}(\mu-\varepsilon^*). \tag{2.9}$$

28. This is the case first studied by Farmer (1990).

29. Looking at the second derivative is also inconclusive.

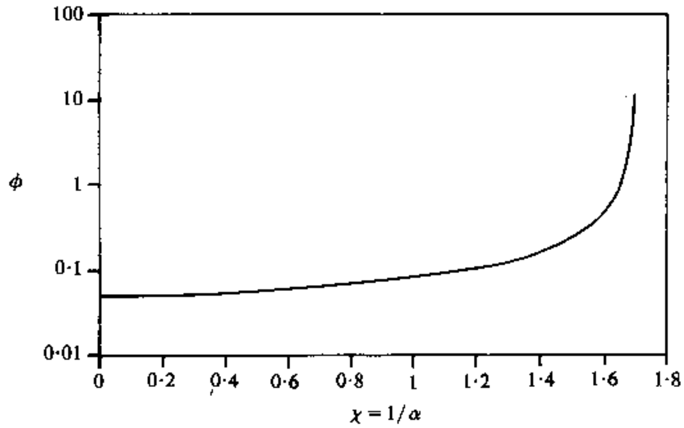

FIGURE 1

$\delta = 0{\cdot}98,\ R = 1{\cdot}05$

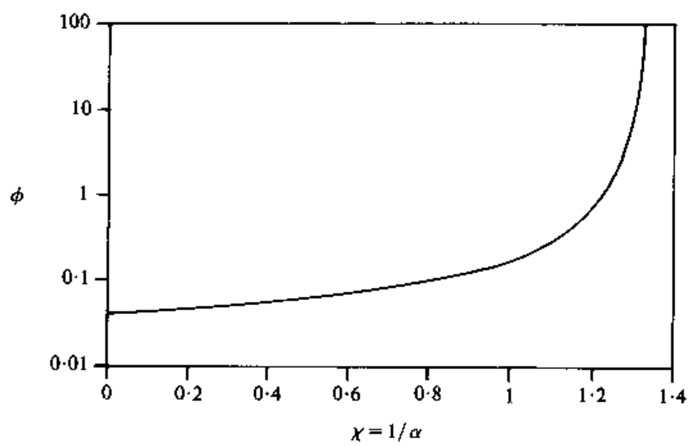

FIGURE 2

$\delta = 0{\cdot}99,\ R = 1{\cdot}04$

The more prudent behaviour, the smaller ε^*, and the larger, on average, the rate of growth of consumption relative to $(\delta R)^{1/\alpha}$—which would be the optimal rate of growth of consumption if labour income uncertainty could entirely be insured away. As a consequence, while more prudent behaviour provides a partial hedge against uncertainty, thus satisfying the risk-averse aspects of our consumer's personality, it also generates on average exceedingly high rates of consumption, which run counter to our agent's desire for consumption smoothing. The weaker this desire for consumption smoothing (i.e., the larger the elasticity of intertemporal substitution), the more prudent risk-averse consumers must be.

It is noteworthy that, as χ tends to $\bar{\chi}$, ϕ becomes infinite, so that consumers become infinitely prudent ($\varepsilon^* \to \underline{\varepsilon}$). This results from the fact that ϕ is the indirect marginal utility of wealth: as is the case under certainty, indirect utility becomes infinitely sensitive to wealth when the elasticity of substitution approaches the upper-bound $\bar{\chi}$ at which, for given δ and R, utility becomes infinite. As a consequence, the given labour income uncertainty which our consumer is facing translates into increasingly risky indirect utility as χ approaches $\bar{\chi}$. In the limit, this phenomenon leads the agents to become infinitely prudent.

Given the absence of conclusive evidence on the magnitude of the elasticity of intertemporal substitution—existing estimates are based on aggregate data and implicitly assume complete markets—it is not possible to discount very prudent behaviour as a theoretical aberration solely on the ground of empirical irrelevance. For the case studied in Figure 2, an elasticity of intertemporal substitution equal to 1·32 yields, for instance, a ϕ approximately equal to 100! A more convincing argument against such a configuration of parameters is, however, that it implies an implausibly low propensity to consume out of lifetime wealth, inferior to 0·001.

