"Exotic" (Recursive) Preferences & Cyclical Properties of US Asset Returns

David Backus (NYU), Bryan Routledge (CMU),
and Stanley Zin (CMU)

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Overview

Time preference

Risk preference

- Chew-Dekel
- Risk premiums

Recursive preferences

Applications of recursive preferences

- Pricing kernels
- Risk sharing
- **Asset returns**

Time preference

Additive preferences

$$U_t \;=\; (1-\beta)u_t + \beta U_{t+1} \;=\; (1-\beta)\sum_{j=0}^{\infty}\beta^j u_{t+j}$$

Time aggregator V

$$U_t \;=\; V(u_t, U_{t+1})$$

(discounting built into V_2)

Why don't we care about this?

Risk preference

Basics: states $s \in \{1, \ldots, S\}$, consumption $c(s)$, probabilities $p(s)$

Certainty equivalent function: μ satisfying

$$U(\mu, \ldots, \mu) \;=\; U[c(1), \ldots, c(S)]$$

Properties:

- Sure things: $\mu(c) = E(c) = c$
- FOSD: $\mu(c + a) \geq \mu(c)$ for constant $a > 0$
- SOSD: $\mu(c + a) \leq \mu(c)$ for mean preserving spread a
- $\Rightarrow \;\; \mu(c) \leq E(c)$

Chew-Dekel preferences

Certainty equivalent function defined by risk aggregator M

$$\mu \;=\; \sum_s p(s) M[c(s), \mu]$$

Recursive definition unavoidable (you'll see why)

Generalization of expected utility (weaker independence axiom)

Chew-Dekel examples

Expected utility

$$M(c,m) = c^{\alpha} m^{1-\alpha}/\alpha + m(1 - 1/\alpha)$$

Weighted utility

$$M(c,m) = (c/m)^{\gamma} c^{\alpha} m^{1-\alpha}/\alpha + m[1 - (c/m)^{\gamma}/\alpha].$$

Disappointment aversion

$$\begin{aligned} M(c,m) &= c^{\alpha} m^{1-\alpha}/\alpha + m(1 - 1/\alpha) \\ &\quad + \delta I(m-c)(c^{\alpha} m^{1-\alpha} - m)/\alpha \\ I(x) &= 1 \text{ if } x > 0, \;\; 0 \text{ otherwise} \end{aligned}$$

Chew-Dekel as adjusted probabilities

Expected utility

$$\mu \;=\; \left(\sum_s p(s) c(s)^\alpha \right)^{1/\alpha}$$

Weighted utility: ditto with

$$\hat{p}(s) \;=\; \frac{p(s)c(s)^\gamma}{\sum_u p(u)c(u)^\gamma},$$

Disappointment aversion: ditto with

$$\hat{p}(s) \;=\; \frac{p(s)(1+\delta I[\mu - c(s)])}{\sum_u p(s)(1+\delta I[\mu - c(s)])},$$

Small risks

Two states $(1+\sigma, 1-\sigma)$, equal probs, Taylor series around $\sigma = 0$

Expected utility

$$\mu(\text{EU}) \;\approx\; 1 - (1-\alpha)\sigma^2/2$$

Weighted utility

$$\mu(\text{WU}) \;\approx\; 1 - [1 - (\alpha + 2\gamma)]\sigma^2/2$$

Disappointment aversion

$$\mu(\text{DA}) \;\approx\; 1 - \left(\frac{\delta}{2+\delta}\right)\sigma - (1-\alpha)\left(\frac{4+4\delta}{4+4\delta+\delta^2}\right)\sigma^2/2$$

Lognormal risks

Let: $\log c \sim \mathrm{N}(\kappa_1, \kappa_2)$, $\mathrm{rp} = \log[E(c)/\mu(c)]$

Expected utility

$$\mathrm{rp(EU)} = (1-\alpha)\kappa_2/2$$

Weighted utility

$$\mathrm{rp(WU)} = [1-(\alpha+2\gamma)]\kappa_2/2$$

Disappointment aversion

$$\mathrm{rp(DA)} = \mathrm{E2C2E}$$

Extreme risks

Let: $\log E \exp(\log c) = \kappa_1 + \kappa_2/2! + \kappa_3/3! + \kappa_4/4!$

Expected utility

$$\text{rp(EU)} = (1-\alpha)\kappa_2/2 + (1-\alpha^2)\kappa_3/3! + (1-\alpha^3)\kappa_4/4!$$

