NBER WORKING PAPER SERIES

AN EMPIRICAL EVALUATION OF THE LONG-RUN RISKS MODEL FOR ASSET PRICES

Ravi Bansal
Dana Kiku
Amir Yaron

Working Paper 15504
http://www.nber.org/papers/w15504

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
November 2009

Yaron thanks the Rodney White Center for financial support. The views expressed herein are those of the author(s) and do not necessarily reflect the views of the National Bureau of Economic Research.

An Empirical Evaluation of the Long-Run Risks Model for Asset Prices
Ravi Bansal, Dana Kiku, and Amir Yaron
NBER Working Paper No. 15504
November 2009
JEL No. E0,G0,G1,G12,G14

ABSTRACT

We provide an empirical evaluation of the forward-looking long-run risks (LRR) model and highlight model differences with the backward-looking habit based asset pricing model. We feature three key results: (i) Consistent with the LRR model, there is considerable evidence in the data of time-varying expected consumption growth and volatility, (ii) The LRR model matches the key asset markets data features, (iii) In the data and in the LRR model accordingly, past consumption growth does not predict future asset prices, whereas lagged consumption in the habit model forecasts future price-dividend ratios with an R2 of over 40%. Overall, our evidence implies that the LRR model provides a coherent framework to analyze and interpret asset prices.

Ravi Bansal
Fuqua School of Business
Duke University
1 Towerview Drive
Durham, NC 27708
and NBER
ravi.bansal@duke.edu

Amir Yaron
The Wharton School
University of Pennsylvania
2256 Steinberg-Dietrich Hall
Philadelphia, PA 19104-6367
and NBER
yaron@wharton.upenn.edu

Dana Kiku
Finance Department
Wharton School
University of Pennsylvania
3620 Locust Walk
Philadelphia, PA 19104-6367
kiku@wharton.upenn.edu

1 Introduction

The economic sources of risks and the magnitude of predictability of asset prices, consumption, dividends, and volatility are topics of considerable interest for financial economists. Cash-flow predictability and asset price fluctuations impose economic restrictions that help evaluate the plausibility of asset pricing models. In this paper, we empirically evaluate the long-run risks (LRR) model of Bansal and Yaron (2004) along these challenging dimensions. In our analysis, we also highlight some key economic and quantitative differences between the LRR model and the external habit model of Campbell and Cochrane (1999).

The Bansal and Yaron (2004) LRR model contains two long-run risk channels: (i) long-run fluctuations in expected growth, and (ii) long-run fluctuations in consumption volatility. The model features an Epstein and Zin (1989) life-time utility function, with investor preference for early resolution of uncertainty. Shocks to expected growth and consumption volatility are long-lasting and alter investor's expected growth and volatility computations for long-horizons. Equity prices in the model are determined by expected growth and consumption volatility and, therefore, may help predict future growth and uncertainty. In this sense, the LRR model is forward-looking. In contrast, asset prices in the external habit model of Campbell and Cochrane (1999), in which consumption and dividends are *i.i.d*, are driven by time-varying risk aversion that moves in response to the entire history of consumption growth. Asset prices in this external habit model are backward-looking as lagged consumption significantly influences their movements. It is evident that the two models have considerable differences in their implications for predictability of asset prices, consumption growth, dividends, and volatility of returns. [1]

We calibrate the LRR model of Bansal and Yaron (2004) and use an improved model solution based on the approximate analytical method as in Bansal, Kiku, and Yaron (2007b). The model configuration we use for the LRR model is taken from the refined calibration provided in Bansal, Kiku, and Yaron (2007a). In terms of the data, we confine our attention to the long annual sample from 1929 to 2008, due to its long span and the fact that this sample experienced various episodes of high turbulence in asset markets, a feature that should not be ignored, as recent financial market events underscore.

[1] Throughout the paper the external habit model refers to the Campbell and Cochrane (1999) specification. Our analysis does not evaluate implications of other variants of habit models.

The key data-features we focus on are: (i) consumption and return predictability, (ii) relation between consumption volatility and asset prices, (iii) predictability of return volatility, and (iv) price-dividend ratio predictability via consumption. We also make observations regarding empirical evidence on the magnitude of preference parameters, in particularly, the elasticity of intertemporal substitution, and discuss model implications for the yield curve and their fit to the observed data. Our statistical analysis is carried out by using the model-based finite sample empirical distribution, which is also the approach pursued in Beeler and Campbell (2009).[2] Our model inferences are robust to using alternative methods to construct standard errors as confirmed by the results reported in Bansal, Kiku, and Yaron (2007a).

We show that in the data consumption growth is highly predictable at both short- and long-horizons. A vector autoregression (VAR) based on consumption growth, price-dividend ratio, and the real risk-free rate implies consumption predictability at the one- and five-year horizons of more than 20%, which is statistically different from zero. The VAR-based predictability of consumption growth in the LRR model is of the same magnitude as in the data. Using a VAR framework, Hansen, Heaton, and Li (2006) also find strong evidence of predictable variations in consumption growth. Our evidence indicates that there can be significant loss of information about variation in expected consumption growth rates in univariate predictive regressions considered in Beeler and Campbell (2009). Even then, we document that if one relies only on the price dividend ratio to forecast future consumption, the regression statistics implied by the LRR model are well within the two standard error (2-SE) from the data.

Consistent with the literature, we find that future equity returns are predictable by current dividend yields. However, the evidence for return predictability in the data is very fragile – confidence bands for predictive R^2's include zero, suggesting lack of predictability. Further, when we predict future returns using the better behaved variable, dividend-price ratio less the real risk-free rate, the level of return predictability declines from 31% to only about 9% at the five-year horizon.[3] After accounting for standard errors, we show that the

[2]Beeler and Campbell (2009) examine several aspects of the LRR model. According to their reported standard errors, the LRR model matches all the data features they focus on within the usual two standard error range.

[3]The difference in the magnitude of R^2's from the dividend yield-based regression and the predictive regression based on the adjusted dividend-price ratio is most likely due to the very high persistence of the dividend yield in the data (see also Hodrick (1992) and Stambaugh (1999)).

LRR model is consistent with the observed predictability of returns.

Bansal and Yaron (2004) show that in the LRR model consumption volatility is a source of systematic risk as shocks to it carry a separate risk premium. They further document that in the data a rise in current consumption volatility lowers the price-dividend ratios and that future consumption volatility can be forecasted by current price-dividend ratios. As highlighted in Bansal and Yaron (2004), this evidence suggests that the elasticity of intertemporal substitution (IES) is larger than one. Bansal, Khatchatrian, and Yaron (2005) document the robustness of the negative relation between consumption volatility and asset price and further confirm that movements in consumption volatility are indeed an important risk channel. Beeler and Campbell (2009) also explore the dynamics of return volatility inside the LRR model. In particular, they consider an integrated return volatility measure and explore its predictability properties. We show that quantitatively, the LRR model matches the sign and the magnitude of the inverse relation between prices and consumption uncertainty and accounts for the observed predictable variation of the integrated volatility of asset returns.

The LRR model and the habit model provide an interesting contrast in the context of predictability of price to dividend ratios. In the data, forecasting future price to dividend ratios with lagged consumption yields an R^2 close to zero. Consistent with the data, in the LRR model, lagged consumption growth rates do not predict future prices. In fact, the R^2 in the LRR model and in the data are almost identical. As mentioned above, asset prices in the Bansal and Yaron (2004) model are determined by expectations of future growth and volatility, and therefore changes in these expectations drives movements in current price-dividend ratios. In contrast, in the habit model of Campbell and Cochrane (1999), asset prices are driven by backward consumption as lagged consumption growth forecasts future price-dividend ratios with an R^2 of more than 40%. In terms of differences in economic implications, the LRR model would attribute the sharp recent decline in equity prices to a decline in future expected growth and a rise in volatility of future growth. The habit model, on the other hand, would attribute the decline in equity prices to past and current reductions in consumption growth and a resulting rise in risk-aversion. In all, the data evidence regarding future price predictability with lagged consumption raises considerable doubt regarding the key channel featured in the Campbell and Cochrane (1999) habit model.

Overall, our results (i) support the view that there is a small long-run predictable component in consumption growth, and that consumption volatility is time-varying, (ii) confirm that the forward-looking LRR model can account for the key dynamic properties of asset market data, and (iii) suggest that there is little empirical support for the key mechanism in the backward-looking habit model, that lagged consumption growth forecasts asset prices. Our conclusions are in sharp contrast to Beeler and Campbell (2009) regarding the fit of the LRR model and the empirical plausibility of the habit model.

