Quantifying Interest Rate Risk

0. Overview

- Examples

- Price and Yield

- Duration

- Risk Management

- Convexity

- Value-at-Risk

- Active Investment Strategies

1. Examples

- Example 1: Bell Atlantic

    – Long or short (floating rate) debt?

    – Level and slope of yield curve

    – Liquidity management

    – Risk exposure

    – Accounting methods

1. Examples (continued)

- Example 2: Banc One

    - Measurement of interest-sensitivity

    - Regulatory reporting

    - Liquidity management

    - Risk management and financial engineering

    - Accounting methods

1. Examples (continued)

- Example 3: Salomon Smith Barney Proprietary Trading

  - Making money:

    * Spotting good deals

    * Taking calculated risks

  - Risk assessment

    * How much risk does the firm face?

    * How much risk does a specific trader or position add?

  - Risk management

    * Should some or all of the risk be hedged?

1. Examples (continued)

- Example 4: Bond Funds

  - Mission:

    * Target specific markets?

    * Target specific maturities?

    * Index or active investing?

  - Risk reporting to customers

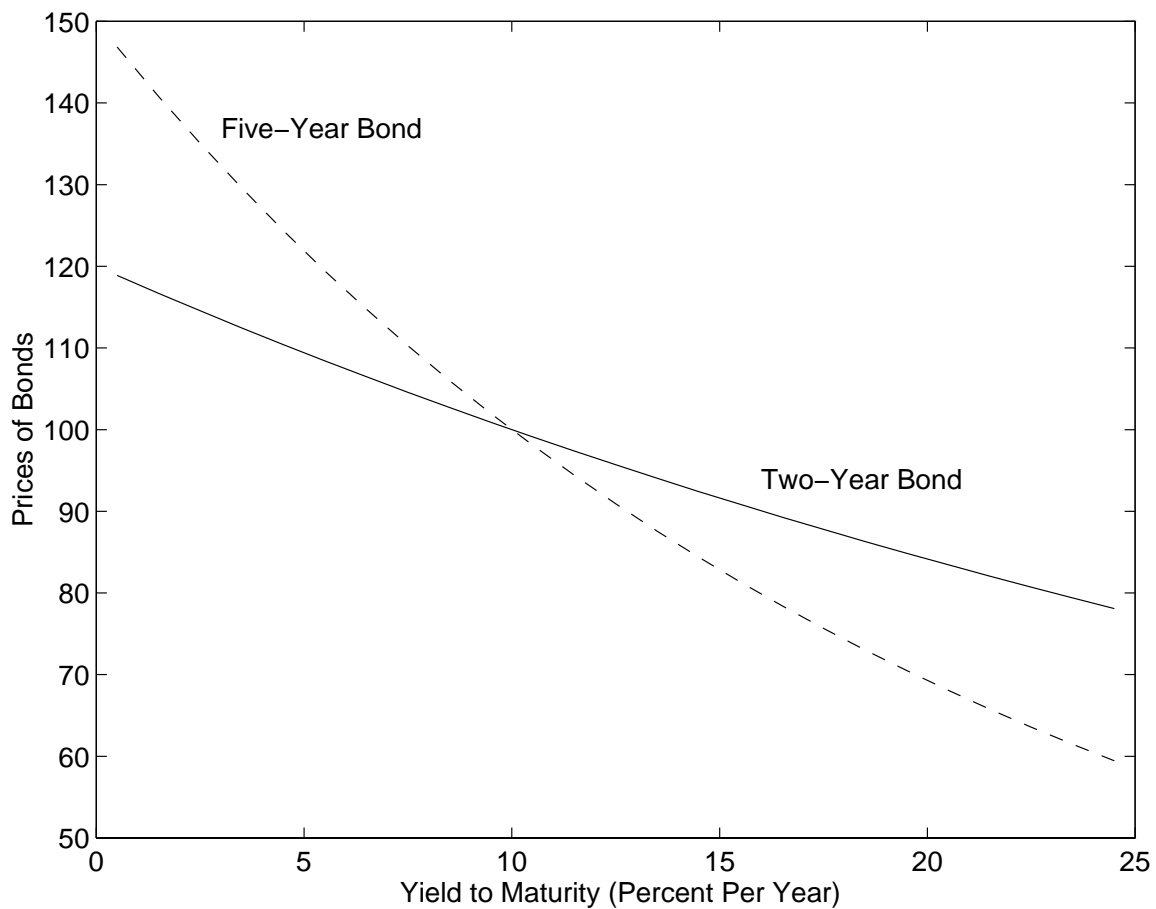 - Risk management

1. Examples (continued)

- Example 5: Dedicated Portfolios

    - Purpose: fund fixed liabilities

    - Example: defined-benefit pensions

    - Objective: minimize cost

    - Approach tied to accounting of liabilities

2. Price and Yield

- Price and yield are inversely related:

