A Simple Empirical Model of Equity-Implied Probabilities of Default*

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Abstract

We approximate the likelihood of default inferred from equity prices using accounting-based measures, firm characteristics and industry-level expectations. Such empirical approximations enable the timely modeling of distress risk in the absence of equity prices or sufficient historical records of defaults. We show, through a series of re-sampling experiments, that our models deliver out-of-sample classification performance comparable to that of default likelihood inferred from equity prices using the Black-Scholes-Merton framework. Further, we document the distinct roles of firm-level and macroeconomic information in capturing time-varying exposure to the risk of financial distress. More generally, our results underscore the importance of treating equity-implied default probabilities and fundamental variables as complementary rather than competing sources of predictive information.

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The option pricing model of Black and Scholes [1973] and Merton [1974] (henceforth BSM) implies a direct linkage between equity values and the risk of credit default. When equity is viewed as a call option on the assets of a limited-liability firm, a probability of default can be inferred from the proximity of the value of a firm's assets to the value of its liabilities, and the volatility of the firm assets. While the characterization of debt and equity as contingent claims on the assets of a firm is not new, the explicit use of this structure for default risk measurement is a more recent development spearheaded by the commercial success of the KMV Corporation's equity based measure of default risk.[1]

The current work takes the perspective of a researcher or analyst who cannot discard an accounting based approach in favor of a market equity based approach to estimating the risk of financial distress. Credit exposures to small to medium enterprises (SMEs) and public firms that have gone private by way of leverage buyouts (LBOs) whose equity is not exchange traded, cannot be modeled directly using a BSM approach.[2] With such considerations in mind, we model the associations between expectations of default extracted from equity prices, accounting-based measures, firm characteristics and industry-level expectations of distress conditions.

Empirical models linking BSM default probabilities to fundamentals are of potential importance for two reasons. First, if the linkage between market implied default expectations and observable fundamentals is understood, then default probabilities may be estimated when firm-level market data is not available – as is typically the case for the bulk of middle-tier bank loan exposures. Second, such models can serve to reduce the volatility of economic capital requirements associated equity-based models of default. Dampening the volatility of equity-implied default risk estimates is especially desirable if equity prices are known to depart from fundamentals ,or, exhibit volatility that is not directly associated with default risk exposure.

Our findings suggest that models mapping fundamental variables to default expectations deliver out-of-sample classification performance that is indistinguishable from that of BSM-default likelihood (DLI) itself.[3] Combining industry-level estimates of DLI with firm-level accounting information greatly improves the in-sample fit and out-of-sample classification performance of traditional scoring-type models. Industry-level expectations of default likelihood parsimoniously summarize macroeconomic conditions relevant to subsequent aggregate default rates.[4]

Firm-Level Default Likelihood and Fundamentals

Since the work of Altman [1968], equity prices are commonly used to construct independent variables in models linking firm-level fundamentals to observed outcomes of default and survival. We use fundamental variables to estimate empirical approximations of default likelihood gleaned from equity prices using the option-theoretic approach. By modeling default likelihood rather than default outcomes, our approach does not rely on a sufficient history of default observations. Further, default likelihoods provide a richer, forward looking set of data to examine the risk of default at a firm or industry level than simple default realizations, enabling the estimation of models over long horizons, and at different levels of aggregation.

We commence our analysis by estimating the relation between default likelihood and fundamental variables: firm attributes, accounting measures of financial position and performance, and systematic risk. Following the approach of Wilson [1997], we model the probability of default of firm i at time t as the logistic function of an index y_{it}. More precisely, the risk neutral probability estimate DLI_{it} can be written as:

$$DLI_{it} = \frac{1}{1+e^{y_{it}}}, \tag{1}$$

where,

$$\begin{aligned} y_{it} &= \hat{\alpha}_0 + \hat{\alpha}_1 ln\left[1-\frac{EBIT}{TA}\right]_{i,t-1} + \hat{\alpha}_2\left[\frac{WC}{TA}\right]_{i,t-1} + \hat{\alpha}_3 ln\left[1-\frac{RE}{TA}\right]_{i,t-1} + \hat{\alpha}_4\left[1+ln\frac{TA}{TL}\right]_{i,t-1} \ldots \\ &+ \hat{\alpha}_5 IR_{i,t-1} + \hat{\alpha}_6 Size_{i,t-1} + \hat{\alpha}_7 CNCL_{i,t-l} + \hat{\alpha}_8 Age_{i,t-1} + e_{it}. \end{aligned} \tag{2}$$

Model (2) incorporates two measures of historical firm-specific performance: the ratio of earnings before interest and taxes ($EBIT$) to total assets (TA) and retained earnings (RE) to total assets. Leverage is measured by the ratio of total assets to total liabilities (TL), the (log) ratio of current liabilities to non-current liabilities ($CNCL$) provides a basic summary of debt maturity structure, and the ratio of working capital (WC) to total assets is a summary of relative liquidity.[5] Firm size ($Size$) is the total book value of assets standardized by the cumulative growth in the S&P500 market index from the commencement of the sample period, and Age is the number of months elapsed since the initial listing date from CRSP. The variable $IR_{i,t-1}$ is the relative risk of the industry to which company i belongs at time $t-1$ as measured by the mean default likelihood of firms in the relevant industry portfolio.[6]

In equation (2), we augment the Altman and Rijken [2006] specification of independent variables to include for each firm a measure of whether it is operating in an industry that is expected to experience a relatively high level of defaults at a particular time. Measurement of industry risk by way of an industry-level default likelihood is attractive to the extent that market prices reflect a broad range of macroeconomic information relevant to the credit risk of an industry, and to the extent that firm specific pricing errors are diversified away.[7]

Finally, our estimates of default likelihood are based on the assumption that the default barrier is equal to short term debt plus one half of long term debt. For this reason we include as an independent variable the ratio of current to non current liabilities $CNCL$ to account for systematic variation in default likelihood that may be attributable to this modeling assumption.

In Exhibit 1 we report the characteristics of the firm-level data used in our monthly regressions over the sample period spanning from January 1978 to December 2007.

Exhibit 1 About Here

The statistics in Exhibit 1 describe the sample over 5-year sub-periods prior to any winsorization or filtering of outliers, hence, the median is in most cases the most appropriate measure of central tendency. For the same reason, we measure sample dispersion by way of the interquartile range.[8]

Two sources of variability in the median ratios over the span of the sample are noteworthy. First, the ratio of $\frac{ME}{TL}$ has varied markedly and increased over time - from a median of 0.97 in the first sub-period to 2.86 in the last, consistent with a decrease in market leverage. The increase in the corresponding accounting measure over the same interval has been modest by comparison with median $\frac{TA}{TL}$ rising from 1.9 to 2.18. Second, the median value of $\frac{RE}{TA}$ has declined from 0.29 in the first sub-period to 0.07 in the last. As measured by the interquartile range, the variability of leverage and $\frac{RE}{TA}$ has also increased dramatically over the sample, though again, more for the market-based measure than its accounting counterpart. These observations aside, the distributions of other ratios appear to exhibit little change from one period to the next.

Default Likelihood Estimates

We estimate DLI on a monthly basis for all firms in the merged CRSP-COMPUSTAT data set from Wharton Research Data Services (WRDS) using data from January 1978 to December 2007. To estimate DLI we utilize historical volatility estimates based on daily closing prices from the CRSP daily data set over the year prior to the month of estimation. As a minimum, we require all firms included in the sample at a given point in time to have at least three prior months of daily closing prices in the CRSP daily data set for purposes of volatility estimation.

Given current equity value E_t together with empirical estimates of equity volatility σ_E and the expected drift rate in the value of firm assets μ, estimates of asset value A_t and asset volatility σ_A can be obtained. Specifically, we estimate A_t and σ_A through simultaneous (numerical) solution of the BSM equation for the value of equity, and the equation linking the volatility of equity to that of (unobservable) asset volatility in the BSM framework.[9] In computing DLI we set the annual drift rate on firm assets equal to the yield on 1-year treasury bonds.

