

Group 8: A Scheduling Problem

Or how not to spend your last
semester at Stern...

Presented by Carlo Guagnano, Stephan Reinhard, Olivier Schlatter, Julien Serdaru and Jeremy Stern

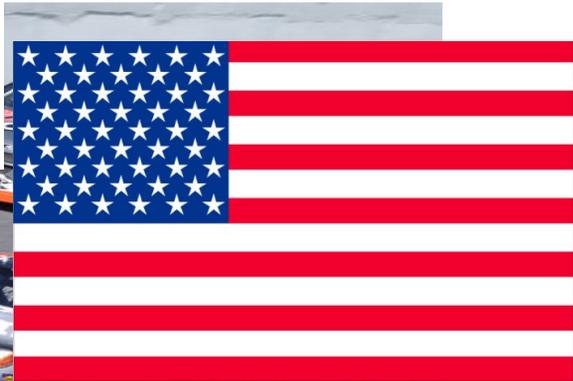
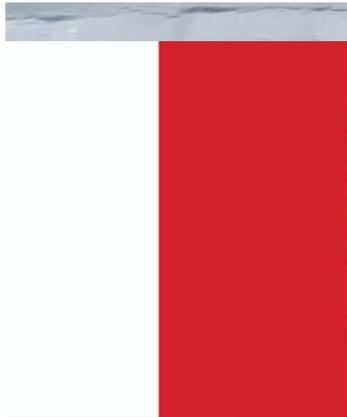
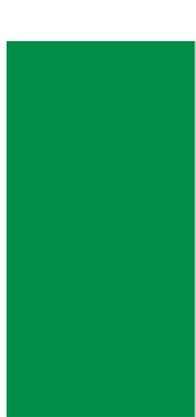
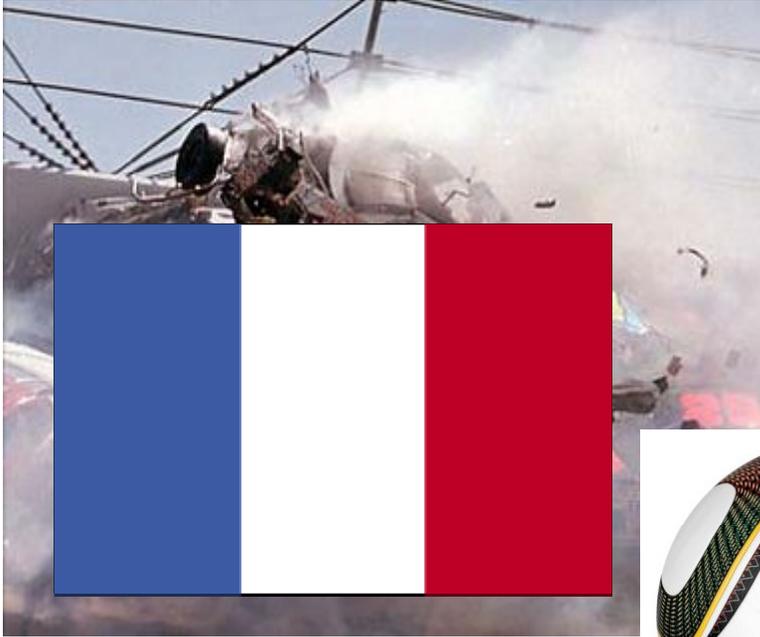


Agenda

- **Our Goal**

- Mathematical formulation of the problem
- The first (unsuccessful) attempts:
 - Week by week
 - An overall approach
- The “American” solution
- Conclusions
- Ways to improve model

Any great love...



...often hide a great challenge



Main objectives

Develop a game scheduler with the following criteria:

- N teams play each other in a league
- Each team needs to play equally at home and away with appropriate breaks (e.g. two away games in a row are to be avoided)

Using the following simplifying assumption:

- “Mirrored double round robin tournament” (you play opponents in the same order for first and second half of the league)