$$p(y) \; = \; \frac{c_1}{(1 + y/2)} + \frac{c_2}{(1 + y/2)^2} + \cdots + \frac{c_n}{(1 + y/2)^n}$$

- Long bonds are more sensitive to yield changes than short bonds

3. DV01

- Our task is to produce numbers that quantify our sense that the price-yield relation is "steeper" for long bonds

- Measure 1 is the DV01: "Dollar Value of an 01"
  (aka Present Value of a Basis Point or PVBP)

- Definition: DV01 is the decline in price associated with a one basis point increase in yield:

$$DV01 = -\text{ Slope of Price-Yield Relation } \times \ 0.01\%$$
$$= -\frac{dp}{dy} \times \ 0.0001$$

- Calculation:
  (a) compute yield associated with (invoice) price
  (b) compute price associated with yield plus 0.01%
  (c) DV01 is difference between prices in (a) and (b)

3. DV01 (continued)

- Example 1 (2-year 10% bond, spot rates at 10%)

    - Initial values: $p = 100$, $y = 0.1000$
    - At $y = 0.1001$ (one bp higher), $p = 99.9823$
    - DV01 $= 100 - 99.9823 = 0.0177$

- Example 2 (5-year 10% bond, spot rates at 10%)

    - Initial values: $p = 100$, $y = 0.1000$
    - At $y = 0.1001$ (one bp higher), $p = 99.9614$
    - DV01 $= 100 - 99.9614 = 0.0386$
    - More sensitive than the 2-year bond

- Example 3 (2-year zero, spot rates at 10%)

    - Initial price: $p = 100/1.05^4 = 82.2702$
    - At $y = 0.1001$ (one bp higher), $p = 82.2546$
    - DV01 $= 82.2702 - 82.2546 = 0.0157$

- Example 4 (10-year zero, spot rates at 10%)

    - Initial price: $p = 100/1.05^{20} = 37.6889$
    - DV01 $= 37.6889 - 37.6531 = 0.0359$

3. DV01 (continued)

- What you need to reproduce Bloomberg's calculation

- Price-yield relation with fractional first period:

$$p(y) = \frac{c_1}{(1+y/2)^w} + \frac{c_2}{(1+y/2)^{w+1}} + \cdots + \frac{c_n}{(1+y/2)^{w+n-1}}$$

- Example 5 (Citicorp 7 1/8s revisited)

  Recall: settlement 6/16/95, matures 3/15/04,
  $n = 18$, p (invoice price) $= 103.056$, $w = 0.494$

  - Compute yield as before: $y = 6.929\%$
  - Price at $y = 6.939\%$ (+1 bp) is $p = 102.991$
  - DV01 $= 103.056 - 102.991 = 0.065$

3. DV01 (continued)

- Similar methods work for portfolios:

$$v = x_1 p_1(y_1) + x_2 p_2(y_2) + \cdots + x_m p_m(y_m)$$

$$= \sum_{j=1}^{m} x_j p_j(y_j)$$

- The DV01 of a portfolio is the sum of the DV01's of the individual positions:

$$\frac{dv}{dy} \times 0.0001 = \sum_{j=1}^{m} x_j \frac{dp_j}{dy_j} \times 0.0001$$

$$\text{DV01 of Portfolio} = \text{Sum of DV01's of Positions}$$

(Mathematically: the derivative of the sum is the sum of the derivatives. The math isn't as precise as it looks, since the yields may differ across securities. The idea is that we raise all of them by one basis point.)