2.5 *Interest rate*

An increase in the interest rate increases, as under certainty, the propensity to consume out of wealth if $\alpha < 1$—i.e. if the income effect dominates the substitution effect.

The impact of an increase in the interest rate on the magnitude of precautionary savings is easily characterized when income is i.i.d.[30] ($\rho = 0$), for it is then easy to show that

$$\frac{\partial \varepsilon^*}{\partial R} \approx -(\mu - \varepsilon^*)\frac{1-\omega}{R\omega} < 0. \tag{2.10}$$

Thus, for an i.i.d. income process, the larger the interest rate, the larger the precautionary savings effect. The rationale for this result is again to be found by considering the implications of an increase in R for the indirect marginal utility of wealth ϕ. When R increases, so does ϕ; as a consequence, labour income risk translates into increased utility risk, and thus into a stronger precautionary savings motive.

2.6. *Summary*

Consumers have a precautionary savings motive. Their prudent behaviour is reinforced by larger income risk, more persistence in the income process, stronger aversion to risk, weaker distaste for intertemporal substitution, and higher interest rates.

3. CONCLUSION

The main limitation of the model of precautionary savings I have introduced in this paper is clear: it does not provide, because of its very simplicity, an explicit representation of the precautionary savings motive for all income processes and interest rates. Yet, for those cases in which it does yield explicit solutions, the model sheds new light on the

30. Algebraic analysis is inconclusive for the non-i.i.d. income case.

determinants of precautionary savings in multi-period economies. Moreover, the characterization of optimal consumption under undiversifiable labour income risk which it supplies is undeniably more satisfactory than those previously available—as it neither neglects the non-negativity constraint on consumption nor is based on the questionable assumption of quadratic utility so often used in the literature.

In this respect, equation (1.24) should prove to be very valuable for empirical students of the permanent income hypothesis, as it offers a much richer representation of that theory—one which allows, for the first time, both the existence of a geometric trend in consumption (time-additive, expected exponential utility predicts only arithmetic trends), and the presence of a precautionary savings motive (ruled out by time-additive quadratic utility). It is only natural, therefore, that the examination of the implications of precautionary savings for the debate over the excess volatility of consumption figures prominently on the agenda for future research, as this debate has so far been conducted mainly on the basis of simplistic, and thus impoverished, quadratic utility characterizations of the permanent income hypothesis.

APPENDIX A

This appendix clarifies the connection between the concept of risk aversion used in this paper (β is the coefficient of relative risk aversion for utility lotteries) and the more traditional definition in terms of consumption.

Consider the following lottery (A) on lifetime consumption: $c_0 = c > 0$ with certainty, and $c_t = c + v$ for all $t > 0$, where v is a random variable with zero mean and variance s^2. Given the specification of utility in (1.4) and (1.6), the utility derived from this lottery by our consumer is

$$U_A = \left\{(1-\delta)c^{1-\alpha} + \delta\left[c + \frac{\ln Ee^{-\beta v}}{-\beta}\right]^{1-\alpha}\right\}^{1/(1-\alpha)}. \tag{A.1}$$

Consider, in addition, the certain (degenerate) lottery (B) which yields $c_0 = c > 0$ in the first period, and a constant consumption level $c + \pi$ with certainty thereafter. From (1.1), the utility derived from this lottery is

$$U_B = \{(1-\delta)c^{1-\alpha} + \delta[c+\pi]^{1-\alpha}\}^{1/(1-\alpha)}. \tag{A.2}$$

Our consumer is indifferent between the lottery on permanent consumption A and the deterministic consumption allocation B if and only if $U_A = U_B$, i.e., if and only if

$$\pi = \frac{\ln Ee^{-\beta v}}{-\beta} < 0. \tag{A.3}$$

The consumption premium π which a consumer is thus willing to pay to avoid facing permanent consumption risk is thus the certainty equivalent, computed at rate β, of the permanent consumption shock v.

