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# Risk Preferences and the Welfare Cost of Business Cycles*

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This paper reexamines the "cost of business cycle" calculations made by Lucas ("Models of Business Cycles," Basil Blackwell, New York, 1987) and İmrohoroglŭ (*J. Polit. Econ.* **97** (1989), 1364–1383) under alternative specifications of individuals' risk preferences and using alternative specifications of the stochastic process for per capita consumption. I find that for a class of preferences used by Epstein and Zin (*J. Monetary Econom.* **26** (1990), 387–407), in an analysis of the equity premium puzzle, which display "first-order" risk aversion, the welfare cost of business cycles is potentially much larger than previous estimates. *Journal of Economic Literature* Classification Numbers: E32, D81. 

## 1. INTRODUCTION

In a simple and thought-provoking exercise, Lucas [8] estimated the welfare cost of business cycle fluctuations, using the preferences of a representative agent. By plugging hypothetical consumption streams into the agent's utility function, Lucas calculated the gain from eliminating business cycles altogether as the minimum percentage increase in the level of consumption in every period needed to render the agent indifferent between a consumption stream with a realistic variance-about-trend and a deterministic one with the same average growth rate. The striking conclusion of Lucas's analysis was that the gain from eliminating business cycles was quite small—eliminating business cycles was equivalent, in Lucas's

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most risk-averse agent's eyes, to at most a 0.17% increase in the level of consumption in all periods. İmrohoroğlu [5] later performed a Lucas-type experiment in an environment with many individual agents facing idiosyncratic, imperfectly insurable income risk. While İmrohoroğlu found costs larger than those calculated by Lucas, her upper bound—for an economy in which the only means of "self-insurance" available to agents is a storage technology, and for the highest level of risk-aversion which she considers —is a 1.3% increase in the average level of consumption needed to compensate agents (in an ex ante sense) for living in an environment with business cycles.

This paper extends Lucas's analysis to the consideration of alternative specifications of individuals' preferences toward risk and intertemporal substitution, as well as alternative specifications of the stochastic process governing consumption growth.

Evidence on consumption growth and asset returns—e.g., the equity premium puzzle—casts at least some doubt on whether the standard isoelastic, expected-utility preference specification employed by both Lucas and İmrohoroğlu is the "correct" one. Whether the quantitative estimates obtained in those earlier exercises are robust to plausible deviations from the standard specification is worth examining, particularly as the earlier estimates, if correct, have rather strong policy implications. To quote Lucas:

> [The exercise] suggests that the main social gains from a deeper understanding of business cycles, whatever form this deeper understanding may take, will be in helping us to see how to avoid large mistakes with policies that have minimally inefficient side-effects, not in devising ever more subtle policies to remove the residual amount of business-cycle risk. (Lucas [8, p. 31])

The class of preferences I consider, employed by Epstein and Zin [2] in a study of the equity premium puzzle, encompasses the standard isoelastic specification as well as several "non-expected-utility" specifications, including preferences which allow a separation of the parameters governing risk aversion and intertemporal substitution, and preferences which display first-order risk aversion. "First-order risk aversion" means that the risk premium associated with a small gamble is proportional to the standard deviation of the gamble, rather than the variance; if the gamble is truly small, the standard deviation will be larger than the variance.

In terms of consumption processes, I consider—in addition to the deterministic-trend-plus-*i.i.d.*-disturbances specification used by Lucas—processes which allow for a greater persistence of shocks to consumption. In particular, I also calculate business cycle costs using consumption processes in which business cycle fluctuations are represented by autocorrelated shocks about a deterministic trend or as stochastic fluctuations in the growth rate of consumption itself. In the years since the work of Nelson and Plosser [10], there has been no clear consensus on whether business cycle fluctuations in macroeconomic aggregates such as consumption are more adequately described as transitory fluctuations about deterministic trends or as disturbances with more permanent consequences. The experiments in this paper show that the costs associated with business cycle fluctuations in consumption can, in fact, vary a great deal with changes in the posited stochastic process for consumption. Moreover, the experiments show that the preference parameters which matter most in determining the cost of fluctuations depend crucially on the degree of persistence of shocks to consumption.

Consistent with the results of Pemberton [11], I find that when one allows preferences which display first-order risk aversion, the welfare cost of business cycle fluctuations in consumption is potentially much higher than the estimates calculated by either Lucas or İmrohoroğlu. However, I find that the magnitude of the welfare cost is potentially much higher than even Pemberton's estimates, depending on whether business cycle fluctuations in consumption are treated as *i.i.d.* disturbances around a linear deterministic trend, autocorrelated disturbances around a linear trend, or fluctuations in the growth rate of consumption. While adding persistence to consumption shocks very naturally raises the welfare cost of business cycles across all utility specifications, the experiments below show that for first-order risk-averse preferences, the effect of adding persistence is particularly dramatic. For the expected utility case considered by Lucas, taking account of the persistence of shocks to consumption can raise the cost estimates from $1/10\%$ of initial consumption to roughly 0.5%. For first-order risk-averse preferences, costs on the order of 1% in the *i.i.d.* case can rise as high as 23% in the stochastic growth case, for extreme amounts of risk aversion, with requisite consumption increases on the order of 2–5% for even a moderate degree of risk aversion. In contrast, the largest cost estimate reported in Pemberton is 1.09% of initial consumption.[1]

The remainder of the paper is organized as follows. The next section presents the preference specification which will be employed in the subse-

[1] This figure is also for a degree of risk aversion much greater than any which we consider below. Pemberton also measures the cost of business cycles simply as the risk premium associated with a timeless binary gamble between two consumption levels, calibrated to have a variance equal to the variance of Hodrick–Prescott detrended per capita consumption. The experiments I report on below solve for the stochastic process for a representative individual's lifetime utility, thus highlighting the interaction between aversion to risk and willingness to substitute intertemporally. The cost of business cycles, as shown below, depends on both.

quent analysis and discusses some of its key features. Section 3 describes the experiment, which is ‘‘Lucas-like’’ in character. Section 4 presents the results. Details of the solution methods are contained in an appendix.

## 2. INTERTEMPORAL RISK PREFERENCES

### 2.1. The General Intertemporal Framework

The preferences which I employ in this paper are recursive, with lifetime utility at each date a function of known current consumption and a ‘‘certainty equivalent’’ value of random lifetime utility from the next period onward. A special case of this form is the standard intertemporal expected utility specification. The treatment here is somewhat heuristic; more details can be found in Epstein and Zin [2].

The most general specification of this class of preferences assumes that lifetime utility from date $t$ onward is a function of known consumption at date $t$ and a certainty equivalent of random future utility,

$$U_t = W\Big(c_t, \mu\big(\tilde{U}_{t+1} : \mathscr{I}_t\big)\Big),$$

where $\mu(\cdot : \mathscr{I}_t)$ is a certainty equivalent operator, conditional on information $\mathscr{I}_t$ available at date $t$, and $W$ is, in the language of Koopmans [6], an ‘‘aggregator.’’ The precise form for the aggregator $W$ which I employ, as in Epstein and Zin [2], has a constant elasticity of substitution between current consumption and certainty-equivalent future utility,

$$W(c, m) = [c^{\rho} + \beta m^{\rho}]^{1/\rho}.$$

Thus, lifetime utility evolves according to

$$U_t = \Big[c_t^{\rho} + \beta u\big(\tilde{U}_{t+1} : \mathscr{I}_t\big)^{\rho}\Big]^{1/\rho}. \tag{1}$$

The parameter $\rho$ is assumed to satisfy $\rho < 1$, $\rho \neq 0$.

The properties of (1) for deterministic consumption streams are well known. For deterministic streams, $U_t$ is ordinally equivalent to

$$u_t \equiv \sum_{j=0}^{\infty} \beta^j c_{t+j}^{\rho} / \rho,$$

where $u_t = U_t^{\rho}/\rho$. Thus, for evaluating deterministic consumption streams, the parameter $\beta$ is simply the individual's utility discount factor, while $1/(1-\rho)$ corresponds to the individual's elasticity of intertemporal substitution.

## 2.2. Risk Preferences

The individual's attitudes toward risk depend on the precise form which is assumed for the certainty equivalent $\mu$.[2] In all cases, $\mu$ is assumed to be linearly homogeneous in the sense that

$$\mu\left(\lambda \tilde{U}_{t+1} : \mathscr{I}_t\right) = \lambda \mu\left(\tilde{U}_{t+1} : \mathscr{I}_t\right)$$

for all constants $\lambda$. As shown by Epstein and Zin [2], for appropriate choices of $\mu$, the utility process (1) subsumes as special cases the standard, isoelastic, expected-utility preferences used by Lucas and İmrohoroğlu, the non-expected-utility preferences used by Epstein and Zin [1], [3] in their study of consumption and asset returns, as well as intertemporal extensions of the sort of risk preferences formulated by Quiggin [12], Yaari [15], and others.

To formulate the different cases precisely, though, requires concrete assumptions about the nature of the stochastic consumption process. In the experiments below, I consider consumption paths which consist of stationary disturbances around a deterministic time trend and paths with growth rates which follow first-order Markov processes, as in the equity premium literature. To proceed immediately to these particular cases, though, would tend to obscure the nature of the risk preferences implied by different specifications of the certainty equivalent $\mu$.