- Example 6: one 2-year bond and three 5-year bonds (examples 1 and 2)

$$\text{DV01} = 1 \times 0.0177 + 3 \times 0.0386 = 0.1335$$

4. Duration

- Measure 2 of interest-sensitivity: duration

- Definition: duration is the *percentage* decline in price associated with a one percent increase in the yield:

$$D = -\frac{\text{Slope of Price-Yield Relation}}{\text{Price}}$$

$$= -\frac{dp/dy}{p}$$

- Formula (with semiannual compounding):

$$D \equiv \frac{dp/dy}{p} = -(1 + y/2)^{-1} \sum_{j=1}^{n} (j/2) \times w_j$$

with

$$w_j = \frac{(1 + y/2)^{-j} c_j}{p}.$$

- Remarks:

    - Weight w_j is fraction of value due to the jth payment
    - Sum is weighted average life of payments

4. Duration (continued)

- Example 1 (2-year 10% bond, spot rates at 10%)

  Intermediate calculations:

Payment (j)	Cash Flow (c_j)	Value	Weight (w_j)
1	5	4.762	0.04762
2	5	4.535	0.04535
3	5	4.319	0.04319
4	105	86.384	0.86384

  Duration:

  $$\begin{aligned} D &= (1 + .10/2)^{-1}(0.5 \times 0.04762 + 1.0 \times 0.04535 \\ &\quad + 1.5 \times 0.04319 + 2.0 \times 0.86384) \\ &= 1.77 \text{ years} \end{aligned}$$

  If y rises 100 basis points, price falls 1.77%.

4. Duration (continued)

- Example 2 (5-year 10% bond, spot rates at 10%)
  $D = 3.86$.

- Example 3 (2-year zero, spot rates at 10%)
  Duration for an n-period zero is

$$D = (1 + .10/2)^{-1}(n/2)$$

Note the connection between duration and maturity.
Here $n = 4$ and $D = 1.90$.

- Example 4 (10-year zero, spot rates at 10%)
  $D = (1 + .10/2)^{-1}(20/2) = 9.51$.

4. Duration (continued)

- What you need to reproduce Bloomberg's calculation

- Duration formula becomes (semiannual compounding)

$$D \equiv \frac{dp/dy}{p} = -(1+y/2)^{-1} \sum_{j=1}^{n} [(w+j-1)/2] \times w_j$$

with

$$w_j = \frac{(1+y/2)^{-w-j+1} c_j}{p}.$$

- Example 5 (Citicorp 7 1/8s revisited again)

  Recall: settlement 6/16/95, matures 3/15/04, $n = 18$, p (invoice price) $= 103.056$, $w = 0.494$, $y = 6.929\%$

  Answer: $D = 6.338$ years (less than maturity of 8.75 years)

- Remark: there's a formula for D, but it's ugly

4. Duration (continued)

- Similar methods work for portfolios:

$$v = x_1 p_1(y_1) + x_2 p_2(y_2) + \cdots + x_m p_m(y_m)$$

$$= \sum_{j=1}^{m} x_j p_j(y_j)$$

- The duration of the portfolio is the value-weighted average of the durations of the individual positions:

$$\frac{dv/dy}{v} = \sum_{j=1}^{m} x_j \frac{dp_j/dy_j}{v}$$

$$= \sum_{j=1}^{m} w_j \frac{dp_j/dy_j}{p_j}$$

with

$$w_j = \frac{x_j p_j}{v}$$

(As with the DV01, the math isn't as precise as it looks, since the yields may differ across securities. The idea is that we raise all of them by the same amount.)

- Remark: weight w_j is fraction of value due to the jth position

4. Duration (continued)

- Example 6: one 2-year bond and three 5-year bonds (examples 1 and 2)

  – Since prices are equal, value weights are 1/4 and 3/4
  – $D = 0.25 \times 1.77 + 0.75 \times 3.86 = 3.38$

- Example 7: combination of 2- and 10-year zeros with duration equal to the 5-year par bond

  – This kind of position is known as a barbell, since the cash flows have two widely spaced lumps (picture a histogram of the cash flows)
  – If we invest fraction w in the 2-year, the duration is

  $$D = w \times 1.90 + (1 - w) \times 9.51 = 3.86$$

  Answer: $w = 0.742$.