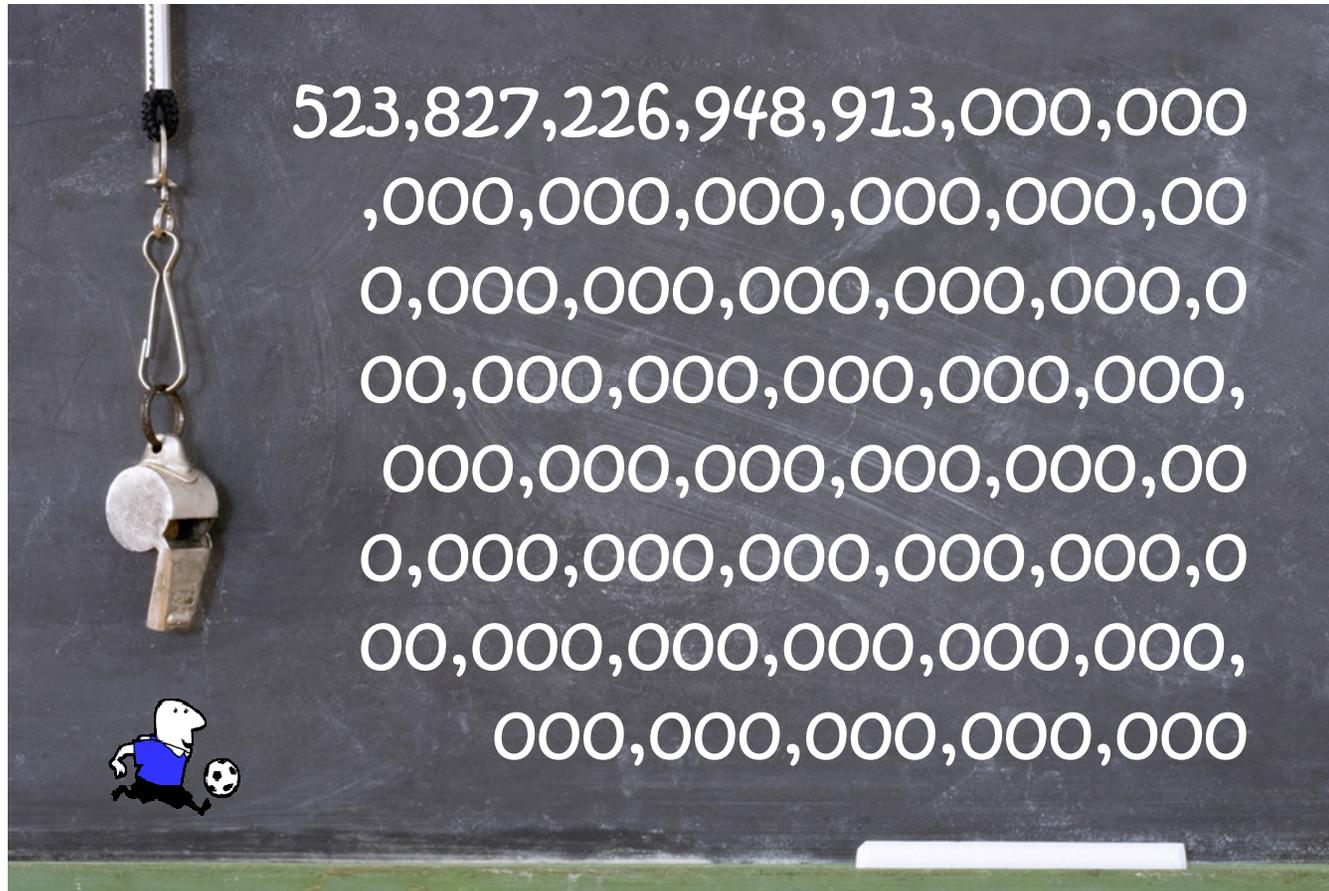
Agenda

- Our Goal

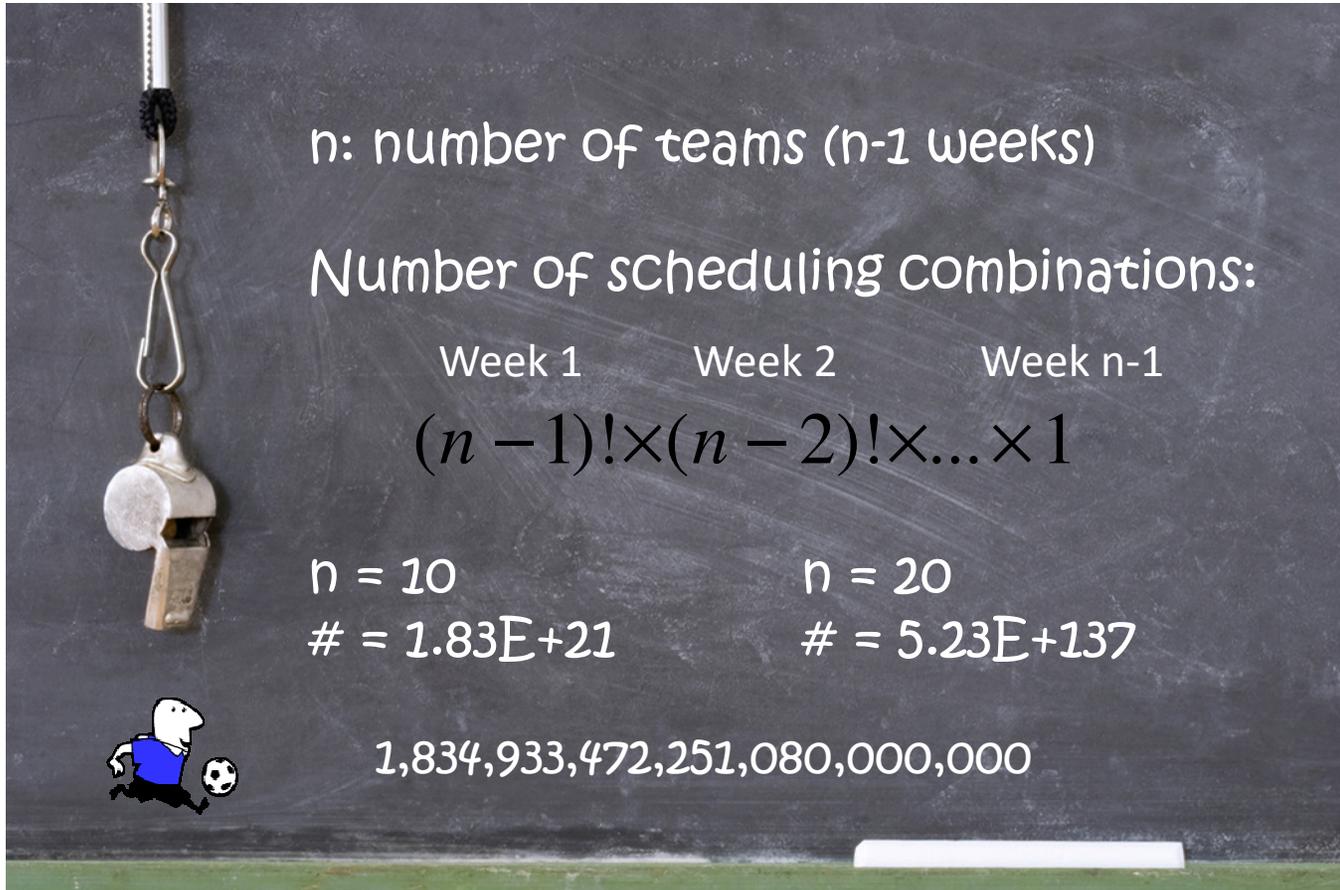
- **Mathematical formulation of the problem**