For simplicity, then, suppose only that there is a "state" variable, $s_t$, which follows an $n$-state Markov chain—precisely, that $s_t \in \{s(i) : i = 1, 2, \dots, n\}$ for all $t$, and there is given a matrix of transition probabilities,

$$\begin{aligned} P &= \left[P_{ij}\right]_{i,j=1,2,\dots,n} \\ &= \left[\Pr\{s_{t+1} = s(j) : s_t = s(i)\}\right]_{i,j=1,2,\dots n}. \end{aligned}$$

Suppose, too, that current consumption $c_t$ is part of $s_t$, and that lifetime utility at each date $t$ can be expressed as a function of $s_t$. In other words, in terms of the utility process (1) above, the information $\mathscr{I}_t$ is precisely the value of the current state $s_t$, and lifetime utility evolves according to

$$U(s_t) = \left[c_t^{\rho} + \beta\mu\left(U(\tilde{s}_{t+1}) : s_t\right)^{\rho}\right]^{1/\rho}.$$

[2] This statement is somewhat heuristic. In an intertemporal context, both $\mu$ and $\rho$ matter for risk aversion, as will become apparent in the experiments below. However, for gambles which are "timeless"—either static or one-shot gambles over constant consumption paths—only the parameters of $\mu$ matter in the measurement of risk aversion.

Suppose also that the states are ordered so that $U(s(1)) \le U(s(2)) \le \cdots \le U(s(n))$, and consider the following certainty equivalent of future utility, conditional on today's state being $s = s(i)$:

$$\mu(U(\tilde{s}) : s(i)) \equiv \left\{ \sum_{j=1}^{n} \left[ \left( \sum_{h=1}^{j} P_{ih} \right)^{\gamma} - \left( \sum_{h=1}^{j-1} P_{ih} \right)^{\gamma} \right] U(s(j))^{\alpha} \right\}^{1/\alpha}, \quad (2)$$

for $\gamma \in [0,1]$ and $\alpha \le 1$. The notation above assumes, naturally, that $\sum_1^{-1} P_{ih} = 0$. When $n = 2$, for example, we have

$$\mu(U(\tilde{s}) : s(i)) = \left[ P_{i1}^{\gamma} U(s(1))^{\alpha} + (1 - P_{i1}^{\gamma}) U(s(2))^{\alpha} \right]^{1/\alpha}. \quad (3)$$

If $\gamma = 1$, we obtain

$$\begin{aligned} \mu(U(\tilde{s}) : s(i)) &= \left[ P_{i1} U(s(1))^{\alpha} + (1 - P_{i1}) U(s(2))^{\alpha} \right]^{1/\alpha} \\ &= \left[ P_{i1} U(s(1))^{\alpha} + P_{i2} U(s(2))^{\alpha} \right]^{1/\alpha} \end{aligned}$$

for the $n = 2$ case; more generally, when $\gamma = 1$, we have

$$\mu(U(\tilde{s}) : s(i)) = \left( E\left[ U(\tilde{s})^{\alpha} : s(i) \right] \right)^{1/\alpha},$$

and the resulting utility process is identical to that considered by Epstein and Zin in their study of asset returns and consumption, [1], [3]—i.e.,

$$U_t = \left[ c_t^{\rho} + \beta \left( E\left[ \tilde{U}_{t+1}^{\alpha} : s_t \right] \right)^{\rho/\alpha} \right]^{1/\rho}.$$

For this specification, the parameter $\alpha$ governs risk aversion—which increases as $\alpha$ shrinks—and the parameter $\rho$ governs intertemporal substitution for deterministic consumption streams.

If, in addition, $\alpha = \rho$, the utility process (1) becomes ordinally equivalent to the standard isoelastic, expected utility formulation,

$$u_t = c_t^{\rho}/\rho + \beta E_t u_{t+1},$$

where $u \equiv U^{\rho}/\rho$, since $\gamma = 1$ and $\alpha = \rho$ imply

$$U_t = \left[ c_t^{\rho} + \beta E\left[ \tilde{U}_{t+1}^{\rho} : s_t \right] \right]^{1/\rho},$$

or

$$U_t^{\rho}/\rho = c_t^{\rho}/\rho + \beta E\left[ \tilde{U}_{t+1}^{\rho}/\rho : s_t \right].$$

If $\gamma < 1$, we have a ‘‘rank-dependent’’ certainty equivalent, which displays first-order risk aversion. The rank dependence is most easily seen in (3)—in a sense, the probability of the worse outcome, $s(1)$, is transformed from $P_{i1}$ to $P_{i1}^{\gamma}$, while that of the better outcome becomes $1 - P_{i1}^{\gamma}$. One cannot ‘‘relabel’’ the states and their associated probabilities—taking state one into state two and state two into state one—without affecting the evaluation. Which outcome is worse matters for the evaluation.

When we also have $\alpha = 1$, $\mu$ is precisely the type of certainty equivalent which derives from Yaari’s [15] ‘‘dual theory’’ of choice under uncertainty,

$$\mu(U(\tilde{s}) : s(i)) \equiv P_{i1}^{\gamma} U(s(1)) + (1 - P_{i1}^{\gamma}) U(s(j))$$

in the $n = 2$ case, and more generally,

$$\mu(U(\tilde{s}) : s(i)) \equiv \sum_{j=1}^{n} \left[ \left( \sum_{h=1}^{j} P_{ih} \right)^{\gamma} - \left( \sum_{h=1}^{j-1} P_{ih} \right)^{\gamma} \right] U(s(j)).$$

Here, $\gamma$ measures risk aversion, with $\gamma = 1$ corresponding to risk neutrality, and risk aversion growing as $\gamma$ declines. In contrast to expected utility, the Yaari certainty equivalent is linear in payoffs—which are here the values of $U$—and nonlinear in probabilities.[3]

In the more general case, where neither $\gamma$ nor $\alpha$ is necessarily equal to one, both parameters matter for risk aversion, which increases as either $\gamma$ or $\alpha$ falls.

As noted by Epstein and Zin [2], when $\gamma < 1$, the certainty equivalent $\mu$ displays ‘‘first-order’’ risk aversion—that is, the risk premium it generates for a small gamble is proportional to the standard deviation of the gamble, rather than the variance.[4] As I discuss below, and as is discussed in Epstein and Zin [2], this is an important feature which allows risk preferences of this class to be calibrated so as to give plausible answers to questions about a wide range of risks; it is also a feature which accounts for their better performance with respect to the equity premium puzzle than the standard isoelastic expected utility specification.

[3] It is worth noting that the axioms for Yaari’s dual theory are identical to those for the expected utility theory, with the exception of the ‘independence axiom’ of the latter. Yaari’s theory replaces this with a ‘dual independence axiom’ which, as Yaari notes, is equivalent to requiring that the ranking of two lotteries with payoff vectors $x$ and $y$ is the same as the ranking of lotteries with payoffs $\alpha x + (1 - \alpha)z$ and $\alpha y + (1 - \alpha)z$ for all $z$’s which are not hedges against either $x$ or $y$.

[4] See Segal and Spivak [13] for a formal definition of first-order risk aversion.

### 2.3. Which Specification Is More Plausible?

If one is willing to use the preferences of a representative agent to gauge the costs associated with business cycles—or, as in İmrohoroğlu, the preferences of many identical agents—one has to be sensitive to the question of whether these preferences are plausible descriptions of individuals' attitudes toward risk. The case for the standard, isoelastic, expected utility specification, I suspect, rests largely on a combination of great tractability and consistency with certain important long-run observations—e.g., fairly constant consumption growth and a lack of any discernible trend in the real return to capital. Yet, all of the specifications above—which have, for deterministic paths of consumption, constant intertemporal elasticities of substitution and fixed discount factors—are consistent with the same long-run observations.

What we do know from the literature on the equity premium puzzle, is that the standard isoelastic specification, when coupled with a frictionless Arrow−Debreu environment, fails to rationalize the data on the variability of consumption growth and the structure of asset returns, particularly the level of the risk-free interest rate and the rate spread between risky and riskless assets. Of course, this failure could be due to the presence in reality of important trading frictions not captured in the Arrow−Debreu framework. On the other hand, Epstein and Zin [2], while maintaining the assumption of a frictionless trading environment, find that the preference specifications above which are first-order risk averse—those with $\gamma < 1$—can come closer to matching the equity premium data than the standard specification.[5]

A key feature of both the standard isoelastic, expected utility class of preferences, as well as the earlier Epstein and Zin specification—the two "second-order" risk averse cases above, in which $\gamma = 1$—is that plausible risk premia for small gambles can only be obtained by setting the risk aversion coefficient so large as to make it the case that an agent whose preferences have been calibrated to give plausible answers to questions about small gambles will almost certainly give implausible answers to questions about large gambles.