4. Duration (continued)

Duration comes in many flavors:

- Our definition is generally called "modified duration"

- The textbook standard is Macaulay's duration
    - Differs from ours in lacking the $(1 + y/2)^{-1}$ term:
$$D \; = \; - \sum_{j=1}^{n} (j/2) \times w_j$$
    - Leads to a closer link between duration and maturity (for zeros, they're the same)
    - Nevertheless, duration is a measure of sensitivity; its link with maturity is interesting but incidental
    - Frederick Macaulay studied bonds in the 1930s

- Fisher-Weil duration: compute weights with spot rates
    - Makes a lot of sense
    - Used in many risk-management systems (RiskMetrics, for example)
    - Rarely makes much difference with bonds

4. Duration (continued)

- Bottom line: duration is an approximation (ditto DV01)
  - Based on parallel shifts of the yield curve
    (presumes all yields change the same amount)
  - Holds over short time intervals
    (otherwise maturity and the price-yield relation change)
  - Holds for small yield changes:

$$\frac{dp}{dy} = -D \times p$$

$$\Rightarrow \quad p - p_0 \cong -D \times p_0 \times (y - y_0)$$
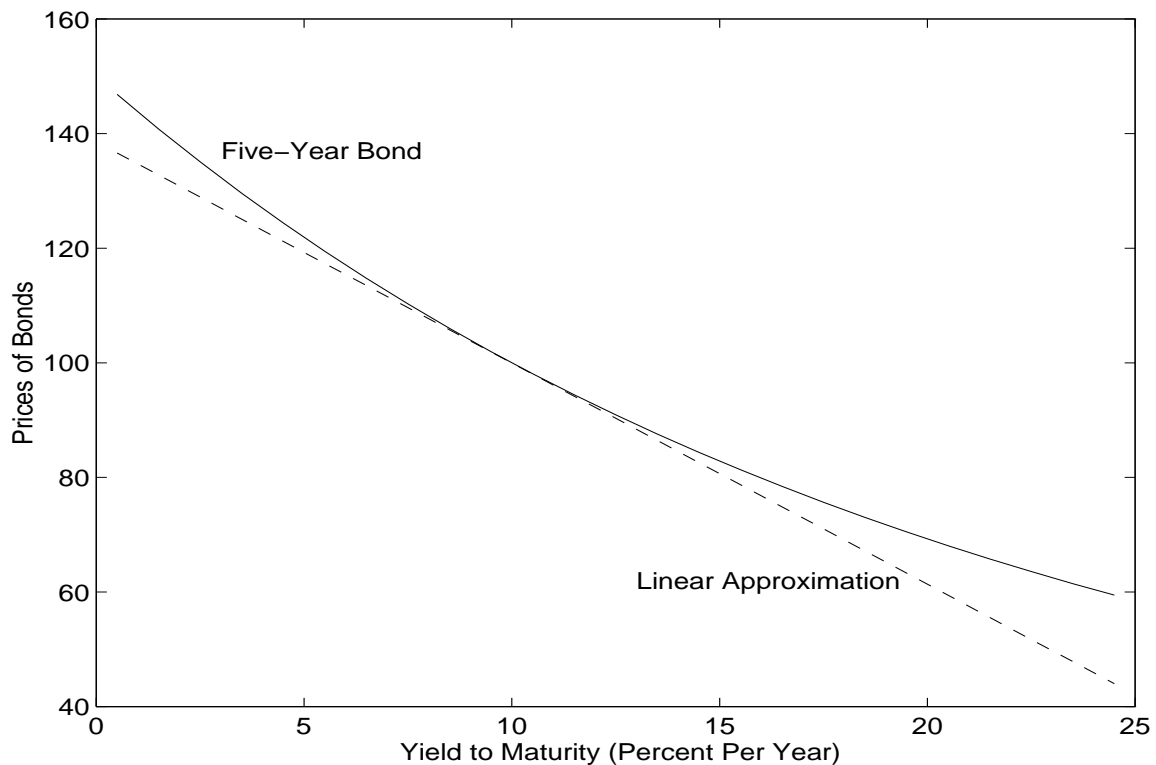
5. Convexity

- Convexity measures curvature in the price-yield relation

- Common usage: "callable bonds have negative convexity" (the price-yield relation is concave to the origin)

- Definition (semiannual, full first period):

$$C \equiv \frac{d^2p/dy^2}{p}$$

$$= (1 + y/2)^{-2} \sum_{j=1}^{n} [j(j+1)/4] \times w_j$$

with

$$w_j = \frac{(1 + y/2)^{-j} c_j}{p}$$

- Example 1 (2-year 10% bond, spot rates at 10%) $C = 4.12$.