- The first (unsuccessful) attempts:
 - Week by week
 - An overall approach
- The “American” solution
- Conclusions
- Ways to improve model

Mathematical Formulation



Mathematical Formulation



n : number of teams ($n-1$ weeks)

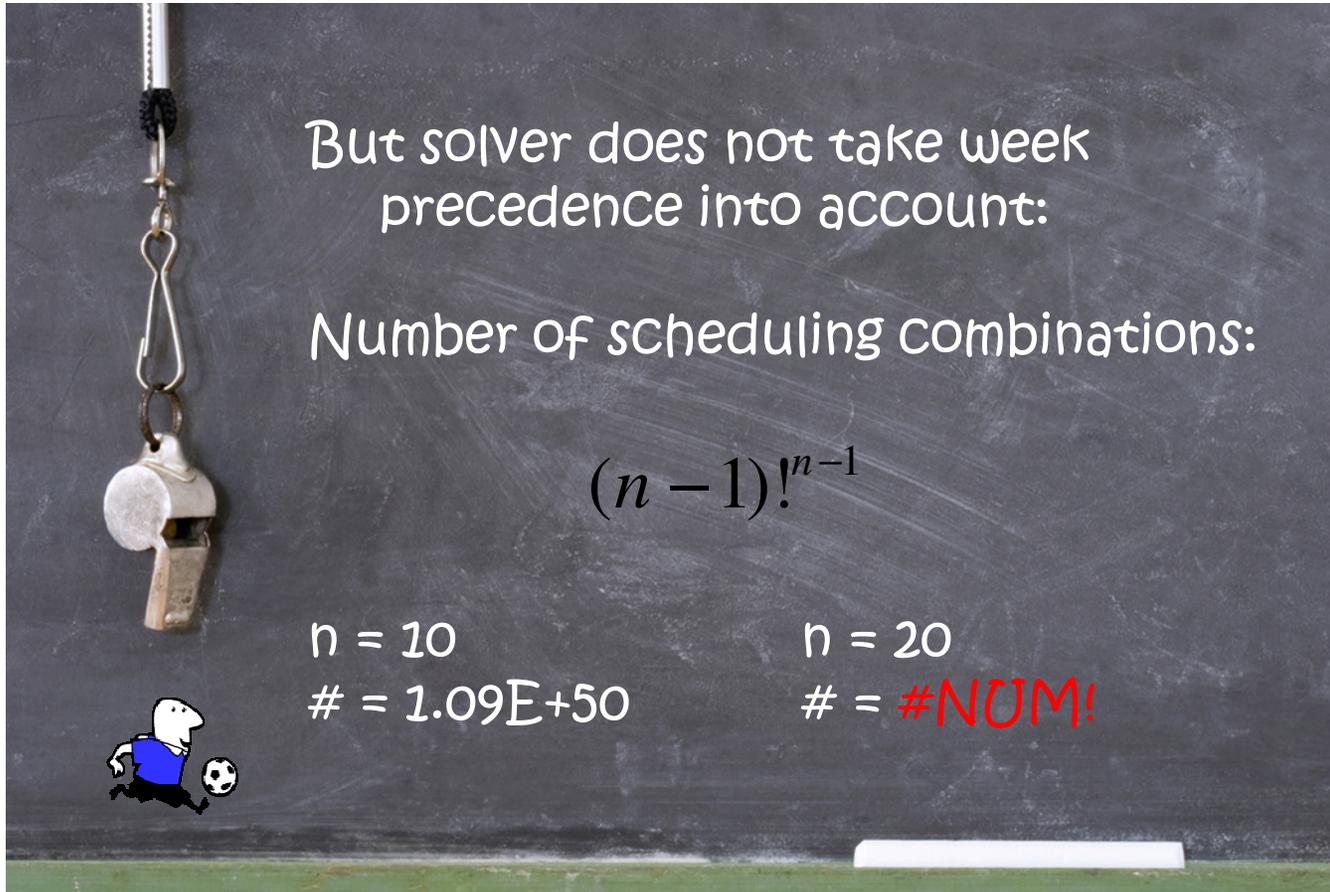
Number of scheduling combinations:

