Time to Produce and the Cyclical Behavior of Productivity

Preliminary and incomplete

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Abstract

Over the business cycle, labor productivity is strongly positively correlated with future output, while its corelation with current output is rather weak. However, in models with technology shocks labor productivity is strongly positively correlated with current and past output. This paper asks whether a multiple-stage production technology (a form of time to build) can account for this anomaly. To this end it introduces such production technology into an otherwise standard business cycle model. The production process is calibrated to sectoral data of groups of 2-digit SIC industries sorted by the degree of fabrication of their products. The model economy is then subjected to both sectoral and aggregate technology shocks. When input-output linkages are strong, labor productivity at the aggregate level exhibits cyclical behavior close to the one observed in the data. While this mechanism accounts for the cyclical behavior of labor productivity, it preserves the ability of the standard model to account for the cyclical behavior of output, consumption, investment, and hours worked. Furthermore, this mechanism generates the real return on capital and the change in input inventories leading aggregate output as in the data.

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1 Introduction

One often cited anomaly in the business cycle literature has been the low contemporaneous correlation between labor productivity and real GDP [McCallum (1989), Benhabib,

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Rogerson, and Wright (1991), Kydland and Prescott (1991), Hansen and Wright (1992), and Christiano and Eichenbaum (1992)]. In the U.S. data the correlation is between 0.2 and 0.5, whereas the basic business cycle model driven by technology shocks [e.g., Hansen (1985)] predicts correlation close to one. The feature of the basic model responsible for this discrepancy is that technology shocks are the only shocks driving the business cycle in the model. Previous attempts to bring the theory in greater conformity with the data therefore introduced into the model shocks that work in the labor market in the opposite direction of productivity shocks. For example, Benhabib et al. (1991) introduce home production shocks while Christiano and Eichenbaum (1992) consider government consumption shocks. Although shocks of reasonable magnitude somewhat reduce the correlation between labor productivity and output, the models' predictions are still substantially off the correlations observed in the data.

One feature of the data that has been broadly overlooked, however, is the phase shift of labor productivity: although the contemporaneous correlation between labor productivity and output is low, labor productivity is strongly correlated with future output; i.e., it leads output. Closely related to this are two other features of the data: labor productivity leads hours worked and is negatively correlated with hours worked contemporaneously; and total factor productivity (measured by Solow residual) leads output. The standard model does not account for these phase shifts.

In this paper we propose a mechanism that accounts for all the above features of the data at the same time. In addition, the mechanism preserves the ability of the standard model to account for the cyclical behavior of other key aggregates, such as consumption, investment, and hours worked. Following the multi-sector business cycle literature [e.g., Hornstein and Praschnik (1997), Horvath (2000), Huffman and Wynne (1999)], we introduce into the basic model a multi-sector production structure with intersectoral input-output linkages. In our framework, however, these linkages are intertemporal: in a multiple-stage production, upstream industries in period t produce intermediate goods for use in downstream industries in period $t+1$. We call this production structure *time to produce* and refer to the input-output linkages as *intertemporal production complementarities (IPC)* - complementarities between production inputs made in period t and value added of capital and labor in period $t+1$. A multiple-stage production technology has been previously used to address questions concerning the cyclical behavior of inventories [Humphreys, Maccini, and Schuh (2001) and Wen (2005)], and as a source of inflation persistence [Blanchard (1983) and Huang and Liu (1999)]. Further, empirical studies document long production/delivery lags in some manufacturing industries [Abel and Blanchard (1986)] and time to build in construction [Koeva (1999)]. [MORE DETAILS ON THIS PLUS MENTION OTHER STUDIES]

We find that when IPC are sufficiently strong, time to produce generates cyclical behavior of labor productivity and the Solow residual quantitatively close to those observed in data. While a positive technology shock increases labor productivity immediately, aggregate output and hours worked peak only a couple of periods later due to the economy's need to build up necessary production inputs.[1] Even for relatively low degree of IPC, however, time to produce brings the model in greater conformity with the data.

The paper proceeds as follows. Section 2 documents the aforementioned anomalies in light of a standard business cycle model due to Hansen (1985) and reviews the existing literature. Section 3 introduces time to produce into the standard model. Section 4 maps the model's variables into data and Section 5 calibrates the model. Section 6 carries out computational experiments and Section 7 concludes. An appendix describes the data.

2 The Anomalies and Related Literature

We consider two measures of labor productivity. The first is obtained by dividing aggregate output from National Income and Product Accounts (NIPA) by total hours from the Establishment Survey.[2] The other measure comes from the DRI Basic Economics Quarterly Database (former Citibase), mnemonic LBOUTU. It measures labor productivity of all persons in non-farm business sector. This measure has been used previously by other researchers [e.g., Stock and Watson (1999)]. Solow residual is computed in the usual fashion from an aggregate Cobb-Douglas production function, using the labor share in aggregate output equal to 0.60.[3] The series for the NIPA measure of labor productivity and the Solow residual are from 1964 Q1 to 2000 Q4; the DRI measure of labor productivity is from 1959 Q1 to 2000 Q4. Before further analysis, all series are detrended by taking logarithms and Hodrick-Prescott filtering.

2.1 Cyclical Behavior of Productivity in Light of Business Cycle Theory

Figures 1 and 2 plot percentage deviations from trend for the two measures of labor productivity and aggregate output. We see that although output and labor productivity move closely together, the comovement is not perfect. Furthermore, labor productivity tends to peak before output. Similar pattern can be also observed in Figure 3, which plots labor productivity against total hours. Again, the two series move together, but

[1] Indirect empirical evidence suggesting that during expansions production of some manufacturers is delayed due to insufficient supplies of intermediate goods comes from the data on unfilled orders and from the vendor performance index. The former measurers the monthly change in the backlog of orders that have been accumulated from previous months for goods that have not yet been delivered. The later represents the percentage of companies receiving slower deliveries. Stock and Watson (1999) compute the correlation coefficients at various leads and lags between the two indexes on one hand, and real GDP on the other. They find that that both indexes lead the cycle by about one to two quarters.

[2] As discussed in Section 4, our measure of aggregate output is calculated as gross domestic product plus the imputed flow of services from consumer durables minus government expenditures on public sector employees.

[3] When computing the labor share in aggregate output, we distribute the proprietors income between capital and labor using the procedure described in Cooley and Prescott (1995).

they are less then perfectly correlated and labor productivity tends to peak before hours. Summary statistics for the two measures of labor productivity are reported in Tables 1a and 1b. The tables also report summary statistics for Solow residual and summary statistics for artificial time series generated by a standard business cycle model; in particular, the indivisible labor model developed by Hansen (1985).[4] We take Hansen's model as a benchmark for our analysis.

The predictions of the benchmark model are in sharp contrast with the data. First, notice that in the data contemporaneous correlation between output and labor productivity is 0.20 for the NIPA series and 0.53 for the DRI series. The benchmark model, on the other hand, predicts a value of 0.89. Furthermore, both data series display a strong lead (the NIPA series displays a lead of four quarters, the DRI series displays a lead of two quarters), whereas the model predicts that labor productivity is more strongly correlated with past output than future output. The benchmark model does not do much better at predicting the cyclical behavior of Solow residual. In the data contemporaneous correlation between Solow residual and output is 0.72, whereas the model predicts a value of 1. Furthermore, in the data Solow residual leads the cycle, whereas the model predicts coincident behavior (i.e., no phase shift). The most dramatic failure of the benchmark model is its prediction for the cyclical behavior of labor productivity with respect to hours worked (Table 1b). In the data labor productivity (NIPA series) leads hours worked by five quarters, whereas the model predicts that labor productivity lags hours worked. More importantly, in the data contemporaneous correlation between the two series is negative, whereas the model predicts highly positive correlation.

2.2 Related Literature

A number of researchers have tried to account for the low degree of comovement between labor productivity and aggregate output observed in the data [Hansen and Wright (1992) and Christiano and Todd (1996) provide a review of the ability of a number of standard business cycle models to account for the weak comovement]. Kydland and Prescott (1991), for example, introduce into a standard business cycle model two margins along which households can adjust total hours worked, hours per worker and the number of workers. This modification decreases contemporaneous correlation between output and labor productivity in their model from 0.90 to 0.77. They suggest that introducing shocks into the model that work in the labor market in the opposite direction of technology shocks would reduce the correlation even further.

Christiano and Eichenbaum (1992) represent probably the most notable attempt

[4] In Hansen's model, a representative agent chooses consumption c_t, hours worked h_t, and capital stock k_{t+1} to solve $\max E_0 \sum_{t=0}^{\infty} \beta^t [\log c_t + b(1-h_t)]$, $\beta \in (0,1)$, $b > 0$; subject to the aggregate resource constraint $c_t + k_{t+1} = A_t k_t^{1-\alpha} h_t^{\alpha} + (1-\delta)k_t$, $\alpha, \delta \in (0,1)$; where $\log A_t = \rho \log A_{t-1} + \varepsilon_t$, $\rho \in (0,1)$, $\varepsilon_t \sim N(0,\sigma)$. The predictions of the model reported in Tables 1a and 1b are for parameterization of the model that results when the model is calibrated to the same long-run averages of the data as the model with time to produce described below.

in this direction. They introduce government consumption shocks into an otherwise standard business cycle model. An unexpected increase in government consumption works like a negative wealth shock in the household's decision problem and induces the household to increase supply of labor. The lowest value for the correlation between labor productivity and output their model predicts is 0.57, which is close to the value observed for the DRI series.[5] Christiano and Eichenbaum, however, assume that government consumption shocks are autocorrelated while technology shocks are not. Such assumption is likely to overestimate the impact of government consumption on the comovement between labor productivity and aggregate output, and under estimate the impact of technology shocks. When we introduce autocorrelated government consumption shocks into the benchmark model, the correlation between labor productivity and aggregate output drops only by nine points, from 0.89 to 0.8 (see Table 1a).[6]

The phase shift of labor productivity has received less attention in the business cycle literature. Fairise and Langot (1994) focus on the phase shift with respect to hours worked. In their model it is costly to adjust the number of workers, but it is costless to increase or decrease their effort. A positive technology shock therefore increases effort first (and thus productivity), and only then hours. The crucial mechanism that workers increase effort during good economic times is, however, problematic. It is equally plausible to assume that workers increase effort during bad economic times because they fear loosing their jobs. Closer to our approach is a paper by Christiano and Todd (1996). They introduce planing into Kydland and Prescott (1982) time-to-build model. Due to a planing stage at the start of an investment project, investment responds to a positive technology shock with a lag. In their model labor productivity leads hours worked but not output, even though it is more strongly correlated with future output than past output. Contemporaneous correlation between labor productivity and aggregate economic activity is still, however, much higher than in the data.

