Asset Prices in Business Cycle Analysis

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Outline

Pictures: leads and lags in US data

Equations: the usual suspects + bells & whistles

Computations: loglinear approximation

More pictures: leads and lags in the model

Extensions

Leads and lags in US data

Cross-correlation functions of GDP with

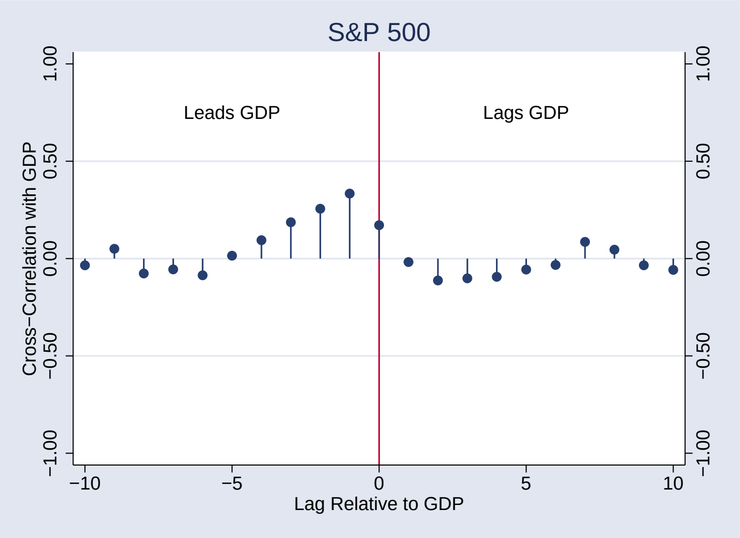
- Stock price indexes
- Interest rates and spreads
- Consumption and employment

US data, quarterly, 1960 to present

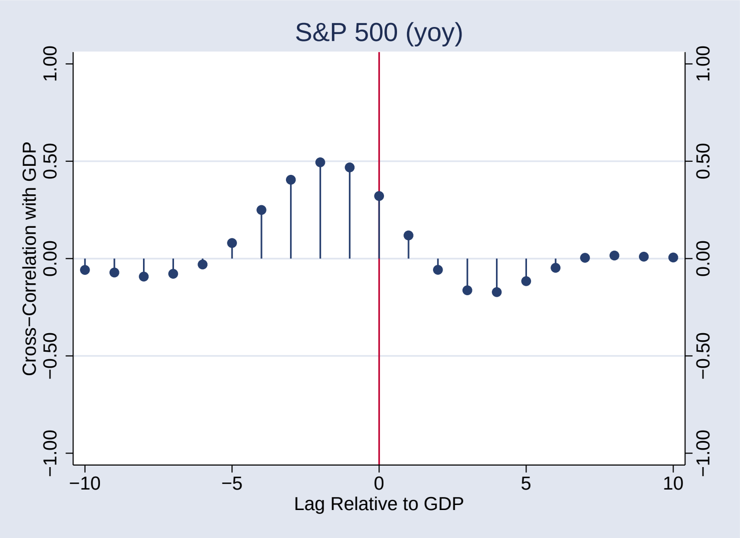
Quarterly growth rates $(\log x_t - \log x_{t-1})$ except

- Interest rates and spreads (used as is)
- Occasional year-on-year comparisons $(\log x_{t+2} - \log x_{t-2})$

