Disaster Risk and Business Cycles

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Plan of attack

Gourio summary

The nature of business cycle risk

- Disasters implied by options
- Cyclical behavior of asset returns

Challenges posed by the model (and met by the paper)

- Computation
- Tallarini's result
- The Barro-King problem

Gourio revisited

Gourio summary

Business cycle model + “disasters” + recursive preferences

Disasters

- Large adverse shock to productivity — and capital
- Probability varies over time
- Magnified by recursive preferences

A disaster generates

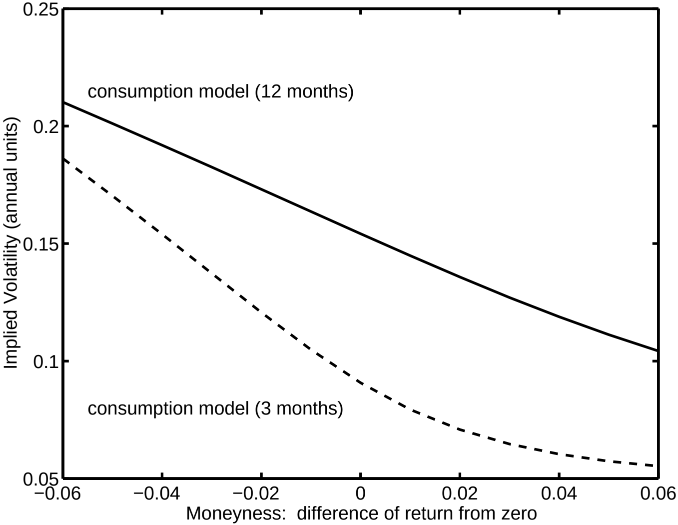
- Sharp declines in investment, consumption, output — and maybe employment

A rise in the disaster probability generates

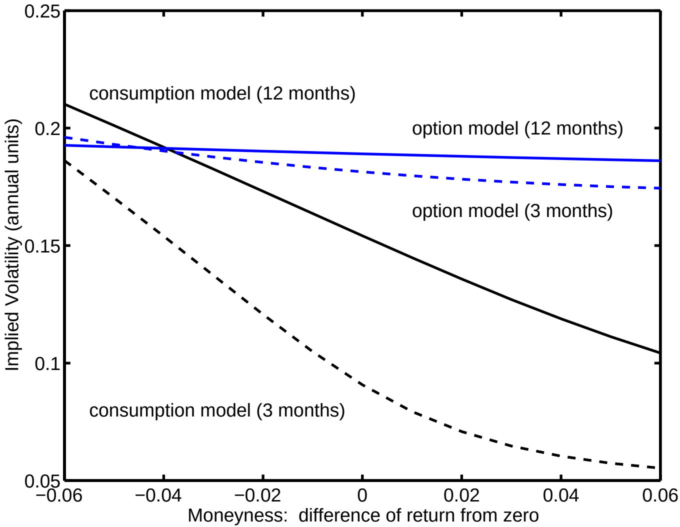
- Lower riskfree rate, higher risk premiums
- Lower investment, employment, output; higher consumption?