In keeping with the approach advocated by Moody's KMV and Vassalou and Xing [2004], we set the strike price of the equity option equal to a weighted average of each firm's long and short term debt, specifically, short term debt plus one half the value of long term debt.[10] In matching market data to information from quarterly financial statements we assume a reporting lag of at least one quarter. For this reason, all but end of quarter market values are matched with financial statement variables reported in the preceding quarter. For example, the equity price at the end of December 2005 would be matched with financial statement information reported during the fourth quarter of 2005, whilst the equity price at the end of November 2005 is matched with information reported during the third quarter of 2005.[11]

To illustrate the properties of our estimates of default likelihood, we present in Exhibit 2 a graphical summary of aggregate time series behavior and cross-sectional variability.

Exhibits 2 and 3 About Here

The dashed line in Exhibit 2 plots the mean of the default likelihood estimates over our sample spanning from the second quarter of 1978 to the end of 2007. The shaded region in Exhibit 2 maps the interquartile range of default likelihood estimates in each sample month. Throughout most of the (almost) 30-year horizon the mean default likelihood lies above the interquartile range, thus indicating the skewness of the distribution. In most years of our sample the majority of companies have risk neutral probabilities of default that are close to zero. As expectations of default increase and a larger number of firms exhibit non-trivial likelihood of default, the interquartile range widens such that during peak periods between 2000 and 2004 the cross-sectional mean falls within the interquartile range.

In Exhibit 3 we plot the mean default likelihood (dashed line) alongside Moody's 12-month trailing issuer-weighted speculative grade default rate, observed one year later (solid line), to provide a visual summary of the extent to which the measure of aggregate expectation tracks the subsequent outcome.[12] At an aggregate level, the mean expectations appear strongly correlated with the subsequent default rate outcome, the sample correlation being 60%. This does suggest we have successfully extracted default related information from equity prices (at an aggregate level) using the BSM framework. The aggregate findings are consistent with what we would expect from a measure of default expectations.

Model Fit

In Panel A of Exhibit 4 we present estimates of model (2) using all firms over the entire sample period.[13] Overall, 40% of the in-sample variation in default likelihood is captured by ratios from financial statements, firm characteristics and an aggregate measure of industry level default expectations. The direction (sign) of the relation between expectations of default and fundamental variables is consistent with economic intuition – the notable exception being the variable $\frac{WC}{TA}$. To the extent that an increase in the ratio of working capital to total assets is consistent with a greater capacity to meet short term financial obligations, one would expect it to be negatively related to default probability (all else being equal).

Exhibit 4 About Here

For purposes of comparison, Exhibit 4 reports two other sets of results. Panel Bcontains estimates of coefficients based on the variables employed by Altman and Rijken [2006] - that is, the coefficients on *Industry Risk* and $ln(CNCL)$ in (2) are restricted to zero. The restrictions diminish the overall explanatory power of the model by approximately a quarter, but the sign and magnitude of the coefficient estimates associated with the remaining variables do not appear to be greatly affected.

Panel C of Exhibit 4 reports reports estimates of model (2) augmented with the variables *Size Risk* and *Age Risk*. The variables *Size Risk* and *Age Risk* reflect the mean DLI of the particular size or age based decile portfolio to which a firm belongs at a particular prediction date, thus capturing the time variation in risk attributable to each particular characteristic. This variation on the estimation approach is beneficial if the DLI premium associated with size and age exhibits cross-sectional stability across exposures with similar characteristics. However, the improvement in model fit is slight relative to Panel A wherein firm-level estimates of size and age are used directly.

In each panel of Exhibit 4 we report estimates of models based on the ratio of total assets to total liabilities (*Accounting Leverage*) and models based on the ratio of the market value of equity to total liabilities (*Market Leverage*). While the primary focus of this work is to model linkages between fundamentals and market estimates of default risk that are applicable to both public and private firms, that is, models based on accounting data at firm level, it nevertheless is of interest to study the difference in model fit and performance attributable to firm level market data. From the results in Exhibit 4 it is apparent that using a market based measure of leverage yields a significant improvement in model fit relative to the accounting based measure: $\bar{R}^2$ increases in each case by approximately 17%. The improvement in model fit is to be expected given that the DLI measures are estimated using market leverage.

Exhibits 5 and 6 About Here

The results in Exhibit 4 represent a conservative estimate of the extent to which fundamental variables capture variation in DLI, as we make the rather restrictive assumption that the coefficients of (2) exhibit no cross-sectional variation. We relax this assumption and present in Exhibits 5 and 6 estimates of model (2) at industry level.

Fitting model (2) to the entire sample using accounting data results in an $\bar{R}^2$ of 40%, while all but three of the industry level regressions reported in Exhibit 5 have $\bar{R}^2$ of 41-46%. With $\bar{R}^2$s of 37% and 32% respectively, industry 6 (Chemicals) and 15 (Financial) exhibit worse in-sample fit than the rest of the sample. Similar improvements in model fit are also evident in Exhibit 6, wherein model (2) is re-estimated at industry level using market data to measure leverage. The industry level $\bar{R}^2$s range from 51-62%. The performance of the model is again weakest in the context of industry 15 (Financial), but the overall fit is significantly better than when accounting data is used as the measure of leverage. Regardless of whether we use accounting or market measures of leverage, the coefficients associated with $ln\left[1-\frac{EBIT}{TA}\right]$ and $\frac{WC}{TA}$ exhibit by far the most cross-sectional variation, while coefficients associated with the remaining variables are relatively stable across industry groups. Overall, the industry-level estimates of model (2) in Exhibit 5 suggest that model fit is improved by estimation at industry level.

The Relative Importance of Fundamental Variables

To gain a better understanding of the relative importance and economic influence of the variables in model (2), we benchmark each coefficient reported in Exhibit 7 using the metric:

$$RW_k = \frac{|\hat{\alpha}_k|\sigma_k}{\sum_{j=1}^{K} |\hat{\alpha}_j|\sigma_j}, \tag{3}$$

where $\hat{\alpha}_k$ is the parameter estimate and σ_k is the standard deviation of variable k pooled cross sectionally and over time. Thus, the measure RW_k summarizes the relative influence of each variable k on y_{it} in terms of the absolute impact of a standard deviation change in independent variable k as a proportion of the total absolute change in the dependent variable, given a standard deviation change in all included variables.

Exhibit 7 About Here

In Exhibit 7 we report the relative influence of the factors in the absence of firm level market data, using the accounting measure of leverage. Based on the pooled estimates of model (2) it appears that variation in leverage and size account for approximately 29% and 31.5% respectively of the explained variation in DLI. Firm age, the ratio of retained earnings to total assets and industry risk each appear to capture approximately 8% of explained variation in DLI, while the remaining variables each contribute 5% or less. However, Exhibit 7 also reports the corresponding results for industry-level estimates of model (2), and these imply that economic inference based on the pooled results should be treated with caution.

Industry level estimates of relative influence reinforce inference based on the measures of regression fit, that restricting the coefficients of model (2) to be constant in cross-section is too restrictive. The latter point is well illustrated by the marked contrast between the role of industry risk in the pooled specification and its role in the industry level estimates. While the pooled model suggests that industry risk accounts for about 8.37% of the explained variation in DLI, at 17.6% the median industry-level measure of its influence is approximately twice as large, and the role of firm size, leverage and age are correspondingly diminished. Further, the industry level estimates of each variable's relative contribution is quite robust: the cross-sectional standard deviation of the industry risk coefficient is approximately 2.5%.

The substance of the results incorporating market measures of leverage is similar. The main point of contrast with the results in Exhibit 7 concerns the relatively heavy influence of market leverage. Market leverage accounts for approximately 42% of the variation in DLI captured by the pooled estimates of model (2) and firm size appears correspondingly less important. Using industry risk as another example, the median estimate of (3) is 13.2% at industry level, but only 5.7% at aggregate level. Thus it can be seen that when the relation between industry risk and DLI is allowed to vary cross-sectionally, the role of leverage appears to diminish correspondingly.

While the measures of model fit and the relative importance of model components provide some statistical and economic insights, it can strongly be argued that model performance is of ultimate interest. For this reason we consider next the out-of-sample predictive properties of the candidate models.