Weighted utility

$$\begin{aligned}\text{rp(WU)} &= [1-(\alpha+2\gamma)]\kappa_2/2 + [1-(\alpha+2\gamma)^2 + \gamma(\alpha+\gamma)]\kappa_3/3! \\ &\quad + [1-(\alpha+2\gamma)^3 + 2\gamma(\alpha+\gamma)(\alpha+2\gamma)]\kappa_4/4!\end{aligned}$$

Disappointment aversion

$$\text{rp(DA)} = \text{Another E2C2E}$$

Recursive preferences

General form

$$U_t = V[u_t, \mu_t(U_{t+1})]$$

Kreps-Porteus/Epstein-Zin/Weil

$$\begin{aligned} V(u_t, \mu_t) &= [(1-\beta)u_t^\rho + \beta\mu_t^\rho]^{1/\rho} \\ \mu_t(U_{t+1}) &= \left(E_t U_{t+1}^\alpha\right)^{1/\alpha} \\ \text{IES} &= 1/(1-\rho) \\ \text{CRRA} &= 1-\alpha \\ \alpha &= \rho \quad \Rightarrow \quad \text{additive preferences} \end{aligned}$$

Applications

Pricing kernels

Risk sharing

Cyclical properties of US asset returns

Kreps-Porteus pricing kernel

Marginal rate of substitution

$$m_{t+1} \;=\; \beta(c_{t+1}/c_t)^{\rho-1}[U_{t+1}/\mu_t(U_{t+1})]^{\alpha-\rho}$$

Note role of future utility

- Allows role for predictable future consumption growth
- Ditto volatility

Kreps-Porteus pricing kernel (continued)

Example: let consumption growth follow

$$\log x_t \;=\; \log x + \sum_{j=0}^{\infty} \chi_j w_{t-j}$$

Pricing kernel

$$\begin{aligned}\log m_{t+1} \;&=\; \text{constant} + [(\rho - 1)\chi_0 + (\alpha - \rho)(\chi_0 + X_1)]w_{t+1} \\ &\quad + (\rho - 1)\sum_{j=0}^{\infty} \chi_{j+1} w_{t-j} \\ X_1 \;&=\; \sum_{j=1}^{\infty} \beta^j \chi_j \quad (\text{“Bansal-Yaron” term})\end{aligned}$$

Kreps-Porteus risk sharing

Pareto problem with two (different) recursive agents

Issues

- Time-varying pareto weights
- Representative agent may look different from individuals
- Possible nonstationary consumption distribution

Cyclical properties of US asset returns

Data: cyclical properties of US asset prices and returns

Theory: numerical example ["Bansal-Yaron plus"]

Cyclical properties of US asset prices and returns

Cross correlations for financial indicators and economic growth

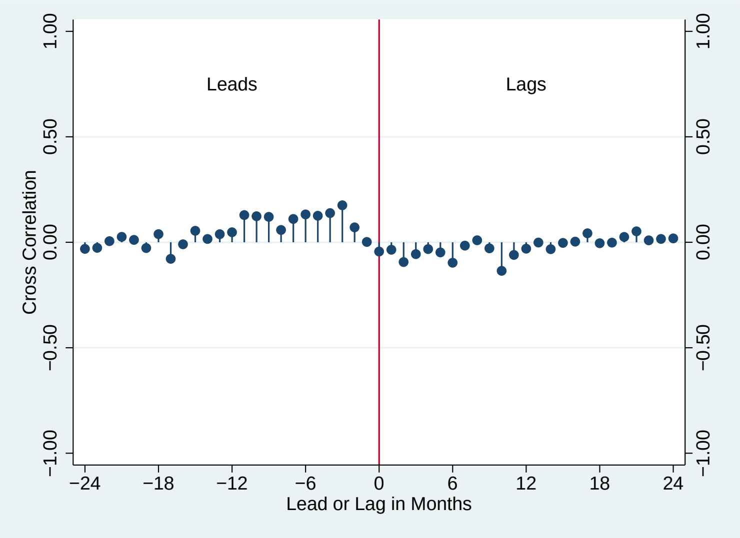
- Returns: logs of gross returns
- Excess returns: differences in logs of gross returns

