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Christopher A. Sims and Vector Autoregressions*

Lawrence J. Christiano

Northwestern University, Evanston, IL 60208, USA
l-christiano@northwestern.edu

Abstract

Three decades ago, Christopher A. Sims suggested that vector autoregressions (VARs) are useful statistical devices for evaluating alternative macroeconomic models. His suggestion has stood the test of time well. In the early days, VARs played an important role in the evaluation of alternative models. They continue to play that role today.

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I. Introduction

The research output of Christopher A. Sims has been nothing less than extraordinary. One of his first published papers continues to be important today.[1] In the four decades since then, Sims has produced a stream of output that has had a profound impact on macroeconomics. Sims continues to shake up the profession. For example, during the decade of the 2000s, we witnessed the wholesale incorporation of the Bayesian perspective into macroeconometric practice. Sims was a major force behind this development. Also, his work on the fiscal theory of the price level and on rational inattention helped to launch these two bodies of literature. A review of all these contributions is well beyond the scope of any one paper. Here, I limit my attention to a discussion of the contributions for which Sims has been honored with the Nobel Memorial Prize in Economic Sciences.

In a series of papers, Sims (1972, 1980a, 1980b) proposed the use of vector autoregressions (VARs). The most comprehensive and influential of these papers is Sims (1980a), 'Macroeconomics and Reality'. Few contributions have withstood the test of time as well as this paper. In the early days, VARs provided key empirical input into substantive economic

*Northwestern University and National Bureau of Economic Research.

[1] For one application of Sims (1971), see Christiano *et al.* (2007).

debates, and they continue to do so today. In addition, research on technical questions raised by VARs proceeds at a brisk pace. Sims continues to be a major force on both the substantive and technical fronts. Some of the best researchers in our profession have also made contributions to the analysis of VARs. The participation of so many prominent economists is eloquent testimony to the importance of VARs.

In Sims (1980a), he suggested that VARs could be of use for three purposes: (1) forecasting economic time series; (2) designing and evaluating economic models; (3) evaluating the consequences of alternative policy actions. Sims provided some preliminary indications of how VARs might serve these three objectives (Sims, 1980a). He explained why the framework that existed at the time for accomplishing the three objectives was inadequate. He argued that the assumptions used to achieve econometric identification were simply "incredible". For example, to identify a structural system with a demand curve and a supply curve, it was standard at the time to assume that one variable shifts the demand curve but not the supply curve, while another variable shifts the supply curve but not the demand curve. The assumption that a variable could be important for one side of the market, but could be excluded from affecting the other is incredible, according to Sims. For example, if the weather in Brazil matters for the supply of coffee, it is also expected to matter for demand. People on the demand side of the market, noticing bad weather in Brazil, would surely stockpile coffee in anticipation of an imminent rise in price. The view taken by Sims was that models based on identification assumptions such as these were not useful. He proposed VARs as an alternative to standard econometric models with their doubtful exclusion restrictions.

We now define a VAR. Let the $N \times 1$ vector y_t denote the set of variables that is of interest in the analysis. The assumption that y_t follows a pth-order VAR means that it can be expressed as

$$y_t = B_0 + B_1 y_{t-1} + \cdots + B_p y_{t-p} + u_t, \qquad E u_t u_t' = V, \tag{1}$$

where u_t is not correlated with $y_{t-1}, \ldots, y_{t-p}$. It is assumed that p is assigned a large enough value so that u_t is not autocorrelated over time. The VAR disturbances, u_t, are assumed to be a linear transformation of the economically fundamental shocks, ε_t:

$$u_t = C\varepsilon_t, \qquad CC' = V. \tag{2}$$

The economic shocks ε_t are assumed to be of independent origins, and therefore to be uncorrelated with each other. In addition, I have adopted the normalization that each economic shock has unit variance. Many objects in equations (1) and (2) are econometrically identified, that is, they can be estimated using data without any further (credible or incredible) assumptions. In particular, B_i and V are econometrically identified. To estimate these, we simply run a series of regressions and we compute the variances and covariances among the regression disturbances. However, C is not identified. This is because the symmetric matrix V has only $N(N+1)/2$ independent elements, while C has $N^2 > N(N+1)/2$ unknowns. For most forecasting purposes, it is enough to have just B_i and V. In this case, no identification assumptions are required. For other purposes, one would like to know the dynamic effects on y_t of one or more elements in ε_t. In this case, the corresponding columns in matrix C are required. Some additional assumptions are necessary to recover these columns of C. It is standard to refer to a VAR together with these additional assumptions as a structural VAR or an SVAR.

The greatest impact of VARs has been in the second area mentioned above (i.e., the construction of economic models). The idea is to make the smallest set of assumptions that allow us to identify the ith column of C. (The advantage of working with a small set of assumptions is that the controversy over assumptions is thereby restricted to essentials.) For example, if the ith element of ε_t corresponds to a monetary policy shock, then identification of the ith column of C allows us to quantify the dynamic effects of a monetary policy shock on inflation, output, and other variables. These effects can then be used to select functional form and other assumptions that allow us to build a fully specified dynamic stochastic general equilibrium (DSGE) model. VARs have been used in this way to guide the construction of New Keynesian models. Indeed, an important reason that New Keynesian economic models have vaulted to center stage in recent years is the discovery that they are capable of mimicking the impulse responses to various shocks that are implied by estimated SVARs. This event has had important consequences. The models provide a widely used, coherent framework for understanding the weakness in aggregate output and employment since the crisis of 2008, and for contemplating appropriate monetary, fiscal and regulatory policy responses. In this way, the success of the New Keynesian model is an important example of the assertion in Sims (1980a) that VARs have a meaningful role to play in selecting and building models.

