# EQUILIBRIUM PRICING THEORIES

Fall 2002

#### A. CAPM

#### **Building Blocks**

- (1) Law of one price.
- (2) Markets dominated by investors with welldiversified portfolios.

Law of one price = two identical items must sell at same price.

Markets dominated by well-diversified portfolios.

- (1) Major portion of trading
- (2) Prices set by information traders
- (3) Small investors have well-diversified portfolios

**Consider three assets:** 

<u>#</u>	R	$\underline{\beta}$
Α	21	2
В	13	1
С	16	1.5

How do all combinations of A & B plot

$$R_{c} = X_{A} (21) + (1 - X_{A}) 13 = 13 + 8X_{A} (1)$$
$$\beta_{c} = X_{A} 2 + (1 - X_{A}) = 1 + X_{A} (2)$$

**Rearranging (2)** 

$$X_A = \beta_c - 1$$

$$\overline{R}_{c} = 13 + 8(\beta_{c} - 1) = 5 + 8\beta_{c}$$

How does it plot?

Any assets off line have violation of law of one price?

**General form:** 

$$\overline{R}_{i} = C_{0} + C_{1}\beta_{i}$$

If you have riskless asset  $\beta_1 = 0$  and you have

$$\overline{R}_{i} = C_{0} = R_{F}$$

$$R_F = C_0$$

Consider market portfolio for this portfolio  $\beta_1 \!=\! 1$  thus

$$\overline{R}_{m} = C_{0} + C_{1}(1) \text{ and } C_{1} = \overline{R}_{m} - C_{0} = \overline{R}_{m} - R_{F}$$

and

$$\overline{R}_{i} = R_{F} + \beta_{i}(\overline{R}_{m} - R_{F})$$

**Standard CAPM** 

$$\beta_{i} = \frac{\operatorname{Cov}\left(\operatorname{R}_{i}\operatorname{R}_{m}\right)}{\operatorname{Var}\left(\operatorname{R}_{m}\right)}$$

$$\overline{R}_{i} = R_{F} + \frac{Cov\left(R_{i}R_{m}\right)}{Var\left(R_{m}\right)}\left(\overline{R}_{m} - R_{F}\right)$$

$$\overline{R}_{i} = R_{F} + \lambda Cov \left( R_{i} R_{M} \right)$$

 $\lambda = \frac{\overline{R}_{m} - R_{F}}{Var[R_{m}]}$ 

reward for time reward for risk market reward risk ratio If riskless asset does not exist then define zero beta portfolios  $\left(\overline{R}_{Z}\right)$ 

$$R_z = C_0 + C_1(0)$$

and

Zero Beta Form

$$\overline{R}_{i} = \overline{R}_{z} + \beta_{i}(\overline{R}_{m} - \overline{R}_{z})$$

### **ARBITRAGE PRICING THEORY**

## A multi-index model explains security returns

$$R_{it} = C_{o} + \beta_{ip} \begin{pmatrix} un \text{ explained} \\ production \\ changes \end{pmatrix} + \beta_{iT} \begin{pmatrix} Term \\ premium \end{pmatrix} + \beta_{iD} \begin{pmatrix} Default \\ premium \end{pmatrix} + e_{it}$$

Systematic risk factors 
$$egin{pmatrix} oldsymbol{eta},oldsymbol{eta}$$

$$\overline{R}_{c} = 8X_{1} + 6X_{2} + (1 - X_{1} - X_{2})9$$
  
$$\beta_{1} = 2X_{1} + 1X_{2} + (1 - X_{1} - X_{2})3$$
  
$$\beta_{2} = 1X_{1} + 1X_{2} + (1 - X_{1} - X_{2})2$$

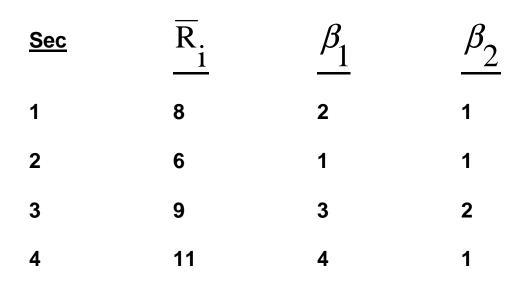
Eliminating 
$$\mathrm{X}_1$$
 and  $\mathrm{X}_2$ 

$$\overline{R}_{c} = 5 + 2\beta_{1} - \beta_{2}$$

and security 4 is mis-priced

Same argument (e.g., Law of One Price) two securities with same risk characteristics must have same expected return. With 3 risk variables, we can match the characteristics of any security with a portfolio of three other securities. Since Betas and expected returns are linear, the relationship is linear.

#### Example



$$\overline{R}_{c} = 8X_{1} + 6X_{2} + (1 - X_{1} - X_{2})9$$
  
$$\beta_{1} = 2X_{1} + 1X_{2} + (1 - X_{1} - X_{2})3$$
  
$$\beta_{2} = 1X_{1} + 1X_{2} + (1 - X_{1} - X_{2})2$$

Eliminating 
$$\boldsymbol{X}_1$$
 and  $\boldsymbol{X}_2$ 

$$\overline{R}_c = 5 + 2\beta_1 - \beta_2$$

and security 4 is mis-priced.

In general, if return generating process:

$$R_{it} = C_0 + \beta_{i1}f_1 + \beta_{i2}f_2 + \beta_{i3}f_3 + e_{it}$$

Expected return is:

$$\overline{R}_{i} = R_{F} + \beta_{i1}\lambda_{1} + \beta_{i2}\lambda_{2} + \beta_{i3}\lambda_{3}$$

$$\lambda_i = \overline{R}_i - R_F$$