| Week 1 | Week 2 | Week $n-1$ |
|----------|----------|------------------|
| $(n-1)!$ | $(n-2)!$ | $\dots \times 1$ |

$n = 10$ $n = 20$
= $1.83E+21$ # = $5.23E+137$

1,834,933,472,251,080,000,000

Mathematical Formulation



But solver does not take week precedence into account:

Number of scheduling combinations:

$$(n - 1)!^{n-1}$$

| | |
|-----------------|----------------------|
| $n = 10$ | $n = 20$ |
| $\# = 1.09E+50$ | $\# = \text{\#NUM!}$ |



Agenda

- Our Goal
- Mathematical formulation of the problem

- **The first (unsuccessful) attempts:**

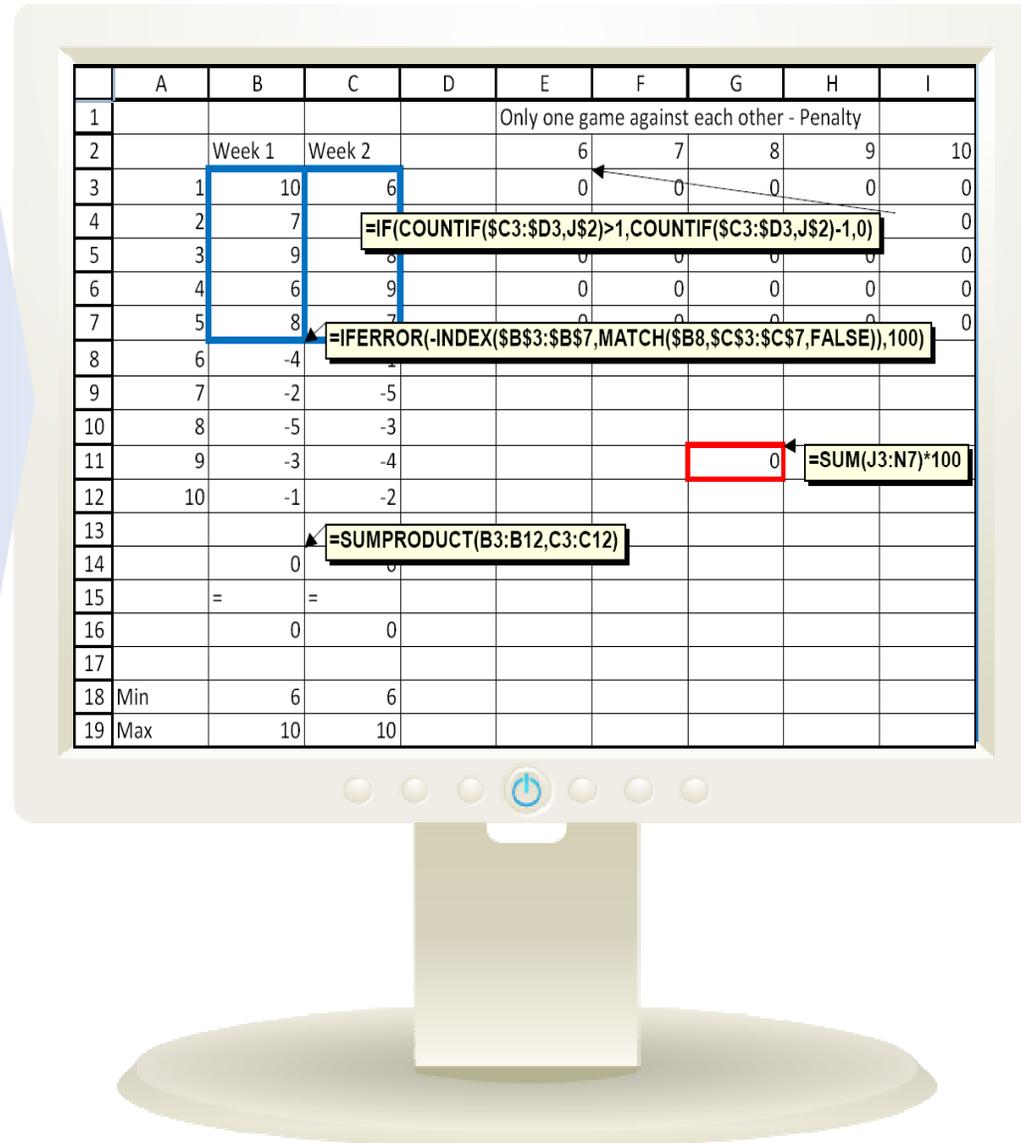
- **Week by week**
 - An overall approach
- The “American” solution
- Conclusions
- Ways to improve model

A first approach... week by week...seems to work...



