Macroeconomic Policy, Exchange-Rate Dynamics, and Optimal Asset Accumulation

Maurice Obstfeld

The Journal of Political Economy, Vol. 89, No. 6 (Dec., 1981), 1142-1161.

Stable URL:
http://links.jstor.org/sici?sici=0022-3808%28198112%2989%3A6%3C1142%3AMPEDAO%3E2.0.CO%3B2-U

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at http://www.jstor.org/about/terms.html. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

The Journal of Political Economy is published by The University of Chicago Press. Please contact the publisher for further permissions regarding the use of this work. Publisher contact information may be obtained at http://www.jstor.org/journals/ucpress.html.

The Journal of Political Economy
©1981 The University of Chicago Press

JSTOR and the JSTOR logo are trademarks of JSTOR, and are Registered in the U.S. Patent and Trademark Office. For more information on JSTOR contact jstor-info@umich.edu.

©2003 JSTOR

http://www.jstor.org/
Wed Sep 24 16:32:29 2003

Macroeconomic Policy, Exchange-Rate Dynamics, and Optimal Asset Accumulation

Maurice Obstfeld

Columbia University

The paper develops a model of exchange-rate and current-account determination for a small economy peopled by infinitely lived, utility-maximizing households. In this setting, a central-bank purchase of foreign exchange has no real effects when central-bank foreign reserves earn interest at the world rate and the proceeds are returned to the public. In contrast, an increase in the monetary growth rate does have real effects, even in the long run. The model developed here implies that an increase in government spending may lead to a surplus on current account. The external adjustment process predicted by the model is one in which consumption, real balances, and external assets all rise or fall simultaneously.

Introduction

This paper studies exchange-rate determination and the external adjustment process in a small economy peopled by infinitely lived, utility-maximizing households. The setting is similar to one proposed by Kouri (1976), in that agents are assumed to consume a single good, to hold domestic money, and to have access to a world market in consumption loans. But the approach departs from those prevalent in

Guillermo Calvo suggested the line of investigation pursued in this paper. I am grateful for his comments and for those of Ralph Braid, Robert Cumby, Ronald Findlay, Duncan Foley, Jacob Frenkel, Michael Mussa, and two anonymous referees. Valuable suggestions were made by participants in seminars at Columbia University, the Federal Reserve Board, the University of Chicago, Stanford University Graduate School of Business, the University of Pennsylvania, and the NBER's 1980 Summer Institute in International Studies. All errors are mine. Financial support from the National Science Foundation is gratefully acknowledged.

[*Journal of Political Economy*, 1981, vol. 89, no. 6]
© 1981 by The University of Chicago. 0022-3808/81/8906-0009$01.50

recent balance-of-payments models by basing saving behavior and money demand on explicit, intertemporal utility maximization.[1]

The analytical framework is adapted from the seminal work of Sidrauski (1967) and employs the perfect-foresight equilibrium concept explored by Brock (1974), Calvo (1979*a*), and Fischer (1979). In contrast to these contributions, however, the model developed below allows agents' subjective rate of time preference to be endogenously determined in a manner suggested by Uzawa (1968) rather than fixed.[2] This modification permits the small economy we study to attain a stationary long-run equilibrium under conditions of perfect capital mobility.[3]

The plan of the paper, and its main results, are as follows. Section I describes the structure of the economy and the typical household's maximization problem. Particular attention is paid to the budget constraints of the central bank and government. Section II closes the model by imposing continuous money-market equilibrium and endowing agents with perfect foresight concerning the future paths of the price level and government transfer payments. The resulting dynamic system possesses a unique stationary position and a single convergent path, along which consumption, real money balances, and the stock of foreign claims all rise or fall simultaneously. This convergent saddlepath is an equilibrium path and is the focus of the analysis that follows.

Section III analyzes the effects of some unanticipated macroeconomic policy actions. A central-bank purchase of foreign exchange is found to have no real effects when official foreign reserves earn interest that is distributed to the public. In contrast, an increase in the monetary growth rate does have real effects: It occasions exchange-rate overshooting (in Flood's [1979] sense) and a current-account surplus and leads, in the long run, to higher levels of consumption and foreign claims. Finally, the model implies that an increase in government spending may induce a surplus on current account, contrary to the usual presumption. A deficit is possible when government spending produces a public good having a high marginal value to consumers. The explicit specification of agents' objective

[1] Research along similar lines is reported in papers by Calvo (1979*b*, 1980), Helpman (1979), Razin (1979), and Stockman (1980). Obstfeld (1980) describes a two-country extension of this paper's model.

[2] Endogenous time preference has been discussed in the trade literature in papers by Calvo and Findlay (1978) and by Findlay (1978). Kouri (1980) adopts a consumption function suggested by Uzawa's (1968) theory. The present approach is applied in a somewhat different context in Obstfeld (1981).

[3] This could be accomplished also by allowing bonds (in addition to real balances) to enter consumers' utility functions, as in Calvo (1980).

functions is particularly useful in Section III, for it permits a natural welfare analysis of alternative macroeconomic policies.

Section IV contains concluding remarks. Appendices supply the technical details justifying some of the paper's assertions.

I. The Model

We consider an open economy inhabited by identical households, each maximizing its welfare over an infinite lifetime. As in Kouri (1976), there is a single, perishable, consumption good, and households' financial wealth is divided between domestic fiat money and internationally traded bonds denominated in foreign currency. The economy is small, and it can therefore influence neither the foreign currency price of the consumption good, P^*, nor the world bond rate, r, both of which are assumed constant. The link between the domestic price level, P, and P^* is provided by the relationship $P = EP^*$, where E denotes the domestic money price of foreign exchange. This exchange rate is allowed to float freely by the monetary authority. On the assumption that foreigners do not hold domestic money, E adjusts to maintain equality between the real money supply and domestic real money demand at each instant. We adopt the normalization $P^* = 1$.

The number of households in the economy is, for convenience, also taken to be 1. The household's instantaneous utility is a separable function $U(c,m) = u(c) + v(m)$ of consumption, c, and real money holdings, m, defined as nominal money holdings, M, deflated by the home price level, $m \equiv M/P$. The functions $u(\cdot)$ and $v(\cdot)$ are nonnegative, strictly concave, and twice continuously differentiable, with the property that

$$\lim_{c \to 0} u'(c) = \lim_{m \to 0} v'(m) = \infty. \tag{1}$$

Assumption (1) eliminates the possibility of corner solutions. It is clear, in addition, that the derivatives $u'(\cdot)$ and $v'(\cdot)$ are everywhere strictly positive.

