MULTI-INDEX

MODELS

Fall 2002

1. **Assumptions that residuals are uncorrelated from a single-index model turns out to be inaccurate. This assumption can be improved by assuming additional indexes.**

2. **Once again multi-index models have developed a life of their own beyond their original purpose of estimating covariances.**

Consider two index model:

$$r_{it} = a_i + b_{i1} I_{1t} + b_{12} I_{2t} + e_{it}$$

Where:

(1). $E\left(I_{1t} I_{2t}\right) = 0$

(2). $E\left(e_{it}\right) = 0$

(3). $E\left(e_{it} I_{1t} \text{ or } I_{2t}\right) = 0$

(4). $E\left(e_{it} e_{jt}\right) = 0$

(5). a_i, b_{i1} and b_{i2} **are constant.**

only real assumption is $E\left(e_{it} e_{jt}\right) = 0$

Mean return:

$$E\left(r_{it}\right) = a_i + b_{i1}\bar{I}_1 + b_{i2}\bar{I}_2$$

$$Var\left(r_{it}\right) = b_{i1}^2\sigma_1^2 + b_{i2}^2\sigma_2^2 + \sigma_{ei}^2$$

in other words:

$$\sigma_{\varepsilon i}^2 = b_{i2}^2\sigma_2^2 + \sigma_{ei}^2$$

$$\operatorname{cov}\left(r_i r_j\right) = b_{i1}b_{j1}\sigma_{I1}^2 + b_{i2}b_{j2}\sigma_{I2}^2$$

Note:

1. **if add** $\mathrm{E}\left(e_i e_j\right)$ **get historical.**

2. **assuming** $\mathrm{E}\left(\varepsilon_i \varepsilon_j\right) = b_{i2} b_{j2} \sigma_{I2}^{2} + 0$

Generalizing to more than two indexes is straight forward.

<u>Types of indexes</u>:

(1). Statistically derived

(2). Portfolios of securities

A.	**S&P, H-L, B-M**	**Fama French**
B.	**S&P, H-L, B-M, Bonds**	**E&G**

(3). Economic Factors

A. ΔIP, surprise in inflation
B. Surprise GNP

(4). Market and Industry

A. S&P
B. Industry factors

In bond area:

$$R_{it} = \overline{R}_i + OAS - D_i \frac{\Delta r}{1+r} - V_i dV_i + C_i(\text{spread}) + e_i$$

<u>Issues</u>:

(1). Real influences

(2). Parsimonious

<u>Uses</u>:

(1). Covariance structure

(2). Selection of exposure

(3). Return attribution

(4). Portfolio evaluation