- Example 3 (2-year zero, spot rates at 10%) Convexity for an n-period zero is

$$C = (1 + .10/2)^{-2}[n(n+1)/4],$$

or 4.53 when $n = 4$.

5. Convexity (continued)

- Bloomberg calculations (fractional first period w)

$$C \ = \ (1+y/2)^{-2} \sum_{j=1}^{n} [(j-1+w)(j+w)/4] \times w_j$$

with
$$w_j \ = \ \frac{(1+y/2)^{-j+1-w} c_j}{p}$$

(Bloomberg divides this number by 100.)

- Example 5 (Citicorp 7 1/8s again)

Answer: $C = 51.3$ (0.513 on Bloomberg)

6. Statistical Measures of Interest Sensitivity

- Measure risk with standard deviations and correlations of price changes

- Fixed income applications

  - Standard deviations and correlations of yield changes
  - Use duration to translate into price changes

- Statistical properties of monthly yield changes

	1-Year	3-Year	5-Year	7-Year	10-Year
A. Standard Deviations of Yield Changes (Percent)					
	0.547	0.441	0.382	0.340	0.309
B. Correlations of Yield Changes					
1-Year	1.000	0.920	0.858	0.800	0.743
3-Year		1.000	0.967	0.923	0.867
5-Year			1.000	0.980	0.930
7-Year				1.000	0.970

Source: McCulloch and Kwon, US treasuries, 1952-91.
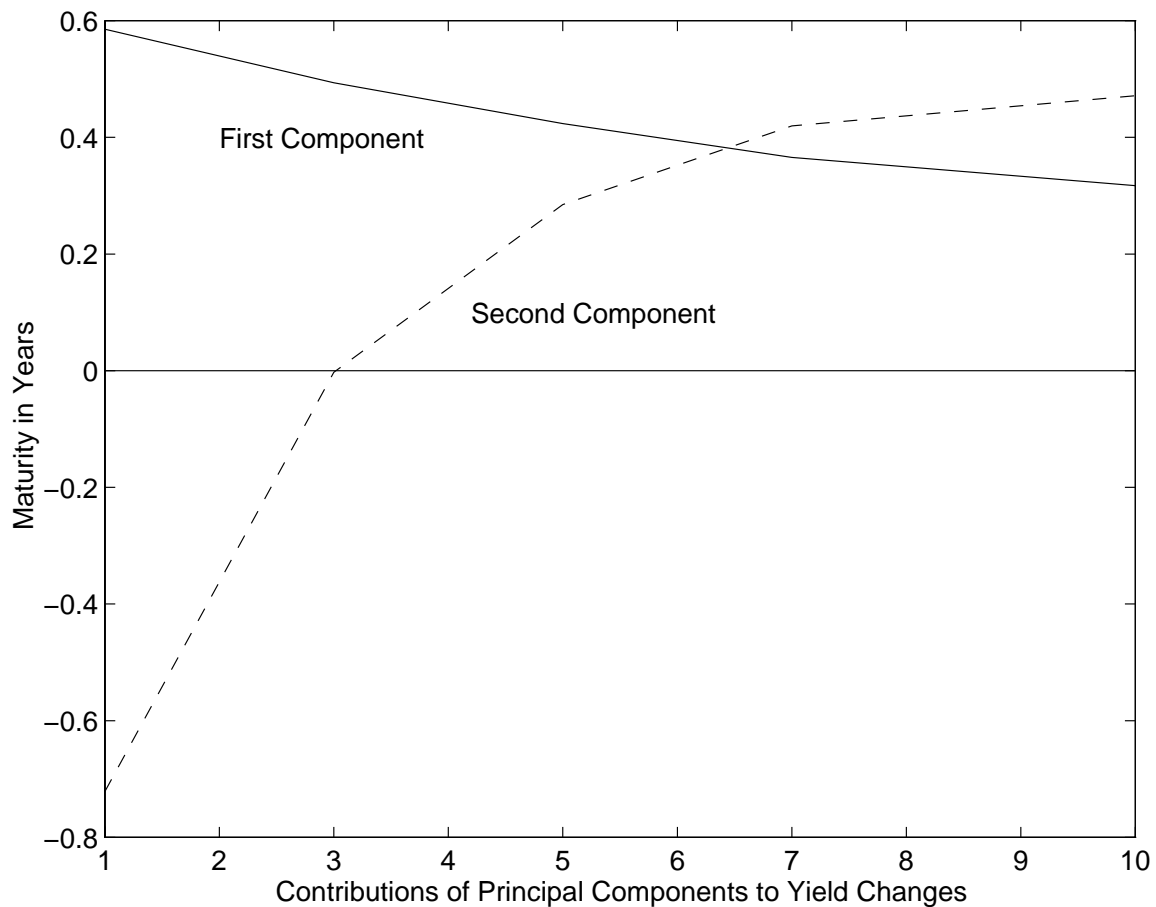
6. Statistical Measures (continued)