Model approach

- 10 teams
- Simulate week by week
- Penalties for playing again with the same team
- Decision variables are only half of the team



...but fail after only 4 weeks



| | B | C | D | E | F | G | H | I | J | K | L | M | |
|----|------------|----------------------------|---|--------|--------|--------|---|--|-----|---|---|----|--|
| 1 | | | | | | | | Only one game against each other - Penalty | | | | | |
| 2 | | Week 1 | Week 2 | Week 3 | Week 4 | Week 5 | | 6 | 7 | 8 | 9 | 10 | |
| 3 | 1 | 6 | 10 | 10 | 7 | 7 | | 0 | 1 | 0 | 0 | 1 | |
| 4 | 2 | 10 | 8 | 7 | 6 | 9 | | 0 | 0 | 0 | 0 | 0 | |
| 5 | 3 | 8 | 7 | 6 | 9 | 10 | | =IF(COUNTIF(\$C3:\$G3,\$I2)>1,COUNTIF(\$C3:\$G3,\$I2)-1,0) | | | | | |
| 6 | 4 | 9 | 9 | 8 | 8 | 6 | | 0 | 0 | 1 | 1 | 0 | |
| 7 | 5 | 7 | 10 | 10 | 7 | 7 | | 0 | 0 | 0 | 0 | 0 | |
| 8 | 6 | -1 | =IFERROR(-INDEX(\$B\$3:\$B\$7,MATCH(\$B8,\$C\$3:\$C\$7,FALSE)),100) | | | | | | | | | | |
| 9 | 7 | -5 | -3 | -2 | -1 | -1 | | | | | | | |
| 10 | 8 | -3 | -2 | -4 | -4 | -5 | | | | | | | |
| 11 | 9 | -4 | -4 | -5 | -3 | -2 | | | | | | | |
| 12 | 10 | -2 | -1 | -1 | -5 | -3 | | | 400 | | | | |
| 13 | | =SUMPRODUCT(B3:B12,C3:C12) | | | | | | | | | | | |
| 14 | Sumproduct | 0 | | | | | | =SUM(I3:M7)*100+C14+D14+E14+F14+G14 | | | | | |
| 15 | | = | = | = | = | = | | | | | | | |
| 16 | | 0 | 0 | 0 | 0 | 0 | | | | | | | |
| 17 | | | | | | | | | | | | | |
| 18 | Min | 6 | | | | | | | | | | | |
| 19 | Max | 10 | | | | | | | | | | | |
| 20 | | | | | | | | | | | | | |
| 21 | | =C3+C4+C5+C6+C7 | | | | | | | | | | | |
| 22 | Subsum | 40 | | | 40 | 40 | | | | | | | |
| 23 | | = | = | = | = | = | | | | | | | |
| 24 | | 40 | 40 | 40 | 40 | 40 | | | | | | | |

WRONG WAY

Agenda

- Our Goal
- Mathematical formulation of the problem
- **The first (unsuccessful) attempts:**
 - Week by week