II. Vector Autoregressions and Forecasting

The primary technical problem in using VARs for forecasting purposes is that they require the estimation of $p \times N^2$ parameters. In practice, this is a large number of parameters, because policy-makers tend to want a large value for N. It is well known that a forecasting model with too many free parameters has large forecast root mean square errors. Indeed, the gains to parameter parsimony in a forecasting model are so great that forecasts can even be improved if false restrictions are used to reduce the size of the parameter space. Sims conjectured that this parsimony principle was the reason econometric models in existence when Sims (1980a) was written had tolerable forecasting properties, despite their incredible identification assumptions. He called for alternative approaches to parameter reduction that did not rely on incredible identification. Sims (1980a) speculated that some sort of Bayesian approach might work better. This was confirmed in thesis work carried out at the University of Minnesota by Robert Litterman under Sims' direction (see Litterman, 1979, 1986a, 1986b). This approach was further articulated and extended in a widely cited paper by Doan *et al.* (1984). The approach to forecasting and policy analysis soon became the analytical foundation for monetary policy briefings given to the president of the Federal Reserve Bank of Minneapolis. Bayesian priors were used to keep the size of the parameter space manageable, even though N was typically large. The prior that Litterman used came to be known as the "Litterman prior", or the "Minnesota prior". This prior specified that each equation in a VAR is a random walk. Depending on how tight the prior uncertainty was specified to be, VAR estimation (better known as BVAR estimation) produced a VAR with all random walk equations, or something closer to an estimated unrestricted VAR. BVARs are now used in many central banks for the purpose of producing national forecasts. They are also used to produce regional forecasts.[2] Early work by Litterman confirmed the high quality of VAR forecasts. Later, Stock and Watson (2001), Bauer *et al.* (2006), and others have shown that Bayesian VARs have solid track records as forecasting devices, and that they compare favorably with alternatives, including the judgemental forecasts of professional forecasters.

The Litterman prior is somewhat arbitrary because it is not well motivated by economic theory. Recent work has taken important steps to correct this shortcoming. The first break came with the work of Ingram and Whiteman (1994). They also used a Bayesian approach, but instead of using a Litterman prior, they used a real business cycle (RBC) model as a prior. Ingram and Whiteman found that a BVAR based on the RBC model prior produced better forecasts than the BVAR with a Litterman prior. Since then, two Yale students of Sims, Marco Del Negro and Frank Schorfheide, have explored this further, by extending the type of DSGE model used as a prior (see Del Negro and Schorfheide, 2004).

From early on (e.g., Sims, 1989), Sims has made it clear that VARs are, in fact, not the ideal model for forecasting. The ideal model to use in forecasting is one based on a fully explicit economic theory. Fully structural models tend to have a small number of parameters, so they perform well on parsimony. Also, with fully structural models, it is possible

[2] An early example is Anderson (1979). More recent examples include Crone and McLaughlin (1999) and Felix and Nunes (2003), and references therein.

to explain recent and prospective data movements in terms of intelligible economic mechanisms and shocks. Finally, with such a model, it is possible to imagine analyzing the effects of changes in policy strategy in ways that are immune from the Lucas critique. The reason VARs were originally proposed was because of the view that a credible structural model was not yet available.

However, as a result of the work of Sims and others, the development of fully specified, empirically estimated DSGE models has proceeded at a rapid pace. We have now reached the point where the claim has been made that DSGE models are superior to VARs in forecasting.[3] Although such claims may be somewhat exaggerated, they certainly do suggest that the time is approaching when structural models can replace VARs for forecasting. However, even then, VARs would be expected to play an important role as benchmarks in evaluating the forecasting abilities of structural models. Moreover, VARs will have played an important role in identifying the models that will ultimately replace them in forecasting. This is the topic I turn to next.

III. Vector Autoregressions and Structural Models

Sims (1980a) suggested that VARs are a fruitful way to organize data because they can be used as a sort of battleground for testing alternative theories. Our experience over the past 30 years has confirmed the wisdom of this suggestion. In some respects, VARs represent a natural statistical tool for economists. Economists are accustomed to thinking of economic models in terms of impulses and propagation mechanisms, and VARs are a device for organizing the data precisely into these categories.[4]

To understand better why VARs are useful, it is convenient to consider two traditions for building dynamic general equilibrium models. One tradition focuses on constructing very simple models that abstract from key shocks and many structural economic details.[5] A classic example is the RBC model of Kydland and Prescott (1991), which is a very simple model driven exclusively by technology shocks. In their analysis, Kydland and

[3] This is a claim made by Smets and Wouters (2003) for a version of the model of Christiano *et al.* (2005).

[4] In terms of equations (1) and (2), the impulses are ε_t. Suppose there is a disturbance to the ith element of ε_t, ε_{it}. The disturbance propagates onto y_t according to $y_t = [I - B_1 L - B_2 L^2 - \cdots - B_p L^p] C_i \varepsilon_{it}$, where C_i denotes the ith column of C, and L denotes the lag operator. Thus, the impulse is ε_{it} and the propagation mechanism is $[I - B_1 L - B_2 L^2 - \cdots - B_p L^p] C_i$.