Recently, there has been growing interest in multisector business cycle models [Hornstein and Praschnik (1997), Horvath (1999) and (2000), and Huffman and Wynne (1999)]. Although the focus of these studies is on other issues, they suggest a mechanism that can potentially resolve all the aforementioned anomalies. In multisector models, there are intersectoral linkages that break up the one-to-one relationship between output and productivity present in one-sector models. An increase in productivity in an intermediate good sector, for example, increases output of a final good sector, even when productivity in the final good sector has not changed. Aggregate output can

[5]Other disturbances that work in the opposite direction of technology shocks in the labor market are shocks to home production. Benhabib, Rogerson, and Wright (1991) show that such shocks are formally equivalent to shocks to preferences. Their model, nevertheless, predicts a value of the correlation coefficient between labor productivity and aggregate output close to the value for a standard business cycle model.

[6]The government consumption shock is assumed to follow an AR(1) process $\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_{gt}$, where $\hat{g}_t$ is a percentage deviation of government consumption from its steady state value. The process is estimated from U.S. data (see Section 4) and the estimates are similar to those in Christiano and Eichenbaum's paper.

thus be fairly uncorrelated with productivity at the aggregate level. Horvath (2000), for example, reports corrrelation between labor productivity and aggregate output in the range from 0.41 to 0.98, depending on the degree of intersectoral linkages. Stronger intersectoral linkages lead to lower correlation. In the next section, we make the intersectoral linkages intertemporal: we consider a vertical chain-of-production structure in which downstream industries are dependent on the output of intermediate goods by upstream industries.

3 The Theoretical Framework

The model economy consists of an infinitely-lived representative household, a representative firm that has access to an aggregate production technology, and a government. The household owns all production inputs in the economy and rents them to the firm at competitively determined rates. Government is introduced into the model in order to make time to produce and intertemporal production complementarities the only deviations from the framework of the benchmark model studied in the previous section.

3.1 The Environment

The aggregate production technology consists of N stages (sometimes we refer to them as a sectors). The first $N-1$ stages, denoted N, $N-1, ..., 3$, 2, produce intermediate goods x_{N-1}, $x_{N-2}, ..., x_2$, x_1, respectively. The index prescribed to a sector refers to the number of stages from completition of the production process. At the final stage, stage 1, the intermediate good of the highest level of fabrication (produced at stage 2) is turned into a final good that can be used for private consumption, c_t, government consumption, g_t, or investment, i_t. At stage N only primary production inputs, labor and capital, are used. A sector $n < N$ uses, in addition to labor and capital, an intermediate good produced in sector $n+1$. Each production stage lasts for one period and intermediate goods cannot be stored for more than one period. That is, an intermediate good produced at stage n at time t can be used only in production at stage $n-1$ at time $t+1$.

At each stage, production is characterized by a production function, which has constant returns to scale in all inputs and is increasing at a decreasing rate in each production input. The aggregate production technology can be summarized as

$$\begin{aligned} \gamma x_{N-1,t+1} &= F_N(A_{Nt}, k_{Nt}, h_{Nt}), \\ \gamma x_{n-1,t+1} &= F_n[G(A_{n,t}, k_{nt}, h_{nt}), x_{nt}], \quad \text{for} \quad n \in \{N-1, ..., 2\}, \\ c_t + i_t + g_t &= F_1[G(A_{1t}, k_{1t}, h_{1t}), x_{1t}]. \end{aligned} \tag{1}$$

Here, A_{nt} is the level of total factor productivity in sector n ($n = N, ..., 1$), γ is a common deterministic growth rate of sectoral productivity, k_{nt} is capital allocated to sector n, and h_{nt} is labor allocated to sector n. The sector-specific levels of total factor productivity follow an exogenous stochastic process specified below.[7] The aggregate production technology can be thought of as capturing both, time to build [e.g., Kydland and Prescott (1982)] and delivery lags [e.g., Abel and Blanchard (1986)].

The interpretation of sector one, is that it represents sectors of the economy that produce both, goods that take only one period to complete (e.g. consumer nondurables) as well as goods that are the last additions to multiple-period production processes (e.g., production of furniture for a newly built office building). Similarly, sector 2 represents sectors that produce goods for the first stages of two-period production processes (e.g., production of heavy industrial machinery) as well as goods for the second-to-last stages of longer production processes. The remaining sectors (sectors 3 to N) have similar interpretation.

The elasticity of substitution between $G(z_{nt}, k_{nt}, h_{nt})$ and x_{nt} $(n = N-1, ..., 1)$ determines flexibility of the production process at stage n. Production at a stage with a high elasticity of substitution can be quickly increased in response to an unexpected increase in A_{nt} or unexpectedly higher demand - it is not dependent on x_{nt} to any considerable degree. On the other hand, production at a stage with a low elasticity of substitution must be delayed until sector $n+1$ generates enough intermediate goods, before it can be increased in order take a full advantage of the positive shock.

The representative household has preferences over private consumption and leisure represented by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t),$$

where $0 < \beta < 1$ is a discount factor, l_t is leisure, and the period utility function $u(.)$ has all the standard properties. The household spends h_t hours working, subject to the time constraint

$$h_t + l_t = 1, \tag{2}$$

where the endowment of time available for work and leisure in period t is normalized to one. In addition, the sum of hours worked across sectors must equal to the total hours worked:

$$\sum_{n=1}^{N} h_{nt} = h_t. \tag{3}$$

At the beginning of period t the household has a capital stock k_t, which it allocates across sectors at zero costs subject to the constraint

$$\sum_{n=1}^{N} k_{nt} = k_t. \tag{4}$$

[7] All variables are in efficiency units: they are normalized by a common deterministic component of total factor productivity so that steady state for the economy is well defined. In steady state all quantities of the original (untransformed) economy, except for hours, are growing at the common constant growth rate γ.

The aggregate capital stock evolves according to the law of motion

$$\gamma k_{t+1} = (1-\delta)k_t + i_t, \tag{5}$$

where $0 < \delta < 1$ is the depreciation rate. Government consumption follows an exogenous stochastic process and is financed by lump sum taxes.

Let $s = [\hat{A}_{1t}, ..., \hat{A}_{nt}, ..., \hat{A}_{Nt}, \hat{g}_t]$ denote the transpose of the $N+1$ vector of the exogenous variables. Here, $\hat{A}_{nt} \equiv \log(A_{nt}) - \log(\overline{A}_{nt})$ is the percentage deviation of total factor productivity in sector n from its steady-state level $\overline{A}_{nt}$, and $\hat{g}_t \equiv \log(g_t) - \log(\overline{g}_t)$ is the percentage deviation of government spending from its steady-state level $\overline{g}_t$. The exogenous variables are assumed to follow an AR(1) process

$$s_t = \Lambda s_{t-1} + \varepsilon_t \tag{6}$$

where Λ is a diagonal matrix, and ε_t is a vector of innovations that have a multivariate normal distribution with zero mean and a covariance matrix Σ. The shocks are observed at the start of the period before any decisions are made.

3.2 Aggregate Output

In a competitive equilibrium, production inputs are paid their respective marginal revenue products. The rental rate of the intermediate good n ($n = N-1, ..., 2$) in period t is therefore given recursively by

$$\begin{aligned} q_{nt} \quad = \quad & \beta E_t \left[\frac{\partial U(c_{t+1}, l_{t+1})/\partial c_{t+1}}{\partial U(c_t, l_t)/\partial c_t} q_{n-1,t+1} \right] \\ & \times \left(\frac{\partial}{\partial x_{n,t}} F_n[G(A_{nt}, k_{nt}, h_{nt}), x_{nt}] \right), \end{aligned} \tag{7}$$

where q_{1t}, the rental rate of the intermediate good 1, is given by

$$q_{1t} = \frac{\partial}{\partial x_{1t}} F_1[G(A_{1t}, k_{1t}, h_{1t}), x_{1t}]. \tag{8}$$

In the pricing function (7), the term on the right-hand side of the first line is the present value of the spot rental rate of the intermediate good $n-1$ in period $t+1$, and the term on the second line is the marginal product of the intermediate good n in period t. The price of the final good is normalized one. And since the final good is used for consumption and investment in the same period in which it is made, the rental rate of the intermediate good 1 in period t is simply equal to its marginal product.

The vector of rental rates for intermediate inputs $q_t = [q_{1t}, ..., q_{N-1,t}]$ is used to construct the gross domestic product for the artificial economy. Notice that output of sector 1 is not the gross domestic product. There are two reasons for this. On one hand it does not include *current* output of the other sectors; on the other hand, it includes the value of their *past* output through x_{1t} in the production function of sector 1. Using the expenditure approach to measuring GDP, GDP for this economy is given by

$$y_t = c_t + i_t + g_t + \Delta m_t, \tag{9}$$

where Δm_t is the change in input inventories given by

$$\Delta m_t \equiv \left(\gamma \sum_{n=1}^{N-1} q_{nt} x_{n,t+1} - \sum_{n=1}^{N-1} q_{nt} x_{nt} \right). \tag{10}$$

Notice that intermediate goods used in production in period $t+1$ are evaluated at rental rates of period t. Such definition of the change in input inventories is consistent with the convention followed by the Bureau of Economic Analysis (BEA) in the National Income and Product Accounts. It incorporates into the change in input inventories the inventory valuation adjustment, known as IVA (see the Bureau's *Guide to the NIPA's*). Finally, we define the total ivestment. e_t as

$$e_t = i_t + \Delta m_t. \tag{11}$$

GDP can also be calculated by summing the value added of capital and labor in each sector (the product approach to measuring GDP). Sectoral value added, a_{nt}, is defined as the total payments to primary factors of production employed in that sector:

$$a_{nt} = w_{nt} h_{nt} + r_{nt} k_{nt},$$

where w_{nt} and r_{nt} are the wage rate and the capital rental rate in sector n, respectively. Since labor and capital can be moved across sectors at zero costs, the wage rates and the capital rental rates across sectors are equalized and given by

$$w_t = \frac{\partial}{\partial G} F_1[G(A_{1t}, k_{1t}, h_{1t}), x_{1t}] \tag{12}$$

and

$$r_t = \frac{\partial}{\partial G} F_1[G(A_{1t}, k_{1t}, h_{1t}), x_{1t}] \frac{\partial}{\partial k_{1t}} G(A_{1t}, k_{1t}, h_{1t}), \tag{13}$$

respectively. Using the product approach, GDP is thus given by

$$a_t = \sum_{n=1}^{N} a_{nt}. \tag{14}$$

This measure of GDP differs from y_t due to IVA by

$$\gamma \left(\sum_{n=1}^{N-1} q_{n,t+1} x_{n,t+1} - \sum_{n=1}^{N-1} q_{nt} x_{n,t+1} \right).$$

3.3 The Social Planner's Problem

Since there are no market failures or distortionary taxes, the First and the Second Theorems of welfare economics can be exploited to obtain the allocations that would result in a dynamic competitive equilibrium of the decentralized economy. The prices of intermediate goods needed to construct the change in input inventories can be then backed out from the pricing functions (7), (8), (12), and (13). We therefore obtain the equilibrium allocations by solving a social planner's problem. Denoting by $d = (c_t, i_t, h_{1t}, ..., h_{Nt}, k_{1t}, ..., k_{Nt})$ the set of decision variables, the social planner's problem involves solving the dynamic programming problem:

$$V(s, k, x_1, ..., x_N) = \max_d u(c, l) + \beta E\left[V(s', k', x'_1, ..., x'_N)|s\right] \tag{15}$$

subject to the aggregate production technology (1); the time constraint (2); the resource constraints for labor and capital (3) and (4), respectively; the law of motion for the capital stock (5); and the law of motion for the exogenous shocks (6).