Stock prices and GDP

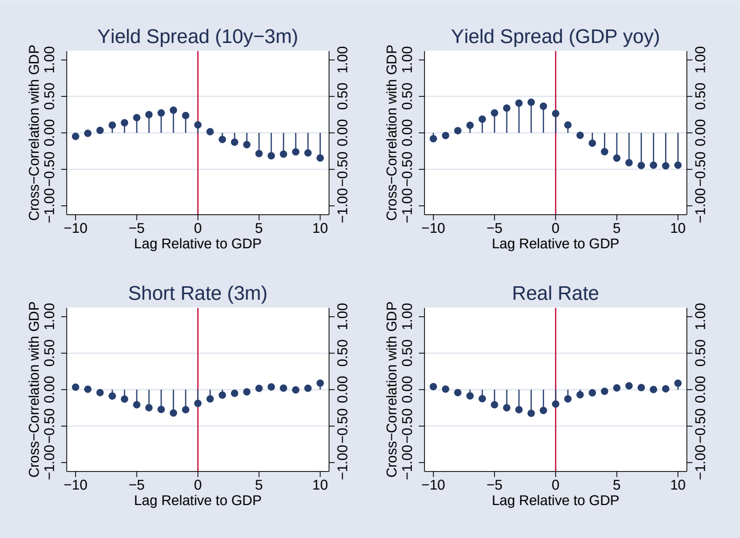


Stock prices and GDP (year-on-year)