[5] The literature on factor models appears to confirm the view that aggregate dynamics is dominated by the effects of a few shocks only (e.g., Sargent and Sims, 1977; Quah and Sargent, 1993; Forni *et al.*, 2003; Giannone *et al.*, 2005).

Prescott work with only one shock, even though, in their opinion, technology shocks account for only 70 percent of business-cycle fluctuations. A conundrum confronted by this modeling tradition is how to empirically evaluate models that contain only a subset of the shocks, when the data are understood to be driven by many shocks.[6] Structural VARs provide a coherent resolution to this challenge by allowing the analyst to isolate the empirical effects of a subset of shocks.

A second tradition for building macroeconomic models incorporates large numbers of shocks and rich detail, in order to provide as complete a characterization as possible of the stochastic processes generating the data (e.g., Smets and Wouters, 2003; Christiano *et al.*, 2012). This tradition avoids the Kydland–Prescott conundrum. Still, for diagnostic purposes, it is useful to assess the implications of these models for particular shocks to the economy. Thus, VAR-based procedures have the potential to provide this sort of diagnostic.

In the following subsection, I explore ways in which VARs have been used as guides for thinking about alternative structural models of aggregate economics.

Granger Causality

At the time that Sims (1980a) was published, Sims and others were interested in a particular hypothesis associated with Milton Friedman and Anna Schwartz. Based on detailed historical analysis, Friedman and Schwartz had argued that monetary disturbances were key factors driving movements in output. They concluded that these disturbances were "causal" because they were convinced by the data that movements in money were not themselves a reaction to movements in past economic activity. What the observed money–income association implied for the ultimate causes of business cycles was a question of great interest when Sims (1980a) was written. According to Bernanke (1986), explaining this association was so important that it "...nearly defined the field [of macroeconomics]".

In (1972), Sims developed the intuitive idea of Friedman and Schwartz into a precise econometric hypothesis, by translating it into a pattern of zero restrictions in a VAR. For this, Sims used a statistical concept

[6] Aiyagari (1994) and Prescott (1991) have drawn attention to the challenge by pointing to a difficulty with the standard RBC strategy for evaluating a model. In this strategy, the second moment properties of the data are compared with the second moment properties of the model. Prescott has famously asserted that if a model matches the data, then that is bad news for the model. The argument is that because good models leave some things out of the analysis, good models should not match the data. Of course, many models do not match the data. This raises the question, how can we use the data to differentiate between good models and bad models?

proposed by Clive Granger. Working with a bivariate VAR in which y_t was composed of US data on money and output, Sims showed that money is useful in forecasting future output, even in the presence of past observations on output (i.e., "money Granger-causes output"). He also showed that output is not similarly helpful in forecasting future money (i.e., "output does not Granger-cause money"). Sims argued that his one-way Granger-causality finding vindicated the hypothesis of Friedman and Schwartz. In Sims (1972), it is clear that Sims understood that, in principle, there are scenarios in which his finding of one-way Granger causality from money to output is consistent with money being completely unimportant for output.[7] However, Sims argued that such scenarios are unlikely.

Later, Sims (1980a, 1980b) reported that when the interest rate is also included in y_t, his earlier one-way Granger-causality result disappears. The presence of the interest rate drains away the predictive power of money for output. In particular, Sims found that an innovation in the nominal rate of interest leads to a decline in output. (King and Watson (1996) later referred to this as the "inverted leading indicator phenomenon".) This VAR work led Sims to retract his view on the Friedman–Schwartz hypothesis. As an "interesting working hypothesis", he adopted the idea that monetary policy actually has little to do with output fluctuations. Instead, he conjectured that the inverted leading indicator phenomenon reflects the operation of real shocks, with monetary disturbances playing only a minor role in fluctuations. The findings of a widely read paper (Litterman and Weiss, 1985) seem to provide support for Sims' hypothesis.

Sims' early work on VARs in the 1970s represented an important step towards placing the influential Friedman–Schwartz hypothesis on a clear scientific basis, so that it could be evaluated using formal statistical tools. The work drew attention to VARs, because it demonstrated their potential to summarize the data in ways that are useful for shedding light on important questions. Interestingly, the substantive conclusion reached in Sims (1980a, 1980b) has stood the test of time well. There is now widespread agreement that although monetary policy and monetary policy shocks are a part of the story of business cycles, they are nowhere near as important as the original Friedman–Schwartz hypothesis appeared to suggest.

Formal economic models played almost no role in early VAR analysis. This reflected two things. First, the structural economic models that were

[7] Sargent (1976) presented an early example that illustrates the kind of concerns that motivated Sims. Sargent's example clarifies the distinction between Granger causality and causality in the economist's sense. In Sargent's example, money does not Granger-cause inflation, while inflation does Granger-cause money. In this model, in fact, money does cause inflation in the economist's sense. In the model, the one-way Granger-causality pattern is a consequence of the nature of monetary policy. The example has the property that a change in monetary policy results in a different Granger-causality pattern.

traditional at the time were not used, because there was a general disenchantment with them. Second, early dynamic equilibrium models were still in their infancy, and they were not yet taken seriously enough to match directly to VARs. Sims (1989) noted the heavy role of "intuition" and "common sense" in VAR analysis up to that time. He was the first to show how useful it can be to use a fully articulated DSGE model to interpret VARs. He did this by constructing a DSGE model, and by showing that it reproduces, qualitatively, both the initial bivariate Granger-causality results reported in (1972), as well as the result reported in Sims (1980a, 1980b) when the interest rate is included in the analysis. Notably, Sims' model reproduced the inverted leading indicator phenomenon he had noticed in Sims (1980b).