Performance Evaluation: Relative Risk Classification

The ability of a model to capture the relative exposure of entities to default over some future horizon is a basic measure of its practical utility. Sobehart, Keenan, and Stein [2000] argue that the most stringent tests of performance are characterized by independence of the estimation and evaluation samples in both time-series and cross-section. That is, there is no overlap between the cohort of firms used to estimate model parameters and the cohort of firms used to evaluate model performance, nor any overlap between the estimation horizon and the evaluation period. Accordingly, in Exhibit 8 we use cumulative accuracy profile (CAP) plots to benchmark the performance of four model specifications in terms of their ability to capture default over the 24-months subsequent to a given estimation period using the following procedure.

1. An estimation and test period are determined by randomly selecting a date between January 1993 and December 2005. The period from the commencement of the sample to the randomly selected date is the estimation period, and the subsequent 24-month interval is the evaluation period.
2. Firms are randomly assigned to an estimation and evaluation cohort.
3. The firms assigned to the estimation cohort are used to estimate model parameters over the estimation period determined in step 1.
4. The model estimates are fitted to the evaluation cohort, as at the end of the estimation horizon, and the fitted values are used to rank firms according to their relative risk exposure.

The CAP curve is estimated based on the performance of the evaluation sample over the 24 months subsequent to estimation, and the results are stored.

Steps 1-4 are repeated 1,000 times to obtain the unconditional (time averaged) empirical distribution of performance outcomes, and we use the mean of these outcomes to plot the CAP curves in Exhibit 8. For the purpose of evaluating predictive performance we avoid the use of retrospective filters to remove outliers. Instead, we utilize in Step 3 a robust regression model that accommodates outliers under the assumption that errors follow a thicker tailed multivariate-T distribution. We use the maximum likelihood estimators derived by Prucha and Kelejian [1984] under the assumption that model errors e_{it} are independent.

In evaluating firm-level classification performance we utilize liquidation and exchange delisting as proxies for default. We identify failed firms as those with CRSP delist codes commencing with 4 or 5.[14] Using this approach we identify 3,324 relevant failure events over our 30-year sample horizon. Using this classification approach we identify firms as having failed if they have been liquidated for any reason, or, if they have been dropped from trading on an exchange due to bankruptcy, or a failure to meet exchange listing requirements. Of the delisted companies, 1684 observations fall in the CRSP delisting code range 550-561, indicating delisting by the exchange due to a failure to meet requirements related to minimal price, capital, shareholder or market-maker interest, and 963 observations fall in the code range 580-91, indicating a failure to meet other technical listing requirements – such as a failure to pay fees. Of the performance related delistings, 274 are attributed directly to bankruptcy or insolvency in category 574.

In interpreting model performance it is worth noting the extreme cases. A perfect model would be mapped by a straight line from the origin to 100% on the vertical axis at the percentile of the distribution that corresponds to the overall default rate indicating that it is one hundred percent accurate in classifying the observations in a given percentile. On the other hand, an uninformative model's cumulative classification performance simply reflects the proportion of delistings that one would capture through random sampling, hence its performance is reflected in the 45-degree solid line that serves as the baseline. Informative but imperfect models will plot between the two extremes, and the more closely a given model approximates the perfect extreme the better.

The CAP curves in Exhibit 8 summarize the out-of-sample classification performance of the BSM default likelihood (dash line); model (2) with firm-level market data (solid line) and without (dash-dot line); as well as two variants of Altman's Z'' model. The Z'' model is an updated variant of the $Z-$score model that is applicable to private firm exposures.[15]

Altman and Hotchkiss [2006] provide the most recent publicly available estimate of the Z'' model as follows:

$$Z'' = 3.25 + 6.56\frac{WC}{TA} + 3.26\frac{RE}{TA} + 6.72\frac{EBIT}{TA} + 1.05\frac{BE}{TL} \tag{4}$$

where BE is the book value of equity. Henceforth we refer to fitted values of (4) as Z''.

To the extent that Z'' is a widely used publicly available measure of private firm bankruptcy risk, it serves as a natural benchmark. From the perspective of private firm credit risk modeling, the classification performance of specifications that do not rely on firm-level market variables is of particular interest.

However, it can also be argued that direct comparison of our results with fitted values of model (4) is inappropriate to the extent that the parameters of model (4) are: (1) constrained to be constant across all industry groupings, (2) estimated using data that are misaligned with our estimation and test samples, and (3) estimated using observations of default and non-default rather than expectations. For these reasons we report the performance of the following specification:

$$y_{it} = a_0 + a_1\frac{WC}{TA}_{i,t-l} + a_2\frac{RE}{TA}_{i,t-l} + a_3\frac{EBIT}{TA}_{i,t-l} + a_4\frac{BE}{TL}_{i,t-l} + g_{it}, \tag{5}$$

and henceforth refer to fitted values of model (5) as Z'' *Re-estimated*. Throughout our analysis of predictive performance we estimate the parameters of Model (5) at industry level using the robust regression estimator described earlier.

Cumulative Accuracy Profiles

Exhibit 8 About Here

The results in Exhibit 8 illustrate the out-of-sample classification performance edge available in situations where firm level prices are available. The performance profiles suggest that DLI and its market-leverage based approximation have the strongest out-of-sample classification performance. The extent to which the fundamental information in model (9) captures the default related information in DLI is remarkable if we recall that the main difference between the firm-level information in model (2) and Altman's original Z-score specification is the inclusion of the characteristics Age and $Size$ together with an industry level measure of market expectations. The out-of-sample classification performance of the approximation to DLI is indistinguishable from DLI itself. This finding suggests that the relative default-risk related information in DLI is well captured by a simple, parsimonious specification along the lines of model (2).

If model (2) incorporates a measure of book leverage in place of market leverage, then classification performance of the model never matches that of DLI. For example, the 10th percentile of highest risk firms, based on model (2), estimated using accounting leverage, captures 40% of the performance based delistngs over the subsequent 24-month period, while both DLI and the market-leverage based approximation capture 47% of the same. At the 50th percentile the corresponding cumulative accuracies are 85.5% and 90% respectively. While it is not in itself surprising that risk estimates incorporating market prices outperform specifications restricted to less timely firm-level information, these findings should be considered in light of the empirical distribution of each model's classification performance outcomes (to be discussed shortly).

Of the five specifications considered in Exhibit 8, Z'' (as mapped by the dotted line) exhibits the weakest overall classification performance, consistent with model inputs being restricted to a parsimonious set of accounting ratios for use in evaluating private firm debt from limited accounting data. However, as discussed earlier, a more reasonable basis of comparison is afforded by re-estimating the model parameters at industry level with respect to DLI. As can be seen from the dashed frontier with diamonds in Exhibit 8, such re-estimation results in a dramatic improvement in out-of-sample classification performance: the CAP curve of Z'' re-estimated is close to that of the *Accounting Leverage* specification up to the 30-th percentile. Again, the magnitude of the performance difference between the models apparent in the CAP curves must be considered in light of the empirical distribution of performance outcomes – which we now consider.

Quantifying Classification Accuracy

While the CAP curves in Exhibit 8 provide a graphical summary of each model's classification performance by risk rank, it is difficult to summarize and make cross model comparisons of overall performance. One metric that enables such comparisons is the AUC statistic (henceforth AUC). The AUC measures the probability that a given model will rank a default observation as higher risk than a randomly selected non-default.[16]

We compute the accuracy associated with each model m, AUC_m, as:

$$AUC_m = \frac{\sum_{j=1}^{def} \sum_{i=1}^{N_T} I_{ij,m}}{ndef \times def}, \tag{6}$$

where $I_{ij,m} = 1$ if the risk score of firm i based on model m is less than that of firm j based on model m, and, firm i did not default. Otherwise $I_{ij,m} = 0$. Further, N_T is the number of test sample firms, $ndef$ is the total number of non-defaulters and def the total number of defaulters in the test sample.