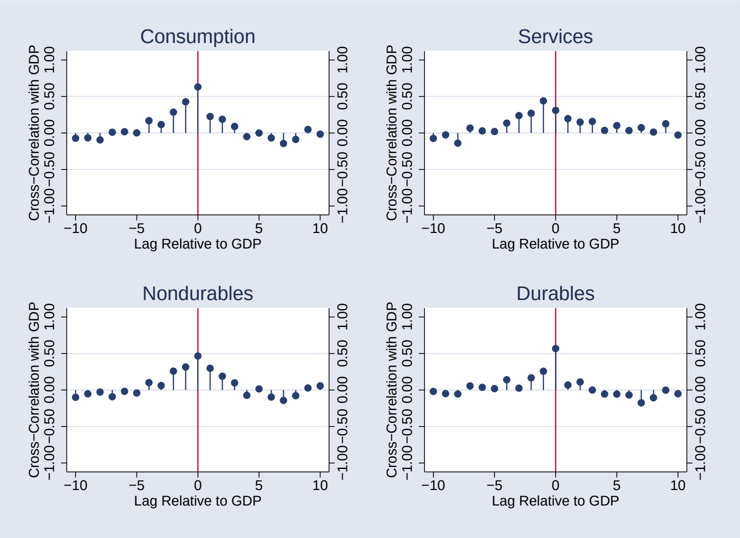


Stock prices and GDP

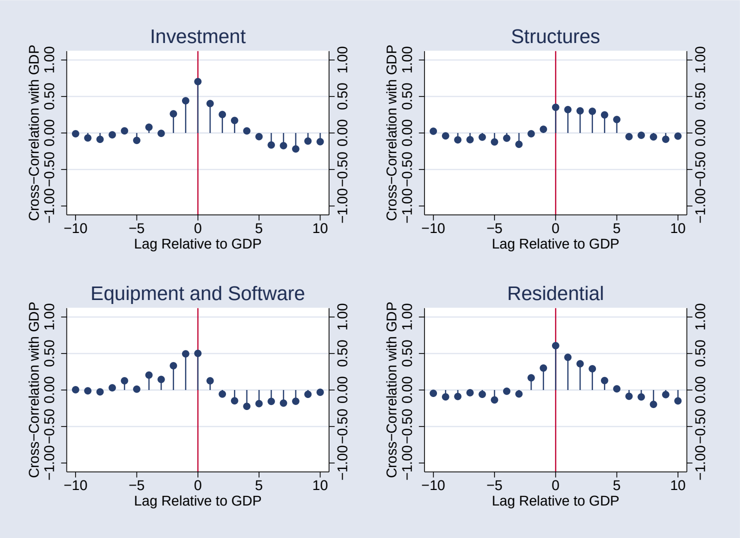
Interest rates and GDP



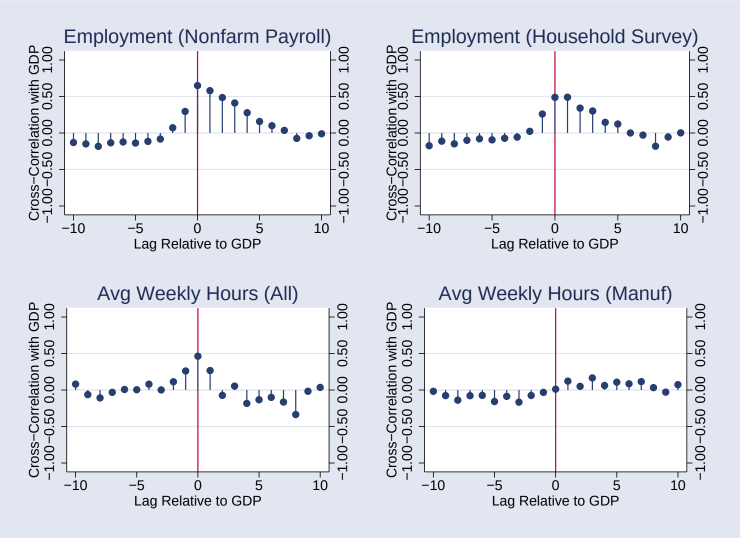
Consumption and GDP



Investment and GDP



Employment and GDP



Lead/lag summary

Things that lead GDP

- Stock prices
- Yield curve and short rate
- Maybe consumption (a little)

Things that lag GDP

- Maybe employment (a little)

Why?

(Almost) the usual equations

Streamlined Kydland-Prescott except

- Recursive preferences (Kreps-Porteus/Epstein-Zin-Weil)
- CES production
- Adjustment costs
- Unit root in productivity
- **Predictable component in productivity growth**

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Preferences

Equations

$$
\begin{aligned}
U_t &= V[u_t, \mu_t(U_{t+1})] \\
u_t &= c_t(1 - n_t)^{\lambda} \\
V(u_t, \mu_t) &= [(1-\beta)u_t^{\rho} + \beta\mu_t^{\rho}]^{1/\rho} \\
\mu_t(U_{t+1}) &= \left(E_t U_{t+1}^{\alpha}\right)^{1/\alpha}
\end{aligned}
$$

Interpretation

$$
\begin{aligned}
IES &= 1/(1-\rho) \\
CRRA &= 1 - \alpha \\
\alpha &= \rho \quad \Rightarrow \quad \text{additive preferences}
\end{aligned}
$$

Technology: production

Equations

$$\begin{aligned} y_t &= f(k_t, z_t n_t) \\ &= [\omega k_t^{\nu} + (1-\omega)(z_t n_t)^{\nu}]^{1/\nu} \\ y_t &= c_t + i_t \end{aligned}$$

Interpretation

$$\begin{aligned} \text{Elast of Subst} &= 1/(1-\nu) \\ \text{Capital Share} &= \omega(y/k)^{-\nu} \end{aligned}$$

Technology: capital accumulation

Equations

$$
\begin{aligned}
k_{t+1} &= g(i_t, k_t) \\
&= (1-\delta)k_t + k_t[(i_t/k_t)^{\eta}(i/k)^{1-\eta} - (1-\eta)(i/k)]/\eta
\end{aligned}
$$

Interpretation

No adjustment costs if $\eta = 1$

Productivity

Equations

$$\begin{aligned} \log x_{t+1} &= (I - A)\log x + A \log x_t + B w_{t+1} \\ \{w_t\} &\sim \text{NID}(0, \text{I}) \\ \log z_{t+1} - \log z_t &= \log x_{1t+1} \quad \text{(first element)} \end{aligned}$$

Interpretation

$$A = [0] \quad \Rightarrow \quad \text{no predictable component}$$

Computation overview

Scaling

- Recast as stationary problem in "scaled" variables

Loglinear approximation

- Loglinearize value function (**not** log-quadratic)
- Loglinearize necessary conditions
- With constant variances, recursive preferences irrelevant to quantities
  A feature, not a bug!