The "willingness to pay" calculations in Epstein and Zin [2] are instructive in this regard. Using the certainty equivalents above, they calculate the amounts an individual with initial wealth of $75,000 would be willing to pay to avoid timeless gambles which add or subtract, with equal probabilities, sums ranging from $250 to $74,000. For cases with $\gamma = 1$, it is

[5] See Mehra and Prescott [9] for evidence on the standard specification. Weil [14] considers "non-expected-utility" preferences of the sort formulated in Epstein and Zin in their earlier work, [1], [3], and shows that the model's ability to match the data on consumption growth and asset returns is not enhanced. First-order risk aversion is important in this respect.

necessary to drive the risk aversion parameter $\alpha$ down to $\alpha = -29$ to generate a willingness to pay even \$12.48 to avoid the \$250 gamble. The problem, of course, is that an individual with $\alpha = -29$ would be willing to pay nearly \$74,000—\$73,975.81, to be exact—to avoid the gamble of size \$74,000. In the first-order risk-averse case, with $\alpha = 0.75$ and $\gamma = 0.5$, these numbers are \$103.64 for the \$250 gamble and \$43,809.83 for the \$74,000 gamble. I find both of these numbers—\$103.64 and \$43,809.83—a bit large for my own tastes, but they are answers which one would not call wildly risk averse, and they both seem about in the right general area. Taking $\gamma = 0.87$ and $\alpha = 0.75$, say, yields numbers around \$24 and \$21,000 for the two gambles—which, if anything, would be called, at most, a moderate aversion to risk. The essential point, though, is that the first-order risk-averse certainty equivalent yields answers which are at least plausible for a wider range of gambles than do the certainty equivalents with $\gamma = 1$.

This feature derives from the fact, noted in Epstein and Zin [2], that both specifications with $\gamma = 1$ are "smooth at certainty." To see what is meant by this, in the context of the framework above, consider the certainty equivalents of random future utility for the case where there are two states, occurring with probabilities $p$ and $1 - p$. For certainty equivalents with $\gamma = 1$, the locus of payoffs $(U_1, U_2)$ which are indifferent to some $\bar{u}$ with certainty, i.e.,

$$\left\{(U_1, U_2) : \left(pU_1^\alpha + (1 - p)U_2^\alpha\right)^{1/\alpha} = \bar{u}\right\},$$

forms a smooth indifference curve in the space of pairs $(U_1, U_2)$, with slope $-p/(1 - p)$ at "certainty," the 45° line. Loosely, one can visualize aversion to a small gamble as the gap, near the 45° line, between this indifference curve and the locus of pairs with expected value equal to $\bar{u}$—in other words, the set of pairs $(U_1, U_2)$ satisfying

$$pU_1 + (1 - p)U_2 = \bar{u},$$

which is tangent to the indifference curve at the certainty point, and represents the indifference curve for a risk-neutral $(\alpha = 1)$ individual. Generating any significant amount of risk aversion near the certainty point requires a large amount of curvature—$\alpha$ large in absolute value—since the indifference curve is smooth at the certainty point. Certainty equivalents with $\gamma = 1$ are, in a sense, locally risk neutral. The upshot of taking the risk aversion parameter $\alpha$ to be large enough in absolute value to generate a plausible aversion to small gambles, though, is extreme aversion for gambles with payoffs further away from the certainty point.

When $\gamma \neq 1$, the indifference curve is kinked at the certainty point—a consequence of the rank dependence which results when $\gamma < 1$, that is, the dependence of the evaluation on whether the state-one payoff is higher than the state-two payoff or vice versa. If $\alpha = 1$, the indifference curve is in fact piecewise linear, with a slope of $-p^{\gamma}/(1-p^{\gamma})$ above the 45° line—outcomes where the state-one payoff is worse—and a slope of $-(1-(1-p)^{\gamma})/(1-p)^{\gamma}$ below—outcomes where the state-two payoff is worse.[6] When $\alpha < 1$, the piecewise linearity is removed, but the kink at certainty remains. The presence of the kink at certainty is the feature of these risk preferences which makes it possible to generate plausible levels of aversion to small gambles without generating implausible levels of aversion to large gambles.[7]

Given that individuals do show some aversion to small gambles but are not so averse to large gambles as to prefer practically nothing with certainty to a gamble with a 50% chance of leaving them with practically nothing, and given the obvious need to parsimoniously parameterize individuals' risk preferences, the rank-dependent, first-order risk-averse certainty equivalents above would seem good candidates for the task of modeling individuals' risk preferences.

## 3. A LUCAS-TYPE EXPERIMENT

As in Lucas's experiment, I consider a representative agent's evaluation of alternative hypothetical stochastic consumption streams. For each posited consumption process, and using different specifications of the agent's intertemporal preferences, I calculate the minimum percentage increase in initial consumption needed to compensate the agent for a switch from a world of certainty—in which the disturbances to consumption have a zero variance—to a world in which the disturbances to consumption have a variance calibrated to be realistic. The subsections below describe in turn the alternative consumption processes, solution methods, and information about model calibration.

[6] See Fig. 1 in Epstein and Zin [2].

[7] As Epstein and Zin [2] discuss, were $\alpha = 1$—so that $\gamma$ alone determined the agent's risk aversion—then calibrating $\gamma$ to give a plausible degree of aversion to small gambles would yield an agent who was not averse enough to large gambles. Thus, the role of $\alpha$ is in generating sufficient aversion to large gambles in conjunction with a plausible aversion to small gambles.

### 3.1. Alternative Consumption Processes

I consider three different models for the consumption process. The first follows Lucas in specifying $c_t = c_0(1+g)^t \exp(\xi_t)$, or

$$\ln c_t = \ln c_0 + t\ln(1+g) + \xi_t, \tag{4}$$

where $\{\xi_t\}$ is an *i.i.d.* process with mean zero. Note, though, that while Lucas assumes consumption paths of the form above, he takes the standard deviation of the disturbance from figures reported in Kydland and Prescott [7], which are for Hodrick–Prescott filtered data.[8] The variance of log U.S. per capita consumption about a linear trend is several times the variance of the HP "cyclical" component which Lucas uses. When I regress the log of annual postwar per capita consumption on a constant and linear time trend, the standard deviation of the residuals from that regression is 0.0271, about twice the standard deviation which Lucas takes from the HP cyclical component of per capita consumption. The residuals are also, as one would expect, highly serially correlated; a first-order autoregression on them yields a coefficient near unity.

The second model maintains the assumption of a linear deterministic trend, but allows for serial correlation in the disturbances $\xi_t$. That is, the logarithm of consumption is assumed to follow (4), and

$$\xi_t = a\xi_{t-1} + u_t, \tag{5}$$

where $\{u_t\}$ is an *i.i.d.* process with mean zero. In the quantitative experiments below, I set $a = 0.9849$, which is the value I obtain from a first-order autoregression on the residuals from a least-squares-estimated version of (4).

Finally, as in much of the equity premium literature, I consider consumption streams with growth rates which follow autoregressive processes —i.e., consumption processes with

$$c_{t+1}/c_t = \xi_{t+1},$$

where $\{\xi_t\}$ is a stationary autoregressive process—in particular,

$$\xi_t = (1-a)(1+g) + a\xi_{t-1} + u_t, \tag{6}$$

[8] This is also the standard deviation used by Pemberton. Of course, there is a subtle issue here as to what constitutes "business cycles"—or, put differently, if business cycles were eliminated, what would be left? One may very well identify business cycle fluctuations in consumption with the Hodrick–Prescott cyclical component, but, except in the case of logarithmic utility, one cannot sensibly evaluate the "cyclical" component of consumption streams separately from the trend component.

where $u_t$ is *i.i.d.* with mean zero. This implies that innovations to the growth rate of consumption have a permanent effect on the level of consumption; in logarithms, consumption would follow a random walk with serially correlated disturbances. This specification, I think, better captures the features of U.S. per capita consumption than the linear time trend specification used by Lucas, although the true data-generating process no doubt lies somewhere between the two extremes.

### 3.2. Solving for the Agent's Lifetime Utility

For computational purposes, I approximate the consumption processes using finite-state Markov chains. In all cases, I derive a solution for the stochastic process of lifetime utility, suitably normalized—either by extracting a deterministic trend in the cases where consumption obeys (4) or by normalizing by the level of current consumption when the consumption process is (6)—by iteratively solving an appropriate functional equation. The normalized lifetime utility levels can then be expressed in terms of the values of the disturbances $\xi$, which take on values in a finite set, $\{\xi_1, \xi_2, \dots, \xi_n\}$.

As discussed in the Appendix, when consumption follows the linear trend specification (4), lifetime utility normalized by $(1+g)^t c_0$—call it $v(\xi)$ in state $\xi$—obeys

$$v(\xi) = \left[\exp(\xi)^{\rho} + \beta(1+g)^{\rho}\mu\left(v(\tilde{\xi}) : \xi\right)^{\rho}\right]^{1/\rho}. \tag{7}$$

Alternatively, when consumption follows (6), lifetime utility, normalized in this case by the level of current consumption, obeys

$$v(\xi) = \left[1 + \beta\mu\left(\tilde{\xi} v(\tilde{\xi}) : \xi\right)^{\rho}\right]^{1/\rho}. \tag{8}$$

Equations (7) and (8) are functional equations—or more precisely, given the finite-state Markov chain assumption, systems of nonlinear equations in $n$ unknowns—which can be solved iteratively for the normalized lifetime utility functions $v(\xi) = (v(\xi_1), v(\xi_2), \dots, v(\xi_n))$ which satisfy them.