- Examples of yield curve shifts (spot rates)
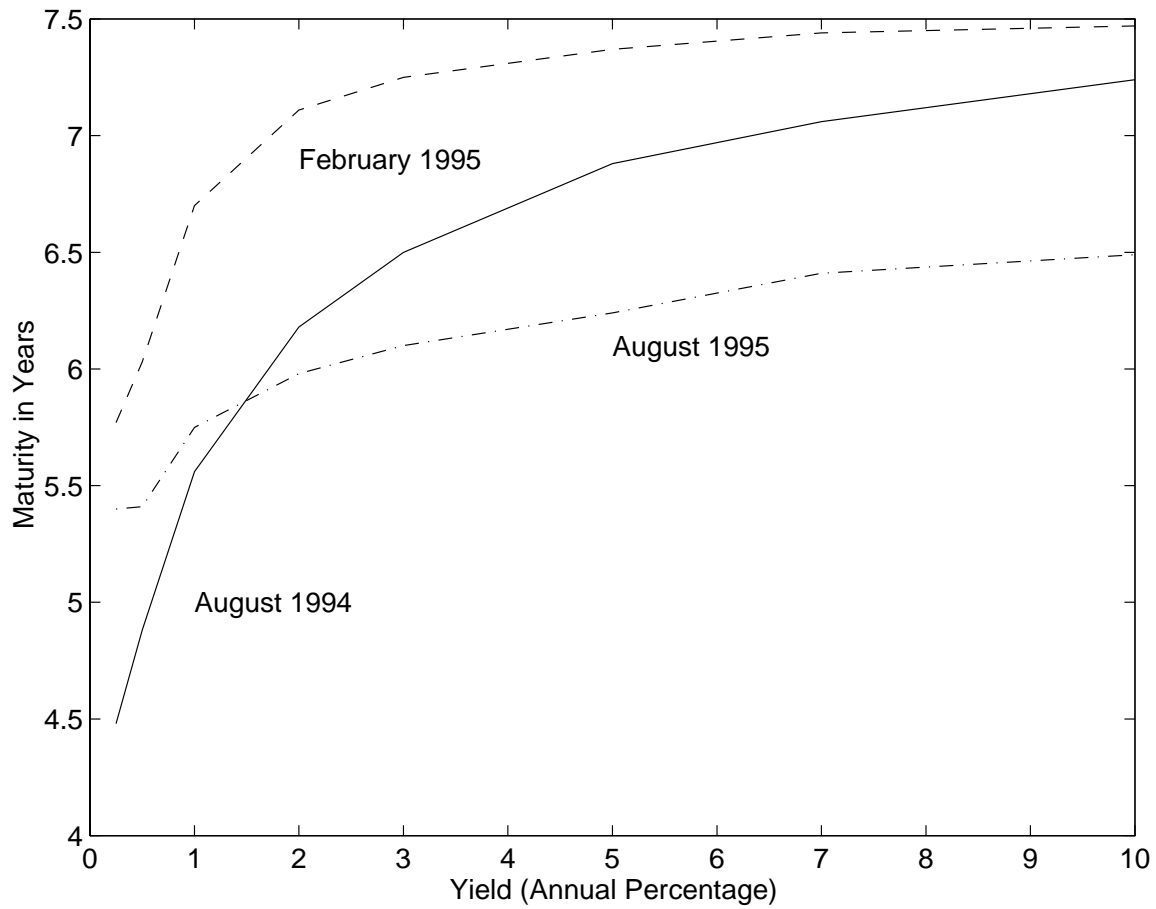
- Remark: not just parallel shifts!