Exhibit 9 About Here

Using the results of the re-sampling scheme described earlier, we estimate the empirical distribution of the AUCs associated with the distribution of the set of CAP curve estimates underlying Exhibit 8. Exhibit 9 presents kernel density estimates of the AUCs associated with each model. It is important to note that each realization of AUC for the set of models under consideration is based on the same (randomly drawn) evaluation sample, hence, the AUCs arising from each iteration of the sampling scheme are directly comparable. The AUC distributions presented in Exhibit 9 reflect the variation in model performance arising from sources of variation that are likely to be encountered in practical application. Specifically: variation in the composition of the estimation and evaluation samples; variation in the length of time series used for estimation; sampling error in model parameters and variation in the overall sample default rate.

Exhibit 10 About Here

The distribution estimates in Exhibit 9 and the descriptive statistics in Exhibit 10 suggest that the overall performance of DLI and the market based approximation is indistinguishable: the mean, median and variability of the respective AUC distributions is very close. Both specifications relying on market leverage have a mean AUC of 85%, with almost identical variability. If we substitute market leverage with an accounting measure, the expected value of AUC is 80% and the variability of the measure increases markedly. While the expected AUC of model (2) is clearly lower when market leverage is not available, the overall performance of the models is close in the sense that the mean of each AUC distribution lies within the 5-10% percentile cut-off point in the respective tail of the other.

Consistent with the inference based on visual inspection of the CAP curves, it is clear that the Z'' benchmark lags the more general specifications. However, when Z'' is re-estimated its classification performance is very close to that of the *Accounting Leverage* variant of model (2) with the mean performance of each model lying well within the interquartile range of the other.

Overall, when we quantify the empirical distribution of out-of-sample classification outcomes, model performances are surprisingly close. More generally, it is also clear that out-of-sample classification performance gains do not necessarily accrue in proportion with measures of in-sample model fit. For example, while *Industry Risk* plays a very significant role in improving the in-sample fit of model (2), the out-of sample classification performance of a specification that ignores *Industry Risk*, such as model (5), is much closer than what the regression evidence may lead one to expect.

Performance Evaluation: Time Variation in Aggregate Default Likelihood

We now examine the extent to which variants of model (2) track the variation in subsequent speculative-grade defaults. In doing this we aggregate the bootstrapped out-of-sample model estimates and match them to the subsequent 12-month speculative grade default rate. Exhibit 11 is a graphical summary of the results.

Exhibit 11 About Here

The aggregate out-of-sample performance of both the *Accounting Leverage* and *Market Leverage* based approximations of default likelihood is approximately equal. Time variation in the mean of both series is strongly correlated with the subsequent year's speculative default rate outcome. This is clear from visual inspection of the series plotted in Exhibit 11, and borne out by correlation estimates. That is, the time series correlation between the fitted value of risk from both models and the subsequent default rate is 79%. Neither model matches the corresponding correlation of mean DLI and the subsequent speculative-grade default rate outcome of 85%.

Exhibit 11 illustrates the close correspondence between variants of model (2) and the subsequent default rate, but Exhibit 11 also reveals that the mean fitted value of Z'' *Re-estimated* does not track the subsequent aggregate default outcome. The correlation of mean fitted values from Z'' *Re-estimated* with the subsequent annual default rate is -17%. While the updated variant of Z'' does well at benchmarking relative (cross-sectional) risk, it seems important to augment the specification with some measure of the underlying time-varying hazard rate – and our current findings suggest that industry-level expectations derived from equity price provide a convenient way to do this.

Summary

Practitioners and academics have exploited the theoretical restrictions developed in Merton [1974] to predict distress based on the risk neutral probability of default inferred from equity prices. Recent empirical studies such as Hillegeist, Keating, Cram, and Lundstedt [2004], and Bharath and Shumway [2008] have advocated the value of the approach relative to widely used alternatives.

We model the associations between expectations of default extracted from equity prices, accounting-based measures, firm characteristics and industry-level expectations of distress conditions Such models can be used to approximate the risk of default on an out-of-sample basis using fundamental variables that are generally observable – even when the requisite firm-level equity prices are not.

Models mapping fundamentals to default expectations capture up to 60% of the time and cross-sectional variation in equity-implied default likelihood. More importantly, we demonstrate through a set of re-sampling experiments the ability of fundamentals-based models to rank the relative exposure of firms to the risk of financial distress. We find that such models deliver out-of-sample classification performance that is essentially indistinguishable from that of BSM-default likelihood itself. Our re-sampling experiments are designed to ensure that our findings are not specific to a particular estimation time or cohort within the sample.

Model performance, as measured by in-sample fit and out-of sample classification performance, deteriorates when market leverage is replaced by an accounting measure by approximately 15% and 5%, as measured by regression R^2 and predictive classification ability respectively. However, these models are of particular practical significance, as they are applicable to both public and private firms. We utilize Altman's Z'' model to benchmark specifications applicable to private firms and demonstrate the benefit of re-estimating such models with reference to default likelihood at industry level. In particular, the out-of-sample classification performance of the model can be dramatically improved even without generalizing it to incorporate firm characteristics or industry expectations. Our findings also suggest that the performance of fundamentals-based specifications is improved by industry-level estimation.

Overall, our findings have practical significance for both practitioners and academics wanting to quantify the risk of distress from fundamentals. Similar to the findings of Das, Hanouna, and Sarin [2009] in their study of CDS spreads, our results suggest that arguments for the displacement of accounting-based measures by those inferred from market prices should be re-considered. Our results demonstrate the potential benefit of treating equity-implied default probabilities and

fundamental variables as complementary sources of information regarding expected default rather than alternative sources of predictive information.

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Notes

[1]The KMV Corporation (now Moody's KMV) pioneered a proprietary approach to credit risk measurement based on the default history of companies with a given risk-neutral probability of default.

[2]At the time of writing, the potential economic importance of credit exposures to highly leveraged private entities arising from a wave of LBOs between 2001 and 2007 is of prominent public concern. For example, over the year of 2008, 49 private equity-backed companies have filed for bankruptcy, followed by a further 74 in 2009, and a further 9 in the first quarter of 2010. *Source: 'PE-Backed Busts: Whats in Store for Q4 and Beyond?', Erin Griffith, www.pehub.com, October 2 2009*, and hand-collected data. Most such filings are based on LBO transactions from 2003-2007.

[3]Henceforth we use the terms 'default likelihood' and 'DLI' interchangeably. Vassalou and Xing [2004] use DLI as an abbreviation for 'Default Likelihood Index', to emphasize that default probabilities derived in the option pricing framework are risk neutral probabilities, and as such, provide an ordinal measure of default risk.

[4]Our approach enables estimation of models at the industry level, while such estimation is generally infeasible in the absence of industry-level default data.

[5]Financial statement data is sourced from COMPUSTAT quarterly files. The definitions used are EBIT (OIBDP), Total Assets (ATQ), retained earnings (REQ), current liabilities (DLCQ), non-current liabilities (DLTTQ), working capital (WC).

[6]The industry portfolio classifications correspond to the 17 industry portfolio groupings provided Kenneth French in his data library at:

`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`

[7]The Altman and Rijken [2006] specification is a variant of Altman's (1968) Z-score approach, modified to make it applicable to non-manufacturing as well as manufacturing industrial firms and augmented with the characteristic variables *Age* and *Size* – important determinants of credit ratings from the major rating agencies.

[8]The data is pre-filtered only to the extent that sufficient daily equity return observations were required to compute volatility.

[9]Equity value E_t is from the monthly closing price of equity at time t from CRSP. The volatility of equity, σ_E, is computed as the volatility of continuously compounded daily returns over the year

to time t. We utilize the 1-month treasury bill rate from Ibbotson and associates as the proxy for the risk free rate.

[10]We acknowledge that this modeling choice is arbitrary. However, in the absence of any compelling *ex-ante* alternative, we maintain the assumption to enhance the comparability of our findings with extant work.

[11]In the absence of information on within-quarter reporting dates over most of the sample, our assumption of at least one quarter reporting lag is intentionally conservative.

[12]The Moody's 12-month trailing issuer-weighted speculative grade default rate is from *Datastream*.