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Scaling the Bellman equation

Key input: (V, μ, f, g) are hd1

Natural version

$$
\begin{aligned}
J(k_t, x_t, z_t) &= \max_{c_t, n_t} V\left\{c_t(1-n_t)^\lambda, \mu_t[J(k_{t+1}, x_{t+1}, z_{t+1}]\right\} \\
\text{subject to:} \quad k_{t+1} &= g[f(k_t, z_t n_t) - c_t, k_t) \\
&\quad \text{plus productivity process \& initial conditions}
\end{aligned}
$$

Scaled version $[\tilde{k}_t = k_t/z_t,\ \tilde{c}_t = c_t/z_t]$

$$
\begin{aligned}
J(\tilde{k}_t, x_t, 1) &= \max_{\tilde{c}_t, n_t} V\left\{\tilde{c}_t(1-n_t)^\lambda, \mu_t[x_{1t+1} J(\tilde{k}_{t+1}, x_{t+1}, 1)]\right\} \\
\text{subject to:} \quad \tilde{k}_{t+1} &= g[f(\tilde{k}_t, n_t) - \tilde{c}_t, \tilde{k}_t]/x_{1t+1} \\
&\quad \text{plus productivity process \& initial conditions}
\end{aligned}
$$

Loglinear approximation

Objective: loglinear decision rules $[\hat{k}_t \equiv \log \tilde{k}_t - \log \tilde{k}$, etc]

$$\begin{aligned} \hat{c}_t &= h_{ck}\hat{k}_t + h_{cx}^\top \hat{x}_t \\ \hat{n}_t &= h_{nk}\hat{k}_t + h_{nx}^\top \hat{x}_t \end{aligned}$$

Key input:

$$\log J(\tilde{k}_t, x_t) = p_0 + p_k \log \tilde{k}_t + p_x^\top \log x_t$$

Solution

- Brute force loglinearization of necessary conditions
- Riccati equation separable: first p_k, then p_x
- Lots of algebra, but separability allows you to do it by hand

Necessary conditions

First-order conditions

$$\begin{aligned} (1-\beta)\tilde{c}_t^{\rho-1}(1-n_t)^{\rho\lambda} &= M_t g_{it} \\ \lambda(1-\beta)\tilde{c}_t^{\rho}(1-n_t)^{\rho\lambda-1} &= M_t g_{it} f_{nt} \end{aligned}$$

Envelope condition

$$J_{kt} = J_t^{1-\rho} M_t (g_{it} f_{kt} + g_{kt})$$

"Massive expression"

$$M_t = \beta\, \mu_t(x_{1t+1} J_{t+1})^{\rho-\alpha} E_t[(x_{1t+1} J_{t+1})^{\alpha-1} J_{kt+1}]$$

Expectations and certainty equivalents

Example: let $\log x \sim \mathrm{N}(\kappa_1, \kappa_2)$

Expectations and certainty equivalents for lognormals

$$\begin{aligned} E(x) &= \exp(\kappa_1 + \kappa_2/2) \\ E(x^\alpha) &= \exp(\alpha\kappa_1 + \alpha^2\kappa_2/2) \\ \mu(x) &= [E(x^\alpha)]^{1/\alpha} = \exp(\kappa_1 + \alpha\kappa_2/2). \end{aligned}$$

Effect of risk is multiplicative (like β)

Approximation: two flavors

Problem: find decision rule $u_t = h(x_t)$ satisfying

$$E_t F(x_t, u_t, w_{t+1}) \;=\; 1, \quad w_t \sim \mathrm{N}(0, \kappa_2)$$

Judd + many others

- Taylor series expansion of F
- nth moment shows up in nth-order term

Us + much of modern finance

- Taylor series expansion of $f = \log F$ in

$$E_t \exp[f(x_t, u_t, w_{t+1})] \;=\; 1$$

- All moments show up even in linear approximation

Approximation: example

Linear "perturbation" method

- Linear approximation of F

$$
\begin{aligned}
F(x_t, u_t, w_{t+1}) &= F + F_x(x_t - x) + F_u(u_t - u) + F_w w_{t+1} \\
E_t F \;=\; 1 \quad \Rightarrow \quad u_t - u &= (1 - F)/F_u - (F_x/F_u)(x_t - x)
\end{aligned}
$$

- Decision rule doesn't depend on variance of w (or higher moments)