4 Measurement

Before the model is calibrated, the variables in the theoretical framework must be mapped into their conceptual counterparts in the U.S. data. This section describes how we construct the empirical counterparts to private consumption, investment, government consumption, and input inventories in the theoretical framework, and how we distribute steady-state value added, capital, and labor across production stages.

4.1 Data

All data series, except for capital stock, are obtained from DRI Basic Economics database (former Citibase); data on capital stock are obtained from the BEA. Total hours come from the Establishment Survey. Except for total hours, which are from 1964 Q1 to 2000 Q4, all data series are from 1959 Q1 to 2000 Q4. The length of the period in the model corresponds to one quarter. Since quarterly data on capital stock are not available, we constructed the series from quarterly data on investment using the procedure described in Cooley and Prescott (1995). Data on value added and employment for 2-digit SIC industries come from DRI Basic Economics - Annual Series database. Disaggregated data on capital are obtained from the BEA.

4.2 Aggregates

We construct private consumption by summing consumer expenditures on nondurable goods and services and the imputed flow of services from consumer durable goods. The latter is constructed using the procedure outlined in Cooley and Prescott (1995). Total investment is constructed by summing fixed private investment, change in private inventories, consumer expenditures on durable goods, and net exports. Government consumption is constructed by summing government consumption expenditures, less compensation to employees, and government investment. The reason for excluding compensation to employees from government spending is that compensation to employees is an accounting item that enters both the expenditure and the income side of GDP. In the theoretical framework, on the other hand, the measure of government expenditures is narrow. It only represents a drain on the economy in the final goods market. Aggregate output is then calculated as a sum the constructed series for private consumption, total investment, and government consumption.

We add unfinished construction to the stock of input inventories (materials and supplies plus work in progress) published by the Bureau of the Census. This broader aggregate better corresponds to the concept of input inventories in the theoretical framework. In the theoretical framework, anything that requires further work before it can be used either for consumption or addition to capital stock is an intermediate input. The Appendix describes the constructed series in more detail.

4.3 Sectoral Variables

There is scattered evidence on production/delivery lags in manufacturing and on time to build in construction. Abel and Blanchard (1986) document delivery lags for fabricated metal, non-electrical machinery, and electrical machinery between two to three quarters, while Mayer and Sonenblum (1955) report that the average time across industries needed to equip plants with new machinery is 2.7 quarters. Further, Mayer (1960) and Koeva (2000) report that it takes on average two years to construct new nonresidential structures, while Gomme, Kydland, and Rupert (2001) cite evidence on time to build of residential structures of three to ten months. We follow Kydland and Prescott (1982) and Gomme, Kydland et al. (2001) and set the length of a period equal to one quarter and N equal to 4. With respect to the cited evidence, four quarters for the maximum production time is not an unreasonable guess and certainly a conservative one.

The distribution of value added across the four stages determines the dynamics of GDP and labor. If, for example, most of the economy's value added is generated at stage 4 (the first stage of the production process) then time to produce and IPC are irrelevant for the dynamics of GDP and the labor input. If, on the other hand, most of the economy's value added is generated at stage 1 (the last stage of the production process), time to produce and IPC generate a delayed response of GDP and labor input to external shocks. It is therefore important that industries are assigned to the four stages of the production process in a plausible way.

A common practice in the multisector business cycle literature [Long and Plosser (1983), Hornstein and Praschnik (1997), Huffman and Wynne (1999), and Horvath (2000)] is to use input-output tables to assign SIC industries to the various sectors of a model economy. This practice, however, cannot be followed here since input-output tables only capture the production structure of an economy within a period. Unfortunately, we are not aware of data that would, in addition to input-output tables, also have information on the order of industries in the production chain. Therefore, we resort to making an educated guess about this aspect of production, using only indirect evidence to judge its plausibility. As we will see, indirect evidence suggests that our guess is reasonable.

The distribution of value added, capital, and labor across the four production sectors in the model economy is obtained in the following two steps. First, we divide 2-digit

SIC industries into four broad groups.[8] Table 2 provides the list of industries in each category. Second, we distribute the value added, labor, and capital of each group across the four production sectors.

Production Categories

The first group of industries includes construction, mining, metal industries, and durable goods manufacturing. We call this group *Core Industries (CI)*. Industries in this group can be easily ranked in terms of the degree of fabrication of their products: mining precedes metal fabrication, which then precedes durable goods manufacturing. As for construction, we follow Kydland and Prescott (1982) and assume that, regardless of the above ranking, one quarter of a structure is built each period.

The second group, which we call *Nondurable Input Industries (NII)*, includes industries that make products usually used as inputs in the production of other goods. The choice of these industries was guided by the US input-output tables. Chemical product industries, petroleum and coal product industries, or rubber and plastic product industries are examples of sectors falling into this category.

The third group, which we refer to as *Production Services (PS)*, consists of industries that provide services mainly used by businesses. Legal services or insurance services, for example, are included in this category. The last group, called *Consumption Goods and Services (CGS)*, consists of nondurable good and service industries primarily used either in private or public consumption, such as entertainment services, or that are naturally at the end of the production process, such as retail trade.

Distribution of Value Added, Capital, and Labor Across Production Stages

First, a guess is made about the distribution of the value added of the CI group. Then, the value added of the NII and PS industries was divided among the four stages proportionally to the value added generated at each stage by the CI industries. The value added of the CGS industries was assigned only to the last stage. This process is summarized in Table 2. A number in front of an industry represents the proportion of the industry's value added assigned to the particular stage. Note that the value added of most industries is assumed to be generated at more than one stage. This captures the notion that while, for example, some metals are used in the production of cars, others are used in the construction of factories. Thus, the output of the metal industry might stay in the production process for periods of different length, depending on the type of the final product in whose production it is used. In the case of metal industries, we assume a distribution across the stages $\{0, 0.5, 0.5, 0\}$.

The constants a, b, and c in Table 2 are the constants of proportionality. For example, the share of value added of chemicals & allied product industries that is assumed to be generated at stage 3 is proportional to the value added generated at

[8] The fact that the number of groups is the same as the number of production stages is just a coincidence.

that stage by the CI industries (denoted in the table as $CI3$). It is assumed that these constants of proportionality are the same across the four stages. They must therefore satisfy

$$c(CI4 + CI3 + CI2) = \begin{bmatrix} \text{Value added of} \\ \text{Oil \& gas extraction + Coal mining} \end{bmatrix},$$

$$b(\text{CI3+CI2+CI1}) = \begin{bmatrix} \text{Value added of} \\ \text{Chemicals \& allied prod. + Petroleum \& allied prod.} \\ \text{+ Rubber \& plast. prod. + Lumber \& wood prod.} \\ \text{+ Stone, clay \& glass prod.} \end{bmatrix},$$

$$a(\text{CI4+CI3+CI2+CI1}) = \begin{bmatrix} \text{Total value added of} \\ \text{Transport \& pub. utilities + Business services} \\ \text{+ FIRE (less housing) + Legal services} \end{bmatrix},$$

where FIRE stands for Finance, Insurance, and Real Estate industry. Notice that the sums of the rows in columns denoted 4, 3, and 2 give the values of input inventories ($q_{nt}x_{nt}$, $n = 3, 2, 1$) made at the respective stages of the production process. Summing the values of input inventories implied by the table gives us the empirical counterpart to the value of input inventories in the model ($q_{3t}x_{3t} + q_{2t}x_{2t} + q_{1t}x_{1t}$). Finally, notice that the sum of the rows in the last column in the table gives the value of the final good ($c_t + i_t + g_t$).

The sectoral distribution of value added in Table 2 is then used to compute the sectoral distribution of capital and labor. For example, if the distribution of value added of metal industries is {0, 0.5, 0.5, 0}, the same distribution is also applied to capital and labor employed in this industry. The estimated distribution of value added, capital, and labor across the four stages of the aggregate production process, obtained by taking the average of the annual distributions from 1959 to 1995, are reported in Table 3. The table also reports the resulting estimates for the stock of input inventories, measured relative to GDP.

One check that we can employ to see whether the measurement of the sectoral value added is reasonable is to compare the series for the stock of input inventories obtained implied by Table 2 ($q_{3t}x_{3t} + q_{2t}x_{2t} + q_{1t}x_{1t}$) with the observed stock (the sum of material and supplies, work in progress, and unfinished construction).[9] The average input inventories to GDP ratios for the two series are 0.35 and 0.34, respectively. More importantly, as we can see Figure 4, the two ratios move closely together.[10]

[9] While the series implied by Table 2 is available only at annual frequency, the Bureau of the Census data on input inventories are available monthly and the BEA data on construction are available quarterly. To make the series comparable, the latter two were transformed into annual series by computing their annual averages.

[10] Our estimate of the ratio of input inventories to GDP is in line with the estimates obtained by other authors. Blinder and Maccini (1991) report that inventories held by manufacturers account on

5 Calibration

This section restricts the theoretical framework to a parametric class of economies and assigns values to the parameters. Where possible, the parameter values are obtained from steady-state relations between the model's variables and parameters.

5.1 Functional Forms

In order to make the model comparable with the benchmark model, we assume that labor is indivisible and households have full insurance against employment risk. The period utility function of the representative household therefore has the form

$$u(c_t, l_t) = \log(c_t) + bl_t \quad b > 0.$$

The production function in sectors 1, 2, and 3 takes the form

$$F_n(.) = \begin{cases} \left[(1-\theta_n)(A_{nt}k_{nt}^{1-\alpha_n}h_{nt}^{\alpha_n})^{-\upsilon_n} + \theta_n x_{nt}^{-\upsilon_n}\right]^{-\frac{1}{\upsilon_n}} & \text{if} \quad -1 < \upsilon_n < 0 \quad \text{or} \quad \upsilon_n > 0 \\ (A_{nt}k_{nt}^{1-\alpha_n}h_{nt}^{\alpha_n})^{1-\theta_n}(x_{nt})^{\theta_n} & \text{if} \quad \upsilon_n = 0 \end{cases}$$

and in sector 4 the form

$$F_4(.) = A_{4t}k_{4t}^{1-\alpha_4}h_{4t}^{\alpha_4},$$

where $\theta_n, \alpha_n \in (0,1)$. The functional forms for the utility function and the production functions are consistent with a balanced growth as well as with the observed behavior of average hours worked in the U.S. data, which have remained roughly constant despite the large increase in real wages.

The parameter υ_n determines the elasticity of substitution between $G(.)$ and x_{nt}. A high υ_n implies a low elasticity of substitution, whereas a low υ_n implies a high elasticity of substitution. In the extreme case when υ_n is equal to infinity, the production function is Leontief. In the other extreme case, when υ_n is equal to -1, the production function is linear.