The paper continues as follows. Section 2 outlines the LRR model and highlights its key features. Section 3 discusses the results of our empirical analysis. Section 4 provides concluding comments.

2 Long-Run Risks Model

In this section we specify a model based on Bansal and Yaron (2004). The underlying environment is one with complete markets and a representative agent has Epstein and Zin (1989) type recursive preferences and maximizes her life-time utility,

$$V_t = \left[(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta\Big(E_t\big[V_{t+1}^{1-\gamma}\big]\Big)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}, \tag{1}$$

where C_t is consumption at time t, $0 < \delta < 1$ reflects the agent's time preference, γ is the coefficient of risk aversion, $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, and ψ is the elasticity of intertemporal substitution (IES). Utility maximization is subject to the budget constraint,

$$W_{t+1} = (W_t - C_t)R_{c,t+1}, \tag{2}$$

where W_t is the wealth of the agent, and $R_{c,t}$ is the return on all invested wealth.

Consumption and dividends have the following joint dynamics:

$$\begin{aligned}
\Delta c_{t+1} &= \mu_c + x_t + \sigma_t \eta_{t+1} \\
x_{t+1} &= \rho x_t + \varphi_e \sigma_t e_{t+1} \\
\sigma_{t+1}^2 &= \bar{\sigma}^2 + \nu(\sigma_t^2 - \bar{\sigma}^2) + \sigma_w w_{t+1} \\
\Delta d_{t+1} &= \mu_d + \phi x_t + \pi \sigma_t \eta_{t+1} + \varphi \sigma_t u_{d,t+1},
\end{aligned} \tag{3}$$

where Δc_{t+1} and Δd_{t+1} are the growth rate of consumption and dividends respectively. In addition, we assume that all shocks are $i.i.d$ normal and are orthogonal to each other. As in the long run risks model of Bansal and Yaron (2004), $\mu_c + x_t$ is the conditional expectation of consumption growth, and x_t is a small but persistent component that captures long run risks in consumption growth. For parsimony, as in Bansal and Yaron (2004), volatility of consumption and dividends is driven by a common time-varying component. As shown in their paper, predictable variations in the conditional second moment of growth rates lead to time-varying risk premia. Dividends have a levered exposure to the persistent component in consumption, x_t, which is captured by the parameter ϕ. In addition, we allow the $i.i.d$ consumption shock η_{t+1} to influence the dividend process, and thus serve as an additional source of risk premia. The magnitude of this influence is governed by the parameter π. Save for this addition, the dynamics are similar to those in Bansal and Yaron (2004).

As in Epstein and Zin (1989), for any asset j, the first order condition yields the following asset pricing Euler condition,

$$E_t \left[\exp\left(m_{t+1} + r_{j,t+1}\right)\right] = 1, \tag{4}$$

where m_{t+1} is the log of the intertemporal marginal rate of substitution (IMRS), and $r_{j,t+1}$ is the log of the gross return on asset j. The log of the IMRS, m_{t+1}, is given by

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1) r_{c,t+1}, \tag{5}$$

where $r_{c,t+1}$ is the continuous return on the consumption asset. To solve for the return on wealth (the return on the consumption asset), we use the log-linear approximation for the continuous return on the wealth portfolio, namely,

$$r_{c,t+1} = \kappa_0 + \kappa_1 z_{t+1} + \Delta c_{t+1} - z_t \ , \tag{6}$$

where $z_t = \log(P_t/C_t)$ is the log of the price to consumption ratio (i.e., the valuation ratio corresponding to a claim that pays consumption) and κ's are log linearization constants which are discussed in more detail below.

To derive the dynamics of asset prices we rely on approximate analytical solutions (instead of the polynomial-based numerical approximation in the original paper of Bansal and Yaron

(2004)), which we find provide a more accurate solution to the model.[4] This easy-to-implement solution technique allows us to better address certain predictability dimensions. Specifically, we conjecture that the price to consumption ratio follows,

$$z_t = A_0 + A_1 x_t + A_2 \sigma_t^2 \tag{7}$$

and solve for A's using the Euler equation (4), the return equation (6) and the conjectured dynamics (7). In solving for the price-consumption ratio we impose model consistency between its mean, $\bar{z}$, and approximation κ's, which themselves depend on the average price-consumption ratio. This allows us to make sure that any change in the model parameters that alters $\bar{z}$ is also incorporated in the approximation constants. The model-based endogenous solution for $\bar{z}$ is thus obtained by solving the equation,

$$\bar{z} = A_0(\bar{z}) + A_2(\bar{z})\bar{\sigma}^2 \ , \tag{8}$$

and recognizing that approximation constants that enter A's are defined by $\kappa_0 = \log(1 + \exp(\bar{z})) - \kappa_1 \bar{z}$ and $\kappa_1 = \frac{\exp(\bar{z})}{1+\exp(\bar{z})}$.

The solutions for A's that describe the dynamics of the price-consumption ratio are determined by the preference and technology parameters as:

$$\begin{aligned}
A_0 &= \frac{1}{1-\kappa_1}\left[\log\delta + \kappa_0 + \left(1-\frac{1}{\psi}\right)\mu_c + \kappa_1 A_2 (1-\nu)\bar{\sigma}^2 + \frac{\theta}{2}\left(\kappa_1 A_2 \sigma_w\right)^2\right] \\
A_1 &= \frac{1-\frac{1}{\psi}}{1-\kappa_1\rho} \\
A_2 &= -\frac{(\gamma-1)(1-\frac{1}{\psi})}{2\ (1-\kappa_1\nu)}\left[1+\left(\frac{\kappa_1\varphi_e}{1-\kappa_1\rho}\right)^2\right]
\end{aligned} \tag{9}$$

Bansal and Yaron (2004) show that solution (9) captures the intuition that, as long as IES is larger than one, the substitution effect dominates the wealth effect. Consequently, high expected growth raises asset valuations, while high consumption volatility lowers the price-consumption (and price-dividend) ratio. This is an important implication of the model as it may help identify the magnitude of IES in the data.

[4]Bansal, Kiku, and Yaron (2007b) evaluate various approaches and find the approximate analytical solution to be the most accurate and easy to implement.

Given the solution for z_t, the innovation to the return to wealth can be derived, which in turn allows us to specify the innovations to the IMRS and facilitates the computation of risk premia of various assets. In particular, it follows that the risk premium on the stock market portfolio is derived from three sources of risks. Specifically,

$$E_t[r_{m,t+1} - r_{f,t} + 0.5\sigma^2_{t,r_m}] = \beta_{\eta,m}\lambda_\eta\sigma^2_t + \beta_{e,m}\lambda_e\sigma^2_t + \beta_{w,m}\lambda_w\sigma^2_w, \tag{10}$$

where $\beta_{m,j},\ j = \{\eta, e, w\}$ are the betas of the market return with respect to the "short-run" risk (η_t), the long-run growth risk (e_t), and the volatility risk (w_t), respectively. The market return betas are determined by the underlying preferences and cash-flow dynamics and are presented in Appendix. λ's represent the corresponding market prices of risks that, as shown in Bansal and Yaron (2004), are given by:

$$\begin{aligned}
\lambda_\eta &= \gamma \\
\lambda_e &= (1-\theta)\kappa_1 A_1 \varphi_e = \Big(\gamma - \frac{1}{\psi}\Big)\frac{\kappa_1\varphi_e}{1-\kappa_1\rho} \\
\lambda_w &= (1-\theta)\kappa_1 A_2 = -\Big(\gamma - 1\Big)\Big(\gamma - \frac{1}{\psi}\Big)\frac{\kappa_1}{2\ (1-\kappa_1\nu)}\Big[1 + (\frac{\kappa_1\varphi_e}{1-\kappa_1\rho})^2\Big].
\end{aligned} \tag{11}$$

Note that, due to separation between risk aversion and IES, each risk carries a separate premium. In power utility framework, where IES equals the reciprocal of risk aversion, only short-run risks receive compensation, while long-run and volatility risks carry no separate risk premia. The market prices of risks in equation (11) show that preference for early resolution of uncertainty (i.e., γ larger than the reciprocal of IES) is required for long-run risks to earn a positive risk premium.

Table I provides the parameter configuration we use to calibrate the model – these are chosen to match several key statistics of consumption and dividend data. The parameter values are identical to those in Bansal, Kiku, and Yaron (2007a) and referred to as BKY parameter configuration in Beeler and Campbell (2009). This calibration refines the Bansal and Yaron (2004) configuration in two directions. First, the persistence of volatility shocks is assumed to be higher; second, dividend shocks are assumed to be correlated with short-run shocks in consumption growth, while in Bansal and Yaron (2004) the correlation between the two is set at zero. These changes enhance the role of the volatility channel relative to Bansal and Yaron (2004).