The household seeks to maximize the discounted sum of future instantaneous utilities,

$$V = \int_0^\infty U(c_t,m_t)e^{-\Delta_t}dt. \tag{2}$$

The discount factor Δ_t is defined by

$$\Delta_t \equiv \int_0^t \delta_s ds, \tag{3}$$

where δ_s is the instantaneous subjective discount rate at time s.[4] Following Uzawa (1968), we postulate that δ_s is a function of utility at time s,

$$\delta_s = \delta[U(c_s, m_s)], \tag{4}$$

and that the function $\delta(\cdot)$ is positive and satisfies

$$\delta'(U) > 0, \delta''(U) > 0, \delta(U) - U\delta'(U) > 0. \tag{5}$$

The first two conditions listed in (5) are in no way implied by the hypothesis of endogenous time preference but are imposed because they are analytically convenient.[5] The last of the conditions ensures that the present discounted value $\bar{U}/\delta(\bar{U})$ of a stationary utility stream $\bar{U}$ is increasing in $\bar{U}$.

At each moment t, the household must allocate its income between current expenditure and the accumulation of real wealth. Real output, y, is taken to be exogenous and fixed. The other components of expected disposable income are interest payments from abroad, equal to the world bond rate, r, times the real value F_t of the family's net foreign claims; expected real transfers from the government, τ_t; and expected capital gains on real balances, equal to $-\pi_t m_t$, where π_t denotes the expected inflation rate. Defining the household's real assets at time t (excluding the capitalized value of expected future transfers) as

$$a_t \equiv \frac{y}{r} + m_t + F_t, \tag{6}$$

the flow constraint linking asset accumulation to saving may be written as

$$\dot{a}_t = ra_t + \tau_t - c_t - (\pi_t + r)m_t, \tag{7}$$

while the intertemporal budget constraint takes the form

$$\int_0^\infty e^{-rt}[c_t + (\pi_t + r)m_t]dt \leq a_0 + \int_0^\infty e^{-rt}\tau_t dt. \tag{8}$$

Restriction (8) is important in the present setting of perfect capital mobility, for it rules out the possibility that the household can attain unbounded utility by borrowing arbitrarily large sums in the world

[4] When $\delta_s = \delta$, a constant, for all s, we have the usual discount factor, $\Delta_t = \delta t$.

[5] The nature of this convenience becomes apparent in applying the maximum principle to solve the household's lifetime problem and in proving the existence of a stable perfect-foresight equilibrium path converging to the economy's stationary state. These points are taken up again in context. A general discussion of the time preference concept is found in Koopmans, Diamond, and Williamson (1964).

capital market and meeting all interest payments through further borrowing. As Appendix B shows, (7) and (8) confine the household to paths along which

$$a_t + \int_t^{\infty} e^{-r(s-t)}\tau_s ds \geqslant 0$$

for all t, so that household net worth is always nonnegative.

The household's problem is to find $\{c_t, m_t\}$ maximizing lifetime welfare V subject to the constraints (3), (7), and (8), given an initial level a_0 of real assets. The calculations are simplified by changing variables from t to Δ ("psychological time") using the fact, implied by (3), that $d\Delta = \delta[U(c_t,m_t)]dt$. The household's problem may then be expressed as that of choosing time paths for consumption and real balances that satisfy (8) and[6]

$$\text{maximize} \int_0^{\infty} \frac{U(c,m)}{\delta[U(c,m)]} e^{-\Delta} d\Delta$$

$$\text{subject to } \frac{da}{d\Delta} = \frac{ra + \tau - c - (\pi + r)m}{\delta[U(c,m)]}.$$

Necessary conditions for an optimal program are given by the maximum principle.[7] These require that c and m be chosen so that for each value of the discount factor Δ, the Hamiltonian

$$H = \frac{U(c,m)}{\delta[U(c,m)]} + \lambda \frac{[ra + \tau - c - (\pi + r)m]}{\delta[U(c,m)]} \tag{9}$$

is maximized. In (9), $\lambda = \lambda_\Delta$ is the co-state variable, which is interpreted as the shadow value, in utility terms, of real assets. The implied first-order conditions (at the interior maximum) are

$$\big(1 - (\delta'/\delta)\{U + \lambda[ra + \tau - c - (\pi + r)m]\}\big)U_c = \lambda, \tag{10}$$

$$\big(1 - (\delta'/\delta)\{U + \lambda[ra + \tau - c - (\pi + r)m]\}\big)U_m = \lambda(\pi + r). \tag{11}$$

An additional necessary condition is that the shadow price of assets evolves according to the law,

$$\frac{d\lambda}{d\Delta} = \lambda\left(\frac{\delta - r}{\delta}\right). \tag{12}$$

Equations (10) and (11) imply the usual necessary condition of static utility maximization,

[6] The psychological time subscript, Δ, is omitted for the sake of notational simplicity.

[7] See Arrow and Kurz 1970. If $U(c,m)$ is strictly concave and both consumption and real balances are normal goods, the conditions listed in (5) imply that the maximand $U(c,m)/\delta[U(c,m)]$ is strictly concave.

$$x(c_t,m_t) \equiv U_m(c_t,m_t)/U_c(c_t,m_t) = \pi_t + r, \tag{13}$$

while (12) implies that the time derivative of λ_Δ is given by[8]

$$\dot{\lambda}_t = \lambda_t\{\delta[U(c_t,m_t)] - r\}. \tag{14}$$

It remains to specify the behavior of the government and the central bank. We consolidate the balance sheets of these two entities and work with a single public-sector budget constraint. A key assumption is that the stock of central-bank foreign reserves earns interest at the world rate r.

The public sector consumes goods and makes (net) transfers, financing any deficit through money creation. Thus, all government debt is purchased by the central bank, none by private citizens. Letting g denote the (constant) level of real government consumption per household, the public sector's budget constraint is $g = \dot{M}_t/P_t + rR - \tau_t = (\dot{M}_t/M_t)m_t + rR - \tau_t$, where R denotes the (constant) level of central-bank foreign reserves, measured in terms of output, and τ_t, as before, denotes net real transfers.

We shall assume that the central bank varies τ_t in such a way as to hold the rate of monetary growth $\dot{M}_t/M_t$ constant at μ.[9] The level of transfers is then determined according to the formula

$$\tau_t = \mu m_t + rR - g. \tag{15}$$

A final assumption is that μ is always chosen so that $\mu + r > 0$. This inequality guarantees the existence of a stationary state.