Because monetary shocks play essentially no role in the model of Sims (1989), his exercise provided additional support for his hypothesis in Sims (1980a, 1980b), that the money–income relationship can be interpreted as reflecting the operation of real shocks, rather than just disturbances to money. At the same time, Sims (1989) had to add a caveat to his conclusion, because he had found other implications of his DSGE model that did not match the data well. Implications such as this are hard to notice using the earlier, intuitive, approach to interpreting VARs. In effect, Sims (1989) provided a concrete illustration of the power of using a DSGE model to interpret VAR findings. A lesson that is of greater significance in the long run is that Sims (1989) also provided an illustration of the power of VARs to help guide the construction of DSGE models.

The use of Granger-causality tests decreased significantly after the 1980s. Evans (1992) is one notable exception. He showed that money, interest rates, and government spending Granger-cause measured Solow residuals. The idea that the Solow residual represents an exogenous shock to technology stood as an important challenge to the hypothesis in the RBC literature at the time. Evans' finding, obtained squarely in a VAR framework, was an important input to further developments of DSGE models (see Burnside *et al.*, 1993).

Impulse Response Functions

The suggestion made by Sims that economists have much to learn from studying VAR-based impulse response functions has stood the test of time well. The implementation of VAR-based estimations of impulse response functions means that certain technical issues have to be addressed. I review some of these in the following subsection, in part as a way of explaining further what impulse response functions are all about. Of course, impulse response functions would be of little interest if they had not been useful for shedding light on the structure of the economy. In fact, they have played a major role as inputs into a variety of important substantive debates. These are reviewed in the second subsection below.

Technical Issues. The major technical hurdle in using impulse response functions has been in understanding the relationship between the VAR disturbances u_t and the objects of interest, ε_t. As noted after equation (1), without additional assumptions, ε_t cannot be uniquely recovered from u_t. These additional assumptions have economic content. The power of the VAR methodology that Sims advocated brought the number of economic assumptions required down to a bare minimum. However, it did not reduce the number to zero.

Looking back at Sims (1980a) now, it is clear that Sims understood the subtleties involved in the relationship between u_t and ε_t. However, many economists who initially read Sims (1980a) clearly missed these subtleties. In subsequent years, the connections have been greatly clarified both by the work of Sims and by that of other leading macroeconomists. This work continues today.

The issue that was perhaps least understood in the early days of VAR analysis is the assumption, implicit in equations (1) and (2), that ε_t can be recovered from the record of current and past y_t (when this is so, ε_t is said to be invertible). Hansen and Sargent (1980, footnote 12) have shown, with a particular DSGE example, that invertibility is not something we can count on in practice.[8] In a recent paper, Fernández-Villaverde *et al.* (2007) have carefully explored invertibility in the special case when the dimensions of u_t and ε_t coincide. They have provided an important and very useful matrix characterization of invertibility. However, a trickier question, which is of greater practical relevance, concerns the connection between u_t and ε_t when the number of shocks in ε_t exceeds the number of variables y_t.[9] In a paper that has not circulated widely (it has now been published as a vintage paper in *Macroeconomic Dynamics*), Sims and Zha (2006) have usefully cast the invertibility question in a Kalman filter framework and are able to make important headway. Although the research on invertibility is still ongoing, the work carried out so far suggests that the news is good.

[8] In their discussion of Blanchard and Quah (1989), Lippi and Reichlin (1993) have presented another important example.

[9] The following argument suggests that this is the case that is likely to be of practical relevance. If the number of elements in ε_t is less than the number of elements in y_t, then the matrix V is singular. In practice, V is far from singular. Moreover, experience suggests that whatever the dimension N of y_t is in any practical application, if the econometrician were to expand the number of variables in y_t to $\tilde{N} > N$, then the resulting V matrix would not be singular. This implies that in the application with N variables, there are at least $\tilde{N} > N$ shocks.

Most of the DSGE models analyzed so far indicate that invertibility holds for the type of shocks considered in existing empirical VAR studies.[10]

If we take it as given that invertibility is satisfied and that the dimensions of ε_t and u_t are the same, there is still an important related problem. This is the problem of estimating the columns of C that are needed to compute impulse response functions. This is another point on which there was substantial confusion in the early days. In his early work on VARs, Sims chose to identify ε_t by selecting the unique lower triangular C consistent with equation (2). Because of the assumed triangular structure, this strategy is sometimes called recursive identification. Initially, users of VARs clearly did not understand that recursive identification was just one of many identification strategies possible. It was poorly understood that the choice of identification strategy is important, and that the choice implicitly or explicitly involves economically substantive assumptions.