Economic growth

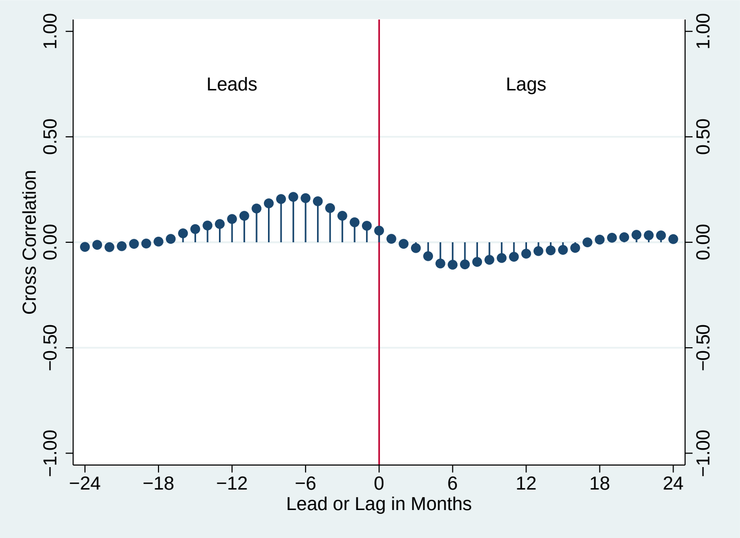
- Monthly: $\log x_t - \log x_{t-1}$
- Or year-on-year: $\log x_{t+6} - \log x_{t-6}$
- Computed from: industrial production, consumption, employment

US data, monthly, 1960 to present

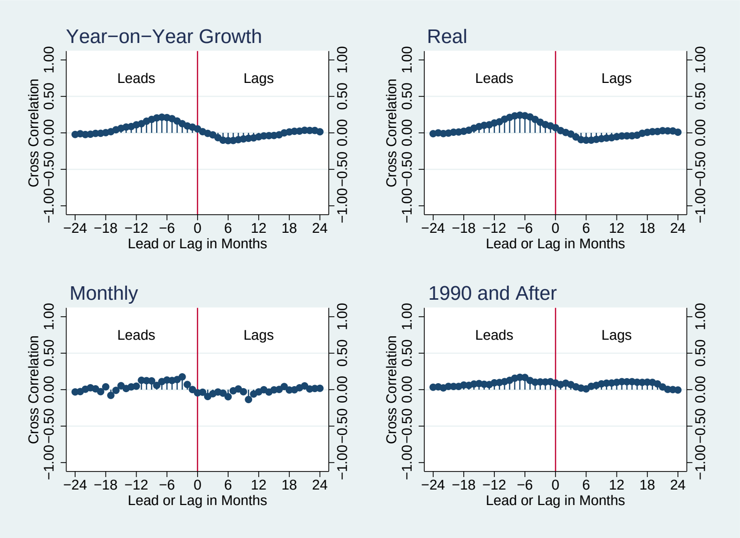
Equity returns (monthly growth)



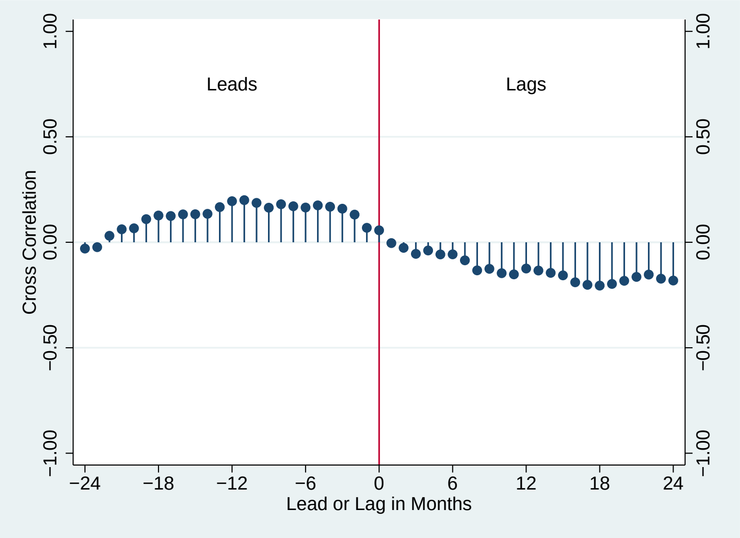
Equity returns (yoy growth)



