Misspecified Recovery

by Jarda Borovicka, Lars Hansen, & Jose Scheinkman

Discussion by Dave Backus

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Ross's recovery theorem

Ross, JF, 2015

> *State prices are the product of risk aversion — the pricing kernel — and the natural probability distribution. The Recovery Theorem enables us to separate them.*

Colleague's textbook

> *Ross shows how to disentangle the probability of future market movements from the degree of risk aversion in the market.*

Recovery logic 1

Environment: Markov chain with states $i, j = 1, \ldots, n$

Premise: We observe state prices Q_{ij}

Can we extract probabilities p and pricing kernel m from Q?

$$\underbrace{Q_{ij}}_{n^2} \quad = \quad \underbrace{p_{ij}}_{n^2-n} \cdot \underbrace{m_{ij}}_{n^2}$$

What if $m_{ij} = \beta u'(c_j)/u'(c_i)$?

Is this plausible?

Recovery logic 2

Environment: Markov with state x

Premise: We observe state prices $Q(x_t, x_{t+1})$

Can we extract probabilities p and pricing kernel m from Q?

What if $m(x_t, x_{t+1}) = \beta v(x_t)/v(x_{t+1})$?

Where does this come from?

Is it plausible?

Alvarez-Jermann/Hansen-Scheinkman decomposition

Think Perron-Frobenius: look for positive $\nu, v(x)$ satisfying

$$E_t\big[m(x_t, x_{t+1})v(x_{t+1})\big] \quad = \quad \nu v(x_t)$$

Define permanent (martingale) and transitory components by

$$\begin{aligned} m^1(x_t, x_{t+1}) &= m(x_t, x_{t+1})v(x_{t+1})/[\nu v(x_t)] \\ m^2(x_t, x_{t+1}) &= \nu v(x_t)/v(x_{t+1}) \\ m(x_t, x_{t+1}) &= m^1(x_t, x_{t+1})m^2(x_t, x_{t+1}) \end{aligned}$$

What if $m^1 = 1$? Then $m(x_t, x_{t+1}) = \nu v(x_t)/v(x_{t+1})$

Is this plausible?

Recovery logic 3

Decomposition implies $E_t[m^1(x_t, x_{t+1})] = 1$

Define new probabilities by $\widetilde{p} = p \cdot m^1$

Value arbitrary cash flow $d(x_{t+1})$ by

$$
\begin{aligned}
q(x_t) &= E_t\left[m(x_t, x_{t+1})d(x_{t+1})\right] \\
&= E_t\left[m^1(x_t, x_{t+1})m^2(x_t, x_{t+1})d(x_{t+1})\right] \\
&= \widetilde{E}_t\left[m^2(x_t, x_{t+1})d(x_{t+1})\right]
\end{aligned}
$$

Recovery theorem recovers $\widetilde{p}$, not p: equal iff $m^1 = 1$

Recovery assessment 1 (AJ proposition)

When are $\widetilde{p}$ and p similar?

Practical issue, not pure theory

Proposition (Alvarez-Jermann). If $m^1 = 1$, then $\widetilde{p} = p$ and the asset with the highest expected (log) return is the long bond.

Evidence: lots of assets have higher mean returns than long bonds

Recovery assessment 2 (iid example)

Pricing kernel

$$\begin{aligned} \log m_{t,t+1} &= \log \beta + \lambda w_{t+1} \\ \{w_t\} &\sim \text{NID}(0,1) \end{aligned}$$

Decomposition

$$\begin{aligned} \log v(x_t) &= 0 \\ \log \nu &= \log \beta + \lambda^2/2 \\ \log m^1(x_t, x_{t+1}) &= -\lambda^2/2 + \lambda w_{t+1} \\ \log m^2(x_t, x_{t+1}) &= \log \nu \end{aligned}$$

Complete recovery failure, m^2 is constant: (im)plausible?

Recovery assessment 3 (Vasicek)

We have lots of estimated pricing kernels

Invariably $\widetilde{p}$ and p are wildly different

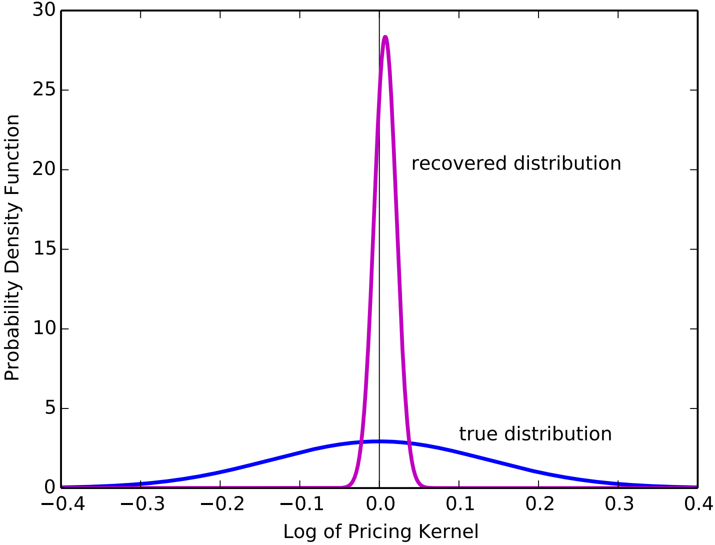
Example (Vasicek)

$$
\begin{aligned}
\log m_{t,t+1} &= \log \beta + x_t + \lambda w_{t+1} \\
x_{t+1} &= \varphi x_t + \sigma w_{t+1}
\end{aligned}
$$

Choose parameters to reproduce

- Mean, variance, and autocorrelation of short rate (time series)
- Mean 10-year bond spread (cross section)

Recovery assessment 3 (Vasicek)



Digression: long forward rates

Well-known that long forward rates are constant in many models (Vasicek, CIR, exponential-affine...)