The important early work of Bernanke (1986) and Blanchard and Watson (1986) clarified these matters greatly (see also Bernanke and Mihov, 1998). These authors clearly showed the logical connections between u_t and ε_t. They highlighted the role and nature of identification in ways that were accessible to a broader audience. In addition, they adopted an alternative to Sims' recursive strategy for identifying C. Many researchers found this alternative style of identification appealing. For example, Sims himself adopted it in Sims (1986) and in some of his later work (Leeper and Sims, 1994; Leeper *et al.*, 1996). Others found the style difficult to interpret economically. Ironically, the approach resembled the exclusion restrictions in traditional econometrics that Sims had found so incredible in Sims (1980a). Under the new identification strategy, u_t was modeled as satisfying a static system of demand and supply curves, in which ε_t denoted the primitive disturbances (this is the reason why I call this style of identification "static identification"). Under the static identification strategy, the elements in C were interpreted as the slope coefficients in a demand-and-supply type system. The numerical value of the slopes were identified by zero (exclusion) restrictions in C. There emerged critics, who were suspicious that static identification was vulnerable to the same critique that Sims had leveled against traditional identification strategies. However, importantly, Sims and Zha (2006) have also shed light on static identification. They have given an explicit DSGE model and they have shown how it rationalizes key aspects of the static identification strategy.

Beyond recursive and static identification, a third strategy has been proposed by Blanchard and Quah (1989). They have shown that the assumption that only one element in ε_t has a permanent impact on some variable is

[10] The recent body of literature on "news" shocks suggests a range of examples in which invertibility is not obtained. For a further discussion, see Blanchard *et al.* (2009).

enough to identify both the shock and its dynamic effects on y_t.[11] The Blanchard–Quah identification strategy was soon applied in many different settings with different types of shocks.[12] I refer to this identification strategy as "long-run identification".

Several technical issues are raised by long-run identification. Intuitively, they stem from the fact that identification is based on an experiment that can never be observed in a finite sample. Under long-run identification, a shock is identified because it is the only one to have a permanent impact on some variable. At a technical level, the difficulties stem from the fact that the sums of the VAR coefficients must be estimated precisely. This is particularly difficult when the true VAR has $p = \infty$, but the econometrician must adopt a finite value of p. This is of more than just theoretical interest. It is well known that, apart from special cases, DSGE models imply $p = \infty$. The primary original discussions of the relevant econometric difficulties are given in two important papers by Sims, written before VARs were invented (see Sims, 1971, 1972). Faust and Leeper (1997) have derived some implications of the early papers by Sims. They have derived some surprising and unappealing sampling properties for VAR-based impulse responses estimated under long-run identification. Efforts to overcome these poor properties of long-run identification represent a current area of research.[13] I discuss long-run identification further in the following subsection.

A final distinct identification strategy has been introduced by Faust (1998) and Uhlig (2002). This uses even less *a priori* information than used in the identification strategies described above. This strategy defines, for example, a monetary policy shock as a disturbance that drives output and price up and interest rates down. The strategy is somewhat more controversial than the others. One problem is that it is difficult to establish standard properties, such as consistency, for it. In a DSGE model driven by many shocks, the probability limit of the impulse response function produced by this strategy will be a combination of the actual responses associated with several shocks.

More technical issues still have to be solved. Although it is understood that there is a connection between the quantity and type of variables in

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[11] If the shock is the ith element in ε_t, then the Blanchard–Quah identification strategy permits the analyst to uniquely recover the ith column of the matrix C. This is what is necessary to determine the dynamic effect of the ith shock in ε_t on y_t.

[12] Later, I discuss applications in which shocks to technology are identified. See Faust and Leeper (1997) for a list of references concerning shocks to money demand and supply, as well as other shocks.

[13] Faust and Leeper (1997) have proposed ways to overcome some of the shortcomings they have identified. Christiano *et al.* (2007) have also attempted to overcome some of the difficulties associated with long-run identification.

y_t and invertibility, this connection is not well understood. However, the framework of Sims and Zha (2006) discussed above has the potential to be very useful in this context. Another issue concerns the number of lags p to be used in the VAR. Cooley and Dwyer (1998) were among the first to draw attention to the fact that standard DSGE models imply $p = \infty$. Because datasets are finite, this means that economists using VARs must, in effect, unavoidably commit a specification error. To determine whether this truncation error is a problem, economists have performed simulation experiments using specific DSGE models. Standard VAR-based impulse response functions are computed in artificial data from these models. The DSGE models considered to date suggest that the lag truncation error does not generate biases large enough to be of concern to the applied econometrician. However, this is true only in the limited set of models studied so far. Much further work on a broader range of models is required before economists can feel secure that the Cooley–Dwyer critique is not a problem in practice.[14]

There are two other econometric issues raised by VAR-based impulse response functions. One of these issues concerns the characterization of sampling uncertainty for VAR-based impulse response functions. In the early days of VARs, sampling uncertainty tended to be ignored. However, sometimes in impulse response analysis, there is not much information in the data about the response of some variables to particular shocks. Of course, this ought to be no surprise. First, very few assumptions are used in VAR-based impulse response functions. Second, if a given shock plays only a negligible role in the dynamics of a particular variable, then we would expect difficulty in statistically isolating the effects on that variable of the shock.[15]

Runkle (1987) was the first to point out that sampling uncertainty can be large with VAR-based impulse response functions.[16] His observation drew attention to the importance of reporting confidence intervals. However, the problem of computing confidence intervals soon ran into an array of complications, which pitted Bayesian procedures against the classical procedures with which macroeconomists are more familiar. Sims and Uhlig (1991) clarified what is at issue in this clash, and they made a powerful and compelling case for the Bayesian approach.[17] The Runkle observation