- **An overall approach**

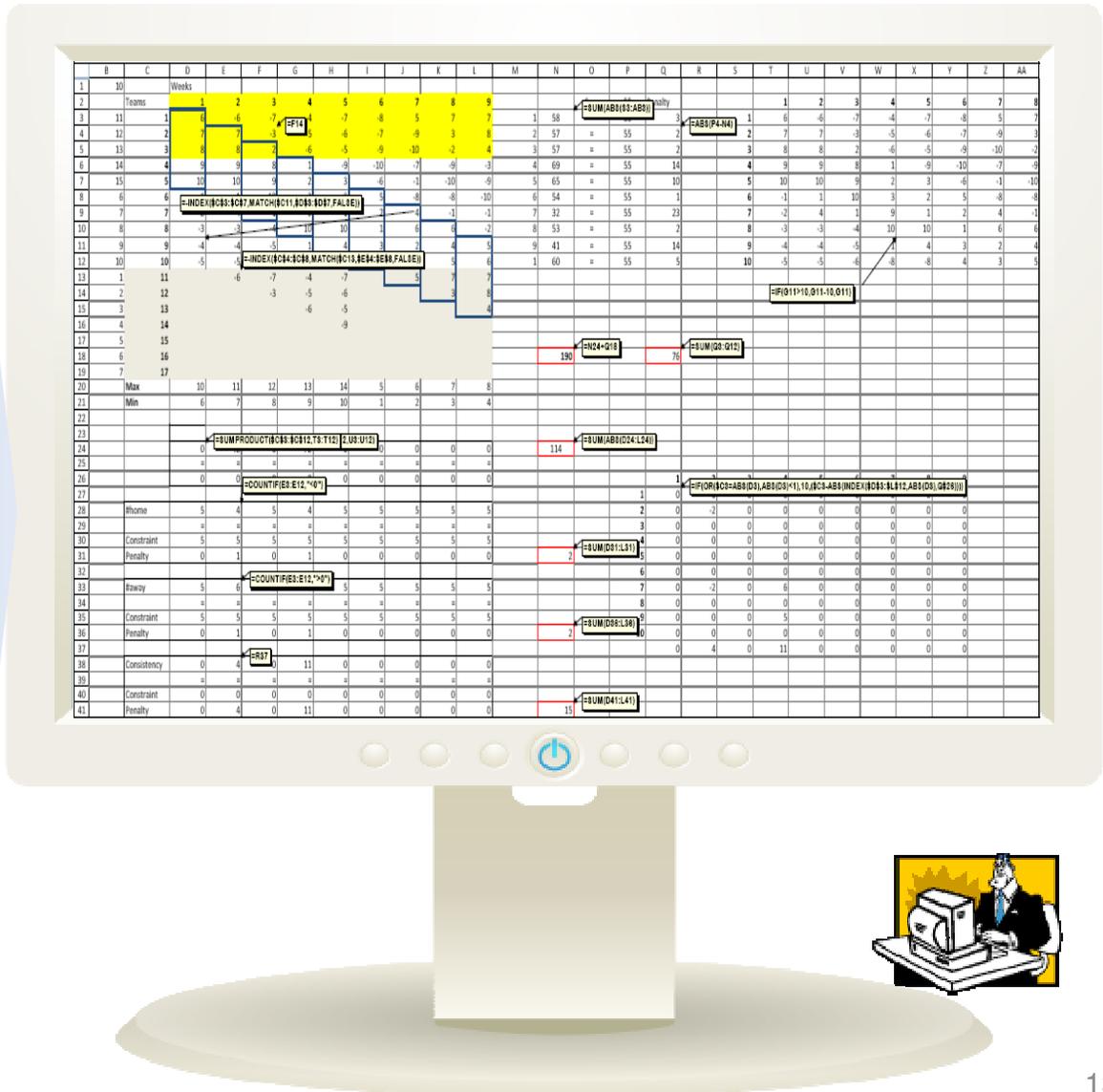
- The “American” solution
- Conclusions
- Ways to improve model



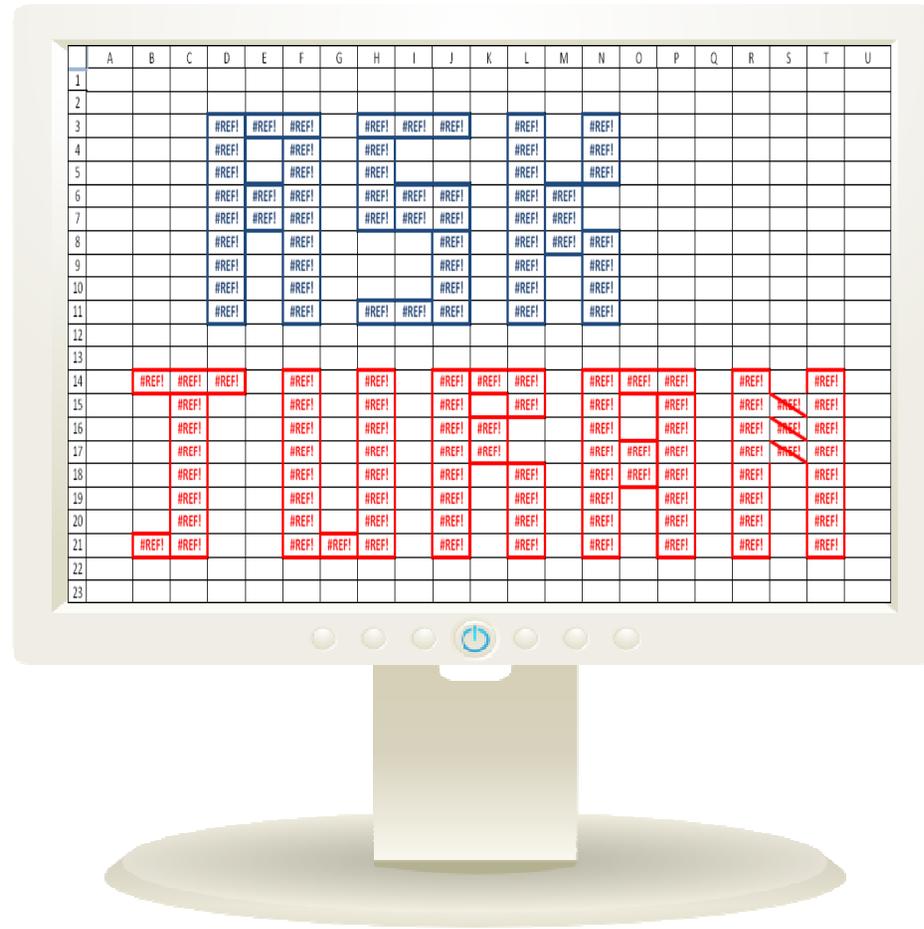
We create a “slightly” more complicated approach...