"Affine" finance method

- Linear approximation of $f = \log F$

$$
\begin{aligned}
f(x_t, u_t, w_{t+1}) &= f + f_x(x_t - x) + f_u(u_t - u) + f_w w_{t+1} \\
E_t \exp(f) \;=\; 1 \quad \Rightarrow \quad u_t - u &= -(f + f_w \kappa_2/2)/f_u - (f_x/f_u)(x_t - x)
\end{aligned}
$$

- Note impact of variance v (higher moments would show up, too)

Loglinear approximation revisited

Decision rules are linear in $s_t = [\hat{k}_t, \hat{x}_t]$

$$\hat{c}_t = h_{cs} s_t, \quad \hat{n}_t = h_{ns} s_t$$

Controlled law of motion is linear

$$s_{t+1} = A_s s_t + B_s w_{t+1}$$

Pricing kernel is loglinear

$$\begin{aligned} \log m_{t+1} &= \log \beta + (\rho - 1) \log(c_{t+1}/c_t) \\ &\quad + (\alpha - \rho) \log [U_{t+1}/\mu_t(U_{t+1})] \\ &\approx \text{constant} + h_{ms} s_t + h_{mw} w_{t+1} \end{aligned}$$

Note: affine bond model a la Ang, Piazzesi, et al.