[14] Chari *et al.* (2005) have recently reiterated the Cooley–Dwyer observation. Christiano *et al.* (2007) have argued that, at least in two standard DSGE models, when biases induced by lag truncation emerge, they are smaller than sampling uncertainty. In this sense, the bias is not large enough to mislead the econometrician using standard practice.
[15] This idea has been explored extensively by Christiano *et al.* (2007).
[16] Erceg *et al.* (2004) have recently made a similar observation.
[17] In later work, Sims and Zha (1999) addressed some important computational issues associated with the Bayesian approach to computing confidence intervals.

also drew attention to the potential value of incorporating more *a priori* information into the VAR-based estimation of impulse response functions. This idea, which was emphasized by Watson (1987) in his discussion of Runkle, deserves more attention than it has received.[18]

VAR-based impulse response functions have been used to select between broad classes of models. They have also been used by economists who are interested in estimating the parameter values of a particular parametric DSGE model. This raises a second set of econometric issues. Rotemberg and Woodford (1997) and Christiano *et al.* (2005) have applied a strategy similar to "estimation by simulation". The strategy selects model parameters in order to make a model's implications for impulse responses as similar as possible to the impulse responses implied by an estimated VAR. Although the asymptotic properties of this estimator are standard, little is known about its small sample properties.

The econometric strategy of Rotemberg and Woodford and Christiano *et al.* has been has been recast in a Bayesian framework by Christiano *et al.* (2010b). A related contribution appears in Del Negro and Schorfheide (2004). By specifying prior odds between the VAR and a particular DSGE model, their strategy uses the DSGE model to learn about VAR-based impulse response functions, and it simultaneously uses the VAR to learn about the parameters of the DSGE model. We can expect further developments in econometric procedures that are inspired by VARs.

Ways in Which Impulse Response Analysis Changed the Way Economists Think. Economists think differently about the aggregate economy because of what they have learned using VARs. The structure of modern monetary DSGE models has been profoundly influenced by VARs. Current views about the relative contribution of different shocks in business fluctuations are, in part, a result of results based on VARs. VAR-based estimates of the response of the economy to government spending shocks have also had a fundamental impact on a range of debates, from how best to model the interaction of government spending and the economy, to the importance of monopoly power.

Perhaps the simplest way to summarize the impact of VAR-based impulse response functions on monetary DSGE models is to refer to the review by Green (2005) of the Woodford (2003) book on monetary economics. This book summarizes the state of the art that was current at the time. Green, who is an outsider to this body of literature, was particularly struck by the abandonment of the style of monetary DSGE models pioneered in early work by Lucas, Prescott, Sargent, and others. Green emphasized how the

[18] See Fisher (2006) for recent work that makes use of more than the minimal assumptions in VARs in order to shrink sampling uncertainty.

models in Woodford (2003) are characterized by sticky prices and wages, and a host of other frictions that did not appear in the early models. Green asked, rhetorically, "how did this shift come about?" His answer is that the model features which Woodford introduced are motivated by VAR-based evidence about the dynamic response of data to identified monetary policy shocks. VAR-based responses suggest that both inflation and real variables respond slowly, and in a hump-shaped fashion, to a monetary disturbance. The design of modern monetary models reflects the attempt to come to grips with these findings.[19]

Blanchard and Quah (1989) is an early paper on the contribution of various shocks to the business cycle. Working with a bivariate system, they concluded that technology shocks (supply shocks) account for a much smaller portion (about one-third) of the variance of business-cycle fluctuations in output than had become conventional wisdom in the RBC literature. Because their paper presented an important technical contribution (they proposed long-run restrictions), as well as an important substantive finding, it is not surprising that it quickly became very influential. Other prominent researchers also applied a variant of the long-run identification of Blanchard and Quah in order to determine the importance of technology shocks in business-cycle fluctuations. For example, Shapiro and Watson (1988) extended the Blanchard–Quah approach by incorporating more variables, and they also concluded that technology shocks account for roughly one-third of the variance of business-cycle fluctuations. King *et al.* (1991) extended the long-run identification strategy of Blanchard and Quah to other types of shocks, such as nominal disturbances. They found that technology shocks are somewhat more important than Blanchard and Quah (1989) and Shapiro and Watson (1988) thought. However, they still concluded (p. 838) that "...the US data are not consistent with the view that a single real permanent shock is the dominant source of business-cycle fluctuations." The results of Blanchard and Quah (1989), Shapiro and Watson (1988), and King *et al.* (1991) are interesting, in part because they conflict with the estimate of the importance of technology shocks emerging from the RBC calibration literature. Using this approach, Prescott (1986) estimated that technology shocks account for 70 percent of business-cycle fluctuations.

In a paper that follows the style of Blanchard and Quah (1989), Galí (1999) also asked "how important are technology shocks in business-cycle fluctuations?" His answer is also, "not very important". The work of Galí

[19] The paper by Eichenbaum and Evans (1995) is important, although it is beyond the scope of this paper. They have analyzed the impact of monetary policy shocks on the exchange rate, and they have provided useful evidence on the nature of the failure of uncovered interest rate parity.

has stirred up considerable controversy, partly because he drew special attention to his finding that hours worked fall in response to a positive technology shock.[20] Because hours worked are procyclical, Galí concluded that technology shocks cannot be important in business cycles. In effect, Galí argued that the standard RBC model (which implies that hours worked rise in the wake of a positive technology shock) can be criticized in two ways: the shock that it emphasizes as a source of business-cycle fluctuations is, in fact, not important, and in any case, it gets the dynamic response of hours worked to that shock exactly wrong. With this sort of provocative conclusion, it is not surprising that Galí has stirred up a great deal of controversy.