For infinitesimal risks,

$$\pi \approx -\beta\frac{s^2}{2}, \tag{A.4}$$

so that, as in standard definitions, the consumption risk premium is approximately equal to the coefficient of absolute risk aversion times half the variance of the lottery.

APPENDIX B

This appendix proves that the solution of the Bellman equation (1.12) and the consumption function given in the text are correct. I indicate, whenever appropriate, the peculiarities of the solution for the logarithmic intertemporal preferences case ($\alpha \to 1$).

Guess that the value function is as written in (1.13). Then,

$$\frac{\ln Ee^{-\beta V[R(a-c)+y', y']}}{-\beta} = \phi R(a-c) + \lambda + (\phi+\psi)[\rho y + (1-\rho)\bar{y} + \varepsilon^*], \tag{B.1}$$

where time subscripts are dropped for ease of notation, y' denotes tomorrow's random labour income, the expectation operator is understood to be conditional on current information, and

$$\varepsilon^* = \frac{\ln Ee^{-\beta(\phi+\psi)\varepsilon'}}{-\beta(\phi+\psi)}. \tag{B.2}$$

Performing the maximization called for by the Bellman equation (1.12), consumption must be equal to

$$c = \theta\{\phi Ra + \lambda + (\phi+\psi)[\rho y + (1-\rho)\hat{y} + \varepsilon^*]\}, \tag{B.3}$$

where

$$\theta = \frac{[\delta\phi R/(1-\delta)]^{-1/\alpha}}{1+\phi R[\delta\phi R/(1-\delta)]^{-1/\alpha}}. \tag{B.4}$$

Substituting this expression into the Bellman equation, and after some straightforward manipulations, one finds that, when $\alpha \neq 1$, the parameters ϕ, ψ, λ and θ must be such that the following relation holds as an identity:

$$\phi a + \psi y + \lambda = [\phi R/(1-\delta)]^{1/(\alpha-1)}\theta^{\alpha/(\alpha-1)}\{\phi Ra + \lambda + (\phi+\psi)[(1-\rho)\hat{y} + \varepsilon^*]\}, \tag{B.5}$$

which requires

$$1 = [\phi R/(1-\delta)]^{1/(\alpha-1)}\theta^{\alpha/(\alpha-1)}R, \tag{B.6}$$

$$\psi = [\phi R/(1-\delta)]^{1/(\alpha-1)}\theta^{\alpha/(\alpha-1)}(\phi+\psi)\rho, \tag{B.7}$$

and

$$\lambda = [\phi R/(1-\delta)]^{1/(\alpha-1)}\theta^{\alpha/(\alpha-1)}\{\lambda + (\phi+\psi)[(1-\rho)\hat{y} + \varepsilon^*]\}. \tag{B.8}$$

Dividing the last two equations term by term by (B.6) yields

$$\frac{\phi+\psi}{\phi} = \frac{R}{R-\rho} \tag{B.9}$$

$$\frac{\lambda}{\phi} = \frac{1}{R-\rho}\frac{R}{R-1}[(1-\rho)\hat{y} + \varepsilon^*], \tag{B.10}$$

so that equation (B.3) can be rewritten as

$$c = \theta\phi R\left\{a + \frac{1}{R-\rho}\rho y + \frac{1}{R-\rho}\frac{R}{R-1}[(1-\rho)\hat{y} + \varepsilon^*]\right\}, \tag{B.11}$$

with, using equations (B.2) and (B.9),

$$\varepsilon^* = -\frac{R-\rho}{\beta\phi R}\ln Ee^{-[\beta\phi R/(R-\rho)]\varepsilon'}. \tag{B.12}$$

The last expression is identical to (1.17).

Now, combining equations (B.4) and (B.6) yields

$$\theta\phi R = 1 - \delta^{1/\alpha}R^{(1-\alpha)/\alpha}, \tag{B.13}$$

which is simply the definition of ω given in (1.19).

To complete the proof for the case $\alpha \neq 1$, it only remains to notice that equation (B.6) implies that $\phi = (\theta R)^{-\alpha}$. But since $\omega \equiv \theta\phi R$, we have

$$\phi = \omega^{\alpha/(\alpha-1)}, \tag{B.14}$$

as claimed in (1.14) in the text. The solution for ψ and λ given in (1.15) and (1.16) follows from equations (B.9) and (B.10).