5.2 Steady-State Relations Between the Model's Variables and Parameters

The model is calibrated to the long-run averages of U.S. data reported in Table 4 and the sectoral distribution of capital and labor reported in Table 3. The discount factor, β, is set equal to 0.99, which yields an annual real interest rate of 6 percent in the steady state. The growth rate, $\gamma - 1$, is set equal to 0.0083, the average quarterly growth rate

average for about 60 percent of all inventories. Humphreys, Maccini, and Schuh (2001) report that about 65 percent of all manufacturers' inventories are input inventories. Since, as reported by Cooley and Hansen (1995), the long-run average ratio of total private inventories to quarterly output is 0.88, the findings of Blinder and Maccini and Humphreys et al. imply ratio of input inventories to GDP equal to 0.34 (excluding unfinished structures).

of our measure of aggregate output. The quarterly depreciation rate, δ, is set to 0.023 in order to make it consistent with the steady-state investment to capital ratio equal to 10. The values of the remaining parameters are obtained from the first-order conditions for the social planner's problem.

There are four ways how to substitute consumption across time in this model. One way is through investment in capital, the other three are through investment in the three intermediate goods. In steady state the intertemporal optimality condition for capital is given by

$$\frac{\gamma}{\beta} = \left[(1-\delta) + \left(\frac{1-\alpha_1}{s_1^k}\right)\left(\frac{c+i+g}{k}\right)\left(1-S_x^1\right)\right], \tag{16}$$

and the intertemporal optimality condition for the intermediate good n $(n = 1, 2, 3)$ is given by

$$\beta^n \left(\prod_{j=1}^{n} S_x^n\right)\left(1-S_x^{n+1}\right)\left(\frac{1-\alpha_{n+1}}{s_{n+1}^k}\right) = \left(1-S_x^1\right)\left(\frac{1-\alpha_1}{s_1^k}\right). \tag{17}$$

Here, $s_n^k \equiv k_n/k$ is the share of aggregate capital in sector n, reported in Table 3, and S_x^n is the share of intermediate good n in the gross product of sector n [given by $F_n(.)$]. It is defined by $S_x^n \equiv MP_{x_n}x_n$, where $MP_{x_n} \equiv \frac{\partial}{\partial x_n}F_n(.)$ is the marginal product of x_n. In equation (16), the expression on the right hand side is the marginal product of capital in sector 1, net of depreciation. The optimality condition requires that it is equal to the intertemporal rate of substitution, which in steady state is equal to γ/β. Similarly, the intertemporal optimality conditions for intermediate goods require that the social planner be indifferent between allocating an additional unit of capital to the production of the final good, or to the production of intermediate good n, which increases output of the final good $n+1$ periods later.

The optimal allocation of capital and labor across sectors requires equalizing the marginal rates of substitution between capital and labor across sectors:

$$\frac{(1-\alpha_1)\, s_1^h}{\alpha_1 s_1^k} = \frac{(1-\alpha_n)\, s_n^h}{\alpha_n s_n^k}, \tag{18}$$

for $n = 1, ..., 4$. Here, $s_n^h \equiv h_n/h$ is the share of labor employed in sector n.

These seven optimality conditions are used to calibrate the labor share in sectoral value added $\{\alpha_1,\ \alpha_2,\ \alpha_3,\ \alpha_4\}$ and the shares of intermediate goods in sectoral gross products $\{S_x^1,\ S_x^2,\ S_x^3\}$. The values of the seven parameters are chosen so that in steady-state the model replicates the sectoral distribution of capital and labor reported in Table 3 and the capital to output ratio reported in Table 4.[11] The optimality

[11]It is easy to show that the system of equations (16)-(18) has a unique solution. The system can be reduced into a single nonlinear equation in one of the parameters. This equation is monotone and crosses the zero line in the range of $[0, 1]$.

condition for the labor-leisure choice determines the utility parameter b. In a steady state this condition takes the familiar form

$$bh = \left(\frac{c+i+g}{c}\right)\left(\frac{\alpha_1}{s_1^h}\right)(1-S_x^1). \tag{19}$$

When h is set equal to 0.31, this condition restricts b to be 3.68.[12]

The values of the parameters obtained so far are reported in Table 4. Notice that their values are independent of the parameter of the elasticity of substitution υ_n. The sectoral shares of labor in value added are somewhat greater the usual measurement for aggregate production functions (the values obtained here are $\alpha_1 = 0.72$, $\alpha_2 = 0.85$, $\alpha_3 = 0.84$, and $\alpha_4 = 0.85$, compared to the usual value in the range of 0.6 to 0.7). They are, however, in the range of the values estimated for 2-digit SIC industries by Horvath (2000).

For a given set of the parameters of the elasticity of substitution $\{\upsilon_1,\ \upsilon_2,\ \upsilon_3\}$, the parameters of the production function $\{\theta_1,\ \theta_2,\ \theta_3\}$ are obtained from the definition of the shares of intermediate goods in sectoral gross products:

$$\begin{aligned} S_x^n &\equiv MP_{x_n}x_n \\ &= \frac{\theta_n x_n^{-\nu_n}}{(1-\theta_n)\left[A\left(s_n^k\right)^{(1-\alpha_n)}\left(s_n^h\right)^{\alpha_n}k^{(1-\alpha_n)}h^{\alpha_n}\right]^{-\upsilon_n}+\theta_n x_n^{-\upsilon_n}}, \end{aligned} \tag{20}$$

where we substitute from the production functions (1), with A_n set equal to A for all n, to for the, yet unknown, values of the quantity indexes x_n.[13] The θ's are obtained by matching the shares of intermediate goods in sectoral gross products reported in Table 4. The procedure utilizes the recursive structure of the economy's production technology, proceeding from calibration of θ_3 to calibration of θ_1. Table 5 reports the values of the θ's for two sets of υ's. The first set, $\{2.75, 4.5, 2.75\}$, which we call *High IPC*, generates a production technology closer to a Leontief form. The second set $\{0, 0, 0\}$, which we call *Low IPC*, gives Cobb-Douglas production functions.[14] While it would be desirable to calibrate the υ's to some measurement from the U.S. data, there is little evidence to guide such procedure. We therefore resort to studying the quantitative properties of the model for the two extreme sets of values, thus generating a lower and upper bound for the model's predictions for the cyclical behavior of productivity [Horvath (2000) proceeds the same way].

[12] Notice that in one sector models S_x^1 is equal zero and $c + i + g \ = y = 1$.

[13] It is not possible to identify the steady-state sectoral technology levels $\{A_1,\ A_2,\ A_3,\ A_4\}$ individually. The reason is that for each sector we only observe the input inventories to output ratio $(q_{n,t}x_{n,t}/y_t)$ and not the price index, $q_{n,t}$, or the quantity index, $x_{n,t}$, individually. Under the assumption of identical A's across sectors, it turns out that the θ's are independent of A. Only the price and quantity indexes depend on A. But since they are indexes, their levels are not irrelevant.

[14] Due to the relatively small values of S_x^n in sectors 1 and 3, the numerical error in computing the planner's problem is relatively large when υ_1 and υ_3 equal to 4.5. Therefore, we set $\upsilon_1 = \upsilon_3 = 2.75$, which reduces the error to an acceptable level.

5.3 Stochastic Processes

Because sectoral data on value added, capital stock, and labor input are only available at annual frequency, calibration of the stochastic processes for technology shocks is complicated. The usual procedure of constructing quarterly Solow residuals and then estimating the parameters of their stochastic processes is not feasible. We therefore proceed in the following way. Horvath (2000) estimates AR(1) processes for sectoral Solow residuals obtained from annual data series for 36 SIC industries. Thirty one out of the thirty six industries have the autocorrelation coefficient of Solow residuals between 0.90 and 0.99. Further, twenty one industries have the autocorrelation coefficient in the range of 0.93 to 0.98. We therefore set ρ_n equal to 0.95 for all n. The autocorrelation coefficient for government consumption is estimated from a linearly detrended series for our measure of government expenditures. The point estimate is 0.95. The off-diagonal elements of Λ are set equal to zero. We consider two extreme structures for the covariance matrix Σ: perfectly correlated productivity shocks and uncorrelated productivity shocks. Government consumption shocks are always assumed to be uncorrelated with productivity shocks. Finally, the standard deviation of the innovation for government consumption, σ_g is estimated from the detrended measure of government expenditures. We obtain an estimate of 0.02. The standard deviations of innovations for productivity shocks, σ_n, are chosen so that the model replicates the standard deviation of detrended annual series for real value added in the four sectors, relative to the standard deviation of GDP (σ_{G_n}/σ_y). Since the artificial data depend on the values of υ and Σ, in Table 6 we report four sets of values for σ_1, σ_2, σ_3, and σ_4.

6 Findings

The aggregate decision rules governing the equilibrium allocation of the model economy are obtained for a linear-quadratic approximation of the social planner's problem (15) [see, for example, Hansen and Prescott (1995)]. The artificial time series are treated the same way as the U.S. data series. That is, they are detrended by taking logarithms and Hodrick-Prescott filtering. The results of the experiments are reported in Table 7 and Tables 8a - 8c. The business cycle moments for the model with time to produce are reported for the two extreme degrees of intertemporal production complementarities (High IPC and Low IPC) and the two alternative assumptions about the covariance matrix of productivity shocks [perfectly correlated shocks (PC) and uncorrelated shocks (UNC)].

6.1 Volatility

Although attention will be focused on the cyclical behavior of labor productivity and the Solow residual, Table 7 reports standard deviations for variables often studied in the business cycle literature. The business cycle moments for the model with time to produce are compared to the moments for the U.S. data and the benchmark model with government consumption.

In terms of volatility, there do not seem to be any major differences between the predictions of the model with time to produce and the benchmark model. There are a few minor ones. First, time to produce reduces volatility of aggregate output. Second, it somewhat increases volatility of labor productivity, measured relative to that of output. The finding that the model with time to produce generates largely the same standard deviations for key variables as the benchmark model is positive. It means that a model with time to produce can potentially account for the cyclical behavior of labor productivity, while at the same time be in line with standard business cycle models in terms of volatility. The degree of IPC and the covariance matrix for sectoral technology shocks do not affect volatility to any considerable degree. They have only a minor effect on volatility of hours worked.

6.2 Anomaly 1: Contemporaneous Correlations

Table 8a and Table 8b report the correlations between aggregate output and labor productivity and aggregate output and Solow residual, respectively, at various leads and lags. Table 8c reports the correlations between total hours worked and labor productivity at various leads and lags. First, we focus on contemporaneous correlations. In Table 8a we see that time to produce generates much lower contemporaneous correlations between aggregate output and productivity than the benchmark model with government. The improvement is most significant for a high degree of IPC. The correlation coefficient decreases by almost thirty points (from 0.80 in the benchmark case to 0.51, when sectoral shocks are perfectly correlated, and to 0.53, when the shocks are uncorrelated). These values are very close to the correlation coefficients found in the U.S. data. Especially, when the DRI measure of labor productivity is used, in which case the model matches the observed correlation coefficient exactly. For a low degree of IPC, aggregate output and labor productivity move more closely together than when IPC are high, but the correlation coefficient is still substantially smaller than in the benchmark model. Time to produce therefore brings the comovement between aggregate output and labor productivity predicted by theory in greater conformity with data than government consumption shocks. From Table 1a, we see that government shocks decrease the correlation between output and labor productivity by nine points. Time to produce decreases it at least by additional twelve points for low IPC and by almost thirty points for high IPC.