[13]Utilities (Fama-French industry group 14 in the 17 industry group classification scheme) is excluded from the current analysis of model fit as the strength and form of the relation between utilities' default likelihood and fundamentals differs markedly from other industries.

[14]Specifically, our sample includes the following codes: 450, 460, 480, 500, 510, 516, 520, 550, 551, 552, 560, 561, 570, 573, 574, 580, 581, 582, 583, 584, 585, 587, & 591. Codes in the 400 range are liquidations. Codes in the 500 range are exchange delistings, for performance-related reasons, including: insufficient number of shareholders (550), price below acceptable level (552), insolvency or bankruptcy (574), and failure to meet listing requirements (580, 581, 584).

[15] For a detailed exposition of Z'' refer to Altman and Hotchkiss [2006].

[16]Refer to Engelmann, Hayden, and Tasche [2003] for a full exposition – including methods to compute confidence intervals. Engelmann, Hayden, and Tasche [2003] demonstrate that this statistic is mathematically equivalent to the Mann-Whitney U statistic. It is also equivalent to a simple transformation of the *accuracy ratio* associated with each CAP curve.

Tables and Figures (for placement in paper)

Exhibit 1: Firm-Month Observations: Descriptive Statistics

Data sources and definitions are provided in Section . The abbreviation IQR refers to interquartile range.

Period		$\frac{EBIT}{TA}$	$\frac{WC}{TA}$	$\frac{RE}{TA}$	$\frac{TA}{TL}$	$\frac{ME}{TL}$	$\frac{CL}{NCL}$	TA ($M)
1978 - 82	Mean	0.04	0.26	0.25	2.54	2.59	4.37	763.96
	Median	0.04	0.26	0.29	1.89	0.97	1.05	111.55
	IQR	0.03	0.29	0.26	0.87	1.66	1.552	379.18
	N	68880						
1983 - 87	Mean	0.02	0.27	0.02	3.00	5.88	8.14	627.50
	Median	0.03	0.27	0.18	1.98	1.69	1.19	44.08
	IQR	0.04	0.35	0.37	1.38	3.60	2.46	188.84
	N	167525						
1988 - 92	Mean	0.02	0.25	-0.16	2.84	5.20	12.12	762.92
	Median	0.03	0.25	0.13	1.92	1.57	1.28	55.94
	IQR	0.04	0.35	0.47	1.44	3.34	3.08	238.45
	N	197391						
1993 - 97	Mean	0.018	0.26	-0.18	3.21	7.28	21.98	873.55
	Median	0.03	0.25	0.08	2.06	2.33	1.37	84.20
	IQR	0.04	0.35	0.47	1.74	4.99	4.08	326.91
	N	241330						
1998 - 02	Mean	0.01	0.25	-0.40	3.30	7.60	25.30	1573.97
	Median	0.03	0.23	0.05	2.05	1.96	1.26	166.89
	IQR	0.05	0.36	0.57	1.87	4.64	4.69	648.52
	N	243675						
2003 - 07	Mean	0.02	0.26	-0.68	3.30	7.29	19.51	2910.91
	Median	0.03	0.23	0.07	2.18	2.86	1.22	329.25
	IQR	0.04	0.35	0.74	1.92	5.55	4.01	1243.68
	N	207605						

Exhibit 2: Mean and Variability of Default Likelihood Estimates: April 1978 - December 2007

This figure plots the default likelihood estimates based on the estimation procedure described in Section **??**. The dashed line is the mean, and the shaded area covers the span of the corresponding (cross-sectional interquartile) range of default likelihood estimates.

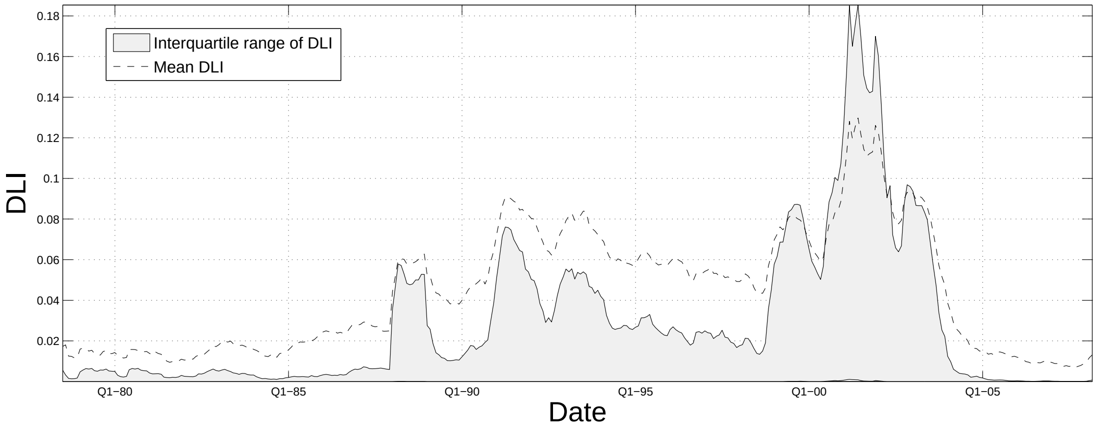

Exhibit 3: Mean Default Likelihood and the Subsequent 12-mth Speculative Grade Default Rate (1978-2008, monthly frequency)

This figure plots the mean default likelihood estimates based on the estimation procedure described in Section **??** (dashed line) and the Moody's 12-mth trailing Issuer-Weighted Speculative Grade Default Rate outcome observed 12 months later (solid line). The plotted series are standardized to have a mean of zero, and scaled to units of standard deviation.

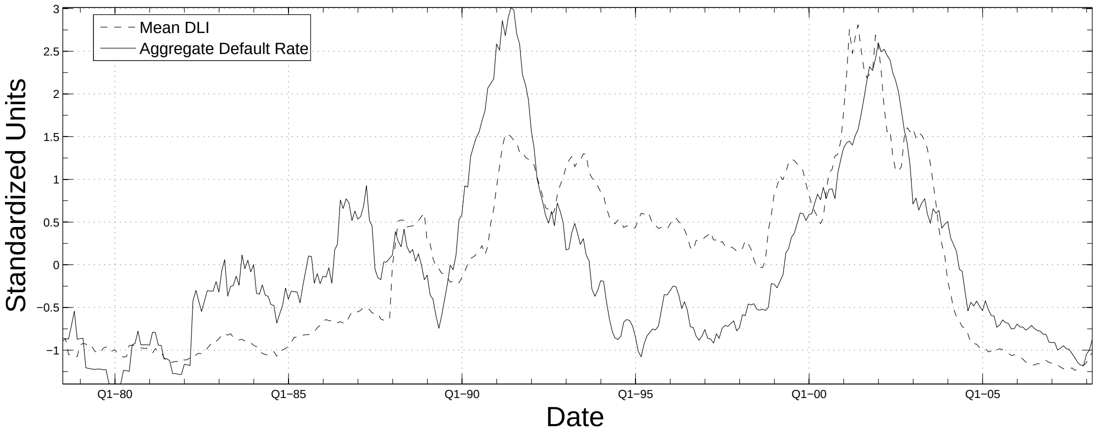

Exhibit 4: Quarterly Predictive Regression: All Firms

Estimates of $y_{it} = \hat{\alpha}_0 + \hat{\alpha}_1 ln\left[1 - \frac{EBIT}{TA}\right]_{i,t-1} + \hat{\alpha}_2\left[\frac{WC}{TA}\right]_{i,t-1} + \hat{\alpha}_3 ln\left[1 - \frac{RE}{TA}\right]_{i,t-1} + \hat{\alpha}_4\left[1 + ln\frac{TA}{TL}\right]_{i,t-1} + \hat{\alpha}_5 IR_{i,t-1} + \hat{\alpha}_6 Size_{i,t-1} + \hat{\alpha}_7 CNCL_{i,t-l} + \hat{\alpha}_8 Age_{i,t-1} + e_{it}$ [Model (2)] presented in this table are based on pooled monthly observations over the sample period April 1978-December 2007, excluding Utilities. Italicized numbers below the coefficient estimates are Newey and West [1987] corrected T-statistics and $\bar{R}^2$ denotes adjusted r-squared. In addition to the variants of equation (2) based on accounting and market measures of leverage, we report estimates of (2) augmented with the variables *Size Risk* and *Age Risk*. The latter variables represent the mean default likelihood of the Size and Age decile portfolios (respectively) to which a firm belongs at time $t-1$.