The LRR model specification as stated in equation (3), for analytical tractability and ease of solution, assumes that volatility shocks are normally distributed. In simulations, we address the possibility of negative realizations by discarding negative draws. We have also evaluated the approach of replacing negative realizations of σ^2 with a small positive number and found the results to be virtually identical in the two cases. Note that the standard deviation of volatility shocks (σ_w) is quite small relative to its mean. The fraction of negative realizations, therefore, is also small, averaging about 0.6% of the draws at our calibrated values. A conceptually cleaner approach is pursued in Bansal and Shaliastovich (2009), who follow Barndorff-Nielsen and Shephard (2001), and assume that volatility shocks have a gamma distribution which ensures positivity of the volatility process. Bansal and Shaliastovich (2009) show that the model implications in the gamma distribution case are similar to the gaussian case presented here.

3 Empirical Findings

We use annual data on consumption and asset prices for the time period from 1929 till 2008. The annual data provides the longest available data span and is arguably the least susceptible to measurement errors. This sample also contains various episodes of crisis in asset markets, a feature, as recent financial market events underscore, that cannot be ignored or excluded from the sample. Focusing on the post-war data, as some papers do, can be misleading as this sample ignores a number of important volatile asset market and macro-economic events.

Consumption data are based on seasonally adjusted per-capita series on real consumption from the NIPA tables available on the Bureau of Economic Analysis website. Aggregate consumption is defined as consumer expenditures on non-durables and services. Growth rates are constructed by taking the first difference of the corresponding log series. Our asset menu comprises the aggregate stock market portfolio on the value weighted return of the NYSE/AMEX/NASDAQ from CRSP and a proxy of a risk-less asset. The real interest rate is constructed by subtracting the trailing 12-month realized annual inflation from the annualized yield on the 3-month Treasury bill taken from the CRSP treasury files. Use of other estimates of expected inflation to construct the real rate does not lead to any significant changes in our results. Descriptive statistics for consumption growth, the return and dividend yield of the aggregate stock market, and the risk-free rate are presented in

Table II. All entries are expressed in real percentage terms.

In calibration and simulations, following the standard in the literature, we assume that the decision interval of the agent is monthly. To make the model-implied data comparable to the observed annual data, we appropriately aggregate the simulated monthly observations and construct annual growth rates and annual asset returns. The price-dividend ratio, as in the data, is constructed by dividing the end-of-year price by the trailing sum of 12-month dividends.

For statistical inference, as in Drechsler and Yaron (2007) and Beeler and Campbell (2009), we sample from the calibrated model and construct the finite-sample empirical distribution for various statistics of interest. Reported statistics are based on 10,000 simulated samples with 79×12 monthly observations that match the length of the actual data. We report the median and tail percentiles of the monte-carlo distributions. In addition, we present population values that correspond to the statistics constructed from a long-sample of 10,000 annualized observations.

3.1 Equity Premium & Risk-free Rate Puzzles

Table II displays the model implications for the unconditional moments of consumption and dividend growth rates, the equity return, price-dividend ratio and the risk-free rate. Overall, the model matches all these dimensions quite well. In particular, our calibration accounts for the first-order (and unreported higher order) autocorrelations of consumption growth. It is worth noting that the first-order autocorrelation of consumption growth in the data is 0.46, which is much higher than the one implied by monthly *i.i.d* growth rates even after accounting for time-aggregation. According to the results of Working (1960), the annual autocorrelation with *i.i.d* growth rates would only be 0.25.[5] The model based finite-sample empirical distribution shows that the model matches the unconditional moments of consumption and dividends data quite well.

The model matches the level and volatility of the equity returns and the risk-free rate quite well. The average excess return in our data set is around 7%. For comparison, the model-implied risk premium of the stock market portfolio averages 6.9%. In the model, as in the data, the volatility of equity returns is about 20%, which is much higher than the

[5] It would be even lower under plausible scenarios of measurement errors in monthly consumption data.

volatility of the underlying cash-flow growth rates. Consistent with the data, the model-impled mean and volatility of the real risk-free rate are around 1% per annum.

An insightful experiment is to evaluate the contribution of various risk sources to the equity risk-premium. In our calibration, short-run risks contribute 25%, long-run growth risks contribute 32%, and long-run volatility risks contribute 43% to the overall risk premium. The two persistent sources combined account for 75% of the equity premium.

It is important to note that the long-run growth risk is critical for explaining the equity risk premium, as it not only accounts for a significant portion of the premium itself but also magnifies the contribution of the volatility risk. In the absence of the long-run growth risk (i.e., if the variance of x_t is zero), the annualized equity premium is only 0.92%. The population value of the volatility of the price-dividend ratio in this case is about 0.19. If, on the other hand, the long-run growth risk is present but the volatility channel is shut down, the annualized equity premium is 3.95% but the variance of the price-dividend ratio drops to 0.09. Thus, the long-run growth risk is important for the level of the equity risk-premium, while the volatility channel is important for the variability of asset prices.

3.2 Consumption, Dividends & Return Predictability

Table III provides evidence on consumption predictability using a VAR with consumption growth, real risk-free rate, and the log price-dividend ratio. The R^2 for consumption predictability starts at 26% at the 1-year horizon and drops only to 22% at the 5-year horizon.[6] Thus, consumption growth in the data is strongly predictable at both short and long horizons, which is consistent with consumption predictability evidence reported in Hansen, Heaton, and Li (2006) and Bansal, Kiku, and Yaron (2007b). Table III further shows that the LRR model fully duplicates the documented pattern of consumption predictability. Note that a monthly *i.i.d* consumption growth process, time-aggregated to the annual frequency, would imply an R^2 of only 6% for the first year and close to zero for the second and subsequent years. Our empirical evidence, therefore, casts doubt on the view that consumption growth is *i.i.d*, as often assumed in the literature (e.g., Campbell and Cochrane (1999)).

Panel A of Table IV provides the results of consumption growth predictability using the

[6] As in Hodrick (1992), R^2 are constructed by exploiting the dynamics of the first-order VAR specification.

log of the dividend-price ratio as the only regressor. Estimates of slope coefficients ($\hat{\beta}$) in these regressions for various horizons are presented in Panel B of the table. In the data, the R^2's in these regressions are 7% at the 1-year horizon and close to zero at the 5-year horizon. The model-implied evidence reveals a similar modest forecasting power of the price-dividend ratio. In particular, the population R^2 in these predictive regressions is only 7% and 4% at the one- and five-year horizons, respectively. Likewise, the model-implied regression slopes, on average and in population, are close to the corresponding point estimates. Note that in the LRR model, variation in price-dividend ratios is driven by two state variables: the conditional mean and volatility of consumption growth. Therefore, the price-dividend ratio by itself may not forecast future growth rates in any significant manner. Consequently, univariate regressions of future consumption growth on current price-dividend ratios, also considered in Beeler and Campbell (2009), may fail to capture all the predictable variation in consumption growth. As shown above, consumption growth in the data is highly predictable when one relies on a multivariate regression setting and a richer information set to learn about predictable variation in expected growth rates.

In all, the model and the data are a close match in terms of short and long-run consumption predictability. Recent work by Kaltenbrunner and Lochstoer (2006) and Croce (2005) shows that consumption and savings decisions of agents in a production economy lead to low-frequency movements in consumption growth, similar to those in the LRR model.

Table V provides evidence on dividend predictability using a VAR with dividend growth, real risk-free rate, and the log price-dividend ratio. In the data, the R^2 in dividend predictive regressions starts at 15% and rises to 28% at the 5-year horizon and then gradually tapers off. The model implications for dividend growth predictability lines up with the data.[7] Table VI documents evidence on short- and long-horizon dividend predictability using only the price-dividend ratio as the regressor. The data feature modest predictability, with an R^2 in the range of 6-10%, and the slope coefficients varying from 0.08 at the 1-year horizon to 0.11 at the 5-year horizon. After accounting for sampling uncertainty, the LRR model matches both the R^2's and the estimated slopes quite well.