II. Perfect-Foresight Equilibrium Dynamics

The model of the previous section is closed by the assumption that the path of the economy is a perfect-foresight equilibrium path, in the sense of Brock (1974). Let $\{\hat{P}_t\}$ be a (differentiable) price-level path and $\{\widehat{P_t\tau_t}\} = \{\mu e^{\mu t}M_0 + \hat{P}_t rR - \hat{P}_t g\}$ the associated path of nominal transfer payments from the government, where M_0 denotes the nominal money stock at $t = 0$. Acting on the belief that these paths of the price level and nominal transfers will prevail, the household can calculate expected real transfers $\{\hat{\tau}_t\}$ and determine optimal paths for

[8] Only necessary conditions (10) and (11) differ from what they would be in the standard intertemporal optimization framework, where $\delta_t = \delta$, an exogenous constant. In the standard setup, (10) is replaced by $U_c = \lambda$. But in the present context, an increase in today's consumption affects not only today's instantaneous utility but also the discount factor applied to all utilities enjoyed in the future. Note also that if the instantaneous subjective discount rate δ were constant, necessary condition (14) would rule out the possibility of a stationary state unless it so happened that δ and r were equal.

[9] This assumption is crucial for the foreign-exchange intervention "neutrality" proposition derived below and differs from the one made by Kouri (1976), who takes government consumption to be the residual item.

its consumption rate and real balances, subject to (7) and (8) with initial real assets $a_0 = (y/r) + F_0 + (M_0/\hat{P}_0)$. The path $\{\hat{P}_t\}$ is a perfect-foresight equilibrium path if the desired path of real balances, $\{\hat{m}_t\}$, equals the actual path, that is, if

$$\hat{m}_t = M_t/\hat{P}_t = e^{\mu t}M_0/\hat{P}^t \tag{16}$$

for all $t \geqslant 0$.

The necessary conditions derived in the previous section may be used to find the collection of price-level paths $\{P_t\}$ consistent with perfect foresight and individual optimality. The differential equations (7) and (14), together with the conditions (6), (10), (13), (16), and

$$\pi_t = \dot{P}_t/P_t, \tag{17}$$

must be satisfied by any perfect-foresight equilibrium trajectory of the economy. We now investigate the restrictions these requirements place on the possible equilibrium paths of bonds F_t, consumption c_t, real balances m_t, and, by implication, the home price level. Recalling our earlier convention that the world price level $P^* = 1$, we shall identify the path of the domestic price level with that of the exchange rate, E.

Differentiating equilibrium condition (16), we find that desired real balances must satisfy the relation

$$\dot{P}_t/P_t = \mu - (\dot{m}_t/m_t) \tag{18}$$

along an equilibrium path. Combining (17) and (18) with necessary condition (13) yields

$$\dot{m}_t = [\mu + r - x(c_t,m_t)]m_t. \tag{19}$$

The differential equation (19) must govern the evolution of real balances in perfect-foresight equilibrium.

The flow constraint (7) may be written in the form $\dot{m}_t + \dot{F}_t = y + rF_t + \tau_t - c_t - \pi_t m_t$. When we use (17), (18), and the public-sector budget constraint (15), this differential equation becomes

$$\dot{F}_t = y + r(F_t + R) - c_t - g, \tag{20}$$

the familiar relation between the current and capital accounts under a floating exchange rate. Equation (20), which equates the home country's foreign lending to the excess of domestic income over absorption by the domestic private and public sectors, describes the equilibrium path of foreign bond holdings. Note that because foreigners do not hold domestic money, the variable F_t is predetermined by the past history of the current account. It can jump instantaneously only through central-bank purchases and sales and must otherwise adjust slowly over time.

It remains to derive the differential equation governing movements in consumption. Using the necessary condition (10), we may solve for the co-state variable λ_t with the help of (15) and (17)–(19):

$$\lambda_t = \lambda(c_t, m_t, F_t + R) =$$

$$\frac{[\delta(U_t) - U_t\delta'(U_t)]u'(c_t)}{\delta(U_t) + \{y + r(F_t + R) + [\mu + r - x(c_t, m_t)]m_t - c_t - g\}\delta'(U_t)u'(c_t)}.$$

Differentiating λ with respect to time and applying (14) yields $\dot{\lambda}_t = \lambda_c\dot{c}_t + \lambda_m\dot{m}_t + \lambda_F\dot{F}_t = \lambda_t\{\delta[U(c_t, m_t)] - r\}$, which may be solved for the time derivative of consumption,

$$\dot{c}_t = \lambda_c^{-1}(\lambda_t\{\delta[U(c_t, m_t)] - r\} - \lambda_m\dot{m}_t - \lambda_F\dot{F}_t) \equiv \psi(c_t, m_t, F_t + R). \quad (21)$$

Equation (21) is the final differential equation of a three-equation system.[10] Together, equations (19)–(21) must describe the evolution of the economy along any perfect-foresight equilibrium path.

Not every path satisfying (19)–(21) is a perfect-foresight equilibrium path, however, for the conditions from which these paths are derived are necessary for optimality, but are not, in general, sufficient. For the balance of this paper, we shall focus attention on paths that bring the economy to the *stationary state* $(\bar{c}, \bar{m}, \bar{F})$, defined by the conditions $\psi(\bar{c}, \bar{m}, \bar{F} + R) = \mu + r - x(\bar{c}, \bar{m}) = y + r(\bar{F} + R) - \bar{c} - g = 0$. As is easily verified, the stationary state exists and is unique when, as we have assumed, $\mu + r > 0$.

Appendix A proves that the system (19)–(21), when linearized in a neighborhood of the stationary state, possesses a single negative characteristic root, and so a unique convergent path. Appendix B proves that this path is a perfect-foresight equilibrium path: The choices of real balances and consumption it dictates are optimal choices from the household's standpoint, given the implied initial value of real assets and the implied anticipated paths of prices and transfers. We therefore assume that for any value of the predetermined stock of foreign claims, the equilibrium price level and consumption rate are those placing the economy on the path converging to the stationary or long-run equilibrium.[11] It should be noted that this convergent saddlepath satisfies the household budget constraint (8), for, along it,

$$\int_0^\infty e^{-rt}[c_t + (\pi_t + r)m_t - \tau_t]dt = a_0 - \lim_{t\to\infty} a_t e^{-rt} = a_0. \quad (22)$$

To find the saddlepath, we let θ_1 denote the system's negative characteristic root and $\vec{\omega} = [\omega_{11}, \omega_{21}, \omega_{31}]'$, an eigenvector belonging to

[10] The equation giving $\dot{c}$ cannot be defined if $\lambda_c = 0$. It is shown in Appendix A that $\lambda_c < 0$ in a neighborhood of the system's stationary state. In fact, it can be shown that $\lambda_c < 0$ globally if hyperdeflations are ruled out.