The nature of risk business cycle risk

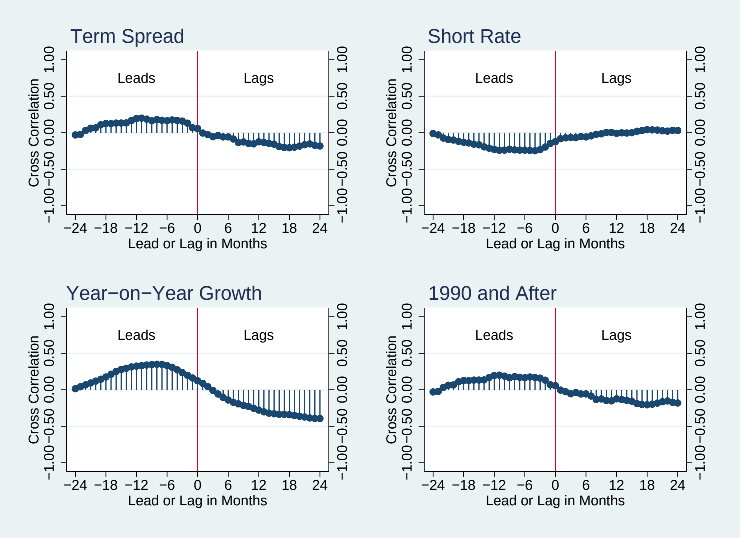
Disasters implied by options



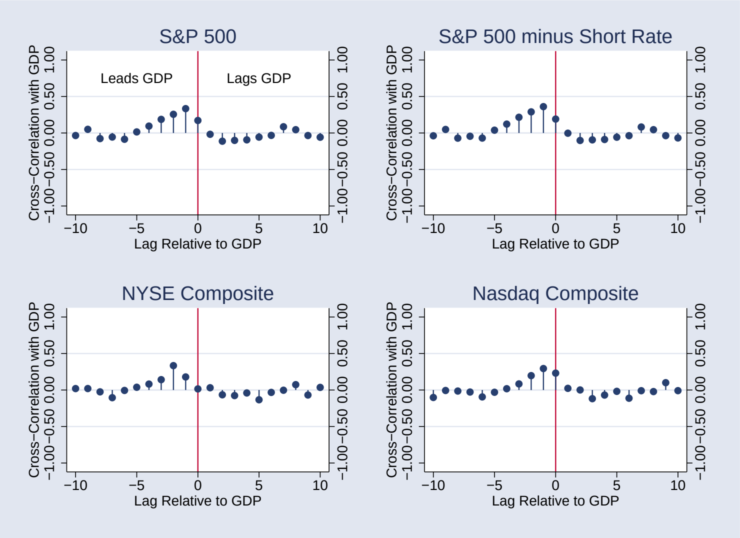
Disasters implied by options



Cyclical behavior of term spread



Cyclical behavior of equity returns



Challenges posed by the model

Computation

Problem: standard LQ methods independent of risk

Ditto (log)-linearization around "deterministic steady state"

Lots of alternatives

- Tallarini uses risk-sensitive control (IES $= 1$)
- Close relative: log-linear approximation of value function
- Fernandez-Villaverde et al., Justiano-Primiceri, Rudebusch-Swanson: perturbation
- Campanale, Castro, and Clementi: splines
- Bloom, Croce, Krueger-Kubler: discrete approximation of state space

Gourio: discrete approximation

Tallarini's result

Tallarini found

- Change in risk or risk aversion had little impact on quantities

Extensions

- Given a log-linear approximation of the value function, any change in risk or risk aversion generates identical decision rules
- Approximately true even in some severely nonlinear environments
- Gourio: exactly true if risk changes productivity and capital proportionately and risk is constant

Risk and risk aversion do affect asset returns, esp risk premiums

Tallarini intuition: the recursive business cycle model

Bellman equation

$$\begin{aligned} J(k_t, z_t) &= \max_{c_t, n_t} V\left\{c_t(1-n_t)^\lambda, \mu_t[J(k_{t+1}, z_{t+1})]\right\} \\ \text{subject to:} \quad & k_{t+1} = g(i_t, k_t) = g[f(k_t, z_t n_t) - c_t, k_t) \\ & \text{plus productivity process \& initial conditions} \end{aligned}$$

Ingredients

- V: time aggregator; eg, $V(x, y) = [(1-\beta)x^\rho + \beta y^\rho]^{1/\rho}$
- μ: risk preference; eg, $\mu(x) = [Ex^\alpha]^{1/\alpha}$
- f: production function
- g: law of motion (allows adjustment costs)

If ingredients hd1 $\Rightarrow$ J is hd1

Tallarini intuition: mechanics of certainty equivalents

Example: let $\log x \sim \mathrm{N}(\kappa_1, \kappa_2)$

Expectations and certainty equivalents for lognormals

$$\begin{aligned} E(x) &= \exp(\kappa_1 + \kappa_2/2) \\ E(x^\alpha) &= \exp(\alpha\kappa_1 + \alpha^2\kappa_2/2) \\ \mu(x) &= [E(x^\alpha)]^{1/\alpha} = \exp(\kappa_1)\exp(\alpha\kappa_2/2) \end{aligned}$$

Effect of risk same as discount factor β

Tallarini intuition: scaling

Bellman equation (reminder)

$$\begin{aligned} J(k_t, z_t) &= \max_{c_t, n_t} V\left\{c_t(1-n_t)^\lambda, \mu_t[J(k_{t+1}, z_{t+1})]\right\} \\ \text{subject to:} \quad k_{t+1} &= g(i_t, k_t) = g[f(k_t, z_t n_t) - c_t, k_t) \end{aligned}$$

Scaled version $[\tilde{k}_t = k_t/z_t,\ \tilde{c}_t = c_t/z_t]$

$$\begin{aligned} J(\tilde{k}_t, 1) &= \max_{\tilde{c}_t, n_t} V\left\{\tilde{c}_t(1-n_t)^\lambda, \mu_t[(z_{t+1}/z_t)J(\tilde{k}_{t+1}, 1)]\right\} \\ \text{subject to:} \quad \tilde{k}_{t+1} &= g[f(\tilde{k}_t, n_t) - \tilde{c}_t, \tilde{k}_t](z_t/z_{t+1}) \end{aligned}$$

