EQUILIBRIUM PRICING THEORIES

Fall 2002

A. CAPM

<u>Building Blocks</u>

(1) Law of one price.

(2) Markets dominated by investors with well-diversified portfolios.

Law of one price = two identical items must sell at same price.

Markets dominated by well-diversified portfolios.

(1) Major portion of trading
(2) Prices set by information traders
(3) Small investors have well-diversified portfolios

Consider three assets:

#	$\bar{R}$	β
A	21	2
B	13	1
C	16	1.5

How do all combinations of A & B plot

$$R_C = X_A(21) + (1 - X_A)13 = 13 + 8X_A \quad \textbf{(1)}$$

$$\beta_C = X_A 2 + (1 - X_A) = 1 + X_A \quad \textbf{(2)}$$

Rearranging (2)

$$X_A = \beta_C - 1$$

$$\overline{R}_C = 13 + 8(\beta_C - 1) = 5 + 8\beta_C$$

How does it plot?

Any assets off line have violation of law of one price?

General form:

$$\overline{R}_i = C_0 + C_1 \beta_i$$

If you have riskless asset $\beta_i = 0$ **and you have**

$$\overline{R}_i = C_0 = R_F$$

$$\therefore R_F = C_0$$

Consider market portfolio for this portfolio $\beta_i = 1$ **thus**

$$\overline{R}_m = C_0 + C_1(1) \text{ and } C_1 = \overline{R}_m - C_0 = \overline{R}_m - R_F$$

and

$$\overline{R}_i = R_F + \beta_i(\overline{R}_m - R_F)$$

Standard CAPM

$$\beta_i = \frac{Cov\left(R_i R_m\right)}{Var\left(R_m\right)}$$

$$\overline{R}_i = R_F + \frac{Cov(R_i R_m)}{Var(R_m)}(\overline{R}_m - R_F)$$

$$\overline{R}_i = R_F + \lambda Cov(R_i R_M) \qquad \lambda = \frac{\overline{R}_m - R_F}{Var(R_m)}$$

reward for time	**reward for risk**	**market reward risk ratio**

If riskless asset does not exist then define zero beta portfolios $\left(\overline{R}_z\right)$

$$\overline{R}_z = C_0 + C_1(0)$$

and

Zero Beta Form

$$\overline{R}_i = \overline{R}_z + \beta_i(\overline{R}_m - \overline{R}_z)$$

ARBITRAGE PRICING THEORY

A multi-index model explains security returns

$$R_{it} = C_o + \beta_{ip}\begin{pmatrix}\text{unexplained}\\ \text{production}\\ \text{changes}\end{pmatrix} + \beta_{iT}\begin{pmatrix}\text{Term}\\ \text{premium}\end{pmatrix} + \beta_{iD}\begin{pmatrix}\text{Default}\\ \text{premium}\end{pmatrix} + e_{it}$$

Systematic risk factors $\left(\beta_{iP}, \beta_{iT}, \beta_{iD}\right)$

$$\bar{R}_c = 8X_1 + 6X_2 + (1 - X_1 - X_2)9$$

$$\beta_1 = 2X_1 + 1X_2 + (1 - X_1 - X_2)3$$

$$\beta_2 = 1X_1 + 1X_2 + (1 - X_1 - X_2)2$$

Eliminating X_1 **and** X_2

$$\bar{R}_c = 5 + 2\beta_1 - \beta_2$$

and security 4 is mis-priced

Same argument (e.g., Law of One Price) two securities with same risk characteristics must have same expected return. With 3 risk variables, we can match the characteristics of any security with a portfolio of three other securities. Since Betas and expected returns are linear, the relationship is linear.

Example

Sec	$\bar{R}_i$	β_1	β_2
1	8	2	1
2	6	1	1
3	9	3	2
4	11	4	1

$$\bar{R}_c = 8X_1 + 6X_2 + (1 - X_1 - X_2)9$$

$$\beta_1 = 2X_1 + 1X_2 + (1 - X_1 - X_2)3$$

$$\beta_2 = 1X_1 + 1X_2 + (1 - X_1 - X_2)2$$

Eliminating X_1 **and** X_2

$$\bar{R}_c = 5 + 2\beta_1 - \beta_2$$

and security 4 is mis-priced.

In general, if return generating process:

$$R_{it} = C_0 + \beta_{i1} f_1 + \beta_{i2} f_2 + \beta_{i3} f_3 + e_{it}$$

Expected return is:

$$\overline{R}_i = R_F + \beta_{i1}\lambda_1 + \beta_{i2}\lambda_2 + \beta_{i3}\lambda_3$$

$$\lambda_i = \overline{R}_i - R_F$$