When $\alpha \to 1$, equation (B.5) is replaced by

$$\phi a + \psi y + \lambda = \delta^{\delta}(1-\delta)^{1-\delta}(\phi R)^{\delta-1}\{\phi Ra + \lambda + (\phi+\psi)[(1-\rho)\hat{y} + \varepsilon^*]\}. \tag{B.15}$$

For (B.6) to hold as an identity, one needs, as claimed in the text, that

$$\phi = (1-\delta)(\delta R)^{\delta/(1-\delta)}, \tag{B.16}$$

with ψ and λ satisfying (B.9) and (B.10). It immediately follows from (B.4) that

$$\omega \equiv \theta\phi R = 1 - \delta, \tag{B.17}$$

so that, using (B.3), (B.9) and (B.10),

$$c = (1-\delta)\left\{a + \frac{1}{R-\rho}\rho y + \frac{1}{R-\rho}\frac{R}{R-1}[(1-\rho)\hat{y} + \varepsilon^*]\right\}. \tag{B.18}$$

Appendix C verifies that the implied consumption process remains always positive if $\delta R > 1$.

APPENDIX C

This appendix establishes that consumption never becomes negative if Assumptions 1 to 4 are satisfied.

It follows from equation (1.24) and the definition of $\underline{\varepsilon}$ that

$$c_{t+1} \geqq (\delta R)^{1/\alpha} c_t + \omega \frac{R}{R-\rho}(\underline{\varepsilon} - \varepsilon^*), \tag{C.1}$$

with equality achieved whenever the worst possible income shock $\underline{\varepsilon}$ is realized.

A deterministic lower bound—which is attained at any finite date with positive probability—on the consumption process is thus given by

$$c_t \geqq x_t, \tag{C.2}$$

where the sequence $\{x_t\}_{t=0}^{\infty}$ is recursively defined by

$$x_0 = c_0 = \omega \frac{R}{R-\rho}\left\{y_0 + \frac{1}{R-1}[(1-\rho)\hat{y} + \varepsilon^*]\right\} > 0 \tag{C.3}$$

$$x_{t+1} = (\delta R)^{1/\alpha} x_t + \omega \frac{R}{R-\rho}(\underline{\varepsilon} - \varepsilon^*). \tag{C.4}$$

Hence we need $x_t \geqq 0$ for all $t \geqq 0$ to guarantee that consumption does not become negative with positive probability in finite time.

Since agents are risk-averse, $\underline{\varepsilon} - \varepsilon^* < 0$ ($\underline{\varepsilon} = \varepsilon^*$ only if consumers are infinitely risk-averse). Therefore, if, contrary to Assumption 3, $\delta R \leqq 1$, Then x_t becomes negative after a long but finite string of bad income realizations. Whence the necessity of Assumption 3 for the solution exhibited in the text to be valid.

Imposing $\delta R > 1$ is however not sufficient to guarantee that $x_t > 0$. We need, in addition, that $x_0 = c_0$ be at least as large as the (now unstable) negative steady state of the difference equation (C.4). Formally one must have

$$\omega \frac{R}{R-\rho}\left\{y_0 + \frac{1}{R-1}[(1-\rho)\hat{y} + \varepsilon^*]\right\} \geqq \omega \frac{R}{R-\rho}\frac{\varepsilon^* - \underline{\varepsilon}}{(\delta R)^{1/\alpha} - 1} > 0. \tag{C.5}$$

For this restriction to hold whichever the initial income realization $y_0 \geqq 0$ and whichever the degree of risk aversion β (i.e., for all $\varepsilon^* \leqq \mu$), it suffices, when Assumptions 1, 2 and 3 hold, that

$$(1-\rho)\hat{y} + \underline{\varepsilon} \geqq \frac{R - (\delta R)^{1/\alpha}}{(\delta R)^{1/\alpha} - 1}(\mu - \underline{\varepsilon}) \geqq 0 \tag{C.6}$$

which is simply Assumption 4.

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