A similar result is observed for correlation between aggregate output and Solow residual. The benchmark model without government predicts perfect correlation, and the version with government a correlation coefficient equal to 0.99. Time to produce reduces the correlation coefficient to 0.85 when IPC are high, and to 0.92 when IPC are low. The lowest value that the model with time to produce can achieve is, however, still more than ten points above the value of 0.72 observed in the U.S. data. Thus, time-to-produce comes about half way between the benchmark model and the U.S. economy.

The most dramatic improvement of time to produce upon the benchmark model is in terms of its predictions for the correlation between labor productivity and total hours. In the data the correlation coefficient is equal to -0.22. While the benchmark model, with or without government shocks, predicts a value of 0.71, the model with time to produce predicts a value of -0.15 for high IPC, and 0.09 for low IPC. This is an improvement by at least eighty points and the values predicted by the model come very close to the observed value of -0.22.

6.3 Anomaly 2: The Lead-Lag Pattern

Regarding the phase shift of labor productivity, we can see from Table 8a that for all four cases considered, labor productivity in the model, as in the U.S. data, is more strongly correlated with future output than with past output. This is in sharp contrast with the cyclical behavior of labor productivity predicted by the benchmark model (with or without government shocks): in the benchmark model, labor productivity is more strongly correlated with past output than with future output. Furthermore, for high IPC labor productivity leads aggregate output by two quarters (the U.S. data display a lead of two quarters for the DRI measure of labor productivity, and a lead of four quarters for the NIPA measure). The phase shift is more pronounced when the technology shocks are perfectly correlated.

A similar improvement upon the benchmark model is also observed in the model's prediction for the cyclical behavior of Solow residual. In the case of high IPC, the model predicts a one to two quarter lead (the U.S data display a lead of one to two quarters, whereas the in benchmark model Solow residual is coincident). As in the case of labor productivity, the lead is stronger for the case of perfectly correlated technology shocks and disappears under low IPC. Nevertheless, even in that case Solow residual in the model is more strongly correlated with future output than with past output.

The most dramatic improvement upon the benchmark model is again observed in the cyclical behavior of labor productivity with respect to total hours. Here, in all four cases considered the model predicts a strong lead of one to three quarters (the U.S. data display a lead of five quarters, whereas in the benchmark model labor productivity is more strongly correlated with past hours). The lead is again most pronounced in the case of high IPC and perfectly correlated technology shocks.

To facilitate the comparisons between the model with time to produce and the data, and between the model with time to produce and the benchmark model, Figure 5 plots the cross-correlations for the case of high IPC and perfectly correlated technology shocks, together with the cross-correlations for the U.S. data and the benchmark model. Figure 6 then shows the cross-correlations for the model with time to produce under both high and low IPC, for the case of perfectly correlated technology shocks.

6.4 Impulse-Responses

Figures 7 and 8 provide insight into our findings. Figure 7 displays the responses of key variables to a 1-percent positive shock to the technology level in sector 1 (the last sector in the production process), while Figure 8 displays responses to a 1-percent positive shock to the technology level in sector 3. The responses are computed for the case of high IPC and are shown in the graphs as percentage deviations from the steady state.

In the upper-left panel of Figure 7 we see that hours in sector 1 decrease on impact in period 1, as labor in sector 1 becomes more productive while production in that sector is constrained by the existing stock of x_{1t}. Labor is therefore allocated to the other three sectors in order to produce more intermediate goods. Labor in sector 4 increase the most on impact since each of the consecutive three sectors is dependent to some extent on output of sector 4.

In period 2, when the economy has more of x_{3t}, labor relocates to sector 3. As production proceeds, labor moves further to sectors 2 and 1. The propagation of the shock through the stages of production can be also seen in the lower-left panel which plots the responses of intermediate goods (measured in quantity indexes). First, output of sector 4 increases (x_3 increases), followed by output of sector 3 (x_2), and output of sector 2 (x_1).

In the upper-right panel we see the effect of time-to-produce on the economy at the aggregate level. Except for consumption, which has its typical response due to the households' desire for smooth consumption over time, all other measures of aggregate economic activity reach its peak four periods after the impact of the shock. The responses of total hours and aggregate output deserve special attention. Since on impact capital and labor become more productive, more intermediate goods can be made with the same amount of labor input. As more intermediate inputs are made, more labor is required due to the high degree of complementarity between value added (and thus labor) and intermediate inputs. While labor input is increasing, the level of total factor productivity in sector 1 is declining back to its steady-state level. This makes labor productivity, and also Solow residual, to go down before total hours and aggregate output reach their peak, as can be seen in the lower-right panel.

Figure 8 displays the responses of the same variables to a 1-percent increase in the technology level in sector 3. Here, the same mechanism as in the previous case makes labor productivity and Solow residual lead aggregate economic activity. The responses at the aggregate level are, however, substantially smaller than in the case of a positive technology shock in sector 1.

6.5 Additional Results: The Cyclical Behavior of Real Return on Capital and Input Inventories

The time-to-produce technology is consistent with two additional features of the data that the standard business cycle model cannot capture: the real return on capital, measured as capital income from NIPA divided by the capital stock (see Gomme, Ravikumar and Rupert (2006)), and the change in input inventories lead the business cycle. The model is consistent with these empirical regularities even when the IPC are relatively week (see Figure 9). [TO BE COMPLETED]

7 Conclusion

This paper provides a mechanism that resolves two important anomalies that have plagued business cycle models driven by technology shocks: the low contemporaneous correlation between labor productivity and real output, and labor productivity's leading real output over the business cycle.

In our attempt to account for the anomalies we have stayed as close as possible to the standard business cycle model. We have introduced into the standard model a multi-sector aggregate production technology that exhibits time to produce: production of downstream sectors is dependent on the output of upstream sectors. The sectoral production functions were calibrated to the value added, capital stock, and employment in groups of 2-digit SIC industries sorted by the degree of fabrication of their products. Such production structure generates delayed responses of aggregate output and hours worked following a total factor productivity shock. While a higher level of total factor productivity immediately increases labor productivity, aggregate output and hours worked increase only a couple of periods later, due to the economy's need to build up the necessary production inputs. While this mechanism accounts for the two anomalies, at the same time it preserves the ability of the standard model to account for the cyclical behavior of other key variables, such as consumption, investment, and hours worked. Our experiments also show that time to produce accounts for the low degree of comovement between labor productivity and real output to larger extent than government consumption shocks, a source of uncertainty previously used to reduce the contemporaneous correlation between productivity and output.

An interesting extension of the current framework would be to disaggregate the aggregate production process into (at least) three sectors with different times to produce: nondurable goods and services, durable goods, and structures. Empirical observations on delivery lags in manufacturing and on time to build in construction industries could then be used to calibrate the multi-stage production structures in these three sectors. Such extension would allow us not only to assess the model's predictions for the cyclical behavior of productivity at the aggregate level, but also in the three sectors.

[MENTION OTHER POSSIBLE USES OF THE MODEL: PROPAGATION OF SHOCKS (AUTOCORRELATED GROWTH RATE OF GDP), CORRELATION OF THE SOLOW RESIDUAL AND GOVERNMENT SPENDING]

Appendix: Data Description

In this appendix we describe how we construct the empirical counterparts to our theoretical variables. All series, except for capital stock series, are taken from the DRI Basic Economic database (former Citibase). Capital stock data are obtained from the Bureau of Economic Analysis web site. All series are from 1959 Q1 - 2000 Q4. The names of the series used here correspond to the names in the databases.

CONSUMPTION = 100[GCN+GCS+ImpCD]/GDPD

GCN: Consumer non-durable goods
GCS: Services
ImpCD: Imputed flow of services from consumer durable goods
GDPD: GDP deflator

GOVERNMENT SPENDING = 100[GC+GI-GCOMP]/GDPD

GC: Government consumption
GS: Government investment
GCOMP: Compensation of government employees

INVESTMENT = Change in broad input inventories
+ Final products investment

Change in broad input inventories = Change in construction in place
+ Change in input inventories

Construction in place = (1/4)0.75CONSTR(t)(100/GDPD(t))
+ (1/4)0.5CONSTR(t-1)(100/GDPD(t-1))
+ (1/4)0.25CONSTR(t-2)(100/GDPD(t-2))

Input inventories = Materials and supplies + Work in progress

Final product investment = 100[GCD+GIPNR+GNET]/GDPD
+ 0.25CONSTR(t)(100/GDPD(t)) + 0.25CONSTR(t-1)(100/GDPD(t-1))
+ 0.25CONSTR(t-2)(100/GDPD)(t-2) + 0.25CONSTR(t-3)(100/GDPD(t-3))
+ Change in inventories - Change in input inventories

GCD: Consumer durable goods
GIPNR: Equipment and software
GNET: Net export
CONSTR: Private residential and non-residential structures

OUTPUT = CONSUMPTION + GOV.SPENDING + INVESTMENT

STOCK OF BROAD INPUT INVENTORIES = Construction in place
+ Input inventories

CAPITAL STOCK = KFPQ + KCDQ - Construction in place

KFPQ: Fixed private capital (Residential and Non-residential structures plus Equipment and software)

KCDQ: Stock of consumer durables

Note: Change in input inventories is corrected for IVA

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Figure 1. Labor Productivity (DRI Data) and Aggregate Output