[11] It has not been demonstrated that perfect-foresight equilibrium is locally unique.

θ_1. Any *convergent* solution of the linearized system must take the form

$$c_t - \bar{c} = \omega_{11} k \exp(\theta_1 t),$$

$$m_t - \bar{m} = \omega_{21} k \exp(\theta_1 t),$$

$$F_t - \bar{F} = \omega_{31} k \exp(\theta_1 t),$$

where k is an arbitrary constant determined by the value of F at time $t = 0$.[12] Differentiating, we find that

$$\dot{c}_t = \theta_1(c_t - \bar{c}) = \theta_1(\omega_{11}/\omega_{31})(F_t - \bar{F}), \tag{23}$$

$$\dot{m}_t = \theta_1(m_t - \bar{m}) = \theta_1(\omega_{21}/\omega_{31})(F_t - \bar{F}), \tag{24}$$

$$\dot{F}_t = \theta_1(F_t - \bar{F}), \tag{25}$$

when the economy approaches the stationary state. The saddlepath of the system is simply the intersection of the two hyperplanes defined by equations (23) and (24).

The importance of equations (23)–(25) is that they allow us to characterize the motion of consumption, real balances, and external asset holdings as the economy converges to long-run equilibrium. As is shown in Appendix A, the components of the eigenvector $\vec{\omega}$ must all be of the same sign, and an immediate consequence is that the three variables must always rise or fall together along the saddlepath.[13] This is a key property, for it allows us to trace the path taken by the economy in response to various policy surprises. However, one particular implication of this result is important enough to be mentioned before we turn to such exercises. When the current account is in surplus ($\dot{F}_t > 0$), real balances must be rising, and when it is in deficit ($\dot{F}_t < 0$), they must be falling. Thus, the exchange rate's depreciation rate will fall short of the rate of monetary growth when there is an external surplus and will exceed it when there is a deficit. This fundamental relationship between exchange-rate depreciation and the current account appears in models developed by Kouri (1976), Calvo and Rodríguez (1977), and others. But it emerges here from the hypothesis of optimizing behavior and not as a consequence of the assumed properties of consumption and money-demand functions.

Figure 1 shows how the economy's long-run equilibrium is found. From (14), stationary-state utility $\bar{U}$ is determined by the equality of the marginal rate of time preference and the world bond rate,[14]

[12] See Hirsch and Smale 1974.

[13] Calvo (1979*a*) provides a similar characterization of the saddlepath arising in the Brock-Sidrauski perfect-foresight model.

[14] From (10), $\bar{\lambda} = \{[\delta(\bar{U}) - \bar{U}\delta'(\bar{U})]/\delta(\bar{U})\}u'(\bar{c})$. The last inequality in (5) therefore implies that $\bar{\lambda}$ is strictly positive. This justifies condition (26).

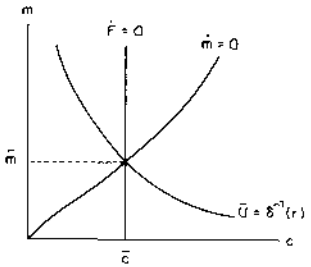

FIG. 1

$$\delta(\bar{U}) = r, \tag{26}$$

and so, stationary-state consumption $\bar{c}$ and real balances $\bar{m}$ must satisfy $U(\bar{c},\bar{m}) = \bar{U}$, as shown in the diagram. The second requirement on the point $(\bar{c},\bar{m})$ is that it lie on the locus of points consistent with $\dot{m} = 0$; the latter is, by (19), the Engel curve associated with an opportunity cost $\mu + r$ of holding real balances. It slopes upward from the origin. Given the value of $\bar{c}$ determined by $\bar{U}$ and $\mu + r$, $\bar{F}$ is determined by the relation $\bar{F} = [(\bar{c} + g - y)/r] - R$. The locus labeled $\dot{F} = 0$ in figure 1 is vertical at the unique private consumption level consistent with current-account balance given stationary-state bond holdings $\bar{F}$. An increase in $\bar{F}$ would increase long-run domestic income and so shift this schedule rightward.

III. The Effects of Macroeconomic Disturbances

This section studies the impact and long-run effects of intervention in the foreign exchange market, an increase in the rate of monetary expansion, and an increase in real government consumption. The macroeconomic disturbances are assumed to take the public by surprise, but they are permanent and lead to no expectation of future policy actions. The economy's initial position is the stationary state.

Foreign Exchange Intervention

The central bank intervenes in the foreign exchange market by purchasing foreign claims from the public with domestic money. The transaction accomplishes a transfer of interest-earning assets from the public to the central bank as well as an increase in the nominal money supply. But the *sum* $\bar{F} + R$ of privately owned bonds and central-bank reserves is unaffected by this transfer.

Now, equations (20) and (21) involve only the economy's aggregate claims on the rest of the world; the distribution of their ownership

between the public and the central bank is irrelevant. Thus, the long-run value of F simply declines from $\bar{F}$ to $\bar{F} - \Delta R$ as a result of the intervention, while $\bar{c}$ and $\bar{m}$ are unchanged. Because the stock of privately owned foreign assets is immediately at its new long-run level, consumption and real balances must, by (23) and (24), remain at their respective stationary-state levels $\bar{c}$ and $\bar{m}$. The monetary authority's action occasions a rise in the price level (a depreciation of the exchange rate) exactly proportional to the increase in the nominal supply of money. It has no real effects. Money creation accomplished through a purchase of foreign exchange has the same impact as a "helicopter" money-supply change of equal magnitude.

It is sometimes asserted that there *is* a difference between these two financial policies, in that an official purchase leads to a current surplus as domestic residents seek to restore their holdings of external claims to the original level.[15] The difference disappears here because foreign assets held by the central bank continue to earn interest at the rate r, while households capitalize all transfers from the government (see Mundell 1971, chap. 1). By the government budget constraint (15), the stream of interest earnings taken away from the public through central-bank intervention is returned in the form of transfers. Households therefore suffer no fall in permanent disposable income when a portion of their bond holdings is transferred to the central bank.

This neutrality result would clearly break down if, for example, reserves were held in some barren, non-interest-bearing form, or if increased central-bank earnings were used to augment government consumption rather than transfer payments. That intervention has real effects in these circumstances is to be expected, for it involves real fiscal measures in addition to a neutral change in the nominal supply of money.