Our evidence of growth rate predictability is robust to alternative measures of asset cash

[7] This evidence is consistent with dividend predictability documented in Bansal, Dittmar, and Kiku (2009). They find that cash-flow growth rates of the aggregate stock market, as well as book-to-market and size sorted portfolios are strongly predictable at both short and long horizons, and highlight the importance of long-run predictable variations in asset cash flows for understanding the term structure of the risk-return trade-off.

flows. In particular, a VAR for earnings growth, price-earnings ratio and risk-free rate yields a predictive R^2 for the earnings growth of 35% at the 1-year horizon and about 40% at the 5-year horizon. Bansal, Khatchatrian, and Yaron (2005) and Ang and Bekaert (2007) also examine predictability of dividend and earnings growth rates in univariate and multivariate regression settings and find similar strong evidence of predictable variation in asset cash flows.

Table VII provides evidence on predictability of multi-period excess returns by the log of the price-dividend ratio. Consistent with evidence in earlier papers, the R^2 rises with maturity, from 4% at the 1-year horizon to about 31% at the 5-year horizon. The model-implied predictability of equity returns is somewhat lower, but the data R^2's are well inside the 2-SE confidence bands. Return predictability is known to be highly uncertain. Not surprisingly, the model-based confidence bands for the R2's are wide and include both zero (indicating lack of predictability) and the sample estimate. As shown in Panel B, the slope coefficients in the multi-horizon return projections implied by the model are of the right sign and magnitude compared to those in the data. Recall that variation in the risk premia in the Bansal and Yaron (2004) model is entirely due to variation in consumption volatility. Shutting down the volatility channel by assuming homoscedastic growth rates will make the conditional risk premia constant.

It is well-known that the return predictability evidence is quite fragile. To highlight this, in Table VIII we run the same multi-horizon return regressions as above but alter the regressor. Instead of the traditional price-dividend ratio, we use the log dividend yield minus the real risk-free rate. Conceptually, subtracting the real risk-free rate from the dividend-price ratio should virtually make no difference to its predictive ability, as only short-horizon risks embodied in the risk-free rate are subtracted from the dividend yield. In the LRR model or the habit-based model of Campbell and Cochrane (1999), the implications for return predictability with the dividend-price ratio or the real-rate adjusted dividend yield are the same. In the data, however, return predictability with the adjusted dividend yield is much weaker than the one implied by the price-dividend ratio. As shown in Tables VII and VIII, the level of the 5-year horizon R^2 drops from 31% to only 9% once the dividend-price ratio is replaced with the adjusted dividend yield. This evidence raises serious concerns about the magnitude of return predictability in the data. The difference in predictability evidence reported in Table VII and Table VIII suggests that much of the ability of the dividend yield to predict future returns might be spurious and due to the very high persistence of the observed price-dividend ratio (e.g., Stambaugh (1999)). Adjusting the dividend-price ratio for the risk-free rate lowers the persistence in the predictive variable and ensures that the regressor is well behaved. This alleviates the possibility of spurious regression and provides more reliable estimates. Therefore, the magnitude of predictability with the adjusted dividend yield of about 5-10% at long horizons, in our view, is more plausible and close to what should be considered realistic. As shown in Table VIII, the LRR model matches the level of predictability and slope coefficients from the regressions based on the adjusted dividend price ratio quite well.

3.3 Forward & Backward Looking Models

Alternative asset pricing models generally match the equity and risk-free rate puzzles, and therefore may be hard to distinguish by focusing only on this dimension. However, it may be possible to learn about the plausibility of different models by evaluating the link between price-dividend ratios and consumption growth. In the LRR model, current price-dividend ratios are determined by time-varying expected growth and consumption volatility. Hence, current prices anticipate the future state of the economy: a drop in current price-dividend ratios, in the model, reflects either a decline in future expected growth and/or a rise in future volatility. In this sense, the LRR model is forward-looking. In contrast, in the habit model, the shock of habit is driven by lagged consumption growth, and a reduction in growth rates raises risk-aversion, the equity premium, and the discount rate leading to a fall in the current price-dividend ratio. That is, backward consumption plays an important role in determining current prices. This important distinction between the two models provides an avenue to evaluate their plausibility in the data. To accomplish this we also solve the habit model. In particular, we simulate cash-flow and asset price data from the habit model, using the same calibration as in Campbell and Cochrane (1999) and relying on their numerical solution methods. As the broader set of model implications for asset returns are already well-reported in their paper, for brevity, we do not repeat them here.

To highlight the key distinction across the two models, we run the following regression:

$$p_{t+1} - d_{t+1} = \alpha_0 + \sum_{j=0}^{L} \alpha_j \Delta c_{t-j} + u_{t+1}$$

In the data and the simulated data we regress the log of price dividend ratio on L lags of consumption growth.

Table IX reports the evidence in the data and the two models for various lag-lengths L.[8] In the data, at all lag-lengths this predictability is close to zero. For example, for the 5-year lag-length, the backward consumption predictability is only 4%. In the LRR model, the backward consumption predictability is close to zero as well. However, in the habit model, the backward consumption predictability is quite large – at the 5-year lag-length, lagged consumption predicts future prices with an R^2 of 42%. At the 10-year horizon, the predictability, in the population, is 50%.[9] This is not surprising as prices in the Campbell and Cochrane (1999) model are driven primarily by the habit stock and, hence, by movements in the lagged consumption. The lack of predictability of price dividend ratios by past consumption growth in the data, presents an important challenge for habit models which emphasize the backward-looking consumption predictability channel for asset price determination. Related evidence regarding the predictability of price-dividend ratios in the LRR and habit model, in a set-up where dividends and consumption are cointegrated, is also presented in Bansal, Gallant, and Tauchen (2007). Yu (2007) also explores the distinction between the forward-looking LRR model and the backward-looking habit model by looking at long-horizon correlations of returns with consumption growth and finds that the LRR model matches the data much better. More recently, Lustig, Nieuwerburgh, and Verdelhan (2009), provide data-driven non-parametric estimates of the wealth to consumption ratio and the risk premium on aggregate wealth and compare the LRR and habit-models; they document that their estimates and findings are quite close to the LRR model.

To highlight the distinction between the two models, consider the sharp decline in asset prices over the 2007-2008 period. According to the LRR model, the decline would be attributed to a decline in expected growth and/or a rise in consumption-volatility. To explain the same decline, the Campbell and Cochrane habit model would argue that a string of past and current negative consumption shocks raises risk aversion and the discount rate leading to a decline in asset prices. As shown in Table IX, there is not much evidence for this channel as lagged consumption does not forecast movements in future prices.

[8] To have a uniform metric for drawing inferences and model comparison, in Table IX, we rely on the data-based standard errors constructed using a block-bootstrap.

[9] The data R^2's are well below the 2.5-percentile of the finite-sample distributions of the habit model for all lag lengths.

3.4 Volatility

As discussed above, Bansal and Yaron (2004) introduce the volatility channel and show that volatility risks are priced and contribute to the equity risk premia. Fluctuations in volatility are the source of time-varying risk premia in the model (that is, risk premia varies as aggregate risk varies). An important implication of the volatility channel in the LRR set-up, with a preference for early resolution of uncertainty, is that higher volatility lowers the price-dividend ratio. Table X reports the evidence on the relation between asset prices and consumption volatility. The annual realized volatility of consumption is measured by fitting an AR(1) process to consumption growth and taking the absolute value of the residuals. At date t, the K-horizon future realized volatility is measured by $\log \sum_{j=1}^{j=K} |u_{t+j}|$, where u_t is the date-t consumption residual. We regress this measure of volatility on the current price-dividend ratio to see how well current asset prices predict future consumption volatility. In the data, the predictive R^2 rises from 4% to 21%, indicating that consumption volatility is indeed predictable and time-varying. The model matches this data dimension very well – the model confidence bands include the data R^2's and, similar to the data, the magnitude of the model-implied R^2 rises with horizon. Panel B reports the slope coefficients from these regressions. In the data, the current price-dividend ratio and volatility at all horizons are negatively related. The size of the slope coefficients is quite large, and the model captures their magnitude quite well. Bansal, Khatchatrian, and Yaron (2005) show that the negative relationship between valuation ratios and future uncertainty is robust to alternative measures of volatility, cash-flow data, and is present in several other countries in addition to the US.

Beeler and Campbell (2009) evaluate the LRR model by also asking how much predictability does the model imply for an integrated return volatility measure. To construct this measure, we regress the monthly returns on the log price to dividend ratio and sum the monthly squared residuals to construct an integrated annual return volatility measure. Table XI shows that in the data, the predictability starts at 10% and declines to 6% – the confidence bands for this R^2 and the slope coefficient reported in Panel B contain the data magnitudes. The model median captures the signs of the slope coefficients in this regression quite nicely. This underscores the economics in the LRR model, that when IES is larger than one, higher consumption volatility (and return volatility) are negatively related to the price-dividend ratio.[10]

In terms of the volatility channel, recent work in Bansal, Kiku, and Yaron (2007b), estimate the LRR model and show that both components of the long-run risk model, long-run expected growth and long-run volatility fluctuations contribute to the cross-sectional dispersion of expected returns. Related evidence showing that the volatility channel is important for the cross-section is also provided by Tédongap (2006).