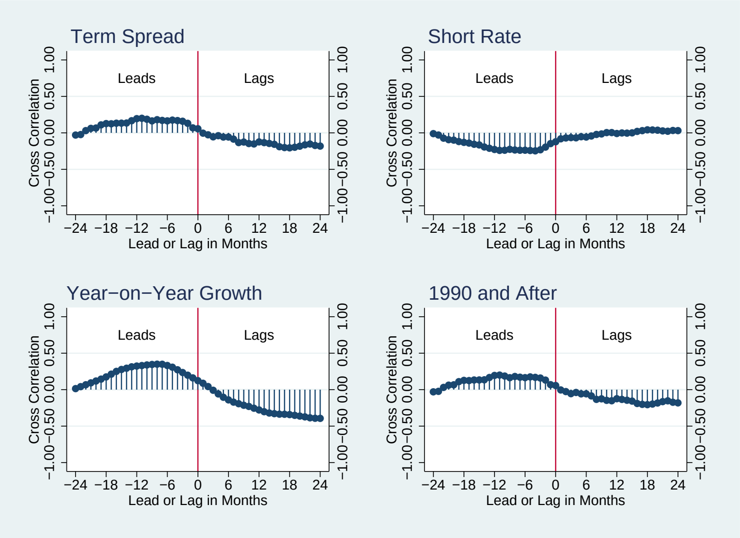
Equity returns (variations)



Term spread (monthly growth)

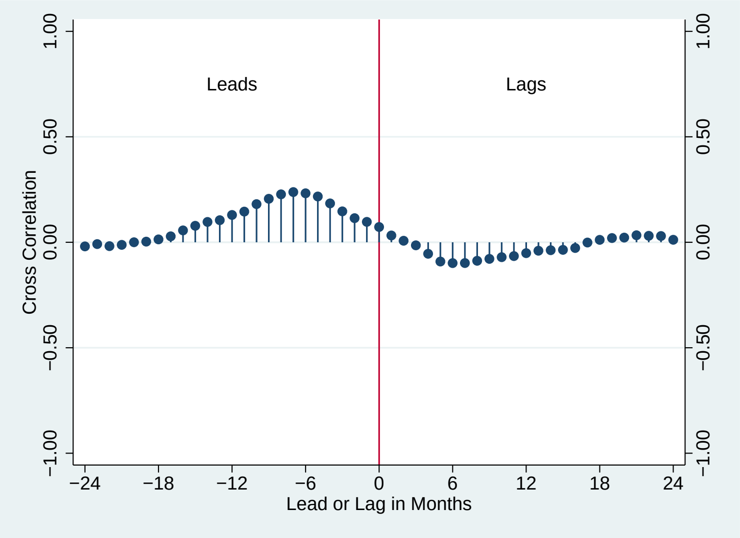


Term spread (variations)

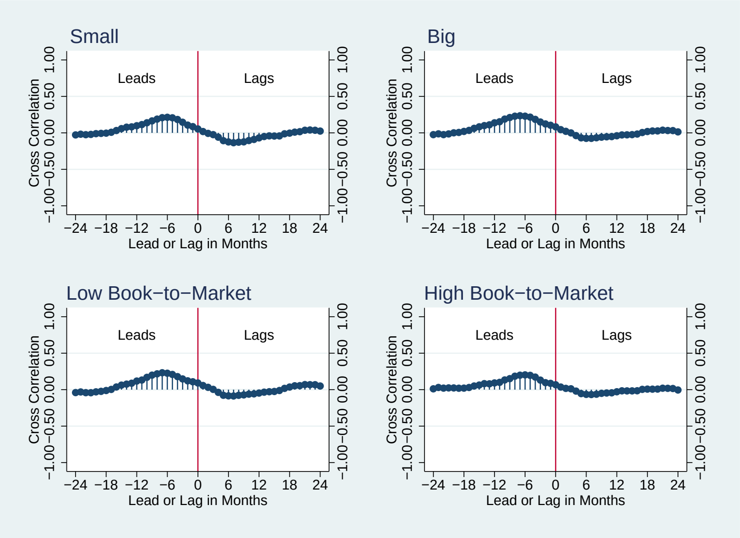


Think about this for a minute...

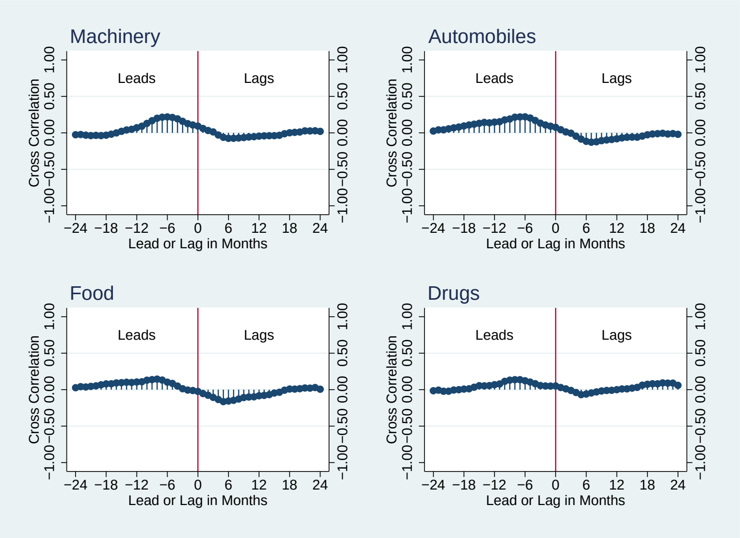
Excess returns: equity (yoy)