The debate between Galí and his critics continues.[21] However, a consensus seems to be emerging concerning one key question: the fraction of business-cycle variance due to technology shocks is less—perhaps even substantially so—than the estimate of 70 percent suggested by Prescott (1986). There is less agreement over whether hours worked rise or fall after a positive technology shock. Still, a very interesting hypothesis has been suggested by Galí *et al.* (2003). They noticed that whatever VAR evidence there is that hours worked fall after a positive technology shock seems to come from the pre-1980s part of the US data. The post-1980s part of the sample suggests that hours worked rise after a positive technology shock. Galí *et al.* suggested that there is a connection between this switch in the response of hours worked and the change in money policy that many believe occurred in the early 1980s. Galí *et al.* hypothesized that the fall in hours worked before 1980 is the outcome of the suboptimal monetary policy in place at the time, while the rise in hours worked after 1980 reflects the switch to a more nearly optimal monetary policy. We do not know how well this hypothesis will stand the test of time. If it does, we will have learned a great detail about the nature of the monetary transmission mechanism. It is an impressive demonstration of what can be learned when creative researchers mix VAR analysis with good economic reasoning.

[20] Interestingly, Blanchard and Quah (1989) and Shapiro and Watson (1988) have obtained similar results. The former have reported that unemployment rises in the immediate aftermath of a positive supply shock, while the latter have shown that hours worked fall after a positive supply shock. However, using more recent data, Christiano *et al.* (2010a) have reported VAR-based evidence in which employment rises and unemployment falls in the aftermath of an expansionary technology shock.

[21] Christiano *et al.* (2003, 2004) have disputed the finding by Galí that hours worked fall based on statistical grounds. Francis and Ramey (2005) have disputed Galí's inference that RBC models must be abandoned in favor of sticky price, monetary models driven by demand shocks. Citing work in, for example, Boldrin *et al.* (2001), they have argued that it is straightforward to construct RBC models in which hours worked fall in the wake of a positive technology shock.

VARs have also played a central role in the huge literature that seeks to shed light on how government spending shocks propagate through the economy. An understanding of how investment, consumption, real wages, etc., respond to fiscal shocks provides valuable information for discriminating between alternative models. Rotemberg and Woodford (1992) were the first to draw attention to this fact. They noted that, according to standard neoclassical theory, a positive government spending shock produces a negative wealth effect. As a result, under normal assumptions about utility, labor supply is predicted to increase, and consumption to decrease. The former implies that the real wage should fall. Rotemberg and Woodford presented a VAR analysis which suggests that the real wage actually increases in the wake of a positive government spending shock. They argued that this evidence is an embarrassment to the neoclassical model, and that we should redirect our attention towards a model with imperfect competition. With imperfect competition, a rise in government spending can result in a reduction in price mark-ups. A decline in these has the effect of increasing labor demand, with the potential that the real wage could rise in equilibrium. The details of the conclusion of Rotemberg and Woodford have not held up under scrutiny. Several authors (see Ramey and Shapiro, 1998; Edelberg *et al.*, 1999) pointed out that the results of Rotemberg and Woodford are not robust to alternative measures of the real wage. Using what they regard as the most suitable measure of the real wage, they argue that the data are, in fact, consistent with the predictions of the basic neoclassical growth model. Also, Ramey and Shapiro showed that a two-sector variant of the neoclassical growth model can account for the fact that the results of Rotemberg and Woodford are not robust to alternative measures of real wages.

Recently, Blanchard and Perotti (2002) and Galí *et al.* (2007) argued that according to VAR-based impulse response functions, consumption rises after a positive shock to government spending not financed by a current rise in taxes. This is consistent with the IS–LM analysis of undergraduate macroeconomic textbooks, which emphasizes the dependence of consumption on current disposable income. However, it is inconsistent with the neoclassical growth model, which implies that consumption falls as higher anticipated taxes generate a negative wealth effect. The basic empirical results are still under dispute, because they are inconsistent with those of Ramey and Shapiro (1998) and Edelberg *et al.* (1999), which favor the neoclassical model. So, an important task is to understand the reasons for the different outcomes of the VAR analyses of Blanchard and Perotti (2002) and Galí *et al.* (2007) on the one hand, and Ramey and Shapiro (1998) and Edelberg *et al.* (1999), on the other. An important argument that supports the neoclassical model view has just been presented in Ramey (2011). Though the argument is a powerful one, presumably the debate over the effects of government spending will continue. Whatever the outcome of the discussion, the role of VARs in this literature has been profound. The analysis of government spending in VARs is an example of what Sims had in mind in Sims (1980a), when he suggested that VARs could serve as a useful battleground for testing alternative theories.