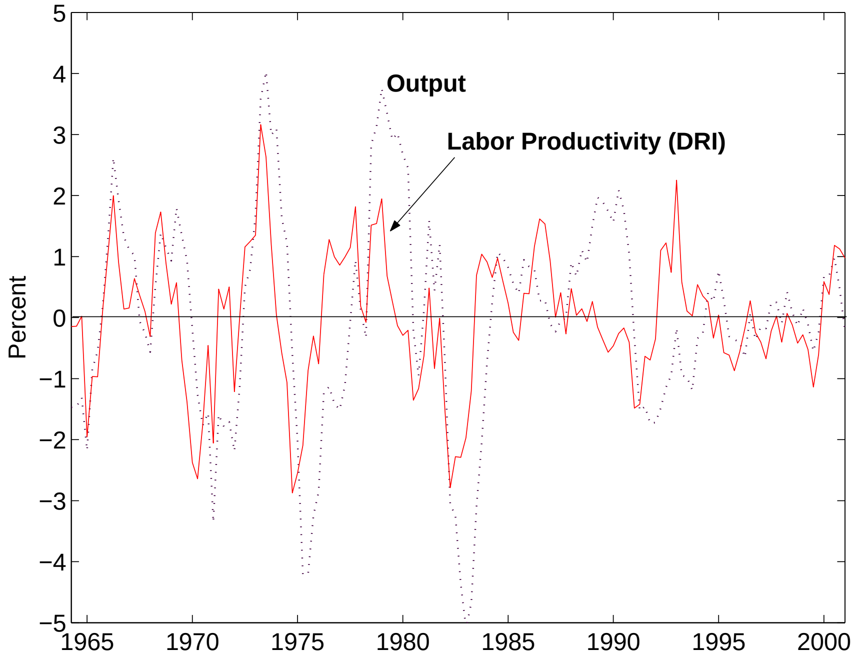


Figure 2. Labor Productivity (NIPA Data) and Aggregate Output

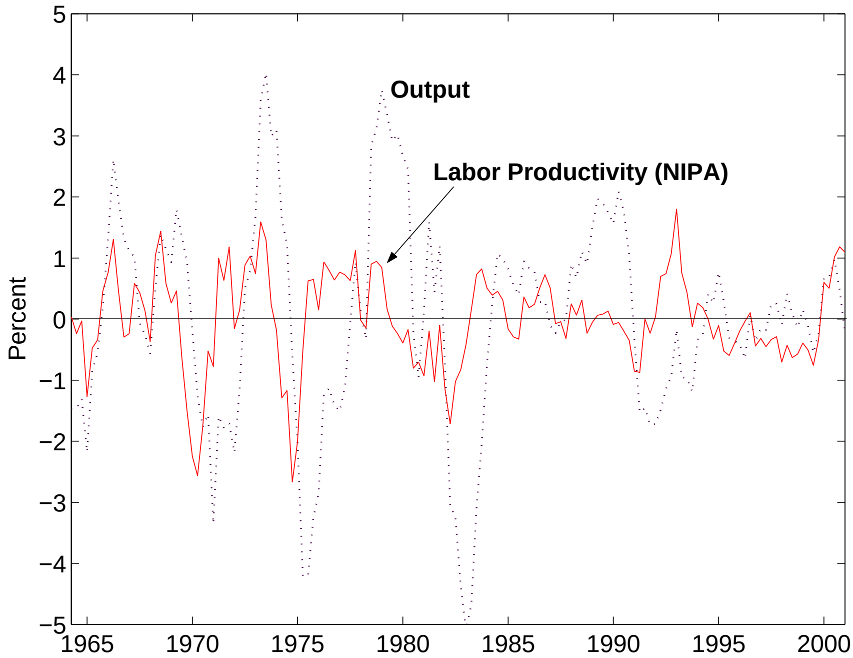


Figure 3. Labor Productivity (NIPA Data) and Total Hours

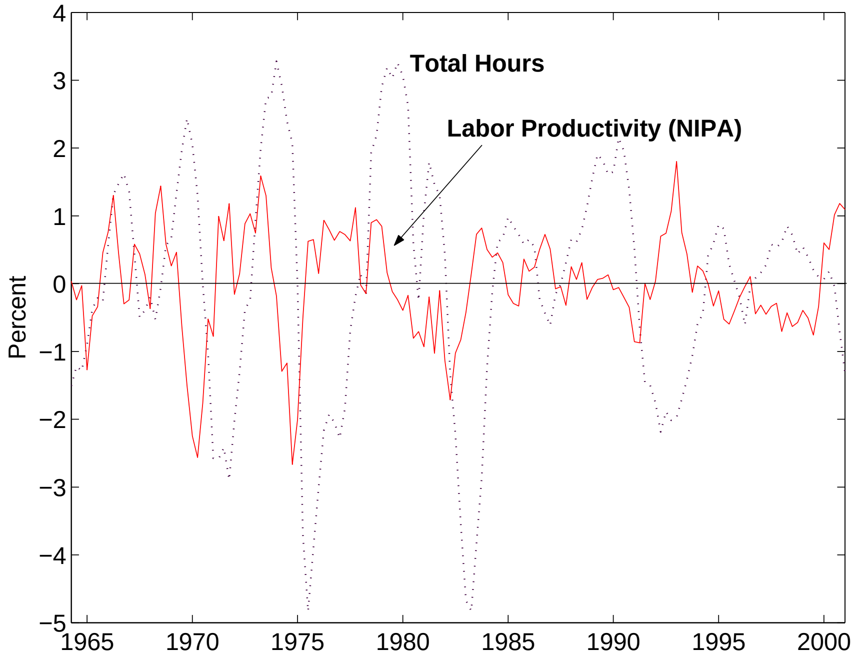


Figure 4. Stock of Broad Input Inventories

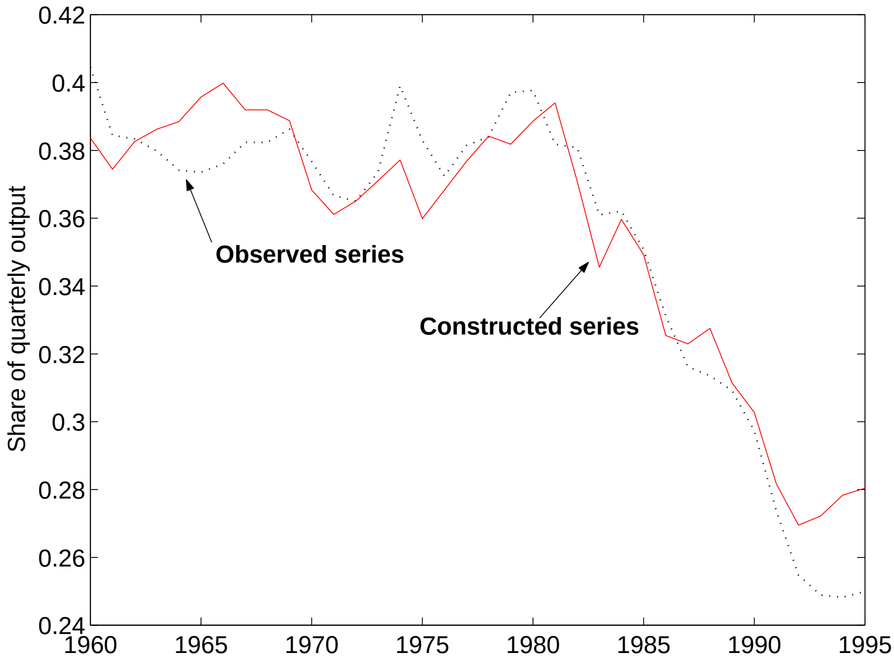


Figure 5. Cross-correlations - High IPC, perfectly correlated sectoral shocks

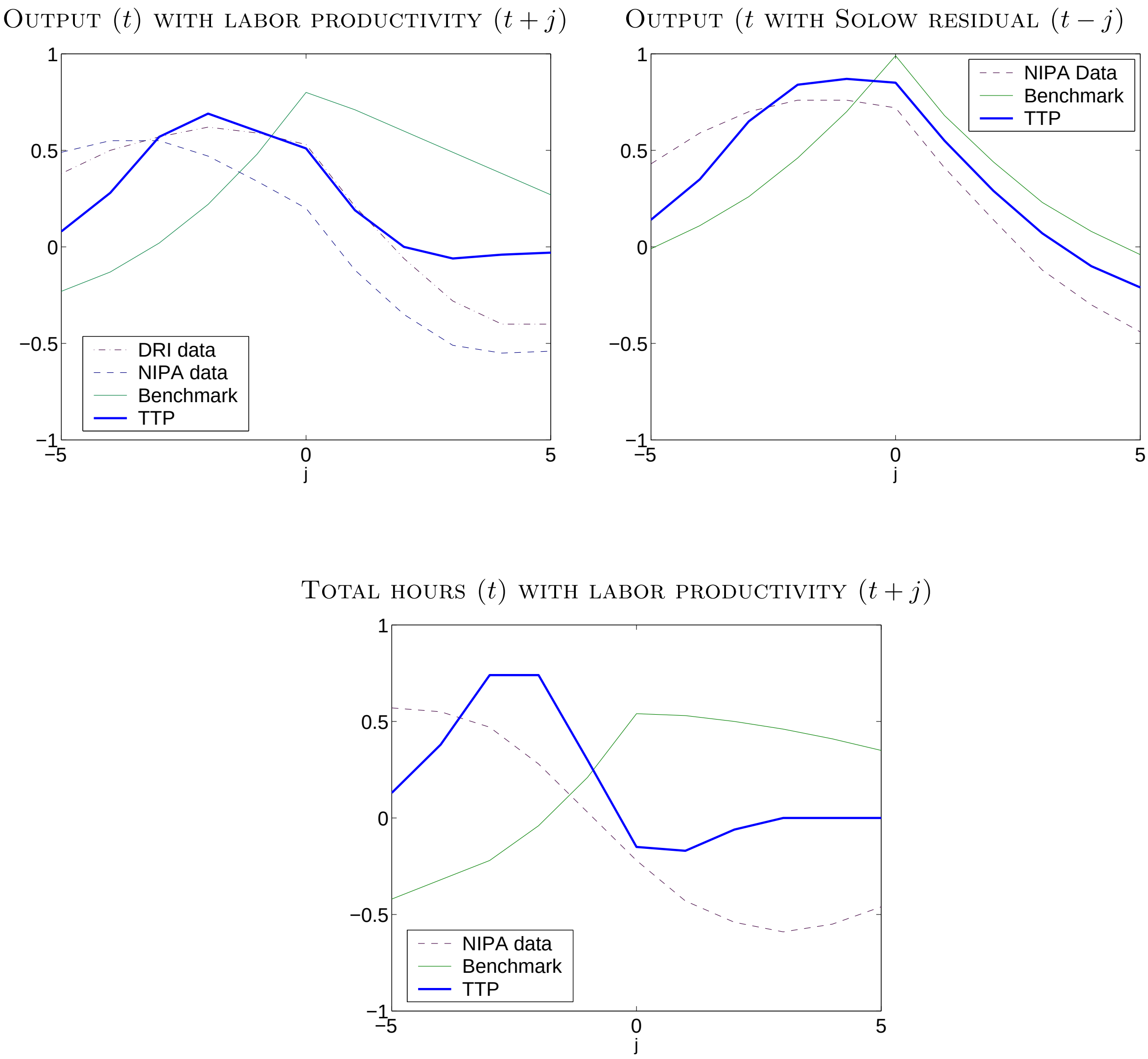
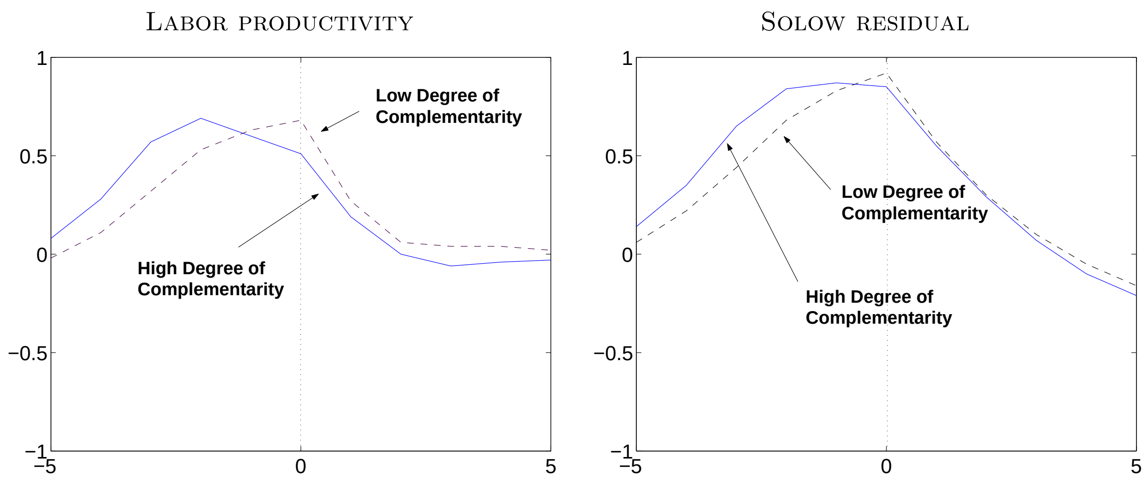


Figure 6. Sensitivity Analysis



Note: Perfectly correlated sectoral productivity shocks.