Empirical evidence presented in Sections 3.1-3.4 is robust to alternative methods of computing standard errors. We have also evaluated the model fit using the data-based (bootstrap) confidence regions for all statistics of interest. We construct empirical distributions by re-sampling the observed data 10,000 times in blocks of 8 years with replacement and find that the inference based on the bootstrap standard errors is virtually unchanged from the one reported above.

3.5 Parameter Magnitudes

In terms of the preference parameters, the magnitude of risk aversion in the LRR model is 10 or below, which is consistent with the magnitudes argued for in Mehra and Prescott (1985). The Campbell and Cochrane habit model, in contrast, relies on extreme risk-aversion that can be as high as 250 in some states.

There is considerable debate about the magnitude of the IES in the data. A large number of papers (Hansen and Singleton (1982), Attanasio and Weber (1989), Beaudry and van Wincoop (1996), Vissing-Jorgensen (2002), Attanasio and Vissing-Jorgensen (2003), Mulligan (2004), Gruber (2006), Guvenen (2006), Hansen, Heaton, Lee, and Roussanov (2007), Engegelhardt and Kumar (2008)) show that the IES is large and indeed greater than one. Hall (1988) and Campbell (1999), however, argue that IES is small and close to zero. There seems to be little agreement on the magnitude of IES in the data, and both high and low magnitudes seem possible.

The Hall (1988) and Campbell (1999) argument for low IES is typically based on

[10] Note that in Table VII of Beeler and Campbell (2009), they use a monthly price-dividend ratio, while in the data the price-dividend is based on the end-of-year price divided by the trailing sum of 12 month dividends. Once this is fixed and the model price-dividend ratio is treated as in the data, their evidence is consistent with ours and, as described in the text, the LRR model also performs quite well along the integrated volatility dimension.

measuring the slope coefficient from regressing consumption growth on the real rate. In the data this slope coefficient is indeed small. However, Bansal and Yaron show that this regression slope coefficient is downward biased and, hence, cannot be a guide for the true value of the IES in the economy, if consumption volatility is time-varying. Beeler and Campbell (2009) question the magnitude of this bias. However, Table VIII in their paper reports that the population magnitude of the IES is 1.5, while the finite sample estimate using Hall's approach of regressing consumption growth on the real rate is only 0.93; this is a big bias in statistical and economic terms, as the estimated value, in contrast to the population value, is below one. Beeler and Campbell also suggest using an instrumental variable approach to circumvent the bias highlighted in Bansal and Yaron (2004). While the bias is smaller in this case, the estimate of IES in the data reported by Beeler and Campbell, is larger than one.

The central issue about IES is its true value in the data and if it is larger than one. It seems to us that a better approach to measure the IES is to use a larger set of model-based moment restrictions, for example, some that exploit the level of the real rate, the consumption volatility effects on price-dividend ratios, and incorporate the conditional version of the Euler equation associated with the real bond. Bansal, Kiku, and Yaron (2007b) pursue the approach of using a larger set of moments and find the estimate the IES to be larger than one, while Hansen, Heaton, Lee, and Roussanov (2007) use the level restrictions and report the IES estimate close to one. In all, a larger than one IES is consistent with the data.

3.6 Yield Curve

Evaluating the model implications for the yield curve, Bansal and Yaron (2004), Piazzesi and Schneider (2007), Bansal and Shaliastovich (2009) show that the real curve is downward sloping. That is, real bonds provide insurance in the model. This implication of the model is consistent with the real yield curve data from the UK (comparable data sample for the US is not available). In particular Evans (1998) shows that the real yield curve in the UK is downward sloping. Hence, the real yield curve implications of the LRR model are consistent with the data.

Piazzesi and Schneider (2007) and Bansal and Shaliastovich (2009) show that the nominal yield curve in the LRR model is upward sloping since the inflation risk-premia in the model increases with maturity. Bansal and Shaliastovich (2009) further show that the model can account for the predictability evidence on bond returns and the violations of the expectations hypothesis documented for nominal bonds.[11]

Beeler and Campbell (2009) argue that, for some calibrations, the price of the real console (a real bond that pays one unit of consumption in every period) is infinity in the LRR model. This is hardly surprising, as even in the standard CRRA model with *i.i.d* consumption growth, the real yield curve is flat and the yield can be negative when risk aversion is sufficiently high or when consumption volatility is high — in this case, the price of a pure discount bond at infinity is infinity. The price of the console will also be infinity. This model implication for the CRRA model and for the LRR model should not be a concern, as a proper equilibrium exists and the price of the consumption claim (i.e., aggregate wealth) in the economy is finite. Also, there are no data counterparts to a real console. Nominal consoles do exist (deliver one dollar each period) and the price of the nominal console is finite in the LRR model, as the nominal yields are positive and the nominal yield curve is upward sloping.

3.7 Additional Considerations

In addition to data features such as the equity-premium and the risk-free rate puzzles, about which which we mostly learn from the time-series, there are additional puzzles, which primarily focus on the cross-sectional differences in expected returns, such as the differences in returns to size sorted, book-to-market sorted, and momentum sorted portfolios (see Fama and French (1992)). These data features also help learn about model dynamics and economic sources of risks. The cross-sectional differences in expected returns on these assets must reflect differences in systematic risks. To evaluate the LRR model, Bansal, Dittmar, and Lundblad (2005) measure the exposure of cash-flows to long-run consumption growth risks for 30 portfolios, sorted by size, book-to-market, and momentum. They show that these long-run cash-flow betas can explain more than 60% of the cross-sectional differences in expected returns of these 30 portfolios. At the same time, exposure to short-term consumption shocks or markets betas have almost no explanatory power in accounting for the cross-sectional differences in expected returns. This evidence in the cross-section is robust to alternative ways of measuring the exposure of cash-flows to long-run consumption shocks. Bansal,

[11] In addition, they document that the model can account for the violations of the expectations hypothesis in currency markets, that is differences in expected returns between foreign and domestic bonds.

Dittmar, and Kiku (2009) measure long-run consumption betas of the cross-section of assets by exploiting the cointegrating relation between aggregate consumption and dividends and show that this long-run cointegration-based dividend beta is critical for explaining both the cross-sectional differences in short-horizon expected returns and the long-horizon differences in expected returns. These papers underscore the importance of LRR consumption-based cash-flow risks in explaining differences in expected returns across assets. Malloy, Moskowitz, and Vissing-Jorgensen (2009) focus on consumption of stock-holders and show that long-run risks in their consumption also accounts for the cross-section of assets returns.

Using simulations from a calibrated model, Kiku (2006) shows that the LRR model can simultaneously account for the differences in value and growth returns and the empirical failure of the standard CAPM betas. Santos and Veronesi (2006) evaluate the ability of the habit-based model to explain the cross-section of book-to-market returns, and show that the benchmark model of Campbell and Cochrane (1999) implies a "growth" premium. They argue that since growth firms are characterized by a relatively long duration of their cash flows, they are more sensitive to discount rate risks than value firms and, consequently, have to carry a high risk premium inside the habit model. As a result the habit-based model cannot account for the cross-sectional differences in expected returns.

Most recent work on LRR models incorporates jumps in the expected growth and/or volatility dynamics. Eraker and Shaliastovich (2008) provide a framework for analyzing jumps in the growth rates and consumption volatility. They show that this can help account for some of the puzzling options markets features. More extensively, Drechsler and Yaron (2007) incorporate jumps in the expected growth and volatility dynamics and show that this augmented LRR framework can explain the volatility premium in options markets. Drechsler (2008) highlights the effect of model uncertainty in the LRR framework with jumps. Bansal and Shaliastovich (2008) incorporate a confidence risk channel in the LRR framework that includes jumps; this extention opens up a channel for jumps in expected returns and yields significantly higher predictability of asset excess returns, relative to the LRR benchmark model without jumps. Shaliastovich (2008) shows that this broader LRR set-up can empirically account for several option market puzzles.