IV. Vector Autoregressions and Policy Analysis

This part of the proposal by Sims (1980a) has been developed the least. In Sims (1980a) and in other places, Sims has argued that despite the relative absence of structure in a VAR, it can nevertheless be used to investigate the type of policy question routinely asked at central banks. A typical question posed is “what will happen if the Federal Funds rate is raised by 25 basis points from its current level, and kept there for two years?” Sims has argued that this policy question can be answered in the following way. In practice, one of the equations in the VAR in equation (1) can be interpreted as a monetary policy rule, and the corresponding element in ε_t is a monetary policy shock.[22] The analysis draws many realizations of ε_t, each extending over two years. The non-monetary policy shocks are drawn from a multivariate random number generator and, conditional on these and on the simulated data, the monetary policy shocks are computed to ensure that the Federal Funds rate is higher than the current value by 25 basis points. This exercise delivers a distribution for all the variables in the VAR, conditional on the event of interest and on the proposition that that event is brought about by the exercise of monetary policy.

This strategy for evaluating policy has been implemented at a variety of Federal Reserve banks, and it is incorporated into the widely distributed VAR software RATS. Nevertheless, it remains controversial. For example, it has been argued that the procedure violates the Lucas critique, because over the sample in which the VAR was estimated, the monetary policy shock was assumed to be drawn from a random number generator. However, in the policy experiment, the monetary policy shock is chosen in a very different way. Critics suspect that this shift in the way of drawing the shocks would—if viewed from the perspective of a fully specified model—cause $B_1, \ldots, B_p$ to change values. Moreover, without a fully structural model, according to the critics, it is not possible to predict exactly how these objects will change value. This criticism has never been fully resolved, and so there remains considerable skepticism over this aspect of Sims’ proposal.

[22] Implicitly, I am speaking of an alternative representation of equation (1) in which the equation is pre-multiplied on both sides by C^{-1}. For our purposes here, it is not necessary for me to explicitly develop this alternative representation.

V. Empirical Macroeconomics

It is useful to examine VARs from the perspective of empirical macroeconomics more broadly. In the following discussion, I first relate VARs to structural estimation of DSGE models using full information methods. I then discuss the relation between the generalized method moments (GMM) procedures proposed by Hansen (1982) and VARs.

In work that began in the 1970s, Thomas Sargent explored full information econometric methods for estimating fully articulated DSGE models (see Sargent, 1978). Later, this work was continued with Lars Hansen (e.g., Hansen and Sargent, 1980). In another important paper, Sargent (1989) presented an ingenious way of addressing the fact that almost all economic data are measured with error. Also, Hansen and Sargent (1983) developed methods for estimating models formulated in continuous time, using sampled, time-averaged data, while Hansen and Sargent (1993) presented an important analysis of the impact of seasonal adjustment on the maximum likelihood estimation of DSGE models.

These contributions complement the VAR work carried out by Sims. For example, diagnostic procedures are required for evaluating a model that has been estimated by full information methods. Comparing model impulse responses and the impulse responses implied by VARs is one useful diagnostic. In addition, Sims (1989) and others have shown how VARs can form the basis of a method for estimating models that is an alternative to full information methods.

The contributions that Sims has made to inference about DSGE models go beyond his work with VARs. For example, he was the key protagonist in a revolutionary development in macroeconometrics in recent years. Before 2000, classical methods of inference were essentially universal in macroeconomics, and these have now been replaced by Bayesian methods.

The body of literature in which fully specified DSGE models are estimated has become vast in recent years. A consensus is forming around the New Keynesian model and VARs played a key role in this development. Because this model incorporates price- and wage-setting frictions, it implies that markets do not work perfectly. Monetary and fiscal policies, if designed properly, have the potential to play important and positive roles in the economy. This class of DSGE models has attracted so much interest that New Keynesian models are now actively under construction in central banks around the world. Formerly, DSGE models were the province of a relatively small group of university academics. Two factors brought them to the notice of practical policy-makers. First, the New Keynesian models have been shown to be capable of reproducing key impulse responses that had been estimated using VARs.[23] Second, New Keynesian DSGE models have

[23] Some of this work is summarized in Christiano *et al.* (2010b).

been shown to be able forecast as well or better than VARs (e.g., Smets and Wouters, 2003). Both factors have elevated New Keynesian models from the status of "toys for academics" to serious tools. Both factors have involved the use of VARs, either directly or indirectly.

I now turn to GMM. In the 1980s and 1990s, GMM had a major substantive impact on empirical macroeconomics. These procedures were used to test particular implications of DSGE models, such as the orthogonality properties of Euler equation errors. Other tests involved comparing a model's implications for various statistics with their empirical analogs. In one set of applications the statistics are second moments (e.g., Christiano and Eichenbaum, 1992; Smith, 1993). In another set of applications, the statistics are impulse reponse functions computed from VARs (e.g., Christiano *et al.* (2010b)).

In recent years, the application of GMM in macroeconomics has begun to wane somewhat, as Bayesian methods of inference have taken the place of classical methods. However, because methods for doing GMM from a Bayesian perspective are being developed, we can expect the use of GMM procedures to resume.[24]

VI. Conclusion

Over 30 years ago, Sims proposed that macroeconomists should include VARs in their kit of econometric tools. He argued that VARs would provide a fruitful statistical framework for comparing alternative models. This view has been largely borne out over the years.[25]

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[24] For a Bayesian version of a GMM moment-matching procedure, see Christiano *et al.* (2010b).

[25] A number of very interesting papers have not been covered. Beaudry and Portier (2006) is one that comes to mind.

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