Figure 7. Responses to a 1-Percent Positive Shock to the Technology Level in Sector 1

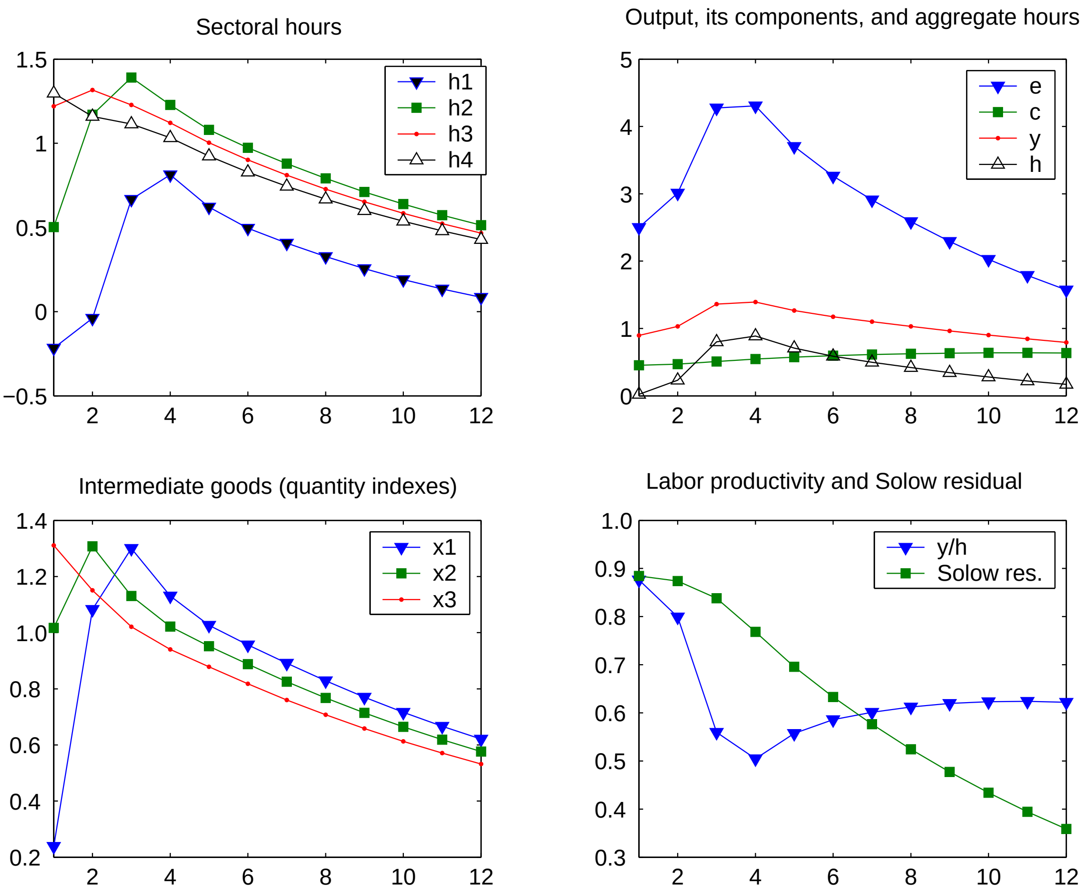


Figure 8. Responses to a 1-Percent Positive Shock to the Technology Level in Sector 3

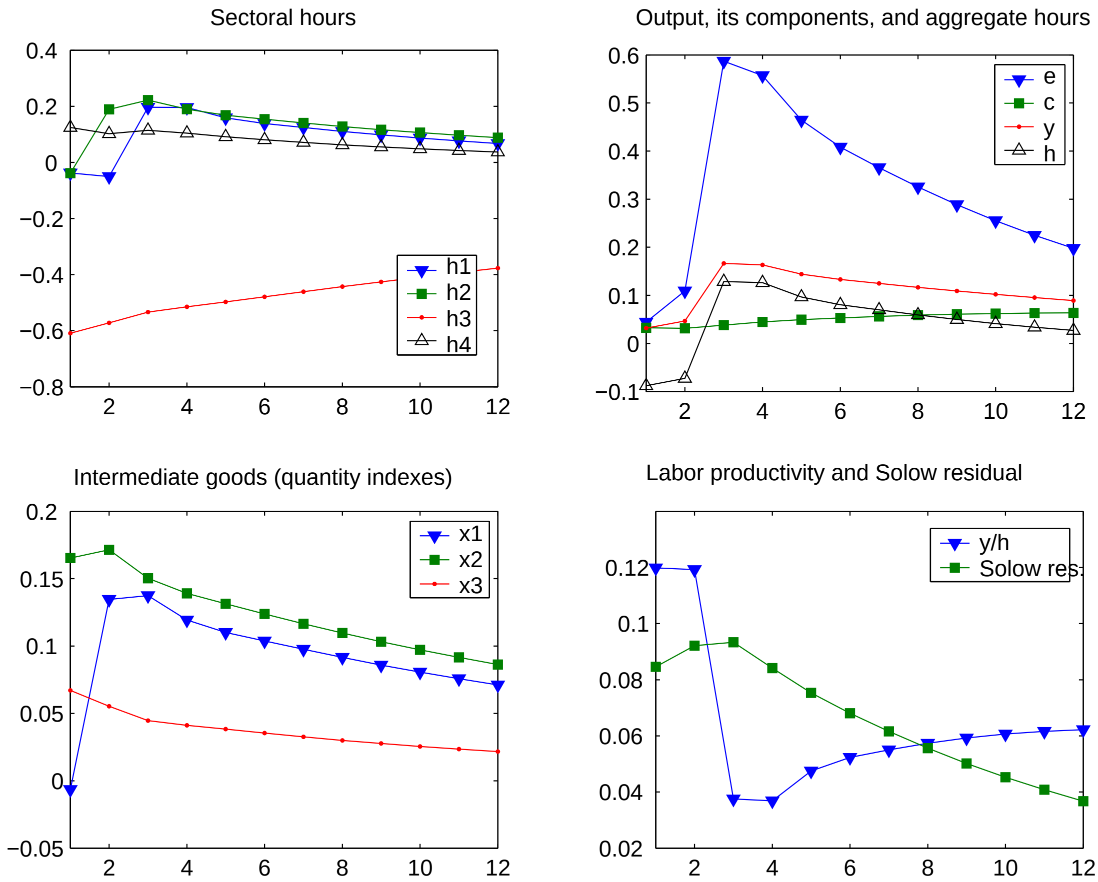
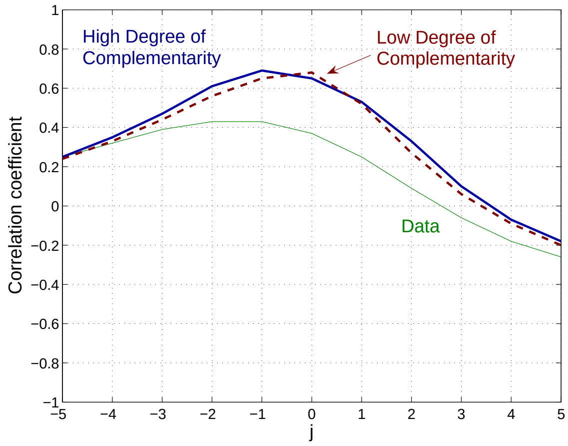


Figure 9. Additional Business Cycle Properties of the Model

Return on capital



Change in Input inventories

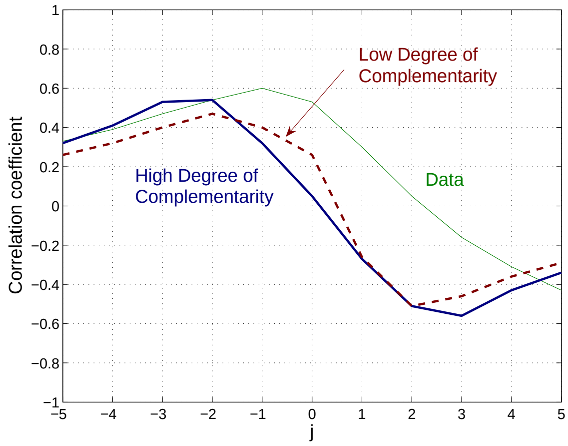


Table 1a. Cyclical Behavior of Labor Productivity and Solow Residual

		Cross-Correlation of Real Output with:										
Variable x		x(t-5)	x(t-4)	x(t-3)	x(t-2)	x(t-1)	x(t)	x(t+1)	x(t+2)	x(t+3)	x(t+4)	x(t+5)
							U.S. Economy					
Labor prod.	NIPA	0.49	0.55	0.55	0.47	0.34	**0.20**	-0.12	-0.35	-0.51	-0.55	-0.54
	DRI	0.38	0.50	0.57	0.62	0.59	**0.53**	0.21	-0.06	-0.28	-0.40	-0.40
Solow res.		0.43	0.59	0.70	0.76	0.76	**0.72**	0.41	0.14	-0.12	-0.30	-0.44
							Benchmark Model					
Labor prod.	(y/h)	-0.26	-0.15	0.01	0.24	0.53	**0.89**	0.77	0.64	0.49	0.36	0.24
Solow res.		-0.05	0.07	0.23	0.45	0.70	**1.00**	0.69	0.43	0.20	0.04	-0.09
							Benchmark Model with Government Consumption					
Labor prod.	(y/h)	-0.23	-0.13	0.02	0.22	0.48	**0.80**	0.71	0.60	0.49	0.38	0.27
Solow res.		-0.01	0.11	0.26	0.46	0.70	**0.99**	0.68	0.44	0.23	0.08	-0.04