In each case, having solved for normalized lifetime utility in the $n$ states, I then calculate the certainty equivalent of normalized lifetime utility according to $\mu$, using the Markov chain's invariant probabilities over the states, $p = (p_1, p_2, \dots, p_n)$. Since we will be interested in variations in the standard deviation of the disturbances, let $V(\sigma)$ denote the certainty equivalent of normalized lifetime utility obtained via the above procedure for a given standard deviation $\sigma$. That is,

$$V(\sigma) = \left[ \sum_{i=1}^{n} \left( \left( \sum_{j=1}^{i} p_j \right)^{\gamma} - \left( \sum_{j=1}^{i-1} p_j \right)^{\gamma} \right) v(\xi_i)^{\alpha} \right]^{1/\alpha},$$

where $v(\xi)$ is the solution for normalized lifetime utility when $\sigma$ is the standard deviation of the disturbances $\xi$.

Because of the linear homogeneity of the aggregator and certainty equivalent, the proportionate increase in initial consumption needed to render the agent indifferent between two environments with standard deviations of $\sigma$ and $\tilde{\sigma} > \sigma$ is the value of $\lambda$ which satisfies

$$(1 + \lambda)V(\tilde{\sigma}) = V(\sigma). \tag{9}$$

Note also that in all cases, scaling up initial consumption by the factor $1 + \lambda$ raises, in an expected sense, consumption in all future periods in the proportion $1 + \lambda$ as well—so $\lambda$ can also be thought of as a proportionate increase, on average, in consumption in every period.

Using the ‘‘long run’’ certainty equivalent of lifetime utility, which uses the Markov chain’s invariant probabilities, to make welfare comparisons requires some justification. In all of the cases I consider, an individual’s lifetime utility from today onward depends on which state the economy is in today.[9] We could ask the agent how much he or she is willing to pay to move from a particular state in one environment—i.e., a world with one $\sigma$ —to a particular state in another environment—a world with another $\sigma$. Evaluating two consumption processes with different $\sigma$’s according to their ‘‘long run’’ certainty equivalents $V$ amounts, as I see it, to asking the agent to evaluate, in an ex ante sense, the choice of living in two different environments. For that question, the proper probabilities to use—from behind a ‘‘veil of ignorance,’’ so to speak—are the invariant probabilities of the $n$ states.

The experiments which I perform, as in Lucas, calculate $\lambda$ for switches from a world of certainty to a world with a realistic $\sigma$, other things constant, for various values of the preference parameters $\rho$, $\gamma$, and $\alpha$. Throughout the experiments, the discount factor $\beta$ will be held constant. The value I use for $\beta$ is $1.03^{-0.25}$, which would imply a 3% steady-state real interest rate.

[9] This is true even in the case of *i.i.d.* disturbances about a linear trend.

### 3.3. Calibrating the Consumption Processes

To perform these experiments, we need to calibrate the parameters of the consumption processes for the "realistic" environment. The data I use are the Citibase quarterly series on consumption of nondurables and services from first quarter 1947 to second quarter 1992. It is natural to exclude durables: purchases of durables—which is what is measured—have high variability, though we know that consumption of the services of durables is probably much smoother. The population data used to obtain per capita consumption are the U.S. Census monthly data on all citizens, averaged over quarters.

An OLS regression of the logarithm of per capita consumption on a constant and a linear time trend—for the cases where consumption follows (4)—yields an estimate of the quarterly growth rate equal to 0.5%, or about 2% annually. The standard deviation of the residuals from this regression is 0.0271. A subsequent $AR(1)$ fit to the residuals—for the case of (5)—gives a coefficient of $a = 0.9849$.

In the case where consumption growth follows an autoregressive process (6), fitting a simple $AR(1)$ to per capita consumption growth yields estimates of $a = 0.2163$ and $(1 - a)(1 + g) = 0.7871$, implying $1 + g = 1.0043$ —a 0.43% quarterly growth rate, or a 1.73% annual growth rate of per capita consumption. The standard deviation of the residuals from this regression is 0.0056, implying that the unconditional standard deviation of $\xi_t$ is $\sigma = 0.0057$.

I use standard $n$-state Markov chain representations which capture unconditional means and standard deviations, as well as autocorrelation.[10] In each case the invariant probabilities over the $n$ states place approximately 70% of the mass of the distribution within $\pm 3/4$th standard deviations from the mean. Thirty percent of the mass is at or outside $\pm 1.10$ standard deviations.

## 4. RESULTS

I divide the results below into three cases corresponding to the three stochastic processes governing consumption—that is, *i.i.d.* disturbances about a linear trend (4), autocorrelated disturbances about a linear trend (4 and 5), or the $AR(1)$ process for consumption growth (6). For each process, I then present welfare cost figures for each specification of risk preferences. The specifications of risk preferences can be divided into two cases, according to whether the certainty equivalent employed is "smooth

[10] See the Appendix for more details.

at certainty'' ($\gamma = 1$) or ''kinked''—i.e., first-order risk averse ($\gamma < 1$). The first, ''smooth'' case encompasses both the standard isoelastic expected utility specification (in which $\alpha = \rho$) and the generalization of the standard specification used by Epstein and Zin ([1], [3]) in their theoretical and empirical analyses of the consumption-based CAPM. The first-order risk-averse case can be divided into what one might call a pure ''Yaari'' specification (in which $\alpha = 1$) and a more general first-order risk-averse specification ($\alpha < 1$).

As will become clear below, for the experiments here, whether $\alpha$ is equal to one or is slightly less than one does not make a great deal of difference, much as in Epstein and Zin [2]. Intuitively, $\alpha$ would figure more prominently if the ''gambles'' involved were larger. It is the kink at certainty, which owes to the fact that $\gamma < 1$, that matters most for small gambles; taking $\alpha < 1$ is important only for combining plausible aversion to small gambles with plausible aversion to large gambles. Consequently, we will make only one reference to the $\alpha < 1$ case, in the first experiment, with the remainder of the $\gamma < 1$ results assuming $\alpha = 1$.

## 4.1. Consumption: a Linear Trend Plus i.i.d. Disturbances

We begin, as in Lucas, with a consumption process which consists, in logarithmic form, of a linear time trend plus *i.i.d.* disturbances—i.e., the process given in (4).

### 4.1.1. *Results for Smooth Certainty Equivalents*

We initially set $\gamma = 1$ and consider ''smooth'' certainty equivalents of the form

$$\mu\left(v(\tilde{\xi}) : \xi_i\right) = \left[\sum_{j=1}^{n} P_{ij} v(\xi_j)^{\alpha}\right]^{1/\alpha}.$$

Of course, in the case where consumption obeys (4) and the disturbances are *i.i.d.*, $P_{ij} = p_j$ for all $i$, and there is then no dependence of $\mu$ on the current state.

The utility process—given in normalized form in (7)—corresponds to that employed by Epstein and Zin ([1], [3]), with the parameter $\rho$ governing attitudes toward intertemporal substitution for deterministic consumption streams and $\alpha$ governing risk aversion for timeless gambles. In fact, $1 - \alpha$ is the agent's Arrow–Pratt coefficient of relative risk aversion for timeless gambles, while $1/(1 - \rho)$ is the agent's elasticity of intertemporal substitution for deterministic consumption streams. In this case, $\alpha = 1$ corresponds to risk neutrality, and risk aversion rises as $\alpha$ decreases. If

$\alpha = \rho$, these preferences correspond to the expected utility preferences employed by Lucas.

The first case we will look at is, in fact, that of expected utility. In this case, the coefficient of relative risk aversion for timeless gambles, $1 - \alpha$, is equal to the inverse of the elasticity of intertemporal substitution, $1/(1 - \rho)$. The welfare cost values are reported in Table I, for coefficients of relative risk aversion equal to 2, 5, 10 and 20—i.e., for values of $\alpha$ (or $\rho$) equal to $-1$, $-4$, $-9$, and $-19$. The entries in the table are the percentage increases in consumption needed to compensate the agent for a switch from a world of certainty to a world with a realistic variance of the disturbances to consumption—i.e., the entries are 100 times the values of $\lambda$ satisfying (9).

The welfare cost values range from just under a 0.04% increase in consumption to just under a 0.70% increase in consumption for a switch from certainty to a realistic variance. In per capita dollar terms—taking U.S. annual per capita consumption to be about $20,000—these costs are on the order of $7.34 to $139 per person per year.

The upper end of this range, which corresponds to a value of $1 - \rho = 1 - \alpha = 20$, is quite a bit larger than what Lucas obtained for the same risk aversion coefficient; for a coefficient of relative risk aversion equal to 20, Lucas obtained a value slightly less than 2/10%. One would expect the numbers reported here to be somewhat larger, however, since the "realistic" standard deviation used in these calculations is the estimate $\sigma = 0.0271$ rather than the $\sigma = 0.013$ used by Lucas and Pemberton.