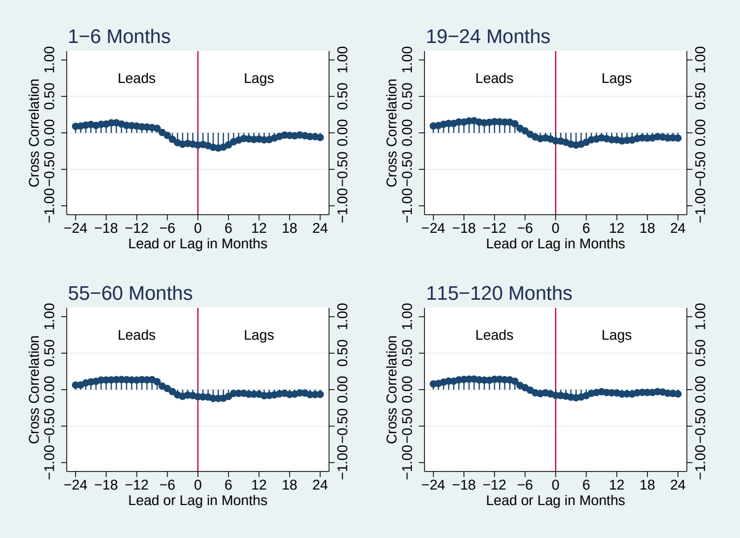
Excess returns: Fama-French portfolios (yoy)



Excess returns: industries (yoy)



Excess returns: bonds (yoy)



Theoretical economy

Take a breath

What do we need?

- Variation in risk and/or price of risk
- ... tied to economic growth

Bansal-Yaron plus

- Representative agent exchange economy
- Recursive preferences (Kreps-Porteus/Epstein-Zin/Weil)
- Loglinear process for consumption growth
- Stochastic volatility
- **Interaction between growth and volatility**

Theoretical economy

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Kreps-Porteus preferences

Equations

$$\begin{aligned} U_t &= \left[(1-\beta)c_t^{\rho} + \beta\mu_t(U_{t+1})^{\rho}\right]^{1/\rho} \\ \mu_t(U_{t+1}) &= \left(E_t U_{t+1}^{\alpha}\right)^{1/\alpha} \\ & \alpha, \rho \leq 1 \end{aligned}$$

Interpretation

$$\begin{aligned} IES &= 1/(1-\rho) \\ CRRA &= 1-\alpha \\ \alpha &= \rho \quad \Rightarrow \quad \text{additive preferences} \end{aligned}$$

Kreps-Porteus pricing kernel

Marginal rate of substitution

$$m_{t+1} \;=\; \beta \left(\frac{c_{t+1}}{c_t}\right)^{\rho-1} \left(\frac{U_{t+1}}{\mu_t(U_{t+1})}\right)^{\alpha-\rho}$$

If $\alpha = \rho$

- Second term disappears
- No roles for volatility or predictable consumption growth

Consumption growth

Consumption growth follows from

$$
\begin{aligned}
\log g_t &= g + e^\top x_t \\
x_{t+1} &= Ax_t + a(v_t - v) + v_t^{1/2} B w_{t+1} \\
v_{t+1} &= (1-\varphi_v)v + \varphi_v v_t + b w_{t+1}
\end{aligned}
$$

Note

- A generates predictable component
- v_t is stochastic
- a generates interaction