4 Conclusions

In this article we provide an empirical evaluation of the Bansal and Yaron (2004) LRR model and compare some key features of the model to the Campbell and Cochrane (1999) habit model. We show that the LRR model matches the key asset market facts quite well, and all the model implications are well within the usual two standard error confidence range from the data. We provide statistical evidence which shows that consumption growth and consumption volatility are predictable both in the short and in the long run, and the LRR model replicates this data feature. Bansal and Yaron (2004) develop and underscore the importance of the volatility channel in their LRR model, this channel leads to volatility shocks receiving separate risk compensation in asset markets. The volatility channel of Bansal and Yaron (2004) also help identify the intertemporal elasticity of substitution, as a large IES is needed to capture an important data feature that higher consumption volatility lowers asset prices. We show that the data and the LRR model are quite consistent in their quantitative implications for the volatility and valuation link.

We provide an important distinction between the Campbell and Cochrane (1999) habit model and the Bansal and Yaron (2004) LRR model. Price-dividend ratios in the habit model are driven by long lags of consumption and, consequently, past consumption growth forecast future price-dividend ratios with an R^2 of up to 40%. In the data this predictability is close to zero. In the LRR model, the price-dividend ratio is forward-looking as it is driven by anticipations of future growth and risk (consumption volatility). Therefore, consistent with the data, lagged consumption growth in the LRR model does not predict future price-dividend ratios. The large predictability of future price-dividend ratios with lagged consumption raises considerable questions about the plausibility of the habit model.

Our evidence calls for estimation procedures, which can incorporate a wide range of data features to evaluate the LRR, habit-based, or other models. An early approach is pursued in Bansal, Gallant, and Tauchen (2007), who use the Efficient Method of Moments (EMM) to empirically test these models. The moments they use in their EMM approach would have to be considerably extended to entertain all the data features discussed in this paper.

Appendix

The price-dividend ratio for the market claim to dividends, $z_{m,t} = A_{0,m} + A_{1,m}x_t + A_{2,m}\sigma_t^2$, where

$$
\begin{aligned}
A_{0,m} &= \frac{1}{1-\kappa_{1,m}}\left[\Gamma_0 + \kappa_{0,m} + \mu_d + \kappa_{1,m}A_{2,m}(1-\nu)\bar{\sigma}^2 + \frac{1}{2}\Big(\kappa_{1,m}A_{2,m} - \lambda_w\Big)^2\sigma_w^2\right] \\
A_{1,m} &= \frac{\phi_m - \frac{1}{\psi}}{1-\kappa_{1,m}\rho} \\
A_{2,m} &= \frac{1}{1-\kappa_{1,m}\nu}\left[\Gamma_2 + \frac{1}{2}\Big((\pi-\lambda_\eta)^2 + (\kappa_{1,m}A_{1,m}\varphi_e - \lambda_\epsilon)^2\Big)\right]
\end{aligned} \tag{12}
$$

where $\Gamma_0 = \log\delta - \frac{1}{\psi}\mu_c - (\theta-1)\left[A_2(1-\nu)\bar{\sigma}^2 + \frac{\theta}{2}\Big(\kappa_1 A_2\sigma_w\Big)^2\right]$ and $\Gamma_2 = (\theta-1)(\kappa_1\nu - 1)A_2$

The risk premium is determined by the covariation of the return innovation with the innovation into the pricing kernel. Thus, the risk premium for $r_{m,t+1}$ is equal to the asset's exposures to systematic risks multiplied by the corresponding risk prices,

$$
\begin{aligned}
E_t(r_{m,t+1} - r_{f,t}) + 0.5\sigma_{t,r_m}^2 &= -Cov_t\Big(m_{t+1} - E_t(m_{t+1}), r_{m,t+1} - E_t(r_{m,t+1})\Big) \\
&= \lambda_\eta\sigma_t^2\beta_{\eta,m} + \lambda_e\sigma_t^2\beta_{e,m} + \lambda_w\sigma_w^2\beta_{w,m}
\end{aligned}
$$

where the asset's βs are defined as,

$$
\begin{aligned}
\beta_{\eta,m} &= \pi \\
\beta_{e,m} &= \kappa_{1,m}A_{1,m}\varphi_e \\
\beta_{w,m} &= \kappa_{1,m}A_{2,m}
\end{aligned}
$$

References

Ang, Andrew, and Geert Bekaert, 2007, Stock Return Predictability: Is it There?, *Review of Financial Studies* 20, 651–707.

Attanasio, Orazio P., and Annette Vissing-Jorgensen, 2003, Stock-Market Participation, Intertemporal Substitution, and Risk-Aversion, *American Economic Review,* Papers and Proceedings, 383–391.

Attanasio, Orazio P., and Guglielmo Weber, 1989, Intertemporal Substitution, Risk Aversion and the Euler Equation for Consumption, *Economic Journal* 99, 59–73.

Bansal, Ravi, Robert F. Dittmar, and Dana Kiku, 2009, Cointegration and Consumption Risks in Asset Returns, *Review of Financial Studies* 22, 1343–1375.

Bansal, Ravi, Robert F. Dittmar, and Christian Lundblad, 2005, Consumption, dividends, and the cross-section of equity returns, *Journal of Finance* 60, 1639–1672.

Bansal, Ravi, Ron Gallant, and George Tauchen, 2007, Rational Pessimism, Rational Exuberance, and Asset Pricing Models, *Review of Economic Studies* 74, 1005–1033.

Bansal, Ravi, Varoujan Khatchatrian, and Amir Yaron, 2005, Interpretable Asset Markets?, *European Economic Review* 49, 531–560.

Bansal, Ravi, Dana Kiku, and Amir Yaron, 2007a, A Note on the Economics and Statistics of Predictability: A Long Run Risks Perspective, Working paper.

Bansal, Ravi, Dana Kiku, and Amir Yaron, 2007b, Risks for the Long Run: Estimation and Inference, Working paper.

Bansal, Ravi, and Ivan Shaliastovich, 2008, Confidence Risk and Asset Prices, Working paper.

Bansal, Ravi, and Ivan Shaliastovich, 2009, A Long-Run Risks Explanation of Predictability Puzzles in Bond and Currency Markets, Working paper.

Bansal, Ravi, and Amir Yaron, 2004, Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles, *Journal of Finance* 59, 1481–1509.

Barndorff-Nielsen, Ole E., and Neil Shephard, 2001, Non-Gaussian Ornstein-Uhlenbeck-based Models and Some of Their Uses in Financial Economics, *Journal of Royal Statistical Society* 63, 167–241.

Beaudry, Paul, and Eric van Wincoop, 1996, The Intertemporal Elasticity of Substitution: An Exploration Using A Panel of State Data, *Economica* 63, 495–512.

Beeler, Jason, and John Y. Campbell, 2009, The Long-Run Risks Model and Aggregate Asset Prices: An Empirical Assessment, *NBER Working paper* 14788.

Campbell, John Y., 1999, *Asset prices, Consumption and the Business Cycle,* Handbook of Macroeconomics, Volume 1, pp. 1231-1303; John B. Taylor and Michael Woodford, editors.

Campbell, John Y., and John H. Cochrane, 1999, By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior, *Journal of Political Economy* 107, 205–255.

Croce, Mariano Massimiliano, 2005, Long-Run Productivity Risk: A New Hope For Production-Based Asset Pricing, Working paper.

Drechsler, Itamar, 2008, Uncertainty, Time-Varying Fear, and Asset Prices, Working paper.

Drechsler, Itamar, and Amir Yaron, 2007, What's Vol Got to Do With It, Working paper.

Engegelhardt, Gary V., and Anil Kumar, 2008, The Elasticity of Intertemporal Substitution: New Evidence from 401(k) Participation, *Working paper* 0812, Federal Reserve Bank of Dallas.

Epstein, Larry G., and Stanley E. Zin, 1989, Substitution, Risk Aversion, and the Intertemporal Behavior of Consumption and Asset Returns: A Theoretical Framework, *Econometrica* 57, 937–969.

Eraker, Bjørn, and Ivan Shaliastovich, 2008, An Equilibrium Guide to Designing Affine Pricing Models, *Mathematical Finance* 18, 519–543.

Evans, Martin, 1998, Real Rates, Expected Inflation and Inflation Risk Premia, *Journal of Finance* 53, 187–218.

Fama, Eugene, and Kenneth French, 1992, The Cross-Section of Expected Stock Returns, *Journal of Finance* 59, 427–465.

Gruber, Jonathan, 2006, A Tax-Based Estimate of the Elasticity of Intertemporal Substitution, *NBER Working paper* 11945.

Guvenen, Fatih, 2006, Reconciling Conflicting Evidence on the Elasticity of Intertemporal Substitution: A Macroeconomic Perspective, *Journal of Monetary Economics* 53, 1451–1472.