Allowing $\alpha$ to differ from $\rho$ yields no cost value greater than the 0.6958% of Table I—at least for $\alpha$ and $\rho$ taking on values in the same range—though one interesting result does come out when we allow the agent's degree of risk aversion and willingness to substitute intertemporally to vary independently. In particular, as Table II below shows, the proper interpretation of the numbers in Table I is not that the welfare cost of business cycles rises as the agent becomes more risk averse—though this is, of course, correct in a technical sense—but, rather, the welfare cost of business cycles rises as the agent becomes less willing to substitute over time.

TABLE I

Business Cycle Costs with Expected Utility, $c_t = c_0(1+g)^t \exp(\xi_t)$, $\xi_t$ *i.i.d.*

| Coefficient of relative risk aversion: | 2 | 5 | 10 | 20 |
|---|---|---|---|---|
| | 0.0367 | 0.1469 | 0.3306 | 0.6958 |

TABLE II
Business Cycle Costs with "Smooth" Certainty Equivalents,
$c_t = c_0(1+g)^t \exp(\xi_t)$, $\xi_t$ *i.i.d.*

| | | Elasticity of intertemporal substitution: 1/2 | 1/5 | 1/10 | 1/20 |
|---|---|---|---|---|---|
| Degree of risk aversion: | 2 | 0.0367 | 0.1440 | 0.3153 | 0.6270 |
| | 5 | 0.0381 | 0.1469 | 0.3210 | 0.6384 |
| | 10 | 0.0403 | 0.1519 | 0.3306 | 0.6575 |
| | 20 | 0.0449 | 0.1619 | 0.3497 | 0.6958 |

Table II shows the percentage increases in consumption needed to compensate the agent for a switch from certainty to a realistic variance, for various combinations of $\alpha$ and $\rho$. The "degree of risk aversion" values shown in the table are values of $1-\alpha$ for $\alpha = -1, -4, -9$, and $-19$. The "elasticity of intertemporal substitution" values are values of $1/(1-\rho)$ as $\rho = -1, -4, -9$, or $-19$. Note that the main "diagonal" of the table —entries where $\alpha = \rho$—simply replicates the numbers in Table I.

As one can see, the largest cost value remains 0.6958, or a little less than a 7/10% increase in initial consumption; this value occurs where the agent is most risk averse and least willing to substitute over time. Looking across the rows and down the columns of the table, one sees the "interesting result" mentioned above: at least for this consumption process, the agent's willingness to substitute intertemporally has a much larger impact on the cost of fluctuations in consumption than does the agent's aversion to risk. In particular, for a given elasticity of intertemporal substitution, the influence of the degree of risk aversion parameter on the cost values is fairly negligible; on the other hand, for a given degree of risk aversion, the costs vary substantially with the agent's elasticity of intertemporal substitution.

### 4.1.2. *Results for First-order Risk-averse Certainty Equivalents*

We now turn to consider the first-order risk-averse preferences which obtain when we take $\gamma < 1$ in the certainty equivalent operator $\mu$. The certainty equivalent of normalized future utility $v(\xi)$ then has the form

$$\mu(v(\xi):\xi_i) = \left[\sum_j \left(\left(\sum_{h=1}^{j} P_{ih}\right)^{\gamma} - \left(\sum_{h=1}^{j-1} P_{ih}\right)^{\gamma}\right) v(\xi_j)^{\alpha}\right]^{1/\alpha}.$$

The utility process is again given by (7), and, in this case of *i.i.d.* disturbances, there is no dependence of $\mu$ on the current state. Setting

$\alpha = 1$ in this certainty equivalent corresponds to what I have called the "Yaari" case. When $\alpha = 1$, $\gamma$ alone governs risk aversion—with smaller $\gamma$ corresponding to greater aversion to risk—while $\rho$ again governs intertemporal substitution for deterministic streams.

Table III below gives percentage increases in consumption needed to compensate the agent for a switch from certainty to a realistic variance for the "Yaari" case of $\alpha = 1$. The "degree of risk aversion" values in this table are values of $\gamma$—in particular, $\gamma$ ranging from 0.99 down to 0.70, with risk aversion increasing as we move down the table, just as in Table II. The "elasticity of intertemporal substitution" values are again values of $1/(1-\rho)$ for $\rho = -1, -4, -9$, and $-19$.

The maximum cost value in the table is roughly 1.6%. The maximum occurs at $\gamma = 0.70$—the highest level of risk aversion shown—and $1/(1-\rho) = 1/20$—the smallest elasticity of intertemporal substitution shown. Even at $\gamma = 0.90$, costs greater than 0.5% and even 0.9% are possible, if the agent is relatively unwilling to substitute intertemporally. In per capita annual dollar terms, costs of this size are in the rough range of $57 to $180 per person per year. If the representative agent is as risk averse as, say, $\gamma = 0.87$, the costs in dollar terms rise to between $74 and $198 per person per year.

How risk averse is an individual with $\gamma = 0.87$? The "degrees of risk aversion" values in the table can be put into perspective by reconsidering the "willingness to pay" calculations discussed in Section 2.3 above. For an individual with initial wealth of $75,000, faced with a (timeless) gamble which adds or subtracts, with equal probability, $250 from the individual's wealth, the individual's willingness to pay to avoid the gamble, for the $\gamma$ values used in Table III, is presented in Table IV. Thus, an individual with risk preferences of the form used in this section, whose degree of risk aversion was $\gamma = 0.87$, would be willing to pay $23.57 to get out of the $250 gamble. While there is certainly room for argument here, I would

TABLE III
Business Cycle Costs with First-order Risk Aversion, $\alpha = 1$,
$c_t = c_0(1+g)^t \exp(\xi_t)$, $\xi_t$ *i.i.d.*

| | | Elasticity of intertemporal substitution: | | | |
|---|---|---|---|---|---|
| | | 1/2 | 1/5 | 1/10 | 1/20 |
| Degree of risk aversion: | 0.99 | 0.0595 | 0.1662 | 0.3366 | 0.6460 |
| | 0.95 | 0.1575 | 0.2664 | 0.4402 | 0.7556 |
| | 0.90 | 0.2873 | 0.3991 | 0.5774 | 0.9005 |
| | 0.87 | 0.3695 | 0.4831 | 0.6642 | 0.9920 |
| | 0.75 | 0.7355 | 0.8568 | 1.0494 | 1.3961 |
| | 0.70 | 0.9084 | 1.0330 | 1.2305 | 1.5847 |

TABLE IV
Willingness to Pay to Avoid a Small Gamble

| Degree of risk aversion: | 0.99 | 0.95 | 0.90 | 0.87 | 0.75 | 0.70 |
|---|---|---|---|---|---|---|
| | \$1.74 | \$8.82 | \$17.94 | \$23.57 | \$47.30 | \$57.79 |

suggest that such a person is not wildly risk averse. For this same individual—depending on his or her elasticity of intertemporal substitution—business cycle costs on the order of \$74 per year to \$198 per year are possible.

To see that there is little consequence to whether $\alpha$ is equal to one or is slightly less than one, as argued above, Table V replicates the experiment in Table III for a value of $\alpha = 0.75$. As is clear, the results are virtually identical. Consequently, in the experiments below, we will restrict our attention to the case of $\alpha = 1$.[11]

Finally, note that while the results in Tables III and V show that with first-order risk aversion there is a greater dependence of the cost of fluctuations on the agent's degree of risk aversion than in the "smooth" case, much of the variation in the cost values even in the first-order risk-averse case is still due to variation in the agent's willingness to substitute intertemporally. This is particularly true if one looks at the rows of Tables III and V, which correspond to values of $\gamma$ other than the two

[11] One may wonder at the choice of $\alpha = 0.75$ in the table. Again referring to the willingness to pay calculations of Section 2.3, if an individual's $\gamma$ is in the range of values in the table, then taking $\alpha$ much below 0.75 would result in an implausibly large aversion to sizable gambles.

TABLE V
Business Cycle Costs with First-order Risk Aversion, $\alpha = 0.75$,
$c_t = c_0(1+g)^t \exp(\xi_t)$, $\xi_t$ *i.i.d.*

| | | Elasticity of intertemporal substitution: 1/2 | 1/5 | 1/10 | 1/20 |
|---|---|---|---|---|---|
| Degree of risk aversion: | 0.99 | 0.0597 | 0.1665 | 0.3371 | 0.6469 |
| | 0.95 | 0.1576 | 0.2666 | 0.4407 | 0.7565 |
| | 0.90 | 0.2874 | 0.3994 | 0.5779 | 0.9015 |
| | 0.87 | 0.3696 | 0.4834 | 0.6647 | 0.9930 |
| | 0.75 | 0.7356 | 0.8571 | 1.0500 | 1.3972 |
| | 0.70 | 0.9086 | 1.0333 | 1.2310 | 1.5857 |

most extreme degrees of risk aversion, $\gamma = 0.75$ and $\gamma = 0.70$. If business cycle fluctuations in consumption are really best described as *i.i.d.* shocks around a linear deterministic trend, then it is not enough simply to get individuals' risk preferences "correct"—the accuracy of one's measure of the cost of those fluctuations will still depend crucially on the accuracy with which one measures individuals' willingness to substitute consumption over time.