Leads and lags in the model: overview

Growth model: no labor or adjustment costs

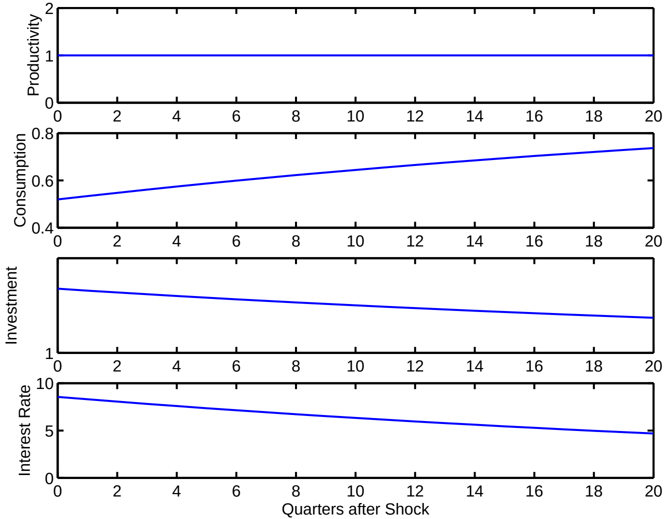
Three processes for productivity growth

- Random walk ($A = 0$)
- Two-period lead
- Small predictable component

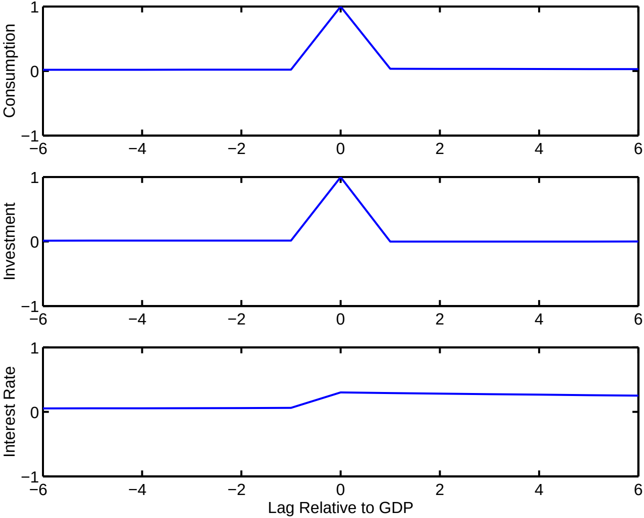
The challenge

- Barro and King

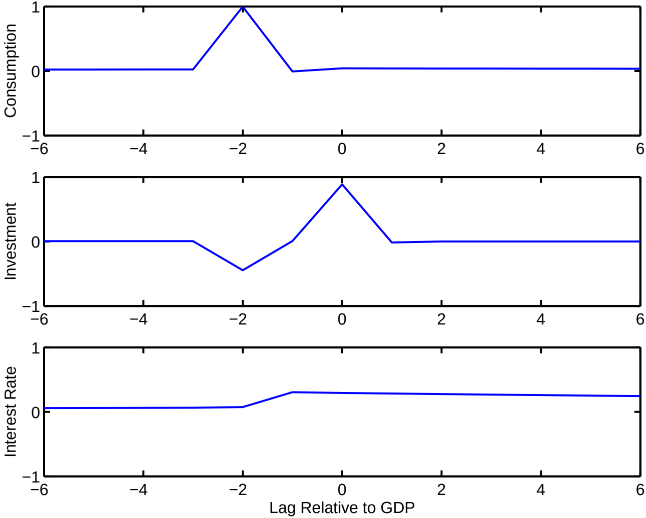
Random walk: impulse responses



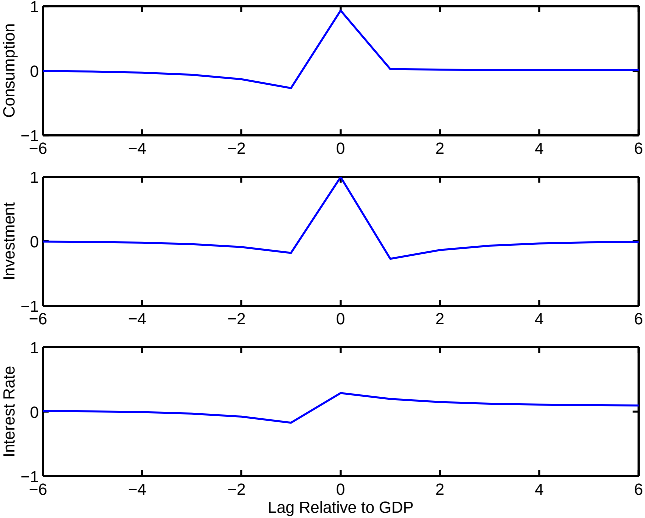
Random walk: cross correlations



Two-period lead: cross correlations



Predictable component: cross correlations



Summary and extensions

Summary

- Data: interest rates lead the cycle
- Model: ditto from predictable component in productivity growth

Extensions

- Labor dynamics: Gali's result?
- Stochastic volatility
- Could this result from endogenous dynamics? Monetary policy?