	Coefficient Estimates: All Firms												
	Constant	$ln\left[1-\frac{EBIT}{TA}\right]$	$\frac{WC}{TA}$	$ln\left[1-\frac{RE}{TA}\right]$	*Leverage*	*Industry Risk*	*Size*	$ln(CNCL)$	*Age*	*Size Risk*	*Age Risk*	$\bar{R}^2$	*N*
Panel A: Firm Characteristics and Industry-Level Expectations													
Accounting Leverage	2.52	-4.25	0.78	-0.81	2.11	-32.25	0.58	-0.13	0.0015			40.0%	1072577
	152	*-55.9*	*49.8*	*-129*	*304*	*-328*	*299*	*-62.6*	*74.8*				
Market Leverage	2.86	-2.12	-0.21	-1.22	1.59	-27.68	0.61	-0.26	0.24			56.9%	1072577
	191	*-39.6*	*-17.8*	*-224*	*722*	*-335*	*361*	*-155*	*86*				
Panel B: Fundamentals based on Altman and Rijken (2006)													
Accounting Leverage	0.90	-4.80	1.16	-0.90	1.91		0.63		0.002			31.90%	1072577
	54.7	*-59.6*	*69.7*	*-127.9*	*259.9*		*291.9*		*82.3*				
Market Leverage	0.85	-2.74	-0.15	-1.25	1.55		0.693		0.30			49.80%	1072577
	53.3	*-46.6*	*-11.8*	*-206*	*696*		*377*		*88.5*				
Panel C: Adding Factors Associated with Size and Age													
Accounting Leverage	3.58	-4.14	0.74	-0.60	2.15	-10.62	0.41	-0.11	0.001	-15.64	-12.55	42.4%	1072577
	207	*-55.8*	*47.8*	*-115*	*313*	*-72.3*	*171.2*	*-57.2*	*31.5*	*-107*	*-112.1*		
Market Leverage	4.09	-2.09	-0.20	-1.11	1.59	-7.37	0.42	-0.25	0.20	-13.62	-12.8	59.1%	1072577
	265	*-40.2*	*-17.5*	*-210*	*731*	*-60.6*	*208*	*-148*	*71.5*	*-118*	*-142*		

Exhibit 5: Quarterly Predictive Regressions based on Industry Groupings (Accounting Leverage)

Estimates of $y_{it} = \hat{\alpha}_0 + \hat{\alpha}_1 ln\left[1 - \frac{EBIT}{TA}\right]_{i,t-1} + \hat{\alpha}_2\left[\frac{WC}{TA}\right]_{i,t-1} + \hat{\alpha}_3 ln\left[1 - \frac{RE}{TA}\right]_{i,t-1} + \hat{\alpha}_4\left[1 + ln\frac{TA}{TL}\right]_{i,t-1} + \hat{\alpha}_5 IR_{i,t-1} + \hat{\alpha}_6 Size_{i,t-1} + \hat{\alpha}_7 CNCL_{i,t-l} + \hat{\alpha}_8 Age_{i,t-1} + e_{it}$ [Model (2)] presented in this table are based on pooled monthly observations over the sample period April 1978-December 2007 within Fama-French industry groups. Italicized numbers below the coefficient estimates are Newey and West [1987] corrected T-statistics and $\bar{R}^2$ denotes adjusted r-squared.

	Coefficient Estimates: By Industry										
Industry	*Constant*	$ln\left[1-\frac{EBIT}{TA}\right]$	$\frac{WC}{TA}$	$\frac{RE}{TA}$	$\left[1+ln\frac{TA}{TL}\right]$	*Industry Risk*	*Size*	*ln(CNCL)*	*Age*	$\bar{R}^2$	*N*
1. Food	2.27	-6.75	1.51	-0.99	1.73	-27.47	0.76	-0.06	0.00		
	24.26	*-14.00*	*14.65*	*-29.92*	*44.79*	*-39.44*	*77.00*	*-4.14*	*8.58*	*0.44*	*36528*
2. Mining, Minerals	1.46	-4.83	1.42	-1.14	2.24	-20.18	0.60	-0.23	0.00		
	21.14	*-16.79*	*17.36*	*-34.63*	*85.16*	*-40.73*	*78.83*	*-23.74*	*21.39*	*0.42*	*66050*
3. Oil & Petroleum	0.15	-14.12	-0.62	-1.04	3.07	-30.19	0.62	-0.10	0.00		
	1.22	*-23.90*	*-4.69*	*-26.43*	*46.38*	*-45.31*	*44.50*	*-6.59*	*15.68*	*0.45*	*27568*
4. Textile, Appareil, Footware	1.21	-4.98	0.55	-1.21	2.55	-29.68	0.67	-0.21	0.00		
	12.68	*-9.86*	*5.42*	*-31.77*	*56.47*	*-48.29*	*57.21*	*-16.65*	*13.38*	*0.42*	*38788*
5. Consumer Durables	2.32	-1.79	2.23	-1.31	1.59	-34.86	0.75	0.09	0.00		
	17.50	*-2.87*	*14.98*	*-29.38*	*24.60*	*-28.39*	*61.29*	*4.81*	*8.49*	*0.46*	*22170*
6. Chemicals	3.85	-0.37	1.64	-0.27	1.41	-44.56	0.61	-0.06	0.00		
	56.22	*-1.50*	*20.68*	*-10.10*	*53.15*	*-57.99*	*62.05*	*-6.00*	*24.05*	*0.37*	*41718*
7. Drugs, Soap, Tobacco	1.13	-10.14	0.37	-1.40	2.27	-26.24	0.73	-0.11	0.00		
	10.88	*-17.71*	*3.72*	*-30.12*	*45.24*	*-40.56*	*68.52*	*-8.30*	*13.20*	*0.42*	*38866*
8. Constuction & Materials	0.82	-12.80	0.43	-1.80	2.80	-27.90	0.39	-0.15	0.00		
	5.18	*-10.06*	*2.50*	*-22.89*	*29.90*	*-38.52*	*23.63*	*-5.31*	*15.34*	*0.41*	*20426*
9. Steel Works	2.40	-8.21	1.83	-1.47	1.32	-22.73	0.70	0.07	0.00		
	14.59	*-7.10*	*9.59*	*-26.40*	*16.48*	*-22.69*	*33.50*	*2.32*	*9.92*	*0.43*	*12001*
10. Fabricated Products	2.88	-4.38	0.27	-0.83	2.11	-38.97	0.47	-0.16	0.00		
	73.82	*-21.60*	*5.55*	*-52.11*	*118.66*	*-140.18*	*99.11*	*-29.47*	*49.33*	*0.40*	*168087*
11. Machinery & Business	2.85	-1.45	2.49	-1.28	1.56	-18.92	0.58	-0.19	0.00		
	23.92	*-3.54*	*22.35*	*-30.73*	*38.72*	*-30.60*	*38.94*	*-11.62*	*0.93*	*0.43*	*19161*
12. Automobiles	1.19	-7.87	0.07	-0.96	2.86	-32.81	0.54	-0.05	0.00		
	9.21	*-8.20*	*0.48*	*-13.59*	*42.72*	*-44.14*	*38.59*	*-2.30*	*7.56*	*0.46*	*16284*
13. Transportation	0.15	-8.07	1.19	-1.43	3.09	-30.92	0.63	-0.04	0.00		
	1.38	*-11.10*	*12.26*	*-25.41*	*57.87*	*-50.85*	*64.51*	*-4.06*	*15.99*	*0.45*	*46428*
14. Retail Stores	0.67	-11.65	-0.84	-1.03	2.97	-33.39	0.76	-0.16	0.00		
	9.62	*-33.22*	*-16.63*	*-38.02*	*96.14*	*-98.02*	*97.74*	*-20.03*	*-0.85*	*0.46*	*83701*
15. Financial	4.47	-7.80	1.82	-0.75	1.33	-25.62	0.43	-0.10	0.00		
	50.37	*-20.42*	*22.49*	*-20.60*	*44.75*	*-40.44*	*36.59*	*-10.87*	*5.14*	*0.32*	*34427*
16. Other	3.15	-3.68	1.05	-0.74	1.93	-35.08	0.58	-0.12	0.00		
	120.29	*-33.49*	*40.63*	*-75.51*	*171.81*	*-235.04*	*175.86*	*-37.35*	*50.98*	*0.41*	*400527*