## 4.2. Consumption: A Linear Trend Plus Autocorrelated Disturbances

In this section, we repeat the previous experiments for the case where the log level of consumption consists of stationary disturbances about a linear trend (4), as before, but with the disturbances following the first-order autoregression described in (5) with coefficient given by the estimate $a = 0.9849$ taken from the data. Normalized lifetime utility follows the same recursion as in the previous section (given in Eq. 7), except that now the certainty equivalent of normalized future utility depends on the realization of today's disturbance.

### 4.2.1. *Results for Smooth Certainty Equivalents*

We again begin by looking at the case where $\gamma = 1$, so that the certainty equivalent operator $\mu$ is "smooth at certainty"; again, $1 - \alpha$ is the agent's risk aversion coefficient for timeless gambles, and $1/(1 - \rho)$ is the agent's elasticity of intertemporal substitution for deterministic consumption streams. Rather than looking at the expected utility case (in which $\alpha = \rho$) and the more general smooth case separately—the former being encompassed by the latter—Table VI repeats the experiment from Table II, giving requisite percentage increases in consumption, for various combinations of the parameters $\alpha$ and $\rho$, for a switch from certainty to a realistic variance of the disturbances to consumption. The results for the expected

TABLE VI
Business Cycle Costs with "Smooth" Certainty Equivalents,
$c_t = c_0(1 + q)^t \exp(\xi_t),\ \xi_t = a\xi_{t-1} + u_t$

| | | Elasticity of intertemporal substitution: | | | |
|---|---|---|---|---|---|
| | | 1/2 | 1/5 | 1/10 | 1/20 |
| Degree of risk aversion: | 2 | 0.0367 | 0.0508 | 0.0517 | 0.0479 |
| | 5 | 0.1135 | 0.1469 | 0.1563 | 0.1563 |
| | 10 | 0.2416 | 0.3072 | 0.3306 | 0.3369 |
| | 20 | 0.4996 | 0.6269 | 0.6769 | 0.6958 |

utility case can be seen by restricting attention to the diagonal elements of the table.

While there is a change in the pattern of the resulting welfare costs as the parameters governing intertemporal substitution and risk aversion vary, the upper bound on the cost figures is essentially unchanged. The maximum value again occurs at the highest degree of risk aversion and smallest elasticity of intertemporal substitution, and is again roughly a 0.7% increase in initial consumption. On the diagonal of the table—where $1 - \alpha = 1/(1 - \rho)$, and expected utility obtains—the costs with autocorrelated disturbances are indistinguishable from their counterparts in the *i.i.d.* case.

What *is* significantly different in Table VI is that now, with a high degree of autocorrelation in the disturbances, the cost values are less sensitive to changes in the elasticity of intertemporal substitution, $1/(1 - \rho)$, and more sensitive to changes in the degree of risk aversion, $1 - \alpha$. For a given value of the risk aversion coefficient, the costs rise only slightly as the agent's elasticity of intertemporal substitution falls from $1/2$ to $1/20$. In contrast, for a given value elasticity of the intertemporal substitution, the costs rise sharply as $1 - \alpha$ moves in the direction of greater risk aversion. Furthermore, compared with the results in Table II, costs "below the diagonal"—where the coefficient of relative risk aversion is greater than the inverse of the elasticity of intertemporal substitution—are higher, while those "above the diagonal"—where the risk aversion coefficient is than the inverse of the elasticity of intertemporal substitution—are smaller.

### 4.2.2. *Results for First-order Risk-averse Certainty Equivalents*

For the first-order risk-averse specification, the effect of added persistence in the disturbances is more dramatic than in the smooth certainty equivalent specification. The results are reported in Table VII.

The maximum cost value is roughly 2.4% for a switch from certainty to a realistic variance. This maximum occurs at the combination of parameters where risk aversion is greatest ($\gamma = 0.70$) and the elasticity of intertemporal substitution is largest ($1/(1 - \rho) = 1/2$). Even for less extreme amounts of risk aversion, though—$\gamma = 0.87$, for example—costs as high as 0.88%, or about \$177 per person per year, are possible.[12] Furthermore, as

[12] By way of contrast, if we restricted attention to binary gambles with equally likely outcomes, as in Pemberton [11], the value of the risk parameter, denoted $\beta$, for which Pemberton calculates a cost of 1.09% in the linear-trend-plus-*i.i.d.*-disturbances case would correspond to a value of $\gamma$ of roughly 0.125 here. Conversely, our extreme of risk aversion ($\gamma = 0.7$) would lie somewhere between risk neutrality ($\beta = 0$) and the next smallest risk parameter Pemberton considers ($\beta = 1$). In Pemberton's framework, values of the parameter $\beta > 0$ "adjust" the probability of the worse outcome upward, much as $\gamma$ does here. A value of $\beta = 1$ would increase the probability of the worse outcome from $1/2$ to $2/3$.

TABLE VII
Business Cycle Costs with First-order Risk Aversion, $\alpha = 1$,
$c_t = c_0(1+g)^t \exp(\xi_t)$, $\xi_t = a\xi_{t-1} + u_t$

| | | Elasticity of intertemporal substitution: 1/2 | 1/5 | 1/10 | 1/20 |
|---|---|---|---|---|---|
| Degree of risk aversion: | 0.99 | 0.0468 | 0.0349 | 0.0211 | 0.0080 |
| | 0.95 | 0.3045 | 0.2393 | 0.1869 | 0.1445 |
| | 0.90 | 0.6565 | 0.5225 | 0.4176 | 0.3332 |
| | 0.87 | 0.8842 | 0.7082 | 0.5699 | 0.4574 |
| | 0.75 | 1.9214 | 1.5801 | 1.2988 | 1.0556 |
| | 0.70 | 2.4104 | 2.0065 | 1.6658 | 1.3624 |

in the smooth case just considered, varying the elasticity of intertemporal substitution, for a given value of $\gamma$, now has relatively less effect on the cost figures than in the *i.i.d.* case.

### 4.3. A Consumption Process with Permanent Innovations

Last, we turn to the case where the growth rate of consumption follows an $AR(1)$ process—i.e., consumption growth obeys (6), implying that in levels, the logarithm of consumption follows a random walk with upward drift. This is the form for per capita consumption used by Mehra and Prescott [9] and Epstein and Zin [2] in their studies of the equity premium puzzle. An implication of this specification is that disturbances to consumption have permanent effects on the level of consumption. The experiments in this section, as in the sections above, consider a change from a world of certainty to a "realistic" environment, within the framework of the posited consumption process. In other words, we consider a switch from a world in which the growth rate of consumption has a zero variance, and is simply $c_{t+1}/c_t = 1 + g$, to one in which of the growth rate obeys $c_{t+1}/c_t = \xi_{t+1}$, where

$$\xi_{t+1} = (1-a)(1+g) + a\xi_t + u_t,$$

with the persistence parameter $a$ and the variance of the *i.i.d.* disturbances $u_t$ set to mimic the first-order autocorrelation and unconditional variance of per capita consumption growth observed in the U.S. data.

#### 4.3.1. *Results for Smooth Certainty Equivalents*

We initially set $\gamma = 1$, and examine the welfare cost of fluctuations in consumption growth, using "smooth" certainty equivalents to describe the agent's risk preferences. Table VIII gives the requisite percentage in-

TABLE VIII
Business Cycle Costs with ‘‘Smooth’’ Certainty Equivalents,
$c_{t+1}/c_t = \xi_{t+1}$, $\xi_{t+1} = (1-a)(1+g) + a\xi_t + u_t$

| | | Elasticity of intertemporal substitution: | | | |
|---|---|---|---|---|---|
| | | 1/2 | 1/5 | 1/10 | 1/20 |
| Degree of risk aversion: | 2 | 0.3469 | 0.1626 | 0.0858 | 0.0444 |
| | 5 | 0.9847 | 0.4606 | 0.2412 | 0.1214 |
| | 10 | 2.0675 | 0.9685 | 0.5064 | 0.2530 |
| | 20 | 4.3151 | 2.0299 | 1.0628 | 0.5301 |

creases in consumption, for various combinations of the risk aversion parameter $\alpha$ and the intertemporal substitution parameter $\rho$, needed to compensate the agent for a switch from a world of certainty to a world with a realistic variance of consumption growth.

Some of the resulting increases in consumption are, in fact, quite large. The maximum value—a roughly 4.3% increases in consumption—occurs at the highest degree of risk aversion, $1 - \alpha = 20$, and the largest elasticity of intertemporal substitution, $1/(1-\rho) = 1/2$. In per capita dollar terms, an increase in consumption of that magnitude corresponds to roughly $863 per person per year. Even at the more moderate degrees of risk aversion of 5 and 10, costs near 1% and 2% of per capita consumption —or $200 to $400 per person—are possible with this consumption process, even in this ‘‘smooth’’ case.

The expected utility case is once again had by looking at the diagonal elements of the table. For this case, the costs are substantially smaller, ranging from about 0.35% to 0.53% of consumption, or, in dollar terms, from about $70 per person per year to about $106 per person per year.