6. Statistical Measures (continued)

- Typical components of yield curve shifts

- Remarks:

    - Component 1: Roughly parallel, but less at long end
    - Component 2: Twist accounts for 10-15% of variance
    - Both indicate weaknesses in duration (ditto DV01)

6. Statistical Measures (continued)

- Industry Practice: JP Morgan's RiskMetrics

- Daily estimates of standard deviations and correlations
  (This is critical: volatility varies dramatically over time)

- Twenty-plus countries, hundreds of markets

- Yield "volatilities" based on proportional changes:

$$\log\left(y_t/y_{t-1}\right) \approx \frac{y_t - y_{t-1}}{y_{t-1}}$$

- Maturities include 1 day; 1 week; 1, 3, and 6 months
  Others handled by interpolation

- Documentation available on the Web
  (useful but not simple)

- Similar methods in use at most major banks

7. Cash-Flow Matching

- Financial institutions typically face risk to the values of both assets and liabilities

- One approach is to buy assets whose cash flows exactly match those of the liabilities

- We say they are immunized: no change in interest rates can change the relative value of assets and liabilities

7. Cash-Flow Matching (continued)

- Example: Balduzzi Insurance and Pension Corporation

Liabilities: projected pensions of retired NYU employees

Time (Yrs)	Payment
0.5	2000
1.0	1900
1.5	1800
2.0	1700

Potential investments:

Bond	Coupon (%)	Mat (Yrs)	Price (Ask)	Yield (%)
A	0	0.5	96.618	7.000
B	0	1.0	92.456	8.000
C	0	1.5	87.630	9.000
D	0	2.0	83.856	9.000
E	4	1.0	96.750	7.432
F	8	2.0	97.500	9.400

Objective: find cheapest way to cover the cash flows

7. Cash-Flow Matching (continued)

- This is an example of *linear programming*
  (something you might see some day, but not from me)

- The problem can be laid out formally with a little effort

  - Bond i (a choice of A, B, C, D, E, or F)
    has cash flows c_{ti} (say) at time t
  - Consider a purchase of x_i units of each asset i
    Total cost is

  $$\text{Cost} \;=\; x_a p_a + x_b p_b + \cdots + x_f p_f \;=\; \sum_i x_i p_i$$

  These choices generate cash flows at date t of

  $$\text{Cash Flow at } t = \sum_i x_i c_{ti}$$

  - The problem is to choose quantities x_i that have the
    lowest costs yet cover the required cash flows (the pen-
    sion liabilities)
  - We may also disallow short sales ($x_i < 0$) or enter
    different bid and ask prices

7. Cash-Flow Matching (continued)

- Solution can be found using (say) Excel

- Guess 1: buy zeros in the appropriate amounts

$$\text{Cost} = 20 \times 96.618 + 19 \times 92.456 + 18 \times 87.630 + 17 \times 83.856$$
$$= 6691.91.$$

- Can we do better?

  - In a frictionless, arbitrage-free world: no!
    Cash flows have the same value regardless of the asset
  - Here: F looks cheap
    Buy as much as possible and make up the difference with zeros:

Bond	Price	Units
A	96.618	19.35
B	92.456	18.35
C	87.630	17.35
D	83.856	0.00
E	96.750	0.00
F	97.500	16.35

  Cost is 6679.19, which is cheaper.

7. Cash-Flow Matching (continued)

- Remarks:

  - This is an example of dedicated portfolio

  - Linear programs are often used to find portfolios

  - In practice, cheapest often leads to junk

  - Cash-flow matching exactly covers cash flows, no risk from interest rate movements

  - In theory, portfolio is static: "Head to the Bahamas and relax" (Elton)

8. Duration Matching

- Another approach to managing risk: match durations

- Example: Foresi Bank of Mahopac (FBM)
  Balance sheet

Assets	25.0
Liabilities	20.0
Shareholders' Equity	5.0

  Assets: duration $D_A = 1.00$
  Liabilities: 10.0 2-year notes with duration 1.77, 10.0 commercial paper with duration 0.48 $\Rightarrow D_L = 1.12$

- Should FBM change its liability mix?