Table 1b. Labor Productivity and Total Hours

		Cross-Correlation of Total Hours with:										
Variable x		x(t-5)	x(t-4)	x(t-3)	x(t-2)	x(t-1)	x(t)	x(t+1)	x(t+2)	x(t+3)	x(t+4)	x(t+5)
							U.S. Economy					
Labor prod.	NIPA	0.57	0.55	0.47	0.28	0.03	**-0.22**	-0.43	-0.54	-0.59	-0.55	-0.46
							Benchmark Model					
Labor prod.	(y/h)	-0.45	-0.35	-0.21	0.01	0.31	**0.71**	0.66	0.60	0.53	0.45	0.36

Table 2. Assignment of two-digit SIC industries to the stages of the production process

STAGE:	**4**	**3**	**2**	**1**
Category				
CI	.25 Construction .50 Metal mining .50 Nonmetalic minerals	.25 Construction .50 Metal mining .50 Nonmetalic minerals .50 Primary metal industries .50 Fabricated metal products .35 Manufacturing (Durables) *less* *Primary metal industries* *Fabricated metal products* *Lumber & wood products*	.25 Construction .50 Primary metal industries .50 Fabricated metal products .40 Manufacturing (Durables) *less* *Primary metal industries* *Fabricated metal products* *Lumber & wood products*	.25 Construction .25 Manufacturing (Durables) *less* *Primary metal industries* *Fabricated metal products* *Lumber & wood products*
	plus	**plus**	**plus**	**plus**
NII	c*CI4 Oil & gas extraction Coal mining	c*CI3 Oil & gas extraction Coal mining b*CI3 Chemicals & allied prod. Petroleum & coal prod. Rubber & plastic prod. Lumber & wood prod. Stone, clay & glass prod.	c*CI2 Oil & gas extraction Coal mining b*CI2 Chemicals & allied prod. Petroleum & coal prod. Rubber & plastic prod. Lumber & wood prod. Stone, clay & glass prod.	b*CI1 Chemicals & allied prod. Petroleum & coal prod. Rubber & plastic prod. Lumber & wood prod. Stone, clay & glass prod.
	plus	**plus**	**plus**	**plus**
PS	a*CI4 Transport & pub. utilities Business services FIRE *less housing* Legal services	a*CI3 Transport & pub. utilities Business services FIRE *less housing* Legal services	a*CI2 Transport & pub. utilities Business services FIRE *less housing* Legal services	a*CI1 Transport & pub. utilities Business services FIRE *less housing* Legal services
				plus
CGS				CGS
		plus	**plus**	**plus**
		Value of stage 4 inp. invntrs (q3x3)	*Value of stage 3 inp. invntrs (q2x2)*	*Value of stage 2 inp. invntrs (q1x1)*
	equals	**equals**	**equals**	**equals**
	Value of stage 4 inp. invntrs (q3x3)	*Value of stage 3 inp. invntrs (q2x2)*	*Value of stage 2 inp. invntrs (q1x1)*	**c+i+g**

Notes: FIRE is an abbreviation for Finance, insurance and real estate industry. CGS includes Agriculture, forestry & fishing, Manufacturing of non-durable goods (less Chemicals & allied products, Petroleum & coal industries, Rubber and misc. plastic products), Wholesale trade, Retail trade, Services (less Business and Legal services), Government less compensation to employees, Housing, and Imputed flow of services from consumer durables.

Table 3. Distribution of Value Added and Production Inputs Across Sectors

	Value added (a_n/y)	Input Inv. ($q_n x_n/y$)	Hours (h_n/h)	Capital (k_n/k)
Stage 1	0.80	0.19	0.79	0.89
Stage 2	0.09	0.10	0.10	0.05
Stage 3	0.09	0.02	0.09	0.05
Stage 4	0.02		0.02	0.01
Total	1.00	0.31	1.00	1.00

Table 4. Long-Run Averages of Data

	Value	Definition
Data		
c/y	0.651	Consumption to output ratio
i/y	0.238	Investment to output ratio
g/y	0.108	Gov. spending to output ratio
k/y	10.00	Capital to output ratio
dm/y	0.003	Input inventories to output ratio
h	0.31	Hours worked
$\gamma - 1$	0.0083	Growth rate of output
r	0.06	Annual real interest rate

Table 5a. Parameters That Do Not Depend on υ

Symbol	Value	Definition
Preferences		
β	0.99	Discount factor
b	3.68	Weight on leisure
Technology		
δ	0.023	Depreciation rate
		Intermediate input share in sectoral gross product:
S_x^1	0.19	sector 1
S_x^2	0.54	sector 2
S_x^3	0.18	sector 3
		Labor income share in sectoral value added:
α_1	0.72	sector 1
α_2	0.85	sector 2
α_3	0.84	sector 3
α_2	0.85	sector 4

Table 5b. Parameters That Depend on υ

Symbol	Value		Definition
$\upsilon =$	$\{2.75, 4.5, 2.75\}$	$\{0, 0, 0\}$	
			Weight on x_n
θ_1	0.00001	0.1889	sector 1
θ_2	0.4198	0.5374	sector 2
θ_3	0.0026	0.1761	sector 3
A	5.28	8.57	Level of total factor productivity
			Quantity index:
x_1	0.0625	0.9	sector 2
x_2	0.0597	0.0783	sector 3
x_3	0.0130	0.0211	sector 2

Table 6. Parameters of the Stochastic Process

Symbol	Value		Definition
Data			
			Standard deviation of
			sectoral value added:
σ_{G1}/σ_y	1.00		sector 1
σ_{G2}/σ_y	2.21		sector 2
σ_{G3}/σ_y	2.20		sector 3
σ_{G4}/σ_y	2.42		sector 4
Parameters			
			Government consumption:
ρ_g	0.95		autocorrelation coefficient
σ_g	2.00		standard deviation $\times$ 100
			Technology shocks:
ρ_A	0.95		autocorrelation coefficient
	Perfectly correlated technology shocks		
$\upsilon =$	$\{2.75, 4.5, 2.75\}$	$\{0, 0, 0\}$	
			Technology shocks:
			standard deviation $\times$ 100
σ_1	0.0045	0.0025	sector 1
σ_2	0.0015	0.016	sector 2
σ_3	0.01	0.014	sector 3
σ_4	0.055	0.015	sector 4
	Uncorrelated technology shocks		
$\upsilon =$	$\{2.75, 4.5, 2.75\}$	$\{0, 0, 0\}$	
			Technology shocks:
			standard deviation $\times$ 100
σ_1	0.0065	0.0065	sector 1
σ_2	0.006	0.02	sector 2
σ_3	0.02	0.025	sector 3
σ_4	0.08	0.027	sector 4

Table 7. Standard Deviations

Statistic	US Data	Benchmark with gov.	High IPC PC	High IPC UNC	Low IPC PC	Low IPC UNC
σ_y	1.82	1.39	1.15	1.24	1.07	1.36
σ_c/σ_y	0.56	0.42	0.36	0.37	0.38	0.38
σ_i/σ_y	2.96	3.17	3.32	3.29	3.18	3.29
σ_h/σ_y	0.87	0.71	0.88	0.85	0.74	0.77
σ_w/σ_y	0.44 0.67	0.42	0.36	0.37	0.38	0.38
$\sigma_{y/h}/\sigma_y$	0.43 0.64	0.42	0.65	0.60	0.61	0.54
σ_{SR}/σ_y	0.48	0.60	0.72	0.69	0.74	0.67

Notes: PC stands for perfectly correlated shocks; UNC stands for uncorrelated shocks. The upper value in the σ_w/σ_y row is based on the NIPA series; the lower value is computed from the DRI Basic Economics LEH77Q series. The upper value in the $\sigma_{y/h}/\sigma_y$ row is based on our measure of output and total hours from the Establishment Survey. The lower value is computed from the DRI Basic Economics LBOUTU series.

Table 8a. Cyclical Behavior of Labor Productivity

		Cross-Correlation of Real Output with:										
Labor prod. (x)		x(t-5)	x(t-4)	x(t-3)	x(t-2)	x(t-1)	x(t)	x(t+1)	x(t+2)	x(t+3)	x(t+4)	x(t+5)
							U.S. Economy					
NIPA		0.49	0.55	0.55	0.47	0.34	**0.20**	-0.12	-0.35	-0.51	-0.55	-0.54
DRI		0.38	0.50	0.57	0.62	0.59	**0.53**	0.21	-0.06	-0.28	-0.40	-0.40
							Time to Produce					
High IPC	PC	0.08	0.28	0.57	0.69	0.60	**0.51**	0.19	0.00	-0.06	-0.04	-0.03
	UNC	0.02	0.19	0.44	0.61	0.59	**0.53**	0.26	0.07	0.03	0.03	0.00
Low IPC	PC	-0.02	0.11	0.32	0.53	0.63	**0.68**	0.27	0.06	0.04	0.04	0.02
	UNC	-0.11	0.02	0.17	0.37	0.51	**0.64**	0.41	0.26	0.22	0.17	0.10

Table 8b. Cyclical Behavior of Solow Residual

		Cross-Correlation of Real Output with:										
Solow res. (x)		x(t-5)	x(t-4)	x(t-3)	x(t-2)	x(t-1)	x(t)	x(t+1)	x(t+2)	x(t+3)	x(t+4)	x(t+5)
							U.S. Economy					
		0.43	0.59	0.70	0.76	0.76	**0.72**	0.41	0.14	-0.12	-0.30	-0.44
							Time to Produce					
High IPC	PC	0.14	0.35	0.65	0.84	0.87	**0.85**	0.55	0.29	0.07	-0.10	-0.21
	UNC	0.10	0.28	0.54	0.77	0.84	**0.87**	0.57	0.30	0.09	-0.07	-0.19
Low IPC	PC	0.06	0.22	0.44	0.68	0.83	**0.92**	0.57	0.30	0.10	-0.05	-0.16
	UNC	0.02	0.16	0.33	0.55	0.73	**0.93**	0.58	0.33	0.14	-0.01	-0.12

Table 8c. Labor Productivity and Total Hours

		Cross-Correlation of Total Hours with:										
Labor prod. (x)		x(t-5)	x(t-4)	x(t-3)	x(t-2)	x(t-1)	x(t)	x(t+1)	x(t+2)	x(t+3)	x(t+4)	x(t+5)
							U.S. Economy					
NIPA		0.57	0.55	0.47	0.28	0.03	**-0.22**	-0.43	-0.54	-0.59	-0.55	-0.46
							Time to Produce					
High IPC	PC	0.13	0.38	0.74	0.74	0.30	**-0.15**	-0.17	-0.06	0.00	0.00	0.00
	UNC	0.04	0.23	0.52	0.64	0.33	**-0.07**	-0.04	0.04	0.09	0.08	0.08
Low IPC	PC	-0.02	0.15	0.43	0.68	0.51	**0.09**	0.01	0.06	0.07	0.05	0.04
	UNC	-0.15	-0.03	0.16	0.4	0.34	**0.15**	0.2	0.22	0.19	0.15	0.11