Related work

Leads and lags in data

- Ang-Piazzesi-Wei, Beaudry-Portier, King-Watson, Stock-Watson

Predictable components in models

- Bansal-Yaron, Jaimovich-Rebelo

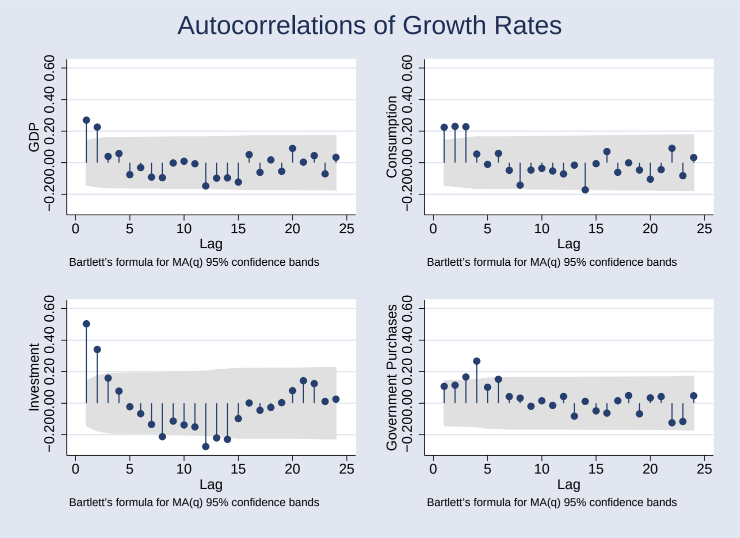
(Log)linear approximation

- Campbell, Hansen-Sargent, Lettau, Tallarini, Uhlig

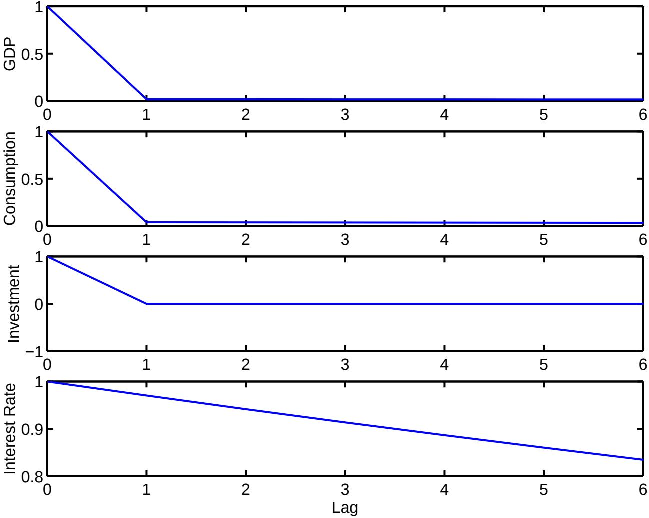
Kreps-Porteus pricing kernels

- Hansen-Heaton-Li, Weil

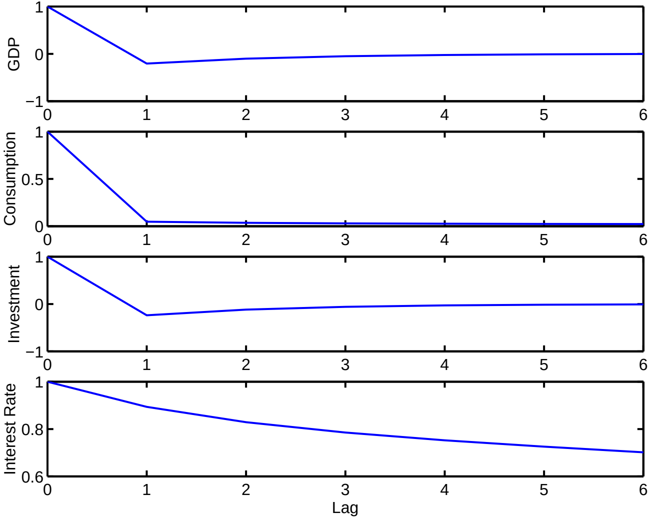
Autocorrelations of quarterly growth rates



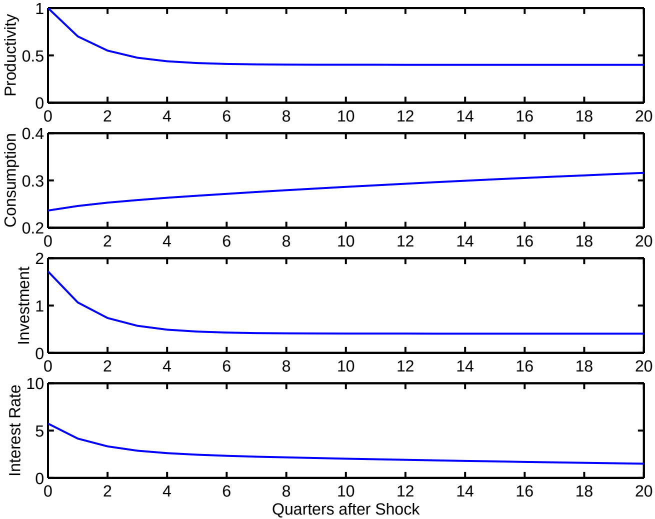
Random walk: autocorrelations



Predictable component: autocorrelations



Predictable component: impulse responses



Theory and reality (circa 1964)

Realist MacLeod:

- Mundell's article [ignores] complications associated with speculation in the forward market. It can only bring discredit on the economics profession to leave unchallenged his attempt to draw from the model policy conclusions that are applicable in the real world.

Theorist Mundell:

- Theory is the poetry of science. It is simplification, abstraction, the exaggeration of truth, a caricature of reality. Dr McLeod calls my assumptions unrealistic. I certainly hope he is right. I left out a million variables that made my caricature of reality unrealistic. At the same time, it enabled me to find fruitful, but refutable, empirical generalizations.

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