An Increase in the Monetary Growth Rate

Figure 2 depicts the effects of an increase in the rate of monetary growth, μ. As (26) shows, the long-run instantaneous utility level of the representative household is unchanged by the disturbance. But as the rate of monetary growth and the inflation rate coincide in the long run, the disturbance raises the opportunity cost of holding money in the stationary state. Thus, the household economizes on real balances and consumes more in the new long-run equilibrium. This is represented in figure 2 by the downward shift of the Engel curve along

[15] See, e.g., Kouri 1978.

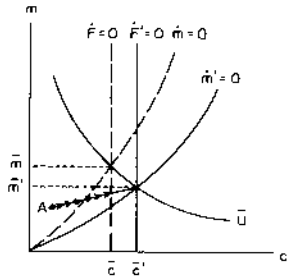

FIG. 2

which real balances are stationary. The rise in long-run consumption entails a rise in long-run bond holdings. Accordingly, the $\dot{F} = 0$ locus shifts to the right, so that it passes through the new asymptotic equilibrium point, $(\bar{c}', \bar{m}')$.

Knowledge of the long-run effects of the policy shift allows us to infer its short-run effects using the properties of the convergent saddlepath described in Section II. Because the eventual level of external assets is higher after the rise in μ, consumption must initially decline to bring about the implied current-account surplus. And because real balances rise during the course of a surplus and $\bar{m}'$ is lower than $\bar{m}$, the policy also entails a sharp initial fall in real balances and so a depreciation of the exchange rate. The economy's short-run, temporary equilibrium at $t = 0$ is thus at a point like A in figure 2.

Both consumption and real balances rise from the levels given by point A as the stock of foreign claims approaches its higher, asymptotic level. This implies, in particular, that the rate of depreciation of the exchange rate during the transition will fall short of the higher rate of monetary expansion. The exchange rate thus exhibits an "overshooting" pattern of adjustment, in that the impact fall in real balances exaggerates the long-run decline. This reproduces, in an optimizing framework, the result obtained by Kouri (1976), Calvo and Rodríguez (1977), and Flood (1979) on the basis of assumed aggregate relationships.

It is of interest to note that the integral of discounted utilities given in equation (2) is lower along the transition path following a rise in μ than it would have been along the original, unperturbed stationary-state path. This follows from the last inequality in (5) and the observation that the household's instantaneous utility falls the moment μ increases, rising to its original level only in the long run. Thus, an increase in the growth rate of money, in the present setting, has an unambiguously negative effect on the welfare of economic agents.

An Increase in Government Consumption

We analyze, finally, the consequences of an increase Δg in real government consumption. It is assumed initially that government consumption is wasteful, in that it does not enter into the household's utility function. An example in which government spending produces a public good complementary to private consumption is analyzed next.

In the absence of any change in private consumption from its initial level $\bar{c}$, an increase in government consumption would entail a current-account deficit at the initial level, $\bar{F}$, of foreign assets. This is shown in figure 3 by the leftward shift of the $\dot{F} = 0$ locus to $\dot{F}' = 0$, which is vertical at the lower level of private consumption compatible with external balance the moment after the increase in g occurs.

The steady-state levels of consumption and real balances are not altered by the government's action, however, for these variables are determined, independently of the level of g, by the two conditions, $\delta[U(\bar{c},\bar{m})] = r$, $x(\bar{c},\bar{m}) = \mu + r$. It follows that the stock of external claims associated with the economy's new stationary state must be higher than the initial stock, $\bar{F}$: In the long-run equilibrium following the increase in public-sector expenditure, the private consumption level $\bar{c}$ can be sustained with no net borrowing from abroad only if the economy's privately owned foreign assets rise by the amount $\Delta g/r$, so that additional interest payments from foreigners finance the additional national absorption.

Because the economy's adjustment path is noncyclical, the current account must go into surplus when the rise in g occurs. Consumption thus declines by an amount *greater* than the increase in government spending. And because consumption, real balances, and foreign bond holdings rise together along the saddlepath, the exchange rate necessarily depreciates on impact, reducing real balances in the short run so that they may subsequently increase toward their long-run level, $\bar{m}$.

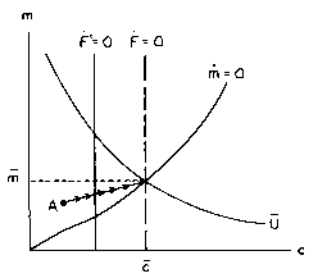

FIG. 3

The point labeled A in figure 3 again represents short-run equilibrium at $t = 0$.

When the monetary growth rate, μ, is zero, the government's action is simply a tax-financed increase in government spending. The model's prediction contradicts the standard Keynesian presumption that the accompanying short-run fall in disposable income must reduce private saving along with private expenditure, pushing the current account into deficit. In the present framework, private saving must actually increase in order that the economy may attain, in the long run, the higher level of national income consistent with the predisturbance private consumption level. Looked at another way, a tax-financed increase in government spending entails a more than complete crowding out of private aggregate demand in the short run.

The preceding discussion has been based on the assumption that the level of government spending does not enter the utility function, as it would if government consumption resulted in the provision of some public good. To illustrate how the analysis might change if this assumption were modified, we study a particular example.

Assume now that the utility function has the form

$$U(c,m,g) = u(c,g) + v(m), u_g, u_{cg} > 0. \tag{27}$$

According to (27), public and private consumption are complementary goods. The stationary values of consumption, real balances, and foreign assets are determined by the equations

$$\delta[U(\bar{c},\bar{m},g)] = r, \tag{28}$$

$$v'(\bar{m})/u_c(\bar{c},g) = \mu + r, \tag{29}$$

$$\bar{F} = [(\bar{c} + g - y)/r] - R. \tag{30}$$

In particular, (28) and (29) imply that $\bar{c}$ and $\bar{m}$ are no longer independent of g, as they were in the case of purely wasteful government spending.

The long-run derivatives of $\bar{c}$, $\bar{m}$, and $\bar{F}$ with respect to g are:

$$\frac{d\bar{c}}{dg} = -rD^{-1}[\bar{u}_g\bar{v}'' + \bar{v}'(\mu + r)\bar{u}_{cg}], \tag{31}$$

$$\frac{d\bar{m}}{dg} = rD^{-1}(\mu + r)(\bar{u}_c\bar{u}_{cg} - \bar{u}_g\bar{u}_{cc}), \tag{32}$$

$$\frac{d\bar{F}}{dg} = D^{-1}[(\bar{u}_c - \bar{u}_g)\bar{v}'' + \bar{v}'(\mu + r)(\bar{u}_{cc} - \bar{u}_{cg})], \tag{33}$$

where $D = r[\bar{u}_c\bar{v}'' + (\mu + r)\bar{u}_{cc}\bar{v}'] < 0$ and functions beneath overbars are evaluated at the initial long-run equilibrium. Only (32) can be

unambiguously signed under the assumption $u_{cg} > 0$.[16] In particular, if the marginal utility of private consumption is sufficiently low relative to that of public consumption, an increase in g may, as (33) shows, occasion a current deficit, contrary to the earlier analysis. In this situation, private consumption falls by an amount smaller than Δg in the short run; however, there is a decline in *long-run* consumption that exceeds Δg.