While some of the costs in this experiment are certainly quite large compared to their counterparts in the previous experiments with trend-stationary consumption—particularly the costs associated with parameter combinations which correspond to higher risk aversion and greater willingness to substitute intertemporally—it is worth noting that in Epstein and Zin's [3] empirical estimation of the parameters of this utility process, they found in none of their estimations a value of $1 - \alpha$ even as large as 1.40. As the table shows, for a degree of risk aversion, $1 - \alpha$, equal even to two, the maximum cost is less than 0.35% of initial consumption, or $70 per person per year.

### 4.3.2. *Results for First-order Risk-averse Certainty Equivalents*

Some of the cost numbers obtained in the previous section are quite large as far as welfare cost estimates go—compared, for example, to

Harberger's estimates of the gains from eliminating monopoly. In the following experiments, we will see that costs of an even greater magnitude are possible when first-order risk aversion combines with a consumption process like (6), in which disturbances to consumption have permanent effects.

Table IX gives the requisite percentage increase in consumption for a switch from certainty to a realistic variance of consumption growth for various values of the risk aversion parameter $\gamma$, ranging again from 0.99 down to 0.70, and values of the elasticity of intertemporal substitution $1/(1-\rho)$ ranging from 1/2 to 1/20.

The maximum cost value, which occurs at $\gamma = 0.70$ and $1/(1-\rho) = 1/2$, is 22.9162—a compensating increase in initial consumption of nearly 23%. In dollar terms, this is about $4600 per person per year. Of course, $\gamma = 0.70$ might be considered a very extreme degree of risk aversion. But, for even small departures from risk neutrality, the costs can become quite large. For example, when $\gamma = 0.87$, the costs, in percentage terms, range from just under 1% at the lowest elasticity of intertemporal substitution to about 7.6% at the highest. In per capita annual dollar terms, these costs are in the rough range of $200–$1500 per person per year. For $\gamma = 0.75$, the costs range from 3.32% to 12.73% of initial consumption, or from about $600 to $2500 per person per year.

If we think back to the ''willingness to pay'' calculations discussed above, taking $\gamma = 0.87$ and $\alpha = 0.75$ implied a willingness to pay about $24 to avoid the $250 gamble and about $21,000 to avoid the $74,000 gamble. For those parameter values, depending on the agent's elasticity of intertemporal substitution, the welfare cost of fluctuations in the growth rate of consumption of the magnitude experienced in the post-war United States is in the range of 1% to 7% of consumption—by no means a negligible cost.

TABLE IX
Business Cycle Costs with First-order Risk Aversion, $\alpha = 1$,
$c_{t+1}/c_t = \xi_{t+1}$, $\xi_{t+1} = (1-a)(1+g) + a\xi_t + u_t$

| | | Elasticity of intertemporal substitution: 1/2 | 1/5 | 1/10 | 1/20 |
|---|---|---|---|---|---|
| Degree of risk aversion | 0.99 | 0.4347 | 0.2050 | 0.1091 | 0.0570 |
| | 0.95 | 2.5903 | 1.2214 | 0.6449 | 0.3279 |
| | 0.90 | 5.5969 | 2.6551 | 1.4048 | 0.7135 |
| | 0.87 | 7.5953 | 3.6194 | 1.9189 | 0.9756 |
| | 0.75 | 17.5319 | 8.5784 | 4.6093 | 2.3645 |
| | 0.70 | 22.9162 | 11.4067 | 6.1878 | 3.1957 |

### 4.3.3. *Business Cycles or Growth*?

When shocks to consumption have permanent effects—as in the experiments just performed—it is natural to wonder to what extent the large cost figures obtained in this case can truly be ascribed to business cycles, rather than to reduced growth. In this section I will present some evidence that the welfare costs obtained in those experiments are in fact distinct from ''growth costs,'' and—in line with one of Lucas's basic points—those costs are still smaller than the costs of reduced growth, despite their large absolute size. In this sense, the results in this paper do not overturn Lucas's conclusion that the costs of reduced growth are large relative to the cost of business cycles; only Lucas's conclusion that the latter costs are actually negligible in an absolute sense is in question.

To explore the cost of reduced growth vis-à-vis the cost of fluctuations, I consider the following experiment: keeping the long-run variance of consumption growth at its realistic level, I ask, what percentage increase in consumption would be needed to compensate the agent for a one percentage point decrease in the long-run growth rate of consumption. For the ''smooth'' certainty equivalent case, the requisite percentage increases in consumption are given in Table X.

As is clear from a comparison of Table X with Table VIII, the costs of slower growth dwarf the costs associated with fluctuation for all combinations of parameters. The costs of reduced growth are particularly large where the agent's elasticity of intertemporal substitution is high, and decline as this elasticity falls. While there is some influence of risk aversion, it is small.

Costs of a similar magnitude are obtained in the first-order risk-averse case (with $\alpha = 1$), the results for which are shown in Table XI.

In fact, except for the two most extreme degrees of risk aversion, $\gamma$ equal to 0.75 and 0.70, the costs of slower growth shown in Table XI are strikingly similar to those shown in Table X. Their pattern is dominated by the agent's elasticity of intertemporal substitution, with requisite consumption increases in the rough neighborhoods of 28%, 15%, 8%, and 4% as

TABLE X
Costs of Slower Growth with ''Smooth'' Certainty Equivalents

| | | Elasticity of intertemporal substitution: | | | |
|---|---|---|---|---|---|
| | | 1/2 | 1/5 | 1/10 | 1/20 |
| Degree of risk aversion: | 2 | 26.7965 | 13.5856 | 7.4503 | 3.8796 |
| | 5 | 27.0218 | 13.8237 | 7.6167 | 3.9788 |
| | 10 | 27.4062 | 14.2404 | 7.9119 | 4.1563 |
| | 20 | 28.2116 | 15.1588 | 8.5813 | 4.5669 |

TABLE XI
Costs of Slower Growth with First-order Risk Aversion, $\alpha = 1$

| | | Elasticity of intertemporal substitution: 1/2 | 1/5 | 1/10 | 1/20 |
|---|---|---|---|---|---|
| Degree of risk aversion: | 0.99 | 26.8245 | 13.6160 | 7.4717 | 3.8924 |
| | 0.95 | 27.5767 | 14.4356 | 8.0530 | 4.2428 |
| | 0.90 | 28.6410 | 15.6898 | 8.9838 | 4.8211 |
| | 0.87 | 29.3583 | 16.6056 | 9.6980 | 5.2810 |
| | 0.75 | 33.0472 | 22.5176 | 15.2693 | 9.5955 |
| | 0.70 | 35.1355 | 27.1875 | 21.7311 | 20.3938 |

the agent's substitution elasticity is either 1/2, 1/5, 1/10, or 1/20, respectively. Compared to the increases required to compensate for fluctuations in the growth rate of consumption, the increases in consumption required to compensate for a one percentage point slower long-run growth rate range from roughly twice as large to 70 times as large.

Numbers of a similar magnitude and pattern obtain if one performs the analogous experiment with either of the other two processes posited for consumption, the one exception being that, for the two deterministic-trend specifications, the costs of slower growth fall with greater risk aversion, although the impact of variation in the risk aversion parameters is even more negligible in those cases. Lucas's conclusion that the welfare costs of slower growth are large in an absolute sense is thus a very robust one—even if individuals are relatively unwilling to substitute consumption over time, whether or not they are not particularly risk averse, and for several possible processes governing aggregate consumption, the welfare costs of slower long-run consumption growth are enormous.

## 5. CONCLUSIONS

What conclusions can be drawn from these experiments? Like Lucas, I do not intend these numbers to be taken as precise estimates of the cost of business cycles. Rather, I want only to show that for what I believe are plausible alternatives to the standard preference specification, and alternative processes for aggregate consumption, large costs are possible. One and one-half percent of current consumption, or two percent—both large numbers—are in the ballpark, in fact in the infield. If one allows that an individual's risk preferences may be better described by something other than the standard, isoelastic expected utility framework, one must be open to at least the possibility that business cycles do have large welfare consequences. More to the point, I think it would be premature to write off stabilization as a possible goal of business cycle theory.

Not surprisingly, the results also show that what one means by ‘‘business cycle fluctuations in consumption’’ has a large impact on the calculated costs imposed by those fluctuations. Whether business cycle fluctuations are essentially transitory disturbances to an otherwise smoothly growing consumption path, or they have permanent effects on the level of consumption, matters a great deal for the size of the welfare costs one obtains. One lesson that might be drawn from this sensitivity of the results is that, while there is clearly something attractive in the simplicity of the sort of ‘‘back-of-the-envelope’’ calculations in Lucas [8] and (albeit on a larger envelope) here, the question of the costliness of business cycles is perhaps not well posed outside of a particular model of business cycles.

One final way of thinking about the results is to think simply in terms of the range of numbers—the one ‘‘safe’’ conclusion is that the cost of business cycles lies somewhere between 1/10% and 23% of annual consumption—which suggests that we need a better understanding of individuals’ attitudes toward risk before we can safely draw conclusions about the cost of business cycles. Whether deviations from the standard specification of risk preferences have a significant impact on positive business cycle theory is a question for further research.[13]

## 6. APPENDIX

The solution technique which I employ—which involves normalizing growing lifetime utility to render it stationary, then iteratively solving a functional equation—is similar under any of the three alternative consumption processes, although the normalizations differ, and the resulting functional equations thus differ somewhat. In all cases the uncertainty is ‘‘discretized’’ down to a finite-state Markov chain, and the functional equations are solved via iterative techniques common in dynamic programming problems.