Exhibit 6: Quarterly Predictive Regressions based on Industry Groupings (Market Leverage)

Estimates of $y_{it} = \hat{\alpha}_0 + \hat{\alpha}_1 ln\left[1 - \frac{EBIT}{TA}\right]_{i,t-1} + \hat{\alpha}_2\left[\frac{WC}{TA}\right]_{i,t-1} + \hat{\alpha}_3 ln\left[1 - \frac{RE}{TA}\right]_{i,t-1} + \hat{\alpha}_4\left[1 + ln\frac{ME}{TL}\right]_{i,t-1} + \hat{\alpha}_5 IR_{i,t-1} + \hat{\alpha}_6 Size_{i,t-1} + \hat{\alpha}_7 CNCL_{i,t-l} + \hat{\alpha}_8 Age_{i,t-1} + e_{it}$ [Model (2)] presented in this table are based on pooled monthly observations over the sample period April 1978-December 2007 within Fama-French industry groups. Italicized numbers below the coefficient estimates are Newey and West [1987] corrected T-statistics and $\bar{R}^2$ denotes adjusted r-squared.

	Coefficient Estimates: By Industry										
Industry	*Constant*	$ln\left[1 - \frac{EBIT}{TA}\right]$	$\frac{WC}{TA}$	$\frac{RE}{TA}$	$\left[1 + ln\frac{ME}{TL}\right]$	*Industry Risk*	*Size*	*ln(CNCL)*	*Age*	$\bar{R}^2$	*N*
1. Food	3.56	-3.16	1.59	-0.83	1.25	-29.01	0.81	-0.16	0.00		
	61.07	*-9.15*	*19.62*	*-27.39*	*92.04*	*-47.38*	*89.35*	*-13.42*	*-1.67*	*0.54*	*36528*
2. Mining, Minerals	2.13	-2.07	0.30	-1.24	1.68	-6.18	0.69	-0.26	0.00		
	47.96	*-9.23*	*4.41*	*-42.80*	*160.54*	*-13.78*	*101.29*	*-31.74*	*14.76*	*0.56*	*66050*
3. Oil & Petroleum	3.88	-5.46	-0.02	-1.18	1.56	-28.67	0.54	-0.24	0.00		
	51.83	*-13.98*	*-0.27*	*-30.38*	*94.96*	*-48.79*	*42.88*	*-18.96*	*25.53*	*0.56*	*27568*
4. Textile, Appareil, Footware	3.65	-2.84	-0.22	-1.36	1.71	-28.22	0.68	-0.33	0.00		
	66.59	*-8.89*	*-2.95*	*-41.22*	*140.50*	*-55.67*	*64.39*	*-33.45*	*18.76*	*0.59*	*38788*
5. Consumer Durables	2.29	-2.15	0.68	-1.08	1.61	-29.73	0.87	-0.14	0.00		
	31.10	*-5.08*	*7.01*	*-28.34*	*86.08*	*-29.95*	*78.45*	*-9.11*	*8.87*	*0.61*	*22170*
6. Chemicals	3.86	-0.44	0.45	-0.88	1.46	-44.58	0.58	-0.17	0.00		
	74.59	*-2.36*	*7.70*	*-37.28*	*137.19*	*-67.34*	*63.91*	*-21.76*	*14.39*	*0.55*	*41718*
7. Drugs, Soap, Tobacco	3.11	-3.77	0.15	-1.71	1.54	-22.46	0.75	-0.24	0.00		
	54.39	*-10.31*	*1.87*	*-48.20*	*115.03*	*-39.64*	*77.08*	*-19.07*	*13.92*	*0.55*	*38866*
8. Constuction & Materials	3.92	-2.06	-0.30	-1.76	1.54	-22.34	0.46	-0.30	0.00		
	47.99	*-2.70*	*-2.41*	*-26.53*	*81.44*	*-34.27*	*31.20*	*-12.10*	*19.79*	*0.54*	*20426*
9. Steel Works	3.12	-3.24	-0.17	-1.32	1.57	-18.79	0.76	-0.11	0.00		
	32.42	*-4.83*	*-1.20*	*-27.70*	*53.43*	*-21.95*	*41.72*	*-4.29*	*11.43*	*0.56*	*12001*
10. Fabricated Products	3.99	-2.55	-0.50	-1.32	1.61	-32.06	0.42	-0.28	0.00		
	143.58	*-17.82*	*-14.17*	*-95.10*	*265.77*	*-122.33*	*95.43*	*-63.53*	*71.25*	*0.55*	*168087*
11. Machinery & Business Equip.	2.87	1.16	2.17	-1.60	1.31	-12.88	0.66	-0.16	0.00		
	38.05	*3.38*	*24.18*	*-42.06*	*95.70*	*-25.16*	*47.53*	*-11.41*	*8.64*	*0.57*	*19161*
12. Automobiles	3.65	-4.06	0.34	-1.07	1.51	-28.57	0.65	-0.19	0.00		
	40.06	*-6.97*	*2.99*	*-18.14*	*75.62*	*-45.91*	*48.12*	*-9.51*	*9.77*	*0.58*	*16284*
13. Transportation	3.25	-1.37	0.23	-1.59	1.77	-25.93	0.66	-0.22	0.00		
	66.83	*-3.54*	*3.19*	*-37.26*	*141.15*	*-50.73*	*80.80*	*-23.44*	*18.63*	*0.61*	*46428*
14. Retail Stores	3.83	-4.30	-0.72	-1.19	1.65	-29.89	0.65	-0.32	0.00		
	111.26	*-19.45*	*-17.56*	*-52.03*	*180.75*	*-102.27*	*101.48*	*-47.83*	*12.76*	*0.62*	*83701*
15. Financial	3.76	-4.12	0.38	-0.89	1.43	-22.37	0.61	-0.19	0.00		
	61.35	*-13.63*	*5.58*	*-28.02*	*115.48*	*-42.79*	*62.98*	*-25.08*	*14.99*	*0.51*	*34427*
16. Other	3.67	-2.68	-0.43	-1.10	1.66	-29.98	0.58	-0.27	0.00		
	225.52	*-34.07*	*-22.51*	*-135.24*	*444.24*	*-241.16*	*201.95*	*-103.96*	*85.65*	*0.59*	*400527*

Exhibit 7: Relative Influence based on Industry Groupings (Accounting Leverage)

This table reports estimates of relative influence $RW_k = \frac{|\hat{\alpha}_k|\sigma_k}{\sum_{j=1}^{8}|\hat{\alpha}_j|\sigma_j}$, for the coefficients of model (2): $y_{it} = \hat{\alpha}_0 + \hat{\alpha}_1 ln\left[1 - \frac{EBIT}{TA}\right]_{i,t-1} + \hat{\alpha}_2\left[\frac{WC}{TA}\right]_{i,t-1} + \hat{\alpha}_3 ln\left[1 - \frac{RE}{TA}\right]_{i,t-1} + \hat{\alpha}_4\left[1 + ln\frac{TA}{TL}\right]_{i,t-1} + \hat{\alpha}_5 IR_{i,t-1} + \hat{\alpha}_6 Size_{i,t-1} + \hat{\alpha}_7 CNCL_{i,t-l} + \hat{\alpha}_8 Age_{i,t-1} + e_{it}$, by industry group, as reported in Table 5, and for the pooled estimates reported in Table 4.