In the event that the level of public-good provision happens to be initially at the Samuelson static optimum, so that $\bar{u}_c = \bar{u}_g$, (32) and (33) imply that the increase in g must occasion a current surplus and a depreciation, as before. Why must stationary consumption rise above $\bar{c} - \Delta g$ in this case? If, in the long run, consumption were to fall by exactly Δg with $\bar{m}$ unchanged, the economy would be at its long-run utility level $\bar{U}$, but with $v'(\bar{m})/u_c(\bar{c} - \Delta g, g + \Delta g) < \mu + r$, in violation of (29). Long-run equilibrium thus requires that real money balances fall below $\bar{m}$ and that consumption rise above $\bar{c} - \Delta g$.

The welfare analysis of an increase in government consumption is straightforward. We note here simply that when $u_{cg} > 0$ and $\bar{u}_c = \bar{u}_g$, an increase in g must decrease the discounted sum of future utilities V below $\bar{U}/\delta(\bar{U})$, for it induces a current surplus and so a short-run decline in private consumption greater than Δg. The implication is that a level of public expenditure optimal from a static standpoint may no longer appear optimal once full account is taken of the dynamic repercussions of a change.

IV. Conclusion

This paper has developed an open-economy model based on household utility maximization over an infinite lifetime. The model answers several questions concerning the effects of macroeconomic policies when exchange rates float freely. In some cases, the answers coincide with results derived previously on the basis of more or less mechanistic behavioral assumptions. But in other cases, the answers contradict these results.

In particular, the model implies that money is not superneutral in either the short or long runs and places both the exchange-rate "overshooting" hypothesis and the association between depreciation and current deficits on a firm microeconomic foundation. However, it

[16] The following considerations imply that $d\bar{m}/dg < 0$. Because $u_{cg} > 0$, the increase in g would entail an increase in the marginal utility of private consumption at the initial level of c. If $\bar{m}$ were to rise, v' would fall, and (29) could not be satisfied in the long run in the absence of a higher stationary consumption level. But with $\bar{c}$, $\bar{m}$, and g all higher, the household's long-run utility level would exceed $\bar{U}$, contrary to (28). Thus, $\bar{m}$ declines and $v'(\bar{m})$ rises.

casts doubt on the validity of inferences drawn from the Keynesian consumption function which, in some form, is central to the classic Mundell-Fleming model and most subsequent work. In addition, the approach highlights the importance of the government and central-bank budget constraints in discussions of the effects of financial policy measures.

Appendix A

This appendix provides the proof of local saddlepath stability for the system (19)–(21) and characterizes the unique path leading to stationary-state equilibrium.

The model involves two state variables, c and m, which may take (unanticipated) discrete jumps, and a third state variable, F, which is predetermined by the past history of external asset accumulation. Thus, to prove saddlepath stability, it must be shown that when linearized about its stationary position, the differential equation system,

$$\dot{c} = \frac{\lambda\{\delta[u(c) + v(m)] - r\} - \lambda_m[\mu + r - x(c,m)]m - \lambda_F[y + r(F + R) - c - g]}{\lambda_c}$$

$$= \psi(c,m, F + R),$$

$$\dot{m} = [\mu + r - x(c,m)]m,$$

$$\dot{F} = y + r(F + R) - c - g,$$

possesses two positive characteristic roots and one negative characteristic root, implying the existence of a unique path converging to long-run equilibrium. Recall that

$$\lambda = \frac{\{\delta - [u(c) + v(m)]\delta'\}u'}{\delta + \{y + r(F + R) + [\mu + r - x(c,m)]m - c - g\}\delta'u'},$$

$$x(c,m) \equiv v'(m)/u'(c).$$

At the system's long-run equilibrium $(\bar{c},\bar{m},\bar{F})$, we have $\dot{c} = \dot{m} = \dot{F} = \dot{a} = \delta[u(\bar{c}) + v(\bar{m})] - r = 0$, and, linearizing in a neighborhood of this point, the system becomes

$$\dot{c} = \bar{\psi}_c(c - \bar{c}) + \bar{\psi}_m(m - \bar{m}) + \bar{\psi}_F(F - \bar{F}),$$

$$\dot{m} = -\bar{x}_c\bar{m}(c - \bar{c}) - \bar{x}_m\bar{m}(m - \bar{m}),$$

$$\dot{F} = -(c - \bar{c}) + r(F - \bar{F}).$$

(Functions beneath overbars are evaluated at $[\bar{c},\bar{m},\bar{F}]$.) Using the definition of long-run equilibrium, we find that

$$\bar{\psi}_c = (\bar{\lambda}/\bar{\lambda}_c)\bar{\delta}'\bar{u}' + (\bar{\lambda}_m/\bar{\lambda}_c)\bar{x}_c\bar{m} + (\bar{\lambda}_F/\bar{\lambda}_c), \tag{A1}$$

$$\bar{\psi}_m = (\bar{\lambda}/\bar{\lambda}_c)\bar{\delta}'\bar{v}' + (\bar{\lambda}_m/\bar{\lambda}_c)\bar{x}_m\bar{m}, \tag{A2}$$

$$\bar{\psi}_F = -(\bar{\lambda}_F/\bar{\lambda}_c)r, \tag{A3}$$

where

$$\bar{\lambda}_c = \frac{[\bar{\delta} - (\bar{u} + \bar{v})\bar{\delta}'][\bar{\delta}\bar{u}'' + \bar{\delta}'(\bar{u}')^2\bar{x}_c\bar{m}] - (\bar{u} + \bar{v})(\bar{u}')^2\bar{\delta}\bar{\delta}''}{\bar{\delta}^2}, \tag{A4}$$

$$\bar{\lambda}_m = \frac{[\bar{\delta} - (\bar{u} + \bar{v})\bar{\delta}'](\bar{\delta}'\bar{u}'\bar{x}_m\bar{m} - \bar{\delta}'\bar{v}')\bar{u}' - (\bar{u} + \bar{v})\bar{u}'\bar{v}'\bar{\delta}\bar{\delta}''}{\bar{\delta}^2}, \tag{A5}$$

$$\bar{\lambda}_F = \frac{-[\bar{\delta} - (\bar{u} + \bar{v})\bar{\delta}']\bar{\delta}'(\bar{u}')^2}{\bar{\delta}} < 0. \tag{A6}$$