Model approach

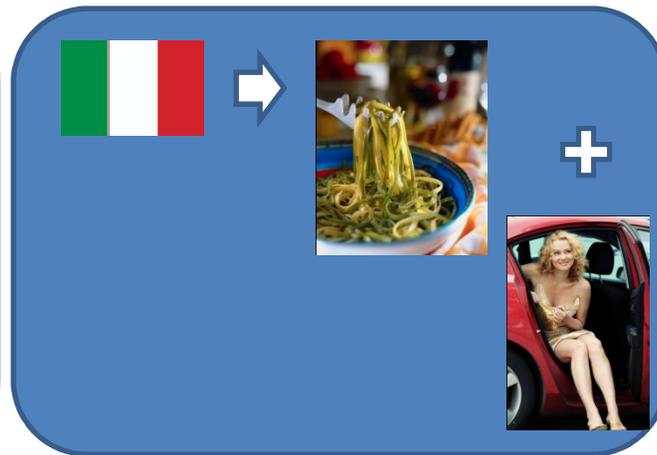
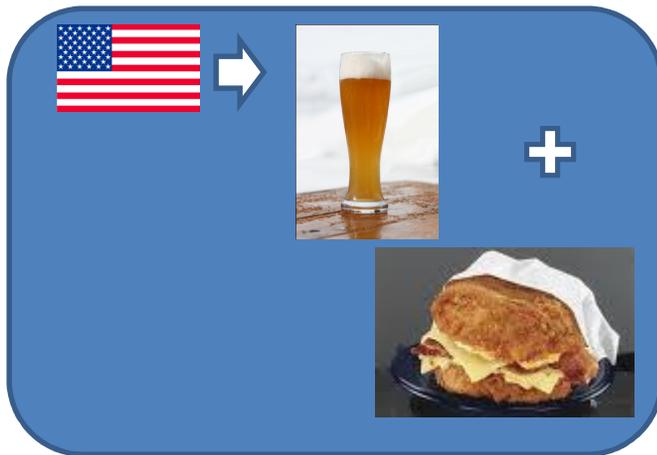
- 10 teams
- Simulate all 10 weeks
- Penalties for:
 - Playing again with the same team (season)
 - All teams play during game day
 - Not playing home/away between 4-5 times
 - Playing home/away consecutively
 - being Brazil...
- Decision variables for only half of the teams



...but solver reacted unexpectedly...



...and our team took it in different ways...



Agenda

- Our Goal
- Mathematical formulation of the problem
- The first (unsuccessful) attempts:
 - Week by week
 - An overall approach

- **The “American” solution**

- Conclusions
- Ways to improve model

But then we got the final idea....

The American way...



...with a little bit of **(hard)** (manual) work...