  – Dollar sensitivity is

  $$\text{Change in Asset Value} = -D_A \times 25.0 \times \Delta y$$
  $$\text{Change in Asset Value} = -D_L \times 20.0 \times \Delta y$$

  – If we want to equate the two, we need

  $$D_L = D_A \times 25/20 = 1.25$$

  – Consider a change to fraction w in 2-year notes:

  $$D_L = w \times 0.48 + (1 - w) \times 1.77$$

  Answer: $w = 0.40$.

8. Duration Matching (continued)

- Remarks:

  - Less restrictive than cash-flow matching

  - Not bullet-proof against non-parallel shifts in yields

  - Durations change as maturities and yields change: strategy requires dynamic updating (no Bahamas)

9. Active Investment Strategies

- Two basic strategies:

  - Bet on the level of yields
  - Bet on the shape of the yield curve (yield spreads)

- Betting on yields

  - If you expect yields to fall, shorten duration
  - If you expect yields to rise, lengthen duration
  - Modications typically made with treasuries or futures (lower transactions costs than, say, corporates)
  - Forecasting yield changes is notoriously difficult
  - Requires a combination of analytics, macroeconomics, and psychology of traders and policymakers
  - Henry Kaufman made two great calls in the 1980s

9. Active Investment Strategies (continued)

- Betting on yield spreads

  - Scenario: Spot rates flat at 10%
  - We expect the yield curve to steepen , but have no view on the level of yields. Specifically, we expect the 10-year spot rate to rise relative to the 2-year.
  - Strategy: buy the 2-year, short the 10-year, in proportions with no exposure to overall yield changes

- Using DV01 to construct a spread trade:

  - We showed earlier that a one basis point rise in yield reduces the price of the 2-year zero by 0.0157 and the 10-year zero by 0.0359 (examples 3 and 4).
  - Intuitively, we buy more of the 2-year than we sell of the 10-year:

  $$\frac{x_2}{x_{10}} = -\frac{0.0359}{0.0157} = -2.29$$

  (the minus sign tells us one is a short position)
  - Mathematically:

  $$\Delta v \times 0.0001 = (x_2 \times \text{DV01}_2 + x_{10} \times \text{DV01}_{10}) \, \Delta y,$$
  $$= 0$$

  for equal changes in yields of both maturities.

9. Active Investment Strategies (continued)

- Using duration to construct a spread trade

  - The only difference is that duration measures *percentage* price sensitivity, and to eliminate exposure to parallel yield shifts we need to construct a portfolio with zero *dollar* sensitivity.

  - The durations of the 2-year and 10-year zeros are 1.90 and 9.51.

  - Dollar sensitivity is duration times price

  - To eliminate overall price sensitivity, set

$$\Delta v = (x_2\, p_2\, D_2 + x_{10}\, p_{10}\, D_{10})\, \Delta y,$$
$$= 0,$$

  which implies

$$\frac{x_2}{x_{10}} = -\frac{p_{10}D_{10}}{p_2 D_2} = -\frac{37.69 \times 9.51}{82.27 \times 1.90} = -2.29$$

- Same answer: both DV01 and duration are based on the slope of the price-yield relation.

10. Interview with Henry Kaufman

- Biographical sketch

  - Started Salomon's bond research group

  - Legendary interest rate forecaster

  - Picked rate rise in 1981, when most thought the "Volcker shock" of 1979 was over

  - Now runs boutique

- Investment strategy

  - Study macroeconomic fundamentals (chart room)

  - Adjust duration depending on view

  - Adjustment through sale and purchase of treasuries

  - Some foreign bonds

  - No derivatives (except occasional currency hedge)

- Thoughts on modern risk analytics

  - What's "market value"?

  - RiskMetrics-like methods look backwards, but history can be a poor guide to the future in financial markets

Summary

- Bond prices fall when yields rise

- Prices of long bonds fall more

- DV01 and duration measure sensitivity to generalized changes in yields — parallel shifts

- Cash-flow matching eliminates all exposure to interest rate movements

- Duration matching eliminates exposure to equal movements in yields of different maturities

- Statistical risk measures allow for different yield changes across the maturity spectrum

- Active investment strategies include bets on the level and slope of the yield curve