Now $\bar{x}_c = -v'(\bar{m})u''(\bar{c})/[u'(\bar{c})]^2 > 0$ and $\bar{x}_m = v''(\bar{m})/u'(\bar{c}) < 0$; and since $U \geqslant 0$, $\delta - U\delta' > 0$, and $\delta'' > 0$ by assumption,

$$\bar{\lambda}_m < 0. \tag{A7}$$

Further, substituting for $\bar{x}_c$ in (A4) yields

$$\bar{\lambda}_c = \frac{[\bar{\delta} - (\bar{u} + \bar{v})\bar{\delta}'](\bar{\delta} - \bar{\delta}'\bar{v}'\bar{m})\bar{u}'' - (\bar{u} + \bar{v})(\bar{u}')^2\bar{\delta}\bar{\delta}''}{\bar{\delta}^2}. \tag{A8}$$

Because $u(\cdot)$ and $v(\cdot)$ are nonnegative and concave, we have, using (5), $\bar{\delta} - \bar{\delta}'\bar{v}'\bar{m} > \bar{\delta} - \bar{v}\bar{\delta}' > \bar{\delta} - (\bar{u} + \bar{v})\bar{\delta}' > 0$, and (A8) implies immediately that

$$\bar{\lambda}_c < 0. \tag{A9}$$

We note, finally, that because $\bar{\lambda} = \{\bar{\delta} - [u(\bar{c}) + v(\bar{m})]\bar{\delta}'\}u'/\bar{\delta}$, (A1) and (A6) imply that

$$\bar{\psi}_c = (\bar{\lambda}_m/\bar{\lambda}_c)\bar{x}_c\bar{m} > 0. \tag{A10}$$

Using (A10), the linearized system becomes

$$\begin{bmatrix} \dot{c} \\ \dot{m} \\ \dot{F} \end{bmatrix} = \begin{bmatrix} \dfrac{\bar{\lambda}_m\bar{x}_c\bar{m}}{\bar{\lambda}_c} & \dfrac{\bar{\lambda}\bar{\delta}'\bar{v}' + \bar{\lambda}_m\bar{x}_m\bar{m}}{\bar{\lambda}_c} & \dfrac{-\bar{\lambda}_F r}{\bar{\lambda}_c} \\ -\bar{x}_c\bar{m} & -\bar{x}_m\bar{m} & 0 \\ -1 & 0 & r \end{bmatrix} \begin{bmatrix} c - \bar{c} \\ m - \bar{m} \\ F - \bar{F} \end{bmatrix}. \tag{A11}$$

By (A10) and the fact that $\bar{x}_m < 0$, the trace of the system's matrix is positive. Since this trace is also equal to the sum of the system's characteristic roots, there must be at least one root with positive real part.

On the other hand, we may write the system's determinant as

$$\begin{aligned} \Omega &= [\bar{x}_m\bar{m}(\bar{\lambda}_F/\bar{\lambda}_c)r] + r(\bar{x}_c\bar{m})(\bar{\lambda}\bar{\delta}'\bar{v}' + \bar{\lambda}_m\bar{x}_m\bar{m})/\bar{\lambda}_c - r(\bar{x}_m\bar{m})(\bar{\lambda}_m/\bar{\lambda}_c)\bar{x}_c\bar{m} \\ &= [\bar{x}_m\bar{m}(\bar{\lambda}_F/\bar{\lambda}_c)r] + r(\bar{x}_c\bar{m})(\bar{\lambda}\bar{\delta}'\bar{v}')/\bar{\lambda}_c < 0. \end{aligned}$$

Because Ω is the product of the system's characteristic roots, it must be the case that there are either three roots with negative real part or one negative root. (Recall that complex roots must occur as conjugate pairs.) Since we already know there cannot be three roots with negative real part, there must be exactly one negative root. This proves that in a neighborhood of long-run equilibrium, our system is saddlepath-stable.

It remains to provide a characterization of the unique stable path. Let θ_1 denote the system's sole negative root, and let $\vec{\omega} = [\omega_{11}, \omega_{21}, \omega_{31}]'$ denote an eigenvector belonging to θ_1. As was shown in the text, c, m, and F must rise or fall together along the saddlepath if the components of $\vec{\omega}$ are all of the same sign. We now prove that this is the case.

First, we note that if one of the components of $\vec{\omega}$ is zero, all must be zero, and so all components must be strictly positive or negative. To see this, suppose $\omega_{11} = 0$. Then, using the last equation of (A11), $-\omega_{11} + r\omega_{31} = \theta_1\omega_{31}$,

and so $\omega_{3t} = 0$. But using the second equation of (A11), we find that $\omega_{2t} = 0$ as well; and so $\vec{\omega} = 0$. Similar arguments dispose of the possibilities that $\omega_{2t} = 0$ and that $\omega_{3t} = 0$.

So suppose $\omega_{3t} > 0$. From (A11), $-\omega_{1t} + r\omega_{3t} = \theta_t\omega_{3t} < 0$, and so it must be the case that $\omega_{1t} > 0$. Again using (A11), we see that $(-\bar{x}_c\bar{m})\omega_{1t} + (-\bar{x}_m\bar{m})\omega_{2t} = \theta_t\omega_{2t}$, implying $\omega_{2t} = [(-\bar{x}_c\bar{m})/(\theta_t + \bar{x}_m\bar{m})]\omega_{1t}$. Thus, it must be true that $\omega_{2t} > 0$. In this case, therefore, all components of $\vec{\omega}$ have the same sign.

In the case $\omega_{3t} < 0$, an identical argument shows that $\omega_{1t}, \omega_{2t} < 0$ as well. This completes the proof.

Appendix B

This appendix contains a proof that the unique convergent path of the system (19)–(21) is a perfect-foresight equilibrium path. The minor technical issue in the proof is that the state variable a_t may assume negative values, so that the sufficiency theorem for optimal controls (see Arrow and Kurz 1970, p. 49) appears inapplicable. We show that the intertemporal budget constraint (8) implies that a_t must be bounded from below when consumers expect the path of prices implied by the convergent solution to (19)–(21). This fact, in the present context, implies that the sufficiency theorem remains applicable.