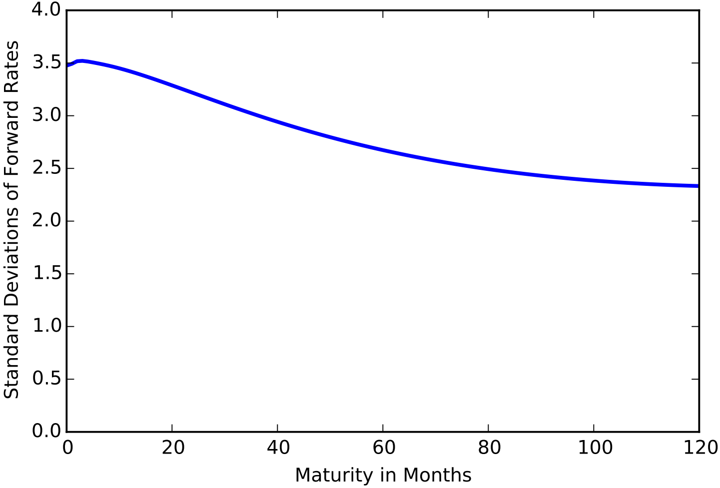
$$\lim_{n\to\infty} f^n(x) \quad = \quad f^\infty$$

AJ/HS clarify and generalize the logic: eg, $f^\infty = -\log \nu$

Evidence less than definitive

- Convergence (very) slow for nominal bonds
- Risk-adjusted persistence greater than true persistence?
- Long memory or unit root?

Standard deviations of forward rates, 1970-2015



Gurkaynak, Sack, and Wright, nominal US Treasuries

This paper

Recovery theorem: "It doesn't mean what you think it means"

Good entry point to AJ/HS literature

- Lots of interesting examples
- Insight into a growing literature on long-maturity assets

Read it!

Reading list (some of many)

Long forward rates, decomposition, and recovery

- Dybvig, Ingersoll, and Ross, JoB, 1996
- Alvarez and Jermann, Econometrica, 2005
- Hansen and Scheinkman, Econometrica, 2009
- Hansen, Econometrica, 2012
- Ross, JF, 2015
- Martin and Ross, ms, 2013

Estimated pricing kernels and returns over different horizons

- Ang and Piazzesi, JME, 2003
- Bansal and Yaron, JF, 2004
- Lettau and Wachter, JF, 2007
- Duffee, RFS, 2011
- Joslin, Singleton, and Zhou, RFS, 2011
- Binsbergen, Brandt, and Koijen, AER, 2012
- Koijen, Lustig, and Van Nieuwerburgh, ms, 2015

Extra slides

Recovery logic 1 (risk-adjusted version)

Environment: Markov chain with states $i, j = 1, \ldots, n$

Premise: We observe state prices Q_{ij}

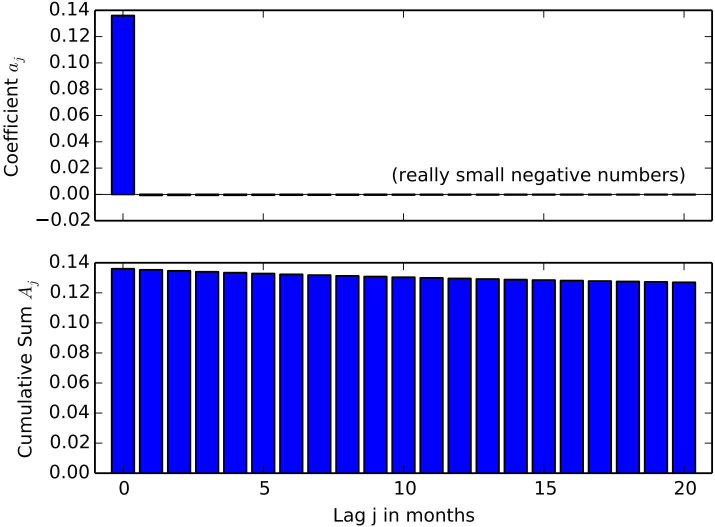
One-period bond prices are $b_i = \sum_j Q_{ij}$

Define risk-adjusted probabilities $p^*_{ij} = Q_{ij}/b_i$

Can we extract probabilities p and pricing kernel m from p^*?

$$\underbrace{b_i p^*_{ij}}_{n^2} \quad = \quad \underbrace{p_{ij}}_{n^2-n} \cdot \underbrace{m_{ij}}_{n^2}$$

Vasicek pricing kernel dynamics



Attachments

Attachments are imbedded in the pdf file of the slides.

Click on the pushpins in Adobe Reader or the equivalent.

Works in most pdf viewers, but not Preview on Macs.

Attachments

- Notes on the recovery theorem:
- Python code for Vasicek calculations: