INTERNATIONAL

DIVERSIFICATION

Fall 2002

International

Examining international diversification serves two purposes:

a. Interesting by itself

b. It's a nice application of prior discussion.

Return and risk of buying foreign securities

Some Underlying Relationships

1. Effects of Exchange Risk

Time	Cost of 1 Mark	Value of German Bond	Value in Dollars
0	.50	920 DM	.50 x 920 = 460
1	.40	1000DM	.4 x 1000 = 400

Return in Germany:

$$\left(1+R_H\right)=\frac{1000}{920} or\ R_H=8.70\%$$

Return in U.S.:

$$\left(1+R_{US}\right)=\frac{400}{600} or\ R_{US}=-13.04\%$$

$$\left(1+R_{US}\right)=\frac{.40\cdot 1000}{.50\cdot 920}=\frac{40}{60}$$

$$=\left(\frac{.40}{.50}\right)\left(\frac{1000}{920}\right)$$

Define Exchange Return As:

$$\left(1+R_X\right)=\frac{.40}{.50}$$

$$R_X=-.20\%$$

$$\left(1+R_{US}\right)=\left(1+R_X\right)\left(1+R_H\right)$$

$$1+R_{US}=1+R_X+R_H+R_X R_H$$

$$R_{US}=1-.20+.0870+\left(-.20\right)\left(.0870\right)$$

Normally drop $R_X R_H$ term arguing it's small.

$$\bar{R}_{US} \cong \bar{R}_X + \bar{R}_H$$

$$\bar{R}_{US} \cong -11.30\%$$

$$Var\left(R_{US}\right) \cong Var\left(R_X + R_H\right)$$

$$Var\left(R_{US}\right) = Var\left(R_X\right) + Var\left(R_H\right) + Cov\left(R_X R_H\right)$$

Normally drop covariance arguing it's small.

$$Var\left(R_{US}\right) \cong Var\left(R_X\right) + Var\left(R_H\right)$$

Effect on variance different than on standard deviation

$$\left[Var\left(R_{US}\right)\right]^{1/2} \cong \left[Var\left(R_X\right) + Var\left(R_H\right)\right]^{1/2}$$

$$Var\left(R_X\right) = .25 \qquad Std.Dev.\left(R_H\right) = .5$$

$$Var\left(R_X\right) = .16 \qquad Std.Dev.\left(R_X\right) = .4$$

$$Var\left(R_{US}\right) = \left[.25 + .16\right]^{1/2}$$

$$Std.Dev.\left(R_{US}\right) = .64$$

Condition for non-U.S. fund to be held by U.S. investors.

Consider constant correlation model:

$$Z_N = \frac{1}{\rho\sigma_N}\left[\frac{\bar{R}_N - R_F}{\sigma_N} - \frac{\rho}{1+\rho-M\rho}\Sigma\frac{\bar{R}_R - R_F}{\sigma_R}\right]$$

Consider two security cases. $Z_i = 0$ **if term in brackets is greater than zero.**

$$\frac{\bar{R}_N - R_F}{\sigma_N} - \frac{\rho}{1-\rho}\left[\frac{\bar{R}_N - R_F}{\sigma_N} + \frac{\bar{R}_A - R_F}{\sigma_A}\right] > 0$$

$$\frac{\bar{R}_N - R_F}{\sigma_N}\left[1 - \frac{\rho}{1-\rho}\right] > \frac{\rho}{1-\rho}\left[\frac{\bar{R}_A - R_F}{\sigma_A}\right]$$

$$\frac{\bar{R}_N - R_F}{\sigma_N}\left[\frac{1}{1-\rho}\right] > \frac{\rho}{1-\rho}\left[\frac{\bar{R}_A - R_F}{\sigma_A}\right]$$

$$\left(\bar{R}_N - R_F\right) > \left[\bar{R}_A - R_F\right]\left(\frac{\sigma_A}{\sigma_N}\right)\rho_{NA}$$

Concepts:

1. **Mean return security & portfolio**

2. **Correlation security & portfolio**

3. **Standard deviation security & portfolio**

4. **Efficient frontier:**
   a. **No lending & borrowing**
   b. **Lending & borrowing at same rate**
   c. **Lending & borrowing at different rates**
   d. **Capital market line**

5. **Estimating correlations or covariances:**
   a. **Single-index model**
   b. **Multi-index model**
   c. **Correlation models**

6. **Solving simple problems:**
   a. **General solution**
   b. **Simple rules**

7. **Added problems when assets in different countries**