LIABILITIES

EFFECTS

Fall 2002

Surplus = Assets - Liabilities

$$S_1 = A_1 - L_1$$ **at one**

$$S_0 = A_0 - L_0$$ **at zero**

$$\frac{S_1}{S_0} = \left(1 + r_s\right) = \frac{A_1}{S_0} - \frac{L_1}{S_0}$$

$$= \frac{A_1}{A_0}\left(\frac{A_0}{S_0}\right) - \frac{L_1}{L_0}\left(\frac{L_0}{S_0}\right)$$

$$= \left(1 + r_A\right)\frac{A_0}{S_0} - \left(1 + r_L\right)\frac{L_0}{S_0}$$

$$R_S = \left(\frac{A_0}{S_0}\right) r_A - \left(\frac{L_0}{S_0}\right) r_L$$

Mean $\overline{R}_S$

$$\overline{R}_S = \left(\frac{A_0}{S_0}\right) \overline{R}_A - \left(\frac{L_0}{S_0}\right) \left(\overline{R}_L\right)$$

Variance r_S

$$\sigma_S^2 = \left(\frac{A_0}{S_0}\right)^2 \sigma_A^2 + \left(\frac{L_0}{S_0}\right)^2 \sigma_L^2 - 2\left(\frac{A_0}{S_0}\right)\left(\frac{L_0}{S_0}\right) \rho_{AL} \sigma_A \sigma_L$$

What can manager influence:

1. **Can not affect** A_0, L_0 or S_0

2. **Can not affect** $\bar{r}_L$ or σ_L

3. **Can only affect** $\rho_{AL}, \bar{R}_A$ and σ_A

Note if $\rho_{AL} = 0$ **can concentrate on asset allocation and ignore liabilities but being conscience that mean return and variance of surplus are affected by liabilities when looking at trade off.**

What if $\rho_{AL} \neq 0$**?**

1. **What are desirable assets?**

Those assets that serve hedge functions.

Unlike normal mean variance, portfolio manager needs to be concerned with $\sigma_A, \bar{R}_A$, and ρ_{AL}.

Can view as three-dimensional with the following properties:

1. **max** $\bar{R}_A$

2. **min** σ_A

3. **max** ρ_{AL}

Efficient set defined over these three.

Consider, however, trade off. Terms that influence variance under managers control are:

$$\sigma_S^2 = \left(\frac{A_0}{S_0}\right)^2 \sigma_A^2 + \left(\frac{L_0}{S_0}\right)^2 \sigma_L^2 - 2\frac{A_0}{S_0}\frac{L_0}{S_0}\rho_{AL}\sigma_A\sigma_L$$

$$\sigma_S^2 = \left(\frac{L_0}{S_0}\right)^2 \sigma_L^2 + \left(\frac{2A_0\sigma_L L_0}{S_0^2}\right)\left[\left(\frac{1}{2\sigma_L}\frac{A_0}{L_0}\right)\sigma_A^2 - \rho_{AL}\sigma_A\right]$$

Note first term and term in front of brackets is not under manager's control, thus manager can only control

$$\left[\left(\frac{1}{2\sigma_L}\frac{A_0}{L_0}\right)\sigma_A^2 - \rho_{AL}\sigma_A\right]$$

as long as this term is constant risk unchanged and this term measures trade-off.

For example, assume:

1. $A_0 = 100$
2. $L_0 = 80$
3. $\sigma_L = 20$

then $\frac{1}{2\sigma_L}\frac{A_0}{L_0} = \frac{1}{40}\frac{100}{80} = \frac{1}{32}$

and I would be equally happy to choose a portfolio with σ_A^2 up 10 if $\rho_{AL}\sigma_A$ was also up $\left(\frac{1}{32}\right)\cdot 10$.

This explicit tradeoff allows me to collapse the choice into mean return and standard deviation.

Example 2:

Shape and TINIC suggests giving less than full credit to the risk reducing aspects of liabilities because, among other reasons, uncertainty in their estimated value.

Asset Classes	ρ_{iL}	σ_i
Intermediate Bonds	**.20**	**8**
Growth Stocks	**.65**	**20**

$$A_0 = 100$$
$$L_0 = 80$$
$$\sigma_L = 20$$

$$\frac{1}{2}\frac{A_0}{L_0}\frac{\sigma_A^2}{\sigma_L}$$

Intermediate $\frac{1}{2}\frac{100}{80}\frac{64}{20}=2$

Growth $\frac{1}{2}\frac{100}{80}\frac{400}{20}=12.5$

and term in paranthesis

(2 - 1.6) = .4

(12.5 - 13) = -.5

and the higher "risk" asset is actually less risky. It is possible, however, that both assets should enter.

Note last term in brackets is:

$$\frac{\operatorname{cov}\left(R_A R_L\right)}{\sigma_A \sigma_L} \sigma_A$$

or

$$\frac{\operatorname{cov}(R_A R_L)}{\sigma_L}$$

But

$$\operatorname{Cov}\left(R_A R_L\right) = \Sigma X_i \operatorname{Cov}\left(R_i R_L\right)$$

so individual assets enter linearly. The affect on last term in brackets can be looked at one term at a time. However, this does not hold for σ_A.