Theoretical excess returns

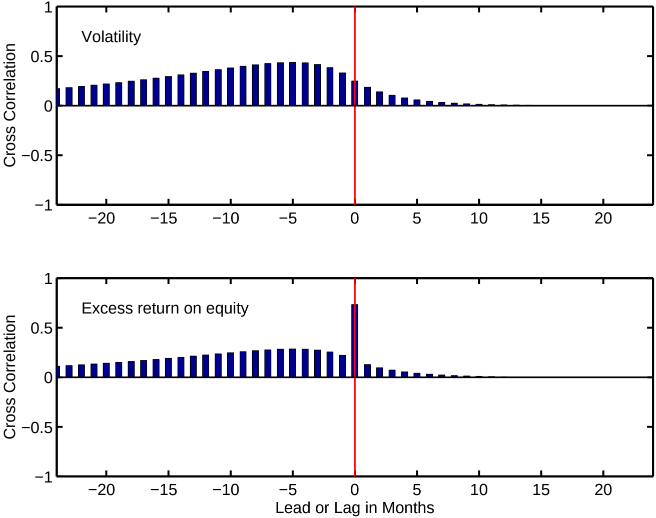
Transparent loglinear solution

- We love this, but won't bore you with the details
- Still needs some work

Excess returns depend on

- Volatility (v_t)
- Innovations in consumption growth and volatility
- Not expected future consumption growth (x_t)!

Excess returns: numerical example



Summary and extensions

Summary

- Data: excess returns correlated with future growth
- Model: ditto via stochastic volatility

Fixups and extensions

- Model dividends explicitly
- Production economies: volatility acts like shock to discount factor, affects consumption and labor supply

Related work (some of it)

Evidence on financial indicators of business cycles

- Ang-Piazzesi-Wei, Estrella-Hardouvelis, King-Watson, Rouwenhorst, Stock-Watson

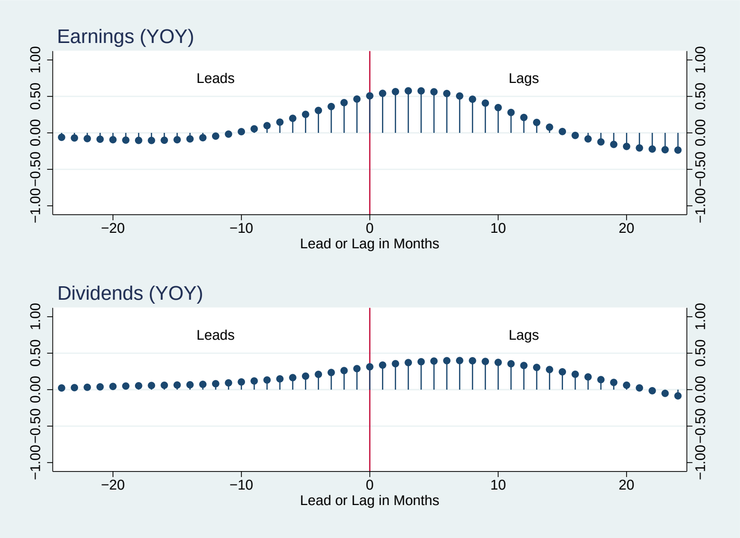
Kreps-Porteus pricing kernel

- Hansen-Heaton-Li, Weil

Stochastic volatility and returns

- Atkeson-Kehoe, Gallmeyer-Hollifield-Zin, Naik, Primiceri-Schaumburg-Tambalotti

Earnings and dividends (yoy)



Approximation: two flavors

Problem: find decision rule $u_t = h(x_t)$ satisfying

$$E_t F(x_t, u_t, w_{t+1}) \;\;=\;\; 1, \quad w_t \sim \mathrm{N}(0, \kappa_2)$$

Judd + many others

- Taylor series expansion of F
- nth moment shows up in nth-order term

Us + much of modern finance

- Taylor series expansion of $f = \log F$ in

$$E_t \exp[f(x_t, u_t, w_{t+1})] \;\;=\;\; 1$$

- All moments show up even in linear approximation

Approximation: example

Linear "perturbation" method

- Linear approximation of F

$$
\begin{aligned}
F(x_t, u_t, w_{t+1}) &= F + F_x(x_t - x) + F_u(u_t - u) + F_w w_{t+1} \\
E_t F = 1 \;\Rightarrow\; u_t - u &= (1 - F)/F_u - (F_x/F_u)(x_t - x)
\end{aligned}
$$

- Decision rule doesn't depend on variance of w (or higher moments)

"Affine" finance method

- Linear approximation of $f = \log F$

$$
\begin{aligned}
f(x_t, u_t, w_{t+1}) &= f + f_x(x_t - x) + f_u(u_t - u) + f_w w_{t+1} \\
E_t \exp(f) = 1 \;\Rightarrow\; u_t - u &= -(f + f_w \kappa_2/2)/f_u - (f_x/f_u)(x_t - x)
\end{aligned}
$$

- Note impact of variance v (higher moments would show up, too)