PORTFOLIO THEORY WITH MULTI-INDEX MODELS

Fall 2002

For illustrative purposes, consider a two-index model. The results clearly generalize

Assume the following return generating process:

$$R_{it} = a_i + \beta_{i1} R_{mt} + \beta_{i2} I_{It} + e_i$$

Where:

(1) R_{mt} **is return on market.**

(2) I_I **is an inflation index.**

Assume investor in question cares about inflation and that the market cares about inflation.

<u>Implications</u>:

(1) Sensitivity to inflation should be priced and we should observe an APT model like:

$$\bar{R}_i = R_{Ft} + \beta_{i1}\lambda_m + \beta_{i2}\lambda_I$$

(2) Investors should be willing to trade off inflation hedging for expected return.

If the investor does not care about β_1 per se, but does care about β_2, the choice set is three-dimensional. It can be represented with these three axies:

$$\bar{R}_p,\ \sigma_p^2,\ \beta_2$$

Separation Theorem.

(1) The efficient frontier can be obtained using three efficient portfolios.

If the investor cares about the β_1, then we have a four-fund theorem and the efficient frontier becomes a combination of:

(1) The riskless asset.

(2) Two portfolios that have a beta of one and minimum risk, e.g., factor replication portfolios.

(3) A special portfolio that maximizes α for any residual risk.

When we have a multi-index model, it is often sensible to assume that residuals are un-correlated. In this case, an extremely easy version of simple rules exists and the optimal proportion in any asset is proportional to:

$$Z_i = \frac{\alpha_i}{\sigma^2_{\varepsilon_i}}$$

Now assume the investor does not care about inflation, but the market does. Then the market's equilibrium model is the capm model and the investor does not pay a cost in expected return by adjusting sensitivity to it.