Hall, Robert, 1988, Intertemporal Substitution in Consumption, *Journal of Political Economy* 96, 339–357.

Hansen, Lars, John Heaton, and Nan Li, 2006, Consumption strikes back?, Working paper, University of Chicago.

Hansen, Lars Peter, John Heaton, Junghoon Lee, and Nikolai Roussanov, 2007, Intertemporal Substitution and Risk Aversion, *Handbook of Econometrics* 6.

Hansen, Lars Peter, and Kenneth Singleton, 1982, Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models, *Econometrica* 63, 767–804.

Hodrick, Robert, 1992, Dividend Yields and Expected Stock Returns: Alternative Procedures for Inference and Measurement, *Review of Financial Studies* 5-3, 357–386.

Kaltenbrunner, Georg, and Lars Lochstoer, 2006, Long-Run Risk through Consumption Smoothing, Working paper.

Kiku, Dana, 2006, Long Run Risks and the Value Premium Puzzle, Working paper, Duke University.

Lustig, Hanno, Stijn Nieuwerburgh, and Adrien Verdelhan, 2009, The Wealth-Consumption Ratio, Working paper, NYU.

Malloy, Christopher, Tobias Moskowitz, and Annette Vissing-Jorgensen, 2009, Long-Run Stockholder Consumption Risk and Asset Returns, *Journal of Finance, forthcoming.*

Mehra, Rajnish, and Edward C. Prescott, 1985, The Equity Premium: A Puzzle, *Journal of Monetary Economics* 15, 145–162.

Mulligan, Casey B., 2004, What do Aggregate Consumption Euler Equations Say about the Capital Income Tax Burden?, *NBER Working paper* 10262.

Piazzesi, Monika, and Martin Schneider, 2007, Equilibrium Yield Curves, Working paper, NBER Macroeconomics Annual 2006.

Santos, Tano, and Pietro Veronesi, 2006, Habit Formation, the Cross Section of Stock Returns, and the Cash-Flow Risk Puzzle, Working paper.

Shaliastovich, Ivan, 2008, Learning, Confidence and Option Prices, Working paper.

Stambaugh, Robert, 1999, Predictive Regressions, *Journal of Financial Economics* 54, 375–421.

Tédongap, Roméo, 2006, Consumption Volatility and the Cross-Section of Stock Returns, *Working paper.*

Vissing-Jorgensen, Annette, 2002, Limited Asset Market Participation and the Elasticity of Intertemporal Substitution, *Journal of Political Economy* 110, 825–853.

Working, Holbrook, 1960, Note on the Correlation of First Differences of Averages in a Random Chain, *Econometrica* 28, 916–918.

Yu, Jianfeng, 2007, The Long and the Short of Asset Prices: Using Long Run Consumption-Return Correlations to Test Asset Pricing Models, Working paper.

Table I

Configuration of Model Parameters

Preferences	δ	γ	ψ			
	0.9989	10	1.5			
Consumption	μ	ρ	ϕ_e	$\bar{\sigma}$	ν	σ_w
	0.0015	0.975	0.038	0.0072	0.999	0.0000028
Dividends	μ_d	ϕ	π	φ		
	0.0015	2.5	2.6	5.96		

Table I reports configuration of investors' preferences and time-series parameters that describe dynamics of consumption and dividend growth rates. All the parameters are taken from Bansal, Kiku, and Yaron (2007a). The model is calibrated on a monthly basis.

Table II

Dynamics of Growth Rates and Prices

Moment	**Data**		**Model**					
	Estimate	(SD)	Median	2.5%	5%	95%	97.5%	Pop
$E[\Delta c]$	1.92	(0.33)	1.80	0.72	0.92	2.73	2.93	1.79
$\sigma(\Delta c)$	2.12	(0.52)	2.47	1.52	1.64	3.60	3.84	2.83
$AC1(\Delta c)$	0.46	(0.15)	0.39	0.15	0.19	0.57	0.60	0.45
$E[\Delta d]$	1.36	(0.88)	1.84	-2.79	-1.96	5.64	6.52	1.45
$\sigma(\Delta d)$	11.05	(2.72)	14.11	8.53	9.19	20.02	21.15	15.83
$Corr(\Delta d, \Delta c)$	0.57	(0.21)	0.46	0.22	0.26	0.62	0.65	0.46
$E[R]$	7.84	(1.97)	8.12	3.59	4.38	13.62	14.96	8.75
$\sigma(R)$	20.16	(2.14)	20.44	12.45	13.62	30.15	31.90	23.37
$E[p-d]$	3.36	(0.13)	3.14	2.79	2.85	3.29	3.32	3.07
$\sigma(p-d)$	0.45	(0.08)	0.18	0.11	0.11	0.28	0.30	0.26
$AC1(p-d)$	0.87	(0.09)	0.62	0.33	0.38	0.79	0.82	0.79
$E[R^f]$	0.86	(0.89)	1.24	0.10	0.31	1.78	1.86	1.05
$\sigma(R^f)$	1.74	(0.33)	0.94	0.54	0.59	1.46	1.59	1.22

Table II presents descriptive statistics for aggregate consumption growth, dividends, prices and returns of the aggregate stock market, and the risk-free rate. Data statistics along with standard deviations of bootstrap distributions (in parentheses) are reported in "Data" panel. The data are real, sampled on an annual frequency and cover the period from 1930 to 2008. "Model" panel presents the corresponding moments implied by the Long-Run Risks model. The first five columns in the right panel represent percentiles of finite sample monte-carlo distributions. Population values (Pop) are computed from a very long sample of simulated data. Means and volatilities of returns and growth rates are expressed in percentage terms.

Table III

VAR-implied Predictability of Consumption Growth

	Data	**Model**					
	Estimate	Median	2.5%	5%	95%	97.5%	Pop
1yr	0.26	0.32	0.10	0.13	0.50	0.54	0.27
3yr	0.24	0.34	0.10	0.13	0.57	0.61	0.30
5yr	0.22	0.31	0.08	0.12	0.55	0.60	0.31
10yr	0.17	0.21	0.05	0.07	0.45	0.50	0.27
15yr	0.13	0.15	0.03	0.05	0.35	0.41	0.22
20yr	0.11	0.11	0.02	0.04	0.28	0.33	0.17

Table III presents predictive R^2's for consumption growth implied by a first-order VAR model for consumption growth, price-dividend ratio of the aggregate stock market portfolio and risk-free rate. Data statistics are reported in "Data" panel. The data employed in estimation are real, compounded continuously, sampled on an annual frequency and cover the period from 1930 to 2008. "Model" panel presents predictability evidence implied by the Long-Run Risks model. The first five columns in the right panel represent percentiles of finite sample monte-carlo distributions. Population values (Pop) are computed from a very long sample of simulated data.

Table IV

Predictability of Consumption Growth by PD-Ratio

Panel A: Predictive R^2's

	Data	Model					
	Estimate	Median	2.5%	5%	95%	97.5%	Pop
1yr	0.07	0.14	0.00	0.01	0.36	0.41	0.07
3yr	0.02	0.10	0.00	0.00	0.37	0.42	0.05
5yr	0.00	0.08	0.00	0.00	0.37	0.43	0.04

Panel B: Predictive Slopes

	Data	Model					
	Estimate	Median	2.5%	5%	95%	97.5%	Pop
1yr	0.01	0.05	0.01	0.01	0.09	0.10	0.03
3yr	0.01	0.10	-0.03	-0.01	0.20	0.22	0.05
5yr	0.00	0.12	-0.09	-0.06	0.28	0.32	0.06

Table IV presents R^2's and slope coefficients from projecting 1-, 3- and 5-year consumption growth onto lagged price-dividend ratio of the aggregate stock market portfolio. Data statistics are reported in "Data" panel. The data employed in estimation are real, compounded continuously, sampled on an annual frequency and cover the period from 1930 to 2008. "Model" panel presents predictability evidence implied by the Long-Run Risks model. The first five columns in the right panel represent percentiles of finite sample monte-carlo distributions. Population values (Pop) are computed from a very long sample of simulated data.