Phase 1



Phase 2

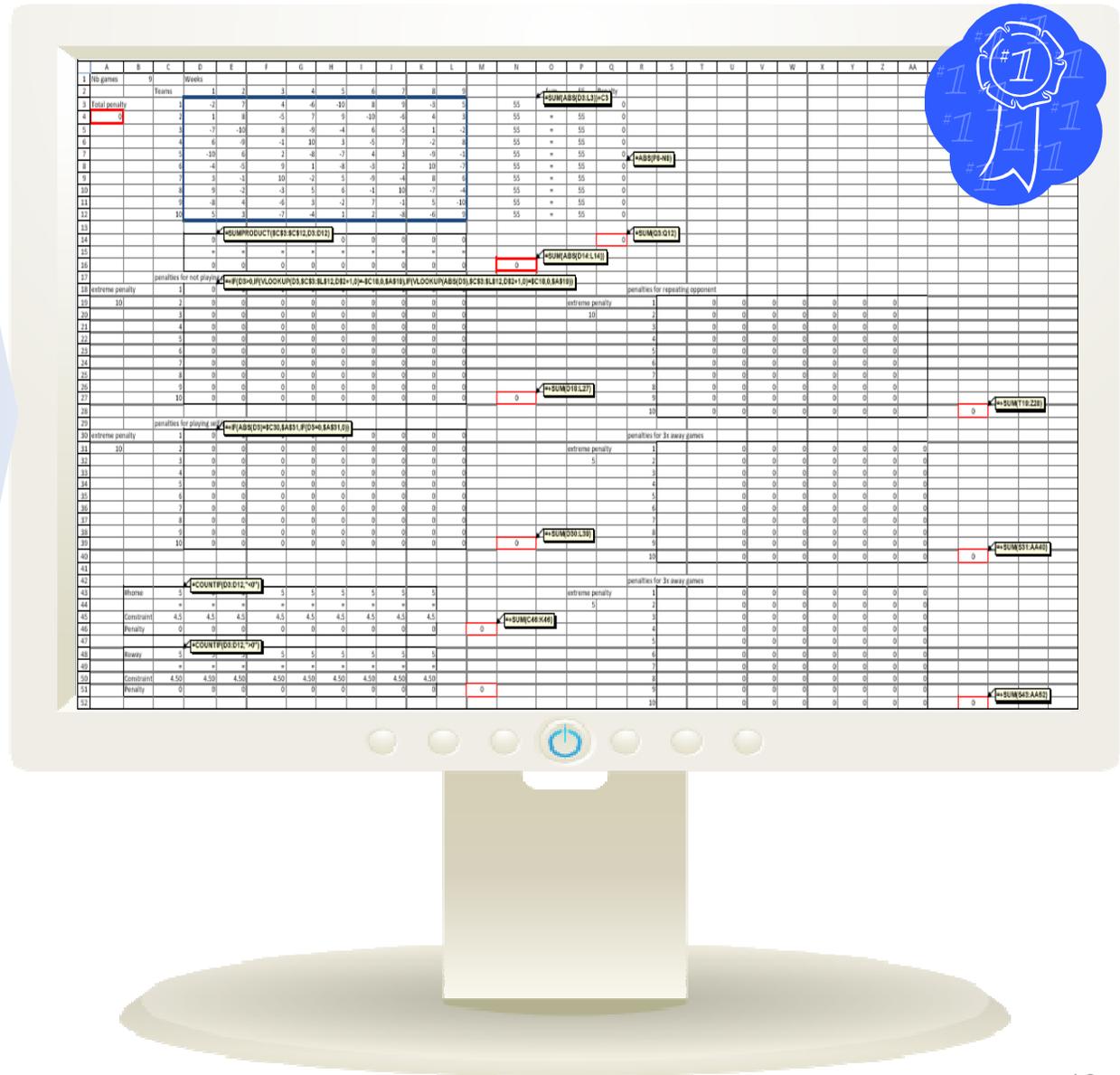


Phase 3



... “Yes, we can!!”

- Model approach
- Little constrain
- A lot of penalties
- Run solver and... “pray”
- Take a bear (or two) and start to solve the remaining discrepancy manually



“...and Yes, we did!!”

| | <u>Weeks</u> | | | | | | | | |
|--------------|--------------|------|-------|-------|------|------|-------|-------|------|
| <u>Teams</u> | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | @ 2 | vs 7 | vs 4 | @ 6 | @ 10 | vs 8 | vs 9 | @ 3 | vs 5 |
| 2 | vs 1 | vs 8 | @ 5 | vs 7 | vs 9 | @ 10 | @ 6 | vs 4 | vs 3 |
| 3 | @ 7 | @ 10 | vs 8 | @ 9 | @ 4 | vs 6 | @ 5 | vs 1 | @ 2 |
| 4 | vs 6 | @ 9 | @ 1 | vs 10 | vs 3 | @ 5 | vs 7 | @ 2 | vs 8 |
| 5 | @ 10 | vs 6 | vs 2 | @ 8 | @ 7 | vs 4 | vs 3 | @ 9 | @ 1 |
| 6 | @ 4 | @ 5 | vs 9 | vs 1 | @ 8 | @ 3 | vs 2 | vs 10 | @ 7 |
| 7 | vs 3 | @ 1 | vs 10 | @ 2 | vs 5 | @ 9 | @ 4 | vs 8 | vs 6 |
| 8 | vs 9 | @ 2 | @ 3 | vs 5 | vs 6 | @ 1 | vs 10 | @ 7 | @ 4 |
| 9 | @ 8 | vs 4 | @ 6 | vs 3 | @ 2 | vs 7 | @ 1 | vs 5 | @ 10 |
| 10 | vs 5 | vs 3 | @ 7 | @ 4 | vs 1 | vs 2 | @ 8 | @ 6 | vs 9 |



Agenda

- Our Goal
- Mathematical formulation of the problem
- The first (unsuccessful) attempts:
 - Week by week
 - An overall approach
- The “American” solution
- **Conclusions**
- Ways to improve model

Conclusions

Our learning...

1. Solver sometimes, is not a solution finder, but a “solution facilitator”
2. This problem is much more complex than initially thought, so next time we will check online before finding out that there is tons of literature about this topic
3. There is always the “lifetime guarantee”!!

...will follow us in our future



Agenda

- Our Goal
- Mathematical formulation of the problem
- The first (unsuccessful) attempts:
 - Week by week
 - An overall approach
- The “American” solution
- Conclusions
- **Ways to improve model**

And if you are not bored yet...



Ideas for improving the model:

Find a working solution!

Add a Brazilian to the team!

Add constraints:

- Minimize breaks
- Don't play strong opponents too often in a row
- Don't play in stadiums that are not available that day
- Hire Juran as a consultant!

Our Questions...



DONATE NOW ON JEREMY STERN'S FACEBOOK PAGE!!!

Your Questions