Note: proportional shock to k and z leaves $\tilde{k}$ unchanged

The Barro-King problem

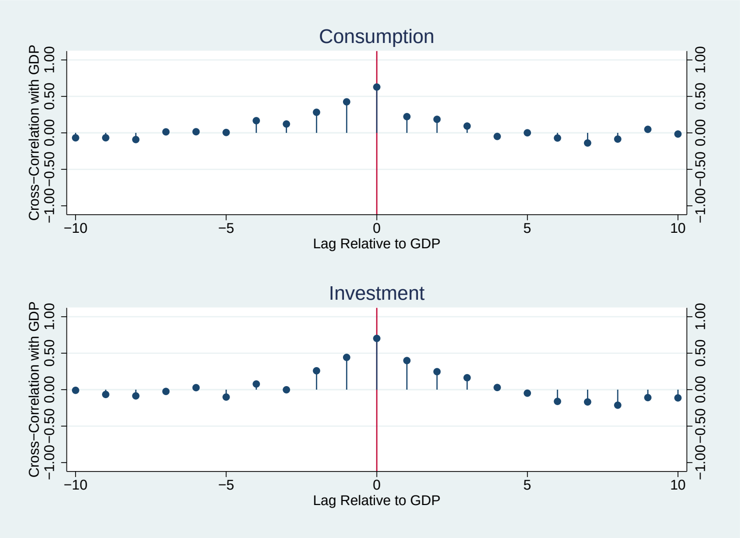
Shocks to anything but current productivity generate opposite movements in some of: (c, i, y, n)

Example: shocks to disaster probability

Resolutions

- Keep shocks small relative to productivity
- Adjustment costs (?)
- These things aren't as highly correlated as you think

Barro-King: comovements in US data



Gourio revisited

Contributions

- Risk is an interesting business cycle shock
- Improves behavior of prices and quantities

Open questions

- Would stochastic volatility do the job?
- Relation to Justiano-Primiceri?
- Relation to Bloom: market price or technology?
- Where does risk come from?
- How should monetary policy respond to risk?

Related work (some of it)

Asset pricing with disasters

- Barro et al., Benzoni-Collin-Dufresne-Goldstein, Dreschler-Yaron, Gabaix et al., Longstaff-Piazzesi, Rietz, Wachter, Backus-Chernov-Martin

Cyclical behavior of asset returns

- Ang-Piassezi-Wei, Atkeson-Kehoe, Barsky, Campbell-Cochrane, Fama-French, Gilchrist-Zakrajsek, King-Watson, Mueller, Backus-Routledge-Zin

Business cycle models with recursive preferences

- Campanale-Castro-Clementi, Croce, Fernandez-Villaverde et al., Kaltenbrunner-Lochstoer, Rudebusch-Swanson, Tallarini, Uhlig, Backus-Routledge-Zin

Other related work

- Bloom, Justiano-Primiceri

Preferences

Equations

$$
\begin{aligned}
U_t &= V[u_t, \mu_t(U_{t+1})] \\
u_t &= c_t(1 - n_t)^{\lambda} \\
V(u_t, \mu_t) &= [(1-\beta)u_t^{\rho} + \beta\mu_t^{\rho}]^{1/\rho} \\
\mu_t(U_{t+1}) &= \left(E_t U_{t+1}^{\alpha}\right)^{1/\alpha}
\end{aligned}
$$

Interpretation

$$
\begin{aligned}
IES &= 1/(1-\rho) \\
CRRA &= 1-\alpha \\
\alpha &= \rho \quad \Rightarrow \quad \text{additive preferences}
\end{aligned}
$$

Kreps-Porteus pricing kernel

Marginal rate of substitution

$$m_{t+1} \;=\; \beta \left(\frac{c_{t+1}}{c_t}\right)^{\rho-1} \left(\frac{U_{t+1}}{\mu_t(U_{t+1})}\right)^{\alpha-\rho}$$

If $\alpha = \rho$

- Second term disappears
- No roles for volatility or predictable consumption growth

Linear approximation: two flavors

Problem: find decision rule $u_t = h(x_t)$ satisfying

$$E_t F(x_t, u_t, w_{t+1}) \;\; = \;\; 1, \quad w_t \sim \mathrm{N}(0, \kappa_2)$$

Judd + many others

- Taylor series expansion of F
- nth moment shows up in nth-order term
- Linear approximations independent of risk

Modern finance

- Taylor series expansion of $f = \log F$ in

$$E_t \exp[f(x_t, u_t, w_{t+1})] \;\; = \;\; 1$$

- All moments show up even in linear approximation

Linear approximation: example

Linear "perturbation" method

- Linear approximation of F

$$\begin{aligned} F(x_t, u_t, w_{t+1}) &= F + F_x(x_t - x) + F_u(u_t - u) + F_w w_{t+1} \\ E_t F = 1 \;\Rightarrow\; u_t - u &= (1 - F)/F_u - (F_x/F_u)(x_t - x) \end{aligned}$$

- Decision rule doesn't depend on variance of w (or higher moments)

"Affine" finance method

- Linear approximation of $f = \log F$

$$\begin{aligned} f(x_t, u_t, w_{t+1}) &= f + f_x(x_t - x) + f_u(u_t - u) + f_w w_{t+1} \\ E_t \exp(f) = 1 \;\Rightarrow\; u_t - u &= -(f + f_w^2 \kappa_2/2)/f_u - (f_x/f_u)(x_t - x) \end{aligned}$$

- Note impact of variance κ_2 (higher moments would show up, too)

Disasters implied by options

	Cons Growth Process Based on	
	Macro Data	Option Prices
Skewness	–11.02	–0.31
Excess Kurtosis	145.06	0.87
Tail prob (≤ -3 st dev)	**0.0090**	**0.0086**
Tail prob (≤ -5 st dev)	**0.0079**	**0.0002**