Table V

VAR-implied Predictability of Dividend Growth

	Data	**Model**					
	Estimate	Median	2.5%	5%	95%	97.5%	Pop
1yr	0.15	0.26	0.09	0.12	0.44	0.47	0.14
3yr	0.25	0.20	0.06	0.08	0.39	0.43	0.18
5yr	0.28	0.16	0.04	0.05	0.36	0.39	0.20
10yr	0.27	0.10	0.02	0.03	0.29	0.32	0.19
15yr	0.22	0.07	0.01	0.02	0.23	0.28	0.15
20yr	0.19	0.05	0.01	0.01	0.18	0.23	0.13

Table V presents predictive R^2's for dividend growth implied by a first-order VAR model for cash-flow growth, price-dividend ratio of the aggregate stock market portfolio and risk-free rate. Data statistics are reported in "Data" panel. The data employed in estimation are real, compounded continuously, sampled on an annual frequency and cover the period from 1930 to 2008. "Model" panel presents predictability evidence implied by the Long-Run Risks model. The first five columns in the right panel represent percentiles of finite sample monte-carlo distributions. Population values (Pop) are computed from a very long sample of simulated data.

Table VI

Predictability of Dividend Growth by PD-Ratio

Panel A: Predictive R^2's

	Data	**Model**					
	Estimate	Median	2.5%	5%	95%	97.5%	Pop
1yr	0.10	0.20	0.04	0.05	0.40	0.42	0.11
3yr	0.08	0.09	0.00	0.00	0.31	0.36	0.05
5yr	0.06	0.06	0.00	0.00	0.32	0.38	0.04

Panel B: Predictive Slopes

	Data	**Model**					
	Estimate	Median	2.5%	5%	95%	97.5%	Pop
1yr	0.08	0.35	0.12	0.16	0.56	0.60	0.19
3yr	0.12	0.45	-0.06	0.02	0.94	1.04	0.26
5yr	0.11	0.49	-0.33	-0.20	1.26	1.43	0.30

Table VI presents R^2's and slope coefficients from projecting 1-, 3- and 5-year dividends growth of the aggregate stock market portfolio onto lagged price-dividend ratio. Data statistics are reported in "Data" panel. The data employed in estimation are real, compounded continuously, sampled on an annual frequency and cover the period from 1930 to 2008. "Model" panel presents predictability evidence implied by the Long-Run Risks model. The first five columns in the right panel represent percentiles of finite sample monte-carlo distributions. Population values (Pop) are computed from a very long sample of simulated data.

Table VII

Predictability of Excess Return by PD-Ratio

Panel A: Predictive R^2's

	Data	Model					
	Estimate	Median	2.5%	5%	95%	97.5%	Pop
1yr	0.04	0.01	0.00	0.00	0.07	0.10	0.01
3yr	0.19	0.03	0.00	0.00	0.18	0.22	0.03
5yr	0.31	0.05	0.00	0.00	0.27	0.32	0.04

Panel B: Predictive Slopes

	Data	Model					
	Estimate	Median	2.5%	5%	95%	97.5%	Pop
1yr	-0.09	-0.09	-0.38	-0.33	0.11	0.15	-0.08
3yr	-0.27	-0.26	-0.96	-0.82	0.25	0.34	-0.24
5yr	-0.43	-0.39	-1.39	-1.21	0.38	0.57	-0.38

Table VII presents R^2's and slope coefficients from projecting 1-, 3- and 5-year excess return of the aggregate stock market portfolio onto lagged price-dividend ratio. Data statistics are reported in "Data" panel. The data employed in estimation are real, compounded continuously, sampled on an annual frequency and cover the period from 1930 to 2008. "Model" panel presents predictability evidence implied by the Long-Run Risks model. The first five columns in the right panel represent percentiles of finite sample monte-carlo distributions. Population values (Pop) are computed from a very long sample of simulated data.

Table VIII

Predictability of Excess Return by Adjusted Dividend Yield

Panel A: Predictive R^2's

	Data	**Model**					
	Estimate	Median	2.5%	5%	95%	97.5%	Pop
1yr	0.04	0.01	0.00	0.00	0.07	0.10	0.01
3yr	0.06	0.03	0.00	0.00	0.18	0.22	0.03
5yr	0.09	0.05	0.00	0.00	0.27	0.32	0.04

Panel B: Predictive Slopes

	Data	**Model**					
	Estimate	Median	2.5%	5%	95%	97.5%	Pop
1yr	0.91	0.93	-1.64	-1.33	3.60	4.44	0.74
3yr	1.59	2.63	-4.93	-3.71	9.89	11.64	2.18
5yr	2.37	4.21	-7.83	-5.37	13.96	16.70	3.72

Table VIII presents R^2's and slope coefficients from projecting 1-, 3- and 5-year excess return of the aggregate stock market portfolio onto lagged dividend yield adjusted by the risk-free rate. Data statistics are reported in "Data" panel. The data employed in estimation are real, compounded continuously, sampled on an annual frequency and cover the period from 1930 to 2008. "Model" panel presents predictability evidence implied by the Long-Run Risks model. The first five columns in the right panel represent percentiles of finite sample monte-carlo distributions. Population values (Pop) are computed from a very long sample of simulated data.

Table IX

Predictability of PD-Ratio by Consumption Growth

	Data					LRR Model		Habit Model	
	Estimate	2.5%	5%	95%	97.5%	Pop	Median	Pop	Median
1yr	0.02	0.00	0.00	0.10	0.12	0.01	0.03	0.19	0.19
3yr	0.03	0.01	0.01	0.15	0.17	0.01	0.06	0.33	0.33
5yr	0.04	0.02	0.03	0.25	0.29	0.01	0.09	0.41	0.42

Table IX presents R^2's in regressions of price-dividend ratio onto 1, 3 and 5 lags of consumption growth. Data statistics along with percentiles of the corresponding bootstrap distributions are reported in "Data" panel. The data employed in estimation are real, compounded continuously, sampled on an annual frequency and cover the period from 1930 to 2008. "LRR Model" panel presents predictability evidence implied by the Long-Run Risks model. "Habit Model" panel shows the corresponding statistics in the habit model of Campbell and Cochrane (1999). Median represents the 50%-percentile of finite sample monte-carlo distributions. Population values (Pop) are computed from a very long sample of simulated data.

Table X

Predictability of Volatility of Consumption Growth by PD-Ratio

Panel A: Predictive R^2's

	Data	Model					
	Estimate	Median	2.5%	5%	95%	97.5%	Pop
1yr	0.04	0.03	0.00	0.00	0.13	0.16	0.07
3yr	0.14	0.08	0.00	0.00	0.32	0.38	0.21
5yr	0.21	0.11	0.00	0.00	0.41	0.46	0.29

Panel B: Predictive Slopes

	Data	Model					
	Estimate	Median	2.5%	5%	95%	97.5%	Pop
1yr	-0.58	-0.95	-3.00	-2.67	0.40	0.68	-1.00
3yr	-0.54	-0.90	-2.35	-2.17	0.14	0.28	-0.95
5yr	-0.54	-0.78	-2.19	-1.96	0.13	0.27	-0.93

Table X presents R^2's and slope coefficients from projecting 1-, 3- and 5-year volatility of consumption growth onto lagged price-dividend ratio of the aggregate stock market portfolio. Data statistics are reported in “Data” panel. The data employed in estimation are real, compounded continuously, sampled on an annual frequency and cover the period from 1930 to 2008. “Model” panel presents predictability evidence implied by the Long-Run Risks model. The first five columns in the right panel represent percentiles of finite sample monte-carlo distributions. Population values (Pop) are computed from a very long sample of simulated data.

Table XI

Predictability of Volatility of Excess Return by PD-Ratio

Panel A: Predictive R^2's

	Data	Model					
	Estimate	Median	2.5%	5%	95%	97.5%	Pop
1yr	0.10	0.20	0.01	0.02	0.47	0.50	0.39
3yr	0.07	0.28	0.02	0.04	0.59	0.65	0.55
5yr	0.06	0.28	0.02	0.04	0.61	0.67	0.58

Panel B: Predictive Slopes

	Data	Model					
	Estimate	Median	2.5%	5%	95%	97.5%	Pop
1yr	-0.04	-0.05	-0.09	-0.08	-0.02	-0.01	-0.08
3yr	-0.07	-0.13	-0.26	-0.23	-0.05	-0.03	-0.23
5yr	-0.07	-0.20	-0.41	-0.38	-0.06	-0.03	-0.38

Table XI presents R^2's and slope coefficients from projecting 1-, 3- and 5-year volatility of excess return of the aggregate stock market portfolio onto lagged price-dividend ratio. Data statistics are reported in "Data" panel. The data employed in estimation are real, compounded continuously, sampled on an annual frequency and cover the period from 1930 to 2008. "Model" panel presents predictability evidence implied by the Long-Run Risks model. The first five columns in the right panel represent percentiles of finite sample monte-carlo distributions. Population values (Pop) are computed from a very long sample of simulated data.