When consumption obeys $c_t = c_0(1 + \gamma)^t \exp(\xi_t)$, with $\xi_t$ either *i.i.d.* or following a first-order autoregressive process, the stochastic process for lifetime utility, which will be growing over time,[14] can be normalized in the

[13] See [4] for a recent exploration along these lines.

[14] Note that ‘‘lifetime utility’’ at date $t$ is utility from date $t$ onward as of date $t$—i.e., undiscounted.

following manner. Let $U_t$ denote lifetime utility from date $t$ onward, so that

$$U_t = \left[c_t^{\rho} + \beta\mu\left(\tilde{U}_{t+1} : \mathscr{I}_t\right)^{\rho}\right]^{1/\rho}.$$

Given the homogeneity of $\mu$ and the CES aggregator, lifetime utility at each date $t$ can be "normalized" by $c_0(1+g)^t$. Let $V_t \equiv U_t / c_0(1+g)^t$ denote normalized lifetime utility; $V_t$ then evolves according to

$$V_t = \left[\exp(\xi_t)^{\rho} + \beta(1+g)^{\rho}\mu\left(\tilde{V}_{t+1} : \xi_t\right)^{\rho}\right]^{1/\rho},$$

where the conditioning information $\mathscr{I}_t$ is now simply the realization of $\xi$ at $t$. I treat this as a functional equation to be solved for a particular function $v(\xi)$. A solution is a $v(\xi)$ such that $\{V_t\}_{t=0}^{\infty} = \{v(\xi_t)\}_{t=0}^{\infty}$ satisfies the above stochastic difference equation. More simply, $v$ is a solution if $v$ satisfies

$$v(\xi) = \left[\exp(\xi)^{\rho} + \beta(1+g)^{\rho}\mu\left(v(\tilde{\xi}) : \xi\right)^{\rho}\right]^{1/\rho}$$

for all $\xi$.

When consumption obeys $c_{t+1}/c_t = \xi_{t+1}$, with $\xi_{t+1}$ a stationary $AR(1)$ process, as in the equity premium literature, lifetime utility can be normalized as follows. Given the nature of the posited consumption processes, and the recursive formula (1) for lifetime utility, one can express lifetime utility from a state with current consumption $c$ and current realization of the growth rate $\xi$ as a function $V(c, \xi)$, satisfying

$$V(c,\xi) = \left[c^{\rho} + \beta\mu\left(V(\tilde{\xi}c, \tilde{\xi}) : c, \xi\right)^{\rho}\right]^{1/\rho}$$

for each $c$ and $\xi$. In fact, given the homogeneity of the aggregator and certainty equivalent in (1), $V$ will be linearly homogeneous in the level of current consumption—i.e.,

$$V(c,\xi) = cv(\xi)$$

—so that lifetime utility normalized by the level of current consumption will obey

$$v(\xi) = \left[1 + \beta\mu\left(\tilde{\xi}v(\tilde{\xi}) : \xi\right)^{\rho}\right]^{1/\rho},$$

which is again a functional equation in $v$.

For computational purposes, the processes for the disturbances $\xi_t$ are approximated by finite-state Markov chains. That is, $\xi_{t+1}$ takes on one of $n$ values—

$$\xi_{t+1} \in \{ \xi_1, \xi_2, \dots, \xi_n \}$$

with probabilities given by a transition matrix $P$,

$$\begin{aligned} P &= \left[ P_{ij} \right]_{i,j=1,2,\dots,n} \\ &= \left[ \Pr\{ \xi_{t+1} = \xi_j : \xi_t = \xi_i \} \right]_{i,j=1,2,\dots,n}. \end{aligned}$$

I take $n$ to be even, and fix probabilities with $p_1 < p_2 < \cdots < p_{n/2}$ and $p_{(n/2)+1} > p_{(n/2)+2} > \cdots > p_n$. The probabilities $p = (p_1, \dots, p_n)$ represent the invariant distribution of $\xi$. If $m$ denotes the mean and $\sigma$ the standard deviation of the invariant distribution of $\xi$, set

$$\xi_i = m - \frac{\sigma}{p_i \sqrt{\sum_{j=1}^{n} (1/p_j)}}$$

for $i = 1, 2, \dots, n/2$ and

$$\xi_i = m + \frac{\sigma}{p_i \sqrt{\sum_{j=1}^{n} (1/p_j)}}$$

for $i = (n/2) + 1, (n/2) + 2, \dots, n$. Then, $\xi_1 < \xi_2 < \cdots < \xi_n$, $\sum_i p_i \xi_i = m$ and $\sum_i p_i (\xi_i - m)^2 = \sigma^2$. If the $p_i$'s are symmetric—i.e., if $p_1 = p_n$, $p_2 = p_{n-1}$, etc.—then the $\xi_i$'s will also be symmetric about $m$.

The Markov transition probability matrix $P$ is then specified as

$$P_{ij} = \Pr\{ x_{t+1} = x_j : x_t = x_i \} = (1 - a) p_j + a \delta_{ij},$$

where $\delta_{ij} = 1$ if $i = j$ and zero otherwise, and $a$ is an autocorrelation parameter. When $a = 0$, the $\xi$ process is *i.i.d.*, with $P_{ij} = P_{kj} = p_j$.

In principle, one would like to construct the Markov chain—which here amounts to selecting $n$ and the probabilities $p_1, p_2, \dots, p_n$—so as to capture a number of salient features of the distributions of the actual variables whose behavior is being approximated. Given the standard formulation which I have followed, taking $n = 2$, with $p_1 = p_2 = 1/2$, is sufficient to capture the unconditional mean and variance, and autocorrelation, in a parsimonious way. In this case, the realizations of $\xi$ are restricted to being one standard deviation above or below the mean. To the extent that other features of the distribution seem relevant for risk calculations—e.g., existence or nonexistence of "two-$\sigma$" or greater events or features of the cumulative distribution function—one would like to incorporate that information. This seems particularly the case when considering, as we are here, a variant on Yaari certainty equivalents, which are nonlinear in probabilities. In practice, though, given the forms specified above for the outcomes, it is difficult to select $n$ and the $p_i$'s so that $(\xi_i)_{i=1}^n$ with probabilities $(p_i)_{i=1}^n$ matches even a rough histogram of a given distribution. For the distributions at hand, a good but by no means perfect approximation was had with $n = 8$ and

$$p = \{0.05, 0.10, 0.15, 0.20, 0.20, 0.15, 0.10, 0.05\}.$$

Experiments with other possible combinations of $n$ and $p$—including $n = 2$ and $p = (\frac{1}{2}, \frac{1}{2})$—indicate that the results reported above are robust, at least in terms of orders of magnitude. Given that our purpose here is not so much to give a precise estimate of the cost of business cycles as it is to provide rough indications of possible magnitudes, robustness to this extent seems satisfactory.

Now, let $G(P) = [G_{ij}(P)]_{i,j=1,2,\ldots,n}$ be the matrix of "Yaari-adjusted" probabilities, defined by

$$G_{ij}(P) = \left(\sum_{h=1}^{j} P_{ih}\right)^{\gamma} - \left(\sum_{h=1}^{j-1} P_{ih}\right)^{\gamma}.$$

Then, given the particular form of the certainty equivalent $\mu$ which I employ, and the $n$-state Markov chains for the $\xi$ processes, the "functional equations" in $v$ to be solved are

$$v(\xi_i) = \left[\xi_i^{\rho} + \beta(1+g)^{\rho}\left[\sum_{j=1}^{n} G_{ij}(P)v(\xi_j)^{\alpha}\right]^{\rho/\alpha}\right]^{1/\rho} \qquad (i = 1, 2, \ldots n)$$

for the case of a linear trend, and

$$v(\xi_i) = \left[1 + \beta\left[\sum_{j=1}^{n} G_{ij}(P)\left[\xi_j v(\xi_j)\right]^{\alpha}\right]^{\rho/\alpha}\right]^{1/\rho} \qquad (i = 1, 2 \ldots n)$$

for the stochastic growth case. Both are simply systems of $n$ equations in $n$ unknowns. Given the recursivity, however, the most efficient way to solve these systems is iteratively, as in dynamic programming problems. I thus begin in each case with an initial guess $v_0 = (v_0(\xi_i))_{i=1}^n$, and update according to either

$$v_{s+1}(\xi_i) = \left[ \xi_i^\rho + \beta(1+g)^\rho \left[ \sum_{j=1}^n G_{ij}(P) v_s(\xi_j)^\alpha \right]^{\rho/\alpha} \right]^{1/\rho}$$

or

$$v_{s+1}(\xi_i) = \left[ 1 + \beta \left[ \sum_{j=1}^n G_{ij}(P) [\xi_j v_s(\xi_j)]^\alpha \right]^{\rho/\alpha} \right]^{1/\rho}.$$

The tolerance criterion I use stops the iterations when $\|v_{s+1} - v_s\|_1 < 10^{-7}$.

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