	Relative Weight Estimates: By Industry							
Industry	$ln\left[1-\frac{EBIT}{TA}\right]$	$\frac{WC}{TA}$	$\frac{RE}{TA}$	$\left[1+ln\frac{TA}{TL}\right]$	*Industry Risk*	*Size*	$ln(CNCL)$	*Age*
1. Food	0.07	0.06	0.12	0.19	0.14	0.36	0.02	0.03
2. Mining, Minerals	0.05	0.05	0.11	0.28	0.11	0.28	0.06	0.06
3. Oil & Petroleum	0.13	0.02	0.11	0.29	0.18	0.19	0.03	0.06
4. Textile, Appareil, Footware	0.05	0.02	0.14	0.28	0.16	0.24	0.06	0.05
5. Consumer Durables	0.02	0.10	0.17	0.18	0.13	0.35	0.03	0.04
6. Chemicals	0.008	0.10	0.05	0.24	0.20	0.30	0.02	0.09
7. Drugs, Soap, Tobacco	0.10	0.02	0.15	0.23	0.14	0.29	0.03	0.05
8. Constuction & Materials	0.10	0.02	0.17	0.24	0.20	0.16	0.03	0.09
9. Steel Works	0.07	0.09	0.19	0.15	0.15	0.28	0.02	0.07
10. Fabricated Products	0.06	0.01	0.11	0.27	0.24	0.18	0.05	0.08
11. Machinery & Business Equip.	0.02	0.12	0.16	0.24	0.18	0.23	0.06	0.004
12. Automobiles	0.07	0.003	0.10	0.30	0.24	0.23	0.01	0.05
13. Transportation	0.07	0.05	0.12	0.27	0.16	0.26	0.01	0.05
14. Retail Stores	0.11	0.03	0.10	0.27	0.20	0.25	0.04	0.002
15. Financial	0.12	0.11	0.11	0.23	0.18	0.20	0.04	0.02
16. Other	0.05	0.05	0.10	0.24	0.25	0.22	0.04	0.05
POOLED	0.05	0.05	0.08	0.29	0.08	0.32	0.05	0.08
Cross Sectional Median	0.07	0.05	0.12	0.24	0.18	0.25	0.03	0.052
Cross Sectional Std	0.04	0.04	0.04	0.04	0.04	0.06	0.02	0.03

Exhibit 8: Cumulative Accuracy Profile: Out of Sample, Out of Time

This figure presents the mean Cumulative Accuracy Profile (CAP) curve estimated using 1,000 draws of the sampling scheme described in Section . The dashed line plots the performance of the unadjusted BSM default likelihood, the solid line plots model (2) employing market leverage, the dash-dot line plots model (2) employing accounting leverage, the dotted line plots Z'', and the dashed line with circles plots Z'' *Re-estimated*. The line of '+' signs is the naive benchmark. All predictive regression models are estimated by industry using robust regression.

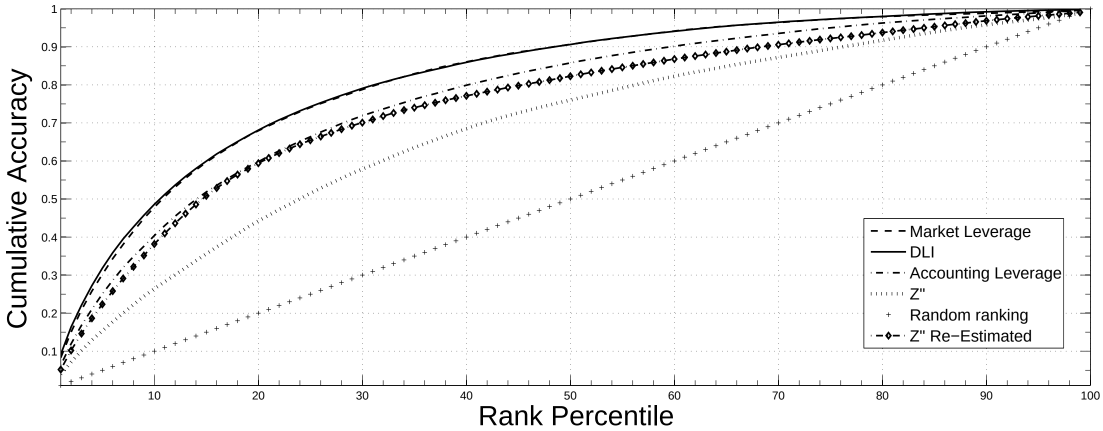

Exhibit 9: Empirical Distribution of AUC Statistics

This figure presents the kernel density estimates of the AUC distributions estimated using 1,000 draws of the sampling scheme described in Section . The dashed line plots the performance of the unadjusted BSM default likelihood, the solid line plots model (2) employing market leverage, the dash-dot line plots model (2) employing accounting leverage, the dotted line plots Z'', and the dashed line with circles plots Z'' *Re-estimated*. The AUC pertaining to each model is computed using equation (6) for each test sample. The density estimates in this figure are based on a Normal kernel function.

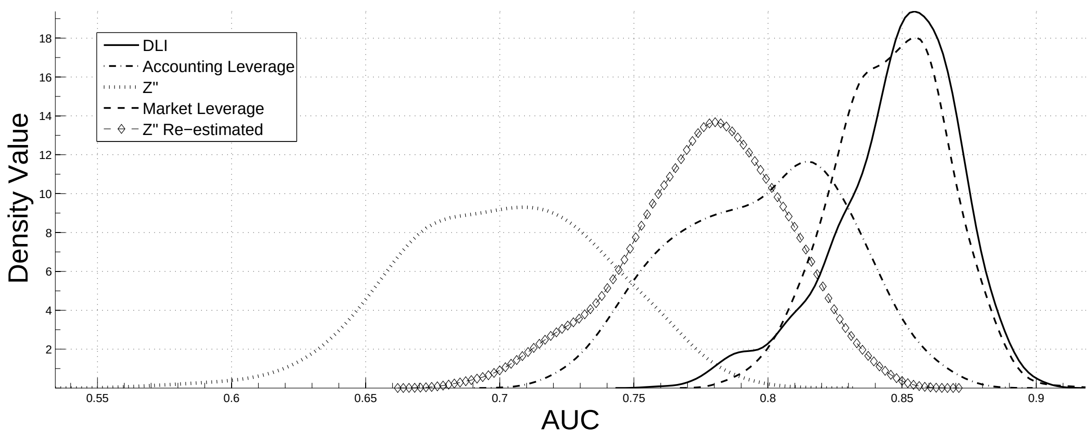

Exhibit 10: Characteristics of AUC Distributions

The statistics in this table describe the characteristics of the AUC distributions estimated using 1,000 draws of the sampling scheme described in Section . The AUC pertaining to each model is computed using equation (6) for each test sample. “IQR” refers to interquartile range. *DLI* is the unadjusted BSM default likelihood, *Market Leverage* refers to model (2) estimated using market leverage, and *Accounting Leverage* is the corresponding estimate based on accounting leverage. Z'' refers to the estimates based on the coefficient estimates reported in Altman and Hotchkiss [2006], while Z'' *Re-estimated* refers to the same model with updated coefficient estimates (mapped to default likelihood).

	DLI	*Market Leverage*	*Accounting Leverage*	Z''	Z'' *Re-estimated*
Mean	0.85	0.85	0.80	0.71	0.77
Median	0.85	0.85	0.80	0.71	0.78
IQR	0.03	0.03	0.05	0.06	0.04
Percentile 10	0.82	0.82	0.76	0.66	0.74
Percentile 90	0.87	0.87	0.84	0.75	0.81

Exhibit 11: Aggregate Predictive Performance: Out of Sample, Out of Time

This figure plots the average estimate of risk based on model (2) employing accounting leverage (dash-dot line), market leverage (solid line) and Z'' *Re-estimated* (dashed line with circles), together with Moody’s 12-mth trailing Issuer-Weighted Speculative Grade Default Rate outcome observed 12 months after the forecast (solid line with crosses). Using the output of the sampling scheme described in Section , the mean value of the fitted forecast formed at each date during the test period is estimated and plotted The plotted series are standardized to have a mean of zero, and scaled to units of standard deviation.