Substituting the flow constraint (7) into (8) and integrating by parts, we obtain the inequality

$$\lim_{t\to\infty} a_t e^{-rt} \geqslant 0. \tag{B1}$$

Condition (B1) may be used to prove the inequality

$$a_t + \int_t^\infty e^{-r(s-t)}\hat{\tau}_s ds \geqslant 0, \text{ for all } t, \tag{B2}$$

where hatted values are those expected to prevail along the convergent path. Assume (B2) fails at time t'. Integration of (7) implies that for $t > t'$, a_t must satisfy

$$a_t = a_{t'}e^{r(t-t')} + \int_{t'}^t e^{-r(s-t)}[\hat{\tau}_s - c_s - (\hat{\pi}_s + r)m_s]ds, \tag{B3}$$

for any path of consumption and real balances. Multiplying both sides of (B3) by e^{-rt} and taking limits as $t \to \infty$ yields the equality

$$\lim_{t\to\infty} a_t e^{-rt} = e^{-rt'}\left\{a_{t'} + \int_{t'}^\infty e^{-r(s-t')}[\hat{\tau}_s - c_s - (\hat{\pi}_s + r)m_s]ds\right\}. \tag{B4}$$

Because consumption and real balances are nonnegative, the right-hand side of (B4) must be negative if (B2) is false at t'.[17] This contradicts (B1), showing that (B2) must hold at each instant.

Define

$$k \equiv \sup_t \left\{\int_t^\infty e^{-r(s-t)}\hat{\tau}_s ds\right\}.$$

The number k is finite as a result of the boundedness of the sequence $\{\hat{\tau}_s\}$ along the convergent path. From (B2),

$$a_t + k \geqslant 0, \text{ for all } t. \tag{B5}$$

[17] Our assumptions ensure that $\hat{\pi} + r > 0$ at all times.

It is now straightforward to apply the sufficiency theorem. Redefine the state variable for the household's problem to be $w_t \equiv a_t + k$. The flow constraint (7) becomes $\dot{w}_t = rw_t + \hat{\tau}_t - c_t - (\hat{\pi}_t + r)m_t - rk$. The differential equation system (19)–(21) is unchanged as a result of this modification, and along its convergent path,

$$\lim_{\Delta \to \infty} w_\Delta \lambda_\Delta e^{-\Delta} = 0, \lim_{\Delta \to \infty} \lambda_\Delta e^{-\Delta} \geqslant 0.$$

Because negative values of the state variable are infeasible, the sufficiency theorem implies that the choices of consumption and real balances implied by the convergent path are indeed optimal choices, given the associated paths of the price level and real transfer payments. The convergent path is therefore a perfect-foresight equilibrium path.

References

Arrow, Kenneth J., and Kurz, Mordecai. *Public Investment, the Rate of Return, and Optimal Fiscal Policy.* Baltimore: Johns Hopkins Univ. Press (for Resources for the Future), 1970.

Brock, William A. "Money and Growth: The Case of Long Run Perfect Foresight." *Internat. Econ. Rev.* 15 (October 1974): 750–77.

Calvo, Guillermo A. "On Models of Money and Perfect Foresight." *Internat. Econ. Rev.* 20 (February 1979): 83–103. (*a*)

———. "Stabilization Rules and the Managed Float: A Search for the Essentials." Discussion Paper no. 23, Columbia Univ., Dept. Econ., 1979. (*b*)

———. "Financial Opening, Crawling Peg and the Real Exchange Rate." Mimeographed. Centro de Estudios Macroeconómicos de Argentina, Buenos Aires, 1980.

Calvo, Guillermo A., and Findlay, Ronald. "On the Optimal Acquisition of Foreign Capital through Investment of Oil Export Revenues." *J. Internat. Econ.* 8 (November 1978): 513–24.

Calvo, Guillermo A., and Rodríguez, Carlos Alfredo. "A Model of Exchange Rate Determination under Currency Substitution and Rational Expectations." *J.P.E.* 85, no. 3 (June 1977): 617–25.

Findlay, Ronald. "An 'Austrian' Model of International Trade and Interest Rate Equalization." *J.P.E.* 86, no. 6 (December 1978): 989–1007.

Fischer, Stanley. "Capital Accumulation on the Transition Path in a Monetary Optimizing Model." *Econometrica* 47 (November 1979): 1433–39.

Flood, Robert P. "An Example of Exchange Rate Overshooting." *Southern Econ. J.* 46 (July 1979): 168–78.

Helpman, Elhanan. "Inflation and Balance of Payments Adjustments with Maximizing Consumers." Seminar Paper no. 129, Inst. Internat. Econ. Studies, Univ. Stockholm, 1979.

Hirsch, Morris W., and Smale, Stephen. *Differential Equations, Dynamical Systems, and Linear Algebra.* New York: Academic Press, 1974.

Koopmans, Tjalling C.; Diamond, Peter A.; and Williamson, Richard E. "Stationary Utility and Time Perspective." *Econometrica* 32 (January/April 1964): 82–100.

Kouri, Pentti J. K. "The Exchange Rate and the Balance of Payments in the Short Run and in the Long Run: A Monetary Approach." *Scandinavian J. Econ.* 78 (May 1976): 280–304.

———. "Balance of Payments and the Foreign Exchange Market: A Dynamic

Partial Equilibrium Model." Cowles Foundation Discussion Paper no. 510, Yale Univ., Dept. Econ., 1978.

———. "Monetary Policy, the Balance of Payments, and the Exchange Rate." In *The Functioning of Floating Exchange Rates: Theory, Evidence, and Policy Implications,* edited by David Bigman and Teizo Taya. Cambridge, Mass.: Ballinger, 1980.

Mundell, Robert A. *Monetary Theory: Inflation, Interest, and Growth in the World Economy.* Pacific Palisades, Calif.: Goodyear, 1971.

Obstfeld, Maurice. "Macroeconomic Policy and World Welfare under Flexible Exchange Rates." Discussion Paper no. 63, Columbia Univ., Dept. Econ., 1980.

———. "Capital Mobility and Devaluation in an Optimizing Model with Rational Expectations." *A.E.R. Papers and Proc.* 71 (May 1981): 217–21.

Razin, Assaf. "Exchange Rates and Price Movements in a World of Capital Mobility." Mimeographed. Foerder Inst., Tel-Aviv Univ., 1979.

Sidrauski, Miguel. "Rational Choice and Patterns of Growth in a Monetary Economy." *A.E.R. Papers and Proc.* 57 (May 1967): 534–44.

Stockman, Alan C. "A Theory of Exchange Rate Determination." *J.P.E.* 88, no. 4 (August 1980): 673–98.

Uzawa, Hirofumi. "Time Preference, the Consumption Function, and Optimum Asset Holdings." In *Value, Capital, and Growth: Papers in Honour of Sir John Hicks,* edited by J. N. Wolfe. Chicago: Aldine, 1968.